The Darwinian Returns to Scale

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Origins of Aggregate Increasing Returns to Scale

- Technical efficiency?
- Allocative efficiency?
- Unresolved theoretically and empirically.

Model Elements

- Monopolistic competition: Kimball demand.
- Heterogeneity: marginal costs, markups, pass-throughs.
- Technical increasing returns: fixed costs (entry, overhead).

CES Non-Starter

- CES (e.g. Melitz, 2003): special case.
- Efficient: only technical efficiency, no allocative efficiency.
- Counterfactual: constant markups, complete pass-throughs.
- Non-starter: need to move away from CES.
- Kimball: allows *any* demand curve, markups, pass-throughs.

Theoretical Results

- Comparative statics in second best (welfare, output, TFP).
- Decomposition into technical and allocative efficiency.
- Dist. to frontier and gains from industrial/competition policy.
- Measurable sufficient statistics.

Empirical and Quantitative Results

- Non-parametric estimation.

- Increasing returns from technical and allocative efficiency.
- Key: heterogeneity $\times$ inefficiency via Darwinian reallocations.

- Large gains from industrial/competition policy.
- Key: heterogeneity $\times$ inefficiency on all margins.

Selected Related Literature

- Chamberlin (33), Robinson (33).
- Spence (76), Dixit-Stiglitz (77), Lancaster (79), Salop (79), Hart (85), Mankiw-Whinston (86), Vives (99), Zhelobodko et al. (12), Dhingra-Morrow (19), Midrigan et al. (19).
- Norman (76), Krugman (79), Dixit-Norman (80), Helpman-Krugman (85), Venables (85), Melitz (03), Arkoloakis et. al (12), Melitz-Redding (14), Midrigan et al. (15), Arkolakis et al. (19), Bartelme et al. (19).
- Harberger (54,61,71), Epifani-Gancia (11), Hsieh-Klenow (09), Baqaee-Farhi (19).

Outline

- Setup.
- Concepts and Solution Strategy.
- Comparative Statics with Homogeneous Firms.
- Comparative Statics with Heterogeneous Firms.
- Dist. to Frontier and Gains from Industrial/Competition Policy.
- Empirical and Quantitative Results.

Households

- Mass L of identical households with unit labor supply.
- Kimball preferences over varieties of consumption goods:

$$\int_0^\infty \Upsilon(\frac{y_\omega}{Y})d\omega = 1.$$

- Maximize utility s.t. budget constraint (wage numeraire):

$$\max_{\{y_\omega\}} Y$$

s.t.

$$\int_0^\infty p_\omega y_\omega d\omega = 1.$$

Demand Curves

- Demand curve for each variety:

$$\frac{p}{P} = \Upsilon'\left(\frac{y}{Y}\right),$$

with "price index" and "demand index" given by

$$P = \frac{\bar{\delta}}{Y} \quad \text{and} \quad \bar{\delta} = \frac{1}{\int_0^\infty \frac{y_\omega}{Y} \Upsilon'\left(\frac{y_\omega}{Y}\right) d\omega}.$$

- Elasticity:

$$\sigma\left(\frac{y}{Y}\right) = \frac{\Upsilon'\left(\frac{y}{Y}\right)}{-\frac{y}{Y}\Upsilon''\left(\frac{y}{Y}\right)}.$$

Producers

- Each variety supplied by single producer.
- Free entry with cost f_e (labor), type realization $\theta \sim g(\theta)$.
- Production with overhead cost f_o, marginal cost $1/A_\theta$ (labor).
- Maximize profits s.t. demand:

$$\max_{\{p_\theta, y_\theta\}} L(p_\theta y_\theta - \frac{1}{A_\theta} y_\theta) - f_o$$

s.t.

$$\frac{p_\theta}{P} = \Upsilon'(\frac{y_\theta}{Y}).$$

Markups, Entry and Exit

- Optimal price and markup:

$$p_\theta = \frac{\mu_\theta}{A_\theta}, \quad \text{where} \quad \mu_\theta = \mu(\frac{y_\theta}{Y}) = \frac{1}{1 - \frac{1}{\sigma(\frac{y_\theta}{Y})}}.$$

- Survival if profits exceed overhead cost:

$$L p_\theta y_\theta \left(1 - \frac{1}{\mu_\theta}\right) \geq f_o.$$

- Entry profitable if expected profits exceed entry cost:

$$\frac{1}{\Delta}\int_0^\infty \max\left\{L p_\theta y_\theta \left(1 - \frac{1}{\mu_\theta}\right) - f_o, 0\right\} g(\theta) d\theta \geq f_e.$$

Equilibrium

- Households maximize utility.
- Firms maximize profits.
- Free entry and exit.
- Markets clear.

"Coordinates" for Equilibrium Allocations

- Sales, markups, and mass of firms:

$$\lambda_\theta = (1 - G(\theta^*))Mp_\theta y_\theta, \quad \mu_\theta, \quad \text{and} \quad M.$$

- Pin down allocation:

$$y_\theta = \frac{\lambda_\theta A_\theta}{\mu_\theta(1 - G(\theta^*))M} \quad \text{and} \quad p_\theta = \frac{\mu_\theta}{A_\theta}.$$

- Use as "coordinates".

Equilibrium Equations

- Consumer welfare:

$$1 = (1 - G(\theta^*))M\mathbb{E}\left[\Upsilon(\frac{\lambda_\theta A_\theta}{\mu_\theta MY})\right].$$

- Free entry:

$$\frac{Mf_e\Delta}{L} = \mathbb{E}\left[\lambda_\theta\left(1 - \frac{1}{\mu_\theta}\right) - \frac{(1 - G(\theta^*))Mf_o}{L}\right].$$

- Selection:

$$\frac{(1 - G(\theta^*))Mf_o}{L} = \lambda_{\theta^*}\left(1 - \frac{1}{\mu_{\theta^*}}\right).$$

Equilibrium Equations

- Markups:

$$\mu_\theta = \mu\left(\frac{\lambda_\theta A_\theta}{\mu_\theta(1 - G(\theta^*))MY}\right).$$

- Variety demand:

$$\frac{\mu_\theta}{A_\theta} = P\Upsilon'\left(\frac{\lambda_\theta A_\theta}{\mu_\theta(1 - G(\theta^*))MY}\right).$$

- Price index and demand index:

$$P = \frac{\bar{\delta}}{Y} \quad \text{and} \quad \bar{\delta} = \frac{1}{M\mathbb{E}\left[\frac{\lambda_\theta A_\theta}{\mu_\theta MY}\Upsilon'\left(\frac{\lambda_\theta A_\theta}{\mu_\theta(1-G(\theta^*))MY}\right)\right]}.$$

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Demand Concepts

- Markups:

$$\mu(\frac{y}{Y}) = \frac{1}{1 - \frac{1}{\sigma(\frac{y}{Y})}} \geq 1.$$

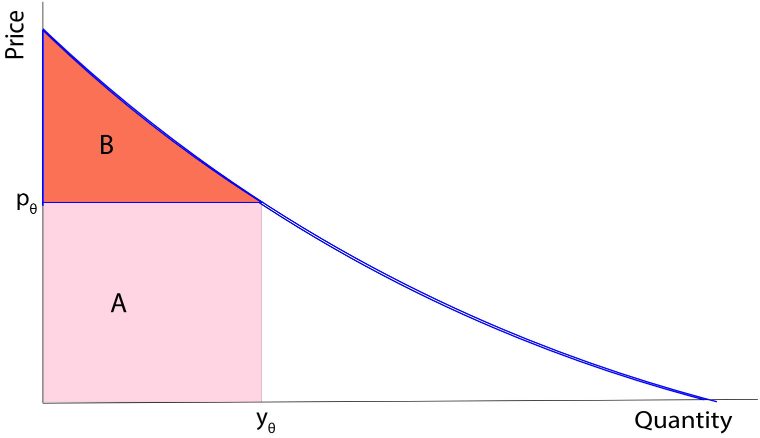
- Pass-throughs:

$$\rho(\frac{y}{Y}) = \frac{1}{1 + \frac{\frac{y}{Y}\mu'(\frac{y}{Y})}{\mu(\frac{y}{Y})}\sigma(\frac{y}{Y})} \lesseqgtr 1.$$

- Infra-marginal surplus ratios (noting $\bar{\delta} = \mathbb{E}_\lambda[\delta_\theta]$):

$$\delta(\frac{y}{Y}) = \frac{\Upsilon(\frac{y}{Y})}{\frac{y}{Y}\Upsilon'(\frac{y}{Y})} \geq 1.$$

Infra-Marginal Surplus Ratio $\delta = \frac{A+B}{A}$



Demand Properties

- Not imposed in theory but verified empirically.
- Marshall's weak second law of demand:

$$\mu'(\frac{y}{Y}) \geq 0 \quad \iff \quad \rho(\frac{y}{Y}) \leq 1.$$

- Marshall's strong second law of demand:

$$\rho'(\frac{y}{Y}) \leq 0.$$

Welfare and Real Output

- Welfare per capita:

$$d \log Y.$$

- Real output per capita (prices, see paper for quantities):

$$d \log Q^p = -\mathbb{E}_\lambda [d \log p_\theta].$$

Technical and Allocative Efficiency

- Allocation and productivity vectors:

$$\mathscr{X} = (l_e, l_o, \{l_\theta\}) \quad \text{and} \quad \mathscr{A} = (L, f_e\Delta, f_o, \{A_\theta\}).$$

- Welfare function:

$$Y = \mathscr{Y}(\mathscr{A}, \mathscr{X}).$$

- Technical and allocative efficiency:

$$d\log Y = \underbrace{\frac{\partial \log \mathscr{Y}}{\partial \log \mathscr{A}} d\log \mathscr{A}}_{\text{technical efficiency}} + \underbrace{\frac{\partial \log \mathscr{Y}}{\partial \mathscr{X}} d\mathscr{X}}_{\text{allocative efficiency}} .$$

- No equivalent for real output capita (prices).

Solution Strategy

- Start at initial equilibrium.
- Shocks: **population**, fixed costs, productivity.
- Changes in welfare and real output.
- Changes in technical and allocative efficiency.
- Sufficient statistics: sales λ_θ, markups μ_θ, pass-throughs ρ_θ, infra-marginal surplus ratios δ_θ.

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Social Inefficiency

- Entry only efficiency margin.
- Excessive entry iff:
$$\delta < \mu.$$

Welfare

- Change in welfare per capita with population shocks:

$$d\log Y = \underbrace{(\delta - 1) d\log L}_{\text{technical efficiency}} + \underbrace{\delta \frac{\xi}{1-\xi} d\log L}_{\text{allocative efficiency}},$$

where

$$\xi = \left(1 - \rho\right)\left(1 - \frac{\delta - 1}{\mu - 1}\right)\frac{1}{\sigma} = \left(1 - \rho\right)\left(1 - \frac{\delta}{\mu}\right).$$

- $\xi > 0$ (increasing returns via allocative efficiency) iff:
  1. $\rho < 1$ (incomplete pass-through);
  2. $\delta < \mu$ (excessive entry).

Real Output

- Changes in real output per capita (prices):

$$d\log Q^p = \frac{1-\rho}{\sigma}(d\log Y + d\log L).$$

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Social Inefficiency

- Three margins of efficiency: entry, selection, relative size.
- Excessive entry iff:

$$\mathbb{E}_\lambda[\delta_\theta] < \frac{1}{\mathbb{E}_\lambda[\frac{1}{\mu_\theta}]}.$$

- Excessive selection iff:

$$\delta_{\theta^*} > \mathbb{E}_\lambda[\delta_\theta].$$

- Excessive relative size θ' vs. θ iff:

$$\mu_{\theta'} < \mu_\theta.$$

Welfare

- Change in welfare per capita:

$$d\log Y = \underbrace{\left(\mathbb{E}_\lambda[\delta_\theta]-1\right) d\log L}_{\text{technical efficiency}}$$
$$+ \underbrace{\frac{\xi^{\varepsilon}+\xi^{\mu}+\xi^{\theta^*}}{1-\xi^{\varepsilon}-\xi^{\mu}-\xi^{\theta^*}}\left(\mathbb{E}_\lambda[\delta_\theta]\right) d\log L}_{\text{allocative efficiency}},$$

where

$$\xi^{\varepsilon} = \left(\mathbb{E}_\lambda[\delta_\theta]-1\right)\left(\mathbb{E}_\lambda[\sigma_\theta]-\mathbb{E}_{\lambda(1-1/\mu)}[\sigma_\theta]\right)\left(\mathbb{E}_\lambda\left[\frac{1}{\sigma_\theta}\right]\right),$$
$$\xi^{\theta^*} = \left(\mathbb{E}_\lambda[\delta_\theta]-\delta_{\theta^*}\right)\left(\lambda_{\theta^*}\gamma_{\theta^*}\frac{\sigma_{\theta^*}-\mathbb{E}_{\lambda(1-1/\mu)}[\sigma_\theta]}{\sigma_{\theta^*}-1}\right)\left(\mathbb{E}_\lambda\left[\frac{1}{\sigma_\theta}\right]\right),$$
$$\xi^{\mu} = \left(\mathbb{E}_\lambda\left[(1-\rho_\theta)\left(1-\frac{\mathbb{E}_\lambda[\delta_\theta]-1}{\mu_\theta-1}\right)\right]\right)\left(\mathbb{E}_\lambda\left[\frac{1}{\sigma_\theta}\right]\right).$$

Understanding ξ^{ε} via Demand Curve $\frac{p}{P} = \Upsilon'(\frac{y}{Y})$

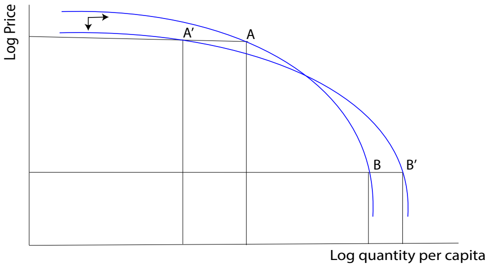

Figure: Reallocation effect due to increased entry (holding fixed markups and the selection cutoff) assuming second Marshall laws of demand.

- $\xi^{\varepsilon} > 0$ (irrespective of shape of demand).
- Assuming second Marshall laws of demand, signs of ξ^{θ^*} and ξ^{μ} ambiguous (too much or too little selection and entry).

Real Output

- Changes in real output per capita (prices):

$$d \log Q^p = \left(\mathbb{E}_\lambda \left[(1 - \rho_\theta) \right] \right) \left(\mathbb{E}_\lambda \left[\frac{1}{\sigma_\theta} \right] \right) \left(d \log Y + d \log L \right).$$

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Efficient Allocation and Industrial/Competition Policy

- Industrial/competition policy implements efficient allocation.
- Regulate markups to equal infra-marginal surplus ratios:

$$\mu_\theta = \delta_\theta > 1.$$

- Introduce production subsidies to offset markups:

$$\tau_\theta = \frac{1}{\delta_\theta} < 1.$$

- Restores efficiency on all margins: entry, selection, relative size.

Dist. Frontier and Gains from Industrial/Competition Policy

- Second-order approximation to distance to frontier and gains from industrial/competition policy:

$$\mathscr{L} \approx \frac{1}{2}\mathbb{E}_{\lambda}\left[\sigma_{\theta}\left(\frac{\mathbb{E}_{\lambda}\left[\mu_{\theta}\right]}{\mathbb{E}_{\lambda}\left[\delta_{\theta}\right]}-1\right)^{2}\right] + \frac{1}{2}\lambda_{\theta^{*}}\gamma_{\theta^{*}}\left(\mathbb{E}_{\lambda}\left[\delta_{\theta}\right]-\delta_{\theta^{*}}\right)^{2} + \frac{1}{2}\mathbb{E}_{\lambda}\left[\sigma_{\theta}\left(\frac{\mu_{\theta}}{\mathbb{E}_{\lambda}\left[\delta_{\theta}\right]}-\frac{\mathbb{E}_{\lambda}\left[\mu_{\theta}\right]}{\mathbb{E}_{\lambda}\left[\delta_{\theta}\right]}\right)^{2}\right].$$

- Separate contributions from inefficients along different margins: entry, selection, relative size.

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Non-Parametric Estimation

- Inputs:
  - λ_θ, ρ_θ, M (data);
  - $\bar{\mu} = 1/[\mathbb{E}_\lambda[1/\mu_\theta]]$ and $\bar{\delta} = \mathbb{E}_\lambda[\delta_\theta]$ (postulates).
- Outputs:
  - μ_θ, σ_θ, A_θ, δ_θ, γ_θ (local counterfactuals);
  - f_e, f_o, $\Upsilon(\cdot)$ (global counterfactuals).

Non-Parametric Estimation (Key Equations)

- Changes in λ_θ with A_θ:

$$\frac{d\log\lambda_\theta}{d\theta} = \frac{\rho_\theta}{\mu_\theta - 1}\frac{d\log A_\theta}{d\theta}.$$

- Changes in μ_θ with A_θ:

$$\frac{d\log\mu_\theta}{d\theta} = (1-\rho_\theta)\frac{d\log A_\theta}{d\theta}.$$

Non-Parametric Estimation (Local)

- Recover μ_θ and A_θ by solving:

$$\frac{d\log\mu_\theta}{d\theta} = \frac{(\mu_\theta - 1)(1-\rho_\theta)}{\rho_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad \frac{1}{\mathbb{E}_\lambda[\frac{1}{\mu_\theta}]} = \bar{\mu},$$

$$\frac{d\log A_\theta}{d\theta} = \frac{\mu_\theta - 1}{\rho_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad A_{\theta^*} = 1.$$

- Recover δ_θ by solving:

$$\frac{d\log\delta_\theta}{d\theta} = \frac{\mu_\theta - \delta_\theta}{\delta_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad \mathbb{E}_\lambda[\delta_\theta] = \bar{\delta}.$$

Non-Parametric Estimation (Global)

- Recover Υ using:

$$\Upsilon(\frac{y}{Y}) = \frac{\delta_{\theta(y)}\lambda_{\theta(y)}}{\bar{\delta}M}.$$

where $\theta(y)$ inverse of $y_\theta = (\lambda_\theta A_\theta)/(M\mu_\theta)$.

- Recover $f_e\Delta$ and f_o using:

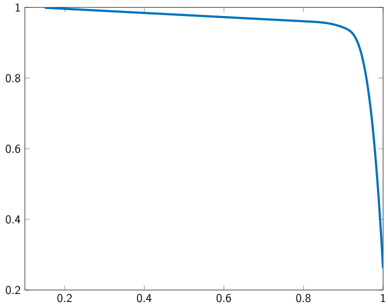
$$\frac{f_e\Delta}{L} + (1 - G(\theta^*))\frac{f_o}{L} = \frac{1}{M}\mathbb{E}\left[\lambda_\theta\left(1 - \frac{1}{\mu_\theta}\right)\right],$$

$$\frac{f_o}{L} = \frac{1}{M}\lambda_{\theta^*}\left(1 - \frac{1}{\mu_{\theta^*}}\right).$$

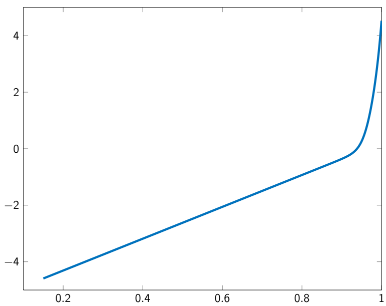
Data

- Belgian data for manufacturing firms.
- Sales and pass-throughs by firm size for ProdCom sub-sample (price and quantity data) from Amiti et al. (19).
- Extrapolate to entire manufacturing sample by matching firms on size.

Data



(a) Pass-through ρ_θ.

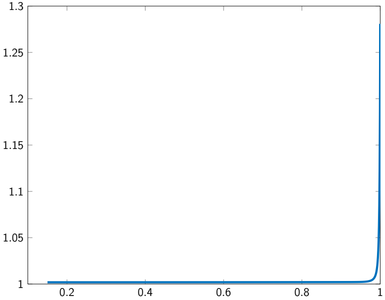


(b) Sales share density $\log \lambda_\theta$.

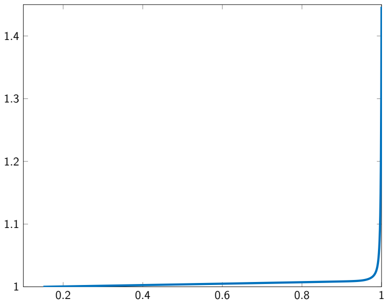
Postulates for Initial Conditions

- Take one of two values for $\bar{\delta}$:
  - $\bar{\delta} = \bar{\mu}$ (efficient entry);
  - $\bar{\delta} = \delta_{\theta^*}$ (efficient selection).
- Take one of two values for $\bar{\mu}$
  - $\bar{\mu} = 1.045$ ($d \log Y / d \log L \approx 0.14$);
  - $\bar{\mu} = 1.09$ ($d \log Y / d \log L \approx 0.3$).

Estimates

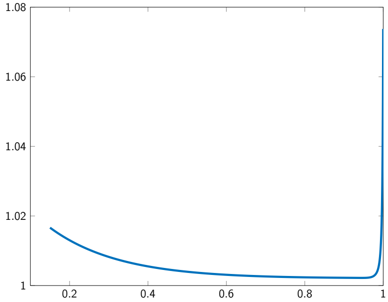


(a) Markup μ_θ ($\bar{\mu} = 1.045$)

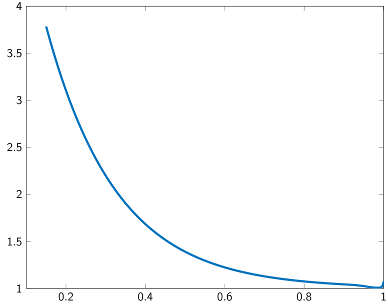


(b) Productivity $\log A_\theta$ ($\bar{\mu} = 1.045$)

Estimates (Efficient Selection vs. Efficient Entry)

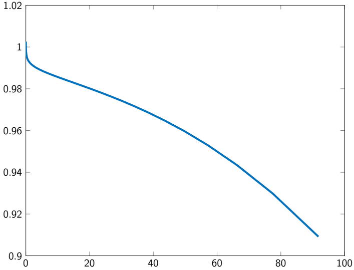


(a) Infra-marginal surplus ratio δ_θ (efficient selection, $\bar{\mu} = 1.045$).

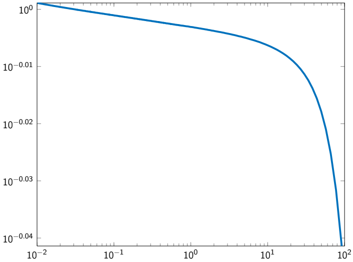


(b) Infra-marginal surplus ratio δ_θ (efficient entry, $\bar{\mu} = 1.045$).

Residual Demand Curve



(a) Residual demand curve (efficient entry, $\bar{\mu} = 1.045$).



(b) Log-log residual demand curve (efficient entry, $\bar{\mu} = 1.045$).

Counterfactual: 1% Population Shock

	$\bar{\mu} = 1.045$		$\bar{\mu} = 1.090$	
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$
Welfare	0.130	0.145	0.293	0.323
Technical efficiency	0.017	0.045	0.034	0.090
Allocative efficiency	0.114	0.100	0.260	0.233
Entry	0.117	0.408	0.272	1.396
Exit	0.000	-0.251	0.000	-1.006
Markups	-0.004	-0.057	-0.012	-0.157
Real GDP per capita	0.024	0.024	0.051	0.052

Table: The elasticity of welfare and real GDP per capita to population with heterogeneous firms.

Counterfactual: 1% Population Shock (Homogenous Firms)

	$\bar{\mu} = 1.045$		$\bar{\mu} = 1.090$	
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$
Welfare	0.030	0.045	0.060	0.090
Technical efficiency	0.017	0.045	0.034	0.090
Allocative efficiency	0.013	0.000	0.026	0.000
Real GDP per capita	0.021	0.022	0.042	0.043

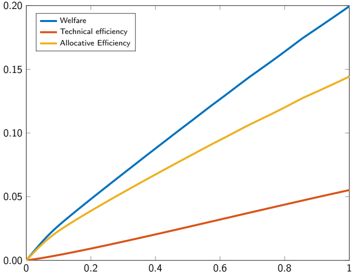
Table: The elasticity of welfare and real GDP per capita to population with homogenous firms.

Counterfactual: 50% Population Shock (Nonlinearities)

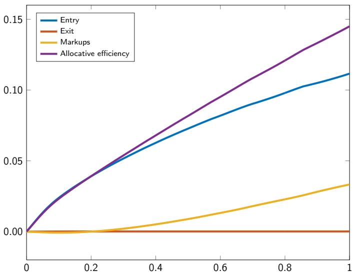
	$\bar{\mu} = 1.045$		$\bar{\mu} = 1.090$	
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$
Welfare	0.100	0.099	0.215	0.216
Technical efficiency	0.025	0.048	0.052	0.098
Allocative efficiency	0.075	0.051	0.162	0.117
Entry	0.066	0.107	0.145	0.272
Exit	0.000	-0.065	0.000	-0.176
Markups	0.008	0.008	0.017	0.021
Real GDP per capita	0.025	0.024	0.054	0.051

Table: The average elasticity of welfare and real GDP per capita to population with heterogeneous firms for a 50% population shock.

Counterfactual: 50% Shock (Nonlinearities)



(a) Welfare: technical and allocative efficiency as functions of $\log L$ (efficient selection, $\bar{\mu} = 1.09$).



(b) Allocative efficiency: entry, exit, and markups as functions of $\log L$ (efficient selection, $\bar{\mu} = 1.09$).

Gains from Industrial/Competition Policy

- Second-order approximation as local counterfactual.
- Exact number as global counterfactual.
- Work in progress.

Conclusion: Summary

- Increasing returns to scale?
- Technical and allocative efficiency.
- Gains from industrial/competition policy.
- Key: heterogeneity $\times$ inefficiency.
- Different for welfare and real output or TFP.

Conclusion: Extensions

- Other demand systems and market structures.
- Open economy.
- Dynamics.
- HAIO.

Back-Up Slides

Welfare and Real Output

- Welfare per capita:

$$d \log Y = \left(\mathbb{E}_{\lambda} [\delta_{\theta}] - 1 \right) d \log M$$
$$+ \left(\mathbb{E}_{\lambda} [\delta_{\theta}] - \delta_{\theta^*} \right) \lambda_{\theta^*} \frac{g(\theta^*)}{1 - G(\theta^*)} d\theta^* + \mathbb{E}_{\lambda} \left[d \log(\frac{A_{\theta}}{\mu_{\theta}}) \right].$$

- Real output per capita (prices):

$$d \log Q^p = \mathbb{E}_{\lambda} \left[d \log(\frac{A_{\theta}}{\mu_{\theta}}) \right].$$