Social Networks 10 (1988) 209–231
North-Holland

THOUGHTS ABOUT ROLES AND RELATIONS: AN OLD DOCUMENT REVISITED

Christopher WINSHIP

Northwestern University *

Historical overview

"Thoughts about roles and relations – Part I: Theoretical considera tions", was written in the fall of 1974 at the beginning of my secon year of graduate study at Harvard. It was a memorandum to Harriso White's research group on blockmodel analysis. "Part II: Methodologi cal considerations" was never written. The major purpose of the mem was to outline a theory of roles that would allow a researcher not onl to identify individuals who where in the same roles in the sam population but individuals who were in similar roles in differen populations.

The main ideas in "Thoughts about roles and relations" were no followed up until Mike Mandel and I started working together in 1977 At the time Mike was an undergraduate at Harvard. This collaboratio resulted in Mike writing an undergraduate senior honors thesis (Mande 1978) that not only extended the theory, but carried out extensiv empirical analyses.

There were, however, considerable personal obstacles to bringing ou work to publication. Mike had decided to pursue a graduate degree i economics at Harvard. I had moved on to post-docs at the Universit of Wisconsin and the University of Chicago, and my interests ha moved away from social networks to social stratification and econo metric modelling. It was not until 1983 that we published two paper (Winship and Mandel 1983; Mandel 1983) that presented most of th core ideas in my original memorandum and our later unpublishe work. By that time though, the pull of different intellectual agenda

* Department of Sociology, Northwestern University, Evanston, IL 60201, U.S.A.

meant that neither of us further pursued research on social networks.

At least as early as Sailer's 1978 article, individuals outside of Harrison White's group had begun to question the robustness of the concept of roles as sets of structurally equivalent individuals. In 1982 I became aware that Doug White quite independently was pursuing a line of analysis similar to that of mine and Mandel's (White 1982; White and Reitz 1983). In fact, the awareness of Doug's work did much to motivate Mandel and me to finish our two papers and submit them for publication. Since that time a number of researchers have worked on a theory of roles that would allow a researcher not only to identify individuals in the same roles in a single population, but individuals who were in similar roles in different populations (Wu 1983; Everett 1985; Breiger and Pattison 1986; Boyd and Everett 1987).

It is, perhaps, worth briefly discussing why I did not pursue at the time it was written the line of research outlined in "Thoughts about roles and relations". It is a somewhat interesting story for the sociology of science.

At the time I wrote my memo the only published paper on blockmodels was Lorrain and White's 1971 paper "Structural equivalence of individuals in social networks". As a close reading of that article will reveal what Lorrain and White were trying to do was to generalize the concept of structural equivalence. They were analyzing homomorphic reductions of graphs as a possible way of arriving at an underlying structure for a network. What they wanted to do was "fold" networks onto themselves (also see Lorrain 1972).

The Lorrain and White approach led to the development of the "blocker" algorithm (Heil and White 1976) and the attempt to identify blocks as homomorphic reductions using the zero block criteria. The problem with blocker was that it often provided many solutions, and data were frequently consistent with several different blockmodels. In short, it was an unsatisfactory data analytic tool.

In the fall of 1973 I began graduate school in sociology at Harvard. Harrison White was visiting the University of Edinburgh and Ron Breiger was busy trying out different ways of analyzing network data into blockmodels. I had already started developing the ideas for "Thoughts about roles and relations".

I believe that it was during the winter of 1974 that Ron discovered that if you did correlations of correlation matrices the result would originally converge into a matrix of one and minus one correlations creating a dichotomous partition of the original variables. By stackir the row and column vectors from relational matrices into variable Breiger found he could generate nice blockmodels. Thus the "Conco algorithm was invented. (The algorithm had actually been invent previously by McQuitty. See McQuitty (1968), and McQuitty ar Clark (1968).)

The invention of Concor pushed blockmodeling in a direction diffe ent than that found in Lorrain and White. Individuals were now p together not because of some abstract similarity in their positions, b because they shared many of the same ties with the same individual Structural equivalence became generalized in a statistical instead homomorphic sense.

Concor resulted in a lot of very good and important work (e.g Breiger *et al.* 1975; White *et al.* 1976; Breiger 1976, 1979). Wh Harrison White came back from Scotland in the fall of 1974, he w presented with this important new tool called Concur, and some ide of mine suggesting that the concept of role as structurally equivale individuals was too narrow. With Concor one could analyze real da and get interesting results. In fact, Breiger had already analyzed number of data sets. My ideas only promised empirical results son time in the future. Thus Concor and the generalization of structur equivalence implicit within it took center stage.

By the late 1970s individuals in Harrison White's group started become interested in the ideas in "Thoughts about roles and relation Philippa Pattison and I did some work together that was never writt up (though see Pattison 1982), and Mandel completed his dissertatio It was, however, too late, at least for Mandel and me, since we h both moved on to pursue research in other areas. It was a struggle write our two papers (Winship and Mandel 1983; Mandel 1983).

Over the years I have occasionally received requests for the origin "Thoughts about roles and relations" paper and I have been pleas that a few individuals have cited my obscure and almost lost docume Thus, from time to time, it had occurred to me that there might some historical interest in having the paper published. I discussed th recently with Malcolm Dow and Martin Everett and with their e couragement I sent it to Lin Freeman and asked if he had any intere as editor, in publishing the paper in *Social Networks*. He said that would, and I am grateful to him for his generous offer.

What follows as the main text of this paper is the original "Thoughts ›out roles and relations" memorandum. I have done some minor :iting in order to correct spelling and grammar and in a few places to ırify meaning. Basically, however, it is the original document.

It is wonderful to have the opportunity to publish an old paper that ıs virtually buried and forgotten. I can no longer claim to be current th the literature in social networks. My sense, though, is that research the field has gone considerably beyond the original ideas in my ɜmorandum. Despite this, my hope is that people will at least find the ›cument of historical interest.

ıoughts about roles and relations, Fall 1974. Part I: Theoretical nsiderations

the past year I have spent a lot of time thinking about roles and lations along the lines of White and his coworkers. My efforts to date ve been to attempt to extend the ideas and methods of White's group a way which would provide for a richer analysis of sociometric ucture. The purpose of this paper is to describe some of the thoughts at have been born of this effort and to examine new directions for search.

Before heading off into the land of mathematical abstraction, I ›uld like to state three of the issues that are a motivating force in this ›rk and to discuss the concept of a "role". All of this will sound like niliar jargon to many of you, especially those in the White group. ıe three issues are:

(1) The need for precise language, perhaps mathematical, in which e is able to give concrete description of the social structure of groups d in which one is able to pose theoretical issues.

(2) The desire to explain how it is that people are in new situations ɜryday, that they have never been in before, yet in which they know w to act (an idea of Chomsky's that the ethnomethodologists have isted into a sociological context).

(3) A desire to gain an explicit understanding of the duality that ɜms to exist between roles and relations: i.e. the fact that if we know : roles people have within a group we should be able to derive the ationships which exist between them; conversely, the relationships

which exist in a group should determine the roles that the people hav
within the group.

These three issues are the motivating force in my attempt to build
"calculus of roles". Before building such a calculus, we need to ex
amine the concept of a role.

I will differentiate between two types of roles and two ways o
looking at a role. My intuition tells me that two people are in the sam
role if the position they maintain in a social structure is identical or a
least similar in some sense. Clearly two people are in the same role
they are in the same relation to the same people. This is precisely wha
Lorrain and White (1971) mean by two individuals being structurall
equivalent. When we think of roles in this sense they are tied into th
relational structure of a group in a very concrete way. They not onl
specify what types of relationships a person has, but also the specif
people that he has those relationships with. I will refer to roles of th
type as "concrete roles". Concrete roles are very important in socia
structure. In fact, it may be reasonable to conjecture that they are th
primitive building blocks of social structure. When we think of some
one being an American, a Republican or a Harvard student, these ar
all examples of concrete roles.

Not all roles are of a concrete nature. In fact, many roles are not of
concrete nature. A role may specify that a person has certain types o
relations, but not with whom a person has those roles. Thus we ma
think of a person as a citizen, a party member or a student. We will ca
roles of this type "abstract roles". The difference between concre
roles and abstract roles is that the former are concerned with th
specific relations that one has with specific people, whereas the latte
are concerned only with the specific types of relations one has. Peop
who have the same concrete roles have identical positions within
social structure, whereas people who have the same abstract roles ar
people who have "similar" positions within the social structure. Thu
we have the difference between two individuals who are students at th
same school and two individuals who are students at two differen
schools.

The above example of students is illustrative of another distinctio
that we need to make. If I am a student at a school I may recogni
that a student from my school and a student at another school hav
from an "objective" point of view similar roles. However, from my ow

ubjective" point of view they have very different roles. I may see one my ally, a classmate or whatever, and the other as a foe, or a ·anger. We need to make a distinction between looking at roles from . objective versus subjective points of view. The fact that two people ɐ subjectively equivalent is stronger than the fact that they are ›jectively equivalent. When we ask whether two people are equivalent ›m a subjective point of view, we need not only ask whether their ›sitions in a social structure are similar, but whether the person whose ›int of view is being taken "plugs into" their social structure in the ame sort of way". We may have two brothers, for instance. The son one will see one brother as a father, and the other as uncle.

athematical considerations

y discussion above has been vague and imprecise. At this point I will ırt to develop a mathematical framework within which to understand me of these ideas. I have already mentioned the Lorrain–White ncept of structural equivalence as being related to the concept of a ncrete role. For the moment I will deal with the concept of structural uivalence in its strictest sense. I will examine extensions of this idea the end of part I. We define the concept:

:finition. Two people, a and b, are structurally equivalent if and only for each relationship R, person a is in relationship R to a person c, if d only if person b is in relation R to c.

$:c \leftrightarrow bRc$ for all R and c.

Our discussion above of concrete and abstract roles suggests that the ncept of structural equivalence is appropriate for concrete roles, but ıt it is inadequate for the abstract roles. Figure 1 illustrates this ıdequacy. Each person in the group clearly has the same abstract .e, but no one is structurally equivalent to any other person in the ›up.

It is clear from its definition that structural equivalence is based on ving identical relationships with *exactly the same people*.

The mathematical concept of an automorphism will be of some help overcoming this problem. An automorphism of a graph is a one to

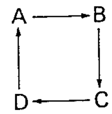


Fig. 1. Nonstructurally equivalent individuals who have similar roles.

one onto mapping of a graph into itself that preserves adjacency. More precisely:

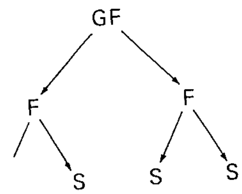
Definition. An automorphism, $f(\cdot)$, is a one to one onto mapping of a graph into itself such that

aRb if and only if $f(a)Rf(b)$.

This is identical to saying that there is a permutation of the matrix of the graph such that the original matrix and the permuted matrix are equivalent.

If we reconsider Figure 1 we will notice that there is an automorphism which equates (i.e. $f(i)=j$) each node with each other node. To equate any two nodes we need only rotate the graph by either 90, 180 or 270 degrees. We will speak of any two people who can be identified through an automorphism as being automorphically equivalent. Two people that are automorphically equivalent are structurally equivalent except that we have had to relabel all the points with the appropriate names. The idea of automorphic equivalence is closely related to what I have meant by an abstract role. Two people that are automorphically equivalent have the same relations in their social structure, though these relations may be with different people. Thus their positions in the social structure are similar, though not identical. To go back to some concrete examples, we might think of two quarterbacks on opposing football teams. Clearly they are not structurally equivalent as they have very different types of relationships with different people. They are however, automorphically equivalent. By interchanging the two quarterbacks, and each of the respective members of the opposing team, the structure of the groups is clearly preserved.

In my discussion of roles I made a distinction between considering roles from an objective point of view and a subjective point of view. Two people are in objectively equivalent roles if they are automorphically equivalent, but when are they subjectively equivalent? If we go



ig. 2. Male lineage hierarchy.

ack to our quarterbacks and consider things from the point of view of he umpire we can see that the umpire considers them to be in quivalent roles with respect to him. Thus, if we are interchanging the espective members of each of the football teams, the umpire can still emain the same umpire.

We can make the above discussion a little more precise by making he following:

efinition. Two people, a and b, are automorphically equivalent with espect to point c if and only if there exists an automorphism such that $(a) = b$ and $f(c) = c$.

he above mathematical characterization is exactly what we have eant by two people being in the same role with respect to another erson. It should be clear, however, that two people may be equivalent ith respect to one person but not with respect to one and another. The ollowing oversimplified example of an "ideal" American male lineage rves to elucidate this point.

The grandfather sees both fathers as being equivalent and all of the ns. Both fathers see each of their own sons as being equivalent, but e the other two sons as nephews. Each of the sons sees the two thers differently – one as a father and one as an uncle. They see their ncle's sons as being equivalent with each other, but not with their rothers.

At this point structural equivalence has been lost from sight. There is very simple relationship between structural equivalence and automor- hic equivalence at a point.

roposition. Two people are structurally equivalent if and only if they e automorphically equivalent with respect to every person in the oup.

his is a reassuring result. In sociological terms it states that if two people are seen as being equivalent in the subjective structure of eac person in the group, then they must also be equivalent in the concret structure.

At this point I have developed some ideas for thinking about role However, I have not yet developed a way of characterizing sociometri structure, nor have I classified what I mean by a relation. We turn t this task now.

Characterization of sociometric structure

The problem of how to characterize sociometric structure is ver important and delicate. In the literature there are essentially tw different ways of looking at sociometric structure: one way represente by the work of Davis, Holland and Leinhardt (see papers by thes authors in Leinhardt 1977), and the other represented by the work c White and his group. The approach of the former group is to conside sociometric structure as consisting of various configurations. Confi urations are isomorphic classes of subgraphs, and the various sets c relations within them. Davis, Holland and Leinhardt have principall been concerned with the analysis of triads and triplets. It would seer reasonable to think of trying to characterize roles as consisting c various configurations. Thus we might think of a role as consisting c various triad types. For example, a leader might be a member of triad in which he only chose people that choose him.

This approach has some inherent difficulties. There is strong ev dence to suggest that there is no configuration of a fixed size that i sufficient to determine whether two people are automorphically equi alent in an arbitrarily large graph. Without going into details, th evidence consists in the fact that there is a one to one correspondenc between the size of a configuration and some n-dimensional binar matrix with fixed marginals. The configurations are sufficient to d termine whether two people are automorphically equivalent if and onl if the marginals of the appropriate dimension matrix are sufficient t determine the matrix. Since there seems to be no hope that there is an n such that any binary matrix of that dimension would be determine by its marginals, this approach seems to have little hope of being ver powerful [1].

[1] At one time I had constructed some examples illustrating this point, but they are now lost.

White's group has thought of relations in terms of paths. We think of graph as consisting of two relations, P, the raw matrix of data, and ^{t}, the transpose of the raw matrix. Other relationships are created by ıultiplying the matrices by themselves and by each other. This multilication can be carried out in two ways – by doing ordinary multipliıtion and binary multiplication. When we do regular multiplication, ıe i-j entry in matrix P^n can be interpreted as the number of paths of ngth n that exist between i and j. When we do binary multiplication zero in the i-j cell means that there does not exist a path of length n ętween i and j. If there is a one in the i-j cell this implies that there is : least one path of length n between i and j. By combining the ırious "primitive" relationships we arrive at different compound :lationships between the people in a group. Using regular matrix .ultiplication gives us a characterization that puts emphasis on the ıantitative differences in relations that exist between people whereas .nary multiplication only puts emphasis on the qualitative differences ıat exist between people.

At this point I have been unable to determine whether the above ıaracterization of relations and sociometric structure is appropriate ır implementing our ideas concerning roles and automorphism. I think .ere is some worth in examining what is involved in this approach. he rest of this paper does this.

he relation box

ɔr the present I want to think about relations and their compounds in rms of regular matrix multiplication. Thus I am interested in quantitive rather than just the qualitative aspects of relationships. Using gular matrix multiplication we generate all the compound relationips that exist between the various relationships. Since we are using gular matrix multiplication, this will generate an infinite, but countae number of matrices. For the moment we will ignore any problems :nerated by this. Now we will line each of these matrices up behind e other, into an infinitely long rectangle. The first n matrices are presented in Figure 3 as the vertical sheets of the box. What we are terested in are the horizontal planes of this rectangle and the columns :thin these planes. Each plane can be associated with one individual the group. A plane represents all the various relationships that a

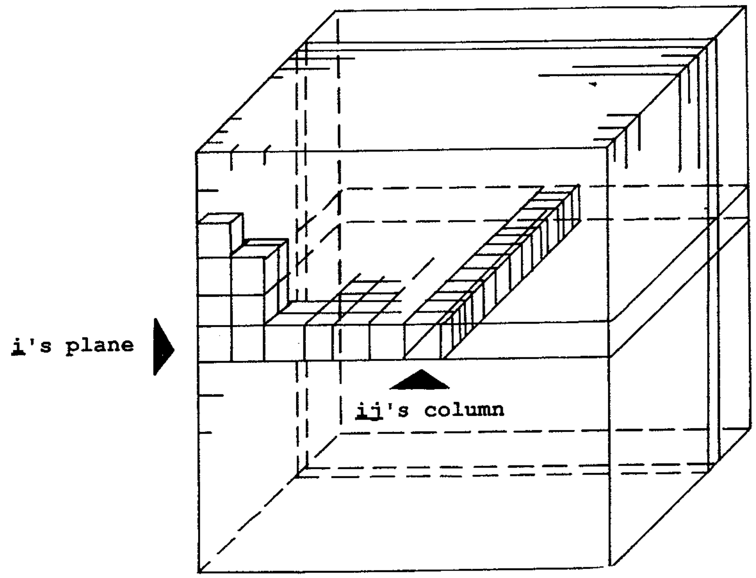


Fig. 3. Box of relations.

person has with the people in a group. Any column can be associated with two specific people. A column represents the relationships a person *i* (whose plane the column is a member of) has with person *j*. Thus a plane represents the relational structure that a person is part of from his point of view, and a column represents the relationships that exist between two people. We will say that two people are congruent or that they have congruent relational structures if their planes are equal to within a permutation of their columns.

At this point I would like to tie the above ideas in with the previous discussion of roles and automorphisms. Unfortunately at this time I can only hypothesize about the interrelationship between these two sets of ideas.

In my discussion of roles and automorphisms, I examined both concepts from two points of view. I described roles from an objective and subjective point of view, and analogously I discussed automorphisms in terms of two people being automatically equivalent, and in terms of them being automorphically equivalent with respect to another person. This duality is also found in our discussion of planes and columns. If a plane truly describes a person's relational structure, it would seem reasonable to hypothesize that two people who have planes that are equivalent to within a permutation of their columns are automorphically equivalent. If two people are not automorphically equivalent then their positions in the relational structure are different and this should be reflected in their planes. This is our first conjecture.

Conjecture 1. Two people are automorphically equivalent if and only if they are congruent.

It would also seem reasonable to think that if a column describes the relationships between two people in an appropriate way that two people that have equivalent columns with respect to another person should be automorphically equivalent with respect to that person.

Conjecture 2. If column *ij* equals column *ik* then *j* and *k* are automorphically equivalent with respect to *i*.

The above two conjectures lead to the obvious corollary:

Corollary. If two people have columns with respect to another person that are equivalent, then they are congruent.

The above conjectures are very powerful ones, and if true would indicate that the compound relationships approach is quite appropriate for the study of roles and automorphisms. For the moment attempts are being made to find either a proof or a counterexample [2].

Roles and automorphism reconsidered

From the above discussion of automorphisms as one to one onto mappings and our definition of congruence in terms of regular multiplication, it is clear that the concept of an automorphism is directly connected to not only the qualitative aspects of a structure, but also to the quantitative. It is not at all clear that roles depend on the quantitative aspects of structure. Thus whether someone is a father or not is not determined by whether he has two sons or three, or analogously whether he is a leader or not is not determined by whether he has ten

I no longer believe that these conjectures are true.

followers or forty. This seems like a major inadequacy of my treatment of roles in terms of automorphic equivalence.

Fortunately our discussion of congruence and its hypothesized relationship with automorphism suggests a natural way out of this dilemma. One of Lorrain and White's (1971) ideas was that matrix multiplication should be done in a binary manner. This has a number of advantages in the present situation. First, it guarantees that there will be only a finite number of compound relational matrices. (There are arguments why even in the case of regular multiplication only a finite number of matrices need be considered.) Second, binary multiplication provides a shift from the quantitative considerations of congruence, to purely qualitative considerations. This can be shown best by defining the concept of similarity.

Definition. Two people are similar if and only if for each column in their respective binary planes there exists an equivalent column in the other's plane.

Note that the mapping we have defined above is not one–one or onto. Two people may have differing numbers of columns of the same type. It is only essential that they have one column of each type that the other person has. The following modified "ideal" American male lineage kinship example illustrates why the concept of similarity is closer to the concept of what we mean by role than that of automorphism.

In the above example, both fathers are similar, although they are certainly not structurally equivalent or automorphically equivalent. Table 2 contains the first ten powers of their planes indicating their

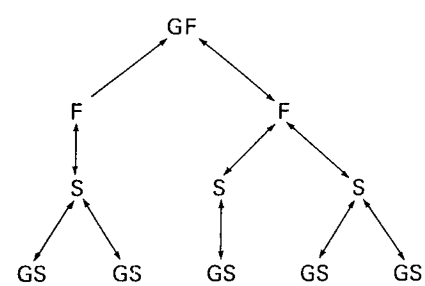


Fig. 4. Modified male lineage hierarchy.

Table 1

Raw data for Figure 4

	GF	F	F	S	S	S	GS	GS	GS	GS	GS
GF	–	1	1	–	–	–	–	–	–	–	–
F	1	–	–	1	–	–	–	–	–	–	–
F	1	–	–	–	1	1	–	–	–	–	–
S	–	1	–	–	–	–	1	1	–	–	–
S	–	–	1	–	–	–	–	–	1	–	–
S	–	–	1	–	–	–	–	–	–	1	1
GS	–	–	–	1	–	–	–	–	–	–	–
GS	–	–	–	1	–	–	–	–	–	–	–
GS	–	–	–	–	1	–	–	–	–	–	–
GS	–	–	–	–	–	1	–	–	–	–	–
GS	–	–	–	–	–	1	–	–	–	–	–

similarity. Put in another way, one could say they have the same type of relationships with people although they may not have the same number of each type of relationship. Similarity is what we mean by two people being qualitatively in the same type of situation in the social structure.

At this point we have defined similarity in terms of an objective view of a role. One would hope that the natural counterpart to the case in congruence with respect to a subjective point of view would hold here. We hypothesize:

Conjecture 3. If the binary columns of j, k with respect to i are equal, then j and k are similar.

Table 3 represents the first n elements for the plane of the grandfather, and indicates that he sees the fathers, sons, and grandsons, each being respectively similar to the others.

It would be nice to relate the idea of similarity back to automorphisms. The only way I can think of to do this is by means of the following conjecture:

Conjecture 4. If two people a, b are similar then there exists a family of isomorphic subgraphs, the union of which equals the graph. In addition there is a set of mappings which map the graph into each subgraph, such that if $f_i(a)$ is automorphically equivalent to $f_i(b)_i$ then $f_i(a)$, $f_i(b)$, a, and b are all similar to each other.

Table 2

First father's plane											
	F	E	B	S	N	N	GS	GS	GN	GN	GN
R	1	–	–	1	–	–	–	–	–	–	–
R2	–	1	1	–	–	–	1	1	–	–	–
R3	1	–	–	1	1	1	–	–	–	–	–
R4	–	1	1	–	–	–	1	1	1	1	1
R5	1	–	–	1	1	1	–	–	–	–	–
R6	–	1	1	–	–	–	1	1	1	1	1
R7	1	–	–	1	1	1	–	–	–	–	–
R8	–	1	1	–	–	–	1	1	1	1	1
R9	1	–	–	1	1	1	–	–	–	–	–
R10	–	1	1	–	–	–	1	1	1	1	1
R11	1	–	–	1	1	1	–	–	–	–	–
Second father's plane											
	F	B	E	N	S	S	GN	GN	GS	GS	GS
R	1	–	–	–	1	1	–	–	–	–	–
R2	–	1	1	–	–	–	–	–	1	1	1
R3	1	–	–	1	1	1	–	–	–	–	–
R4	–	1	1	–	–	–	1	1	1	1	1
R5	1	–	–	1	1	1	–	–	–	–	–
R6	–	1	1	–	–	–	1	1	1	1	1
R7	1	–	–	1	1	1	–	–	–	–	–
R8	–	1	1	–	–	–	1	1	1	1	1
R9	1	–	–	1	1	1	–	–	–	–	–
R10	–	1	1	–	–	–	1	1	1	1	1
R11	1	–	–	1	1	1	–	–	–	–	–

E = Ego F = Father B = Brother S = Son N = Nephew GS = Grandson GN = Grandnephew

The above conjecture is awkward to say the least, but it does captu the fact that when two people are similar it means that there is som underlying structure from which the graph has been built within whic the two people are automorphically equivalent.

Another issue we have examined in the past is the relationshi between abstract structure and concrete structure. The notion of sim larity seems like a relatively weak way of looking at abstract structur Still the relationship between abstract and concrete that existed f automorphism exists here also.

Proposition. If two people are similar with respect to every person in group, then they are structurally equivalent.

ıble 3

:andfather's plane

	E	S	S	GS	GS	GS	GG	GG	GG	GG	GG
	–	1	1	–	–	–	–	–	–	–	–
[illegible]	1	–	–	1	1	1	–	–	–	–	–
[illegible]	–	1	1	–	–	–	1	1	1	1	1
[illegible]	1	–	–	1	1	1	–	–	–	–	–
[illegible]	–	1	1	–	–	–	1	1	1	1	1
[illegible]	1	–	–	1	1	1	–	–	–	–	–
[illegible]	–	1	1	–	–	–	1	1	1	1	1
[illegible]	1	–	–	1	1	1	–	–	–	–	–
[illegible]	–	1	1	–	–	–	1	1	1	1	1
.0	1	–	–	1	1	1	–	–	–	–	–
.1	–	–	–	–	–	–	1	1	1	1	1

= Ego F = Father B = Brother S = Son GS = Grandson GG = Greatgrandson
ıce I have drawn the graph symmetrically, ego and grandsons are indistinguishable.

ıis allows us to again conclude that what abstract structure is doing is lowing us, or perhaps better, people within a group, to see similarities here others see differences. When there is no difference in any of the ›stract structures then there can be no differences in the concrete ructure.

ow are we to think of roles?

aving shown that the concept of automorphism is too strong an idea r what we mean by a role, we should consider the adequacy of the ea of similarity. I think it is clear that this is too strong an idea also. ıplicit in the discussion so far has been the idea that a role is sociated with a single type of position in a social structure and that ıy position in the social structure is descriptive of only one role. In y discussions it has been implicit that there is only one sort of role at exists between two different people. It is certainly clear that people ay assume more than one role in their social structures, and that they ay have more than one role with respect to each other. Thus a person ay be a doctor, a parent, and a friend. Or two people may be both ıssmates, roommates, and friends.

How should we define a role? If we are thinking about things from a bjective point of view it seems that we should most simply define a role as a set of specific relationships that exist between two people. Fo: objective roles it is clear that this is just a specific set of subjectiv roles. One must conclude that this is a surprisingly simple definition o what one means by a role. Given that we have arrived at such a simpl conception, why have we gone through such elaborate pains to ge here? The point of the paper is not just to suggest that roles can b defined analytically, but that conceptualizing roles in this manne points to a deep way of thinking about social structure. I have tried t argue that as relations add up into roles, roles can be added up int similarities, which in turn can be looked at in terms of congruences automorphisms, and structural equivalence. What I am trying to sug gest is that the above discussion reveals that we have a simple tool tha is capable of pointing out similarities in social structure when desired yet is also able to make distinctions when they are important. The rea proof will come when we analyze real social structures in part II.

Algebra

Most of the work in kinship studies has emphasized the importance o looking at things algebraically. Up until this point I have adopted a algebraic method to examine social structures at a very specific level the patterns of relationship that exist between two people. It would b of some use to examine whether we could use an algebraic method t analyze social relations at a higher level of aggregation. A first step a aggregation would be to consider the properties that are common to al the relationships a single person has with the other people in a group A second and final level of aggregation would be to consider propertie that are common to all relationships in a group. It is from this latte level of aggregation that most algebraic considerations of sociometri structure have been made, most prevalently in kinship studies. Typi cally one performs binary multiplication, and is interested in whethe different matrices are equal to each other or whether they are subsets o each other. For instance, White (1963) in *An Anatomy of Kinship* ha shown that a bilateral kinship system is defined by the conditio $W^2 = I$ and $WC = CW$. These two statements imply for *everyone* in th group that (1) one's categorical wife's categorical's brother's categorica wife is one's categorical sister, and (2) my categorical wife's brother' children must be married by my categorical children. In sociometr ıere has been much concern with the transitivity of positive sentiment. ılgebraically this can be represented as $P^2 = P$. This can be interpre-:d as saying that whenever someone is a friend of a friend, then he is lso a friend.

It seems wise to consider first properties that are true of all relation-ıips of a single individual's relations, before trying to discuss the roperties of all relations. We can do this by thinking of each individ-al as having his own semigroup.

The semigroup is generated by multiplying the primitive relational ıatrices in the usual binary fashion, but the equating the elements 'hose rows are equal for the individual under consideration. Clearly ıen the semigroup of the individual will just be a homomorphic :duction of the semigroup of the relational matrices. The semigroup is s required, associate and closed. Multiplication however is defined nly with respect to left-hand substitution. That is, if $A = B$ then $C = BC$, but it is *not* the case that $CA = CB$ (right-hand substitution). ' we are going to use this "algebra of rows", it makes sense to define ', U, I as the appropriate rows of the zero matrix, the one matrix and ıe identity matrix. For the present I will give some simple illustrations f how these equations might be used to think of roles. For instance if is a matrix of positive sentiment we might want to think of a leader s someone for whom $P \subset P^t$ is true. That is he likes only people that ke him. An isolate is clearly someone for whom $P = P^t = Z$. Finally e might want to think of a "hanger-on" as $P \neq P^t = Z$.

In thinking of equality of relations for a person we need only to ıink in terms of the sets of relations that are equal. The subset relation a bit more complicated. We need to think of there being a set of :lations associated with each set such that each relation within the set a superset of that relation. We will call this the superset of a relation.

The question that needs to be answered at this point is what does all ıis consideration of semigroups have to do with roles. For the moment t us consider the case where $P = P^2$. This tells us that any role that ›ntains either P or P^2 as one of its defining relations must contain the :her. In a sense then this forms an element of a basis for constructing ıy role out of those relations. If we instead have the case where $^2 \subset P$, then we only know that any role that contains the relation P^2 ıust also contain the relation P. Thus the equality sets of a role and ıe supersets of a role tell us which relations *must* be included in a role hen others already have been included.

If we are willing to make some stronger assumptions about tl matrices then we can derive some more interesting results. First let ı look at the case where we have two or more people that have identic semigroups. Then we can think of the sets of equivalent relations a representing a minimal set of roles that is part of the relation structure of each person.

Another interesting case to look at is when the elements of tl semigroup are disjoint, or equivalently rows of a person's plane a either equal or disjoint. First we need to conclude that if the rows a either equal or disjoint, then the columns of the plane are also eith equal or disjoint. Assume that this is not the case. Take two columı that are neither disjoint nor equal. Then there is some relation fo which they are both 1 and some relation for which one is 1 and tl other is 0. But then these two relations are neither equal nor disjoi which contradicts our assumption. Given this result we then know th the algebra of the group or the algebra of the subset of the grou represents a set of maximal set of roles such that each person's ro structure is a subset of this structure.

The above two results imply the following lemma which is of son interest.

Lemma. Given two individual semigroups for a, b, such that the el ments of the semigroup are disjoint, if their semigroups are identic then the two individuals are similar.

From the above discussion it is arguable that similarity is tl concept that it really makes sense to talk about in algebraic terms. Th is because algebraic considerations are blind to differences of quanti (since we are using binary multiplication) and to differences of orie tation (something that structural equivalence is sensitive to). Tl surprising result though is that algebraic considerations only seem really pin a structure down when we have disjoint elements in tl semigroup. This also leads to the observation that algebraic consider tions only classify structure when we have disjoint roles, i.e. in cas where we are thinking of roles as either columns or subsets of columı

ther notions of structural equivalence

the preceding discussion of structural equivalence I mentioned that uctural equivalence was being used in its strongest sense. The idea of uctural equivalence has been used in other less restrictive ways. It is ly fair that I attempt to compare these methods with the ones that ve been developed above.

The principal idea of Lorrain and White (1971) is that one should entify a number of various elements of a semigroup, then look for ucturally equivalent individuals in the graph that has been formed taking the union of the various relations. I think the above discuson of algebra and similarities suggests one approach that should make me sense.

If we could partition a semigroup into sets of elements such that the ments between sets were disjoint, then it would make sense to entify all the elements within sets. This type of disjointness is present the kinship examples where the White–Lorrain technique has been ore successful, although I admit I have not done any sort of thorough amination of the matter. It should be pointed out though that this pe of disjointness only implies similarity of the people, which is a ich weaker idea than structural equivalence. Certainly our analysis n not yet give any comprehensive evaluation of applicability of rrain and White's hypothesis, although I think the above analysis ggests that it may be applicable under fairly special conditions.

The other approach that has been considered for structural equivnce is the lean fit idea. In this approach two people are structurally uivalent if there is a set of individuals such that they both are not ated to these individuals. The idea in this approach is that people o are in similar roles are structurally equivalent by this weaker inition. For instance, if we have two hierarchies within a group then could identify each of the two leaders and the appropriate subdinates all the way down the hierarchies. Figure 5 illustrates this idea.

There are a number of problems with this type of approach. First re are a whole class of models that are resistant to this type of proach. These are models that include cycles. For instance, if we go ck to Figure 1, each person is automorphically equivalent, but there no identification which will provide the appropriate sort of mapping. cond, this approach is dependent on there being a lack of relations in y crucial places. For instance, in the above example, if there was

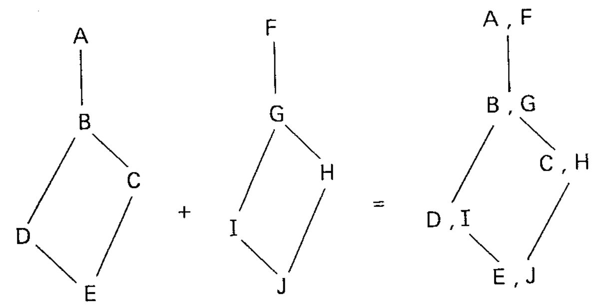


Fig. 5. Identification of two cliques.

animosity between the two cliques then in the reduced picture \ would have a single hierarchy in which there is animosity betwe everyone. Admittedly if we have the ability to know which relationshi to ignore and which to consider in certain cases this approach mig work, but this seems more like guess work than anything else. The fin problem with this approach is, that it is sensitive to small differences the structure. For instance, if one hierarchy had one more level th: another then this type of identification can not work.

Conclusion

"Part II: Methodological considerations" has for the most part be thought out, and awaits for me to find time to do some examples analysis of data and to write it up. I hope that I have at least suggest if not convinced you of the deep richness of approach that has be discussed in this paper. Personally I feel that what has been discussed only the surface, and that there are many more deeper and rich developments to come from this approach. Finally I should say th even though this work departs in many ways from the work of Whi and his associates, it should be clear that many of my ideas are heavi indebted to theirs.

Acknowledgements

I want to thank Malcolm Dow for suggestions on the Historic Overview section, Nancy Kolack Winship for comments on the curre ıd earlier manuscripts, and Richard Ries for typing the manuscript. f course any mistakes that remain are alone my responsibility.

eferences

yd, J.P. and M.G. Everett
1987 "Block structures of automorphism groups of social relations". *Social Networks*, forthcoming.

ꞩiger, R.L.
1976 "Career attributes and network structure: A blockmodel study of a biomedical research specialty". *American Sociological Review 41*: 117–135.
1979 "Toward an operational theory of community elite structures". *Quality and Quantity 13*: 21–57.

ꞩiger, R.L., S.A. Boorman and P. Arbie
1975 "An algorithm for clustering relational data with applications to social network analysis and comparison with multidimensional scaling". *Journal of Mathematical Psychology 12*: 328–383.

ꞩiger, R.L. and P. Pattison
1986 "Cumulated social roles: The duality of persons and their algebras". *Social Networks 8*: 215–56.

ꞩrett, M.G.
1985 "Role similarity and complexity in social networks". *Social Networks 7*: 353–59.

il, G.H. and H.C. White
1976 "An algorithm for finding simultaneous homomorphic correspondances between graphs and their image graphs". *Behavioral Science 21*: 26–35.

ınson, S.C.
1967 "Hierarchical clustering schemes". *Psychometrica 32*: 241–254.

nhardt, Samuel
1977 *Social Networks: A Developing Paradigm*. New York: Academic Press.

rain, F.P.
1972 *Social Research and Social Classification*. Ph.D. Dissertation. Harvard University.

rain, F.P. and H.C. White
1971 "Structural equivalence of individuals in social networks". *Journal of Mathematical Sociology 1*: 49–80.

ndel, M.J.
1978 *Roles and Networks: a Local Approach*. Senior honors thesis. Harvard University.
1983 "Local roles and social networks". *Americal Sociological Review 48*: 376–385.

Quitty, L.L.
1968 "Multiple clusters, types, and dimensions from interactive intercolumnar correlation analysis". *Multivariate Behavioral Research 3*: 465–477.

Quitty, L.L. and J.A. Clark
1968 "Clusters from interactive, intercolumnar correlation analysis". *Educational and Psychological Measurement 28*: 211–238.

er, L.D.
1978 "Structural equivalence: Meaning and definition, computation and application". *Social Networks 1*: 73–90.

ite, D.R.
1982 "Rethinking the role concept: Homomorphisms on social networks", in: L. Freeman, D. White and A.K. Romney, *Research Methods in Social Network Analysis*.

White, D.R. and K. Reitz
1983 "Graphs and semigroup homomorphisms on networks of relations". *Social Networks 5* 143–234.
White, Harrison
1963 *An Anatomy of Kinship*. Englewood Cliffs, N.J.: Prentice-Hall.
White, H.C., S.A. Boorman and R.L. Breiger
1976 "Social structure from multiple networks. I. Blockmodels of roles and positions" *American Journal of Sociology 81*: 730–780.
Winship, C. and Mandel, M.
1983 "Roles and positions: A critique and extension of the blockmodeling approach", in Samuel Leinhardt (ed.) *Sociological Methodology 1983–1984*. San Francisco: Jossey-Bass
Wu, L.L.
1983 "Local blockmodel algebras for analyzing social networks", in: Samuel Leinhardt (ed. *Sociological Methodology 1983–1984*. San Francisco: Jossey-Bass.