

# THE ALLOCATION OF TIME AMONG INDIVIDUALS

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For most of us time is a scarce resource. In our daily lives we are constantly allocating time among various activities and people. Allocating time among people is different from allocating it among activities. In allocating time among activities I need only consider my own preferences. In allocating my time among people I need to consider my own preferences and the willingness of others to spend time with me. People's decisions about whom they are going to spend their time with are interdependent. I may

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want to go to the movies with John, but he may be willing to go only if he cannot go out with Mary.

Economists have dealt extensively with how individuals allocate their time among different activities (see Becker, 1965; Linder, 1970; Ghez and Becker, 1975). A principal concern has been how individuals divide up their time between work and leisure. Some work has been done with respect to how individuals allocate their time among one another. Granovetter (1973) briefly discusses the problem. Boorman (1975) uses time in his job search model. Becker's (1973 and 1974) development of a theory of marriage parallels the discussion in this chapter.

Boorman (1975) has developed a model in which individuals allocate their time among one another while competing for job information. Each individual has two types of relationships: "strong ties" and "weak ties" (see Granovetter, 1973). Strong ties take more time than weak ties. Each individual has the same probability of being unemployed. Each also has the same probability of hearing of a new job. When an individual hears of a job, three things can happen: (1) If he is unemployed, he keeps the information; (2) if he is employed, he randomly gives the information to a strong tie who is unemployed; (3) if he and his strong ties are employed, he randomly gives the information to an unemployed weak tie. Once information is passed from one person to another, it is not passed any further. Individuals try to maximize the probability of getting a job when they are unemployed by choosing the optimal combination of strong and weak ties. Boorman goes on to examine stability and Pareto optimality for different employment conditions.

The model developed here is also concerned with how people allocate their time with each other. There are a number of important differences between Boorman's model and the model developed here. First, this model is not probabilistic. Second, the precise reason why one person prefers to spend more time with another is left unspecified. Individuals may have different reasons for preferring to spend time with one person rather than another. This is contrary to Boorman's assumption that individuals have the same probabilities of needing jobs and hearing of jobs. Boorman's assumptions guarantee anonymity to persons

with whom one spends one's time. This is not assumed here. Finally, it is assumed that each person's desire to spend time with another person is not dependent upon that person's allocation of time to others. This assumption differs from Boorman's model, where there is a priority rule between strong and weak ties. In summary this model is simpler than Boorman's in that it is non-probabilistic and assumes no interdependence in the importance of other's relationships to a person. On the other hand, it assumes a much more general situation in terms of people's preferences for spending time with each other.

Becker's marriage model is very similar to the time-allocation model in this chapter. In a sense marriage is very much like allocating all of one's time to one other person. In the simplest model Becker assumes there is a production function associated with each potential couple. The production function summarizes the product of the couple's relation—"quality of meals, the quality and quantity of children, prestige, recreation, companionship, love, and health status" (Becker, 1973, p. 816). Much of Becker's analysis consists of looking at the way final output changes with different inputs and different production functions. Much attention is given to who should marry whom within this framework. Our concern, however, is in the structure of the marriage market that results from this formulation. The market can most easily be represented in terms of an  $n + 1$  by  $m + 1$  payoff matrix with  $n$  males associated with the rows and  $m$  females associated with the columns. The last row and column represent the payoff to remaining single for males and females respectively. The  $ij$ th entry in the matrix represents the product that will result from the  $i$ th male marrying the  $j$ th female. The product of the marriage is assumed to be divided among each member. We can think of this division as a second payoff matrix in which there are two entries in the  $ij$ th cell. There is the share of the product the male receives and the share of the product the female receives. The specific share each spouse receives is in part determined by the marriage market itself. One person gives up a larger share of the product in order to induce another person to marry. Each person marries that person who is willing to marry them and from whom she or he receives the largest absolute share

of product. Becker goes on to show that an equilibrium will exist in this situation and that any equilibrium will maximize the total product in all marriages.<sup>1</sup> Thus any equilibrium is necessarily Pareto optimal. Becker briefly considers marriages in which the product is not perfectly divisible and interprets these situations in terms of love and caring.

We shall delay further discussion of the Becker model until we have given a full description of the time-allocation model. At that point we shall compare the time model to the Becker model and to more traditional graph-theory models of network theory.

### THE MODEL

Our goal is to describe the structure of a time-allocation market. In particular we shall be interested in the existence of equilibria, the efficiency of the allocation, and stability properties. Because of this we shall give a very simple description of how an individual allocates time and how two individuals decide to spend time together. We shall only consider *how much* time two individuals decide to spend with each other. We shall not attack the problem of where, when, and how people spend their time. The model does not consider issues of persuasion and inducement. The mechanisms by which one person convinces another to spend more time with him or with her are not made explicit. Although these problems are significant, they are beyond the scope of this investigation. Becker does deal with many of these issues in his two articles on a theory of marriage, and the interested reader is referred there. It should be pointed out, however, that failure to make these mechanisms explicit in the time model does not mean that we have restricted our model in any important sense. These mechanisms are implicit in our description. Since we shall be giving a simple description of how individuals allocate their time, there is no need for a comparative statics analysis. Within the present context such an analysis would not be very revealing.

To simplify the notation and exposition of the model we

<sup>1</sup>The actual proof is due to Koopmans and Beckman (1957).

assume that individuals spend time together only in pairs—that is, two at a time. Individuals are assumed to act individually: They do not form coalitions in deciding how to allocate their time.<sup>2</sup> Individuals will limit the amount of time they spend with others. Individuals see the limits imposed upon them as given. They cannot change the limits imposed upon them by changing the limits they impose upon others.

Besides making these general assumptions, we need to make a number of mathematical assumptions about the set of feasible allocations and people's preferences with respect to them. These assumptions are similar to those frequently used in the theory of the consumer (see Malinvaud, 1972, Chap. 2).

We assume that the set of physically feasible allocations is closed, bounded, nonnegative, and convex. The assumption that the set is convex is substantively important. A set is convex if for any two elements of the set  $X_1, X_2$  and for  $0 \leq a \leq 1$ ,  $X = aX_1 + (1 - a)X_2$  is also a member of the set. Convexity guarantees that for any two points, the points between them are also in the set. This assumption may not always be realistic. I may be able to spend 3 hours with Jim by spending an hour alone driving West to his place, or I may be able to spend 3 hours with John by spending an hour alone driving East to his place. To spend  $1\frac{1}{2}$  hours with each I must drive an additional 2 hours by myself. Clearly this situation is not convex— $(3,0,1)$  and  $(0,3,1)$  are both physically feasible, but  $(1\frac{1}{2}, 1\frac{1}{2}, 1)$  is not.

In the exposition of the model we make a stronger assumption. We assume that the total amount of time each person has is 1. We assume that the set of feasible allocations is the set of  $n \times n$  (where  $n$  is the number of people being considered) matrices that are nonnegative and have row sums of 1. We let  $X_{ij}$  represent the amount of time  $i$  allocates to  $j$ .

Why have we not assumed that the set of physically feasible allocations is symmetric—that  $X_{ij} = X_{ji}$ ? We think of  $X_{ij}$  not as the actual amount of time that  $i$  spends with  $j$  but instead as the amount of time  $i$  puts aside to spend with  $j$ . The actual

<sup>2</sup>In general the equilibrium found in the time model will not be in the core of possible allocations.

amount of time that  $i$  and  $j$  spend together is the minimum of  $X_{ij}$  and  $X_{ji}$ . If  $i$  allocates more time to  $j$  than  $j$  allocates to  $i$ , then  $i$  will spend the difference alone.

We also need to make assumptions about people's preferences. We assume that their preferences can be represented by a utility function that is continuous, twice differentiable, with continuous first derivatives. We assume that it is only dependent on  $X_i$ : the amount of time  $i$  allocates to others. The importance of this is taken up later. Finally we assume that the utility function is strictly quasi-concave. This is a strong assumption.  $U_i(X_i)$  is strictly quasi-concave, if for two allocations of time  $X_{i1}$  and  $X_{i2}$ ,  $U_i(X_{i1}) \geq U_i(X_{i2})$  then  $U_i(X_i) > U_i(X_{i2})$  for all allocations  $X_i = aX_{i1} + (1 - a)X_{i2}$  for  $0 < a < 1$ .

This assumption implies that an individual indifferent to two different allocations of time will prefer any allocation between the two. There are cases where this assumption does not hold. I may be indifferent between spending 2 hours with Jack at the movies or 2 hours with John at a play, but I will certainly not want to spend an hour with Jack and only see half a movie and then an hour with John and see only half a play.

### Individual Allocation

An individual's allocation process consists of three parts: (1) a set of preferences represented by a utility function  $U_i$ ; (2) a set of upper limits on how much time  $i$  can spend with each other person represented by a vector  $T_{.i}$ ; (3) a set of allocations of  $i$ 's time to other people represented by a vector  $X_i$ . Our goal in this section of the chapter is to derive a function that relates both  $i$ 's preferences and the limits imposed upon him to how  $i$  allocates his time to others. We assume that the amount of time  $i$  allocates to any other person is less than or equal to the upper limit on how much time  $i$  can spend with that person. Thus  $X_i \leq T_{.i}$ . Substantively this means that association is voluntary. One person cannot force another to spend more time with him than that person is willing to.

We assume that  $i$  will choose that allocation of time he prefers the most, which is less than or equal to  $T_{.i}$  and is physi-

cally feasible. Given the assumptions made in the last section, there will be a unique allocation  $X_i$  that will fulfill these criteria:  $X_i$  will vary continuously with  $T_i$ .<sup>3</sup> We let  $D_i$  be the function describing the relationship between  $X_i$  and  $T_i$ :  $X_i = D_i(T_i)$ .

If we hold all the  $T_{ki}$  constant except  $T_{ji}$ , the relationship between  $T_{ji}$  and  $X_{ij}$  will take a very specific form. It will start at the origin following the 45-degree line and then at some point become horizontal. Figure 1 shows an example.

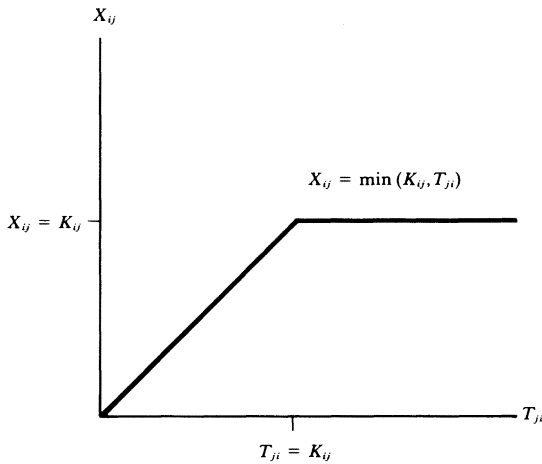


Figure 1. Typical demand function.

If we let  $K_{ij}$  be the point at which the curve bends (later we show that  $K_{ij} = T_{ji}$ ), then the equation for the curve is just  $\min(K_{ij}, T_{ji})$ . Figure 1 is easily interpreted. When  $K_{ij} > T_{ji}$ ,  $j$  is limiting the amount of time that  $i$  and  $j$  are spending together. In this case,  $j$  only wants to spend  $T_{ji}$  with  $i$ , even though  $i$  would be willing to spend  $K_{ij}$  with  $j$ . When  $T_{ji} = K_{ij}$ , then  $i$  and  $j$  both want to spend exactly the same amount of time together. The proof that the function  $D_{ij}$  takes this particular form is given in the mathematical appendix to this chapter.

<sup>3</sup>Proof of either of these statements is identical to the proofs for well-definedness and continuity of individual demand functions in the theory of the consumer (see Malinvaud, 1972, Chap. 2).

## Limits

In the last section we defined a function that related how an individual allocated time to the limits imposed by others. In that section the limits were simply given. In this section we show where these limits come from.

The limits that one individual imposes upon another represent the amount of time that individual is willing to spend with the other. The amount of time one person is willing to spend with another depends on how much time others are willing to spend with him. Harry may be willing to go to the movies with John only if he cannot go out with Mary. In limiting others  $i$  needs to consider how much time others are willing to spend with him. How much time should  $i$  be willing to spend with another person? Since his associations with people are voluntary, it makes sense to assume that  $i$  would be willing to spend that amount of time with someone that maximizes his utility, given the limits imposed by others.

Mathematically we can formulate this notion in a way that is almost identical to the way we set up the allocation problem. In this case we define a function that relates a person's preferences ( $U_i$ ) and the limits that are imposed upon him by others ( $T_i$ ) to the limits that he imposes on others ( $T_i$ ).<sup>4</sup> We let  $T_{ij}$  equal the amount of time that  $i$  would allocate to  $j$  if  $j$  did not impose any limit on  $i$ ; that is,  $T_{ji} = 1$ . Thus we have the same maximization process as in the allocation problem, except that in deciding how much time  $i$  is willing to spend with  $j$ ,  $i$  does not consider how much time  $j$  is willing to spend with him. In determining  $X_{ij}$  we assume that the  $X_{ij} \leq T_{ji}$  constraint does not hold. We define a function  $T_i = K_i(T_i)$ . From our previous discussion of the allocation problem it should be clear that  $T_{ij} = K_{ij}$ . That is,  $T_{ij}$  is just the amount of time that  $i$  would allocate to  $j$  if  $i$  were not constrained by  $j$  in how much time  $i$  could spend with him. Since  $D_i$  is well defined and continuous,  $K_i$  is also.

<sup>4</sup> People impose only upper limits on the amount of time they will spend with others because we have assumed that their preferences are strictly quasi-concave. No other type of limit would be consistent with this assumption.



### Partial Equilibrium

We are now in a position to determine how much time two people will spend together. We assume that two individuals,  $i$  and  $j$ , take the limits imposed upon them by others as given. From these limits they can calculate how much time they would be willing to spend with each other,  $T_{ij}$  and  $T_{ji}$  respectively. From the last section we know that  $K_{ij} = T_{ij}$ . From the section before that we know that  $X_{ij}$ , the amount of time  $i$  allocates to  $j$ , is equal to the minimum of  $K_{ij}$  and  $T_{ji}$  or equivalently  $T_{ij}$  and  $T_{ji}$ . Thus the amount of time that  $i$  allocates to  $j$  will be the minimum of the amount of time that  $i$  is willing to spend with  $j$ — $T_{ij}$ —and the amount of time  $j$  is willing to spend with  $i$ — $T_{ji}$ . And  $j$  will allocate the same amount of time to  $i$ . Two individuals will spend that amount of time together which is the minimum of the amount each would like to spend together.

This “minimum principle” is closely related to “the principle of least interest” (Homans, 1974; Waller and Hill, 1951). The person who has the least interest in a relationship is able to determine the conditions under which it will continue. Thus if one person would like to be good friends but the other wants only a passing acquaintance, the two will in all probability have only a passing acquaintance. This is exactly the idea that lies behind our model. It is the person who wants to spend the *least* amount of time together who determines how much time the two will spend together.

In this framework there are three possible relations that can exist between two people: (1) The first person would like to spend more time with the second than the second person is willing to spend with the first; (2) the second person would like to spend more time with the first than the first is willing to spend with the second; (3) both are satisfied with the amount of time they are spending with each other.<sup>5</sup> Are all three possibilities realistic?

<sup>5</sup>In terms of Lagrangian analysis, the specific multiplier associated with a constraint will be positive only when that person would like to spend more time with the other person than that person is willing to spend with him. Otherwise the multiplier will be zero.

The last one clearly is. What about the first two? In mathematical terminology each is a “corner” solution. One person has been unable to do anything to induce the other to spend any more time with him. Is this realistic? It is clear that there are cases where it is. An ordinary middle-class American might like to spend the next month with the president of the United States. Given middle-class resources there is probably nothing this person could do to induce the president to spend the next month with him. There are certainly similar examples. It is a matter of debate as to how often these situations occur in everyday life.

In terms of the first two relationships we have specified here, we can think of one person as having control over another. One person has control over the relationship by limiting the amount of time the other person can spend with him. What is interesting about this notion of control is that the control of one person over another is not derived from any intrinsic difference in their abilities to determine the outcome of events. Rather it is due to their different interests in terms of how they would like to spend their time. It is, by definition, the fact that one person has less interest in spending time with another that gives that person control. We shall define a relationship  $C$  between two people such that  $iCj$  if  $i$  has control over  $j$ —for example,  $j$  would like to spend more time with  $i$  than  $i$  is willing to spend with  $j$ . By definition  $C$  is asymmetric. We shall have more to say about  $C$  when we consider the problem of Pareto optimality.

### General Equilibrium Mechanisms

In the last three sections we have considered how each individual allocates time separately and how two individuals allocate their time with each other. Now we consider how a whole group of people simultaneously allocate their time. In particular we want to examine how a group of people arrive at an equilibrium. By a (general) equilibrium we mean a situation in which for every pair of people, neither of them would like to spend any less time together and at least one person in each pair would not like to spend any more time together. This just states that every pair of persons is in partial equilibrium—for example,  $X_{ij} = D_{ij}(T_i) = D_{ji}(T_j) = \min(T_{ij}, T_{ji})$  for all  $i, j$ .

Another way of explaining an equilibrium is to define it as a stationary or fixed point in some process: a point the process never leaves once it has arrived there. The simplest process we can describe is  $X_{t+1} = D(X_t)$ . Each individual examines how much time people allocated to him in the past period and then, interpreting this information as his constraints, reallocates his time for the next period. This process will have many fixed points, not all of which will be general equilibriums. The simplest example is the case where  $X$  is the identity matrix—each person is only spending time with himself. The identity matrix will be a fixed point of this process even if there are pairs of individuals who would like to spend more time together. Each individual will assume that no one wants to spend any time with him since no one allocated any time to him in the previous time period.

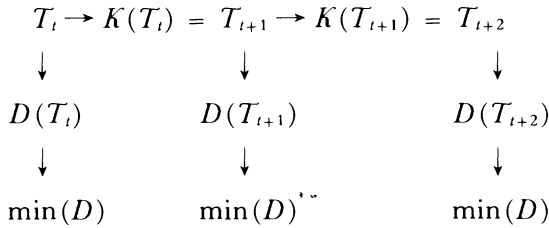
There is another way to look at this: There are two different meanings to how much time one person allocates to another. The amount of time one person allocates to another may indicate the amount of time that person would like to spend with the other. On the other hand, it may signify that person's perception of how much time the other is willing to spend with him. For a general equilibrium to be obtained, individuals need to be able to distinguish between the two different situations. And for this to be the case, people must communicate about the amount of time they are willing to spend together.

Instead of using a process in which people's allocations in the last time period determine how they will allocate their time in the next, we consider a process in which people's communications about their willingness to spend time with others in the last time period determine how willing they are to spend time with others in the future. We can describe this process as  $T_{t+1} = K(T_t)$ . The willingness of others to spend time with  $i$  in time  $t$  determines  $i$ 's willingness to spend time with others in time  $t + 1$ . This process will have a fixed point. This fixed point will be an equilibrium.<sup>6</sup>

The actual manner in which people allocate their time can be thought of in two different ways, both unrealistic. First, it can be assumed that individuals do not spend any time together until

<sup>6</sup>Proof of the existence of an equilibrium follows from the continuity of  $K$  and the Brouwer fixed-point theorem.

the process is at equilibrium.<sup>7</sup> In this case the  $X_{ij} = X_{ji} = \min \cdot (T_{ii}, T_{ji})$  will define a feasible general equilibrium. The second way that people could allocate their time is with respect to the constraints existing at that time. Since people may not allocate the same amount of time to each other, the actual amount of time they spend together will be the minimum of the amount that each allocated. We can describe this process as follows:



$\text{Min}(D)$  is defined as  $\min(D)_{ij} = \min(D_{ij}, D_{ji})$  for all  $i, j$ .

We illustrate this process by an example. We have three people: 1, 2, 3. Their utility functions are as follows:

$$\begin{aligned}
 U_1 &= X_{11}^{.2} X_{12}^{.7} X_{13}^{.1} \\
 U_2 &= X_{21}^{.1} X_{22}^{.2} X_{23}^{.7} \\
 U_3 &= X_{31}^{.7} X_{32}^{.1} X_{33}^{.2}
 \end{aligned}$$

The exponents represent the proportion of a person's time he would like to spend with each other person, assuming that there were no constraints imposed on how much time he could spend with any specific person. Thus person 1 would like to spend two tenths of his time by himself, seven tenths with person 2 and one tenth with person 3; similarly for persons 2 and 3. If a person is constrained with respect to how much time he spends with one particular person, he will try to allocate his remaining time to other people in the ratio of the other exponents. Thus if person 1 cannot spend seven tenths of his time with person 2, he will spend as much time with 2 as he can and then allocate the rest of his time between  $X_{11}$  (time by himself) and  $X_{13}$  (time spent with 3) in the ratio 0.2 to 0.1 or 2:1. The demand functions for person 1 are as follows:

<sup>7</sup>This assumption is identical to the assumption in economics that no trading takes place outside of equilibrium.

$$\begin{aligned}
 X_{12} &= \min\left[T_{21}, \frac{7}{9} - \frac{7}{9} \min(T_{31}, 0.1)\right] \\
 X_{13} &= \min\left[T_{31}, \frac{1}{3} - \frac{1}{3} \min(T_{21}, 0.7)\right] \\
 X_{11} &= 1 - X_{12} - X_{13}
 \end{aligned}$$

Thus if person 2 is willing to allocate only half his time to 1 ( $T_{21} = \frac{1}{2}$ ), person 1 would allocate his time as follows:  $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$ . The demand functions for persons 2 and 3 are similar to those of 1. The constraint functions for person 1 are as follows:

$$\begin{aligned}
 T_{11} &= 1 \\
 T_{12} &= \frac{7}{9} - \frac{7}{9} \min(T_{31}, 0.1) \\
 T_{13} &= \frac{1}{3} - \frac{1}{3} \min(T_{21}, 0.7)
 \end{aligned}$$

Thus if person 3 is willing to spend no time with 1, person 1 would be willing to spend seven ninths of his time with 2. A comparison of formulas for  $X_{12}$  and  $T_{12}$  illustrates the point made early:  $X_{12} = \min(T_{21}, T_{12})$ . Similarly for  $X_{13}$ . The constraint functions for persons 2 and 3 are similar.

Table 1 shows how the process evolves over time when

TABLE 1  
Example Using Cobb-Douglas Utility Functions

Time	Constraints			Allocations			Actual Time Spent Together		
1	1	0	0	1	0	0	1	0	0
	0	1	0	0	1	0	0	1	0
	0	0	1	0	0	1	0	0	1
2	1	$\frac{7}{9}$	$\frac{1}{3}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{5}{9}$	$\frac{2}{9}$	$\frac{2}{9}$
	$\frac{1}{3}$	1	$\frac{7}{9}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{5}{9}$	$\frac{2}{9}$
	$\frac{7}{9}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{5}{9}$
3	1	$\frac{7}{10}$	$\frac{2}{9}$	$\frac{14}{27}$	$\frac{2}{9}$	$\frac{7}{27}$	$\frac{5}{9}$	$\frac{2}{9}$	$\frac{2}{9}$
	$\frac{2}{9}$	1	$\frac{7}{10}$	$\frac{7}{27}$	$\frac{14}{27}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{5}{9}$	$\frac{2}{9}$
	$\frac{7}{10}$	$\frac{2}{9}$	1	$\frac{2}{9}$	$\frac{7}{27}$	$\frac{14}{27}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{5}{9}$
4	1	$\frac{7}{10}$	$\frac{7}{27}$	$\frac{40}{81}$	$\frac{7}{27}$	$\frac{20}{81}$	$\frac{41}{81}$	$\frac{20}{81}$	$\frac{20}{81}$
	$\frac{7}{27}$	1	$\frac{7}{10}$	$\frac{20}{81}$	$\frac{40}{81}$	$\frac{7}{27}$	$\frac{20}{81}$	$\frac{41}{81}$	$\frac{20}{81}$
	$\frac{7}{10}$	$\frac{7}{27}$	1	$\frac{7}{27}$	$\frac{20}{81}$	$\frac{40}{81}$	$\frac{20}{81}$	$\frac{20}{81}$	$\frac{41}{81}$
$\infty$	1	$\frac{7}{10}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
	$\frac{1}{4}$	1	$\frac{7}{10}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
	$\frac{7}{10}$	$\frac{1}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

persons 1, 2, 3 start off spending all their time by themselves;  $T_\infty$  is the equilibrium set of constraints and  $D_\infty$  is the corresponding allocation. Convergence of the process is very rapid. By the fourth time period each entry in the constraint matrix is to within  $\frac{1}{54}$  of the equilibrium solution. Each entry in the allocation matrix is to within  $\frac{1}{161}$  of the equilibrium solution. If we examine the relationship between the amount of time that people allocate to each other, and the actual amount of time that people spend together, we can see that when the system is out of equilibrium, the two do not necessarily correspond. At time 2, for instance, person 1 allocates  $\frac{1}{3}$  of his time to 2 but ends up only spending  $\frac{2}{9}$  with 2. At time 3 the situation is reversed. Person 2 allocates  $\frac{7}{27}$  of his time to person 1 but can only spend  $\frac{2}{9} = \frac{6}{27}$  of it. Being out of equilibrium causes inefficiencies in terms of how people would like to allocate their time.

The foregoing process is of course unrealistic. Most importantly it assumes that people's preferences for spending time with others at one point in time are not affected by the amount of time they have spent with these people in the past. In most circumstances this is not the case. If John has recently spent a great deal of time with Jim, he may well not want to spend much time with him now. Alternative processes could be described, but we need not consider them here.

### Pareto Optimality

An important question in evaluating an allocation is whether it is efficient. Pareto optimality is one way to think about efficiency.

*Definition:* An allocation  $X^*$  is Pareto optimal if there exists no other allocation  $X$  such that, for all  $i$ ,  $U_i(X_i) \geq U(X_i^*)$  with one inequality holding as a strict inequality.

If an allocation is Pareto optimal, there is no way to reallocate things so that each person is at least as well off as before and some people's positions are improved.

In general the equilibrium in the time model will not be a Pareto optimum. Consider the example in Table 1. The equi-

librium allocation was for each person to spend half his time by himself and a quarter of his time with each of the other two people. In this case each person's utility would be 0.287. Alternatively if each person spent a third of his time by himself and a third with each of the other two people, each person's utility would be 0.33. In this situation person 1 has agreed to spend more time with 3 in order to spend more time with 2; person 2 has agreed to spend more time with 1 in order to spend more time with 3; person 3 has agreed to spend more time with 2 in order to spend more time with 1. The situation represents a three-way trade. The situation is not, however, in equilibrium. Each person is in a position to increase his utility by breaking the agreement. For instance, person 1 can increase his utility by spending less than a third of his time with 3. Of course, once one person breaks the agreement there is no reason for others not to do so also. The situation then reverts to the original equilibrium.

We need to understand why this situation is not Pareto optimal. One way we can think about the situation is to regard it as an economy in which there are fixed prices.<sup>8</sup> Within the time model all the prices are fixed at 1. In having one person spend a certain amount of time with another, the other person is forced by the very definition of the problem to spend the same amount of time with that person. People would like to trade their time at different prices. In particular they would like to trade the time one person is willing to spend with them for time spent with another person. Since time is not transferable in the same sense that goods are, such a transfer could only take place if three or more people formed a coalition and collectively decided how to allocate their time. We have already ruled out such possibilities in our model.

Since in general the equilibrium is not Pareto optimal, it is natural to ask what the necessary and sufficient conditions are for an equilibrium to be Pareto optimal. At this point only a sufficient condition is known. In the section on partial equilibrium we defined a relation  $C_i C_j$  if  $i$  controls his relationship with  $j$ . Now  $i$  controls his relationship with  $j$  in the sense that  $j$  wants to

<sup>8</sup>The model developed in this chapter is closely related to fixed-price models that are being developed in neo-Keynesian economics. The interested reader should see Benassy (1973) and Grandmont (1975).

spend more time with  $i$  than  $i$  is willing to spend with  $j$ . It seems natural to think of a control relation as defining a hierarchy among a group of people, where people higher up in the hierarchy have control over those lower down. We think of hierarchies here in a very weak sense: A hierarchy exists if the relation  $C$  is acyclic.<sup>9</sup> We can induce a partial ordering on  $C$  by taking its transitive closure. If  $C$  is acyclic, it may contain a number of hierarchies. Between hierarchies there will be no control relations. Within hierarchies people lower down will be controlled by some, though not necessarily all, of the people above them. No one higher in the hierarchy will be controlled by anyone lower.

If the relation  $C$  associated with an equilibrium is hierarchical, the equilibrium will be Pareto optimal. Proof is given in the mathematical appendix. It should be pointed out that the situation in which no one constrains anyone else is included in this theorem. In this case each person is satisfied with the amount of time he spends with the others. Each person is his own hierarchy and the  $C$  matrix is trivially acyclic.

### *BECKER'S MARRIAGE MODEL*

In the introduction we gave a brief description of what was termed Becker's simple marriage model. In that model Becker derived very different results from those we have derived for the time model. In particular Becker showed that the marriage market maximized the total output over all marriages. In the time model the equilibria will not in general even be Pareto optimal. Where does the difference lie? The models differ in a number of ways, all of which are important in accounting for the difference in results.

Not all situations can be characterized by Becker's simple marriage model. Becker recognizes this and discusses the problem briefly in terms of rigidities in the division of the product. The situation that interests us in particular is when there is some sort of "cyclic" pattern of preferences among people. Imagine two males and two females sitting at a table: male, female, male,

<sup>9</sup>A relationship is acyclic if there is not a directed path from  $j$  to itself for all  $j$ ; that is, it is not the case that  $jPk, kPL, LPm, \dots, sPj$  for any  $j$ .



female. Each person would prefer to marry the person to his or her left and would rather remain single than marry the person to the right. These preferences or payoffs are represented in Figure 2.

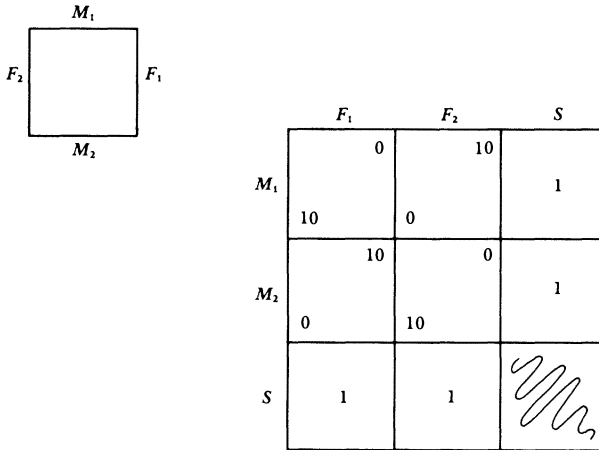


Figure 2. Marriage Market with Cycle Structure

These preferences cannot be represented in Becker's simple marriage model. For any person the minimum product from the relationship to the left must be greater than the maximum product from the relationship to the right. This can occur only if two numbers are both strictly greater than and less than each other. Clearly this is impossible. Becker's simple model rules out such cyclic patterns of preference and thus guarantees a certain type of consistency among people's preferences.

The situation we have described does have an equilibrium—everyone remains single. At any point in time the equilibrium is Pareto optimal. It is not, however, Pareto optimal over time. We assume that people's payoff over time is the average of their payoffs at each particular time. In Figure 2, it will be optimal for each person to be married to the person to the left half the time and to the person to the right the other half. In this case the payoff for each person is 5 whereas when they remained single it was 1. Allowing people to split their marriages is similar to allowing people to split up the time they spend with others. In the marriage situation we have described, the four people would

have to make an agreement collectively. How such an agreement would be enforced is not clear. The situation here is similar to the three-person situation described in the section on Pareto optimality.

Becker does not show that an equilibrium will exist in the more general case. If an equilibrium does exist, it certainly does not have to maximize the total output over all marriages as the example above indicates. If an equilibrium does exist in the more general case, its existence depends crucially on the fact that the marriage market has two sides: male and female. Consider the case where people marry within rather than across sex. Assume that we have three males, John, Joe, and Harry. Each would prefer to be married than to remain single. John would prefer to “marry” Joe, Joe would prefer to “marry” Harry, and Harry would prefer to “marry” John. Here there will be no equilibrium. If John and Joe are “married,” then both Joe and Harry could improve their situation by “marrying” each other. Similarly for any other situation. The example is equivalent to a situation in which people are allocating their time but are not allowed to divide it up. If they are allowed to divide their time, there will be an equilibrium.

The situation we have described is also equivalent to Condorcet’s paradox. There are three people and three alternatives ( $X, Y, Z$ ) to be chosen by the group. One person’s preferences are  $X, Y, Z$ ; the next person’s preferences are  $Y, Z, X$ ; the third person’s preferences are  $Z, X, Y$ . In this case there will be no “reasonable” way to decide on which alternative to choose without knowing the cardinal utilities of the people involved. (See Arrow, 1951, for the classic discussion of this issue and definition of what we mean by “reasonable.”) In order to have a “reasonable” decision rule, there must be some consistency among people’s preferences.

In both the Becker marriage model and the time model, as in the collective decision problem, there needs to be some type of consistency in people’s preferences in order to obtain efficient equilibria. In particular, preferences that are cyclic in structure either prohibit the existence of equilibria or prevent them from being Pareto optimal. Becker’s simple marriage model by assumption prohibits cyclic preferences. The way it does so may

well be very plausible. The assumptions that he constructs, however, need to be much more fully explored.

### *NETWORK MODELS*

The model presented in this chapter differs considerably from the usual graph-theory models that dominate social network theory. The typical approach in networks has been to assume that relationships should form certain patterns. This theme is found in balance theory (Cartwright and Harary, 1956; Heider, 1958); its extension in the work of Davis, Holland, and Leinhardt (Davis, 1967, 1970; Davis and Leinhardt, 1972; Holland and Leinhardt, 1970, 1971, 1972, 1973, 1975a, 1975b, 1976) and in the block models of White, Boorman, and Breiger (1976; Boorman and White, 1976). The fact that someone likes someone else is explained by the pattern of relations of which that relationship is part. Thus balance theory stipulates that a friend of a friend should be a friend. This is both very similar to and very different from the model offered here.

The similarity is that patterns—balanced graphs, for example—represent “equilibrium” positions. In defining what a relationship should be in this equilibrium, it is necessary to refer to other relationships that exist in the group. In a similar way, in defining constraints—that is, how much time people are willing to spend with others—it is necessary to know how much time others are willing to spend with different people. Thus the interdependence of people’s choice is common to all the models. In the present model the interdependence is simpler than in the usual network model. In the time model willingness to spend time with others is only dependent on their willingness to spend time with me, not on their willingness to spend time with others. According to balance theory, liking someone is not just dependent on whether that person likes me but also on whether someone I like likes that person.

Perhaps the major difference between the present model and the usual network model is this: Whereas the usual network model seeks to explain why people like each other (in terms of some pattern of relationships), the present model takes people’s preferences for others as given. There are both good and bad as-

pects of this characteristic. On the bad side, the model is weaker because it does not explain this aspect of relations. On the good side, taking people's preferences as given has allowed us to look at the relationship between people's preferences (likes) and the actual behavior that exists between them (the amount of time they spend together). In general, people's preferences do not uniquely determine the way they allocate their time. People's preferences uniquely determine how people allocate their time only if there is a globally stable equilibrium.<sup>10</sup> In general this will not be the case. To understand why a certain equilibrium is arrived at, both the properties of the equilibrium and the prior history of the group need to be known. This can be shown by example.

The following are the utility functions for a group of four people:

$$U_1 = -(X_{12} - 2X_{13})^2 - (X_{11} + 10)^2 - (X_{14} + 10)^2$$

$$U_2 = -(X_{23} - 2X_{21})^2 - (X_{22} + 10)^2 - (X_{22} + 10)^2$$

$$U_3 = -(X_{31} - 2X_{34})^2 - (X_{33} + 10)^2 - (X_{32} + 10)^2$$

$$U_4 = -(X_{43} - 2X_{42})^2 - (X_{44} + 10)^2 - (X_{41} + 10)^2$$

Although these functions are not strictly quasi-concave over the set of real numbers, they are strictly quasi-concave over the set of feasible allocations. In fact they are strictly concave over this set. All the functions are of similar form, although people have different roles in each function. Let us examine the utility function of person 1.

The first term in  $U_1$  represents a preference on 1's part for having the amount of time he spends with 2 be in a 2:1 ratio to the amount of time he spends with 3. The second term represents a strong aversion on 1's part to spending time alone. The third term represents a strong aversion to spending time with 4.

This set of utility functions will have an infinite number of equilibria and cycles. Two of the equilibria and one of the cycles are shown in Table 2. What is interesting about the two equilibria shown is that the roles of people are reversed. In the first equilibrium, persons 1 and 4 are at unconstrained optima:

<sup>10</sup>If, starting at any point, a process converges to the same equilibrium, that equilibrium is said to be globally stable.

TABLE 2  
 Example Using Quadratic Utility Functions  
 Two Equilibria Constraint Matrices

<i>A</i>				<i>B</i>			
1	$\frac{2}{3}$	$\frac{1}{3}$	0	1	$\frac{2}{3}$	$\frac{2}{3}$	0
$\frac{2}{3}$	1	0	$\frac{2}{3}$	$\frac{1}{3}$	1	0	$\frac{2}{3}$
$\frac{2}{3}$	0	1	$\frac{2}{3}$	0	0	1	$\frac{1}{2}$
0	$\frac{1}{3}$	$\frac{2}{3}$	1	0	$\frac{2}{3}$	$\frac{2}{3}$	1

One Cycle							
Odd				Even			
1	$\frac{2}{3}$	$\frac{2}{3}$	0	1	$\frac{2}{3}$	$\frac{1}{3}$	0
$\frac{2}{3}$	1	0	$\frac{2}{3}$	$\frac{1}{3}$	1	0	$\frac{2}{3}$
$\frac{2}{3}$	0	1	$\frac{2}{3}$	$\frac{2}{3}$	0	1	$\frac{1}{3}$
0	$\frac{2}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$	$\frac{2}{3}$	1

Person 3 would like to spend more time with 1 and person 2 would like to spend more time with 4. In the other equilibrium, persons 2 and 3 are at unconstrained optima: Person 1 would like to spend more time with 2 and person 4 would like to spend more time with 3.

The equilibrium the process arrives at depends on where it starts. If the process in the preceding example starts with no one thinking he can spend time with anyone else, then it will immediately start into the cycle in Table 2. Other initial points will cause the process to head off into another cycle or to some equilibrium. The history of a process as well as people's preferences are essential to understanding the actual way time is allocated among different people.

Besides making a distinction between people's preferences (their utility functions) and the actual amount of time they spend together, the model makes a similar distinction between the amount of time people communicate as wanting to spend together and the actual time spent together. Where most models are just concerned with one type of relationship (see White, Boorman, and Breiger, 1976, and Boorman and White, 1976, for exceptions to this rule), in the present model there are two different types of relationships: the amount of time you actually spend with someone and the amount of time you would like to spend with them. In essence there are two processes going on in the model: a communication process in which people tell each other how much

time they would like to be spending with each other and an allocation process which determines the amount of time that people actually spend together. This type of distinction is an advantage absent from most network models.

Within the model we have also specified a dynamic process in terms of how people allocate their time. The process is admittedly a crude one, but it leaves much room for elaboration and further work. Dynamic models are new to sociometry (see Holland, and Leinhardt, 1977, for example, and Hunter, 1974), and it is a strength of this model that it already includes one. The fact that we assume that people's preferences are given weakens the applicability of the dynamics we have described. This assumption is adequate if we are looking at a group that has been together for a while. In the case where we are trying to examine how relationships form from the beginning of the group, the description is much less adequate. We must assume that people have certain preferences for spending time with each other even before they meet. This seems to relegate people's preferences to personality. People with compatible personalities would want to spend a great deal of time together, and people with antagonistic personalities would want to spend little time together. Our model shows, however, that even under this extreme psychological assumption, people's preferences alone are not adequate to explain the allocation of time arrived at. The development of a group's relations needs to be understood in order to explain why one equilibrium is arrived at instead of another.

Before becoming too enamored of our model we should examine some of its peculiarities. The model treats time like a good that can be consumed. The notion of time as a sequence of events is absent. This treatment can lead to some rather striking anomalies when a small number of people are considered. For instance consider three people: John, Jim, and Harry. Consider the case where only two people spend time together at any one time. Assume that no one spends any time alone and that each person spends half his time with each of the other two. From the point of view of our model this seems like a most reasonable allocation. In physical time, of course, it is not possible. If John and Jim are spending time together, Harry must necessarily spend time alone. Thus the sequential nature of time imposes con-

straints we have not included in the model. Certainly the sequential nature of time should be accounted for in any future elaborations of this model. This might be done by modifying the model to include not only how much time people spend together but also when they spend that time. Alternatively one might find a clever way to define the set of feasible allocations that would avoid the problem. These are matters for future research.

### MATHEMATICAL APPENDIX

We need to show that  $D_{ij}(T_i)$  has the property illustrated in Figure 1 by proving the following two propositions.

*Proposition 1:*  $D_{ij}(T_i) = K_{ij}$  for  $K_{ij} \leq T_{ji} \leq 1$ .

*Proof.*  $K_{ij}$  and its associated vector of allocations is feasible for  $K_{ij} \leq T_{ji} \leq 1$ . Assume that  $D_{ij} = X_{ij} \neq K_{ij}$  for  $T_{ji}$  in this range. Certainly  $X_{ij}$  is feasible for  $T_{ji} = 1$ . But we know that  $K_{ij}$  and its associated vector is maximum for this set, in particular that  $i$ 's utility is greater with  $K_{ij}$  than  $X_{ij}$ . But this must then also be true for  $T_{ij}$  for which both  $K_{ij}$  and  $X_{ij}$  are both feasible. So  $D_{ij}(T_i) = K_{ij}$ .

*Proposition 2:*  $D_{ij}(T_i) = T_{ji}$  for  $T_{ji} \leq K_{ij}$ .

*Proof.* Let  $Y_i = D_i(T_i)$  but  $Y_{ij} < T_{ji}$  for some  $T_{ji} \leq K_{ij}$ . Since the allocation associated with  $K_{ij}$ , call it  $A_i$ , is a maximum for  $T_{ji} = 1$ , then  $A_i$  must be preferred to  $Y_i$ . By strict quasi concavity we know that any vector in between  $Y_i$  and  $A_i$  is strictly preferred to  $Y_i$ . Since  $Y_i < T_{ji}$  we can find an  $a$  such that  $0 < a < 1$  and  $X_i = aY_i + (1 - a)A_i$  is feasible for  $T_{ji}$ . But then  $Y_i$  cannot be a maximum for this set of feasible allocations, since  $X_i$  is feasible in this set and is also preferred to  $Y_i$ . Thus  $D_{ij}(T_i) = T_{ji}$ .

We also need to show that an equilibrium is Pareto optimal if the relation  $C$  associated with it is acyclic.

*Theorem:* If the relation  $C$  associated with an equilibrium is acyclic, then the equilibrium is Pareto optimal.

*Proof.* If  $C$  is acyclic there will be a set of people  $Y_0$  who are not controlled by anyone: the people at the top of the hierarchy. For these people the equilibrium allocation  $X_i$  represents a global maximum. By quasi concavity this maximum must be unique. Thus there is no other allocation these people would prefer or be indifferent to relative to  $X$ . Then for  $i$  an element  $Y_0$ ,  $X_{ij} = X_{ji}$  for all  $j$ . Now consider the set of all the other people  $(N - Y_0)$ . We have already determined how they have allocated their time to the people in  $Y_0$ . Consider the set of people who are constrained only by those people in  $Y_0$ . Call these people  $Y_1$ . In terms of how these people allocate the rest of their time to those not in  $Y_0$  they are at a global (unconstrained) maximum. By quasi concavity the maximum will be unique. Thus for  $i$  an element of  $Y_1$ ,  $X_{ij} = X_{ji}$  for all  $j$ . We can continue with this procedure by choosing  $Y_2$ , then  $Y_3$ , and so on until we have included everyone in the group. Thus  $X$  must be Pareto optimal.

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