Abstract

We provide a framework to study bail-in policies that have been adopted in the financial regulatory regimes in the US and EU. In our model, the optimal bank capital structure is a combination of standard debt, which liquidates the bank, and bail-in debt, which is written down to restore solvency. In the presence of fire sale externalities from bank failures and moral hazard from bailouts, banks privately use too much standard debt relative to bail-in debt. A bail-in regime is the optimal regulatory policy and fully replaces bailouts. Bail-ins can generate self-fulfilling crises in long-term debt markets, leading to bank runs. Debt guarantees and an expanded notion of lender of last resort can prevent these crises, and should complement bail-ins as parts of the crisis resolution toolkit.
1 Introduction

In the aftermath of the 2008 financial crisis, the question of orderly bank resolution has received significant attention on both sides of the Atlantic. In many advanced economies, governments employed bailouts to stem financial turbulence in late 2008 and early 2009.\(^1\) Bailouts were arguably very effective at stabilizing financial markets, but have been criticized for leading to moral hazard and perverse redistribution.\(^2\) As a result, the US (Title II of the Dodd-Frank Act) and the EU (Bank Recovery and Resolution Directive) have introduced “bail-ins,” which allow the government to impose haircuts on (long-term) debt holders. The Dodd-Frank Act lists ensuring that “creditors and shareholders will bear the losses of the financial company” as one of the primary purposes of bail-ins, and requires that “[n]o taxpayer funds shall be used to prevent the liquidation of any financial company under [Title II].”\(^3\)

There is limited formal economic analysis on bail-ins, both from an optimal contracting perspective and from a crisis resolution perspective. Two natural concerns arise. The first is that if bank solvency can be improved by replacing standard debt with bail-in debt, then what prevents banks from efficiently doing so using private contracts?\(^4\) Second, during a crisis, there is concern that the possibility of haircuts may destabilize financial markets. In the words of former Treasury Secretary Timothy Geithner, “the consequence of the haircuts imposed on creditors in the case of Lehman and Washington Mutual was a dramatic escalation in the scope and intensity of the run” (Geithner (2016)). More recently in the resolutions of Veneto Banca and Banco Popolare di Vicenza in 2017, the Italian government spared senior debt holders from a bail-in in part due to concerns about investor confidence in other fragile Italian banks.\(^5\)

To understand these issues, we provide a framework for an optimal bank contracting model based on an incentive problem. Banks must monitor the quality of their loans both

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1Two examples in the US are the Troubled Asset Relief Program (TARP), which authorized the government to buy toxic bank assets, and the Temporary Liquidity Guarantee Program (TLGP), which provided for guarantees of bank debt.

2The Dodd-Frank Wall Street Reform and Consumer Act (Dodd-Frank Act) lists “protect[ing] the American taxpayer by ending bailouts” as one of its main objectives, and lists “minimiz[ing] moral hazard” (Section 204) as one of the purposes of bail-ins.

3Dodd-Frank Act Sections 204 and 214.

4For example, banks could use contingent convertible (CoCo) securities that have gained traction in Europe, which are an internal recapitalization instrument with a trigger event (for example, the bank’s capital ratio falling below some threshold) for either a principal write-down or a conversion into equity.

5For example, then Bank of Italy Deputy Governor Fabio Panetta stated “I think resolution would have been very costly not just in monetary terms but also in terms of confidence.” Political concerns and guarantee obligations were also important factors in the decision. Financial Times, “Why Italy’s €17bn bank rescue deal is making waves across Europe,” June 26, 2017.
at the onset of the lending relationship and in its continuation. Because monitoring is not contractible at either stage, the optimal contract must incentivize monitoring, which involves the bank keeping a sufficient stake ("agency rent") in its loan performance not only ex ante but also in continuation. Banks write optimal liability contracts in a complete markets setting so that the liability structure of the bank is endogenous. The optimal bank contract combines two debt instruments: non-bail-inable, or standard, debt and bail-in debt. Standard debt has a face value that does not depend on the idiosyncratic state of the bank and leads to insolvency and liquidation when bank returns are low. This provides strong monitoring incentives to the bank for initial monitoring, by eliminating the bank’s agency rent from the continuation monitoring problem. Bail-in debt provides weaker incentives, by holding the bank to the continuation agency rent but not liquidating the bank. Bail-in debt is useful because it combines the loss-absorbing capability of equity with a payoff profile that is closer to standard debt.

Before the introduction of bail-ins, the private use of bail-in debt instruments by banks had been limited to nonexistent. In our model, banks prefer not to issue bail-in debt for two reasons. First, standard debt becomes a more valuable instrument to manage the incentive problem when the cost of liquidation is sufficiently low. The bank reduces or eliminates its use of bail-in debt in this case. Second, if the bank expects to be bailed out by the government ex post, it will not issue bail-in debt. Issuing standard debt rather than bail-in debt allows the bank to remain solvent while capitalizing on the bailout subsidy.

We then study the design of socially optimal bank regulation. To motivate the social planning problem, we introduce a fire sale spillover into the model – more bank liquidations reduce the recovery value to any individual bank in liquidation. We study the optimal liability contracts that would be written by a social planner who internalizes the fire sale spillover, but is subject to the same set of constraints as the bank and must, therefore, respect the monitoring incentive problem. The socially optimal contract also combines standard and bail-in debt. Relative to the privately optimal bank contract, the social planner has an incentive to reduce the use of standard debt to mitigate the fire sale. A bail-in regime can implement this socially optimal contract by changing the contingency properties of bank debt ex post.

When faced with the possibility of time-inconsistent bailouts, optimal regulation ensures that there is sufficient bail-in debt to recapitalize the banks without engaging in bailouts: bail-ins fully replace bailouts. Bailouts are a socially costly resolution method since they have to be financed by distortionary taxes. The costs of bail-ins, however, are efficiently priced into bank contracts ex ante. As a result, the planner prefers recapitalization via bail-ins over bailouts. This coincides with a core principle of post-crisis resolution, that
the costs of bank resolution should be borne by bank investors and not by taxpayers.\textsuperscript{6}

The model sheds light on the difference between the pre- and post-crisis worlds, and the role of bail-ins. Prior to the crisis, private use of bail-in debt was limited in good times due to high liquidation values, and in bad times due to fire sale spillovers and moral hazard from bailouts. Regulation is necessary because the social costs of bankruptcy are higher than the private costs of bankruptcy, due to fire sales and bailouts. Bail-ins reduce the social cost of bank failures while respecting the underlying private incentive problem that gave rise to debt contracts in the first place, and so constitute optimal regulation in the model.

Drawing an analogy to debt restructuring mechanisms, such as Chapter 11 of the US Bankruptcy Code, the model further helps to explain why bail-ins may be a particularly effective mechanism to regulate banks as opposed to non-financial corporates. Non-financials may have higher average liquidation discounts than banks, but are less exposed to fire sales and less likely to receive bailouts in crisis states. Our model predicts that non-financials adopt capital structures that are easier to resolve, and make use of existing debt restructuring mechanisms. By contrast, banks adopt capital structures that make them more difficult to resolve. These capital structure choices can be seen as relatively socially efficient in the case of non-financials, but as inefficient in the case of banks due to fire sales and bailouts. As a result, bail-in regulation may be especially suitable for banks.

In the final sections of the paper, we study whether bail-ins can destabilize bank refinancing efforts. In order to do so, we distinguish between short- and long-term debt, and model one of the important institutional features of bail-ins: they subordinate long-term debt to short-term debt in the event of bank failure. We demonstrate the existence of self-fulfilling rollover crises for fundamentally solvent banks. A rollover crisis occurs when long-term debt purchasers believe they are about to be bailed in and become unwilling to purchase newly issued long-term debt, so that the bank cannot refinance itself and is forced into bankruptcy and liquidation. Since the outstanding stock of short-term debt is senior to long-term debt in resolution, long-term debt indeed receives no payoff at this point, justifying the equilibrium beliefs.

We study policy options to bolster market stability and prevent rollover crises, as a complement to an effective bail-in regime. First, we show that an expanded lender of last resort facility (LOLR), which promises to lend long-term debt to banks, can eliminate rollover crises. Second, we show that an extension of temporary guarantees to new long-

\textsuperscript{6}See Sections 204 and 214 of the Dodd-Frank Act as cited above. The Dodd-Frank Act further states that “[t]axpayers shall bear no losses from the exercise of any authority under this title” (Section 214). See also e.g. French et al. (2010).
term debt during crisis times can help stabilize the market and prevent rollover crises, even though these guarantees are not fulfilled in equilibrium. These proposals have precedent in programs used by the US government during the 2008 financial crisis; for example, the Temporary Liquidity Guarantee Program (TLGP), which extended debt guarantees to new issuances of long-term debt.

**Related Literature.** First, we relate to a growing literature on bail-ins. Dewatripont and Tirole (2018) explore how bail-ins can complement liquidity regulation. Keister and Mitkov (2017) show that banks may not write down their (deposit) creditors if they anticipate government bailouts. Chari and Kehoe (2016) use a costly state verification model and show that bail-ins are not required in the optimal regulatory regime. In their model, costly state verification implies that standard debt contracts are the only renegotiation-proof contracts, so that the possibility of bail-ins leads to a reduction in standard debt issuance but not the use of bail-in debt. Mendicino et al. (2018) numerically explore the optimal composition of bail-in debt and equity in the presence of both private benefit taking and risk shifting, taking contracts as given and with a regulatory objective of protecting insured deposits. Pandolfi (2018) studies a related incentive problem to ours, but takes standard debt contracts as given. The paper argues that bailouts may be desirable in conjunction with bail-ins when bail-ins limit investment scale by weakening bank incentives. Walther and White (2018) show that precautionary bail-ins of long-term debt can signal adverse information about a bank’s balance sheet and cause a bank run, leading to an overly weak bail-in regime. Our contribution is to derive an optimal bank contract that combines standard and bail-in debt, to rationalize bail-ins as optimal regulation, and to study rollover crises and their implications for the crisis resolution toolkit.

Second, we relate to a literature on contingent convertible debt securities as a recapitalization instrument, which documents the possibility of multiple equilibria arising from the conversion trigger. Multiplicity in our model arises from the relationship between short-term and long-term debt, and results in a bank run. We further study the role of lender of last resort and debt guarantees in preventing multiplicity.

Third, we relate to the literature on theories of debt in both the banking and corporate finance contexts. Our model incorporates an incentive theory of debt and combines two

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7In addition to the papers described below, see also e.g. Bolton and Oehmke (2019) and Colliard and Gromb (2018).
8See e.g. Sundaresan and Wang (2015). See also Flannery (2014) for a broader overview of the literature.
views of the role of debt. In the corporate finance context, debt is valuable from a cash flow transfer perspective: a firm manager forfeits the entire cash flow of the firm to investors when returns are low, which generates good incentives. In the banking context, illiquid assets are backed by standard debt (often demand deposits) as a threat to liquidate the bank. Our model combines these views, with standard debt generating liquidations and bail-in debt generating cash flow transfers.

Fourth, we relate to the literature on macroprudential regulation. This literature features two common motivations for macroprudential regulation – pecuniary externalities (e.g. fire sales) and fiscal externalities (bailouts) – and studies optimal ex ante regulation, possibly in conjunction with ex post bailouts. Our focus is on ex post bail-ins as an optimal policy, and whether it is a complement or substitute to macroprudential regulation and bailouts.

Finally, we connect to the literature on debt dilution. A large finance literature has focused on different methods of debt dilution, including issuance of new senior debt and maturity shortening. Brunnermeier and Oehmke (2013) show that maturity shortening can result from a deliberate desire to dilute longer-maturity creditors. He and Milbradt (2016) show that multiple equilibria can arise if long-term creditors expect to be diluted by future short-term creditors, resulting in a gradual maturity shortening. In our model, multiplicity arises because bail-ins imply existing short-term debt dilutes new long-term debt and results in an immediate (fundamental) bank run.

2 Model

We develop a three-period model with two economic agents: banks and investors. Banks sign contracts with investors to raise investment funds. We tailor the model to address the core trade-off of bail-ins: between standard debt and bail-in debt. Our baseline model will have no role for instruments such as equity, or for other trade-offs that affect the use of debt (e.g. tax benefits).11

The three-period economy, \( t = 0, 1, 2 \), has a unit continuum of banks and investors. Banks invest in a project of variable scale \( Y_0 = A_0 + I_0 > 0 \) by using their own funds, \( A_0 > 0 \), and funds \( I_0 \geq 0 \) from (date 0) investors. Investors are deep-pocketed at date 0 and can finance any investment scale.

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11In the appendix, we extend the baseline model to incorporate a role for equity in the capital structure and additional trade-offs such as tax benefits of debt.
Banks and investors are risk-neutral and do not discount the future. We denote bank consumption by \((c_0, c_1, c_2)\), so that bank expected utility is given by \(E_0 [c_0 + c_1 + c_2]\). We denote the payments to investors by \((x_1, x_2)\). \(x_t\) is the actual amount received by investors, and is distinct from the face value of liabilities. Investor expected utility from the bank contract is \(E_0 [-I_0 + x_1 + x_2]\). Contracts are subject to limited liability constraints, given by\(^\text{12}\)

\[
c_0, c_1, c_2, x_1, x_2 \geq 0
\] (1)

Banks need to refinance any liabilities that mature at date 1. They raise these funds from a set of (date 1) risk-neutral, no-discounting, deep-pocketed investors.

There is an aggregate state \(s \in S\) of the economy that realizes at date 1. For expositional simplicity, we assume that \(S\) is a finite set, with probability measure \(\pi(s)\).

### 2.1 Bank Projects

Banks extend financing to firms, thereby establishing a lending and monitoring relationship with those firms. When first extending funds to firms, banks monitor their borrowers, ensuring that the projects undertaken are of good quality. In doing so, banks develop specialized knowledge of that firm, and are uniquely able to monitor and collect from the firm in continuation. This relationship is the foundation of banking in our model. We refer to these relationships as bank projects.\(^\text{13}\)

The bank project experiences a stochastic quality shock \(R\) at date 1, adjusting its scale to \(Y_1 =RY_0\), at which point all uncertainty is resolved. The shock \(R\) is idiosyncratic with a density \(f_e(R|s)\) that has support over \([R, \bar{R}]\). The aggregate state \(s\) may capture simple “total factory productivity” shocks (e.g. shifting all returns upwards or downwards) or other higher-moment shocks (e.g. increasing the dispersion of returns). Both \(R\) and \(s\) are contractible. However, the distribution of \(R\) depends on the bank’s non-contractible monitoring effort \(e \in \{H, L\}\), where \(e = H\) is high monitoring effort and \(e = L\) is low monitoring effort. \(f_e(R|s)\) satisfies the monotone likelihood ratio property (MLRP), conditional on the aggregate state \(s\), that is \(\frac{\partial}{\partial R} \left[ \frac{f_L(R|s)}{f_H(R|s)} \right] < 0\). MLRP is a standard assumption in generating debt contracts,\(^\text{14}\) and implies that high (low) returns are a signal that the bank exerted high (low) monitoring effort. We assume throughout the paper that optimal contracts induce high monitoring effort, so that \(e = H\).

Because monitoring effort is non-contractible, the bank chooses \(e\) to maximize bank

\(^{12}\)In our model, these are equivalent to the limited liability constraints \(c_1 + c_2 \geq 0\) and \(x_1 + x_2 \geq 0\).

\(^{13}\)For simplicity, firms in our model earn zero surplus from the lending relationship.

\(^{14}\)See e.g. Innes (1990).
utility after contracts have been signed. Given the consumption profile under the contract signed, the bank exerts high monitoring effort if

\[
E [c_1(R, s) + c_2(R, s)|e = H] \geq E [c_1(R, s) + c_2(R, s)|e = L] + BY_0
\]

where \( B > 0 \) is a private benefit of exerting low monitoring effort ("shirking"). We rearrange this incentive compatibility constraint to obtain the representation

\[
E \left[ (c_1(R, s) + c_2(R, s)) \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \right]|e = H \geq BY_0. \tag{2}
\]

Higher payoffs \( c_1(R, s) + c_2(R, s) \) in states where the likelihood ratio \( \frac{f_L(R|s)}{f_H(R|s)} \) is low relax incentive compatibility because these states signal that monitoring effort was high.

Although the quality shock \( R \) is realized at date 1, the project does not mature until date 2 and yields no dividend at date 1. If the project survives to date 2, it generates 1 unit of the consumption good per unit of final scale, \( Y_2 = Y_1 \). Only a portion \((1-b)Y_2\) is pledgeable to investors, while the remaining portion \( bY_2 \) is retained by the bank. This non-pledgeable portion \( bY_2 \) is an agency rent in the continuation monitoring or collection problem of the bank.\(^{15}\) Holding projects to maturity implies a maximum pledgeability constraint \( c_2 \geq bY_2 \).

Projects can be liquidated prematurely at date 1, in which case they yield \( \gamma(s)Y_1 < Y_1 \) units of the consumption good at date 1 and nothing at date 2, with the proceeds accruing entirely to investors.\(^{16}\) We assume that \( \gamma(s) < 1 - b \) for all \( s \), so that liquidating the project is not desirable from an investor repayment perspective and so are not desirable ex post. Liquidations may be ex ante efficient for incentive reasons, since the bank can be paid 0 when liquidated but must be paid the agency rent \( bY_2 \) when not liquidated.

Finally, because banks are risk-neutral and do not discount the future, banks are indifferent to whether they consume at date 1 or date 2. We set \( c_1(R, s) = 0 \) to ease the exposition going forward.

### 2.2 Bank Liabilities

In order to raise investment \( I_0 \geq 0 \), banks pledge state-contingent liabilities to investors at date 0, which promise a face value of \( L_t(R, s) \geq 0 \) to be paid in period \( t \). Banks may

\(^{15}\)See e.g. Holmstrom and Tirole (1997).

\(^{16}\)We think of the liquidation discount as arising from selling projects to second-best users, who have not developed the knowledge of the firm lending relationship that the bank has. See Appendix B.1.
pledge a face value of liabilities in excess of pledgeable income. In such states, the bank is then unable to repay its liabilities in full. It enters bankruptcy and liquidates its assets.

Without loss of generality, we assume that banks pledge one-period liabilities contracts, so that \( L_2(R, s) = 0 \). This is without loss of generality in our model since investors are indifferent to the period in which repayment occurs. Given a liability structure \( L_1(R, s) \), the resulting payoff profiles of banks and investors are

\[
(c_2(R, s), x_1(R, s)) = \begin{cases} 
(Y_0 - L_1(R, s), L_1(R, s)) & \text{if } L_1(R, s) \leq (1 - b)Y_0 \\
(0, \gamma(s)Y_0) & \text{if } L_1(R, s) > (1 - b)Y_0
\end{cases}
\]  

(3)

where \( c_1(R, s) = x_2(R, s) = L_2(R, s) = 0 \). To understand this payoff profile, if \( L_1(R, s) \leq (1 - b)Y_0 \), the bank can roll over its face value of liabilities by raising money from date 1 investors, who break even at the same face value \( L_1(R, s) \). Date 0 investors receive \( x_1(R, s) = L_1(R, s) \), while the bank receives \( c_2(R, s) = Y_2 - L_1(R, s) \). If instead \( L_1(R, s) > (1 - b)Y_0 \), the face value of liabilities exceeds pledgeable income and the bank is liquidated, yielding payoffs \( x_1(R, s) = \gamma(s)Y_0 \) and \( c_2(R, s) = 0 \).

The voluntary investor participation constraint states that investors must at least break even in expectation on the contract they signed, and it is given by

\[
Y_0 - A \leq E[x_1(R, s) | e = H].
\]  

(4)

where \( I_0 = Y_0 - A \) is the amount financed by investors.

Finally, we assume that liabilities \( L_1(R, s) \) must be monotone, conditional on the aggregate state \( s \). In other words,

\[
R \geq R' \Rightarrow L_1(R, s) \geq L_1(R', s).
\]  

(5)

Monotonicity is a common assumption in many settings of optimal contracts or security design. It generates the flat face value of liabilities in high-return states.\(^{17}\)

\(^{17}\)For example, one justification offered is that banks would be incentivized to pad their returns, for example by secretly borrowing from a third party. Without monotonicity, optimal contracts in our model have the “live-or-die” feature (see Innes (1990)) above the thresholds \( R_u(s) \) defined in Proposition 1, so that \( L_1(R, s) = 0 \) for \( R > R_u(s) \). The form of banks’ liability structure below \( R_u(s) \) would be the same.
2.3 Fire Sales and Liquidation Values

The liquidation value \( \gamma(s) \) is endogenous and given by a function

\[
\gamma(s) = \gamma(s, \Omega(s)), \quad \Omega(s) = \int_R a(R, s) R f_H(R|s) dR
\]  

(6)

where \( a(R, s) \in \{0, 1\} \) indicates whether or not a bank liquidates in state \((R, s)\), with \( a(R, s) = 1 \) denoting liquidation. The function \( \gamma \) is nonincreasing and weakly concave in \( \Omega(s) \). \( \Omega(s) \) is the economy-wide fraction of bank projects being liquidated at date 1.\(^{18}\)

When \( \frac{\partial \gamma}{\partial \Omega} < 0 \), there is a fire sale spillover from bank liquidations: more liquidations reduce the liquidation value. We may think of states with \( \frac{\partial \gamma}{\partial \Omega} = 0 \) as “normal times” and states with \( \frac{\partial \gamma}{\partial \Omega} < 0 \) as times of market stress or financial crisis.\(^{19}\)

2.4 Bank Optimal Contracting and Equilibrium

Every bank takes the equilibrium liquidation values \( \gamma(s) \) as given, and signs a contract \((L_1, Y_0)\) with investors. Banks maximize their own expected utility

\[
\max_{L_1, Y_0} E[c_2(R, s)|e = H],
\]

subject to incentive compatibility (2), investor participation (4), monotonicity (5), and limited liability (1), where \( c_2 \) and \( x_1 \) are given by equation (3). Figure 1 presents a simple timeline underlying this contracting problem.

Since all banks are identical ex ante, all banks sign the same equilibrium contract. Therefore, \( a(R, s) = 1_{L_1(R, s) > (1-b)RY_0} \), where \( 1_{\cdot} \) is the indicator function. An equilibrium of the economy is a set of liquidation values \( \gamma(s) \) such that the contract \((L_1, Y_0)\) is optimal, given \( \gamma(s) \), and such that liquidation values are determined by equation (6), given \((L_1, Y_0)\).

3 Optimal Contracts and Optimal Regulation

In this section, we characterize the optimal contracts written by banks that take as given the liquidation value \( \gamma(s) \). We compare these privately optimal contracts to the contract written by the social planner who internalizes the fire sale spillover. In both cases, the capital structure of the bank is determined by the optimal contract, and consists of two debt instruments. The first, which we call standard debt, has a face value that does not

\(^{18}\) In Appendix C.9, we extend the model to allow for bank size \( Y_0 \) to affect the liquidation discount.

\(^{19}\) In Appendix B.1, we provide a foundation for this liquidation function from limits to arbitrage.
depend on the idiosyncratic state $R$, and liquidates the bank in low-return states. The second, which we call bail-in debt, has a face value that can be written down based on the idiosyncratic return, and restores bank solvency when total debt exceeds the pledgeable income. Although the bank and planner both agree that the optimal liability structure combines standard and bail-in debt, they disagree on the relative use of the two instruments. The planner wishes to use less standard debt than the bank, internalizing the fire sale spillover.

3.1 Privately Optimal Contracts

We begin by characterizing the privately optimal bank contract in terms of two thresholds, $R_l(s)$ and $R_u(s)$, that are contingent on the aggregate state $s$. We then associate these two thresholds with the two debt instruments. These thresholds are sufficient statistics for the privately optimal liability structure of the bank.

Proposition 1. A privately optimal bank contract has a liability structure

$$L_1(R, s) = \begin{cases} 
(1 - b)R_l(s)Y_0, & R \leq R_l(s) \\
(1 - b)RY_0, & R_l(s) \leq R \leq R_u(s) \\
(1 - b)R_u(s)Y_0, & R_u(s) \leq R 
\end{cases}$$

where $0 \leq R_l(s) \leq R_u(s) \leq \bar{R}$ are aggregate-state-contingent thresholds. The bank is liquidated if and only if $R \leq R_l(s)$. These thresholds, when interior and not equal, are given by

$$
\mu b \left( \frac{f_L(R_l(s)|s)}{f_H(R_l(s)|s)} - 1 \right) = b + \lambda (1 - b - \gamma(s)) \tag{7}
$$

$$
0 = E \left[ \lambda - 1 - \mu \left( \frac{f_L(R|s)}{f_H(R|s)} \right) \left| R \geq R_u(s), s, e = H \right] \tag{8}
$$

For the remainder of the paper, we assume that the thresholds are interior and not equal, except when explicitly stated otherwise. Although $R \in [\bar{R}, \bar{R}]$ with $\bar{R} > 0$, it is possible that when $\bar{R} > 0$, the bank finds it optimal to set $R_l, R_u < \bar{R}$, in which case there is only standard debt but the debt level is low enough that the bank is always solvent. Generally speaking, $R_l(s)$ will be interior when the likelihood ratio $\frac{f_L(R|s)}{f_H(R|s)}$ is sufficiently large, that is when $\bar{R}$ is a sufficiently good signal of low effort. $R_u(s)$ will be interior when $\frac{f_L(R|s)}{f_H(R|s)}$ is sufficiently small and $\mu > \lambda - 1$, that is when $\bar{R}$ is a sufficiently good signal of high effort.
where $\mu > 0$ is the Lagrange multiplier on incentive compatibility (2) and $\lambda > 1$ is the Lagrange multiplier on investor participation (4).

Proof. All proofs are contained in the appendix.\textsuperscript{21}

Before discussing the properties of the optimal contract, we first associate these two thresholds $R_l(s)$ and $R_u(s)$ with the two debt instruments that we discussed before. We associate $R_l(s)$ with standard debt and $R_u(s)$ with bail-in debt.

To understand this terminology, suppose that there is no aggregate risk ($|S| = 1$). Figure 2 illustrates the optimal liability contract in this case. There are three regions of the liability structure. The first region, where $R \leq R_l(s)$, is one where the total face value of liabilities is constant, but exceeds the pledgeable income of the bank. In this region, the bank is liquidated, and investors only receive partial repayment on the face value of their debt contracts. This is a standard debt contract.

In the third region, above $R_u(s)$, investors receive a constant payoff equal to the total face value of liabilities, so that $(R_u(s) - R_l(s))Y_0$ corresponds to the total level of bail-in debt. What distinguishes bail-in debt from standard debt is that in the second region, where $R_l(s) \leq R \leq R_u(s)$, the face value of bail-in debt is written down to $((1 - b)R - R_l(s))Y_0$. This recapitalizes the bank and allows it to continue operating, rather than being liquidated. We refer to this as bail-in debt because its face value can be written down (“bailed in”) based on the idiosyncratic state.

In the general model with aggregate risk ($|S| > 1$), the optimal contract consists of the same instruments as above, conditional on a realized aggregate state. The only difference is that the levels of both standard and bail-in debt may depend on the aggregate state. Standard and bail-in debt therefore refer to whether or not the debt instrument is made contingent on the idiosyncratic state, appealing to the case where $|S| = 1$. We adopt this terminological convention throughout the rest of the paper in order to ease exposition.

**Corollary 2.** The optimal contract can be implemented with a combination of standard debt with face value $(1 - b)R_l(s)Y_0$, which cannot be written down contingent on the idiosyncratic state $R$, and bail-in debt with face value $(1 - b)(R_u(s) - R_l(s))$, which can be written down contingent on the idiosyncratic state.

\textsuperscript{21}In the proof of this proposition, see Appendix A.1.1 for a comment on non-uniqueness of face value of liabilities $L_1(R, s)$ below the threshold $R_l(s)$. Non-uniqueness arises in this region because any face value of liabilities above $(1 - b)RY_0$ results in bank liquidation. We have chosen the face value of liabilities that correspond to standard debt, which seems most natural in the context of banks and bail-ins. Moreover, uniqueness is restored if there is an $\epsilon \to 0$ premium for standard debt, for example due to tax benefits of debt. The face value of liabilities is unique above $R_l(s)$.
For the remainder of the paper, we associate standard and bail-in debt with the thresholds \( R_l(s) \) and \( R_u(s) \), respectively, rather than writing out their associated (face value) liabilities.

The core property of standard debt is that it forces liquidations in low-return states. Equation (7) describes the marginal trade-off the banker faces in replacing a unit of bail-in debt with a unit of standard debt. On the one hand, liquidating the bank results in a total resource loss \( b + \lambda(1 - b - \gamma(s)) \) to the bank and investors. On the other hand, pledging to liquidate the bank provides higher-powered monitoring incentives at date 0, reflected in the term \( \mu b \left( \frac{f_L(R_l(s)|s)}{f_H(R_l(s)|s)} - 1 \right) \), by depriving the bank of the non-pledgeable income \( bRY_0 \). In particular, the liquidation threshold features \( \frac{f_L(R_l(s)|s)}{f_H(R_l(s)|s)} > 1 \). That is, at \( R_l(s) \) the likelihood ratio is greater than 1, implying that the state provides a stronger signal that low effort may have been exerted.

By contrast, for a given level of standard debt, an additional unit of bail-in debt does not change liquidations but does transfer value from banks to investors. The marginal trade-off is summarized in equation (8). On the one hand, the binding investor participation constraint implies this transfer is valuable \((\lambda - 1 > 0)\), as it allows the bank to increase project scale. On the other hand, increasing the total debt level reduces bank consumption in high-return states, where the likelihood ratio \( \frac{f_L(R|s)}{f_H(R|s)} \) is low and the signal of high effort is stronger. This weakens bank monitoring incentives and tightens the incentive compatibility constraint (2). The optimal level of bail-in debt equalizes these two effects on the margin.

Both instruments are contingent on the aggregate state, reflecting that the terms of bank contracts adjust to verifiable events that are beyond a bank’s control. For example, if all else equal a state \( s \) has lower returns due to an aggregate (TFP) shock, equation (7) implies it should have a lower liquidation threshold.\(^{22}\)

Our model features three ingredients that are jointly necessary to generate contracts that consist of combinations of standard and bail-in debt: the ex ante incentive problem \((B > 0)\), limited pledgeability \((b > 0)\), and costly liquidations \((\gamma(s) < 1)\). In the absence of any one of these elements, contracts in our model would not combine standard and contingent debt. In Appendix C.1, we show that in the absence of the ex ante incentive problem optimal contracts would feature only equity, in the absence of limited pledgeability optimal contracts would feature only bail-in debt, and in the absence of costly liquidations optimal contracts would feature only standard debt.

\(^{22}\)See Dewatripont and Tirole (2012) for a related argument.
3.1.1 Bail-in Debt or Equity?

Proposition 1 highlights why standard debt can be a valuable loss-absorbing instrument for banks, relative to equity. Bail-in debt combines the incentive properties of standard debt with the loss-absorbing properties of equity. It generates a cash flow transfer below $R_u(s)$ and a flat investor payoff above $R_u(s)$, similar to standard debt, but does so without liquidating the bank. Bail-in debt therefore achieves a capital structure that standard debt and equity combined cannot. Under the incentive problem of the baseline model, banks prefer bail-in debt to equity as a loss-absorbing instrument.

Our baseline model was set up specifically to isolate the trade-off between standard debt and bail-in debt. Reflecting this, Proposition 1 does not feature equity. However, equity is an important instrument in practice for banks. In Appendix C.5, we add a role for equity in the model by incorporating risk aversion and risk shifting, and show that the core trade-off between standard debt and bail-in debt exists as in the baseline model.

3.1.2 Interpretation as Contingent Convertibles

Bail-in debt in our model can be interpreted as a form of contingent convertible (CoCo) debt, a form of contractual bail-in instrument that has gained prominence in Europe. The most natural interpretation in this context is that bail-in debt in our model is a principal write-down CoCo debt security that applies at the point of non-viability. Conditioning the level of bail-in debt on both the idiosyncratic state (i.e. individual bank health) and aggregate state (i.e. banking sector health) resembles a dual price trigger.

3.1.3 Why Didn’t Banks Issue Bail-in Debt before 2008?

Although Proposition 1 states that privately optimal bank contracts combine standard and bail-in debt, bail-in debt is largely a post-crisis innovation that was “introduced” by bail-in regulation: it places contingencies into debt contracts where few (if any) had previously existed. This leads to the question: Under what conditions would the privately optimal contract feature no bail-in debt? 

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23 See Avdjiev et al. (2017) and Flannery (2014) for more background on CoCos.

24 In Appendix D.2, we show that it can also be implemented using a debt-equity conversion.

25 Unfavorable tax or regulatory treatment may also have contributed to a lack of bail-in debt issuance. Prior to the crisis, regulatory requirements were generally equity requirements, which bail-in debt would not count towards. We see some support for this force mattering in the fact that CoCos have grown in use post-crisis in the EU, where they often count towards regulatory capital requirements, but not in the US, where they do not count towards regulatory capital requirements.
The case where banks do not issue bail-in debt, that is \( R_u(s) = R_l(s) \), is a corner solution of the model. This case is summarized in the following corollary to Proposition 1.

**Corollary 3.** Suppose that the solution \( R_l(s) \) to equation (7) satisfies

\[
E \left[ \lambda - 1 - \mu \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \bigg| R \geq R_l(s), s, e = H \right] \leq 0
\]  
(9)

Then, banks do not issue bail-in debt in state \( s \), that is \( R_l(s) = R_u(s) \).

To understand Corollary 3, the left-hand side of equation (9) is the marginal value of increasing \( R_u(s) \) above \( R_l(s) \), as in equation (8) from Proposition 1. When this marginal value is negative, the incentive costs of increasing the total debt level outweigh the investor repayment benefits, and the bank chooses \( R_l(s) = R_u(s) \).

A key determinant of the trade-off between standard and bail-in debt is the cost of liquidations, \( \gamma(s) \). When liquidation values are high, the resource loss from liquidations is low and \( R_l(s) \) increases,\(^{26}\) pushing the bank towards \( R_l(s) = R_u(s) \). Banks are less likely to issue bail-in debt when liquidation values are high.

It is not clear that the model so far offers a suitable explanation for limited contingencies against aggregate (crisis) risk, where liquidation values may be lower due to fire sale spillovers.\(^{27}\) We provide two explanations going forward. In Section 3.2, we show that banks use too much standard debt, not internalizing the fire sale spillover. In Section 4 we introduce ex post bailouts of insolvent banks, and show that banks do not issue bail-in debt when they anticipate being bailed out instead. Combined with Corollary 3, these results can help illustrate why banks may have chosen not to incorporate contingencies on either the idiosyncratic or the aggregate state into their capital structures prior to the crisis.

### 3.2 Socially Optimal Contracts

The social planner internalizes the fire sale spillover and writes bank contracts to maximize social welfare. Since all investors in our model receive no surplus in expectation, social welfare is equal to bank utility.\(^{28}\) The only difference between the private bank contracting
problem and the social planning problem is that the planner internalizes the fire sale spillover.\textsuperscript{29} We assume away the possibility of bailouts until Section 4.

The social planner writes contracts subject to the same conditions as banks, namely incentive compatibility (2), investor participation (4), monotonicity (5), and limited liability (1), with \( c_2 \) and \( x_1 \) given by equation (3). In other words, any socially optimal contract must also be privately feasible.

**Proposition 4.** The socially optimal contract can be implemented by a combination of standard and bail-in debt. These debt levels are associated with endogenous thresholds \( R_l(s) \) and \( R_u(s) \). Whenever \( R_l(s) < R_u(s) \) is interior, \( R_l(s) \) is given by

\[
\mu b \left( \frac{f_L(R_l(s)|s)}{f_H(R_l(s)|s)} - 1 \right) = b + \lambda ((1 - b) - \gamma(s)) + \lambda \left[ \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \right] \int_{R_l(s)}^{R_u(s)} f_H(R|s) dR
\]

while \( R_u(s) \) is given as before by equation (8).

Even though the planner uses the same debt instruments as the bank, the term \( \frac{\partial \gamma}{\partial \Omega} < 0 \) generates an additional social cost of liquidation in the planner’s optimality condition for \( R_l(s) \). This liquidation cost term represents the only difference between the private and social optimality conditions in equations (7) and (10), respectively. Fire sale spillovers generate an additional social cost: the project liquidations of one bank increase the resource loss to all other banks that liquidate projects at the same depressed prices. By contrast, there is no additional wedge in the determination of \( R_u(s) \), since a greater total debt level arising from more bail-in debt does not change total liquidations. Relative to private banks, the planner prefers relatively less use of standard debt in favor of more use of bail-in debt.

In the case of Corollary 3, where banks had not found it optimal to issue bail-in debt, the additional social cost of bankruptcy implies optimal regulation can introduce bail-in debt into the bank’s capital structure. The planner’s motivation to do so does not result from an incentive to complete markets by writing contracts that banks could not write on their own. Rather, private contracts feature too little or no bail-in debt because banks do not internalize the fire sale spillover.

\textsuperscript{29}In characterizing the liquidation value function \( \gamma \), we appealed to a microfoundation from limits to arbitrage (Appendix B.1). In this context, setting social welfare to be bank welfare implies the planner places a welfare weight of 0 on the arbitrageurs. In Appendix C.3, we use the arbitrageur model of Appendix B.1 and derive a sufficient condition under which the socially optimal contracts in this section are Pareto efficient.
In Appendix C.2, we show one decentralization of this contract that uses positive ex ante taxes on standard debt. These taxes lead banks to internalize the additional fire sale cost and to increase their use of bail-in debt.

3.3 Implementing Optimal Regulation with Bail-ins

So far, the planner has used ex ante regulation to issue sufficient bail-in debt, which specifies write-down provisions contractually. Bail-ins are also associated with an ex post resolution authority: the planner takes a debt contract with a fixed face value, and writes down that face value ex post. Both forms of authority are used in practice. In the US, banks are required to maintain a certain level of total loss-absorbing capital (TLAC), principally long-term debt and equity, to safeguard the bank against poor returns. Debt used to satisfy TLAC requirements must be plain-vanilla, implying a fixed face value, while debt with contractual contingencies cannot generally be used to satisfy TLAC requirements.\footnote{82 FR 8266 (January 24, 2017). In particular, “eligible external LTD [is] prohibited from including contractual triggers for conversion into or exchange for equity.”} However, US regulation does grant ex post bail-in authority as well, so that the US implementation is essentially entirely via an ex post resolution authority. By contrast, the European regulatory regime is more accommodating of ex ante contractual provisions.\footnote{See Avdjiev et al. (2017).}

We now discuss the equivalence between these two implementations.

In our model, there is a straightforward equivalence between the two regulatory methods when there is no aggregate risk ($|S| = 1$).\footnote{Appendix D.3 provides an equivalence result when there is aggregate risk ($|S| > 1$).} Under the ex ante (contractual) implementation, the planner caps bank issuance of standard debt at $R_l$ and requires all remaining debt $R_u - R_l$ to have contractual write-down provisions that restore solvency to the bank. Under the ex post (resolution authority) implementation, the planner caps the amount of non-bail-inable debt at $R_l$ and designates the remaining $R_u - R_l$ to be bail-inable. The resolution authority can write down bail-inable debt ex post to recapitalize the bank. The planner will bail-in banks whenever possible ex post using the bail-in debt, yielding the same result as under the ex ante contractual implementation.

Since the model predicts that these two methods are equivalent, it is not necessarily surprising that we see both methods used in practice.\footnote{It is not necessarily surprising to see some preference for ex post implementation, for example due to a regulatory complexity problem associated with contractually pre-arranging write-downs. Formal analysis of a regulatory complexity problem is beyond the scope of this paper.}
3.4 Relationship to Debt Renegotiation and Restructuring

In practice, bail-in debt is generally associated with banks. However, the core (private) optimal contracting model of the paper could also be applied to non-financial corporates, some of whom may have high liquidation discounts even in the absence of fire sales. In principle, this suggests that non-financial corporates might also wish to use bail-in debt.

One interpretation in this spirit can be provided in the context of debt renegotiation and restructuring. Chapter 11 of the US Bankruptcy Code provides a reorganization and debt restructuring process for non-financials, allowing them to avoid liquidation under Chapter 7.\(^{34}\) Our model predicts that if failures of non-financial firms are not associated with externalities such as fire sales or bailouts, they will use Chapter 11 efficiently.

In comparison to non-financials, banks take on sufficiently more short-term debt and do not make use of a Chapter 11 type process. In our model, one way to understand this difference is as follows. Banks and other financials may be relatively easier to liquidate during normal times, but are also associated with externalities and bailouts in crises. Our model suggests that both of these forces encourage banks to adopt capital structures that leave them with larger quantities of short-term, or non-resolvable, debt, and therefore do not make use of Chapter 11. By contrast, non-financials may have greater liquidation discounts on average, but may not be as vulnerable to fire sales or as likely to receive bailouts.\(^{35}\) Our model suggests that they adopt capital structures that make them easier to resolve ex post, and make use of Chapter 11.\(^{36}\)

One important consideration is that the design of Chapter 11 may simply not be appropriate for banks due to the financial nature of their activities.\(^{37}\) Reflecting this, the US Treasury Department has adopted a proposal for a Chapter 14 bankruptcy process, with the aim of creating a process in the spirit of Chapter 11 that is tailored to banks.\(^{38}\) Our model predicts that banks would privately under-utilize Chapter 14 relative to the social optimum, leaving a role for a bail-in regime.\(^{39}\)

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\(^{34}\)To the extent non-financials can control bankruptcy decisions ex ante, for example with their capital structure choices, a process like Chapter 11 provides an ex post alternative to ex ante contractual write-down arrangements. This is in the spirit of the equivalence between ex ante contractual provisions and ex post bail-ins in our model.

\(^{35}\)In this sense, we might think of financial companies as having a high average liquidation value \(\gamma\), but also having a high sensitivity \(\frac{\partial \gamma}{\partial \Omega}\). By contrast, non-financials may have a lower average liquidation value, but also a much lower sensitivity \(\frac{\partial \gamma}{\partial \Omega}\).

\(^{36}\)For example, the percent of nonfinancial corporate debt that is short-term is approximately 32%, while the ratio of nonfinancial corporate debt to the market value of equities is approximately 34%. (US Flow of Funds)

\(^{37}\)See French et al. (2010).

\(^{38}\)See US Department of Treasury (2018).

\(^{39}\)For example, by over-issuing difficult-to-resolve short-term debt.
3.5 Macroprudential Regulation and Bail-ins

In the baseline model, the fact that banks have a single investment project means that liability-side regulation is sufficient. In practice, banks asset allocations also affect their risk profiles. We now show that macroprudential (asset-side) regulation is a necessary complement to bail-ins when banks can affect risks using both sides of their balance sheet.

We augment the model as follows. Banks choose a contractible vector \( \theta = (\theta_1, ..., \theta_N) \) of asset allocations. The total return \( R \) on bank scale \( Y_0 \) follows a density \( f_e(R|s, \theta) \), which depends on the allocation \( \theta \). \( f_e(R|s, \theta) \) satisfies MLRP (conditional on \( (s, \theta) \)) over the relevant range of allocations \( \theta \). To simplify exposition, the support of \( R \) is an interval \([R_L, R_U]\) that does not depend on \( \theta \) or \( s \). Otherwise, the setup is the same as before.\(^{40}\)

As before, optimal liability contracts combine contingent and standard debt, and the trade-off between standard and bail-in debt reflects the same forces as before.\(^{41}\) We now characterize the optimal asset allocation rule under the socially optimal contract.

**Proposition 5.** The socially optimal contract has FOC for \( \theta_n \)

\[
0 = E \left[ (\lambda x(R, s) + c_2(R, s)) \left( 1 + \mu \left( 1 - \frac{\partial f_L(R|s, \theta) / \partial \theta_n}{f_H(R|s, \theta) / \partial \theta_n} \right) \right) \frac{\partial f_H(R|s, \theta) / \partial \theta_n}{f_H(R|s, \theta)} \right]
\]

\[
+ \lambda E \left[ \frac{\partial \gamma(s)}{\partial \Omega(s)} \Omega(s) Y_0 \int_{R_L}^{R(s)} R \frac{\partial f_H(R|s, \theta)}{\partial \theta_n} dR \right]
\]

The first line of Proposition 5 reflects the private trade-off to banks of a change in asset composition, corresponding to changes in the return distribution. These changes are weighted by the (weighted) sum of payoff to investors in those states, and to banks in those states, where the weighting reflects both the direct value of payoffs, and the incentive value of payoffs. The second line of Proposition 5 reflects the social cost of changes in asset composition. The social cost arises when changes in the return distribution affect the magnitude of the fire sale spillover, by altering the measure \( \Omega(s) \) of bank liquidations.

When an asset increases the probability that the banks’ total return is lower than \( R_L(s) \),

\(^{40}\)In Appendix B.2, we show how a standard asset allocation problem generates a density function of this form. If the shirking benefit \( B(\theta) \) depended on the allocation, e.g. because riskier assets are more difficult to monitor, the planner and banker would agree on how \( \theta \) affects \( B \). Assets in our model all sell at the same discount and generate the same fire sale spillover. If they differed in terms of liquidation discounts and fire sale spillovers, there would be an additional regulatory incentive on this margin.

\(^{41}\)Given that \( \theta \) is contractible, the proof follows the same steps as Proposition 1.
larger allocations to that asset result in more severe fire sale spillovers. The social cost term penalizes investment in such assets. The social cost term exists whenever \( R_l(s) > R \), that is whenever liability-side regulation has not completely eliminated bank failures.

Proposition 5 illustrates that macroprudential (asset) regulation is a necessary complement to bail-ins (liability regulation). Macroprudential regulation and bail-ins co-exist in the regulatory regime because they control fire sales in different manners. For a given level of asset risk, bail-ins mitigate fire sales by reducing the liquidation threshold. For a given liquidation threshold, macroprudential regulation mitigates fire sales by reducing the probability that a bank will fall below that threshold.\(^{42}\) These two aspects of regulation are not generally perfect substitutes, so they co-exist under the optimal regulatory regime.

Even though macroprudential regulation and bail-ins are not perfect substitutes, Proposition 5 suggests that bail-ins are a partial substitute for macroprudential regulation. Stronger liability regulation pushes the magnitude of the additional wedge in the asset allocation decision towards zero, by reducing the size of the liquidation region.

### 3.6 Extensions: Bank Capital Structure

Our baseline model focuses on the trade-off between standard debt and bail-in debt, which revolves around an incentive problem. In the appendix, we include two extensions to the bank’s problem to incorporate additional elements in the capital structure decision problem.

In Appendix C.5, we extend the model to incorporate equity-like claims into the bank’s capital structure. We do so by adding bank risk aversion and risk shifting. In this case, the disagreement between the bank and planner is over the use of standard debt versus loss-absorbing capital (bail-in debt + equity), and not over the composition of bail-in debt. Bail-in debt is valuable in the capital structure as a component of a bank’s loss-absorbing capital because it provides better incentives to the bank. This suggests that a planner may find it optimal to allow for the use of bail-in debt to satisfy regulatory requirements.

In Appendix C.6, we allow for standard debt to command a premium over other instruments, including bail-in debt.\(^{43}\) This further encourages the bank to use standard debt and helps to explain why, in practice, the level of standard debt banks employ is so high. The marginal trade-off for banks is still influenced by the incentive problem. In this sense, our model emphasizes the incentive story in determining the marginal trade-off

\(^{42}\)Macroprudential regulation in our model closely risk weights on loss-absorbing capital.

\(^{43}\)For example due to either tax benefits of debt or a liquidity premium associated with deposits.
between standard and bail-in debt, rather than in determining the total debt level of the bank. Moreover, the pure premium story (i.e. with no incentive problem) would predict that banks should hold a combination of standard debt and equity in an optimal capital structure, and has trouble explaining why banks would use illiquid assets to back standard debt, rather than backing them with safe assets such as government bonds (see Appendix C.6 for details). Our model respects one of the fundamental paradigms of banking: illiquid assets are used to back standard debt.

4 Bailouts and Time Consistency

We now introduce bailouts to the model and study their role in the optimal regime. Bailouts may be welfare-enhancing in our model, even if they are chosen ex post without commitment, because they can mitigate the fire sale spillover. However, a core principle of post-crisis regulation is that banks and bank investors, not taxpayers, should bear the costs of bank resolution.

We first study the private bank contracting problem and show that bailouts can completely eliminate banks’ incentives to issue bail-in debt. This offers another explanation for why banks wrote few contingencies into their debt contracts prior to the crisis. We then show that optimal regulation limits the use of standard debt so that no ex post bailouts occur: bail-ins fully replace bailouts. A planner who could tie her hands and never engage in bailouts would always prefer to do so.

The contracting problem is the same as before, except that banks understand they may be bailed out when insolvent.

4.1 Ex Post Bailout Authority

The planner can bail out insolvent banks in order to prevent liquidations and fire sales. Bailouts are chosen ex post according to a utilitarian welfare function, and are financed by taxpayers.

The bailout required to recapitalize an insolvent bank is \( T_1(R, s) = L_1(R, s) - (1 - b)RY_0 \). Bailouts are associated with two costs. First, bailouts have an ex post (date 1) political or administrative cost \( \kappa Y_1 \), which we think of as corresponding to a political backlash against bailouts. Second, bailouts have an ex ante (date 0) cost \( (\tau - 1)T_1(R, s) \).

Another view (Chari and Kehoe (2016)) is that a planner may be tempted ex post to bail out banks to prevent resource losses from liquidation, even in the absence of fire sale spillovers. The results of this section also hold in this case.

The political cost scales with bank size in order to prevent banks from “outgrowing” the cost.
to taxpayers, which we think of as corresponding to distortionary costs of taxation. The distortionary cost $\tau - 1$ is sufficiently large that transfers from taxpayers to banks are not welfare-enhancing for redistributive reasons alone.\textsuperscript{46} The timing of the two costs in our model generates simple bailout rules.

We conjecture a threshold rule for bailouts, so that insolvent banks with $R \geq R^{BO}(s)$ are bailed out, and then verify the rule is optimal. The optimal threshold $R^{BO}(s)$ is the solution to the ex post bailout problem

$$\max_{R^{BO}(s) \leq R_l(s)} \int_{\mathbb{R}} \left( (\gamma(s, \Omega(s)) - 1) RY_0 f_H(R|s) dR - \int_{R^{BO}(s)} \kappa RY_0 f_H(R|s) dR \right)$$

where the transfer $T_1(R, s)$ does not appear in the objective function because the planner is utilitarian and the distortionary cost arises ex ante. The ex post bailout decision trades off losses from bank liquidation against the political cost of bailing out banks. The optimal bailout rule, when interior,\textsuperscript{47} is given by

$$1 - \gamma(s, \Omega(s)) - \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{\mathbb{R}} R f_H(R|s) dR = \kappa$$

Equation (11) implies a threshold bailout rule, as conjectured, with a unique solution $R^{BO}(s)$.\textsuperscript{48} If the planner continued bailing out banks beyond $R^{BO}(s)$, concavity of $\gamma$ implies that total loss falls below the political cost of bailouts. As a result, we have a threshold rule.

Even though $R^{BO}(s)$ is unique, equation (11) implies strategic complementarities in bank risk taking: the planner only has an incentive to engage in bailouts ex post if the equilibrium contract features $R_l(s) > R^{BO}(s)$.\textsuperscript{49} If the equilibrium contract sets $R_l(s) < R^{BO}(s)$, but a single bank instead writes an alternative contract $R'_l(s) > R^{BO}(s)$,

\textsuperscript{46}See the proof of Proposition 7 for a formal condition. Even if redistribution were desirable, they could be done with ex ante lump-sum rather than ex post bailouts. We can interpret bank resources $A$ as including any desirable redistribution.

\textsuperscript{47}The optimal bailout rule may be a corner solution at $R$ if the left-hand side of equation (11) is lower than $\kappa$ at $R$, or a corner solution at $R_l(s)$ if the right-hand side of equation (11) is greater than $\kappa$ at $R$.

\textsuperscript{48}Uniqueness follows since since $\gamma(s)$ is non-decreasing and weakly concave in $\Omega(s)$. To ensure strict (rather than weak) optimality of the threshold rule, we can add a distortionary cost $\tau_1 \to 0$ of transfers at date 1.

\textsuperscript{49}See e.g. Farhi and Tirole (2012). To see the complementarity in our model, the left hand side of equation 11 is increasing in the marginal bank that is liquidated in equilibrium. When there are bailouts, the marginal bank that is liquidated has return $R = R^{BO}(s)$. When $R'_l(s) < R^{BO}(s)$, the marginal bank that is liquidated has $R = R'_l(s)$, and so the value of rescuing any failing bank is below the cost $\kappa$. 

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that bank will not be bailed out. As a result, there may be multiple equilibria of the date 0 private bank contracting problem. Our main result on the private contracting problem provides a generic result, and is agnostic to the equilibrium that is actually selected.

### 4.2 Privately Optimal Contracts

Optimal bank contracts take the same form as before, combining standard and bail-in debt. The only difference is that insolvent banks are now bailed out when the equilibrium contract sets \( R_l(s) \geq R^{BO}(s) \).\(^{50}\) We show that whenever the equilibrium contract sets \( R_l(s) > R^{BO}(s) \), then \( R_u(s) = R_l(s) \) and banks issue no bail-in debt for state \( s \). Bailouts crowd out bail-ins.

**Proposition 6.** Suppose there are time-inconsistent bailouts. If the equilibrium private contract sets \( R_l(s) \geq R^{BO}(s) \), then it also sets \( R_u(s) = R_l(s) \). There is bail-in debt in state \( s \) only if there are no bailouts in state \( s \).

To understand Proposition 6, suppose that the equilibrium contract features \( R_u(s) > R_l(s) \geq R^{BO}(s) \) in state \( s \). In states \( R_l(s) \leq R \leq R_u(s) \), bail-in debt is written down and the bank consumes \( c_2 = b R Y_0 \). Suppose that a single bank deviated by writing a contract that set \( R'_l(s) = R_u(s) \), and otherwise left contract terms unchanged. The bank now receives bailouts over the range \( R_l(s) \leq R \leq R_u(s) = R'_l(s) \), meaning that standard debt holders are fully repaid, while the bank consumes \( c_2 = b R Y_0 \). The investor participation constraint is relaxed, so that this contract strictly dominates the equilibrium contract, contradicting that \( R_u(s) > R_l(s) \geq R^{BO}(s) \) was an equilibrium optimal contract.\(^{51}\)

Proposition 6 provides a moral hazard view of limits to the private use of bail-in debt: banks do not use bail-in debt when they expect to be bailed out. The moral hazard view is particularly strong in states with fire sale spillovers, where resource losses are larger and bailout incentives stronger. This suggests why banks would not insure themselves against crisis states by using bail-in debt.\(^{52}\)

The moral hazard perspective is complementary to the high liquidation values and fire sale views in Section 3. High liquidation values help explain limited contingencies against

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\(^{50}\)Bank contract incentives do not change in the region of the contract above \( R_l(s) \). Below \( R_l(s) \), banks want to maximize on the bailout subsidy whenever possible. Due to liability monotonicity (5), this implies standard debt.

\(^{51}\)This result does not rely on fire sale spillovers. If \( 1 - \gamma(s) > \kappa \) but \( \gamma(s) \) does not depend on \( \Omega \), then \( R^{BO}(s) = R \) and every insolvent bank is bailed out in state \( s \). Proposition 6 implies that no bail-in debt is issued for state \( s \).

\(^{52}\)It also highlights why banks limit contingencies on the aggregate state, since increasing \( R_l(s) \) increases the size of the bailout subsidy.
“normal” times, while the fire sale and moral hazard views help explain limited contingencies against crisis states. Taken together, these views provide a more comprehensive account of the lack of pre-crisis contingencies in bank debt contracts.

4.3 Socially Optimal Contracts With Bailouts

The moral hazard problem of Proposition 6 generates an additional role for bank regulation. We study the socially optimal contract in the presence of ex post bailouts, and show that it sets \( R_l(s) \leq R^{BO}(s) \) so that no bailouts occur. This is true whether or not there are fire sale spillovers.

**Proposition 7.** Suppose there are time-inconsistent bailouts, and suppose that \( \tau \) is sufficiently large that ex ante transfers from taxpayers to banks are not welfare enhancing. The socially optimal contract sets \( R_l(s) \leq R^{BO}(s) \) \( \forall s \), and there are no bailouts. Bank welfare is non-decreasing in \( R^{BO}(s) \).

When banks fail ex post, the planner is tempted to bail them out to mitigate fire sales and prevent resource losses. However, bailouts are costly to taxpayers in a socially undesirable way. Although bail-ins are costly to investors ex post, this cost is priced into bank contracts ex ante. The planner prefers to recapitalize failing banks using bail-ins rather than bailouts, avoiding perverse redistribution from taxpayers to banks and bank investors. A planner would therefore prefer to commit never to bail out failing banks and instead use bail-ins to mitigate fire sales.

In the absence of commitment power, bailouts are chosen in a time-inconsistent manner. A planner that prefers ex ante to liquidate a bank for incentive reasons may prefer ex post to bail out the bank to mitigate fire sales. Since liquidation is not time consistent, the planner must choose between bail-in debt and bailouts to recapitalize the bank. Since bailouts are socially costly, the planner chooses bail-ins. Bail-ins are an imperfect but time-consistent substitute for a commitment against bailouts. Nevertheless, since the planner is forced to increase the use of bail-in debt to prevent bailouts, the optimal bank contract is distorted relative to the optimal commitment contract. Bank welfare increases in the strength of the commitment against bailouts.\(^{53}\)

Proposition 7 reflects the principle that banks and bank investors, not taxpayers, should bear the cost of bank recapitalization. Optimal regulation fully replaces bailouts with bail-ins.

\(^{53}\)We can think of the cost \( \kappa \) as reflecting the strength of commitment against bailouts.
4.4 Optimality of Bailouts in Extended Settings

Proposition 7 shows that optimal regulation fully replaces bailouts with bail-ins. In the appendix, we study two settings in which bailouts can be desirable in equilibrium.

In Appendix C.7, the planner lacks commitment on both bailouts and bail-ins, meaning that standard debt cannot be made contingent on the aggregate state. The planner may prefer to allow for bailouts in crisis states to allow for incentive-providing liquidations in non-crisis states.

In Appendix C.8, we allow for insured deposits at banks. The planner faces a trade-off between greater deposit insurance (i.e. taxpayer) losses when liquidating the bank, and worse bank incentives when bailing out the bank. Bailouts may be desirable to lessen the taxpayer burden of deposit insurance. All non-deposit investors are fully bailed in whenever the planner bails out the bank. This motivates the possibility of having a deposit guarantee scheme, even in the absence of other bailouts.

5 Rollover Crises

We now turn to the second part of the paper, in which we characterize rollover crisis equilibria that can arise due to the prospect of a bail-in. In these equilibria, bail-in debt holders believe they are about to be written down, and refuse to refinance an otherwise healthy bank at date 1. The bank becomes too fragile to recapitalize itself and suffers an immediate bank run. We first show the existence and properties of rollover crises, and then in Section 6 consider potential solutions to the problem.

To generate rollover crises, we need to introduce a notion of fragility in the date 1 economy. To do so, we extend the model to four periods, and incorporate uncertainty at date 2 via a second quality shock. We incorporate standard short-term debt and bail-inable long-term debt. Given the uncertainty in continuation, banks will try to refinance the maturing short-term debt by replacing it with long-term debt in order to avoid future liquidations. The model is otherwise the same, and will generate the same optimal contracts as in Section 3.

We extend the model to four periods, $t = 0, 1, 2, 3$. We assume there is no aggregate uncertainty ($|S| = 1$) for expositional simplicity. Figure 3 provides an illustration of the extended timeline. Banks sign complete markets contracts at date 0 and experience a quality shock $R$ at date 1, with the same properties as in the baseline model. Banks also

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54 In practice, bail-in debt is typically long-term debt. See Appendix D.1.
55 Results in this section can be interpreted as being conditional on a realized aggregate state.
experience a quality shock $R_2$ at date 2, with $R_2 \sim F_2$ on $[R, \bar{R}]$ and $E[R_2] = 1$, after which uncertainty resolves. Projects mature at date 3 and pay off $Y_3 = Y_2 = R_2 Y_1$ units of the consumption good, but may be liquidated prior to that. The date 1 limited pledgeability constraint takes the form of a maximum date 1 debt level $R_b Y_1$, where $R_b \in [R, \bar{R}]$. This debt limit implies the maximum date 1 pledgeable income is

$$\int_{R \leq R_b} R f_2(R_2)dR_2 + \int_{R \geq R_b} R_b f_2(R_2)dR_2 \equiv 1 - b$$

(12)

so that, as before, $(1 - b)RY_0$ is the maximum pledgeable (expected) repayment to investors. Liquidations are not necessary for incentive provision in the continuation problem, meaning that all liquidations on the equilibrium path occur at date 1. However, the liquidation value $\gamma(\Omega)$ at date 1 is persistent and applies also at date 2. We think of dates 1 and 2 as subsequent stages of a crisis, where fire sale prices remain depressed throughout.

Optimal contracts are the same as in Section 3 and combine standard and bail-in debt. In the best equilibrium, the bank refines itself when it is solvent at date 1 using a combination of standard and bail-in debt. It uses sufficient bail-in debt to ensure that it never becomes insolvent at date 2. We focus now on formally characterizing the refinancing problem.

Let $D_1$ denote the initial level of standard (short-term debt) and $Y_0$ the project scale. We assume all bail-in debt has already been written down preemptively at the beginning of date 1 to ease exposition.\textsuperscript{56} The bank refines itself using a combination of short-term debt $D_2$ and long-term debt $L_3$. We incorporate an institutional feature of bail-ins, which is that long-term (bail-in) debt is subordinated to short-term (non-bail-inable) debt in all resolution proceedings.\textsuperscript{57}

Throughout this section, we study the refinancing problem of a fundamentally solvent bank, with $(1 - b)RY_0 < D_1 < (1 - b)RY_0$. Fundamentally insolvent banks are always liquidated. We will also rule out all equilibria except two: the best equilibrium with successful refinancing (as in the baseline model), and the rollover crisis equilibrium.

### 5.1 Best Equilibrium with Successful Refinancing

At date 1, the bank operates in a Walrasian market, where $q_1^D$ is the price of a new unit of short-term debt, and $q_1^L$ is the price of a new unit of long-term (bail-in) debt.

\textsuperscript{56}Existence of rollover crises does not depend on this assumption.

\textsuperscript{57}This is consistent with the No Creditor Worse Off principle of resolution. See Appendix D.1. See Appendix C.12 for the case where bail-in debt is not fully subordinated.
We assume all contracts are fully visible to all creditors to rule out conventional rat race dynamics.\textsuperscript{58} This assumption is reflected in the following pair of constraints

\begin{align}
q_1^D D_2 &\leq \int_{D_2 \geq R_2 Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2 \leq R_2 Y_1} D_2 f_2(R_2) dR_2 \tag{13} \\
q_1^L L_3 &\leq \int_{D_2 \leq R_2 Y_1} \min\{R_2 Y_1 - D_2, L_3\} f_2(R_2) dR_2 \tag{14}
\end{align}

which state that the expected payoff to debt holders must be at least as high as the amount they pay, given market prices.\textsuperscript{59} They imply the bank cannot deliberately dilute new bank creditors. Under these constraints, there is a unique best equilibrium under which the bank successfully refines itself and never liquidates after date 1.

**Lemma 8.** There is a unique best equilibrium with no liquidations after date 1. It has prices $q_1^D = 1$ and $q_1^L < 1$. It has short-term debt issuance $D_2 = \bar{R} R Y_0$ and long-term debt issuance $L_3 = \frac{D_1 - D_2}{q_1^L}$.

The best equilibrium of Lemma 8 is the refinancing problem outcome associated with the optimal contracts considered in the first part of the paper.

### 5.2 Rollover Crisis Equilibrium

We now study the existence of the worst “rollover crisis” equilibrium, where the bank fails to refinance itself and is immediately liquidated, despite being fundamentally solvent. To be clear, we are not looking for traditional bank run equilibria, and we assume that a costless lender of last resort stands ready to stop a sunspot bank run. Rollover crises will result in a bank run: short-term debt alone cannot refinance the bank.

In what follows, we assume that the maximum amount pledgeable to investors is higher in continuation than in liquidation even with short-term debt,

\[
\sup_{d_2 \leq R_2} \int_{\mathbb{R}} d_2 \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{\infty} d_2 f_2(R_2) dR_2 > \gamma
\]

where we define $d_2 = D_2 / Y_1$ to be the continuation debt-to-asset ratio. Repayment to short-term debt holders therefore cannot be increased by liquidating the bank. As a result, if the bank cannot refinance itself, there is sufficient short-term debt to liquidate the bank.

\textsuperscript{58}We could also interpret these as bond covenants.

\textsuperscript{59}We use equations (13) and (14), which are stronger conditions than needed, in order to guarantee the debt issuances of the best equilibrium are unique. Rollover crises are not affected by using these stronger conditions. See Appendix B.3.
We now show that a rollover crisis equilibrium generically exists in a region above the short-term debt level $R_l$.

**Proposition 9.** Let $D_1 = (1 - b)R_lY_0$ be the short-term debt level, with $R_l > R$. A rollover crisis equilibrium exists for all date-1 returns $R \in [R_l, R^*)$, where $R^*$ is given by

$$
\frac{D_1}{R^*Y_0} = \sup_{d_2 \leq R_l} \int_{R_l}^{d_2} \gamma R_2 f_2(R_2)dR_2 + \int_{d_2}^{R^*} d_2 f_2(R_2)dR_2
$$

In the rollover crisis equilibrium, the bank fails to refinance itself at date 1 and is liquidated. The equilibrium price of new long-term debt is $q^L_1 = 0$.

To understand the intuition behind the rollover crisis equilibrium, consider the threshold bank for which $D_1 = (1 - b)Y_1$. This bank is just able to recapitalize itself in the best equilibrium. It can survive only by issuing the securities packages with $D_2 \leq RY_1$ and $L_3 = R_3 Y_1 - D_2$ that pledges away all of its pledgeable income without any liquidations in continuation. In particular, there are no securities packages $(D_2, 0)$ and price $q_1$ that would allow the bank to generate enough revenue and refinance itself in a Walrasian equilibrium. As a result, this threshold bank, if faced with the only option of refinancing itself with short-term debt, would suffer a run by its short-term creditors.\(^{60}\) This generates the rollover crisis equilibrium: holders of long-term debt expect to be bailed in and quote a price of 0, leading to a bank run and liquidation. Since new long-term debt is subordinated to short-term debt, it would receive no recovery value in the liquidation, justifying the equilibrium price of 0 and completing the equilibrium.

Rollover crisis equilibria result from sunspots in long-term debt markets unlike conventional bank runs which are the result of sunspots in short-term debt markets. They are justified by the bail-in regime, which promises to subordinate long-term debt to short-term debt in resolution, and rely on the fundamental fragility of short-term debt relative to long-term debt. A bank on the brink of non-viability has an excessively high short-term-debt-to-assets ratio which it needs to unwind in order to remain viable. If it cannot, it becomes non-viable and subject to a run.

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\(^{60}\)A lender of last resort also cannot break even on any securities package $(D_2, 0)$ below $R^*$. Above $R^*$, there exists a price $q^D_1$ and debt issuance $D_2$ such that $q^D_1 D_2 = D_1$, in other words the bank can refinance itself from a short-term lender of last resort.
5.3 Propagation of Rollover Crises

The existence of rollover crises in Proposition 9 relies only on the presence of liquidation discounts, and not on fire sale spillovers. However, we show that fire sale spillovers contribute to the propagation of rollover crises, leading to more frequent crises and larger fire sales.

Because we have multiple equilibria, we must adopt an equilibrium selection rule that determines which equilibrium a bank in the rollover crisis region experiences. We adopt a simple selection rule: banks experience a rollover crisis with probability \(0 < p < 1\) and experience the best equilibrium with probability \(1 - p\). By the law of large numbers, equilibrium liquidations and the liquidation value are a solution to

\[
\gamma^* = \gamma(\Omega^*), \quad \Omega^* = \epsilon + \int_{R_i}^{R_f} R f_H(R) dR + p \int_{R_i}^{R_f} R f_H(R) dR
\]

where \(\epsilon\) is an exogenous liquidation shock that we use to illustrate the feedback loop. The rollover crisis threshold \(R^*\) depends on the equilibrium liquidation value \(\gamma^*\), as illustrated in Proposition 9, so that we have a fixed point problem. We characterize the feedback loop by starting from \(\epsilon = 0\), and then study the equilibrium response of total liquidations \(\Omega^*\) with respect to \(\epsilon\). If there is no feedback loop, then \(\frac{\partial \Omega^*}{\partial \epsilon} = 1\).

**Proposition 10.** Starting from an equilibrium of the date-1 economy with \(\epsilon = 0\), an exogenous increase in liquidations \(\epsilon\) generates a total increase in equilibrium liquidations

\[
\left. \frac{\partial \Omega^*}{\partial \epsilon} \right|_{\epsilon=0} = 1 + \frac{p \left| \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \right| \left| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \right| R^* f_H(R^*)}{1 - p \left| \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \right| \left| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \right| R^* f_H(R^*)} \bigg|_{\epsilon=0}
\]

The core of the feedback loop in Proposition 10 is the sensitivity of the liquidation discount to liquidations, \(\frac{\partial \gamma}{\partial \Omega}\). When this sensitivity is higher, \(\frac{\partial \Omega^*}{\partial \epsilon}\) increases, and the exogenous shock to liquidations is amplified by the propagation of rollover crises. The increase in liquidations lowers the liquidation value, expanding the region of crises. More banks become subject to rollover crises, pushing down liquidation values further. By contrast, if liquidation values are not sensitive to liquidations (\(\frac{\partial \gamma}{\partial \Omega} = 0\)) then there is no feedback loop and no propagation.

In normal times, when fire sale spillovers are limited, rollover crises may therefore be relatively contained. By contrast, during crises, when spillovers may be more severe, they can propagate, thus increasing bank failures and exacerbating fire sales. This propagation effect reflects the concern that even though bail-ins may have beneficial properties for bank
resolution during relatively normal times or for the resolution of individual banks, they may generate adverse effects during times of systemic crises.\textsuperscript{61}

5.4 Impact of Covenants

A natural conjecture is that bond covenants, which are a common tool for addressing typical debt dilution or rat race incentives, would be effective in preventing rollover crises. We show that bond covenants do not prevent rollover crises.

A bond covenant is an arbitrary set of restrictions on the refinancing structures available to the bank at date 1,

$$C_1 (D_2, L_3 | D_1, Y_1, q_1) \leq 0,$$

where $C_1$ is some vector-valued function.\textsuperscript{62} Define the \textit{vacuous covenant} by $C_1 (D_2, L_3 | D_1, Y_1, q_1) = 0$, which places no restrictions on the bank. All previous results have assumed the vacuous covenant.

\textbf{Proposition 11. If a rollover crisis equilibrium exists under the vacuous covenant $C_1 (D_2, L_3 | D_1, Y_1, q_1) = 0$, then it also exists under any other covenant $C_1 (D_2, L_3 | D_1, Y_1, q_1)$.}

Proposition 11 shows that covenants are not a solution to rollover crises. To understand why, rollover crises are justified in equilibrium by outstanding short-term debt, not by new short-term debt. As a result, covenants cannot rule out rollover crises.

6 Policy Responses to Rollover Crises

In Section 5, we showed that the socially optimal contracts written by the planner in Section 3.2 were susceptible to rollover crises at date 1. We now consider ex post policies the planner could adopt to prevent rollover crises.\textsuperscript{63} We consider two policies that have precedent in policies employed during the 2008 financial crisis. The first is an expanded lender of last resort that extends both short- and long-term loans to banks. The second is temporary guarantees of new issuances of long-term debt.

\textsuperscript{61}In Appendix C.11, we show that rollover crises can generate multiple aggregate equilibria of the date 1 economy, even for a fixed equilibrium selection rule $p$, due to the propagation effect.

\textsuperscript{62}For example, the covenant $C_1 (D_2, L_3 | D_1, Y_1, q_1) = D_2 - RY_1$ restricts short-term debt issuance at date 1.

\textsuperscript{63}The planner could also limit rollover crises by adjusting ex ante contract terms, but this would not rule out rollover crises unless the planner set $R_t \leq R$ and would distort the optimal contract.
6.1 Expanded Lender of Last Resort

A common solution to sunspot bank runs by short-term debt holders (i.e. depositors) is a lender of last resort (LOLR), which extends short-term loans to fundamentally solvent banks faced with a sunspot bank run.\textsuperscript{64} Although rollover crisis equilibria in our model are a form of coordination failure, the coordination failure arises from long-term debt, not short-term debt. A conventional LOLR extending short-term loans is therefore not a sufficient policy in this case,\textsuperscript{65} but a LOLR facility might be successful if it provided long-term debt loans to distressed banks.\textsuperscript{66} A facility of this form is not without precedent. For example, the Capital Purchase Program (CPP) implemented under TARP in 2008, made $250 billion available for the purchase of preferred stock in banks, with the goal of making funding available in a fragile market. The nature and goals of CPP were similar to those of our proposal.\textsuperscript{67}

We model the extended LOLR facility as follows. At the same time as the bank accesses private markets, it can also access the LOLR which is willing to make available to the bank any package \((D_{LOLR}^L, L_{LOLR}^L)\) at prices \(q_{LOLR}^L\) so that it breaks even in expectation. LOLR claims rank \emph{pari passu} with private sector claims of the same instrument.\textsuperscript{68} Provided that the bank successfully refinances with issuances \((D_2, L_3, D_{LOLR}^L, L_{LOLR}^L)\), the break-even prices of the LOLR are given by

\begin{align}
q_{D_{LOLR}}^L &= 1 - \int_{D_2 + D_{LOLR}^L \geq R_1 Y_1} \left(1 - \frac{\gamma R_2 Y_1}{D_2 + D_{LOLR}^L} \right) f_2(R_2) dR_2 \quad (19) \\
q_{LOLR}^L &= \int_{D_2 + D_{LOLR}^L \leq R_1} \min \left\{ \left( \frac{R_2 Y_1 - D_2 - D_{LOLR}^L}{L_3 + L_{LOLR}^L} \right) \frac{1}{L_3 + L_{LOLR}^L}, 1 \right\} f_2(R_2) dR_2 \quad (20)
\end{align}

\textsuperscript{64}For example, the Federal Reserve set up the Term Securities Lending Facility (TSLF) in March 2008 to provide liquidity to financial institutions and restore credit market functioning, allowing them to swap illiquid collateral for liquid Treasury securities, which could then act as collateral in new funding agreements in credit markets.

\textsuperscript{65}A conventional LOLR facility could provide a temporary stopgap while the government attempted to orchestrate a public or private sector rescue program.

\textsuperscript{66}These loans would be subordinated to short-term debt, and would be potentially bail-inable and uncollateralized. The LOLR would break even in expectation but not on every realized path. This LOLR facility differs starkly from the usual facility and violates Bagehot best practice principles.

\textsuperscript{67}The US Treasury Department, in describing CPP, states that it “helped bolster the capital position of viable institutions of all sizes and built confidence in these institutions and the financial system as a whole.” https://www.treasury.gov/initiatives/financial-stability/TARP-Programs/bank-investment-programs/cap/Pages/default.aspx.

\textsuperscript{68}In practice, the government tends to give itself high priority on claims in resolution. See 12 CFR §380.21.
Given market prices $q_1$ for private sector debt, the bank can refinance itself if there are a securities package $(D_2, L_3, D_2^{LOLR}, L_3^{LOLR})$ and corresponding LOLR prices $q_1^{LOLR}$ that allow the bank to raise $D_1$ in revenue, while satisfying the private sector no-rat-race conditions.69

Because the bank can always refinance itself successfully by borrowing best equilibrium quantities from the LOLR at best equilibrium prices, rollover crises are eliminated. Since LOLR debt and private sector debt rank pari passu, private sector prices must be the best equilibrium prices. The bank can then refinance itself entirely from the private sector.

**Proposition 12.** Suppose there is an extended LOLR facility. Suppose private sector and LOLR claims rank pari passu. Rollover crises are eliminated. The bank can refinance itself entirely in private markets at best equilibrium prices.

Proposition 12 is similar in spirit to the standard LOLR solution to sunspot bank runs: by making loans available to the bank, the LOLR prevents the onset of the crisis. Although these loans only happen off equilibrium in the model, in practice such a facility would likely be used to some degree by banks during crises. This leads to two natural practical concerns. The first is that because the LOLR extends uncollateralized, long-term loans and would lose money on some realized paths (but not in expectation), the operating principles are different from those of a standard LOLR. The second is that the facility would be subject to familiar moral hazard concerns: it may unintentionally lend to insolvent banks, or lend at prices lower than break-even prices.70 This would lead the LOLR to subsidize distressed banks, amounting to a partial bailout.

### 6.2 Debt Guarantees

Deposit insurance is a second common solution to sunspot bank runs. In its idealized form, deposit insurance prevents the run and is not filled on the equilibrium path. In a rollover crisis, although insurance would allow the bank to roll over its short-term debt and avoid liquidation, insurance would be filled on the equilibrium path.

We consider instead an extension of temporary guarantees to new issuances of long-term debt, and show that these guarantees can successfully rule out rollover crises without being filled on the equilibrium path. This has precedent in the Temporary Liquidity

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69The proof of Proposition 12 restates these conditions when LOLR debt is included.

70One particular manifestation of this is that the break-even prices of long-term debt are bank-dependent, because it is not guaranteed to be repaid in full. A LOLR operating during stress times may not wish to price discriminate between banks to avoid reputational damage to distressed banks.
Guarantee Program (TLGP) instituted in 2008, under which the US government provided guarantees to new issuances of senior unsecured debt with the goal of “preserving confidence in the banking system and encouraging liquidity.” These guarantees were “temporary” in that they expired no later than June 2012.

We model guarantees as follows: at the beginning of date 1, the government extends a temporary guarantee to all new issuances of long-term debt. Guarantees oblige the government to cover any losses relative to face value during the guarantee period. Guarantees expire at the end of date 1, after which the debt is once again subject to the bail-in regime. In other words, if at date 1 the bank enters bankruptcy and liquidates, the government is obligated to pay $1 - q_{LB1}$ to a holder of new long-term debt, where $q_{LB1} \leq 1$ is the recovery value of new long-term debt in liquidation. This guarantee extension eliminates rollover crises.

**Proposition 13.** A temporary guarantee of new long-term debt that expires at the end of period 1 eliminates rollover crises. Guarantees are not filled on the equilibrium path.

Proposition 13 is closely related to the rationale for deposit insurance. Since debt is guaranteed, investors cannot expect a price of 0, ruling out the rollover crisis and allowing the bank to refinance itself. Guarantees are never filled on the equilibrium path because they expire the end of date 1. This “temporary” aspect is consistent with the principle of TLGP, where the guarantees expired after a certain time frame.

It is too strong in practice to assume that guarantees can be timed perfectly to never be filled. Some guarantees would be filled on the equilibrium path, leading to moral hazard concerns. Proposition 13 is an idealized result that helps explain why debt guarantee programs such as TLGP may be a valuable part of a crisis resolution toolkit.

### 6.3 Bailouts versus Debt Guarantees

Sections 6.1 and 6.2 show that expanded LOLR facilities and long-term debt guarantees can be valuable components of a planner’s crisis resolution toolkit. However, post-crisis

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72 We have assumed away existing long-term debt. Guarantees would not need to be extended to existing long-term debt. This is consistent with TLGP.
73 The fact that long-term debt does not mature until the final period increases its value relative to short-term debt even in the absence of future bail-ins, as it cannot trigger early liquidations. The results that follow would apply, possibly in a weaker form, even if the government cannot bail-in this debt in the future either.
74 The Debt Guarantee Program (DGP) portion of TLGP, which involved guarantees of new loans, at its peak guaranteed approximately $345 billion in debt. Approximately $153 million in guarantees were filled. A second component of TLGP, the Transaction Account Guarantee Program, provided guarantees for noninterest-bearing transaction accounts and resulted in estimated losses of $2.5 billion. These numbers are reported by the FDIC as of February 2019 (https://www.fdic.gov/regulations/resources/tlgp/index.html).
regulation has limited the ability of the US government to engage in such policies as part of a commitment against future bailouts.\footnote{See Geithner (2016).} Our model suggests an important distinction between bailouts of existing debt and protection of new debt issued during stress times, even when such protection is “bailout-like.” Optimal regulation in Section 4 replaces bailouts of existing debt with bail-ins. Guarantees serve a different function and can be a valuable stabilization tool during a crisis. A commitment against bailouts of existing debt need not preclude protection of new debt during a crisis.

7 Extensions

In Appendix C, we provide several extensions to the optimal contracting problem, to the bailout problem, and to the rollover crisis problem. In Appendix D, we discuss the relationship between our model and bail-in implementations in practice.

In Appendix C.1, we discuss the role of the agency problems in generating optimal contracts that combine standard and bail-in debt. We show that in the absence of both the ex ante and interim agency problems ($B, b > 0$), contracts would not combine standard and bail-in debt.

In Appendix C.2, we characterize the (Pigouvian) tax wedges needed to decentralize the socially optimal contract. Tax wedges are needed only on standard debt, and not on bail-in debt.

In Appendix C.3, we adopt the arbitrageur model of Appendix B.1, and characterize sufficient conditions under which the socially optimal contract in Section 3.2 is Pareto efficient relative to the privately optimal contract.

In Appendix C.4, we extend the model to allow for heterogeneous investors with different risk tolerances and different exposures to the banking sector, and discuss which investors bear which risk exposures. This allows us to consider the allocation of bail-in securities. We show that retail investors, who maintain greater exposures to individual banks, and institutional investors who experience spillovers from fire sales should hold safer (non-bail-inable) claims.

In Appendices C.5 and C.6, we study the capital structure extensions discussed in Section 3.6. In Appendices C.7 and C.8, we consider the two bailout extensions discussed in Section 4.4.

In Appendix C.9, we allow liquidation discounts to depend on bank size, and show that the planner taxes both short-term debt and bank size under optimal regulation. In
Appendix C.10, we allow the monitoring effort choice to be continuous rather than binary, and show that this generates a motivation for the planner to control the use of both standard and bail-in debt.

In Appendix C.11, we characterize conditions under which rollover crises can generate multiple equilibria of the date 1 economy.

In Appendix C.12, we study rollover crises, early triggers, and the relative roles of \textit{de jure} and \textit{de facto} seniority in rollover crises. First, we argue that the existence region of rollover crises defined in Proposition 9 is invariant to early triggers. Second, we argue that affording \textit{de jure pari passu} status to a set of non-bail-inable long-term debt claims during a crisis can potentially alleviate rollover crises, but may be problematic.

8 Conclusion

We characterize optimal bank contracts under a monitoring incentive problem. Privately optimal bank contracts combine standard and bail-in debt. Banks’ private use of contingencies on both the idiosyncratic state and aggregate state is limited when liquidation values are high, when there are fire sale spillovers, and when there is moral hazard from bailouts. Optimal regulation increases state-contingencies in debt contracts, on both idiosyncratic and aggregate states, and so resembles the bail-in regimes that have been implemented in both the US and EU. Optimal regulation replaces bailouts with bail-ins.

We also show that the prospect of bail-ins can have destabilizing effects on bank refinancing during times of market stress, leading to rollover crises and bank failures. Rollover crises can be addressed using policies such as extended lender of last resort facilities and debt guarantees. These crisis resolution tools complement an effective bail-in regime.

References


Figure 1: This figure presents a simple timeline for the model.
Figure 2: This figure provides an illustration for the privately optimal contract when \(|S| = 1\) (no aggregate risk). Up to a threshold \(R_l\), bank liabilities are constant and exceed pledgeable income, leading to liquidations (“standard debt”). Between \(R_l\) and \(R_u\), the face value of liabilities is written down to coincide with pledgeable income (“bail-in” or “write-down”). Above \(R_u\), the face value of liabilities is constant (“bail-in debt”).
Figure 3: This figure presents a simple timeline for the extended model (Section 5).
A Proofs

A.1 Proof of Proposition 1

Consider the program

\[
\max_{L_1,Y_0} E [c_2(R,s)|e = H]
\]

Subject to

\[
E \left[ c_2(R,s) \left( 1 - \frac{f_L(R,s)}{f_H(R,s)} \right) \right] \geq BY_0
\]

\[
Y_0 - A = \sum_s \pi(s) \left[ \int_{R|\alpha=1} \gamma(s)RY_0f_H(R|s)dR + \int_{R|\alpha=0} L_1(R,s)f_H(R|s)dR \right]
\]

\[
R \geq R' \Rightarrow L_1(R,s) \geq L_1(R',s)
\]

\[
c_2 \geq (1 - \alpha(R,s))bRY_0
\]

\[
L(R,s) \geq 0
\]

and recall the second to last constraint is limited pledgeability. It is helpful to redefine the problem in the investor payoff space, and then to define the implementing liability structure \(L_1(R,s)\). Total investor payoff \(x(R,s)\) is given by

\[
x(R,s) = \alpha(R,s)\gamma(s)RY_0 + (1 - \alpha(R,s))RY_0 - c_2(R,s)
\]

where \(\alpha(R,s) \in \{0,1\}\) is the liquidation rule. We treat \(\alpha(R,s)\) as a choice variable, and then back out the liability structure that implements it. Note that because banks are repaid 0 when \(\alpha(R,s) = 1\), it is irrelevant whether we multiply \(c_2(R,s)\) by \(1 - \alpha(R,s)\). Given this characterization, investor voluntary participation can be rewritten as

\[
Y_0 - A = E [\alpha(R,s)\gamma(s)RY_0 + (1 - \alpha(R,s))RY_0 - c_2(R,s)|e = H].
\]

We begin by studying the optimization problem not subject to liability monotonicity, and show that it generates a non-monotone contract. The Lagrangian of this relaxed
problem is
\[ L = E [c_2(R, s)|e = H] + \mu \left[ E \left[ c_2(R, s) \left(1 - \frac{f_L(R|s)}{f_H(R|s)}\right) \right| e = H \right] - BY_0 \]
\[ + \lambda \left[ E[\alpha(R, s)\gamma(s)RY_0 + (1 - \alpha(R, s))RY_0 - c_2(R, s)|e = H \right] + A - Y_0 \]
\[ + E[\chi(R, s) (c_2(R, s) - (1 - \alpha(R, s))bRY_0) |e = H] \]
\[ + E[\xi(R, s) ((\alpha\gamma(s)RY_0 + (1 - \alpha)RY_0 - c_2(R, s)))|e = H] \]

From here, first order condition for bank consumption as

\[ 0 = f_H(R|s) + \mu \left(1 - \frac{f_L(R|s)}{f_H(R|s)}\right) f_H(R|s) - \lambda f_H(R|s) + \chi(R, s)f_H(R|s) - \zeta(R, s)f_H(R|s) \]
\[ = \left[1 - \lambda + \mu \left(1 - \frac{f_L(R|s)}{f_H(R|s)}\right)\right] f_H(R|s) + \chi(R, s) - \zeta(R, s) \]

By MLRP, there is a threshold \( R^*(s) \) such that \( \chi(R, s) > 0 \) for \( R \leq R^*(s) \) and \( \zeta(R, s) > 0 \) for \( R \geq R^*(s) \), implying that \( x(R, s) = L_1(R, s) = 0 \) for all \( R \geq R^*(s) \). This threshold is given by
\[ 1 - \lambda + \mu \left(1 - \frac{f_L(R^*(s)|s)}{f_H(R^*(s)|s)}\right) = 0. \] (21)

However, this contract violates liability monotonicity unless \( L_1(R, s) = 0 \) for all \( R \) in state \( s \). Therefore, we have an upper pooling region in the optimal contract, where liabilities and investor repayment are constant.\(^{76}\)

It is worth remarking that the contract not subject to monotonicity is of the live-or-die form.\(^{77}\) It implies that banks will be either liquidated or held to the agency rent when \( R < R^*(s) \), with all remaining repayment going to investors. By contrast, the bank receives the full resources of the bank when \( R > R^*(s) \). This contract is optimal because it provides strong incentives to the bank. Because all agents are risk-neutral, they are willing to accept this extreme payoff structure. However, this payoff structure violates liability monotonicity, and so is not implementable.

We now characterize the optimal contract using the following strategy. First, we conjecture pooling thresholds \( R_u(s) \) with corresponding liabilities \( x_u(s) \equiv x(R_u(s), s) = L_1(R_u(s), s) \), so that \( x(R, s) = x_u(s) \) for all \( R \geq R_u(s) \). The live-or-die result of the

\(^{76}\)If \( L_1(R, s) = 0 \) for all \( s \), then the entire contract is pooled in state \( s \). If \( R^* = \bar{R} \), then the results that follow apply setting \( R_u(s) = \bar{R} \) to be the pooling threshold.

\(^{77}\)See Innes (1990).
contract not subject to monotonicity implies such a pooling threshold exists.\textsuperscript{78} We then solve for the optimal contract below $R_u(s)$, taking as given $R_u(s)$ and $x_u(s)$, subject to a relaxed monotonicity constraint $x(R, s) \leq x_u(s) \forall R \leq R_u(s)$, and verify that the resulting contracting is monotone. In doing so, we characterize the space of implementable contracts (that satisfy monotonicity). Finally, we optimize over the choice of $R_u(s)$ and $x_u(s)$.

Conjecture pooling thresholds $R_u(s)$ with liabilities $x_u(s)$. The associated Lagrangian is given by

$$
\mathcal{L} = E[c_2(R, s)|e = H] + \mu \left[ E \left[ c_2(R, s) \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) |e = H \right] - BY_0 \right] + \lambda \left[ E \left[ (\alpha(R, s)\gamma(s)RY_0 + (1 - \alpha(R, s))RY_0 - c_2(R, s)|e = H + A - Y_0 \right] + E \left[ (\chi(R, s) c_2(R, s) - (1 - \alpha(R, s)) bRY_0) |e = H \right] + E \left[ (\nu(R, s) (x_u(s) - (\alpha\gamma(s)RY_0 + (1 - \alpha)RY_0 - c_2(R, s))) |e = H \right]
$$

where the final line is the relaxed monotonicity constraint, and where we have anticipated that limited liability $x(R, s) \geq 0$ does not bind below $R_u(s)$ for feasible contracts. Taking the derivative in consumption $c_2(R, s)$ for $R \leq R_u(s)$, we obtain

$$
0 = 1 + \mu \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) - \lambda + \chi(R, s) + \nu(R, s).
$$

Observe that the resulting contract is non-monotone if $R_u(s) > R^*(s)$ (we would have $\zeta(R, s) > 0$ so that $x(R, s) = 0$), by the same logic as above. Therefore, we can discard candidate contracts with $R_u(s) > R^*(s)$. This implies that $1 + \mu \left( 1 - \frac{f_L(R_u(s)|s)}{f_H(R_u(s)|s)} \right) - \lambda < 0$ among the set of viable contracts.

Now, consider the derivative in liquidations $\alpha(R, s)$, given by\textsuperscript{79}

$$
\frac{\partial \mathcal{L}}{\partial \alpha(R, s)} \propto \lambda (\gamma(s) - 1) + \chi(R, s)b + \nu(R, s) (1 - \gamma(s))
$$

When $\alpha(R, s) = 1$, $\nu(R, s) = 1$ is possible at at most a single point, in particular at $\gamma(s)RY_0 = x_u(s)$. $\alpha(R, s) = 1$ therefore generically implies $\chi(R, s) > 0$ and $\nu(R, s) = 0$.

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\textsuperscript{78}Note that this is without loss, since the pooling threshold could be $R_u(s) = R$ if $R^*(s) = R$.

\textsuperscript{79}Implicitly, we are treating $\alpha(R, s)$ as a continuous variable in performing the differentiation. To do so, we implicitly incorporate the constraint $\alpha(R, s)(1 - \alpha(R, s)) = 0$, which ensures that implementable contracts must set $\alpha(R, s) \in \{0, 1\}$. The logic below is unaffected.
From the FOC for $c_2(R, s)$, we have (almost everywhere) that when $\alpha(R, s) = 1$

$$\chi(R, s) = \lambda - 1 - \mu \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right)$$

which, combined with the liquidation rule, yields

$$\frac{\partial L}{\partial R}(R, s) \propto \lambda (\gamma(s) - 1) + \left( \lambda - 1 - \mu \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \right) b.$$ 

By MLRP, there is a threshold rule $R \leq R_l(s)$ for liquidations.

Finally, in the region (if non-empty) between $R_l(s)$ and $R_u(s)$, by MLRP we have

$$1 + \mu \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) - \lambda < 1 + \mu \left( 1 - \frac{f_L(R(s)|s)}{f_H(R(s)|s)} \right) - \lambda < 0$$

so that we have either $\chi(R, s) > 0$ or $\nu(R, s) > 0$. This implies that $x(R, s) = \min \{(1 - b)RY_0, x_u(s)\}$ for all $R_l(s) \leq R \leq R_u(s)$.

As a result, the optimal contract is a three-part liability structure. First, there is a threshold $R_l(s)$ such that $\alpha(R, s) = 1$ and $x(R, s) = \gamma(s)RY_0$ for $R \leq R_l(s)$, and $\alpha(R, s) = 0$ for $R \geq R_l(s)$. Second, there is a threshold $R_u(s) \geq R_l(s)$ such that $x(R, s) = \min \{(1 - b)RY_0, x_u(s)\}$ for $R \leq R_u(s)$ and $x(R, s) = x_u(s)$ for $R \geq R_u(s)$. Note finally that there cannot be a discontinuity in liabilities at $R_u(s)$. If there were a discontinuity, we would have

$$x_u(s) > \lim_{R \uparrow R_u(s)} x(R, s) = (1 - b)R_u(s)Y_0$$

and liabilities would exceed pledgeable income at $R_u(s)$. The capital structure is therefore continuous at $R_u(s)$.

Finally, the above capital structure can be implemented by a liabilities contract $L_1(R, s) = (1 - b)R_l(s)Y_0$ for $R \leq R_l(s)$ and $L_1(R, s) = x(R, s)$ for $R > R_l(s)$. This liability structure is monotone, and so we have implementable contracts.

In sum, the optimal contract lies within a class of contracts characterized by thresholds $R_l(s)$ and $R_u(s)$ and corresponding liability structure above. This proves the first part of the proposition.

Now, we characterize the optimal thresholds $R_l(s)$ and $R_u(s)$. Considering the case where these thresholds are interior, $R < R_l(s) \leq R_u(s) \leq \bar{R}$ we have the optimization
We selected the contract with a flat face value below which reduces to which reduces to which completes the proof.

Similarly, the optimality condition for $R_u(s)$ is

$$0 = \int_{R_u(s)}^{R} \left[ -(1-b)Y_0 f_H(R|s) - \mu(1-b)Y_0 (f_H(R|s) - f_L(R|s)) + \lambda(1-b)Y_0 f_H(R|s) \right] dR,$$ 

which reduces to

$$0 = E \left[ \lambda - 1 - \mu \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \left| R \geq R_u(s) \right. \right].$$

This completes the proof.

A.1.1 A Remark on Contract Uniqueness

The optimal contract is not generally unique in the following sense. In the region $R \leq R_l(s)$, the bank only needs a liability face value that is sufficient to liquidate the bank, and so any contract with monotone face value $L_1(R,s) > (1-b)Y_0$ in this region is optimal. We selected the contract with a flat face value below $R_l(s)$ due to its correspondence to standard debt. The face value of liabilities above $R_l(s)$ is uniquely determined. Moreover,
in the presence of an $\epsilon \rightarrow 0$ premium for standard debt (e.g. as in Appendix C.6), the implementation using standard debt becomes uniquely optimal.

A.2 Proof of Corollary 2

Consider the proposed liability structure. The amount $(1 - b)R_l(s)Y_0$ of standard debt liquidates the bank when $R \leq R_l(s)$, generating the lower region. $(1 - b)(R_u(s) - R_l(s))$ is written down in the region $R_l(s) \leq R \leq R_u(s)$, so that the bank is always held to the agency rent over this region. The full debt level $(1 - b)R_u(s)Y_0$ is repaid above $R_u(s)$. Therefore, we replicate the contract in Proposition 1.

A.3 Proof of Corollary 3

Characterizing the value of $R_l(s)$ from equation (7), we then take its solution and plug it into the RHS of equation (8). The RHS of equation (8) corresponds to the value of increasing $R_u(s)$ on the margin. Supposing that it is $\leq 0$ at $R_l(s)$, then the value of increasing $R_u(s)$ above $R_l(s)$ on the margin is non-positive. Moreover, by MLRP the RHS of equation (8) is non-positive for all $R > R_l(s)$. As a result, we move to a corner solution where $R_u(s) = R_l(s)$. Note that $R_l(s)$ may fall below the value implied by equation (7), but over the region below this value, the bank prefers to issue standard debt, per the optimality condition of equation (7).

A.4 Proof of Proposition 4

Consider the program of the social planner

$$\max_{L_1, Y_0} E [c_2(R, s) | e = H]$$

subject to

$$E \left[ c_2(R, s) \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) | e = H \right] \geq BY_0$$

$$Y_0 - A = \sum_s \pi(s) \left( \int_{R|\alpha=1} \gamma(s, \Omega(s))RY_0f_H(R|s)dR + \int_{R|\alpha=0} L_1(R, s)f_H(R|s)dR \right)$$

$$R \geq R' \Rightarrow L_1(R, s) \geq L_1(R', s)$$

$$c_2 \geq (1 - \alpha(R, s))bRY_0$$

$$\Omega(s) = \int \alpha(R, s)Rf_H(R|s)dR$$
The proof follows as in the proof of Proposition 1. Redefine the payoff space over \( \chi(R, s) \) and solve for the optimal contract without imposing monotonicity. The first order condition for \( c_2(R, s) \) is the same as in the proof of Proposition 1, since \( c_2(R, s) \) does not directly affect \( \Omega(s) \). This implies as before that we obtain a pooling region at the top.

As before, take \( R_u(s) \) and \( x_u(s) \) as given, and solve for the optimal contract for \( R \leq R_u(s) \). The same steps imply that implementable contracts must satisfy \( R_u(s) < R^*(s) \). The FOC for optimal liquidations \( \alpha(R, s) \) is now

\[
\frac{\partial L}{\partial \alpha(R, s)} = \alpha \lambda (\gamma(s, \Omega(s)) - 1) RY_0 f_H(R|s) + \chi(R, s) b RY_0 f_H(R|s) + \nu(R, s) (1 - \gamma(s)) RY_0 f_H(R|s)
\]

\[
+ \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \frac{\partial \Omega(s)}{\partial \alpha(R, s)} \lambda \int_{R'} \alpha(R', s) R' f_H(R'|s) dR'
\]

Substituting in the derivative \( \frac{\partial \Omega(s)}{\partial \alpha(R, s)} = R f_H(R|s) \), we obtain

\[
\frac{\partial L}{\partial \alpha(R, s)} = \alpha \lambda (\gamma(s) - 1) + \chi(R, s) b + \nu(R, s) (1 - \gamma(s))
\]

\[
+ \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \lambda \int_{R'} \alpha(R', s) R' f_H(R'|s) dR'
\]

The additional wedge \( \frac{\partial \gamma(s, \Omega(s))}{\partial \alpha(s)} \lambda \int_{R'} \alpha(R', s) R' f_H(R'|s) dR' \) is negative and independent of \( R \). The same steps apply as in the proof of Proposition 1, yielding a liquidation threshold rule \( R_l(s) \). Because as before \( R_u(s) < R^*(s) \), we have \( \chi(R, s) = \min\{(1 - b) RY_0, x_u(s)\} \) in the region \( R_l(s) \leq R \leq R_u(s) \). Thus, the set of candidate optimal contracts is the same as in the private equilibrium, and the implementation of Corollary 2 holds.

Lastly, we characterize the optimal choices of \( R_l(s) \) and \( R_u(s) \) for interior solutions. The optimality condition for \( R_u(s) \) is identical to the private optimality condition, since it does not affect the liquidation value. By contrast, the social optimality condition for \( R_l(s) \) satisfies

\[
b + \lambda ((1 - b) - \gamma(s)) = \mu b \left( \frac{f_L(R_l(s)|s)}{f_H(R_l(s)|s)} - 1 \right) + \frac{\lambda \frac{\partial \Omega(s)}{\partial R_l(s)}}{R_l(s) Y_0 f_H(R_l(s)|s)} \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l(s)}^{R_u(s)} RY_0 f_H(R|s) dR.
\]

Substituting in \( \frac{\partial \Omega(s)}{\partial R_l(s)} = R_l(s) f_H(R_l(s)|s) \) and rearranging, we obtain

\[
\mu b \left( \frac{f_L(R_l(s)|s)}{f_H(R_l(s)|s)} - 1 \right) = b + \lambda ((1 - b) - \gamma(s)) - \lambda \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l(s)}^{R_u(s)} R f_H(R|s) dR.
\]
This completes the proof.

### A.5 Proof of Proposition 5

Consider the optimal contract of the social planner. Holding fixed the debt levels $R_l(s)$ and $R_u(s)$, the derivative of the planner’s Lagrangian in $\theta_n$ is given by

$$0 = E \left[ c_2(R,s) \frac{\partial f_H(R|s,\theta)}{f_H(R|s,\theta)} \right] + \mu E \left[ c_2(R,s) \left( \frac{\partial f_H(R|s,\theta)}{f_H(R|s,\theta)} - \frac{\partial f_L(R|s,\theta)}{f_H(R|s,\theta)} \right) \right]$$

$$+ \lambda E \left[ x(R,s) \frac{\partial f_H(R|s,\theta)}{f_H(R|s,\theta)} \right]$$

$$+ \lambda E \left[ \int_{R_l(s)}^{R_u(s)} \frac{\partial \gamma(s,\Omega(s))}{\partial \Omega(s)} \frac{\partial \Omega(s)}{\partial \theta_n} R f_H(R|s,\theta) dR \right]$$

where the first two lines reflect the private bank trade-off, and the last line reflects the social trade-off. Liquidations are given by

$$\Omega(s) = \int_{R_l}^{R_u(s)} R f_H(R|s,\theta) dR$$

so that we have

$$\frac{\partial \Omega(s)}{\partial \theta_n} = \int_{R_l}^{R_u(s)} R \frac{f_H(R|s,\theta)}{\partial \theta_n} dR.$$

Substituting in above, we obtain

$$0 = E \left[ \lambda x(R,s) + c_2(R,s) \left( 1 + \mu \left( 1 - \frac{\partial f_L(R|s,\theta)}{\partial f_H(R|s,\theta) / \partial \theta_n} \right) \right) \frac{\partial f_H(R|s,\theta)}{f_H(R|s,\theta)} \right]$$

$$+ \lambda E \left[ \frac{\partial \gamma(s)}{\partial \Omega(s)} \Omega(s) Y_0 \cdot \left[ \int_{R_l}^{R_u(s)} R \frac{\partial f_H(R|s,\theta)}{\partial \theta_n} dR \right] \right]$$

### A.6 Proof of Proposition 6

The proof proceeds by contradiction. Suppose that the equilibrium optimal contract sets $R_u(s) > R_l(s) \geq R^{BO}(s)$ in a state $s$. Because $R_l(s) \geq R^{BO}(s)$, banks are bailed out when insolvent in any state $R \geq R^{BO}(s)$. Consider any single bank writing a contract, and suppose that bank offers an alternate contract that sets $R'_l(s) = R_u(s)$ in state $s$, but otherwise leaves the liability contract otherwise unchanged. Since the bank is now bailed
out in states $R_l(s) \leq R \leq R_u(s)$ rather than bailed in, investor repayment increases, the participation constraint is relaxed, and investors increase date 0 payment to the bank. Suppose the bank immediately consumes that additional payment, so that project scale is unaffected. The consumption profile of the bank after date 0 under the revised contract is identical to the original, so that the contract remains incentive compatible. Moreover, bank welfare is strictly higher under the revised contract than the original, because the bank consumes the additional date 0 payment from investors. But then this contract yields higher utility than the equilibrium contract, contradicting that it the equilibrium contract was optimal. As a result, if in equilibrium $R_l(s) \geq R^{BO}(s)$, then $R_u(s) = R_l(s)$.\footnote{Note that it is possible that the bank may, in the presence of bailouts, no longer find it optimal to offer a contract that enforces high effort, due to the bailout guarantee. The above argument is unaffected. We can generally rule out the possibility that bailouts induce low effort by assuming that the probability of bailout states is not too high.}

### A.7 Proof of Proposition 7

We adopt the following proof strategy. We will consider a contract that results in bailouts, and show that it is equivalent to a contract that: (1) features bail-ins (rather than bailouts) ex post; and, (2) implements an ex ante lump sum transfer from taxpayers to the bank. Because ex ante transfers from taxpayers to banks are assumed undesirable, it follows immediately that the same (bail in) contract without the ex ante lump sum transfer dominates the bailout contract. Finally, we will derive the required condition on $\tau$ so that ex ante transfers from taxpayers to banks are indeed not optimal.

Define contracts over a space $\{R_l, R_u, T\}$, where $T$ is an ex ante lump-sum transfer from the government (via taxpayers) to banks, above and beyond any bailouts. We use these transfers as an accounting tool to compare contracts.

Consider an implementable contract $\Gamma = \{R_l, R_u, 0\}$ with $R_l(s) > R^{BO}(s)$ for some $s \in S$. Let $T(R_l, R_u)$ be the (ex ante) value of the bailout transfer under this contract, and define an alternative contract $\Gamma'$ by $R'_l(s) = \min\{R_l(s), R^{BO}(s)\}$, $R'_u(s) = R_u(s)$, and $T' = T(R_l, R_u)$. The contract $\Gamma'$ is implementable: bank consumption is identical, and investor repayment is satisfied by ex ante transfers rather than ex post (bailout) transfers. As a result, welfare under $\Gamma'$ is at least as high as under the contract $\Gamma$.\footnote{We will not have to consider political costs of bailouts for the proof, but we state “at least as high” for formality.}

Now, consider an alternate contract $\Gamma'' = \{R'_l, R'_u, 0\}$. This contract is implementable because it generates the same consumption-to-asset ratio $c_2(R, s)/Y_0$ as contract $\Gamma'$, with only the project scale $Y_0$ being different. In order to compare the welfare of these contracts, note that for any incentive compatible bank contract with thresholds $(R_l, R_u)$ and transfers
$T$, equilibrium bank welfare is given by

$$V(R_l, R_u) Y_0(R_l, R_u, T|R^*_l, R^*_u)$$

where we have defined

$$V(R_l, R_u) = \sum \pi(s) \left[ \int_{R_l}^{R_u} b R f_H(R|s) dR + \int_{R_u}^{\bar{R}} (R - (1 - b) R_u) f_H(R|s) dR \right].$$

Total investor repayment under the contract $\Pi$ per unit of scale is given by

$$\Pi = \sum \pi(s) \left[ \int_{R_l}^{R_l(s)} \gamma(s) R f_H(R|s) dR + \int_{R_u(s)}^{R_u} (1 - b) R f_H(R|s) dR + \int_{R_u(s)}^{\bar{R}} (1 - b) R_u(s) f_H(R|s) dR \right]$$

so that the project scale is given by

$$Y_0(R_l, R_u|R^*_l, R^*_u) = \frac{A + T}{1 - \Pi(R_l, R_u|R^*_l, R^*_u)}.$$

Substituting in, we obtain equilibrium bank welfare under the contract as

$$\frac{V(R_l, R_u)}{1 - \Pi(R_l, R_u|R^*_l, R^*_u)} (A + T).$$

Social welfare at date 0, given a lump sum tax cost $\tau > 1$, is given by

$$\frac{V(R_l, R_u)}{1 - \Pi(R_l, R_u|R^*_l, R^*_u)} (A + T) - \tau T$$

Now, we can compare contracts $\Gamma'$ and $\Gamma''$. Because these two contracts differ only in the date 0 lump sum transfer but feature the same thresholds, a sufficient condition for the welfare gain from switching to contract $\Gamma''$ from contract $\Gamma'$ is

$$\tau T' - \frac{V(R'_l, R'_u)}{1 - \Pi(R'_l, R'_u|R'_l, R'_u)} T' \geq 0$$

or in other words

$$\tau \geq \max_{R_l, R_u} \frac{V(R_l, R_u)}{1 - \Pi(R_l, R_u|R_l, R_u)}.$$

Under this condition, contract $\Gamma''$ is preferable to the contract $\Gamma'$. Because contract $\Gamma'$ is

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82We can ignore the political costs of bailouts.
yields at least as high welfare as contract $\Gamma$, then contract $\Gamma''$ is preferable to contract $\Gamma$. In other words, there are no bailouts.

Note that because $R_l(s) \leq R^{BO}(s)$ acts as a constraint on the socially optimal contract, it immediately follows that bank welfare is non-decreasing in $R^{BO}(s)$. In particular, if any contract $\Gamma = \{R_l, R_u, 0\}$ that is implementable given $R^{BO}$ is also implementable for any $R^{BO'}$ satisfying $R^{BO'}(s) \geq R^{BO}(s)$, implying that social welfare cannot be lower. Lastly, recall that social welfare and bank welfare are the same in the absence of bailouts.

**A.8 Proof of Lemma 8**

If the best equilibrium is to have no liquidations, it must set $D_2 \leq RY_1$. Short-term debt is then always repaid in full, and $\eta_1^D = 1$, since it is senior. Because $RY_1 < D_1$, the bank cannot refinance itself with short-term debt $D_2 \leq RY_1$ alone, and therefore must raise some long-term debt. Long-term debt will not be fully repaid, and has price $\eta_1^L < 1$. Given the lower long-term debt price, the “no rat race” condition of equation 13 binds and we have $D_2 = RY_1$. The total funds raised from long-term debt must satisfy the repayment condition

$$D_1 - RY_1 = \int_{\mathbb{R}} \min\{L_3, R_2 Y_1 - RY_1\} f_2(R_2) dR_2.$$

Noting that the RHS is increasing in $L_3$, there is a unique solution $\overline{L}_3$ to this equation. From here, $\eta_1^L$ is given by the definition $\eta_1^L = \frac{D_1 - RY_1}{L_3}$. The prices and quantities satisfy the no rat race conditions by construction. As a result, the best equilibrium has unique prices and quantities, and does not feature liquidations.

**A.9 Proof of Proposition 9**

Suppose that markets quote $q_1^L = 0$. The bank can only raise funds by issuing short-term debt, with maximum raisable funds

$$\sup_{D_2 \leq Rb Y_1} \int_{\mathbb{R}} \frac{D_2}{Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{\overline{D}_2 / Y_1}^{\mathbb{R}} D_2 f_2(R_2) dR_2$$

Define $d_2 = D_2 / Y_1$, then the bank cannot refinance itself if

$$\frac{D_1}{Y_1} > \sup_{d_2 \leq Rb} \int_{\mathbb{R}} d_2 \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{\mathbb{R}} d_2 f_2(R_2) dR_2.$$
The right-hand side is a constant that does not depend on $D_1, Y_1, \text{or } R$. As a result, we obtain a threshold rule in $Y_1$ (given $D_1$), which implies a threshold rule in $R^*$, so that rollover crises may exist when $R \leq R^*$. Lastly, note that equation (15) guarantees that the existing stock $D_1$ is sufficient to liquidate the bank whenever there is a rollover crisis. As a result, a hypothetical unit of new long-term debt receives no recovery value in liquidation, completing the equilibrium.

Finally, at $R = R_l$ we have

$$
\frac{D_1}{R_l Y_0} = (1 - b) > \sup_{d_2 \leq R_b} \int_{d_2}^R \gamma R_2 f_2(R_2)dR_2 + \int_{d_2}^R d_2 f_2(R_2)dR_2
$$

so that a rollover crisis equilibrium always exists at $R = R_l$. Hence, there is always an interval of existence $R \in [R_l, R^*)$.

A.10 Proof of Proposition 10

Consider the fixed point problem

$$
\gamma^* = \gamma \left( \epsilon + \int_{R_l}^R R f_H(R)dR + p \int_{R_l}^{R^*(\gamma^*)} R f_H(R)dR \right)
$$

Totally differentiating in $\epsilon$ and evaluating at $\epsilon = 0$, we obtain

$$
\frac{\partial \gamma^*}{\partial \epsilon} = \frac{\partial \gamma (\Omega^*)}{\partial \Omega^*} \left( 1 + p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \epsilon} R^* f_H(R^*) \right)
$$

which rearranges to

$$
\frac{\partial \gamma^*}{\partial \epsilon} = \frac{\partial \gamma (\Omega^*)}{\partial \Omega^*} \frac{\partial \gamma (\Omega^*)}{\partial \Omega^*} p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} R^* f_H(R^*)
$$

Next, evaluating the derivative in $\frac{\partial \Omega^*}{\partial \epsilon}$ and substituting in, we obtain

$$
\frac{\partial \Omega^*}{\partial \epsilon} = 1 + \frac{p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \epsilon} R^* f_H(R^*)}{1 - \frac{p \frac{\partial \gamma (\Omega^*)}{\partial \Omega^*} \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} R^* f_H(R^*)}{\partial \gamma^*} R^* f_H(R^*)}.
$$
Finally, differentiating equation (16) in $\gamma^*$

$$\frac{\partial R^*}{\partial \gamma^*} = -\frac{(R^*)^2 Y_0}{D_1} \int_R^d R_2 f_2(R_2) dR_2$$

and substituting in, we obtain the final result

$$\frac{\partial \Omega^*}{\partial \epsilon} = 1 + p \left| \frac{\partial \gamma(\Omega^*)}{\partial \epsilon} \right| \left| \frac{\partial R^*(\gamma^*)}{\partial \Omega^*} \right| R^* f_H(R^*)$$

Noting that $\frac{\partial R^*}{\partial \gamma^*}$ and $\frac{\partial \gamma}{\partial \Omega^*}$ are negative, we have adopted the absolute value notation for clarity.

**A.11 Proof of Proposition 11**

Suppose that markets quote $q^L_1 = 0$. Let $C^0$ be the space of tuples $C = (q_1, D_2, L_3)$ with $q^L_1 = 0$. A rollover crisis equilibrium exists under the vacuous covenant if there does not exist any tuple $C \in C^0$ such that $q^D_1 D_2 \geq D_1$ and that is consistent with the no rat race conditions (13) and (14).

Now, consider an alternate covenant $C_1$. When markets quote $q^L_1 = 0$, the set of tuples that satisfy covenant $C_1$ is $C^1 \subset C^0$. But since no tuple $C \in C^0$ refinances the bank, no tuple $C \in C^1$ refinances the bank either. As a result, existence of rollover crises under the vacuous covenant implies existence of rollover crises under any other covenant.

**A.12 Proof of Proposition 12**

The modified private sector no rat race conditions, given pari passu claims, are

$$q^D_1 D_2 \leq \int_{D_2 + D^L_{LOLR} \geq R_2 Y_1} \gamma R_2 Y_1 \frac{D_2}{D_2 + D^L_{LOLR}} f_2(R_2) dR_2 + \int_{D_2 + D^L_{LOLR} \leq R_2 Y_1} D_2 f_2(R_2) dR_2$$ (22)

$$q^L_1 L_3 \leq \int_{D_2 + D^L_{LOLR} \leq R_2 Y_1} \min \left\{ \left( R_2 Y_1 - D_2 - D^L_{LOLR} \right) \frac{L_3}{L_3 + L^L_{LOLR}} L_3 \right\} f_2(R_2) dR_2$$ (23)

Conjecture a rollover crisis equilibrium with $q^L_1 = 0$. Suppose the bank borrows best equilibrium quantities from the LOLR, and does not borrow from the private sector. The
break-even LOLR debt prices are given by

\[
q_{D,LOLR} = \frac{1}{D_2} \int_{D_2 \geq R_2 Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2 \leq R_2 Y_1} D_2 f_2(R_2) dR_2 = \overline{q}_1^D
\]

\[
q_{L,LOLR} = \frac{1}{L_3} \int_{D_2 \leq R_2 Y_1} \min \{ (R_2 Y_1 - D_2), L_3 \} f_2(R_2) dR_2 = \overline{q}_1^L
\]

which are the best equilibrium prices, given that \( D_2 = R_2 Y_1 \). Therefore, the bank raises total funds \( \overline{q}_1^D D_2 + \overline{q}_1^L L_3 = D_1 \) from the LOLR, and successfully refines itself. The rollover crisis equilibrium is eliminated.

Finally, because the LOLR and private sector rank pari passu, in equilibrium \( q_{D,LOLR}^1 = q_1^D \) and \( q_{L,LOLR}^1 = q_1^L \). Equilibrium market prices are therefore best equilibrium prices, and the bank can refinance itself entirely from the private sector using best equilibrium issuances. The best equilibrium is restored.

A.13 Proof of Proposition 13

Suppose that a rollover crisis equilibrium occurs, and consider the value of a hypothetical unit of new private sector long-term debt. The government guarantee implies the unit is repaid its full face value of 1, so that \( q_{L}^1 = 1 \). But then the bank can refinance itself with long-term debt, and a rollover crisis equilibrium does not exist.\(^{83}\)

Consider now instead the best equilibrium prices \( \overline{q}_1^L \). The bank can always refinance itself with the best equilibrium quantities \( (D_2, L_3) \), and therefore survives until period 2. Guarantees expire at the end of period 1, and so are never filled, regardless of the refinancing package chosen by the bank. As a result, the bank indeed chooses the best equilibrium quantities \( (D_2, L_3) \), and the best equilibrium is restored.

B Microfoundations

B.1 Fire Sales and Arbitrageurs

We provide a foundation for the fire sale function \( \gamma(s, \Omega(s)) \) from limits to arbitrage.

\(^{83}\)If \( q_{L}^1 = 1 \), then the stronger no rat race conditions we have applied in the main paper will be violated. Instead, we need to use the weaker form no rat race conditions in Appendix B.3. Under these conditions, we have the quoted price \( \overline{q}_1^L = 1 \). The bank chooses issuance \( (D_2, L_3) = (0, \overline{D}_2 + \overline{L}_3) \), which lowers the price to the level in equation (26) and gives us an equilibrium.
There is a large mass $M_0$ of arbitrageurs, of whom $2Y_0$ develop the skills necessary to purchase and manage bank projects. Due to the scaling $2Y_0$, the arbitrageur assumptions below are expressed per unit measure of arbitrageurs, and we normalize to total mass of 1 from here on.

Arbitrageurs have no funds at date 0, but have 1 unit of an existing project, which we call the arbitrageur project. Arbitrageurs cannot commit to make payments to investors and therefore cannot raise any external financing at any date.

At date 1, the arbitrageur project yields 1 unit of the consumption good at date 1. The project also has a continuation value $y \in \{0, \bar{y}\}$. Half of arbitrageurs are skilled ex post, and receive the positive continuation value $\bar{y}$, while the other half are unskilled ex post and receive no continuation value. Arbitrageurs experience a perfectly correlated liquidity shock $\rho(s) \in \{0, 1\}$, which they are required to pay to continue the project. We assume that $\bar{y}$ is high enough that when the liquidity shock is 1, skilled arbitrageurs use their 1 unit of the consumption good to continue their project. By contrast, unskilled arbitrageurs never continue.

Arbitrageurs can also purchase and manage bank projects. Skilled arbitrageurs have a constant value $\gamma(s)$ from managing bank projects. Unskilled arbitrageurs have a second ex post type $i \in [0, 1]$, distributed by $f_i(i)$, which indexes their ability to manage bank projects. They receive $\gamma_i(s) < 1 - b$ units of the consumption good at date 2 from managing a unit of the bank project purchased at date 1, which WLOG is increasing in $i$. We assume that $0 < \gamma \leq \bar{y}(s)$, so that $\gamma$ is bounded away from 0.

When arbitrageurs experience the correlated shock $\rho(s) = 0$, skilled arbitrageurs have funds available for arbitrage. As long as the arbitrage sector has sufficient mass, the marginal pricing agent is a skilled arbitrageur, and the liquidation value is the constant $\gamma(s) = \bar{y}(s)$. We can ensure that skilled arbitrageurs are the marginal pricing agent by assuming that $\frac{1}{\bar{y}(s)} \geq \int_R R f_H(R|s)dR$. We think of this as times of market depth, where there may be a liquidation discount but there is no fire sale.

When arbitrageurs experience the correlated shock $\rho(s) = 1$, skilled arbitrageurs pay to continue their project and have no funds for arbitrage, so that unskilled arbitrageurs purchase liquidated projects. Given an equilibrium price $\gamma(s)$, types $\gamma_i(s) \geq \gamma(s)$ purchase the project, yielding total demand $\int_{\gamma_i \geq \gamma(s)} \frac{1}{\gamma(s)} f_i(i) di$. The equilibrium price equates supply

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84 We think of the scale effect as arising because a larger scale banking sector implies more agents have the capacity to become arbitrageurs. We can think of the remaining $M_0 - 2Y_0$ arbitrageurs as receiving value of 0 from managing bank projects. In Appendix C.9, we study the case where the size of the arbitrage sector does not scale with $Y_0$. 
and demand, that is
\[
\int_{\gamma_i(s) \geq \gamma(s)} \frac{1}{\gamma(s)} f_i(i) di = \int_R a(R, s) R f_H(R|s)dR = \Omega(s)
\]
The left-hand side is a non-increasing function of the price \(\gamma(s)\). Inverting this equation, we obtain a function \(\gamma(s, \Omega(s))\).

**B.2 Multiple Assets Density Function**

Suppose that there are \(N + 1\) assets between which the bank allocates its funds. Denote \(\omega \in [\omega, \bar{\omega}]\) to be the underlying idiosyncratic state of the bank, with associated density \(f_\omega(\omega|s)\), where \(e \in \{H, L\}\). Suppose that \(f_\omega(\omega)\) satisfies MLRP, so that \(\frac{\partial}{\partial \omega} \left(\frac{f_H(\omega)}{f_L(\omega)}\right) > 0\).

Asset \(n \in \{1, ..., N + 1\}\) generates a return \(R_n(\omega, s)\) per unit in state \((\omega, s)\). Let \(\theta = (\theta_1, ..., \theta_{N+1})\) be a vector that determines the asset allocations \(\theta_1 Y_0, ..., \theta_{N+1} Y_0\). Allocations \(\theta\) satisfy a technological restriction \(F(\theta) = 0\), for example there may be a concave technology. Note that to coincide with the previous parts, we assume the technology is linear in the scale \(Y_0\), and only (potentially) concave in the asset weights. If \(F(\theta) = \sum_{n=1}^{N+1} \theta_n - 1\), we have a simple linear technology with equal cost of investment across assets.

We invert \(\theta_{N+1}\) from \((\theta_1, ..., \theta_N)\) via \(F\), so that we can internalize the constraint. We denote the total return to the bank, given an asset allocation vector \(\theta\), by
\[
R(\omega|s, \theta) = \sum_{n=1}^{N+1} \theta_n R_n(\omega, s) Y_0
\]
where \(\theta_{N+1}\) is derived from the technology \(F(\theta) = 0\), given \(\theta_1, ..., \theta_N\).

Suppose that conditional on \((s, \theta)\), there is an injective mapping between \(\omega\) and \(R\). In this case, \(R\) identifies \(\omega\), given \((s, \theta)\), and we can write contracts on \((R, s)\). We assume that the mapping is injective over the relevant range of asset allocations \(\theta\). For example, this will be the case if asset allocations are non-negative \((\theta_n \geq 0)\) and individual asset returns are monotone in \(\omega\). Without loss of generality, we assume the injective mapping is monotone increasing: high states \(\omega\) identify high returns \(R\), consistent with the interpretation of \(e = H\) as “high effort.”

Denote \(R^{-1}(R|s, \theta)\) to be the inverse function mapping the total return \(R\) into the idiosyncratic state \(\omega\). The inverse function does not depend directly on \(e\), but rather the

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\(^{85}\)Since \(\gamma_i(s) \leq \gamma(s)\), we can again guarantee an interior solution by assuming \(\frac{1}{\gamma_i(s)} \geq \int_R R f_H(R|s)dR\).
density will depend on $e$. We now derive the density of $R$, conditional on $(s, \theta)$. We have

$$F_e(R|s, \theta) = \Pr(R(\omega|s, \theta) \leq R|e) = \Pr(\omega \leq R^{-1}(R|s, \theta)|e) = F_e^\omega(R^{-1}(R|s, \theta)).$$

Differentiating in $R$, we obtain the density function:

$$f_e(R|s, \theta) = f_e^\omega(R^{-1}(R|s, \theta)) \frac{\partial R^{-1}(R|s, \theta)}{\partial R}.$$

We impose the simplifying assumption that the support $[R, \bar{R}]$ of the density is invariant to the allocation $\theta$. If the support depended on the portfolio allocation, we would have boundary terms in derivatives. The principal term of relevance would be how the lower boundary of the support moves in the asset allocation, which reflects changes in the measure of the liquidation region. These effects are qualitatively the same as the direct effects of changing the measure from changes in the density. For simplicity, we keep the support fixed.

Finally, we can show that this function satisfies monotone likelihood. Differentiating the likelihood ratio in $R$, we obtain

$$\frac{d}{dR} \left( \frac{f_H(R|s, \theta)}{f_L(R|s, \theta)} \right) = \frac{d}{dR} \left( \frac{f_e^\omega(R^{-1}(R|s, \theta))}{f_e^\omega(R^{-1}(R|s, \theta))} \right)$$

$$= \frac{\partial f_e^\omega}{\partial \omega} \frac{\partial R^{-1}(R|s, \theta)}{\partial R} f^\omega_L - \frac{\partial f_e^\omega}{\partial \omega} \frac{\partial R^{-1}(R|s, \theta)}{\partial R} f^\omega_H$$

$$= \frac{\partial}{\partial \omega} \left( \frac{f^\omega_H}{f^\omega_L} \right) \frac{\partial R^{-1}(R|s, \theta)}{\partial R}$$

$$> 0$$

where in the last line, we have used MLRP on $f_e^\omega$ combined with monotonicity of $R^{-1}$.

As a result, we obtain a representation of the problem as a density $f_e(R|s, \theta)$. Implicitly, we differentiate in $(\theta_1, ..., \theta_N)$, where we have internalized $\theta_{N+1}$ as arising from the technology.

### B.3 No Rat Race Conditions

In the main paper, we have used equations (13) and (14) to rule out rat race dynamics. These conditions are stronger than is necessary as we discuss in this section. Suppose instead that the market quotes debt prices $\bar{q}_1^L$ and $\bar{q}_1^D$. We define the equilibrium prices
associated with an issuance \((D_2, L_3)\) by

\[
q_1^D = \min\{q_1^D, \hat{q}_1^D\}
\]

\[
q_1^L = \min\{q_1^L, \hat{q}_1^L\}
\]

where \(\hat{q}_1^D\) and \(\hat{q}_1^L\) are the payoff-neutral prices, given by

\[
\hat{q}_1^D = \frac{1}{D_2} \left[ \int_{D_2 \geq R_2 Y_1} \gamma R_2 Y_1 f_2(R_2)dR_2 + \int_{D_2 \leq R_2 Y_1} D_2 f_2(R_2)dR_2 \right]
\]

\[
\hat{q}_1^L = \frac{1}{L_3} \left[ \int_{D_2 \leq R_2 Y_1} \min\{R_2 Y_1 - D_2, L_3\} f_2(R_2)dR_2 \right]
\]

Under these conditions, equilibrium prices are the maximum between the payoff-neutral prices, and the prices quoted by the market. In other words, the bank internalizes that certain issuances may lower market prices for its debt, but can never increase them.

This distinction is not relevant for the majority of the analysis, and so we present the simpler conditions of equations (13) and (14). The simpler conditions have the benefit that the best equilibrium is unique, whereas it is not unique under the extended conditions. To illustrate why it is not unique, suppose that \((\bar{D}_2, \bar{L}_3)\) are the best equilibrium quantities. Then, there is also a best equilibrium associated with quantities \((0, \bar{D}_2 + \bar{L}_3)\). The prices are \(q_1^D = 1\) and

\[
q_1^L = \frac{1}{\bar{D}_2 + \bar{L}_3} \left[ \int_{R} \min\{R_2 Y_1, \bar{D}_2 + \bar{L}_3\} f_2(R_2)dR_2 \right]
\]

However, this is not an equilibrium under the no rat race conditions (13) and (14), since the bank always prefers to issue \(D_2 = R Y_1\) to capitalize on the higher price of short-term debt.

Rollover crisis equilibria are not affected by this definition, because given a market price \(\bar{q}_1^L = 0\), we have \(q_1^L = 0\) regardless of the payoff-neutral price \(\hat{q}_1^L\).

## C Extensions

### C.1 Role of Agency Problems and Liquidation Costs

Our model features three ingredients that are jointly necessary to generate contracts that consist of combinations of contingent and standard debt. First, there is an ex ante incentive problem, that is \(B > 0\), which implies a conventional incentive-based deviation from Modigliani-Miller. Second, there is a limited pledgeability problem (e.g. a continuation
incentive problem), that is \( b > 0 \). Finally, there are costly liquidations, that is \( \gamma(s) < 1 \). In the absence of any one of these elements, contracts in our model would not combine contingent and standard debt.

**Corollary 14.** Optimal contracts do not consist of both contingent and standard debt if \( B = 0 \), \( b = 0 \), or \( \gamma(s) = 1 \). In particular, considering each deviation:

1. If \( B = 0 \), then optimal contracts feature only equity (without loss of generality).
2. If \( b = 0 \), then optimal contracts feature only bail-in debt.
3. If \( \gamma(s) = 1 \), then optimal contracts feature only feature standard debt.

When \( B = 0 \), a standard Modigliani-Miller logic applies. The bank can ensure incentive compatibility with any monotone consumption policy \( c_2(R, s) \), and in particular has no need of or desire for liquidations. As a result, without loss of generality the bank uses entirely equity financing. However, the second and third cases show that \( B > 0 \) is not sufficient to generate contracts that combine contingent and standard debt. When \( B > 0 \), optimal contracts employ some debt instrument for ex ante incentive reasons. If \( b = 0 \), then all income is pledgeable to investors, and the bank can set \( c_2(R, s) = 0 \) without liquidating. Banks use only bail-in debt. If \( \gamma(s) = 1 \) but \( b > 0 \), there is a limit to pledgeable income, but no bankruptcy costs from liquidation. Banks can repay any amount \( x_1(R, s) \leq RY_0 \) by liquidating bank projects, and the pledgeability constraint ceases to be relevant. Banks use only standard debt.

In both the second and third cases, the key property of debt is the full transfer of value of the bank from the bank to investors in low-return states. This corresponds to a common understanding of debt in the optimal contracting literature: the core property of debt is its payoff profile \( x_1(R, s) = \min\{RY_0, R_u(s)Y_0\} \),\(^{86}\) which corresponds to a full value transfer in low-return states. In the absence of pledgeability limitations, this value transfer is achieved with bail-in debt. In the absence of bankruptcy costs, this value transfer is achieved with standard debt.

If there are incentive problems (\( B > 0 \)), limited pledgeable income (\( b > 0 \)), and bankruptcy costs (\( \gamma(s) < 1 - b \)), then bail-in debt cannot enact full value transfer, whereas standard debt can enact full value transfer, but comes at a resource cost. A role emerges for both forms of debt in the optimal contract.

This suggests why banks are a natural candidate for this hybrid capital structure. Bank liquidations tend to be costly from an investor recovery perspective. Banks are also

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\(^{86}\)For example, see Hébert (2018).
likely to face incentive problems in their lending that limit ability to pledge full returns to investors.

C.1.1 Proof of Corollary 14

We split the proof into the different cases.

Case 1: Suppose first that \( B = 0 \), but \( b > 0 \) and \( \gamma(s) < 1 - b \). We impose \((1 - b)E[R] < 1\) to obtain a finite solution.

The result is a Modigliani-Miller type result. Incentive compatibility is now

\[
\mathbb{E} \left[ c_2(R, s) \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \mid e = H \right] \geq 0.
\]

Let \( c_2(R, s) \) be some monotone consumption rule. We have

\[
\mathbb{E} \left[ c_2(R, s) \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \mid e = H \right] = \text{cov} \left( c_2(R, s), 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \geq 0
\]

where the inequality follows from MLRP. As a result, any monotone consumption rule is implementable. We can span the frontier of expected repayment splits between banks and investors, with \( \Pi \in [0, (1 - b)E[R]] \) to investors and \( (1 - b)E[R] - \Pi \) to bankers, with monotone consumption rules (e.g. equity). Because all agents are risk-neutral, all that matters is the expected revenue division, and there is no need to liquidate the bank. Equity allocation rules \( E \in [0, 1 - b] \), with investors receiving shares \( E \) and banks retaining equity \( 1 - E \), generate monotone consumption profiles and so are incentive compatible. They also span the range of possible surplus divisions. As a result, pure equity constitutes an optimal contract.

Case 2: Consider next \( b = 0 \). The RHS of (7) then collapses to \( \lambda(1 - \gamma(s)) \) while the LHS collapses to 0, and so banks never choose to liquidate. Optimal contracts use only bail-in debt.

Case 3: Consider finally \( \gamma(s) = 1 \forall s \). Any face value \( L_1(R, s) \leq RY_0 \) can then be repaid by liquidating assets, so that bank consumption is \( c_2(R, s) = RY_0 - L_1(R, s) \) for any \( L_1(R, s) \leq RY_0 \). Therefore for any liability structure \( L_1(R, s) \), we can define

\[
(c_2(R, s), x(R, s)) = \begin{cases} 
(RY_0 - L_1(R, s), L_1(R, s)) , & L_1(R, s) \leq RY_0 \\
(0, RY_0), & L_1(R, s) \geq RY_0 
\end{cases}
\]
where the relevant liquidation function $\alpha(R, s) \in [0, 1]$ is defined from the liability structure. For example, without loss of generality we could define $\alpha(R, s) = \frac{x(R, s)}{R_Y 0}$. As a result, minimum pledgeability never binds.

Defining the problem in the repayment space, we then have

$$\max \sum_s \pi(s) \int_R [R_Y 0 - x(R, s)] f_H(R|s) dR,$$

subject to

$$\sum_s \pi(s) \int_R [R_Y 0 - x(R, s)] (f_H(R|s) - f_L(R|s)) dR \geq BY_0$$

$$Y_0 - A = \sum_s \pi(s) \int_R x(R, s) f_H(R|s) dR$$

$$R \geq R' \Rightarrow x(R, s) \geq x(R', s)$$

with $0 \leq x(R, s) \leq R_Y 0$. Relaxing monotonicity, the FOC for $x(R, s)$ is given by

$$\frac{\partial L}{\partial x(R, s)} = \left[ -1 - \mu \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) + \lambda \right] \pi(s) f_H(R|s)$$

yielding a threshold rule $R^*(s)$ such that $x(R, s) = R_Y 0$ for $R \leq R^*(s)$ and $x(R, s) = 0$ for $R \geq R^*(s)$. This results in an upper pooling region $R_u(s)$ with liabilities $x_u(s)$. Because $R_u(s) < R^*(s)$ as in the proof of Proposition 1, we have $x(R, s) = R_Y 0$ for all $R \leq R_u(s)$. Continuity implies $L_1(R, s) = R_u(s) Y_0$ for all $R$, and so the contract is standard debt.

### C.2 Decentralizing the Socially Optimal Contract

Proposition 4 describes the structure and optimality conditions of the socially optimal contract. We decentralize the optimal contract with wedges $\tau_l(s)$ and $\tau_u(s)$ on debt issuance, and show that tax wedges $\tau_l(s)$ on issuance of standard debt alone are sufficient to decentralize the socially optimal contract. In other words, the planner sets $\tau_u(s) = 0$ for all $s$.

The total tax burden $T$ of the bank is

$$T (R_l, R_u|R^*_l, R^*_u) = \sum_s \pi(s) \left[ \tau_l(s) (R_l(s) - R^*_l(s)) + \tau_u(s) (R_u(s) - R^*_u(s)) \right]$$

where $\{R_l, R_u\}$ are the contracts of an individual bank, and $\{R^*_l, R^*_u\}$ are the terms of the socially optimal contract. Equilibrium taxes are remitted lump-sum to banks ex ante. The
modified investor participation constraint is

\[ Y_0 = A - T \left( R_l, R_u | R^*_l, R^*_u \right) + E [x_1(R, s)|e = H]. \]

Recall that \((1 - b)R_l(s)Y_0\) is the level of standard debt. \(R_l(s)\) is therefore a measure of the equilibrium standard-debt-to-asset ratio of the bank. Similarly, \(R_u(s)\) is a measure of the total-debt-to-asset ratio. Tax wedges are thus defined against debt-to-asset ratios. Indeed in practice, many regulatory requirements are expressed as ratios rather than as levels.

From here, we characterize the tax wedges that align private and social incentives on the margin and so decentralize the socially optimal contract.

**Proposition 15.** The marginal tax wedges on contingent and standard debt that decentralize the social optimum are given by

\[ 0 \leq \tau_l(s) = -R_l(s)f_H(R_l(s), s)\frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int R_l(s) RY_0 f_H(R|s) dR \]  

(27)

\[ \tau_u(s) = 0 \]  

(28)

**C.2.1 Taxes on Other Liabilities**

Proposition 15 characterizes taxes for standard and bail-in debt, but admits a Lucas critique: banks may find it optimal to include a liability other than standard or bail-in debt in its capital structure as a result of the taxes. Issuance of other (non-debt) liabilities would amount to a form of regulatory arbitrage. We now characterize these wedges, which are only required for alternate liabilities that generate additional liquidations.

Starting from the socially optimal contract \(\{R_l, R_u\}\), consider some alternate liability with a continuous and monotone (in \(R\)) face value \(L_1^{alt}(R, s) \geq 0\). The privately and socially optimal contracts feature no issuance of this liability. Its issuance cannot be negative because it is a liability. \(^{87}\)

The optimal tax on \(L_1^{alt}\) is a simple combination of the tax rates on standard debt for the states where \(a\) causes the threshold bankruptcy state to increase. In particular, we

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\(^{87}\) To guarantee sufficient smoothness, we assume that bail-in debt is (on the margin) written down to restore solvency in response to a marginal increase in issuance of liability \(a\). As a result, issuance of \(a\) has the effect of increasing the threshold bankruptcy state, rather than causing bankruptcies over the entire region \((R_l(s), R_u(s))\). This is consistent with the notion that bail-in debt is written down to restore solvency whenever possible. As a result, the total value of liabilities is constant in the write-down region, with only the distribution of payoffs between holders of liability \(a\) and bail-in debt holders changing. Because all these investors are risk neutral, these changes are efficiently priced into contracts.
define the tax burden resulting from \( \epsilon > 0 \) units of issuance of liability \( L_{alt}^{alt} \) by \( \tau^{alt}\epsilon \). We obtain the following result.

**Proposition 16.** Let \( \{R_l, R_u\} \) be the socially optimal contract, and \( \{\tau_l\} \) the implementing tax wedges. Consider a liability with a face value \( L_{alt}^{alt}(R, s) \geq 0 \) that is continuous and monotone nondecreasing in \( R \). Then, the required tax rate on issuance of \( L_{alt}^{alt} \) is

\[
\tau^{alt} = \sum_{s \in S} \pi(s) \frac{\tau_l(s) L_{alt}^{alt}(R_l(s), s)}{(1 - b) Y_0}
\]

Proposition 16 tells us that the tax on any other liability \( a \) is related to the extent to which that liability moves the liquidation thresholds across states. Any alternate instrument for which \( L_{alt}^{alt}(R_l(s), s) = 0 \forall s \in S \), that is which does not increase the liquidation threshold, does not require any regulatory tax. As a result, if standard debt is the only instrument the bank issues that generates bankruptcies, then it is the only instrument that needs to be taxed.

**C.2.2 Proof of Proposition 15**

Because there is no disagreement between the bank and planner on \( R_u(s) \), we can set \( \tau_u(s) = 0 \). By contrast for \( R_l(s) \), the additional wedge in the planner’s FOC relative to the bank’s is the additional term is

\[
\lambda \frac{\partial \Omega(s)}{\partial R_l(s)} \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l(s)}^{R_l(R_l(s)|s)} R Y_0 f_H(R|s) dR
\]

as derived in Proposition 4. Substituting \( \frac{\partial \Omega(s)}{\partial R_l(s)} = R_l(s) f_H(R_l(s)|s) \) and setting equal to the bank tax burden \( -\lambda \tau_l(s) \pi(s) \), we obtain

\[
-\lambda \tau_l(s) \pi(s) = \lambda \pi(s) R_l(s) f_H(R_l(s)|s) \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l(s)}^{R_l(R_l(s)|s)} R Y_0 f_H(R|s) dR
\]

which rearranges to

\[
\tau_l(s) = -R_l(s) f_H(R_l(s)|s) \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l(s)}^{R_l(R_l(s)|s)} R Y_0 f_H(R|s) dR
\]

giving us our result. Non-negativity follows since \( \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \leq 0 \).
C.2.3 Proof of Proposition 16

Consider an alternate liability \( L^\text{alt}_1(R, s) \) satisfying the conditions of the proposition. From Proposition 4, any such liability is not used under the optimal contract, that is \( L^\text{alt}_1(R, s) = 0 \). As a result, the first order condition for optimal use of \( L^\text{alt}_1(R, s) \) is satisfied with potential inequality.

Since \( L^\text{alt}_1(R, s) \geq 0 \) satisfies monotonicity conditional on \( s \), a marginal increase in its issuance generates a monotone liability structure. Suppose that the bank issues an amount \( \epsilon \) of \( L^\text{alt}_1(R, s) \). The marginal increase in the bankruptcy threshold for increasing \( \epsilon \) marginally above 0 is given by

\[
\frac{\partial R_l(s)}{\partial \epsilon} = \frac{1}{(1-b)Y_0} L^\text{alt}_1(R_l(s), s).
\]

As a result, the planner’s FOC is given by

\[
\sum_s \frac{\partial L^\text{SP}}{\partial R_l(s)} \frac{\partial R_l(s)}{\partial \epsilon} + \sum_s \pi(s) \int_{R \geq R_u(s)} L^a_1(R, s) \left[ \lambda - 1 - \left(1 - \frac{f_L(R \mid s)}{f_H(R \mid s)} \right) \right] f_H(R \mid s) dR \leq 0
\]

where we have \( \frac{\partial L^\text{SP}}{\partial R_l(s)} \) given as before. Consider instead the FOC of the bank, which is given by

\[
-\lambda \tau^\text{alt} + \sum_s \frac{\partial L^B}{\partial R_l(s)} \frac{\partial R_l(s)}{\partial \epsilon} + \sum_s \pi(s) \int_{R \geq R_u(s)} L^a_1(R, s) \left[ \lambda - 1 - \left(1 - \frac{f_L(R \mid s)}{f_H(R \mid s)} \right) \right] f_H(R \mid s) dR \leq 0.
\]

The required tax rate that aligns the private and social incentives is

\[
-\tau^\text{alt} = \sum_s \pi(s) \frac{\partial L^\text{SP}}{\partial R_l(s)} \frac{\partial L^B}{\partial R_l(s)} \frac{\partial R_l(s)}{\partial \epsilon} \lambda \pi(s) \frac{\partial R_l(s)}{\partial \epsilon}.
\]

By construction, \( -\lambda \pi_l(s) \pi(s) = \frac{\partial L^\text{SP}}{\partial R_l(s)} - \frac{\partial L^B}{\partial R_l(s)} \), giving

\[
\tau^a = \sum_s \pi(s) \pi_l(s) \frac{\partial R_l(s)}{\partial \epsilon}.
\]

Finally, substituting in \( \frac{\partial R_l(s)}{\partial \epsilon} = \frac{1}{(1-b)Y_0} L^\text{alt}_1(R_l(s), s) \), we obtain in equilibrium

\[
\tau^a = \sum_s \pi(s) \pi_l(s) \frac{L^\text{alt}_1(R_l(s), s)}{(1-b)Y_0}.
\]
giving us the required tax.

C.3 Pareto Efficiency

We now study whether the socially optimal contract in Section 3 is Pareto efficient relative to the privately optimal contract. Because investors receive zero surplus from bank contracts, contracts are always (weakly) Pareto efficient from their perspective. We only need to consider arbitrageurs, who purchase liquidated bank project.

To ease exposition, we set $|S| = 1$ and drop the $s$ notation. Starting from our model of arbitrageurs in Appendix B.1, suppose that the shock is $\rho = 1$ so that skilled arbitrageurs have no cash available for arbitrage. As a result, the marginal pricing agent is an unskilled arbitrageur.

The construct a Pareto efficient allocation, we will require transfers at date 0 to arbitrageurs. Although arbitrageurs are penniless at date 0 in the baseline model, we need a notion of what they would do with funds, if given them via a transfer. We now develop that notion.

Arbitrageurs have the ability to finance a project at date 0, that has a date 2 payoff $y_2 > 1$. The project has no dividend at date 1 and cannot be liquidated prior to maturity. In other words, if arbitrageurs receive a transfer $T > 0$, they receive additional utility $Ty_2$ from the transfer via project investment, but the problem is otherwise unaffected.

We denote super script $P$ ($S$) to be equilibrium objects under the privately (socially) optimal contract. Suppose for simplicity that $\gamma_1 = \bar{\gamma}$, that is the best unskilled arbitrageur has the same value to managing bank projects as skilled arbitrageurs. We obtain the following sufficient condition for the socially optimal contract in Proposition 4 to be Pareto efficient relative to the privately optimal contract in Proposition 1.

**Proposition 17.** Let $|S| = 1$ and let $\frac{\partial \gamma}{\partial \Omega} < 0$.

1. A sufficient condition for the socially optimal contract to be a Pareto improvement over the privately optimal contract is

$$y_2 A \left[ 1 - \frac{V^P}{V^S} \right] \geq \left( \bar{\gamma}_B - \gamma^P \right) \Omega^P Y^P$$

where $V^P$ and $V^S$ are bank welfare under the privately and socially optimal contracts, respectively, and where $\gamma^P$ is the private equilibrium liquidation price (and so on).

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88 Arbitrageurs cannot scale up in the arbitrageur project.
2. A Pareto efficient allocation is achieved by combining bank contract regulation with an ex ante lump sum transfer from banks to arbitrageurs.

Pareto efficient improvements arise from the same source as the fire sale: external financing constraints on arbitrageurs. If arbitrageurs were able to raise external financing against arbitrage, then any positive premium \( \gamma(s) > \gamma(s) \) would be bid away and we would have a constant liquidation price. However, it is also this inability to raise external financing that creates a wedge between the marginal ex post value of funds to arbitrageurs, and the marginal ex ante value. Arbitrageurs would prefer funds ex ante, which they can use in their investment projects, but due to limits to external financing are unable to transfer profits from arbitrage from period 1 into period 0. The planner generates a Pareto improvement by coordinating a reduction in liquidations with an ex ante resource transfer to arbitrageurs. Banks are better off because liquidation prices are higher. Arbitrageurs are better off because they are able to invest in valuable projects.

C.3.1 Proof of Proposition 17

Using incentive compatibility, we associate a level of \( R_l \) with a level of \( R_u \)

\[
\int_{R_l}^{R_u(R_l)} bR (f_H(R) - f_L(R)) dR + \int_{R_u(R_l)}^{R} \left[ R - (1 - b)R_u(R_l) \right] (f_H(R) - f_L(R)) = B.
\]

Define investor repayment per unit of investment scale by

\[
\Pi(R_l) = \left[ \int_{R_l}^{R_u(R_l)} \gamma(R_l)R f_H(R)dR + \int_{R_l}^{R_u(R_l)} (1 - b)R f_H(R)dR + \int_{R_u(R_l)}^{R} (1 - b)R_u(R_l) f_H(R)dR \right].
\]

Let \( T \) be a lump sum tax on banks. The scale of the bank is then given by

\[
Y_0 = A - T + \Pi(R_l) Y_0 \Rightarrow Y_0 = \frac{A - T}{1 - \Pi(R_l)}.
\]

From here, we can obtain bank utility

\[
V(R_l, T) = V(R_l) \frac{A - T}{1 - \Pi(R_l)}
\]

where \( V \) is given by

\[
V(R_l) = \int_{R_l}^{R_u(R_l)} bR f_H(R)dR + \int_{R_u(R_l)}^{R} \left[ R - (1 - b)R_u(R_l) \right] f_H(R)dR.
\]

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Now, let $R^S_i$ be the socially optimal contract and let $R^P_i$ be the privately optimal contract. Define $T(R^P_i, R^S_i)$ by

$$V(R^S_i, T(R^P_i, R^S_i)) = V(R^P_i, 0).$$

which is the maximum transfer that can be taken from banks, while moving from the private to social contract, that leaves them indifferent to the contract change. Solving for $T(R^P_i, R^S_i)$, we obtain

$$T(R^P_i, R^S_i) = A \left[ 1 - \frac{V(R^P_i, 0)}{V(R^S_i, 0)} \right] = A \left[ 1 - \frac{V^P}{V^S} \right].$$

Now, let $L^A(R^P_i, R^S_i, T(R^P_i, R^S_i))$ be the ex post losses to arbitrageurs from moving from the contract $(R^P_i, 0)$ to the contract $(R^S_i, T(R^P_i, R^S_i))$. These losses can be bounded above by the surplus they receive from purchasing liquidated projects in the private equilibrium, that is

$$\left| L^A \right| \leq Y^P \left| \int_{\gamma_i \geq \gamma^P} \frac{\gamma_i - \gamma^P}{\gamma^P} f_i(i) \, di \right|$$

$$\leq Y^P \int_{\gamma_i \geq \gamma^P} \frac{\overline{\gamma} - \gamma^P}{\gamma^P} f_i(i) \, di$$

$$= Y^P \left( \overline{\gamma} - \gamma^P \right) \Omega^P$$

where the last line follows from the equilibrium condition for the liquidation price. Suppose that we transfer $T(R^P_i, R^S_i)$ to arbitrageurs to compensate them for the losses $L^A$. The total change in arbitrageur welfare can be bounded below as follows

$$\Delta^A = y_2 T(R^P_i, R^S_i) - |L|$$

$$\geq y_2 T(R^P_i, R^S_i) - \left( \overline{\gamma} - \gamma^P \right) \Omega^P Y^P$$

$$= y_2 A \left[ 1 - \frac{V^P}{V^S} \right] - \left( \overline{\gamma} - \gamma^P \right) \Omega^P Y^P$$

This gives us the sufficient condition

$$y_2 A \left[ 1 - \frac{V^P}{V^S} \right] \geq \left( \overline{\gamma} - \gamma^P \right) \Omega^P Y^P$$

for achieving Pareto efficiency when moving from the privately optimal contract to the socially optimal contract.
C.4 Heterogeneous Investors and the Allocation of Securities

In the baseline model, investors are homogeneous and risk neutral, so that the distribution of standard and bail-in debt among investors is irrelevant. A key practical concern is what investors should hold what form of debt, since bail-in debt holders will experience losses when it is written down. Particular concern has been expressed about protecting retail investors from losses that are large relative to their wealth, and to preventing institutional investors who are potentially exposed to fire sales from bearing losses from bail-ins.

To capture these elements, we extend the model to include two classes of bank investors, “institutional” and “retail.” Institutional investors are able to invest across all banks, but still retain exposure to the aggregate state and have preferences that may depend on bank liquidation discounts. Retail investors are only able to invest in a single bank and retain exposure to the idiosyncratic return of that bank. For simplicity, we abstract away from other potential components of these investors’ portfolio choice problems, instead allowing for state dependent preferences. All investors are price takers, and purchase state-contingent payoffs from the banks they invest in. Nevertheless, we show that in equilibrium all investors purchase a combination of the standard and bail-in debt contracts issued by banks.

Denote \( q(R, s) \) the (endogenous) probability-normalized price of a unit of payoff from a bank that realizes state \((R, s)\). Institutional investors are indexed by \( i \in I \), have initial wealth \( w_i^0 \), and preferences \( u_i^0(c_i^0) + E[u_i^1(c_i^1|s, \gamma(s))] \). Retail investors are indexed by \( j \in J \), have initial wealth \( w_j^0 \), and preferences \( u_j^0(c_j^1) + E[u_j^1(c_j^1|s)] \). Both \( I \) and \( J \) are finite sets, and we interpret each investor type as corresponding to a continuum of (atomistic) agents of that type. Both types of agents have period-0 budget constraints given by

\[
c_k^0 + \sum_s \pi(s) \int_R q(R, s)x^k(R, s)f_H(R|s)dR = w_k^0, \quad k \in I \cup J.
\]

However, they differ in their choice of \( c_1 \). Institutional investors are able to diversify across banks, so that \( c_i^1(s) = \int_R x^i(R, s)f_H(R|s)dR \). Retail investors are not able to diversify across banks, and so have \( c_j^1(R, s) = x^j(R, s) \). Given the contract payoff \( x(R, s) \) from the

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89 The resolution of four Italian banks in 2015 sparked a political backlash due to losses to retail investors. Financial Times, “Italy bank rescues spark bail-in debate as anger at Renzi grows,” December 22, 2015.

90 Article 44 of BRRD states that “[m]ember states shall ensure that in order to provide for the resolvability of institutions and groups, resolution authorities limit...the extent to which other institutions hold liabilities eligible for a bail-in tool.”

91 Note that the bank will go bankrupt in some states, implying not all liabilities are repaid at full face value. For simplicity, we price units of payout directly, rather than face value.
bank, market clearing for liabilities is given by

$$\sum_{k \in I \cup J} \mu^k x_1^k(R, s) = x(R, s)$$

where $\mu^k$ is the mass of investors of type $k \in I \cup J$.

We focus on the case where the mass of retail investors is sufficiently small that it does not exhaust the returns of the bank in any state $(R, s)$. That is, $\sum_j \mu^j x_1^j(R, s) < x(R, s)$. As a result, both retail and institutional investors price bank liabilities on the margin. We now characterize the equilibrium of the private economy without government intervention.

**Proposition 18.** Suppose that in equilibrium $\sum_j \mu^j x_1^j(R, s) < x(R, s)$. In the private equilibrium:

1. The price $q(R, s) = q(s)$ depends only on the aggregate state $s$.
2. Optimal bank contracts combine standard and bail-in debt.
3. Retail investors only purchase standard debt, and their consumption profile $c_1^i(R, s) = c_1^i(s)$ only depends on the aggregate state $s$. Consumption profiles of retail investors are given by

   $$\frac{\partial u_1^i(c_1^i(s)|s)}{\partial c_1^i(s)} = q(s) \frac{\partial u_0^i(c_0^i)}{\partial c_0^i}$$

4. Institutional investors purchase both standard and bail-in debt. Consumption profiles of institutional investors are given by

   $$\frac{\partial u_1^i(c_1^i(s)|s, \gamma(s))}{\partial c_1^i(s)} = q(s) \frac{\partial u_0^i(c_0^i)}{\partial c_0^i}$$

Even though retail investors are tied to a specific bank, their equilibrium consumption profile does not depend on the idiosyncratic state. This implies not only that retail investors exclusively purchase standard debt, but also that retail investors are first in line for repayment in the event of bank liquidation. In other words, in equilibrium they purchase claims that have the highest priority for repayment. Since retail investors are often depositors, one natural interpretation of this result is that of deposit priority.\(^92\) However, it extends beyond deposits, and furthermore suggests that retail bondholders may also

\(^92\)These deposits are not insured in this section, but are repaid due to their priority. In Appendix C.8, we consider deposit insurance.
benefit from priority. This suggests a role for non-bail-inable long-term debt, as a way to codify protection for retail investors.

Institutional investors are not exposed to the idiosyncratic state due to their ability to diversify, but are exposed to the aggregate state. Institutional investors face greater losses on the aggregate state when either they are more risk tolerant, or less exposed to bank fire sales. This suggests that the ideal holders of bail-in debt will be institutional investors with limited risk aversion (or ability to diversify using other securities) and limited commonality with the banking sector, so that they are not affected by fire sales.

Finally, consider what would happen if we relaxed the assumption $\sum_{j} \mu_j x^i_j(R, s) < x(R, s)$. Consider an aggregate state $s$ where $\sum_{j} \mu_j x^i_j(R, s) = x(R, s)$ for a range of returns $R \leq R^*$. For $R > R^*$, institutional investors are the marginal pricing agent, and $q(R, s) = q(s)$ is a constant. For $R < R^*$, retail investors are the marginal pricing agents, and $q(R, s) \geq q(s)$. Given monotone liabilities contracts, $q(R, s)$ will be falling in $R$. Contracts will still be debt, but the optimal thresholds are affected by the fact that retail investors suffer larger losses in liquidation, pushing $q(R, s)$ higher above $q(s)$. This generates an additional trade-off for the bank in deciding the optimal composition of standard and bail-in debt.

C.4.1 Proof of Proposition 18

Suppose that there is a state-contingent Arrow price $q(R, s) = q(s)$ that depends only on the aggregate state. Contracts still take the form of standard and bail-in debt, following the same steps as in the proof of Proposition 1.

Now, consider the investor side. Begin first with institutional investors, whose Lagrangian is given by

$$L^i = u^i_0 \left(c_0^i\right) + \sum_{s} \pi(s) u^i_1 \left(c_1^i|s, \gamma(s)\right) + \lambda^i \left[w^i_0 - c_0^i - \sum_{s} \pi(s) \int_{R} q(R,s) x^i(R,s)f_H(R|s) dR\right]$$

$$+ \sum_{s} \pi(s) \mu^i(s) \left[\int_{R} x^i(R,s)f_H(R|s)dR - c_1^i(s)\right].$$

Given the non-negativity constraint $x^i(R, s) \geq 0$, we have

$$\frac{\partial L^i}{\partial x^i(R, s)} = - \left[\lambda^i q(R,s) - \mu^i(s)\right] \pi(s)f_H(R|s) \leq 0.$$

This equation holds with equality only at the lowest value of $q(R, s)$ in state $s$. In other
words, investors only purchase $x^i(R, s) > 0$ if $q(R, s) = q(s)$, where $q(s)$ is defined to be the lowest price of a state-contingent security for some return state $R$ in state $s$.

Suppose then that in equilibrium $\sum_j \mu^j x^j(R, s) < x(R, s)$. Then, at least one institutional investor $i$ is purchasing $x^i(R, s) > 0$. As a result, we have $q(R, s) = q(s)$ for all $R$ in state $s$, that is the price is constant in aggregate state $s$. Moreover, $q(s) \lambda^i = \mu^i(s)$.

From here, we can obtain $\lambda^i$ from the FOC for $c^i_0$ and $\mu^i(s)$ from the FOC for $c^i_1$. Substituting in, we obtain

$$\frac{\partial u^i_1(c^i_1(s)|s, \gamma(s))}{\partial c^i_1(s)} = \frac{\partial u^i_0(c^i_0)}{\partial c^i_0} q(s).$$

giving us the characterization of the consumption rules of institutional investors.

Finally, consider type-$j$ (retail) investors. Given the constant price $q(s)$, their Lagrangian is

$$\mathcal{L}^j = u^j_0(c^i_0) + E \left[ u^j_1(c^i_1(R, s)|s) \right] + \lambda^j \left( w^j_0 - c^j_0 - \sum_s \pi(s) \int_R q(s)c^j_1(R, s)f_H(R|s)dR \right),$$

so that we have optimality condition for $c^i_1(R, s)$

$$\frac{\partial u^j_1(c^i_1(R, s)|s)}{\partial c^i_1(R, s)} = \lambda^j q(s).$$

As a result, $c^i_1(R, s) = c^i_1(s)$ is constant within state $s$. The indifference condition follows immediately by combining with the FOC for $c^i_0$. This concludes the proof.

### C.5 Risk Aversion and Risk Shifting

The baseline model featured no role for equity-like instruments in the bank’s capital structure. We extend the model to incorporate risk aversion and risk shifting, ingredients known to generate a role for equity-like claims. Optimal contracts still feature a region of liquidations and a region of “bail-ins,” where the bank is held to its continuation agency rent. Above the bail-in region, the contract involves equity-like claims. We set $|S| = 1$ for expositional simplicity, and drop the $s$ notation.

Banks are risk averse and have utility $u(c_1 + c_2)$ from consumption, while investors are risk averse and have utility $v(x_1 + x_2)$. Bank utility and marginal utility are finite at 0, and we normalize $u(0) = 0$. We incorporate risk shifting by extending the bank’s
monitoring decision to \( e \in \{L, H, RS\} \), where \( e = RS \) is “risk shifting” and \( e \in \{L, H\} \) are the high and low monitoring choices from before. Risk shifting does not generate a private benefit but affects the return density, \( f_{RS}(R) \).\(^{93}\) Define the likelihood ratios \( \lambda_{L,H}(R) = \frac{f_{L}(R)}{f_{H}(R)} \) and \( \lambda_{RS,H}(R) = \frac{f_{RS}(R)}{f_{H}(R)} \). Risk shifting inefficiently pushes mass towards the extremes of the distribution, which we formalize by defining a point \( R_{RS} \in [R, \bar{R}] \) such that \( \frac{\partial \lambda_{RS,H}(R)}{\partial R} < 0 \) for \( R < R_{RS} \) and \( \frac{\partial \lambda_{RS,H}(R)}{\partial R} \geq 0 \) for \( R \geq R_{RS} \).

As before, we assume optimal contracts enforce \( e = H \). The no-risk-shifting constraint is
\[
\int_{R} u(c(R)) \left( f_{H}(R) - f_{RS}(R) \right) dR \geq 0 \tag{29}
\]
while the incentive constraint is the same as before, except with \( u(c(R)) \). Investor participation is given by
\[
Y_{0} - A = \int_{R} v(x(R)) f_{H}(R) dR.
\]

Define \( \bar{\lambda}_{H}(R) = \frac{\mu_{L}}{\mu} \lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu} \lambda_{RS,H}(R) \) and \( \mu = \mu_{L} + \mu_{RS} \).

To simplify exposition, we will assume that the characterization that follows satisfies both consumption monotonicity for the bank and liability monotonicity for investors.\(^{94}\) Characterization of contracts in settings that do not satisfy monotonicity is beyond the scope of this paper. Moreover, we assume that the region \( 1 + \mu(1 - \bar{\lambda}_{H}(R)) < 0 \) is a connected set. This simplifies exposition.

**Proposition 19.** Let \(|S| = 1\). Suppose that the region \( 1 + \mu(1 - \bar{\lambda}_{H}(R)) < 0 \) is a connected set. The privately optimal contract is as follows.

1. In the region where \( 1 + \mu(1 - \bar{\lambda}_{H}(R)) < 0 \), there are liquidations and bail-ins.

2. In the region where \( 1 + \mu(1 - \bar{\lambda}_{H}(R)) \geq 0 \), there are bail-ins and “equity.” The equity sharing rule is
\[
u'(c(R)) \left( 1 + \mu(1 - \bar{\lambda}_{H}(R)) \right) = \lambda v'(RY_{0} - c(R))
\]

The motivations behind the liquidation region and the bail-in region are as in the baseline model. Consider next the “equity” region. First, bank risk aversion moderates payouts to the bank, smoothing the bank consumption profile on the upside and so giving away some of the equity value to investors. Second, bank consumption decreases with the average likelihood \( \bar{\lambda}_{H}(R) \). In the region \( R \leq R_{RS}, \bar{\lambda}_{H}(R) \) is decreasing in \( R \) and so banker

\(^{93}\)We could incorporate a private benefit or cost of risk shifting without qualitatively changing results.

\(^{94}\)Note that because both agents are risk averse, there is less scope for live-or-die contracts.
consumption is increasing. However, when $R \geq R_{RS}$, $\lambda_{L,H}$ is falling while $\lambda_{RS,H}$ is rising. This second effect, which comes from the risk shifting motivation, moderates payoffs to banks in high return states, which signal a higher likelihood that the bank engaged in risk shifting.

We could also derive the socially optimal contract, which would internalize the fire sale spillover cost of liquidations. However, conditional on not liquidating, bank and planner incentives are aligned, suggesting that the planner needs only to control the trade-off between liquidations and non-liquidations, and not the trade-off between bail-ins and "equity."\textsuperscript{95}

C.5.1 Proof of Proposition 19

Given the assumption of consumption monotonicity, if there is a liquidation region, it satisfies a threshold rule $R \leq R_l$. We define the optimal contract in terms of this threshold rule and in terms of liabilities $x(R)$ above this threshold. The bank’s Lagrangian is given by

$$\mathcal{L} = \int_{R \geq R_l} u(c(R)) f_{H}(R) dR$$

$$+ \mu_L \left[ \int_{R \geq R_l} u(c(R)) (f_{H}(R) - f_{L}(R)) dR - BY_0 \right] + \mu_{RS} \left[ \int_{R} u(c(R)) (f_{H}(R) - f_{RS}(R)) dR \right]$$

$$+ \lambda \left[ A + \int_{R \leq R_l} \nu(\gamma RY_0) f_{H}(R) dR + \int_{R \geq R_l} \nu(RY_0 - c(R)) f_{H}(R) dR - Y_0 \right]$$

$$+ \int_{R \geq R_l} \chi(R) [c(R) - bRY_0] f_{H}(R) dR$$

\textsuperscript{95}If effort were a continuous choice variable that affected bank returns, there would be an incentive to govern this margin. See Mendicino et al. (2018) for a numerical study of this problem.
Define $\lambda_H(R) = \frac{\mu}{\mu_{RS}} \lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu} \lambda_{RS,H}(R)$ and $\mu = \mu_L + \mu_{RS}$. We can combine the second line and obtain

$$\mathcal{L} = \int_{R \geq R_l} u(c(R)) f_H(R) dR$$

$$+ \mu \left[ \int_{R \geq R_l} u(c(R)) [1 - \lambda_H(R)] f_H(R) dR - \frac{\mu_L}{\mu} BY_0 \right]$$

$$+ \lambda \left[ A + \int_{R \leq R_l} \nu(\gamma Y_0) f_H(R) dR + \int_{R \geq R_l} \nu(R Y_0 - c(R)) f_H(R) dR - Y_0 \right]$$

$$+ \int_{R \geq R_l} \chi(R) [c(R) - b Y_0] f_H(R) dR$$

The derivative in $R_l$ is given by

$$\frac{1}{f_H(R_l)} \frac{\partial \mathcal{L}}{\partial R_l} = -u(c(R_l)) \left[ 1 + \mu (1 - \lambda_H(R_l)) \right] + \lambda \left[ \nu(\gamma Y_0) - \nu(R Y_0 - c(R_l)) \right]$$

so that liquidations may be optimal when $1 + \mu (1 - \lambda_H(R_l)) < 0$, that is when the average likelihood ratio is high. At low values of $R_l$, both the risk shifting and shirking problems have high likelihoods, so that $\lambda_H$ is large. As a result, bank consumption contributes negatively to welfare. Provided that this negative contribution outweighs the resource cost to investors, we have $R_l > R$.

Next, consider the region above $R_l$. The FOC for consumption $c(R)$ is

$$0 = u'(c(R)) \left( 1 + \mu (1 - \lambda_H(R)) \right) - \lambda v'(R Y_0 - c(R)) + \chi(R)$$

so that we have $\chi(R) > 0$ when $1 + \mu (1 - \lambda_H(R)) < 0$. As a result, for all values $1 + \mu (1 - \lambda_H(R)) < 0$, we either have liquidation or bail-in.

Finally, for $1 + \mu (1 - \lambda_H(R)) > 0$, we either have bail-in or an interior consumption value. When consumption is interior, it satisfies a risk sharing rule

$$u'(c(R)) \left( 1 + \mu (1 - \lambda_H(R)) \right) = \lambda v'(R Y_0 - c(R))$$

giving us an "equity" sharing rule.

Finally, the only role of assuming $1 + \mu (1 - \lambda_H(R)) < 0$ is a connected set in the proof is to ensure that it there are no points with $1 + \mu (1 - \lambda_H(R)) \geq 0$ below $R_l$. 

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C.6 Premium for standard debt

In the baseline model, the incentive problem is the only motivation for issuance of standard debt. In practice, standard debt can enjoy a premium relative to all other instruments, meaning it can pay a lower rate of return to investors. There are two natural stories for such a premium. The first is that standard debt takes the form of demand deposits, which enjoy a liquidity premium and require a lower rate of return. The second is that standard debt enjoys preferential tax treatment. We show that contracts still feature standard and bail-in debt, and that the trade-off is largely the same up to the consideration of the return premium. We then discuss potential issues with a pure premium story for standard debt.

Let $|S| = 1$. Suppose that standard debt has required return $1 + r$, where $r > 0$. We obtain the following result.

**Proposition 20.** Suppose $|S| = 1$. Suppose the model is extended to include a premium for standard debt. Optimal contracts combine standard and bail-in debt. The private optimality condition for standard debt is

$$
\mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda [(1 - b) - \gamma] + r \left[ \lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R_l)}{R_l f_H(R_l)} \right].
$$

while the optimality condition for bail-in debt is the same as in Proposition 1. The tax on $R_i$ that decentralizes the socially optimal contract is

$$
\tau_i = -(1 + r) R_l f_H(R_l) \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^R Y_0 f_H(R) dR
$$

while the tax on bail-in debt is $\tau_u = 0$.

Relative to the baseline case where $r = 0$, when $r > 0$ we have the term

$$
r \left[ \lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R_l)}{R_l f_H(R_l)} \right]
$$

in the private optimality condition, reflecting an additional cost/benefit trade-off of increasing use of standard debt. This term contains two additional effects of the presence of the liquidity premium. On the one hand, the higher liquidity premium implies that the costs of liquidation go up, because the resources lost would have been repaid to investors who have a high willingness to pay. On the other hand, replacing bail-in debt with standard debt increases payoff to investors with high willingness to pay in non-liquidation states.
The bank privately trades off these two forces in choosing the optimal standard debt level, in addition to the incentive forces.

C.6.1 Premium versus Incentive Problems

If \( r > 0 \), then the bank is willing to issue standard debt even in the absence of an incentive problem, that is if \( B = b = 0 \) and hence \( \mu = 0 \). The premium story alone can generate use of standard debt in the bank’s capital structure. However, in the absence of the incentive problem the logic of Corollary 14 applies. The bank (without loss of generality) uses equity as its other instrument.\(^{96}\) The planner can implement optimal regulation with an equity requirement. By including the incentive problem, our model provides a role for bail-in debt in optimal contracts.

What if instead \( B > 0, b = 0, \) and \( r > 0 \), so that standard debt has value from a premium perspective, but not from an incentive perspective (relative to bail-in debt). In this case, the optimal contract would combine standard and bail-in debt. However, this story on its own is problematic for two reasons.

The first is that because bail-ins typically apply to long-term debt, which were also non-contingent prior to the crisis, the premium story has to revolve around premiums on long-term debt, which is likely due to tax incentives. But if the government is subsidizing (non-contingent) long-term debt, this suggests it must provide some fundamental economic benefit. Our model provides a fundamental economic benefit of non-contingent long-term debt.

A second and closely related way to understand this issue is that in the event that \( b = 0 \), banks have strong incentives to protect themselves against liquidations by backing their non-contingent claims with liquid assets such as treasuries. This relates to a fundamental question in the banking literature: why are illiquid assets paired with fragile (often deposit) financing? Our model endogenously pairs illiquid assets with fragile (non-contingent) financing, rather than exogenously imposing it. Optimal regulation in our model respects the fundamental activity of banks: backing illiquid assets with fragile funding. A model that relies exclusively on a standard debt premium naturally lends itself to a “narrow bank” type result, where not only the planner but also banks prefer to use safe treasuries to keep the bank from ever failing. Indeed in Appendix C.6.2, we show a narrow banking result where the bank fully backs its non-contingent debt with liquid treasuries, eliminating the link between illiquid assets and fragile bank financing.

\(^{96}\)As a technical aside, of course a bank with no incentive problem and an expected return greater than 1 would, given linear technology, scale up to infinity. This issue is fixed simply by assuming that banks operate a concave technology \( Y_0 = f(I_0) \) to produce projects.
We could nevertheless adopt this view. The main result that would change is the non-optimality of bailouts (Proposition 7), which would no longer generically hold. We would be back into an incomplete markets world, in which bailouts may be desirable to mitigate fire sales, in a standard way. Moreover in the case of deposit insurance, the planner would always prefer to bail out the bank, rather than liquidating and repaying depositors. Bailing out the bank would save resources without distorting bank incentives, and so would be strictly preferred to liquidation.

C.6.2 A Narrow Banking Result

We show a “narrow banking” result as a simple extension of this section. Suppose that the bank can purchase safe assets (“treasuries”), which yield a deterministic return of 1. Treasuries do not need to be monitored. To avoid earning infinite profits, suppose the bank can issue a maximum of $D$ in non-contingent “deposits” that pay the premium $r > 0$.

First, note that if $r = 0$ but $b > 0$, we are back in the baseline model. The bank has no incentive to purchase treasures because liquidations are optimal for incentive provision.

Consider next the case where $r > 0$ but $b = 0$, so there is no incentive benefit for liquidations. The bank always issues $D$ in deposits, since it can always back an additional unit of deposit with $\frac{1}{1+r}$ treasuries and immediately consume the surplus without otherwise affecting its contract. Conjecture an implementable contract with treasury purchases $T < D$, bail-in debt $L$, and project scale $Y_0$. The threshold bankruptcy state of the bank is $R, Y_0 + T = D$, while the payoff profile to the bank is $c(R) = \max \{RY_0 + T - D - L, 0\}$. Suppose the bank increases $T$ and $L$ both by $\epsilon$. The bank consumption profile does not change, so the contract is still incentive compatible. However, the bankruptcy threshold falls and investor repayment increases, meaning the bank can do strictly better by implementing this change and immediately consuming the surplus at date 0. Hence, $T = D$ under the privately optimal contract.

The condition $T = D$ tells us that banks privately find it optimal to back their deposits entirely with safe treasuries, so that there is no risk to their depositors. This resembles a “narrow banking” proposal in that non-contingent debt holders are shielded completely from risks of illiquid lending. This applies not only to insured deposits, but to any deposit-like activity (e.g. wholesale deposits). Illiquid lending is no longer associated with standard debt.
C.6.3 Proof of Proposition 20

Relative to the baseline model, the only change is that the participation constraint becomes

\[ Y_0 - A = \int_R^{R_l} (1 + r) \gamma Y_0 f_H(R) dR + \int_{R > R_l}^{R_u} [(1 + r)(1 - b) R_l Y_0 + x_1(R)] Y_0 f_H(R) dR \]

where \( x_1(R) \) is repayment pledged to other investors. Note that it is immediate that standard debt enjoys priority over other liabilities, since it has the lower required rate of return. The proof that optimal contracts combine standard and bail-in debt follows as in the proof of Proposition 1. As a result, the optimization problem that determines \( R_l \) and \( R_u \) is the same as before, except that the participation constraint is now

\[ Y_0 - A = \int_R^{R_l} (1 + r) \gamma Y_0 f_H(R) dR + \int_{R_l}^{R_u} [(1 + r)(1 - b) R_l + (1 - b)(R - R_l)] Y_0 f_H(R) dR \]

\[ + \int_{R > R_u} [(1 + r)(1 - b) R_l + (1 - b)(R_u - R_l)] Y_0 f_H(R) dR \]

This yields the private optimality condition for \( R_l \)

\[ 0 = -b R_l Y_0 f_H(R_l) - \mu b R_l Y_0 \left(1 - \frac{f_L(R_l)}{f_H(R_l)}\right) f_H(R_l) \]

\[ + \lambda [(1 + r) \gamma R_l Y_0 f_H(R_l) - (1 + r)(1 - b) R_l Y_0 f_H(R_l)] + \lambda \int_{R_l}^R r(1 - b) Y_0 f_H(R) dR \]

which rearranges to

\[ \mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1\right) = b + \lambda [(1 - b) - \gamma] + r \left[ \lambda [(1 - b) - \gamma] - \lambda \frac{1 - H(R_l)}{R_l f_H(R_l)} \right] \]

Because \( R_u \) is not directly impacted by the liquidity premium, the optimality condition for \( R_u \) is as before, assuming that \( R_u > R_l \).

The planning problem features a wedge of the same form as before. The only difference is that the wedge is now weighted by \( 1 + r \), reflecting the higher liquidation losses. In other words, the planning problem is decentralized by the tax

\[ \tau_l = -(1 + r) R_l f_H(R_l) \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^{R_l} R Y_0 f_H(R) dR. \]

As before, \( R_u \) does not contribute to liquidations, and therefore \( \tau_u = 0 \).
C.7  Time-Inconsistent Bail-ins and Bailouts

Optimal regulation in the model implies the level of standard debt is contingent on the aggregate state, or in other words that the bail-inability of debt is contingent on the aggregate state. To the extent that bail-ins are ex post policies, they may be subject to similar time consistency problems as bailouts that limit the ability to make the degree of bail-inability $R_l(s)$ contingent on the aggregate state. We consider the possibility of both time-inconsistent bailouts and time-inconsistent bail-ins, so that the planner can only set a single threshold $R_l$ which is not contingent on the aggregate state. We may interpret $R_l$ as consisting of short-term debt (e.g. wholesale deposits) that is difficult in practice to bail-in due to its ability to run. Under time-inconsistent bail-ins, the planner writes down long-term debt whenever it is possible to restore bank solvency by doing so. Only short-term debt $R_l$ credibly liquidates the bank.

To ease exposition, suppose that there are only two aggregate states: a crisis state $s_L$ with fire sales, and a non-crisis state $s_H$ without fire sales. Suppose that the socially optimal contract in the presence of time-inconsistent bailouts but with time-consistent bail-ins features $R_l(s_L) \leq R^{BO}(s_L) < R_l(s_H) < R^{BO}(s_H) = \overline{R}$. Provided that probability of the crisis state is sufficiently low, the optimal contract sets $R_l > R^{BO}(s_L)$, and there are bailouts in the crisis state.

**Proposition 21.** Suppose that the socially optimal contract $\Gamma^*$ with bail-in commitment sets $R^*_l(s_L) \leq R^{BO}(s_L) < R^*_l(s_H) < R^{BO}(s_H) = \overline{R}$ and suppose that $\pi(s_L)$ is sufficiently low.\(^{97}\) Suppose that there are time-inconsistent bailouts and bail-ins. Then, $R_l > R^{BO}(s_L)$ and there are bailouts in the crisis state.

When bail-ins feature the same time-consistency problem as bailouts, the planner faces a trade-off between liquidations in the high state, which provide incentives, and bailouts in the low state, which are costly to taxpayers. If the crisis state is not too likely, the planner is willing to accept some costly bailouts in crisis times to allow incentive-improving liquidations in normal times. Bailouts re-emerge in equilibrium. Bailouts are not desirable per se, but are a necessary consequence of a lack of bail-in commitment.

\(^{97}\)We also need to impose two regularity conditions. The first is that in a neighborhood of $\pi(s_L) = 0$, we have $R^*_l(s_L) \leq R^{BO}(s_L) + \delta < R^*_l(s_H) < R^{BO}(s_H) = \overline{R}$. The second is that in a neighborhood of $\pi(s_L) = 0$, welfare $|V(\Gamma^*) - V(\Gamma)| > \delta$ for some $\delta > 0$, for all contracts $\Gamma$ with $R_l(s_L) = R_l(s_H) \leq R^{BO}(s_L)$. 79
C.7.1 Proof of Proposition 21

Consider the bank welfare function. Following the logic of the proof of Proposition 7, let us take any contract defined by its representation \( \Gamma = \{ R_l, R_u, T \} \), and identify the bank welfare function by

\[
V(\Gamma) = V(R_l, R_u) \; Y_0 (R_l, R_u, T | R_l, R_u)
\]

\[
= \frac{V(R_l, R_u)}{1 - \Pi(R_l, R_u | R_l, R_u)} (A + T)
\]

where \( V \) and \( \Pi \) are defined as in the proof of Proposition 7.

Let \( \Gamma^* \) be the optimal contract under bail-in commitment, which features \( R_l^*(s_L) \leq R^{BO}(s_L) < R_l^*(s_H) < R^{BO}(s_H) = R \). In absence of bail-in commitment, \( \Gamma^* \) is no longer implementable.

Consider any alternate implementable contract \( \Gamma' = \{ R'_l, R'_u, 0 \} \) with \( R'_l(s_L) = R'_l(s_H) \leq R^{BO}(s_L) \), so that there are no bailouts but so that the contract features a constant level of standard debt (not contingent on the aggregate state). Because \( \Gamma^* \) was chosen optimally with bail-in commitment, we have \( V(\Gamma^*) > V(\Gamma') \) for any alternate contract \( \Gamma' \) defined as above. Moreover, we assume that in a neighborhood of \( \pi(s_L) = 1 \), there exists a value \( \delta > 0 \) so that we can bound \( |V(\Gamma^*) - V(\Gamma')| > \delta \) for any alternative contract \( \Gamma' \).

Now, we will construct an alternate contract \( \Gamma'' \) as follows. Let \( R''_l(s) = R^*_l(s_H) \), \( R''_u(s) = R^*_u(s_H) + \epsilon \), for some value \( \epsilon \). Given bailouts, the contribution of the low state to (IC) under contract \( \Gamma'' \) is

\[
\pi(s_L) \left[ \int_{R^{BO}(s_L)}^{R_l^*(s_H)} b R (f_H(R | s) - f_L(R | s)) \, dR + \int_{R_l^*(s_H)}^{R} (R - (1 - b) R^*_l(s_H)) (f_H(R | s) - f_L(R | s)) \, dR \right]
\]

The contribution of the low state to IC under the optimal contract \( \Gamma^* \) (given time consistent bail-ins) was instead

\[
\pi(s_L) \left[ \int_{R^*_l(s_L)}^{R_u(s_L)} b R (f_H(R | s) - f_L(R | s)) \, dR + \int_{R^*_u(s_L)}^{R} (R - (1 - b) R^*_u(s_L)) (f_H(R | s) - f_L(R | s)) \, dR \right]
\]

so that we can write the difference between these two as \( \pi(s_L) \Delta_{L}^{IC} \). Let us now conduct a similar exercise for the contribution of the high state to (IC). The only difference here is in

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\(^{98}\) In other words, in a neighborhood of \( \pi(s_L) = 0 \), we do not converge to \( R_l^*(s_L) = R^{BO}(s_L) \).

\(^{99}\) Here, we make use of the fact that contracts in our model scale with \( Y_0 \), so that we can normalize IC by the asset scale.
the upper threshold, so that we have $\pi(s_H)\Delta_H^{IC}(\epsilon)$ defined by

$$
\Delta_H^{IC}(\epsilon) = \int_{R_H(s_H)}^{R_u(s_H)+\epsilon} bR (f_H(R|s) - f_L(R|s)) dR
+ \int_{R_H(s_H)}^{R} (R - (1 - b) (R_u(s_H) + \epsilon)) (f_H(R|s) - f_L(R|s)) dR
- \int_{R_L(s_H)}^{R} bR (f_H(R|s) - f_L(R|s)) dR
- \int_{R_L(s_H)}^{R} (R - (1 - b) (R^*_u(s_H))) (f_H(R|s) - f_L(R|s)) dR
$$

Because contract $\Gamma^*$ is incentive compatible, then contract $\Gamma''$ is incentive compatible provided that

$$
\pi(s_L)\Delta_L^{IC} + \pi(s_H)\Delta_H^{IC}(\epsilon) \geq 0.
$$

Supposing we take $\pi(s_L) \to 0$, then we can always ensure incentive compatibility holds with an appropriate change in $\epsilon$. Let $\epsilon(\pi(s_L))$ be the required innovation, defined for sufficiently small values of $\pi(s_L)$, so that incentive compatibility holds with equality. This defines the contract $\Gamma''$.

Let $T''$ be the bailout transfer in state $s_L$ with contract $\Gamma''$. Social welfare associated with this contract is

$$
V(\Gamma'') - \pi(s_L)T''
$$

Taking $\pi(s_L) \to 0$, we have $|V(\Gamma'') - V(\Gamma^*)| \to 0$ and $-\pi(s_L)T'' \to 0$. But since $|V(\Gamma^*) - V(\Gamma')| > \delta$, then $V(\Gamma'') > V(\Gamma')$, and the contract with bailouts dominates contracts without bailouts.

### C.8 Insured Deposits and Bailouts

In addition to fire sale spillovers and moral hazard, another goal of bail-ins is to reduce the costs of protecting insured deposits. We consider the addition of a group of insured deposits, and explore how the planner chooses to protect depositors.

For simplicity, we set $|S| = 1$ and assume that $\gamma < 1 - b$ does not depend on liquidations (no fire sale spillover). We further allow for the planner to commit ex ante to the desired combination of bailouts and insurance, so that the planner can always tie their hands and commit to no bailouts if desired. As a result, bailouts in this section will only

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Note that implicitly, the contracts $\Gamma^*$ and $\Gamma''$ are changing with $\pi(s_L)$. 

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occur if they are ex ante optimal.

The bank is constrained to issue insured deposits as a fixed fraction of its total assets, that is it issues \((1 - b)R_dY_0\) in insured deposits for some fixed threshold \(R_d > R\). We abstract away from the socially optimal determination of \(R_d\), instead focusing on how the planner chooses to protect a given set of depositors.\(^{101}\) The bank is always insolvent if \(R < R_d\), absent intervention, regardless of its other liabilities. Because deposits are insured, the planner is liable for any shortfall relative to the face value \((1 - b)R_dY_0\). Insured deposits are always at the top of the creditor hierarchy in liquidation.\(^{102}\)

Because the bank chooses bailouts with commitment, we set the political cost \(\kappa = 0\). When the planner bails out the bank, the bailout cost is

\[
\text{Cost}_{\text{No Liquidation}} = \tau ((1 - b)R_dY_0 + x(R) - (1 - b)RY_0)
\]

where \(x(R)\) is any liabilities in excess of \((1 - b)R_dY_0\) that the planner does not write down. When the planner instead allows the bank to fail, the creditor hierarchy implies the cost to deposit insurance is

\[
\text{Cost}_{\text{Liquidation}} = \min\{\tau ((1 - b)R_dY_0 - \gamma RY_0), 0\}
\]

When \(x(R) = 0\), the cost of rescuing the bank with a bailout is lower than the cost of rescuing the bank under liquidation, due to the loss of pledgeable income in liquidation.

The planner solves for the optimal contract, which includes the rescue decision (either via bailout or via liquidation and repayment by insurance).\(^{103}\) We constrain bank consumption to be monotonic, that is \(c(R)\) must be nondecreasing in \(R\),\(^{104}\) which was satisfied by optimal contracts in the baseline model. This implies that bailouts must be monotonic: if an insolvent bank \(R\) is bailed out, then all insolvent banks \(R' \geq R\) must also be bailed out. This rules out the possibility that the planner bails out a bank with \(R < R_d\) to protect depositors but liquidates a bank with \(R > \frac{1-b}{\gamma}R_d\) for incentive reasons.

**Proposition 22.** Suppose that \(c(R)\) must be monotonic, there are no fire sales, and there are insured deposits. The socially optimal contract consists of insured deposits \(R_d\), standard debt \(R_l \geq R_d\), and bail-in debt \(R_u \geq R_l\). The following are true regarding the use of deposit insurance

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\(^{101}\) For example, the planner may use deposit insurance to backstop risk averse depositors.

\(^{102}\) In practice, banks may issue wholesale funding which is not insured but runs prior to resolution.

\(^{103}\) A technical aside is that it is possible that the planner does not find it optimal to allow the bank to scale up as much as possible due to the cost of insuring deposits. We assume this is not the case, for example if \(R_d\) is close to \(R\).

\(^{104}\) If \(c(R) > c(R')\) but \(R < R'\), the bank could increase its payoff ex post by destroying assets to bring its return down to \(R\). We look for contracts where value destruction is not ex post optimal.
and bailouts.

1. If \( R_l > R_d \), there is deposit insurance but no bailouts. The bank is liquidated when \( R \leq R_l \).

2. If \( R_l = R_d \), there is a threshold \( R_L \leq R_d \) such that the bank is liquidated when \( R \leq R_L \) and bailed out when \( R_L \leq R \leq R_d \). The indifference condition is for bailouts (when interior) is

\[
b + \tau (1 - b - \gamma) = \mu b \left( \frac{f_L(R_L)}{f_H(R_L)} - 1 \right).
\]

Proposition 22 illustrates the trade-off between two mechanisms for protecting insured deposits. Bailing out the bank reduces the taxpayer cost of deposit insurance, but provides worse incentives for the bank. Whenever the planner allows use of standard debt in excess of insured deposits, that is \( R_l > R_d \), then necessarily the planner will commit to rescue depositors but not the bank. In this case, there is deposit insurance but no bailouts.

If \( R_l = R_d \) and \( R_L < R_d \), the planner uses bailouts ex post in order to reduce the cost of protecting depositors. This may or may not imply that the planner wishes to restrict use of insured deposits ex ante to avoid bailouts, depending on the motivation for deposit insurance. If deposit insurance is a way to provide a backstop to risk-averse depositors that the bank cannot provide itself, or if it is a way to stop sunspot runs, the planner may wish to allow enough insured deposits that it sometimes engages in bailouts.

C.8.1 Proof of Proposition 22

Due to consumption monotonicity, there is a threshold \( R_L \geq R \) for bank liquidation, with \( R_L = R \) corresponding to no liquidations. As in the proof of Proposition 7, there are no bailouts above \( R_d \), due to the taxpayer burden. We can thus split the problem into two parts.

First, suppose that the liquidation threshold satisfies \( R_L > R_d \), and suppose that the planner finds it optimal to engage in bailouts in a states \( R < R_d \). By consumption monotonicity, there are also bailouts for \( R_d \leq R \leq R_L \). But then because transfers to regular investors are wasteful, it is optimal to set \( R_L = R_d \), as in the proof of Proposition 7. The optimal contract does not feature both \( R_L > R_d \) and bailouts.

Consider then the form of the optimal contract when \( R_L > R_d \). Because there are no

\[^{105}\]If bailouts are chosen in a time-inconsistent manner and if \( R_d > R^{BO} \), there will have a mixture of bailouts and insurance independent of whether or not it is desirable. The planner will optimally set \( R_l = R_d \).
bailouts, the social objective function is

\[ \int c_2(R)f_H(R)dR - \int_{R\leq R_L} \tau \max\{(1-b)R_d - \gamma R, 0\} Y_0 f_H(R)dR \]

while the corresponding investor participation constraint is

\[ Y_0 - A = \int_{R = R_L}^{R_d} \max\{(1-b)R_d, \gamma R\} Y_0 f_H(R)dR + \int_{R \geq R_L} ((1-b)R_d + x(R)) f_H(R)dR \]

and where incentive compatibility is the same as in the baseline model. From here, note that the trade-off above \( R_L \) is the same as in the baseline model. The model again combines standard and bail-in debt, as in the baseline model.

Consider next the optimal contract when \( R_L < R_d \). \( R_L \) then also corresponds to the bailout threshold, such that there are bailouts when \( R_L \leq R \leq R_d \), and where \( R_L = R_d \) corresponds to no bailouts. The resulting social objective function is

\[ \int c_2(R)f_H(R)dR - \int_{R = R_L}^{R_d} \tau [(1-b)R_d - \gamma R] Y_0 f_H(R)dR - \int_{R_L}^{R_d} \tau (1-b)(R_d - R) Y_0 f_H(R)dR \]

while investor repayment is given by

\[ Y_0 - A = (1-b)R_d Y_0 + \int_{R_d}^{R} x(R) f_H(R)dR \]

reflecting that depositors are always repaid. Finally, incentive compatibility is as in the baseline model. Optimal contracts again combine standard and bail-in debt.

Consider the choice of the liquidation threshold \( R_L \). The trade-off is the same as in the baseline model, expect that an increase in the liquidation threshold leads to a tax burden on taxpayers rather than a cost to investors. That is, the FOC for the liquidation threshold is

\[ 0 = -bR_L - \mu bR_L \left( 1 - \frac{f_L(R_L)}{f_H(R_L)} \right) - \tau \left[ (1-b)R_d - \gamma R_L - (1-b)(R_d - R_L) \right] \]

which simplifies to

\[ b + \tau (1-b - \gamma) = \mu b \left( \frac{f_L(R_L)}{f_H(R_L)} - 1 \right) . \]

The only change is that the effective costs of liquidations has risen, due to the greater burden on taxpayers (\( \tau > \lambda \)). If the solution to this equation features \( R_L < R_d \), then there are bailouts in states \( R_L \leq R \leq R_d \).
C.9 Bank Size and Liquidation Discounts

In the baseline model, the size of the arbitrage sector scaled with the banking sector. A concern is that if the bank sector grows too large, it may be increasingly difficult for the rest of the economy to absorb bank asset sales, contributing to greater fire sales. We extend the baseline model to allow the liquidation discount to depend on bank size: \( \gamma (s, \Omega(s)Y_0) \).

In the arbitrageur model, this can be guaranteed by assuming that the size of the arbitrage sector is fixed and does not scale with the banking sector.

**Proposition 23.** Suppose that the liquidation discount depends on \( Y_0 \). Optimal contracts combine standard and bail-in debt. To decentralize the socially optimal contract, the planner uses the taxes of Proposition 15 combined with a tax on bank size

\[
\tau_Y = \frac{1}{Y_0} E \left[ \frac{\Omega(s)}{R_I(s)f_H(R)} \tau_I(s) \right]
\]  

(30)

The tax on bank size is the cumulative effect on fire sale costs across date 1 states. Because \( \tau_I(s) \) measures this fire sale cost, \( \tau_Y \) can be evaluated from \( \tau_I \).

The tax on bank size could also be implemented by additional taxes on liabilities. However, these restrictions would have needed to apply to all possible liabilities instruments, not only standard and bail-in debt, to avoid regulatory arbitrage. The simplest implementation is to directly tax bank size.

C.9.1 Proof of Proposition 23

The proof of the contract form follows as in the proofs of Proposition 1 and 4. Previously, there was no disagreement between the planner and banker on bank size, so that the wedges \( \tau_I(s) > 0 \) were enough. In other words, previous results implicitly set \( \tau_Y = 0 \), where \( \tau_Y \) is a tax on bank size. As in the proof of Proposition 15, we can consider the first order condition for bank scale \( Y_0 \) and characterize the additional term corresponding to the tax. The uninternalized impact of bank size on utility of the social planner, relative to banks, is given by

\[
\lambda \sum_s \pi(s) \int_R^{R_I(s)} \frac{\partial \gamma(s, \Omega(s))}{\partial Y_0} RY_0 f_H(R) dR
\]
which reflects the uninternalized fire sale impact. Setting the size tax to compensate for this impact, we obtain

\[-\lambda \tau_Y = \lambda \sum_s \pi(s) \int_R^{R_i(s)} \frac{\partial \gamma(s, \Omega(s))}{\partial Y_0} R Y_0 f_H(R) dR\]

which gives

\[\tau_Y = -\sum_s \pi(s) \frac{\partial \gamma(s, \Omega(s) Y_0)}{\partial \Omega(s)} \int_R^{R_i(s)} R Y_0 f_H(R) dR \geq 0\]

Now, given the form of \(\gamma\), we have

\[\frac{\partial \gamma(s, \Omega(s) Y_0)}{\partial \Omega(s)} = \frac{Y_0}{\Omega(s)} \frac{\partial \gamma(s, \Omega(s) Y_0)}{\partial Y_0}\]

Recall the short-term debt tax wedges are

\[\tau_l(s) = -R_l(s) f_H(R_l(s) | s) \frac{\partial \gamma(s, \Omega(s) Y_0)}{\partial \Omega(s)} \int_R^{R_l(s)} R Y_0 f_H(R) dR \]

\[= -\frac{R_l(s) f_H(R_l(s) | s) Y_0}{\Omega(s)} \frac{\partial \gamma(s, \Omega(s))}{\partial Y_0} \int_R^{R_l(s)} R Y_0 f_H(R) dR.\]

Substituting in above, we obtain

\[\tau_Y = \sum_s \pi(s) \frac{-\tau_l(s) \Omega(s)}{R_l(s) f_H(R_l(s) | s) Y_0} \int_R^{R_l(s)} R Y_0 f_H(R) dR \int_R^{R_l(s)} R Y_0 f_H(R) dR\]

\[= -\frac{1}{Y_0} E \left[ \frac{\Omega(s)}{R_l(s) f_H(R_l(s) | s) \tau_l(s)} \right]\]

completing the proof.

\section*{C.10 Continuous Effort}

We extend the model to feature a continuous effort choice, rather than a binary one.

Assume that \(|S| = 1\). The bank exerts an effort level \(e \in [\bar{e}, \overline{e}]\) at utility cost \(-v(e) Y_0\), where \(v', v'' > 0\), which induces a return distribution \(f(R | e)\) satisfying MLRP, \(\frac{\partial}{\partial R} \left( \frac{\partial f(R | e)/\partial e}{f(R | e)} \right) > 0\). We assume that \(e\) is interior under the optimal contract.

Effort is chosen to maximize bank utility, given the optimal contract, that is to say

\[\max_e \int_R c(R) f(R | e) dR - v(e) Y_0\]
which yields the IC constraint

\[ v'(e)Y_0 = \int_R c(R) \frac{\partial f(R|e) / \partial e}{f(R|e)} f(R|e) dR \]  

(31)

Relative to the baseline model, equation (31) specifies the effort level \( e \in [\underline{e}, \bar{e}] \), rather than requiring that high effort be exerted. In other words, equilibrium effort is now endogenous. However, the IC constraint takes essentially the same before, and so optimal contracts again combine standard and bail-in debt.

**Proposition 24.** The privately optimal bank contract combines standard and bail-in debt.

Consider now a social planner trading off between use of standard and bail-in debt. The key innovation relative to the baseline model is that liquidations now depend on equilibrium effort \( e \), that is

\[ \Omega = \int_R R f(R|e) dR \]

However, \( e \) is non-contractible and cannot be taxed directly. As a result, we have the derivative

\[ \frac{\partial \Omega}{\partial R_u} = \int_R R \frac{\partial f(R|e)}{\partial e} \frac{\partial e}{\partial R_u} dR \]

so that aggregate liquidations are now affected by the level of bail-in debt \( R_u \) via its effect on \( e \). The planner now has a motivation to control the total debt level.

**C.10.1 Proof of Proposition 24**

The private bank contracting problem is

\[
\max_{L_1, Y_0} E [c_2(R)|e = H] - v(e)Y_0
\]

subject to:

\[
Y_0 - A = \left[ \int_{R|\alpha=1} \gamma R Y_0 f(R|e) dR + \int_{R|\alpha=0} L_1(R) f_H(R|e) dR \right]
\]

\[ R \geq R' \Rightarrow L_1(R) \geq L_1(R') \]

\[ c_2 \geq (1 - \alpha(R)) b R Y_0 \]

\[ L(R) \geq 0 \]
The only modification relative to before is that in finding the critical points of the Lagrangian, there is also a first order condition for $e$. Given MLRP, the proof of the privately optimal contract follows the same steps as in the proof of Proposition 1.

C.11 Rollover Crises and Aggregate Multiplicity

Rollover crises can generate multiple aggregate equilibria of the economy for a fixed equilibrium selection rule $p$ for any individual bank, provided that $\gamma$ is sufficiently sensitive to additional liquidations. This reflects another form of fragility during crisis times, similar to the feedback loop.

**Proposition 25.** Fix $p \in (0, 1)$ and suppose that $\frac{\partial \gamma}{\partial \Omega} < 0 \forall \Omega \geq 0$. For any aggregate equilibrium $\gamma^*$, $\exists c > 0$ such that if at $\gamma^*$ we have

$$\left| \frac{\partial \gamma}{\partial \Omega} \right| > c,$$

then there also exist at least two additional equilibria: one with lower $\gamma$ and one with higher $\gamma$.

When liquidation values fall rapidly in response to sell-offs, our economy becomes more fragile and subject to multiple equilibria. Crisis times (where $\gamma$ may be highly responsive) are likely characterized by such heightened sensitivity. A bail-in regulatory regime is therefore more likely to contribute to fragility in long-term debt markets during crises. Finally, we note that Proposition 25 is generic: there is always a sufficiently high sensitivity that produces multiple aggregate equilibria. The only question is how high that elasticity is.

C.11.1 Proof of Proposition 25

Consider equation (17)

$$\gamma^* = \gamma \left( \int_R^{R_t} R f_H(R) dR + p \int_{R_l}^{R_t(\gamma^*)} R f_H(R) dR \right)$$

and define the gap

$$\Delta(\gamma^*) = \gamma^* - \gamma \left( \int_R^{R_t} R f_H(R) dR + p \int_{R_l}^{R_t(\gamma^*)} R f_H(R) dR \right)$$

which is the gap between the equilibrium price, and the implied equilibrium price from the liquidation function. Given an inherited contract with $R_t > R$, take the range of values
\( \gamma \in [\gamma, \bar{\gamma}] \) that can be obtained (where \( \gamma = \gamma(E[R]) \) and \( \bar{\gamma} = \gamma(0) \). Since \( R_I > R \), we have \( \Delta(\bar{\gamma}) > 0 \). Since \( p < 1 \), we have \( \Delta(\gamma) < 0 \).

Every zero of \( \Delta \) is an equilibrium of the economy at date 1. Suppose that we have an equilibrium at \( \gamma^* \), that is \( \Delta(\gamma^*) = 0 \), and suppose that \( \Delta'(\gamma^*) < 0 \). Then, for sufficiently small \( \epsilon > 0 \) we have \( \Delta(\gamma^* + \epsilon) < 0 < \Delta(\gamma^* - \epsilon) \). Given that \( \Delta(\bar{\gamma}) < 0 \) and \( \Delta(\bar{\gamma}) > 0 \), then by continuity we must have two additional zeros, one at a point above \( \gamma^* \), and one at a point below \( \gamma^* \). Given these points are zeros of \( \Delta \), they are also equilibria.

Finally, we characterize when we have \( \Delta'(\gamma^*) < 0 \). Differentiating, we obtain the required condition

\[
1 < \frac{\partial \gamma}{\partial \Omega} \frac{\partial \Omega}{\partial \gamma} \frac{\partial R^*}{\partial \gamma}.
\]

We have \( \frac{\partial \Omega}{\partial R^*} = pR^* f_H(R^*) \). Differentiating equation (16), we obtain

\[
- \frac{D_1}{(R^*)^2 Y_0 \partial \gamma} \frac{\partial R^*}{\partial \gamma^*} = \int_{R}^{d_2^*} R_2 f_2(R_2) dR_2
\]

where \( d_2^* \) is the solution to the supremum problem in equation (16). Observe that both \( \frac{\partial \Omega}{\partial R^*} \) and \( \frac{\partial R^*}{\partial \gamma^*} \) depend on \( \gamma^* \), but not on \( \frac{\partial \gamma}{\partial \Omega} \). Note that both of these depend on \( \gamma \), but not on \( \frac{\partial \gamma}{\partial \Omega} \). As a result, we have multiple aggregate equilibria if at \( \gamma^* \) we have

\[
\left| \frac{\partial \gamma}{\partial \Omega} \right| > c
\]

where \( c \) is given by

\[
\frac{1}{c} = \left| \frac{\partial \Omega}{\partial R^*} \frac{\partial R^*}{\partial \gamma} \right| = pR^* f_H(R^*) \frac{(R^*)^2 Y_0}{D_1} \int_{R}^{d_2^*} R_2 f_2(R_2) dR_2
\]

concluding the proof.

**C.12 Early Triggers and De Jure vs De Facto Seniority**

In our model, we abstracted away from existing long-term debt, and considered bail-ins as granting short-term debt de jure seniority over long-term debt. We now discuss the impact of early triggers, which write down existing stocks of long-term debt, and discuss the differences between de jure and de facto seniority of short-term debt over long term debt. Our discussion is high-level and preliminary, and leaves open a set of considerations for further research.
C.12.1 Early Triggers

First we ask whether early triggers - writing down existing long-term debt prior to resolution - can help recapitalize the bank and prevent rollover crises. An early trigger can be likened to a precautionary bail-in, and writes down existing long-term debt at the beginning of period 1, prior to accessing markets. Early triggers are a property of Contingent Convertible (CoCo) instruments, where the trigger (for example, based on a sufficient drop in the bank’s stock price) results in an automatic write-down or conversion from debt to equity with the goal of alleviating debt overhang in a distressed bank.\footnote{A second possible implementation of early triggers leverages the institutional structure of bail-in resolution as currently practiced in the U.S. under Title II. Write-downs of debt during a bail-in are applied at the level of the top-tier bank holding company (BHC). Long-term debt of operating subsidiaries (and intermediate BHCs) in the company structure are only eligible for write-downs once the debt of the top-tier BHC has been fully bailed in. If a BHC issues new long-term out of operating subsidiaries instead of out of the top-tier BHC, that debt will be senior to the top-tier BHCs liabilities in a bail-in resolution. This effect is similar to an early trigger.}

Equation \ref{eq:rollover_crisis} in Proposition 9 is a condition that defines existence of rollover crises when there is no existing long-term debt, in other words early triggers have in effect already been applied. Once the bank is in the rollover crisis region defined in Proposition 9, it is already too late for early triggers. This is not to say early triggers have no benefits in mitigating rollover crises, as they may be able to help prevent the bank from drifting into a rollover crisis region. For example, early triggers may help a bank with weak covenants. If there is a long-term debt overhang, the bank will be tempted to roll over short-term debt rather than refinance using new long-term debt, pushing itself towards the region of rollover crises. In this case, early triggers can help by alleviating debt overhang and reducing the debt dilution incentive.

C.12.2 De Jure vs De Facto Seniority

Second, we consider the difference between \textit{de jure} and \textit{de facto} seniority. Since rollover crises result from the fact that short-term debt is \textit{de jure} senior to long-term debt under bail-ins, one possible solution would be to allow for issuance of non-bail-inable long-term debt. Non-bail-inable long-term debt would rank \textit{pari passu} short-term debt in insolvency proceedings, but could not be used as a recapitalization tool. Non-bail-inability could be contractually designated, or could be implemented (e.g. in the US) by differentiating between top-tier BHC debt that can be bailed in (and is subordinated), and operating subsidiary debt which cannot be bailed in and is \textit{de jure} \textit{pari passu} with subsidiary short-term debt.
Because de jure pari passu status increases recovery values to long-term debt in resolution, hypothetical liquidation values of long-term debt will be positive, which may allow the bank to refinance itself even at the low prices. This relies fundamentally on the stability of long-term debt relative to short-term debt: even if the two instruments carried the same price, the bank may be able to refinance itself with long-term debt even when it cannot with short-term debt, because long-term debt does not induce future liquidations. To reflect this, we assume that long-term debt does not liquidate the bank at date 2, even if the bank is insolvent then. Banks refinanced at date 1 with non-bail-inable long-term debt always survive to the final period, and there are no liquidation discounts.

Formally, suppose that the liquidation value of new long-term debt is \( q_{L,B} > 0 \). The bank can issue at most \( L_3 \leq R_b Y_1 \) in new long-term debt while maintaining its agency rent, so that the most the bank can raise using long-term debt alone is \( q_{L,B} R_b Y_1 \). This implies that the bank can refinance itself provided that \( q_{L,B} \geq D_1 R_b Y_1 \).

**Proposition 26.** Rollover crises are avoided if the bankruptcy value of new long-term debt satisfies \( q_{L,B} \geq \frac{D_1}{R_b Y_1} \).

Suppose that new, non-bail-inable long-term debt ranks pari passu with short-term debt. In this case, its price in a hypothetical date-1 bankruptcy is \( q_{L,B} = \frac{\gamma Y_1}{D_1} \), so that the bank is guaranteed to avoid a rollover crisis if \( \frac{1}{\gamma} \left( \frac{D_1}{Y_1} \right)^2 \leq R_b \). Higher liquidation values \( \gamma \) and lower short-term-debt-to-asset ratios \( D_1 / Y_1 \) are associated with greater ability of the bank to refinance itself using long-term debt. Banks that are closer to insolvency or face large liquidation discounts (e.g. due to a crisis) are less likely to be able to avoid rollover crises this way.

A practical issue with this approach is that even if the planner opens up issuance of non-bail-inable long-term debt during stress times, short-term debt still enjoys an extent of de facto seniority over legally pari passu long-term debt, due to its shorter maturity. Even if long-term debt is de jure pari passu with short-term debt, the maturity subordination may push the effective value of \( q_{L,B} \) below the threshold for viability.

Finally, we have assumed that non-bail-inable long-term debt does not contribute to future solvency problems by studying a case where the bank refinances entirely with long-term debt. In practice, the bank will also roll over a potentially substantial amount of short-term debt. In this case, non-bail-inable long-term debt contributes to future solvency problems, reducing the future value to the bank. This may make this solution non-viable.

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107 The proof of Proposition 26 uses the weaker (true form of the) no rat race conditions of Appendix B.3.

108 See e.g. Brunnermeier and Oehmke (2013) and He and Milbradt (2016).
or leave the bank with a trade-off between trying to avoid a current rollover crisis by issuing non-bail-inable debt, and trying to avoid future solvency issues.

In sum, non-bail-inable debt may help to alleviate the rollover crisis problem, but faces a set of issues and trade-offs. Future research exploring these trade-offs may be valuable.

C.12.3 Proof of Proposition 26

The proposition is immediate, since the bank can issue \( L_3 = R_b Y_1 \) and obtain at least \( D_1 \), except that we have to verify that the no-rat-race conditions hold. In particular, suppose there is a hypothetical price \( q^{L,B}_1 > 0 \), and suppose that the bank issues long-term debt \( L_3 = R_b Y_1 \). For long-term debt to be fairly priced under this issuance, we must have

\[
q^{L}_1 R_b Y_1 \leq \int_{\mathbb{R}} \min\{R_2 Y_1, R_b Y_1\} f_2(R_2) dR_2 = (1 - b) Y_1
\]

or in other words, we must have \( q^{L}_1 \leq \frac{1 - b}{R_b} \).

The fact that the no rat race conditions are violated when \( q^{L}_1 > \frac{1 - b}{R_b} \) owes to the fact that we defined stronger no rat race conditions than are necessary, since they were easier to work with in the main part of the paper. Instead, apply the weaker conditions of Appendix B.3. Now, suppose that \( q^{L,B}_1 > \frac{1 - b}{R_b Y_1} \). The payoff neutral price associated with the issuance \((D_2, L_3) = (0, R_b Y_1)\) is given by \( q^{L}_1 = \frac{1 - b}{R_b} \). As a result, under the true no rat race conditions, banks issuing \((D_2, L_3) = (0, R_b Y_1)\) results in a price

\[
q^{L}_1 = \min\{q^{L,B}_1, \frac{1 - b}{R_b}\} = \frac{1 - b}{R_b}
\]

and so the bank can successfully refinance itself for any \( q^{L,B}_1 \geq \frac{D_1}{R_b Y_1} \).

D Mapping the Model into Existing Implementations

D.1 Short- and Long-Term Debt

In the main body of the paper, we implement the optimal contract using exclusively short-term (one-period) liabilities \( L_1(R, s) \). These claims are contingent on both the idiosyncratic and the aggregate state despite their short maturity of only one period. In practice, bail-in
regimes focus on write-downs of long-term debt. Short-term debt is generally given priority over long-term debt in resolution as part of bail-in regimes.

In the absence of aggregate risk (\(|S| = 1\)), the optimal contract can naturally be implemented by issuing \(R_l\) in non-bail-inable short-term debt along with \(R_u - R_l\) in bail-inable long-term debt. Short-term debt is never bailed in and liquidates the bank when \(R \leq R_l(s)\), whereas long-term debt is bailed in whenever the bail-in restores the solvency of the bank. More generally, when \(|S| > 1\), the short-term debt level would be \(R(s_1)\) (where \(s_1\) is the state with lowest \(R_l(s)\)) and never be bail-inable, while the bail-inability of long-term debt would be contingent on the aggregate state. In other words, our model admits the more standard implementation of bail-ins that apply to long-term debt.

Our model also predicts that short-term debt should enjoy absolute priority over long-term debt in bankruptcy and liquidation, given this implementation. In the threshold state \(R_l(s)\), long-term debt is fully bailed in while short-term debt is fully repaid. Suppose the bank underwent normal bankruptcy proceedings and liquidation instead of a bail-in. If short- and long-term debt were equal claimants in bankruptcy, long-term debt would receive positive repayment, and so would be better off under liquidation than under bail-in. This is inconsistent with a No Creditor Worse Off principle of bank resolution, which requires that resolution be ex post Pareto efficient relative to insolvency.

D.2 Contingent Convertibles (CoCos)

Although bail-in debt is most naturally expressed as a principal write-down in our model, it can also be expressed as a debt-equity conversion. Assume that in the conversion states at date 1, all bail-in debt is converted to equity. The equity value of the bank at date 1 is \((R - (1 - b)R_l(s))Y_0\) while the equity stake retained by the bank is \(bRY_0\) (the agency rent). Bail-in debt therefore converts into a fraction \((1-b)(R-R_l(s))\) of the (post-conversion) total equity of the bank. This debt-equity conversion achieves the same payoff profiles for the bank and its investors as the principal write-down.

As a result, our model does not speak to the optimal form of CoCos (principal write-down and debt-equity conversion). This question revolves around the roles of debt and equity.

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\(^{109}\) For example, in the US the top-tier bank holding company is subject to a “clean holding company” requirement, which bars it from issuing short-term debt to external investors. See 12 CFR §252.64.

\(^{110}\) In practice, short-term debt priority has three implementations. The first is contractual: bail-in debt is junior to short-term debt. The second is organizational: short-term debt is issued at the operating subsidiary, whereas long-term debt is issued at the top-tier holding company. The third is legal: national bankruptcy law confers priority to short-term debt in the case of banks. The three are equivalent in our model, which may help understand why the method of guaranteeing short-term debt priority varies across countries.

\(^{111}\) See e.g. Article 73 of BRRD.
equity in the continuation capital structure of the bank, which we have trivialized by assuming no uncertainty in continuation. In a richer setting, standard debt-equity trade-offs (incentive problems, risk shifting, etc.) would influence the optimal form of CoCos.

D.3 Bail-in Equivalence with Aggregate Risk

When there is aggregate risk (|S| > 1), the planner can still implement the optimal contract with bail-ins, but the implementation is more complicated because $R_l(s)$ depends on the aggregate state. Without loss of generality, order the states such that $R_l(s_1) \leq ... \leq R_l(s_{|S|})$. For exposition, let us assume that $R_u(s) = R_u$ is constant.$^\text{112}$ Under a bail-in implementation, the bail-inability of debt depends on the aggregate state. To implement the optimal contract, take $R_u$ to be a bank’s total debt. In state $s$, a portion $R_u - R_l(s)$ of debt is bail-inable, while $R_l(s)$ is non-bail-inable. If we interpret $s_1$ as a crisis and $s_{|S|}$ as a boom, decentralizing the optimal contract with ex post bail-ins implies that a larger fraction $(R_u - R_l(s_1))$ of debt is eligible to be bailed in by the planner during a crisis, while a smaller fraction $(R_u - R_l(s_{|S|}))$ is eligible to be bailed in by the planner during a boom. As a result, implementing the socially optimal contract requires a set of rules defining whether debt is bail-inable, depending on the aggregate state. Such rules either must be contractually pre-written into debt contracts, or must be written into the rules governing the operations of the bail-in authority.

In the US, such rules could be implemented using the organizational structure of the bank. Bank holding companies are required to maintain an amount of loss-absorbing debt at the level of the top-level holding company. The goal is to resolve the top-level holding company while allowing operating subsidiaries to continue operations without being affected by the resolution of the holding company. In principle, however, if a full write-down of the liabilities at the holding company level is not sufficient to recapitalize the bank, recapitalization would require bail-ins of debt at the operating subsidiaries. One could structure the governing rules of the bail-in authority to condition the ability of that authority to resolve operating subsidiaries based on the state of the economy. Operating subsidiaries could be resolved by the bail-in authority in crises, but not in normal times.

It is not clear whether aggregate state contingent rules governing the bail-in authority could credibly be implemented and followed. A bail-in authority is likely to be tempted to recapitalize a bank if there is enough long-term debt available to do so, suggesting the potential for time inconsistency in bail-ins. This helps to motivate the time consistency

$^\text{112}$For example, this can arise if the return distribution $f(R|s)$ does not depend on $s$ but the liquidation function $\gamma(s)$ does.
problem in Appendix C.7.