Bequest Taxation and $r - g$

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We consider the role of the gap $r - g$ between the return to capital r and the growth rate of the economy g in a political economy model of bequest taxation. Higher values of $r - g$ lead to higher wealth inequality, resulting in higher and more progressive taxes on bequest. These conclusions hold only under specific but reasonable assumptions regarding the bequest motive, the relative magnitudes of bequest and labor income inequality and the nature of the political economy process.

1 Introduction

Recently, Piketty (2014) has hypothesized that the gap between the rate of return on capital r and the growth rate of the economy g is a crucial determinant of the long-run evolution of wealth inequality. His central argument is that the higher is $r - g$, the stronger the inexorable tendency of wealth to become more concentrated over time. Looking towards the future, he argues that $r - g$ is likely to be high and that wealth is likely to become more concentrated. He views this scenario as deeply problematic and argues that the only was to prevent its occurrence is to put in place global progressive wealth and bequest taxes.

Even if one accepts the positive prediction that $r - g$ leads to higher wealth inequality over time, the normative implications and associated policy proposals do not logically follow without theoretical elaboration. Is wealth inequality bad per se? How much wealth inequality should societies tolerate? Why are progressive wealth and bequest taxation the right policies to reign its rise? If so how should they depend on $r - g$?

In this paper, we propose a political economy model of bequest taxation to attack these questions. The model builds and combines on our previous work on bequest taxation Farhi and Werning (2010, 2013) and the political economy considerations from Farhi, Sleet, Werning and Yeltekin (2012). It provides a theoretical benchmark that is broadly consistent with the aforementioned positive and normative arguments of Piketty (2014).

Indeed, in the model, higher values of $r - g$ lead to higher wealth inequality, and higher and more progressive optimal taxes on bequest.

Our results hold under rather specific assumptions regarding the bequest motive, the relative magnitudes of bequest and labor income inequality and the political economy process. Namely, an altruistic bequest motive that resembles warm-glow altruism; bequest heterogeneity driven by differences in altruism; bequests are more unequally distributed than income; taxes are decided sequentially with limited commitment; radical tax reforms are costly.

The empirical validity of these assumptions should be appropriately scrutinized. Identifying these assumptions and their consequences is the key contribution of this paper. Our goal is not to offer a justification or a falsification of Piketty (2014), but rather to develop a framework where the validity of his policy proposals can be considered formally.

2 Setup

Preferences and technology. There are two generations, parents born at $t = 0$ and children born at $t = 1$, each living for one period. Parents are altruistic and each has exactly one offspring. Parents have an endowment e_0. Children have an endowment $e_1 = Ge_0$. There is a storage technology between periods with constant rate of return R. We define the time period to be T and write $R = e^{rT}$ and $G = e^{gT}$. We will state our results using the returns R and G. It is straightforward to translate them into results about r and g. For example, a central object in our theory will turn out to be RG^{-1}, which can be expressed as a function of the much discussed difference between the rate of return on capital r and the growth rate of the economy g through the formula $RG^{-1} = e^{(r-g)T}$.

Parents are heterogenous with respect to their type θ which is distributed according to the probability density function $f(\theta)$. A parent of type θ has preferences

$$(1-\theta)\log(c_0) + \theta\log(c_1 - e_1),$$

where c_0 is consumption of the parent and c_1 is consumption of the offspring. The preferences of children are

$$\log(c_1).$$

The parameter θ indexes altruism. Higher types are more altruistic, lower types more selfish. We assume that the type is private information to the parent, introducing a tradeoff between redistribution and incentives.

Our formulation of altruism is that the parent cares about the consumption of the offspring net of its own endowment $c_1 - e_1$. This allows us to capture the realistic economics of a model with warm glow altruism where parents care about net of taxes bequests, without the normative drawbacks of a warm glow formulation.[1]

Taxes. We will consider two approaches: a Mirrleesian approach with no arbitrary restrictions on tax instruments, and a simple tax approach where we restrict tax instruments to a linear tax one bequests coupled with a lump sum rebate.

With unrestricted taxes, the most general tax instrument is a nonlinear tax of bequests, a result similar to Mirrlees (1971). Parents are subject to the budget constraints

$$c_0 + B + T(B) = I \tag{1}$$

$$c_1 = e_1 + RB \tag{2}$$

where T is a nonlinear tax on bequests. At points where T is differentiable, the marginal tax rate on bequests equals the implicit marginal tax rate on bequests $T'\left(\frac{c_1(\theta)-I_1}{R}\right) = \tau(\theta)$, defined by $(1+\tau(\theta))\frac{1-\theta}{c_0} = R\frac{\theta}{c_1}$.

With simple taxes, we restrict tax instruments to a tax on bequests, the proceeds of which are rebated lump-sum to parents. This amounts to restricting T to be linear so that $\tau(\theta) = \tau$ is independent of θ.

Implementability conditions. We can derive the implementability conditions corresponding to the different tax systems. The following definitions will prove useful. We say that an allocation $\{c_0(\theta), c_1(\theta)\}$ is resource feasible if

$$\int (c_0(\theta) - e_0 + \frac{c_1(\theta) - e_1}{R}) f(\theta) d\theta \leq 0. \tag{3}$$

An allocation is incentive compatible if

$$(1-\theta)\log(c_0(\theta)) + \theta \log(c_1(\theta) - e_1) \geq (1-\theta)\log(c_0(\hat{\theta})) + \theta \log(c_1(\hat{\theta}) - e_1). \tag{4}$$

We start with the unrestricted tax approach with nonlinear taxes on bequests. An allocation can be implemented with a nonlinear tax on bequest if and only if it is resource

[1] In fact, when we confine ourselves to simple taxes (linear tax cum lump sum rebate to parents) this model becomes isomorphic to a model with warm glow altruism where parents care about bequests net of taxes. With unrestricted tax instruments, the model with warm glow altruism has some unattractive properties, because the notion of bequest depends on the particular implementation of a given allocation. Our altruistic formulation does not suffer from these drawbacks, because preferences are defined directly over the allocation that is achieved, independently of the implementation that supports it.

feasible and incentive compatible.

We continue with the simple tax approach with linear taxes on bequests (together with a lump-sum rebate). An allocation can be implemented with a linear tax on bequests if and only if there exists an income I and an tax rate on bequests τ such that

$$c_0(\theta) = (1-\theta)I \qquad \text{and} \quad c_1(\theta) = e_1 + \frac{R}{1+\tau}\theta I,$$

where

$$I \int (1 - \frac{\tau}{1+\tau}\theta) f(\theta) d\theta = e_0.$$

3 No Political Economy Frictions

We begin with the case where there are no political economy frictions. In this case, taxes are decided in period 0 to maximize a Utilitarian objective. As we will show, in this case, under the unrestricted tax approach, optimal nonlinear taxes on bequests are equal to zero. This also immediately implies that under the simple tax approach, optimal linear taxes on bequests are zero.

Optimal nonlinear taxes on bequests are characterized by the following planning problem

$$\max_{\{c_0(\theta), c_1(\theta)\}} \int [(1-\theta)\log(c_0(\theta)) + \theta \log(c_1(\theta) - e_1)] f(\theta) d\theta,$$

s.t. resource feasibility (3) and incentive compatibility (4).

Proposition 1. *With no political economy friction, the implicit tax on bequests is $\tau(\theta) = 0$. The optimal allocation can be implemented with zero taxes on bequests $T(B) = 0$.*

This result is driven by a feature of our preference specification. The marginal utility of income is equalized across parents at any level of income and interest rate. Indeed, define the indirect utility function

$$V^p(I, \hat{R}; \theta) = \max_{c_0, c_1} (1-\theta)\log(c_0) + \theta \log(c_1 - e_1) \qquad \text{s.t.} \quad c_0 + \frac{1}{\hat{R}}(c_1 - e_1) = I.$$

We have

$$V_I^p(I, \hat{R}; \theta) = V_I^p(I, \hat{R}; \theta') \qquad \text{for all } \theta, \theta', \text{ and } I.$$

In particular, because the marginal utility of income is equalized across types at $I = e_0$ and $\hat{R} = R$, redistribution cannot increase the value of the Utilitarian objective starting from zero bequest taxes. Our specification of preferences therefore provides an attractive benchmark where there is no desire to redistribute across parents per se, and bequest taxes are zero in the absence of political economy frictions.

4 Political Economy Frictions

We now depart from the assumption of no political economy friction. Without political economy friction, taxes are set at $t = 0$ when parents make their bequest decisions. There is full commitment, in the sense that any temptation to revise taxes after bequest decisions are made is automatically resisted. Our political economy frictions relax this assumption of full commitment. We explicitly model the temptation to revise taxes to mitigate inequality after bequest decisions have been made. Our political economy friction imposes an additional restriction on allocations. We call this new restriction the credibility constraint.

The credibility constraint is motivated as follows. When period $t = 1$ comes along the original plan calls for the consumption assignment $c_1(\cdot)$ to be carried out. Imagine, however, that this plan can be reformed in favor of an alternative assignment $\tilde{c}_1(\cdot)$. To determine whether a reform takes place, the original assignment is compared to the reformed one using a utilitarian criterion, $\int \log(c_1(\theta))f(\theta)d\theta$ versus $\int \log(\tilde{c}_1(\theta))f(\theta)d\theta$. This captures a preference for equality that is key for our results.

To avoid trivial solutions, we assume a reform costs a fraction $(1 - e^{-\kappa})$ of the available resources where $\kappa \geq 0$, implying the resource constraint

$$\int \tilde{c}_1(\theta)\, f(\theta)d\theta \leq e^{-\kappa} \int c_1(\theta)\, f(\theta)d\theta. \tag{5}$$

If a reform takes place, the criterion $\int u(\tilde{c}_1(\theta))f(\theta)d\theta$ is maximized by a constant consumption level:

$$\tilde{c}_1(\theta) = e^{-\kappa} \int c_1(\theta)\, f(\theta)d\theta.$$

Comparing the two alternatives, it follows that a reform can be avoided if and only if

$$\int \log(c_1(\theta))\, f(\theta)d\theta \geq \log(\int c_1(\theta)\, f(\theta)d\theta) - \kappa. \tag{6}$$

One may interpret the fixed cost literally, perhaps as the opportunity cost of timely legislative procedures. However, its real purpose here is to allow for a simple form of limited commitment. At one extreme, the case with $\kappa = \infty$ effectively delivers full commitment, as in the previous section. Indeed, the same outcome obtains for finite but high enough values of κ. At the other extreme, when $\kappa = 0$ there is no commitment and reform is imminent. Intermediate values of κ capture intermediate levels of commitment.

We say that allocations are credible if they satisfy inequality (6). An allocation that does not satisfy this inequality is not credible in the sense that it can be anticipated that a reform would take place in period $t = 1$. Because reforms are costly, it is best to avoid them. Thus allocations must satisfy an additional implementability condition, namely the credibility constraint (6).

4.1 Simple Taxes

We start with the simple tax approach. This approach focuses on the dependence of the overall level of bequest taxes on RG^{-1}.

The allocation is entirely determined by the income I of parents and the bequest tax τ. Indeed we have

$$c_0(\theta) = (1-\theta)I,$$

$$c_1(\theta) = \theta I \frac{R}{1+\tau} + e_1.$$

Consider first the case where the reform cost κ is large enough that the the credibility constraint is not binding at the optimum, then the results in Proposition 1 applies and the optimal tax on bequest is zero.

Consider now the interesting case where the reform cost κ is low enough that the credibility constraint is binding at the optimum. Then I and τ are entirely pinned down by the resource constraint (3) and the credibility constraint (6):

$$I \int (1 - \frac{\tau}{1+\tau}\theta) f(\theta) d\theta = e_0,$$

$$\int \log(I \frac{R}{1+\tau}\theta + e_1) f(\theta) d\theta = \log(I \frac{R}{1+\tau} \int \theta f(\theta) d\theta + e_1) - \kappa.$$

These form a system of two equations in two unknowns I and τ. Defining $\hat{I} = \frac{I}{e_0}$, we can then rewrite these conditions as

$$\hat{I} \int (1 - \frac{\tau}{1+\tau}\theta) f(\theta) d\theta = 1,$$

$$\int \log(\frac{1}{1+\tau} RG^{-1} \hat{I}\theta + 1) f(\theta) d\theta = \log(\frac{1}{1+\tau} RG^{-1} \hat{I} \int \theta f(\theta) d\theta + 1) - \kappa.$$

This formulation makes it clear that the tax rate τ depends on the gross rate of return on capital R and the gross rate of growth of the economy G only through the sufficient statistic RG^{-1}. In fact, the solution is most easily characterized by first defining the following decreasing transformation of τ as follows

$$x(\tau) = \frac{1 - \frac{\tau}{1+\tau}}{\int (1 - \frac{\tau}{1+\tau}\theta) f(\theta) d\theta},$$

as well as the constant $\hat{x}$ given by

$$\int \log(\hat{x}\theta + 1) f(\theta) d\theta = \log(\hat{x} \int \theta f(\theta) d\theta + 1) - \kappa.$$

Note the important property that $\hat{x}$ depends on the amount of altruism heterogeneity and on the reform cost κ but is independent of the rate of return on capital R and of the growth rate G.

Proposition 2. *With political economy frictions, the credibility constraint binds at the optimum if and only if $\hat{x} \leq RG^{-1}$. If the credibility constraint binds at the optimum, then the optimal tax rate τ is given by* the implicit equation $x(\tau) = \frac{\hat{x}}{RG^{-1}}$. It is increasing in RG^{-1}.

The intuition for Proposition 2 can be broken down into three parts. First, wealth inequality (in a proportional sense) among the generation of children is increasing in RG^{-1}. The wealth of children is composed of two parts, the bequest that they receive from their parents and their income. Inequality arises only because of heterogeneity in bequests. The overall magnitude of bequests increases with the income of parents and with the rate of return on capital R. The growth rate of the economy G determines by how much the income of children is higher than that of parents. Taken together, for a given tax rate on bequests, wealth inequality among children is therefore increasing in the rate of return on capital R and decreasing in the growth rate of the economy G. With our specific functional forms, the dependence of wealth inequality on these two variables is captured by the sufficient statistic RG^{-1}.

Second, since the temptation to undertake wealth equalizing reform increases increases with wealth inequality among children, it increases with RG^{-1}, and hence why the credibility constraint binds for high enough values of RG^{-1} .

Third, when the credibility constraint binds, then taxes on bequests have to be adjusted so that the credibility constraint is not violated. Wealth inequality among children is decreasing in the tax rate on bequests. Given the discussion above, the tax rate on bequest must increase when RG^{-1} increases.

4.2 Unrestricted Taxes

We now turn to the approach with unrestricted taxes. This approach allows to study the dependence of the progressivity (and not simply the overall level) of bequest taxes on RG^{-1}.

If the reform cost κ is high enough that the credibility constraint does not bind at the optimum, then the results in Proposition 1 applies and optimal taxes on bequest are zero. The credibility constraint binds at the optimum when κ is low enough. In this case, optimal taxes on bequests are nonzero just as in the simple tax case. With unrestricted taxes however, there are enough degrees of freedom that taxes are not entirely pinned down by the constraints. Their characterization therefore requires studying a non-degenerate planning problem. We define $v(\theta)$ to be the utility of a parent of type θ so that

$$v(\theta) = (1-\theta)\log(c_0) + \theta\log(c_1 - e_1).$$

It is more convenient to formulate the planning problem using the variables $v(\theta)$ and $c_1(\theta)$ than $c_0(\theta)$ and $c_1(\theta)$.

The planning problem can be written as follows

$$\max_{\{v(\theta), c_1(\theta)\}} \int v(\theta) f(\theta) d\theta,$$

s.t.

$$\int [\exp(\frac{v(\theta) - \theta\log(c_1(\theta) - e_1)}{1-\theta}) + \frac{c_1(\theta) - e_1}{R}] f(\theta) d\theta \leq e_0,$$

$$\dot{v}(\theta) = \log(c_1(\theta) - e_1) - \frac{v(\theta) - \theta\log(c_1(\theta) - e_1)}{1-\theta} \quad \text{and} \quad \dot{c}_1(\theta) \geq 0,$$

$$\int \log(c_1(\theta)) f(\theta) d\theta \geq \log(\int c_1(\theta) f(\theta) d\theta) - \kappa.$$

The objective is to maximize Utilitarian welfare. The first constraint is the resource constraint (3). The second constraint is the incentive compatibility constraint (4) expressed in differential form using an envelope argument along the lines of Milgrom and Segal (2002). Because our preferences satisfy the single crossing condition, incentive compatibility boils down to two conditions, the envelope equation, as well as the monotonicity condition that $c_1(\theta)$ be increasing in θ. When the monotonicity constraint binds, there is bunching. The third constraint is the credibility constraint (6).

It is convenient to perform the following change of variables:

$$c_1(\theta) - e_1 = Re_0\hat{c}_1(\theta),$$

$$\hat{v}(\theta) = v(\theta) - (1-\theta)\log(e_0) - \theta\log(Re_0),$$

$$\dot{\hat{v}}(\theta) = \dot{v}(\theta) - \log(R).$$

We can then write the problem as

$$\max \int \hat{v}(\theta) f(\theta) d\theta,$$

s.t.

$$\int [\exp(\frac{\hat{v}(\theta) - \theta \log(\hat{c}_1(\theta))}{1-\theta}) + \hat{c}_1(\theta)] f(\theta) d\theta \leq 1,$$

$$\dot{\hat{v}}(\theta) = \frac{\log(\hat{c}_1(\theta)) - \hat{v}(\theta)}{1-\theta} \quad \text{and} \quad \dot{\hat{c}}_1(\theta) \geq 0,$$

$$\int \log(RG^{-1}\hat{c}_1(\theta) + 1) f(\theta) d\theta \geq \log(RG^{-1} \int \hat{c}_1(\theta) f(\theta) d\theta + 1) - \kappa.$$

Since we can back out the implicit marginal tax rate on capital using

$$\tau(\theta) = \frac{\theta}{1-\theta} \frac{1}{\hat{c}_1(\theta)} \exp(\frac{\hat{v}(\theta) - \theta \log(\hat{c}_1(\theta))}{1-\theta}) - 1,$$

this formulation shows that just as in the simple tax case, R and G influence the optimal tax rate on bequests only through the sufficient statistic RG^{-1} .

This is an optimal control problem with an integral control constraint. Let $\gamma > 0$ be the multiplier on the resource constraint, $\nu \geq 0$ the multiplier on the credibility constraint, and $\mu(\theta)$ the co-state for $v(\theta)$. Assuming that there is no bunching on an interval around θ, the first-order conditions are

$$\begin{aligned} 0 = \gamma \frac{\theta}{1-\theta} \frac{1}{\hat{c}_1(\theta)} \exp(\frac{\hat{v}(\theta) - \theta \log(\hat{c}_1(\theta))}{1-\theta}) - \gamma + \frac{\mu(\theta)}{f(\theta)} \frac{1}{1-\theta} \frac{1}{\hat{c}_1(\theta)} \\ + \nu RG^{-1} [\frac{1}{RG^{-1}\hat{c}_1(\theta) + 1} - \frac{1}{\int RG^{-1}\hat{c}_1(\theta) f(\theta) d\theta + 1}], \end{aligned}$$

$$\dot{\mu}(\theta) = -f(\theta) + \gamma \frac{1}{1-\theta} \exp(\frac{\hat{v}(\theta) - \theta \log(\hat{c}_1(\theta))}{1-\theta}) f(\theta),$$

$$\mu(\underline{\theta}) = \mu(\overline{\theta}) = 0.$$

We can re-express the first condition to solve for the implicit marginal tax rate $\tau(\theta)$ on bequests.

Proposition 3. *Assume that the reform cost κ is low enough that the credibility constraint binds* *at the optimum. If there is no bunching on an interval around θ. The optimal marginal tax rate on bequests is given by the following formula*

$$\tau(\theta) = -\frac{\mu(\theta)}{\gamma f(\theta)} \frac{1}{1-\theta} \frac{1}{\hat{c}_1(\theta)} + \frac{\nu}{\gamma} RG^{-1} [\frac{1}{\int RG^{-1}\hat{c}_1(\theta) f(\theta) d\theta + 1} - \frac{1}{RG^{-1}\hat{c}_1(\theta) + 1}].$$

where $\mu(\underline{\theta}) = \mu(\bar{\theta}) = 0$. If there is no bunching at the extremes, then $\tau(\underline{\theta}) < 0 < \tau(\bar{\theta})$.

The tax on capital $\tau(\theta)$ depends on R and G only through the sufficient statistic RG^{-1} (so do the multipliers γ, ν, and the co-state $\mu(\theta)$) and is the sum of two terms. The first term is zero at the top and at the bottom. In between, its sign depends on the direction in which the ICs are binding. The second term is increasing, negative at the bottom and positive at the top. This illustrates a precise sense in which optimal taxes on bequests are progressive, since they are smaller (in fact negative) for the smallest bequests than for the largest bequests (where they are positive).

It is harder to derive precise theoretical comparative statics with respect to RG^{-1}. For this reason, we perform some illustrative simulations. These are not meant to represent a real calibration but to provide some illustration for the underlying forces in our theory. We assume that types θ are uniformly distributed between 0.1 and 0.9. We take the length of a generation T to be 35 years. Because bequest taxes depend only on RG^{-1}, we choose to illustrate the comparative statics of the model with respect to G, keeping R constant. We assume that the annual rate of return on capital is 5%. We vary the annual growth rate between 1% and 5%.

Figure 1 plots the marginal tax rate on bequests $\tau(\theta)$ as a function of parent type θ. The different lines correspond to different values of the annual growth rate g. Steeper lines correspond to lower values of g. The credibility constraint binds except when g takes its highest value of 5%, in which case optimal bequest taxes are equal to zero. For lower values of g, the credibility constraint binds. Marginal taxes on bequests are negative for the least altruistic parents (who leave the smallest bequests) and positive for the most altruistic parents (who leave the largest bequests), and broadly increases monotonically in between (there is a small decreasing portion towards the top type $\bar{\theta}$). Hence taxes on bequests are basically progressive, the more so, the lower is the growth rate g of the economy. The main takeaway from this simple simulation is therefore that optimal bequest taxes range from negative to positive and are progressive, the more so, the larger is RG^{-1}.

5 Conclusion

In this paper, we provide a theoretical benchmark that is broadly consistent with some of the central positive and normative arguments of Piketty (2014): higher values of $r - g$ lead to higher wealth inequality, and higher and more progressive optimal taxes on bequest. These conclusions hold only under certain specific assumptions regarding the bequest motive, the relative magnitudes of bequest and labor income inequality, and the nature of the political economy process. The empirical validity of these assumptions should be scrutinized. We hope that this explicit formalism will contribute to promoting further scientific work on this important topic.

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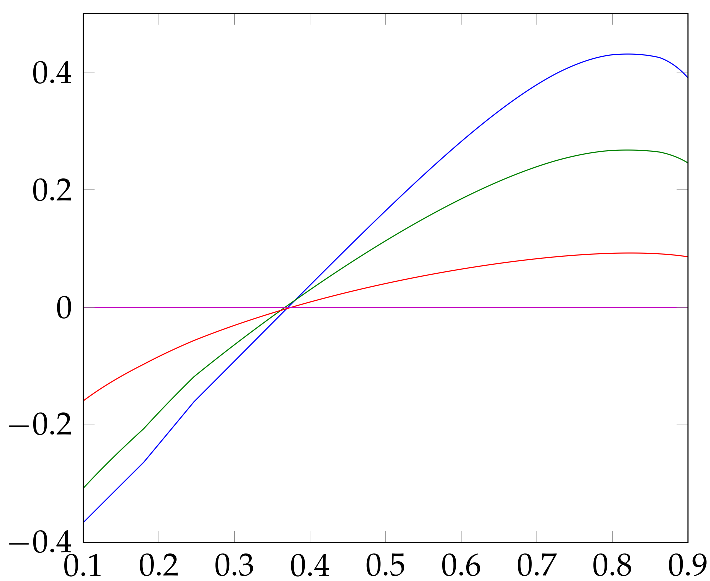

Figure 1: Bequest tax $\tau(\theta)$ as a function of parent type θ for four equally spaced values of the annual growth rate g ranging from 1% to 5%. The annual rate of return on capital is taken to be 5%. Types θ are uniformly distributed 0.1 and 0.9. The length of a generation T is taken to be 35 years.