1 Definitions

Exercise 1.1. Give precise definitions of the following

1. a function (5 points)
2. an equivalence relation (5 points)
3. a field of characteristic p (do not define a field) (5 points)
4. a subspace (5 points)
5. Linearly independent vectors (5 points)

2 Computations

For these computations, you do not have to justify your answer. (5 points each)

Exercise 2.1. Write out the addition and multiplication table for $\mathbb{F}_3$.

Exercise 2.2. How many vectors are there in $\mathbb{F}_2^{\times 2}$?

Exercise 2.3. Define the equivalence relation in $\mathbb{Z}$ by $a \simeq b$ if and only if there exists an $m \in \mathbb{Z}$ such that $a + m = b$. How many equivalence classes are there?

Exercise 2.4. Let k be a field. Let $U = \{(x, x) : x \in k\} \subset k^{\times 2}$. Find a subspace V such that $U + V = k^{\times 2}$ which is not all of $k^{\times 2}$.

Exercise 2.5. Consider the vectors $(1, 0)$ and $(1, 1)$ in $\mathbb{F}_2^{\times 2}$ over $\mathbb{F}_2$. Compute the span of these vectors (describe the subspace it spans or write out all the vectors).

3 True or False

True or false. Explain in one line. (5 points each)

Exercise 3.1.

1. Suppose that X is any set equipped with an equivalence relation. Then there are finitely many equivalence classes.
2. Consider $\mathbb{C}$ as a vector space over itself. Then $\mathbb{R} \subset \mathbb{C}$ is a subspace.
3. The dimension of a vector space over a finite field must be finite.
4. Let k be a field and V a vector space. Then for *any* scalar α, multiplication by α is always injective.
5. Any function between two sets of the same size is a bijection.

4 Problems

Exercise 4.1. Prove that every subspace of a finite dimensional vector space is finite dimensional. (15 points)

Exercise 4.2. Let k be a field and V a vector space. Suppose that we are given

1. $v_1, v_2 \in V$ vectors,
2. $U_1, U_2 \subset V$ subspaces.

For a subspace U of V and a vector $v \in V$ write $v + U := \{v + u : u \in U\}$. Prove that $U_1 = U_2$ if and only if $v_1 + U_1 = v_2 + U_2$. (10 points)