Fiscal Multipliers: Liquidity Traps and Currency Unions[☆]

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Abstract

We provide explicit solutions for government spending multipliers during a liquidity trap and within a fixed exchange regime using standard closed and open-economy New Keynesian models. We confirm the potential for large multipliers during liquidity traps. For a currency union, we show that self-financed multipliers are small, always below unity, unless the accompanying tax adjustments involve substantial static redistribution from low to high marginal propensity to consume agents, or dynamic redistribution from future to present non-Ricardian agents. But outside-financed multipliers which require no domestic tax adjustment can be large, especially when the average marginal propensity to consume on domestic goods is high or when government spending shocks are very persistent. Our solutions are relevant for local and national multipliers, providing insight into the economic mechanisms at work as well as the testable implications of these models.

Keywords

Currency unions, Non-Ricardian effects, Open economy model, Liquidity traps, New Keynesian effects

JEL Classification Code

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1. INTRODUCTION

Economists generally agree that macroeconomic stabilization should be handled first and foremost by monetary policy. Yet monetary policy can run into constraints that impair its effectiveness. For example, the economy may find itself in a liquidity trap, where interest rates hit zero, preventing further reductions in the interest rate. Similarly, countries that belong to currency unions, or states within a country, do not have the option of an independent monetary policy. Some economists advocate for fiscal policy to fill this void,

increasing government spending to stimulate the economy. Others disagree, and the issue remains deeply controversial, as evidenced by vigorous debates on the magnitude of fiscal multipliers. No doubt, this situation stems partly from the lack of definitive empirical evidence, but, in our view, the absence of clear theoretical benchmarks also plays an important role. Although various recent contributions have substantially furthered our understanding, to date, the implications of standard macroeconomic models have not been fully worked out. This is the goal of this chapter. By clarifying the theoretical mechanisms in a unified way, we hope that it will help stimulate more research to validate or invalidate different aspects of the models.

We solve for the response of the economy to changes in the path for government spending during liquidity traps or within currency unions using standard New Keynesian closed and open-economy monetary models. A number of features distinguish our approach and contribution. First, our approach departs from the existing literature by focusing on *fiscal multipliers* that encapsulate the effects of spending for any path for government spending, instead of solving for a particular multiplier associated with the expansion of a single benchmark path for spending (eg, an autoregressive shock process to spending). Second, we obtain simple closed-form solutions for these multipliers. The more explicit and detailed expressions help us uncover the precise mechanisms underlying the effects of fiscal policy and allow us to deliver several new results.

Third, our analysis confirms that constraints on monetary policy are crucial, but also highlights that the nature of the constraint is also important. In particular, we draw a sharp contrast between a liquidity trap, with a binding zero-lower bound, and a currency union, with a fixed exchange rate.

Finally, in addition to nominal rigidities and constraints on monetary policy, we stress the importance of incorporating financial frictions for the analysis of fiscal policy. We do so by extending the benchmark models to include both incomplete markets and non-Ricardian borrowing constrained consumers, allowing for high and heterogeneous marginal propensities to consume out of current income. These financial market imperfections may be especially relevant in the aftermath of a financial crisis, situations where fiscal stimulus is often considered.

Our analysis has obvious implications for the interpretation of recent empirical studies on national and local multipliers. The empirical literature adopts different definitions of summary fiscal multipliers. For example, one popular notion used in many empirical studies consists in computing the ratio of some (discounted or not) average of the impulse responses of output and government spending in response to an innovation in government spending, up to some horizon (in practice 2 or 3 years). We show how our results can be used to compute such numbers analytically, and also discuss alternative definitions of summary fiscal multipliers.

Our results confirm that, in these standard models, fiscal policy can be especially potent during a liquidity trap. In the standard Ricardian model, the multiplier for output is always greater than one. We explicit the way in which the mechanism works through inflation. Higher government spending during a liquidity trap stimulates inflation. With fixed

nominal interest rates, this reduces real interest rates which increases current private consumption. The increase in consumption in turn leads to more inflation, creating a feedback loop. The fiscal multiplier is increasing in the degree of price flexibility, which is intuitive given that the mechanism relies on the response of inflation. We show that in the model, backloading spending leads to larger effects; the rationale is that inflation then has more time to affect spending decisions.

For a country or region in a currency union, by contrast, government spending is less effective at increasing output. In particular, in the standard Ricardian model, we show that private consumption is crowded out by government spending, so that the multiplier is less than one. Moreover, price flexibility diminishes the effectiveness of spending, instead of increasing it. We explain this result using a simple argument that illustrates its robustness. Government spending leads to inflation in domestically produced goods and this loss in competitiveness depresses private spending.

It may seem surprising that fiscal multipliers are less than one when the exchange rate is fixed, contrasting with multipliers above one in liquidity traps. We show that even though in both cases the nominal interest rate is fixed, there is a crucial difference: a fixed exchange rate implies a fixed nominal interest rate, but the reverse is not true. Indeed, we prove that the liquidity trap analysis implicitly combines a shock to government spending with a one-off devaluation. The positive response of consumption relies entirely on this devaluation. A currency union rules out such a devaluation, explaining the difference in the response of consumption.

In the context of a country in a currency union, our results uncover the importance of transfers from outside—from other countries or regions. In the short run, when prices have not fully adjusted, positive transfers from outside increase the demand for home goods, stimulating output. We compute "transfer multipliers" that capture the response of the economy to such transfers. We show that these multipliers may be large when there is a high degree of home bias (ie, low degree of openness).

Note that the analysis of outside transfers requires some form of market incompleteness. Otherwise, with complete financial markets, any outside transfer would be completely undone by private insurance arrangements with outsiders. Such an extreme offset is unlikely to be realistic. Thus, we modify the standard open-economy model, which assumes complete markets, to consider the case with incomplete markets.

Understanding the effect of outside transfers is important because such transfers are often tied to government spending. This is relevant for the literature estimating local multipliers, which exploits cross-sectional variation, examining the effects of government spending across regions, states, or municipalities, within a country. In the US federal military spending allocated to a particular state is financed by the country as a whole. The same is true for exogenous differences, due to idiosyncratic provisions in the law, in the distribution of a federal stimulus package. Likewise, idiosyncratic portfolio returns accruing to a particular state's coffers represent a windfall for this state against the rest.

When changes in spending are financed by such outside transfers, the associated multipliers are a combination of self-financed multipliers and transfer multipliers. As a result, multipliers may be substantially larger than one even in a currency union. This difference is more significant when the degree of home bias is large, since this increases the marginal propensity to spend on home produced goods.

The degree of persistence in government spending is also important. Because agents seek to smooth consumption over time, the more temporary the government spending shock, the more the per-period transfer that accompanies the increase in spending is saved in anticipation of lower per-period transfers in the future. As a result, the difference in the effects on current output between outside-financed and self-financed government spending can be large for relatively persistent shocks, but may be small if shocks are relatively temporary. However, as we shall see, this distinction is blurred in the presence of liquidity constraints.

We explore non-Ricardian effects from fiscal policy by introducing hand-to-mouth consumers in addition to permanent income consumers. We think of this as a tractable way of modeling liquidity constraints. Both in a liquidity trap and in a currency union, government spending now has additional effects because of the differences in marginal propensities to consume of both groups of agents.

First, the incidence of taxes across these two groups matters, and redistribution from low marginal propensity to consume permanent-income agents to high marginal propensity to consume hand-to-mouth agents increases output. Second, since the model is non-Ricardian, the timing of taxes matters.

Both these effects can play a role independently of government spending. Indeed, one may consider tax changes without any change in government spending. However, changes in government spending must be accompanied by changes in taxes. As a result, whether government spending is, at the margin, debt-financed or tax-financed matters. Likewise, the distributional makeup of tax changes, across marginal propensities to consume, also matters. These effects can potentially substantially increase fiscal multipliers, both in liquidity traps and for countries or regions in a currency union. In particular, they may raise the multipliers above one for a region within a currency union.

Most importantly, liquidity constraints significantly magnify the difference between self-financed and outside-financed fiscal multipliers for temporary government spending shocks. Intuitively, a higher marginal propensity to consume implies that a greater part of the outside transfer is spent in the short run, contributing towards an increase in fiscal multipliers.

Overall, this discussion brings back the old Keynesian emphasis on the marginal propensity to consume. In particular, for temporary government spending shocks, the difference between self-financed and outside-financed fiscal multipliers is large when the average marginal propensity to consume on domestic goods is large—either due to a large number of liquidity constrained agents or due to a high degree of home bias in spending.

Finally, we show how to bridge our results for small open economies in a currency union and closed economies in a liquidity trap by simultaneously considering the effects government spending in all the countries within a currency union, depending on whether the currency union is in a liquidity trap or whether the central bank of the union can target inflation by adjusting interest rates.

Related Literature

Our chapter is related to several strands of theoretical and empirical literatures. We will discuss those that are most closely related.

We contribute to the literature that studies fiscal policy in the New Keynesian model in liquidity traps. Eggertsson (2011), Woodford (2011), and Christiano et al. (2011) show that fiscal multipliers can be large at the zero lower bound, while Werning (2012) studies optimal government spending with and without commitment to monetary policy. Gali and Monacelli (2008) study optimal fiscal policy in a currency union, but they conduct an exclusively normative analysis and do not compute fiscal multipliers. The results and simulations reported in Corsetti et al. (2011), Nakamura and Steinsson (2011), and Erceg and Linde (2012) show that fiscal multipliers are generally below one under fixed exchange rates yet higher than under flexible exchange rates (away from the zero bound), somewhat validating the conventional Mundell–Flemming view that fiscal policy is more effective with fixed exchange rates (see, eg, Dornbusch, 1980). Our solutions extend these results and help sharpen the intuition for them, by discussing the role of implicit devaluations and transfers. Gali et al. (2007) introduce hand-to-mouth consumers and study the effects of government spending under a Taylor rule in a closed economy. Our setup extends such an analysis to liquidity traps and currency unions in an open economy. Cook and Devereux (2011) study the spillover effects of fiscal policy in open economy models of the liquidity trap. We also examine this question but focus on a different context, that of a currency union, depending on whether it is or not in a liquidity trap.

Our chapter is also related to a large empirical literature on fiscal multipliers. Estimating national fiscal multipliers poses serious empirical challenges. The main difficulties arise from the endogeneity of government spending, the formation of expectations about future tax and spending policies, and the reaction of monetary policy. Most of the literature tries to resolve these difficulties by resorting to Structural VARs. Some papers use military spending as an instrument for government spending. The relevant empirical literature is very large, so we refer the reader to Ramey (2011) for a recent survey. Estimating fiscal multipliers in liquidity traps is nearly impossible because liquidity traps are rare. The closest substitute is provided by estimates that condition of the level of economic activity. Some authors (see, eg, Gordon and Krenn, 2010; Auerbach and Gorodnichenko, 2012) estimate substantially larger national multipliers during deep recessions, but the magnitude of these differential effects remains debated (see, eg, Barro and Redlick, 2009).

States or regions within a country offer an attractive alternative with plausible exogenous variations in spending. Indeed the literature on local multipliers has recently been very active, with contributions by Clemens and Miran (2010), Cohen et al. (2010), Serrato and Wingender (2010), Shoag (2010), Acconcia et al. (2011), Chodorow-Reich et al. (2011), Fishback and Kachanovskaya (2010), and Nakamura and Steinsson (2011). These papers tend to find large multipliers. Our chapter helps interpret these findings. Government spending at the local level in these experiments is generally tied to transfers from outside. It follows that these estimates may be interpreted as combining spending and transfer multipliers, as we define them here.

2. MULTIPLIERS AND SUMMARY MULTIPLIERS

We first set the stage by taking a purely statistical perspective and use it discuss the connection between theory and empirical work.

Suppose one has isolated a relationship between output and government spending encoded in the dynamic response of both variables to a particular structural shock of interest. One may then summarize this relationship into a single "fiscal multiplier" number in a number of ways. Of course, the entire impulse response contains strictly more information, but the multiplier may be a convenient way to summarize it. In the rest of this chapter, we derive the response of output to *any* spending shock for a set of standard macroeconomic models. The implications of each model are encoded in a set of coefficients or loadings, which can be mapped into dynamic responses to output for any impulse from spending.

2.1 Responses and Shocks

2.1.1 Impulse Responses

Suppose we have two time series $\{\hat{g}_t, \hat{\gamma}_t\}$ for government spending and output respectively and that these series (after detrending) are stationary. Assume we can write these two series as a linear function of current and past shocks

$$\hat{g}_t = \hat{A}^g(L)\hat{\varepsilon}_t = \sum_{j=1}^{J} A^{gj}(L)\varepsilon_t^j = \sum_{j=1}^{J}\sum_{k=0}^{\infty} \psi_k^{gj}\varepsilon_{t-k}^j$$

$$\hat{\gamma}_t = \hat{A}^y(L)\hat{\varepsilon}_t = \sum_{j=1}^{J} A^{yj}(L)\varepsilon_t^j = \sum_{j=1}^{J}\sum_{k=0}^{\infty} \psi_k^{yj}\varepsilon_{t-k}^j$$

where the vector of shocks $\hat{\varepsilon}_t = (\varepsilon_t^1, \varepsilon_t^2, \ldots, \varepsilon_t^J)'$ have zero mean and are uncorrelated over time, $\mathbb{E}[\varepsilon_t] = 0$ and $\mathbb{E}[\hat{\varepsilon}_t\hat{\varepsilon}_s'] = 0$ for $t \neq s$. Let us next isolate the effect of one particular shock $j \in J$ and define the components $\{g_t, \gamma_t\}$ explained by this shock. Dropping the j subscript we write this as

$$g_t = A^g(L)\varepsilon_t = \sum_{k=0}^{\infty} \psi_k^g \varepsilon_{t-k} \tag{1a}$$

$$y_t = A^y(L)\varepsilon_t = \sum_{k=0}^{\infty} \psi_k^y \varepsilon_{t-k} \tag{1b}$$

where ε_t is a scalar shock with zero mean and is uncorrelated over time, $\mathbb{E}[\varepsilon_t] = 0$ and $\mathbb{E}[\varepsilon_t \varepsilon_s] = 0$ for $t \neq s$. The natural interpretation is that this particular shock, ε_t, is an exogenous structural shock to government spending. The coefficients $\{\psi_k^i\}$ are the impulse response functions (IRFs) to this shock. The responses can then be interpreted as encompassing a causal relationship. Strictly speaking, however, most of the discussion below does not require this interpretation.

2.1.2 VARs and Instruments

One way to obtain the decomposition of the series described above is using a structural VAR approach. To see this, suppose the original variables $\hat{g}_t$ and $\hat{y}_t$ are part of a VAR, which may include $J - 2$ other variables (eg, inflation and interest rates). Suppose ε_t is one of the shocks. By definition, this shock is white noise and is orthogonal to the remaining $J - 1$ shocks in the VAR at all leads and lags. In practice, the shock ε_t may be identified using structural assumptions, such as short-run or long-run restrictions. Under appropriate conditions, the shock may then acquire the economic interpretation of a fiscal shock and the response to output can be interpreted as an estimate of the causal relationship between spending and output.

Alternatively, the decomposition may result from an external instrumental variable. Suppose we have a scalar time series $\{z_t\}$ and let the Wold representation of z_t be[a]

$$z_t = A^z(L)\varepsilon_t = \sum_{k=0}^{\infty} \psi_k^z \varepsilon_{t-k}.$$

Thus, the shock ε_t is defined and identified as the innovation from the Wold representation of the instrument z_t. Now project $(\hat{g}_t, \hat{y}_t)$ linearly onto contemporaneous and lagged values of z_t, obtaining the predictors g_t and y_t (with residuals $\tilde{g}_t$ and $\tilde{y}_t$). These can then be represented as in (1). Once again, if the instrument is deemed exogenous to other economic fundamental shocks, then this shock may acquire economic interpretation as a fiscal shock and the response of output and spending can be interpreted as an estimate of the causal relationship between these variables.

2.2 Summary Multipliers

The sequences $\{\psi_k^g, \psi_k^y\}$ provide a full characterization of the joint behavior of $\{y_t\}$ and $\{g_t\}$, with respect to the shock $\{\varepsilon_t\}$. Suppose one insists on summarizing this

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[a] Abstracting from the deterministic component.

relationship by a single number, called a "fiscal multiplier." First define the contemporaneous multiplier

$$m_k = \frac{\psi_k^\gamma}{\psi_k^g}$$

indexed by $k = 0, 1, \ldots$ A general summary multiplier may take a ratio of the form

$$M^\gamma = \frac{\sum_{k=0}^\infty \lambda_k^\gamma \psi_k^\gamma}{\sum_{k=0}^\infty \lambda_k^g \psi_k^g} = \frac{\sum_{k=0}^\infty \lambda_k^\gamma \psi_k^g}{\sum_{k=0}^\infty \lambda_k^g \psi_k^g} \sum_{k=0}^\infty m_k \omega_k$$

where $\omega_k = \lambda_k^\gamma \psi_k^g / \sum_{k=0}^\infty \lambda_k^\gamma \psi_k^g$ is a weight that adds up to unity. A simple case is to add up the unweighted the reaction over the first N periods,

$$M^\gamma = \frac{\sum_{k=0}^N \psi_k^\gamma}{\sum_{k=0}^N \psi_k^g} = \sum_{k=0}^N m_k \omega_k,$$

where $\omega_k = \psi_k^g / \sum_{k=0}^N \psi_k^g$.

2.2.1 Regression Based Summary Multipliers: OLS and IV
Another popular way to proceed in obtaining a summary fiscal multiplier is regress output on spending and to take the coefficient on spending as a summary multiplier. Consider the relationship

$$\hat{y}_t = \beta^{OLS} \hat{g}_t + u_t^{OLS},$$

where $\mathbb{E}[\hat{g}_t u_t^{OLS}] = 0$ and

$$\beta^{OLS} \equiv \frac{\mathbb{E}[\hat{g}_t \hat{y}_t]}{\mathbb{E}[\hat{g}_t^2]} = \frac{\sum_{j=1}^J \sum_{k=0}^\infty \psi_k^{\gamma j} \psi_k^{gj}}{\sum_{j=1}^J \sum_{k=0}^\infty \left(\psi_k^{gj}\right)^2} = \sum_{j=1}^J \sum_{k=0}^\infty m_k^j \omega_k^j.$$

where

$$m_k^j = \frac{\psi_k^{\gamma j}}{\psi_k^{gj}}, \qquad \omega_k^j \equiv \frac{\left(\psi_k^{gj}\right)^2}{\sum_{l=0}^\infty \left(\psi_l^{gj}\right)^2}.$$

Thus, the population regression recovers a weighted average of the k-multipliers associated with each shock j.

Consider next an instrumental variable regression

$$\hat{\gamma}_t = \beta^{IV} \hat{g}_t + u_t^{IV}$$

where $\mathbb{E}[z_t u_t^{IV}] = 0$ and

$$\beta^{IV} \equiv \beta^{OLS} \equiv \frac{\mathbb{E}[\gamma_t z_t]}{\mathbb{E}[g_t z_t]} = \frac{\sum_{k=0}^{\infty} \psi_k^{\gamma} \psi_k^{\tilde{z}}}{\sum_{k=0}^{\infty} \psi_k^{g} \psi_k^{\tilde{z}}} = \sum_{k=0}^{\infty} m_k \omega_k,$$

with weights

$$\omega_k \equiv \frac{\psi_k^{g} \psi_k^{\tilde{z}}}{\sum_{l=0}^{\infty} \psi_l^{g} \psi_k^{\tilde{z}}}.$$

These weights are positive if ψ_k^{g} and $\psi_k^{\tilde{z}}$ take the same sign.[b]

2.3 Connection to Models

As we will show, the implications of a model for fiscal spending can be encoded in a sequence of theoretical multipliers $\{\alpha_{t,k}\}$, where the element $\alpha_{t,k}$ represents the predicted response of output in period t to government spending in period k. This response is calculated as the first-order effect by linearizing the model.

What is the connection between $\{\alpha_{t,k}\}$ and the impulse responses $\{\psi_k^{g}\}$ and $\{\psi_k^{g}\}$ discussed above? Suppose we can interpret ε_t as an exogenous shock to the path for spending as summarized by $\{\psi_k^{g}\}$ and we can interpret the change in spending as a having causal endogenous response in output summarized by $\{\psi_t^{\gamma}\}$. In the model both responses would be related by

$$\psi_k^{\gamma} = \sum_{k'=0}^{\infty} \psi_{k'}^{g} \alpha_{k,k'},$$

for all $t = 0, 1, \ldots$ Given the theoretical multipliers, this relationship give us the output response $\{\psi_k^{\gamma}\}$ for any given government spending response $\{\psi_t^{g}\}$.

Under what conditions can we invert this relationship and identify the theoretical multipliers $\{\alpha_{t,k}\}$ from the responses $\{\psi_k^{g}\}$ and $\{\psi_k^{\gamma}\}$? For a single pair of $\{\psi_k^{g}\}$ and $\{\psi_k^{\gamma}\}$ the answer is generally negative. For any given k the $\alpha_{k,\cdot}$ sequence is not identified: we can only identify the value of the sum $\sum_{k'=0}^{\infty} \psi_{k'}^{g} \{\alpha_{k,k'}\}$.

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[b] In some cases, for example, Nakamura and Steinsson (2011), the IV regressions are run in differences. It is straightforward to adjust the calculations above in this case.

Without further information identification would only be possible if we had multiple responses, $\{\psi_k^g\}$ restrictions, $\{\psi_k^\gamma\}$, that is, multiple spending shocks.

A special case obtains if the response is purely forward looking, as is the case in some of the simplest macroeconomic models. To see this, assume that $\alpha_{t,k} = \alpha_{0,k-t}$ for $k = t, t+1, \ldots$ and $\alpha_{t,k} = 0$ for $k = 1, 2, \ldots, t-1$. Then we have

$$\psi_t^\gamma = \sum_{k=t}^{\infty} \psi_k^g \alpha_{0,k-t}.$$

Then we can identify the entire sequence $\{\alpha_{0,k-t}\}$ from the pair of sequences $\{\psi_k^g\}$ and $\{\psi_k^\gamma\}$, provided we satisfy a standard rank condition (so that the set of sequences $\{\psi_{k-t}^g\}$ for $t \in \{0, 1, \ldots\}$ are linearly independent).

3. A CLOSED ECONOMY

We consider a one-time shock to the current and future path of spending that is realized at the beginning of time $t = 0$ that upsets the steady state. To simplify and focus on the impulse response to this shock, we abstract from ongoing uncertainty at other dates.[c] We adopt a continuous time framework. This is convenient for some calculations but is completely inessential to any of our results.

The remainder of this section specifies a standard New Keynesian model environment; readers familiar with this setting may wish to skip directly to Section 4.

Households

There is a representative household with preferences represented by the utility function

$$\int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right] dt,$$

where N_t is labor, and C_t is a consumption index defined by

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $j \in [0, 1]$ denotes an individual good variety. Thus, ϵ is the elasticity between varieties produced within a given country. We denote by $P_t(j)$ is the price of variety j, and by

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$$

the corresponding price index.

[c] Since we are interested in a first order approximation of the equilibrium response to shocks, which can be solved by studying the log-linearized model, the presence of ongoing uncertainty would not affect any of our calculation or conclusions (we have certainty equivalence).

Households seek to maximize their utility subject to the budget constraints

$$\dot{D}_t = i_t D_t - \int_0^1 P_t(j) C_t(j) dj + W_t N_t + \Pi_t + T_t$$

for $t \geq 0$ together with a no-Ponzi condition. In this equation, W_t is the nominal wage, Π_t represents nominal profits and T_t is a nominal lump sum transfer. The bond holdings of home agents are denoted by D_t and the nominal interest rate for the currency union is denoted by i_t.

Government

Government consumption G_t is an aggregate of varieties just as private consumption,

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

For any level of expenditure $\int_0^1 P_t(j) G_t(j) dj$, the government splits its expenditure across these varieties to maximize G_t. Spending is financed by lump-sum taxes. Ricardian equivalence holds, so that the timing of these taxes is irrelevant.

Firms

A typical firm produces a differentiated good with a linear technology

$$Y_t(j) = A_t N_t(j),$$

where A_t is productivity in the home country.

We allow for a constant employment tax $1 + \tau^L$, so that real marginal cost is given by $\dfrac{1 + \tau^L}{A_t} \dfrac{W_t}{P_t}$. We take this employment tax to be constant in our model, as in standard in the literature. The tax rate is set to offset the monopoly distortion so that $\tau^L = -\dfrac{1}{\varepsilon}$. However, none of our results hinge on this particular value.

We adopt the standard Calvo price-setting framework. In every moment a randomly flow ρ_δ of firms can reset their prices. Those firms that reset choose a reset price P_t^r to solve

$$\max_{P_t^r} \int_0^\infty e^{-\rho_\delta s - \int_0^s i_{t+z} dz} \left(P_t^r Y_{t+s|t} - (1 + \tau^L) W_t \frac{Y_{t+s|t}}{A_t} \right),$$

where $Y_{t+k|t} = \left(\dfrac{P_t^r}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}$, taking the sequences for W_t, Y_t and P_t as given.

3.1 Equilibrium Conditions

We now summarize equilibrium conditions for the home country. Market clearing in the goods and labor market requires that:

$$Y_t = C_t + G_t,$$

$$N_t = \frac{Y_t}{A_t}\Delta_t,$$

where Δ_t is an index of price dispersion $\Delta_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon}$. The Euler equation

$$\sigma\frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho$$

ensures the agents' intertemporal optimization, where $\pi_t = \dot{P}_t/P_t$ is inflation.

The natural allocation is a reference allocation that prevails if prices are flexible and government consumption is held constant at its steady state value G. We denote the natural allocation with a bar over variables.

We omit the first-order conditions for the price-setting problem faced by firms here. We shall only analyze a log-linearized version of the model which collapses these equilibrium conditions into the New Keynesian Phillips curve presented below.

4. NATIONAL MULTIPLIERS IN A LIQUIDITY TRAP

To obtain multipliers, we study the log-linearized equilibrium conditions around the natural allocation with constant government spending. Define

$$c_t = (1 - \mathcal{G})(\log(C_t) - \log(\bar{C}_t)) \approx \frac{C_t - \bar{C}_t}{Y},$$

$$y_t = \log Y_t - \log \bar{Y}_t \approx \frac{Y_t - \bar{Y}_t}{Y} \qquad g_t = \mathcal{G}(\log G_t - \log G) \approx \frac{G_t - G}{Y},$$

where $\mathcal{G} = \frac{G}{Y}$. So that we have, up to a first order approximation,

$$y_t = c_t + g_t.$$

The log linearized system is then

$$\dot{c}_t = \hat{\sigma}^{-1}(i_t - \pi_t - \bar{r}_t), \tag{2}$$

$$\dot{\pi}_t = \rho\pi_t - \kappa(c_t + (1 - \xi)g_t), \tag{3}$$

where $\hat{\sigma} = \frac{\sigma}{1 - \mathcal{G}}$, $\lambda = \rho_\delta(\rho + \rho_\delta)$, $\kappa = \lambda(\hat{\sigma} + \phi)$ and $\xi = \frac{\hat{\sigma}}{\hat{\sigma} + \phi}$. Eq. (2) is the Euler equation and Eq. (3) is the New Keynesian Philips curve. Here, $\bar{r}_t$ is the natural rate of interest, defined as the real interest rate that prevail at the natural allocation, ie, Eq. (2) with $c_t = 0$ for all $t \geq 0$ implies $i_t - \pi_t = \bar{r}_t$ for all $t \geq 0$.

It will prove useful to define the following two numbers ν and $\bar{\nu}$ (the eigenvalues of the system):

$$\nu = \frac{\rho - \sqrt{\rho^2 + 4\kappa\hat{\sigma}^{-1}}}{2} \qquad \bar{\nu} = \frac{\rho + \sqrt{\rho^2 + 4\kappa\hat{\sigma}^{-1}}}{2}.$$

If prices were completely flexible, then consumption and labor are determined in every period by two static conditions: the labor consumption condition and the resource constraint. Spending affects the solution and gives rise to the neoclassical multiplier $1 - \xi$, which is positive but less than 1 and entirely due to a wealth effect on labor supply.

From now on, we take as given a path for the interest rate $\{i_t\}$ summarizing monetary policy. To resolve or sidestep issues of multiplicity one can assume that there is a date T such that $c_t = g_t = \pi_t = 0$ and $i_t = \bar{r}_t$ for $t \geq T$.[d] A leading example is a liquidity trap scenario where $i_t = 0$ and $\bar{r}_t < 0$ for $t < T$. However, although this is a useful interpretation but is not required for the analysis below.

Remark 1 Suppose $c_T = 0$ for some date T, then

$$c_t = \int_t^T (i_{t+s} - \pi_{t+s} - \bar{r}_{t+s}) ds,$$

so that given the inflation path $\{\pi_t\}$ the consumption path $\{c_t\}$ is independent of the spending path $\{g_t\}$.

This remark highlights that the mechanism by which government spending affects consumption, in the New Keynesian model, is inflation which affects the real interest rate. One can draw two implications from this. First, other policy instruments that affect inflation, such as taxes, may have similarly policy effects. Second, empirical work on fiscal multipliers has not focused on the role inflation plays and it may be interesting to test the predicted connection between output and inflation present in New Keynesian models.

4.1 Fiscal Multipliers Solved

Since the system is linear it admits a closed form solution. We can express any solution with government spending as

$$c_t = \tilde{c}_t + \int_0^\infty \alpha_s^c g_{t+s} ds, \tag{4a}$$

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[d] Note that T may be arbitrarily large and will have no impact on the solution provided below. Indeed, the characterization of the equilibrium is valid even without selecting an equilibrium this way: one just interprets c^* and π^* below any equilibrium in the set of equilibrium attained when $g_t = 0$ for all t. The solution then describes the entire set of equilibria for other spending paths $\{g_t\}$.

$$\pi_t = \tilde{\pi}_t + \int_0^\infty \alpha_s^\pi g_{t+s} ds, \tag{4b}$$

where $\{\tilde{c}_t, \tilde{\pi}_t\}$ are equilibria with $g_t = 0$ for all t. We focus on the integral term $\int_0^\infty \alpha_s^i g_{t+s} ds$ for $i = c, \pi$ as a measure of the effects of fiscal policy $g \neq 0$. We assume the integrals are well defined, although we allow and discuss the case where it is $+\infty$ or $-\infty$ below.

Focusing on consumption, we call the sequence of coefficients $\{\alpha_s^c\}$ *fiscal multipliers*. It is crucial to note that these are *total private consumption multipliers* and not *output multipliers*. Indeed, output is given by

$$y_t = \tilde{y}_t + g_t + \int_0^\infty \alpha_s^c g_{t+s} ds.$$

Whereas the natural benchmark for consumption multipliers is 0, that for output multipliers is 1.

The coefficients α_s^c do not depend on calendar time t, nor do they depend on the interest rate paths $\{i_t\}$ and $\{r_t\}$. Thus, the impact on consumption or output, given by the term $\int_0^\infty \alpha_s^c g_{t+s} ds$, depends only on the future path for spending summarized weighted by $\{\alpha_s^c\}$.

There are two motivations for adopting $\int_0^\infty \alpha_s^c g_{t+s} ds$ as a measure of the impact of fiscal policy, one more practical, the other more conceptual.

1. The more practical motivation applies if the economy finds itself in a liquidity trap with nominal interest rates immobilized at zero, at least for some time. Fiscal multipliers $\{\alpha_s^c\}$ can then be used to predict the effects of fiscal policy. To see this, suppose the zero lower bound is binding until T so that $i_t = 0$ for $t < T$; suppose that after T monetary policy delivers an equilibrium with zero inflation, so that $\pi_t = 0$ for $t \geq T$. As is well known, the resulting equilibrium without government spending ($g_t = 0$ for all t) features a negative consumption gap and deflation: $\tilde{c}_t, \tilde{\pi}_t < 0$ for $t < T$ (see, eg, Werning, 2012).

    Now, consider a stimulus plan that attempts to improve this outcome by setting $g_t > 0$ for $t < T$ and $g_t = 0$ for $t \geq T$. Then $\int_0^\infty \alpha_s^c g_{t+s} ds = \int_0^{T-t} \alpha_s^c g_{t+s} ds$ is precisely the effect of the fiscal expansion on consumption c_t, relative to the outcome without the stimulus plan $\tilde{c}_t$.

    More generally, suppose that after the trap spending may be nonzero and that monetary may or may not be described as securing zero inflation. Even in this case, we may still use fiscal multipliers to measure the impact of fiscal policy during the liquidity trap: one can write $c_t = c_T + \int_0^{T-t} \alpha_s^c g_{t+s} ds$ for $t < T$, where the c_T encapsulates the combined effects of fiscal and monetary policy after the trap $t \geq T$.

2. More conceptually, our fiscal multipliers provide a natural decomposition of the effects of the fiscal policy, over what is attainable by monetary policy alone.

    Eqs. (4a) and (4b) characterize the entire set of equilibria for $g \neq 0$ by providing a one-to-one mapping between equilibria with $g = 0$. Both $\tilde{c}_t$ and $\tilde{\pi}_t$ are equilibria with

$g = 0$ and are affected by monetary policy, as summarized, among other things, by the interest rate path $\{i_t\}$.

We can represent these facts as a relationship between the set of equilibria with and without government spending,

$$\mathcal{E}_g = \mathcal{E}_0 + \alpha \cdot g,$$

where $\mathcal{E}_0$ represents the set of equilibria when $g_t = 0$ for all t, while $\mathcal{E}_g$ is the set of equilibria for a given path for spending $g = \{g_t\}$. Here $\alpha = \{\alpha_s^c, \alpha_s^\pi\}$ collects the fiscal multipliers and the cross product $\alpha \cdot g$ represents the integrals $\int_0^\infty \alpha_s^i g_{t+s} ds$ for $i = c, \pi$. The set $\mathcal{E}_g$ is a displaced version of $\mathcal{E}_0$ in the direction $\alpha \cdot g$. Each equilibrium point in $\mathcal{E}_0$ is shifted in parallel by $\alpha \cdot g$ to another equilibrium point in $\mathcal{E}_g$ and it shares the same nominal interest rate path $\{i_t\}$. This last fact is unimportant for this second conceptual motivation, since the focus is on comparing the two sets, not equilibrium points. Instead, the important issue is that $\alpha \cdot g$ measures the influence of government spending on the set of equilibria. This provides a conceptual motivation for studying the multipliers α, since they summarize this influence. In other words, without spending one can view monetary policy as selecting from the set $\mathcal{E}_0$, while with government spending monetary policy can choose from $\mathcal{E}_g$. The effects of fiscal policy on the new options is then precisely determined by the shift $\alpha \cdot g$. Fig. 1 represents this idea pictorially.[e]

Our first result delivers a closed–form solution for fiscal multipliers. Using this closed form one can characterize the multiplier quite tightly.
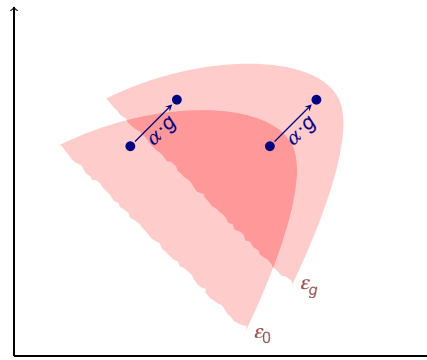
Fig. 1 A schematic depiction of the set of equilibria without government spending and the set of equilibria for a given spending path $\{g_t\}$.

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[e] The figure is purposefully abstract and meant to convey the notion of a parallel shift only, so we have not labeled either axis and the shape of the sets is purely for illustrative purposes.

Proposition 1 (Closed Economy Multipliers) *The fiscal multipliers are given by*

$$\alpha_s^c = \hat{\sigma}^{-1}\kappa(1-\xi)e^{-\bar{\nu}s}\left(\frac{e^{(\bar{\nu}-\nu)s}-1}{\bar{\nu}-\nu}\right).$$

The instantaneous fiscal multiplier is zero $\alpha_0^c = 0$, *but the fiscal multipliers are positive, increasing and convex for large* s *so that* $\lim_{s\to\infty}\alpha_s^c = \infty$.

The left panel of Fig. 2 displays these consumption multipliers α_s^c as a function of s for a standard calibration. The proposition states that current spending has no effect on consumption: $\alpha_0^c = 0$. By implication, changes in spending that are very temporary are expected to have negligible effects on consumption and have an output multiplier that is near unity. As stated earlier, the effects of government spending on consumption work through inflation. Current spending does affect the current inflation rate and thus affects the growth rate of consumption. However, since this higher inflation is so short lived the lower growth rate for consumption has no significant stretch of time to impact the level of consumption.

In contrast, spending that takes place in the far future can have a very large impact. The further out into the future, the larger the impact, since α_s^c is increasing in s. Indeed, in the limit the effect becomes unbounded, since $\lim_{s\to\infty}\alpha_s^c = \infty$. The logic behind these results is that spending at $s > 0$ increases inflation over the entire interval of time $[0, s]$. This then lowers the real interest over this same time interval and lowers the growth rate of consumption. Since the long–run consumption level is fixed, the lower growth rate raises the level of consumption. This rise in consumption in turn leads to higher inflation, creating a feedback cycle. The larger the interval $[0, s]$ over which these effect have time to act, the larger is the effect on consumption.
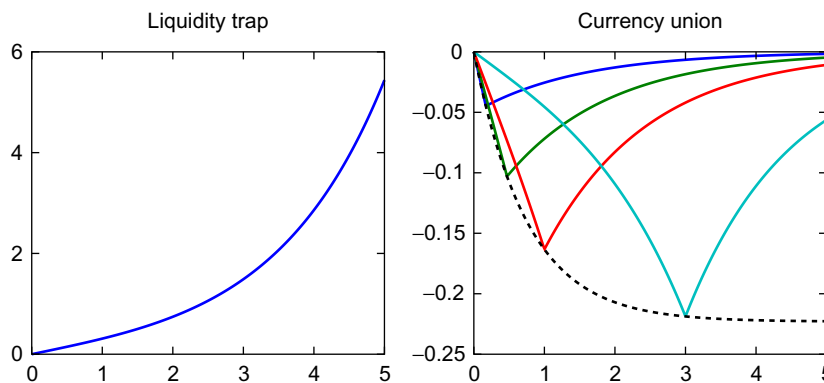
Fig. 2 Liquidity trap and currency union consumption multipliers α_s^c and $\alpha_{s-t}^{c,t,CM}$ as a function of s. Each curve for $\alpha_{s-t}^{c,t,CM}$ is plotted for different values of $t \in \{0.25, 0.5, 1, 3\}$. The black dashed line shows the lower envelope. Parameters are $\sigma = 1$, $\eta = \gamma = 1$, $\varepsilon = 6$, $\phi = 3$, $\lambda = 0.14$, and $\alpha = 0.4$.

The fact that fiscal multipliers are unbounded as $s \to \infty$ stands in strong contrast to the zero multiplier at $s = 0$. It also has important implications. For example, a positive path for spending $\{g_t\}$ that is very backloaded can create a very large response for consumption. This is the case if the shock to spending is very persistent.

Example 1 (AR(1) Spending) Suppose $g_t = g e^{-\rho_g t}$, then if $\rho_g > -\nu > 0$ the response of consumption c_t is finite and given by

$$\int \alpha_s^c g e^{-\rho_g(t+s)} ds = \frac{\hat{\sigma}^{-1} \kappa (1 - \xi)}{(\rho_g + \nu)(\rho_g + \bar{\nu})} g e^{-\rho_g t}.$$

The condition $\rho_g > -\nu > 0$ requires spending to revert to zero fast enough to prevent the integral from being infinite.

Some paths for spending imply an infinite value for $\int_0^\infty \alpha_s^c g_s ds$. For instance, this is the case in the example above when $\rho_g < -\nu$. How should one interpret such cases? Technically, this may invalidate our approximation. However, we think the correct economic conclusion to draw is that spending will have an explosive positive effect on consumption. One way to see this is to truncate the path of spending $\{g_t\}$, by setting $g_t = 0$ for all $t \geq T$ for some large T. This ensures that $\int_0^T \alpha_s^c g_s ds$ is finite but the response is guaranteed to be very large if the cutoff is large.

Next, we ask how fiscal multipliers are affected by the degree of price stickiness. Departures from the neoclassical benchmark, where the consumption multiplier is negative, require some stickiness in prices. Perhaps surprisingly, the resulting Keynesian effects turn out to be decreasing in the degree of price stickiness.

Proposition 2 (Price Stickiness) *The fiscal multipliers* $\{\alpha_s^c\}$
1. *are zero when prices are rigid* $\kappa = 0$;
2. *are increasing in price flexibility* κ;
3. *converge to infinity,* $\alpha_s^c \to \infty$, *in the limit as prices become fully flexible so that* $\kappa \to \infty$.

The logic for these results relies on the fact that spending acts on consumption through inflation. At one extreme, if prices were perfectly rigid then inflation would be fixed at zero and spending has no effect on consumption. As prices become more flexible spending has a greater impact on inflation and, hence, on consumption. Indeed, in the limit as prices become perfectly flexible, inflation becomes so responsive that the effects on consumption explode.

Recall that our fiscal multipliers are calculated under the assumption that the path for interest rates remains unchanged when spending rises. These results seem less counterintuitive when one realizes that such a monetary policy, insisting on keeping interest rates unchanged, may be deemed to be looser when prices are more flexible and inflation reacts more. Of course, this is precisely the relevant calculation when the economy finds itself in a liquidity trap, so that interest rates are up against the zero lower bound.

We capture backloading by a first order dominant shift in the cumulative distribution of spending for a given net present value of output. Backloading leads to a higher path of consumption at every point in time. This is simply because backloading gives more time to the feedback loop between output and inflation to play out.

When applied in a liquidity trap setting it is important to keep in mind the correct interpretation of this result. Our calculations compare spending paths at constant interest rates. In a liquidity trap, this translates to changes in spending before the end of the liquidity trap. If spending is delayed past the liquidity trap this affects consumption differently. For example, if after the end of the trap T monetary policy targets zero inflation, then government spending lowers consumption at T. This feeds back to consumption at $t = 0$, according to $c_t = c_T + \int_0^{T-t} \alpha_s^c g_{t+s} ds$ for $t < T$, lowering the impact on consumption and potentially reversing it. We conclude that backloading spending within the trap increases summary multipliers, but delaying spending past the trap reduce it.

4.2 Summary Fiscal Multipliers Again

Up to now we have discussed properties of fiscal multipliers $\{\alpha_s^c\}$. Usually, fiscal multipliers are portrayed as a single number that summarizes the impact of some change in spending on output or consumption, perhaps conditional on the state of the economy or monetary policy. This requires collapsing the entire sequence of fiscal multipliers $\{\alpha_s^c\}$ into a single number $\bar{\alpha}$, which we shall call a *summary fiscal multiplier,* such as

$$M^c = \frac{\int_0^\infty \lambda_t^c \int_0^\infty \alpha_s^c g_{t+s} ds \, dt}{\int_0^\infty \lambda_t^g g_t dt},$$

where $\{\lambda_t^c\}$ and $\{\lambda_t^g\}$ are weights. It is most natural to consider symmetric weights, with $\lambda_t^g = \lambda_t^c = \lambda_t$, which assume from now on. The simplest weight sets $\lambda_t = 1$ for $t \leq \tau$ and $\lambda_t = 0$ for $t > \tau$, which then computes the ratio of the total responses over the interval $[0, \tau]$. Another possibility is to set $\lambda_t = e^{-\rho t}$, to compute the ratio of the present value responses over the entire horizon.[f]

Note that since $y_t = c_t + g_t$ we have that the output multiplier (defined analogously to the consumption multiplier) is simply[g]

$$M^y = M^c + 1.$$

As this discussion makes clear there are many possibilities for summary multipliers and no universal criteria to select them. Instead, one can adapt the summary multiplier to the

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[f] The empirical counterpart of such an infinite-horizon calculation is, however, impractical.

[g] That is, we define

$$M^y = \frac{\int_0^\infty \lambda_t \left(\int_0^\infty \alpha_s^c g_{t+s} ds + g_t \right) dt}{\int_0^\infty \lambda_t g_t dt}.$$

application and relevant policy at hand. The characterizations provided in the previous section have implications for any of these measures. Namely,

  i if spending $\{g_t\}$ converges to being concentrated at $t = 0$ then $M^c \to 0$;
  ii the more backloaded is government spending for a given net present value, the higher is M^c;
 iii the multiplier M^c is increasing in flexibility, it is zero with rigid prices $\kappa = 0$ and goes to infinity in the limit of flexible prices $\kappa \to \infty$.

Example 2 Suppose we have an autoregressive spending path $g_t = g e^{-\rho_g t}$ for $\rho_g > 0$. The summary multiplier is independent of g_0 and given by

$$M^c = \frac{\int_0^\infty \lambda_t \int_0^\infty \alpha_s^c g_{t+s} ds\, dt}{\int_0^\infty \lambda_t g_t dt} = \frac{\int_0^\infty \lambda_t \int_0^\infty \alpha_s^c e^{-\rho_g(t+s)} ds\, dt}{\int_0^\infty \lambda_t e^{-\rho_g t} dt} = \int_0^\infty \alpha_s^c e^{-\rho_g s} ds.$$

Higher values of ρ_g shift weight towards the future. More persistence leads to higher summary multipliers.

4.3 Endogenous Spending: Policy Shocks vs Policy Rules

Up to now we have considered exogenous changes in government spending and their impact on output—a fiscal policy shock. Many stimulus policies, however, are best thought of as responding endogenously to the state of the economy—a fiscal policy rule.

Since the state of the economy depends on the model parameters, this implies that model parameters may play a double role when evaluating fiscal policy rules, as opposed to evaluating fiscal policy shocks.

In this short section we briefly touch on this issue using two examples. Formally, a change in parameters may affect both the structural fiscal multipliers $\{\alpha_t^c\}$, as we have discussed, and the path for government spending $\{g_t\}$. Both may have effects on output and summary fiscal multipliers.

Example 3 Christiano et al. (2011) compute summary fiscal multipliers in a liquidity trap. They assume a policy for government spending that increases spending by a constant amount as long the economy remains in the liquidity trap. They vary the degree of price flexibility and the duration of the liquidity trap and compute the fiscal multiplier (see Fig. 2).

Their summary multiplier is equivalent to computing the initial output response divided by the initial spending increase. Their results suggest that parameter values that make the recession worse also lead to larger multipliers. In some cases, this follows because the parameters affect the fiscal multipliers $\{\alpha_s^c\}$ directly. For example, this is the case for the degree of price flexibility κ. Higher price flexibility makes the recession worse and leads to higher fiscal multipliers, as shown in Proposition 2.

However, in other cases their conclusion rely on the indirect effects that these parameters have on the policy experiment $\{g_t\}$ itself. Indeed, this may affect summary multipliers even when our multipliers $\{\alpha_s^c\}$ are unchanged. Their setup features Poisson uncertainty regarding the length of the trap, but the same logic applies in a deterministic setting, when the liquidity trap has a known duration T.[h]

Suppose the economy is in a liquidity trap with zero interest rates for $t \leq T$ and returns to the natural allocation $c_t = g_t = 0$ for $t \geq T$. Consider fiscal policy interventions that increase spending during the trap, $g_t = g$ for $t \leq T$ and $g_t = 0$ for $t > T$. Higher T then leads to a deeper recession (see Werning, 2012) but has no effect on fiscal multipliers $\{\alpha_t^c\}$. However, the summary impact multiplier computed as

$$\frac{\int_0^T \alpha_s^c g \, ds}{g} = \int_0^T \alpha_s^c \, ds,$$

is increasing and convex in T. A longer liquidity trap increases this summary multiplier even though spending at any point in time is equally effective (α_s^c unchanged). It would be wrong to conclude that a stimulus plan with a fixed duration $\tau \leq T$ (a policy *shock*), such as a year or two, becomes more powerful when T increases. Rather, if $g_t = g$ for all $t \leq T$ (a policy rule) when T increases, then the effect on output is larger simply because the increase in T extends the time frame over which a fixed increase in spending g takes place, leading to an increase in the cumulative change in spending, Tg. Since cumulative spending increases, the impact effect would be larger even if, counter to the model, α_s^c were constant. Moreover, this effect is amplified because the extension backloads spending, and Proposition 1 shows that this is particularly effective since α_s^c is increasing in s.

Example 4 Another perspective is provided when g_t is set as a linear function of current consumption

$$g_t = -\Psi c_t,$$

for some $\Psi > 0$. Then the Phillips curve becomes

$$\dot{\pi}_t = \rho \pi_t - \kappa(c_t + (1 - \xi)g_t) = \rho \pi_t - \kappa(1 - (1 - \xi)\Psi)c_t.$$

Suppose further that $\Psi = (1-\xi)^{-1}$, so that spending "fills the gap" and $c_t + (1 - \xi)g_t = 0$. We maintain the assumption that $c_t = g_t = 0$ for $t \geq T$. Inflation is then zero for all $t \geq 0$ and the outcome for consumption is *as if* prices were completely rigid. Now, with this fiscal policy in place, consider different values for price flexibility κ. Neither the outcome for consumption $\{c_t\}$ nor the spending path $\{g_t\}$ depend on κ. Thus, in this special case, for given T, the fiscal rule can be interpreted as a fiscal shock, since it is independent of κ.

[h] Their parameter p, which represents the probability of remaining in the trap, has an effect similar to T in our deterministic setting.

However, the benchmark equilibrium outcome without spending, ie, $g_t = 0$, is decreasing in price flexibility κ (see Werning, 2012). Thus, fiscal policy has a greater effect on consumption when prices are more flexible. This is consistent with Proposition 2 regarding the effects of price flexibility on $\{\alpha_s^c\}$.

5. AN OPEN ECONOMY MODEL OF A CURRENCY UNION

We now turn to open economy models similar to Farhi and Werning (2012a,b) which in turn build on Gali and Monacelli (2005, 2008).

The model focuses on a continuum of regions or countries that share a common currency. One interpretation is that these regions are states or provinces within a country. Our analysis is then directly relevant to the literature estimating "local" multipliers, exploiting cross-sectional variation in spending behavior across states in the United States to estimate the effects on income and employment. Another interpretation is to member countries within a currency union, such as the European Monetary Union (EMU). Our analysis then sheds light on the debates over fiscal policy, stimulus vs austerity, for periphery countries.

For concreteness, from now on we will refer to these economic units (regions or countries) simply as countries. We focus on the effects around a symmetric steady state after a fiscal policy is realized in every country. A crucial ingredient is how private agents share risk internationally. We consider the two polar cases: (i) incomplete markets, where agents can only trade a risk-free bond; and (ii) complete markets with perfect risk sharing. These two market structures have different implications for fiscal multipliers.

5.1 Households

There is a continuum measure one of countries $i \in [0, 1]$. We focus attention on a single country, which we call "home" and can be thought of as a particular value $H \in [0, 1]$. We will focus on a one time shock, so that all uncertainty is realized at $t = 0$. Thus, we can describe the economy after the realization of the shock as a deterministic function of time.

In every country, there is a representative household with preferences represented by the utility function

$$\int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right] dt,$$

where N_t is labor, and C_t is a consumption index defined by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $C_{H,t}$ is an index of consumption of domestic goods given by

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $j \in [0, 1]$ denotes an individual good variety. Similarly, $C_{F,t}$ is a consumption index of imported goods given by

$$C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}},$$

where $C_{i,t}$ is, in turn, an index of the consumption of varieties of goods imported from country i, given by

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Thus, ϵ is the elasticity between varieties produced within a given country, η the elasticity between domestic and foreign goods, and γ the elasticity between goods produced in different foreign countries. An important special case obtains when $\sigma = \eta = \gamma = 1$. We call this the Cole–Obstfeld case, in reference to Cole and Obstfeld (1991).

The parameter α indexes the degree of home bias, and can be interpreted as a measure of openness. Consider both extremes: as $\alpha \to 0$ the share of foreign goods vanishes; as $\alpha \to 1$ the share of home goods vanishes. Since the country is infinitesimal, the latter captures a very open economy without home bias; the former a closed economy barely trading with the outside world.

Households seek to maximize their utility subject to the budget constraints

$$\dot{D}_t = i_t D_t - \int_0^1 P_{H,t}(j) C_{H,t}(j) dj - \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + W_t N_t + \Pi_t + T_t$$

for $t \geq 0$. In this equation, $P_{H,t}(j)$ is the price of domestic variety j, $P_{i,t}$ is the price of variety j imported from country i, W_t is the nominal wage, Π_t represents nominal profits and T_t is a nominal lump-sum transfer. All these variables are expressed in the common currency. The bond holdings of home agents is denoted by D_t and the common nominal interest rate within the union is denoted by i_t.

We sometimes allow for transfers across countries that are contingent on shocks. These transfers may be due to private arrangements in complete financial markets. or due to government arrangements. These transfers can accrue to the government or directly to the agents. This is irrelevant since lump-sum taxes are available. For example, we sometimes consider the assumption of complete markets where agents in different countries can perfectly share risks in a complete set of financial markets. Agents form international portfolios, the returns of which result in international transfers that are contingent on the realization of the shock. A different example is in

Section 8 where we consider government spending in the home country paid for by a transfer from the rest of the world. In this case, we have in mind a direct transfer to the government of the home country, or simply spending paid for and made by the rest of the world.

5.2 Government

Government consumption G_t is an aggregate of different varieties. Importantly, we assume that government spending is concentrated exclusively on domestic varieties

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

For any level of expenditure $\int_0^1 P_{H,t}(j)G_t(j)dj$, the government splits its expenditure across varieties to maximize government consumption G_t. Spending is financed by lump-sum taxes. The timing of these taxes is irrelevant since Ricardian equivalence holds in our basic model. We only examine a potentially non-Ricardian setting in Section 7 where we introduce hand-to-mouth consumers into the model.

5.3 Firms

5.3.1 Technology

A typical firm in the home economy produces a differentiated good using a linear technology

$$Y_t(j) = A_{H,t}N_t(j),$$

where $A_{H,t}$ is productivity in the home country. We denote productivity in country i by $A_{i,t}$.

We allow for a constant employment tax $1 + \tau^L$, so that real marginal cost deflated by Home PPI is $\dfrac{1 + \tau^L}{A_{H,t}} \dfrac{W_t}{P_{H,t}}$. We take this employment tax to be constant and set to offset the monopoly distortion so that $\tau^L = -\dfrac{1}{\varepsilon}$, as is standard in the literature. However, none of our results hinge on this particular value.

5.3.2 Price-Setting Assumptions

We assume that the Law of One Price holds so that at all times, the price of a given variety in different countries is identical once expressed in the same currency.

We adopt the Calvo price setting framework, where in every period, a randomly flow ρ_δ of firms can reset their prices. Those firms that get to reset their price choose a reset price P_t^r to solve

$$\max_{P_t^r} \int_0^\infty e^{-\rho_\delta s - \int_0^s i_{t+z} dz} \left(P_t^r Y_{t+s|t} - (1+\tau^L) W_t \frac{Y_{t+s|t}}{A_{H,t}} \right),$$

where $Y_{t+k|t} = \left(\dfrac{P_t^r}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k}$, taking the sequences for W_t, Y_t, and $P_{H,t}$ as given.

5.4 Terms of Trade and Real Exchange Rate

It is useful to define the following price indices: the home Consumer Price Index (CPI) is

$$P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_t^{*1-\eta}]^{\frac{1}{1-\eta}},$$

the home Producer Price Index (PPI)

$$P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}},$$

and P_t^* is the price index for imported goods. The terms of trade are defined by

$$S_t = \frac{P_t^*}{P_{H,t}}.$$

Similarly let the real exchange rate be

$$\mathcal{Q}_t = \frac{P_t^*}{P_t}.$$

5.5 Equilibrium Conditions

We now summarize the equilibrium conditions. For simplicity of exposition, we focus on the case where all foreign countries are identical. Because agents face the same sequence of interest rates optimal consumption satisfies

$$C_t = \Theta C_t^* \mathcal{Q}_t^{\frac{1}{\sigma}},$$

where Θ is a relative Pareto weight which might depend on the realization of the shocks, and C_t^* is union-wide consumption. The goods market clearing condition is

$$Y_t = (1-\alpha)C_t \left(\frac{\mathcal{Q}_t}{S_t} \right)^{-\eta} + \alpha S_t^\gamma C_t^* + G_t.$$

We also have the labor market clearing condition

$$N_t = \frac{Y_t}{A_{H,t}} \Delta_t,$$

where Δ_t is an index of price dispersion $\Delta_t = \int_0^1 \left(\dfrac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon}$ and the Euler equation

$$\sigma \frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho,$$

where $\pi_t = \dot{P}_t/P_t$ is CPI inflation. Finally, we must include the country-wide budget constraint

$$\dot{NFA}_t = (P_{H,t}Y_t - P_tC_t) + i_tNFA_t,$$

where NFA_t is the country's net foreign assets at t, which for convenience, we measure in home numeraire. We impose a standard no-Ponzi condition, $e^{-\int_0^t i_s ds}NFA_t \to 0$ as $t \to \infty$.

Absent transfers or insurance across countries NFA_0 must be equal to zero. Instead, when markets are complete we require that $\Theta = 1$. We then solve for the initial value of NFA_0 that is needed, for each shock realization. This value can be interpreted as an insurance transfer from the rest of the world.

Finally with Calvo price setting we have the equations summarizing the first-order condition for optimal price setting. We omit these conditions since we will only analyze a log-linearized version of the model.

6. NATIONAL AND LOCAL FISCAL MULTIPLIERS IN CURRENCY UNIONS

To compute local multipliers, we study the log-linearized equilibrium conditions around a symmetric steady state with zero inflation. We denote the deviations of total private consumption (by domestic and foreigners), output, and public consumption on domestic goods relative to steady state output by

$$c_t = (1-\mathcal{G})(\log(Y_t - G_t) - \log(Y - G)) \approx \frac{Y_t - G_t - (Y-G)}{Y},$$

$$y_t = \log(Y_t) - \log(Y) \approx \frac{Y_t - Y}{Y} \qquad g_t = \mathcal{G}(\log G_t - \log G) \approx \frac{G_t - G}{Y},$$

where $\mathcal{G} = \frac{G}{Y}$ denotes the steady state share of government spending in output. Then we have, up to a first order approximation,

$$y_t = c_t + g_t,$$

Note that c_t does not represent private domestic total consumption (of home and foreign goods); instead it is private consumption (domestic and foreign) of domestic goods. In a closed economy the two coincide, but in an open economy, for our purposes, the latter is more relevant and convenient.

The log linearized system can then be written as a set of differential equations

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \kappa(c_t + (1-\xi)g_t) - \lambda\hat{\sigma}\alpha(\omega-1)c_t^* - (1-\mathcal{G})\lambda\hat{\sigma}\alpha\omega\theta, \qquad (5)$$

$$\dot{c}_t = \hat{\sigma}^{-1}(i_t^* - \pi_{H,t} - \rho) - \alpha(\omega - 1)\dot{c}_t^*, \tag{6}$$

with an initial condition and the definition of the variable θ,

$$c_0 = (1 - \mathcal{G})(1 - \alpha)\theta + c_0^*, \tag{7}$$

$$\theta = (1 - \mathcal{G}) \int_0^{+\infty} e^{-\rho s} \rho \frac{(\omega - \sigma)}{\omega + (1 - \alpha)(1 - \sigma)} c_s ds + (1 - \mathcal{G}) \frac{1 - \alpha + \alpha\omega}{\omega + (1 - \alpha)(1 - \sigma)} \frac{\rho}{\alpha} nfa_0, \tag{8}$$

and either

$$nfa_0 = 0 \tag{9}$$

if markets are incomplete or

$$\theta = 0 \tag{10}$$

if markets are complete, where $nfa_0 = \dfrac{NFA_0}{Y}$ is the normalized deviation of the initial net foreign asset position from ($nfa_0 = 0$ at the symmetric steady state) and $\theta = \log\Theta$ is the wedge in the log-linearized Backus–Smith equation ($\theta = 0$ at the symmetric steady state). In these equations, we have used the following definitions: $\lambda = \rho_\delta(\rho + \rho_\delta)$, $\kappa = \lambda(\hat{\sigma} + \phi)$, $\xi = \dfrac{\hat{\sigma}}{\hat{\sigma} + \phi}$,

$$\omega = \sigma\gamma + (1 - \alpha)(\sigma\eta - 1),$$

$$\hat{\sigma} = \frac{\sigma}{1 - \alpha + \alpha\omega} \frac{1}{1 - \mathcal{G}}.$$

Eq. (5) is the New Keynesian Philips Curve. Eq. (6) is the Euler equation. Eq. (7) is derived from the requirement that the terms of trade are predetermined at $t = 0$ because prices are sticky and the exchange rate is fixed. Finally Eq. (8) together with either (9) or (10) depending on whether markets are incomplete or complete, represents the country budget constraint. In the Cole–Obstfeld case $\sigma = \eta = \gamma = \Omega = 1$, so that the complete and incomplete markets solutions coincide. Away from the Cole–Obstfeld case, the complete and incomplete markets solutions differ. The incomplete markets solution imposes that the country budget constraint (8) with $nfa_0 = 0$, while the complete markets solution solves for the endogenous value of nfa_0 that ensures that the country budget constraint (8) holds with $\theta = 0$. This can be interpreted as an insurance payment from the rest of the world.

These equations form a linear differential system with forcing variables $\{g_t, g_t^*, i_t^*\}$. It will prove useful to define the following two numbers ν and $\bar{\nu}$ (the eigenvalues of the system):

$$\nu = \frac{\rho - \sqrt{\rho^2 + 4\kappa\hat{\sigma}^{-1}}}{2} \qquad \bar{\nu} = \frac{\rho + \sqrt{\rho^2 + 4\kappa\hat{\sigma}^{-1}}}{2}.$$

6.1 Domestic Government Spending

We first consider the experiment where the only shock is domestic government spending, so that $i_t^* = \rho$, $g_t^* = y_t^* = c_t^* = 0$. Note that if $g_t = 0$ throughout then $\theta = 0$ and $y_t = c_t = 0$. We shall compute the deviations from this steady state when $g_t \neq 0$.

The assumptions one makes about financial markets can affect the results. We consider, in turn, both the cases of complete markets and incomplete markets.

6.1.1 Complete Markets

We start by studying the case where markets are complete. This assumption is representative of most of the literature, and is often adopted as a benchmark due to its tractability. The key implication is that consumption is insured against spending shocks. In equilibrium, private agents make arrangements with the rest of the world to receive transfers when spending shoots up and, conversely, to make transfers when spending shoots down. As a result, government sending shocks to not affect consumption on impact. Formally, we have $\theta = 0$, so the system becomes

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \kappa(c_t + (1 - \xi)g_t),$$
$$\dot{c}_t = -\hat{\sigma}^{-1}\pi_{H,t},$$

with initial condition

$$c_0 = 0.$$

Because the system is linear, we can write

$$c_t = \int_{-t}^{\infty} \alpha_s^{c,t,CM} g_{t+s} ds,$$
$$\pi_{H,t} = \int_{-t}^{\infty} \alpha_s^{\pi,t,CM} g_{t+s} ds,$$

where the superscript CM stands for complete markets. Note two important differences with the closed economy case. First, there are both forward- and backward-looking effects from government spending; the lower bound in these integrals is now given by $-t$ instead of 0. At every point in time, consumption is pinned down by the terms of trade which depend on past inflation. Second, the multipliers depend on calendar time t.

It is important to remind the reader that the sequence of coefficients $\{\alpha_s^{c,t,CM}\}$ represents a notion of fiscal multiplier for total private consumption of domestic goods (by domestic and foreigners) and not for domestic output, which is given by

$$y_t = g_t + \int_{-t}^{\infty} \alpha_s^{c,t,CM} g_{t+s} ds.$$

Whereas the natural benchmark for consumption multipliers is 0, that for output multipliers is 1.

Proposition 3 (Open Economy Multipliers, Complete Markets) *Suppose that markets are complete, then the fiscal multipliers are given by*

$$
\alpha_s^{c,t,CM} =
\begin{cases}
-\hat{\sigma}^{-1}\kappa(1-\xi)e^{-\nu s}\dfrac{1-e^{(\nu-\bar{\nu})(t+s)}}{\bar{\nu}-\nu} & s < 0, \\[3ex]
-\hat{\sigma}^{-1}\kappa(1-\xi)e^{-\bar{\nu}s}\dfrac{1-e^{-(\bar{\nu}-\nu)t}}{\bar{\nu}-\nu} & s \geq 0.
\end{cases}
$$

It follows that
1. *for $t = 0$ we have $\alpha_s^{c,t,CM} = 0$ for all s;*
2. *for $t > 0$ we have $\alpha_s^{c,t,CM} < 0$ for all s;*
3. *for $t \to \infty$ we have $\alpha_{s-t}^{c,t,CM} \to 0$ for all s;*
4. *spending at zero and infinity have no impact: $\alpha_{-t}^{c,t,CM} = \lim_{s\to\infty}\alpha_s^{c,t,CM} = 0$.*

The right panel of Fig. 2 displays consumption multipliers for a standard calibration. Consumption multipliers are very different in an open economy with a fixed exchange rate. For starters, part (1) says that the initial response of consumption is always zero, simply restating the initial condition above that $c_0 = 0$. This follows from the fact that the terms of trade are predetermined and complete markets insure consumption.

Part (2) proves that the consumption response at any other date is actually negative. Note that the Euler equation and the initial condition together imply that

$$
c_t = -\hat{\sigma}^{-1}\log\frac{P_{H,t}}{P_H}.
$$

Government spending increases demand, leading to inflation, a rise in $P_{H,t}$. In other words, it leads to an appreciation in the terms of trade and this loss in competitiveness depresses private demand, from both domestic and foreign consumers. Although we have derived this result in a specific setting, we expect it to be robust. The key ingredients are that consumption depends negatively on the terms of trade and that government spending creates inflation.

It may seem surprising that the output multiplier is necessarily less than one whenever the exchange rate is fixed, because this contrasts sharply with our conclusions in a closed economy with a fixed interest rate. They key here is that a fixed exchange rate implies a fixed interest rate, but the reverse is not true. We expand on this idea in the next section.

Part (3) says that the impact of government spending at any date on private consumption vanishes in the long run. This exact long run neutrality relies on the assumption of complete markets; otherwise, there are potential long-run neoclassical wealth effects from accumulation of foreign assets.

Part (4) says that spending near zero and spending in the very far future have negligible impacts on consumption at any date. Spending near zero affects inflation for a trivial

amount of time and thus have has insignificant effects on the level of home prices. Similarly, spending in the far future has vanishing effects on inflation at any date.

Example 5 (AR(1) Spending) Suppose that $g_t = g e^{-\rho_g t}$ and that markets are complete. Then

$$
c_t = -g e^{\nu t} \frac{1 - e^{-(\nu + \rho_g)t}}{\nu + \rho_g} \frac{\hat{\sigma}^{-1} \kappa (1 - \xi)}{\bar{\nu} + \rho_g}.
$$

For $g > 0$, this example shows that c_t is always negative. In other words, in the open economy model with complete markets, output always expands less than the increase in government spending. The intuition is simple. Because the terms of trade are predetermined, private spending on home goods is also predetermined so that $c_0 = 0$. Government spending initially leads to inflation because the total (public and private) demand for home goods is increased in the short run. With fixed nominal interest rates, inflation depresses real interest rates, leading to a decreasing path of private consumption of domestic goods, so that c_t becomes negative. The inflationary pressures are greatest at $t = 0$ and they then recede over time as public and private demand decrease. Indeed at some point in time, inflation becomes negative and in the long run, the terms of trade return to their steady state value. At that point, private consumption of domestic goods $\hat{c}_t$ reaches its minimum and starts increasing, returning to 0 in the long run. The crucial role of inflation in generating $c_t < 0$ is most powerfully illustrated in the rigid price case. When prices are entirely rigid, we have $\kappa = 0$ so that $c_t = 0$ throughout.[i]

An interesting observation is that the openness parameter α enters Proposition 3 or Example 5 only through its effect on $\hat{\sigma}$.[j] As a result, in the Cole–Obstfeld case $\sigma = \eta = \gamma = 1$ and the private consumption multipliers $\alpha_s^{c,t,CM}$ are completely independent of openness α. Away from the Cole–Obstfeld case, $\alpha_s^{c,t,CM}$ depends on α, but its dependence can be positive or negative depending on the parameters.[k]

Next, we ask how fiscal multipliers are affected by the degree of price stickiness.

Proposition 4 (Price Stickiness) *The fiscal multipliers $\{\alpha_s^{c,t,CM}\}$ depend on price flexibility as follows:*

1. *when prices are rigid so that $\kappa = 0$, we have $\alpha_s^{c,t,CM} = 0$ for all s and t;*

---

[i] Note that the above calculation is valid even if $\rho_g < 0$, as long as $\bar{\nu} + \rho_g > 0$. If this condition is violated, then c_t is $-\infty$ for $g > 0$ and $+\infty$ for $g < 0$.

[j] Recall that $\hat{\sigma} = \dfrac{\sigma}{1 + \alpha[(\sigma\gamma - 1) + (\sigma\eta - 1) - \alpha(\sigma\eta - 1)]} \dfrac{1}{1 - \mathcal{G}}$.

[k] For example, when $\sigma\eta > 1$ and $\sigma\gamma > 1$, $\alpha_s^{c,t,CM}$ is increasing in α for $\alpha \in [0, \min\{\dfrac{(\sigma\gamma - 1) + (\sigma\eta - 1)}{2(\sigma\eta - 1)}, 1\}]$ and decreasing in α for $\alpha \in [\min\{\dfrac{(\sigma\gamma - 1) + (\sigma\eta - 1)}{2(\sigma\eta - 1)}, 1\}, 1]$.

2. *when prices become perfectly flexible* $\kappa \to \infty$, *then for all t, the function* $s \to \alpha_s^{c,t,CM}$ *converges in distributions to* $-(1 - \xi)$ *times a Dirac distribution concentrated at* $s = 0$, *implying that* $\int_{-t}^{\infty} \alpha_s^{c,t,CM} g_{t+s} ds = -(1 - \xi) g_t$ *for all (continuous and bounded) paths of government spending* $\{g_t\}$.

Unlike in the liquidity trap, fiscal multipliers do not explode when prices become more flexible. In a liquidity trap, government spending sets into motion a feedback loop between consumption and inflation: government spending increases inflation, which lower real interest rates, increases private consumption, further increasing inflation, etc. ad infinitum. This feedback loop is nonexistent in a currency union: government spending increases inflation, appreciates the terms of trade, reduces private consumption, reducing the inflationary pressure. Instead, the allocation converges to the flexible price allocation $c_t = -(1 - \xi) g_t$ when prices become very flexible. At the flexible price allocation, private consumption is entirely determined by contemporaneous government spending. Hence the function $\alpha_s^{c,t,CM}$ of s converges in distributions to $-(1 - \xi)$ times a Dirac function at $s = 0$. This implies that fact that for $s = 0$, $\lim_{\kappa \to \infty} \alpha_s^{c,t,CM} = -\infty$ and for $s \neq 0$, $\lim_{\kappa \to \infty} \alpha_s^{c,t,CM} = 0$.

One can reinterpret the neoclassical outcome with flexible prices as applying to the case with rigid prices and a flexible exchange rate that is adjusted to replicate the flexible price allocation. The output multiplier is then less than one. The first result says that with rigid prices but fixed exchange rates, output multipliers are equal to one. In this sense, the comparison between fixed with flexible exchange rates confirms the conventional view from the Mundell–Flemming model that fiscal policy is more effective with fixed exchange rates (see, eg, Dornbusch, 1980). This is consistent with the simulation findings in Corsetti et al. (2011).

6.1.2 Incomplete Markets

We now turn our attention to the case where markets are incomplete. Although the complete market assumption is often adopted for tractability, we believe incomplete markets may be a better approximation to reality in most cases of interest.

A shock to spending may create income effects that affect consumption and labor responses. The complete markets solution secures transfers from the rest of the world that effectively cancel these income effects. As a result, the incomplete markets solution is in general different from the complete market case. One exception is the Cole–Obstfeld case, where $\sigma = \eta = \gamma = 1$.

With incomplete markets, the system becomes

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \kappa \big(c_t + (1 - \xi) g_t \big) - (1 - \mathcal{G}) \lambda \hat{\sigma} \alpha \omega \theta,$$
$$\dot{c}_t = -\hat{\sigma}^{-1} \pi_{H,t},$$

with initial condition

$$c_0 = (1 - \mathcal{G})(1 - \alpha)\theta,$$

$$\theta = (1 - \mathcal{G}) \int_0^{+\infty} e^{-\rho s} \rho \frac{(\omega - \sigma)}{\omega + (1 - \alpha)(1 - \sigma)} c_s ds.$$

We denote the consumption multipliers with a superscript IM, which stands for incomplete markets. We denote by $\hat{t}$ the time such that

$$\frac{e^{\nu \hat{t}}}{1 - e^{\nu \hat{t}}} = \omega \frac{\hat{\sigma}}{\hat{\sigma} + \phi} \frac{\alpha}{1 - \alpha}.$$

We also define

$$\hat{\Sigma} = (1 - \mathcal{G})(1 - \alpha)\frac{1}{\bar{\nu}} + (1 - \mathcal{G})\frac{\hat{\sigma}}{\hat{\sigma} + \phi} \alpha \omega \frac{1}{\rho} \frac{\nu}{\bar{\nu}}.$$

Note that $\bar{\Omega} = 0$ in the Cole–Obstfeld case.

Proposition 5 (Open Economy Multipliers, Incomplete Markets) *Suppose that markets are incomplete, then fiscal multipliers are given by*

$$\alpha_s^{c,t,IM} = \alpha_s^{c,t,CM} + \delta_s^{c,t,IM},$$

where $\alpha_s^{c,t,CM}$ *is the complete markets consumption multiplier characterized in* Proposition 3 *and*

$$\delta_s^{c,t,IM} = \rho \left[\frac{1 - \alpha}{\alpha} e^{\nu t} - \lambda \hat{\sigma} \omega \kappa^{-1} (1 - e^{\nu t}) \right]$$

$$\times \frac{\alpha \dfrac{\omega - \sigma}{\omega + (1 - \alpha)(1 - \sigma)}}{1 - \hat{\Sigma} \dfrac{1}{1 - \mathcal{G}} \rho \dfrac{\omega - \sigma}{\omega + (1 - \alpha)(1 - \sigma)}} (1 - \xi) e^{-\rho(t + s)} \left(1 - e^{\nu(t + s)} \right).$$

The difference $\delta_s^{c,t,IM}$ *is 0 in the Cole–Obstfeld case* $\sigma = \eta = \gamma = 1$. *Away from the Cole–Obstfeld case, the sign of* $\delta_s^{c,t,IM}$ *is the same as the sign of* $\left(\dfrac{\omega}{\sigma} - 1 \right)(t - \hat{t})$; *moreover,* $\delta_{-t}^{c,t,IM} = 0$ *and* $\lim_{s \to \infty} \delta_s^{c,t,IM} = 0$.

The difference between the complete and incomplete market solution vanishes in the Cole–Obstfeld case. Although, away from the this case $\delta_s^{c,t,IM}$ is generally nonzero, it necessarily changes signs (both as a function of s for a given t, and as a function of t, for a given s). In this sense, incomplete markets cannot robustly overturn the conclusion of Proposition 3 and guarantee positive multipliers for consumption.

With complete markets

$$\theta = 0,$$

while with incomplete markets

$$\theta = \int_0^{+\infty} e^{-\rho s} (1 - \mathcal{G}) \rho \frac{\omega - \sigma}{\omega + (1 - \alpha)(1 - \sigma)} c_t ds.$$

This means that with complete markets, home receives an endogenous transfer nfa_0 from the rest of the world following a government spending shock. In the Cole–Obstfeld case, this transfer is zero, but away from this case, this transfer is nonzero. The difference between these two solutions can then be obtained as the effect of this endogenous transfer.

6.2 Understanding Closed vs Open Economy Multipliers

Fig. 2 provides a sharp illustration of the difference between a liquidity trap and a currency union. In a liquidity trap, consumption multipliers are positive, increase with the date of spending, and become arbitrarily large for long-dated spending. By contrast, in a currency union, consumption multipliers are negative, V-shaped and bounded as a function of the date of spending, and asymptote to zero for long-dated spending.

Before continuing it is useful to pause to develop a deeper understanding of the key difference between the closed and open economy results. The two models are somewhat different—the open economy features trade in goods and the closed economy does not—yet they are quite comparable. Indeed, we will highlight that the crucial difference lies in monetary policy, not model primitives. Although a fixed exchange rate implies a fixed nominal interest rate, the converse is not true.

To make the closed and open economies more comparable, we consider the limit of the latter as $\alpha \to 0$. This limit represents a closed economy in the sense that preferences display an extreme home bias and trade is zero. To simplify, we focus on the case of complete markets so that $\theta = 0$. Even in this limit case, the closed and open economy multipliers differ. This might seems surprising since, after all, both experiments consider the effects of government spending for a fixed nominal interest rate. To understand the difference, we allow for an initial devaluation.

Consider then the open economy model in the closed-economy limit $\alpha \to 0$ and let e_0 denote the new value for the exchange rate after the shock in log deviations relative to its steady-state value (so that $e_0 = 0$ represents no devaluation). The only difference introduced in the system by such one-time devaluation is a change the initial condition to[1]

$$c_0 = \hat{\sigma}^{-1} e_0.$$

---

[1] The full system allowing for a flexible exchange rate and an independent monetary policy i_t is (with $\theta = 0$ and $c_t^* = 0$)

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \kappa(c_t + (1 - \xi)g_t),$$
$$\dot{c}_t = \hat{\sigma}^{-1}(i_t - \pi_{H,t} - \rho),$$
$$\dot{e}_t = i_t - i_t^*,$$

with initial condition

$$c_0 = \hat{\sigma}^{-1} e_0.$$

If we set $i_t = i_t^*$ then $\dot{e}_t = 0$ so that $e_t = e_0$, which amounts to a one-time devaluation.

The exchange rate devaluation e_0 depreciates the initial terms of trade one for one and increases the demand for home goods through an expenditure switching effect. Of course, this stimulative effect is present in the short run, but vanishes in the long run once prices have adjusted. A similar intuition for the effect of fiscal policy on the exchange rate in a liquidity trap is also discussed in Cook and Devereux (2011).

Now if in the closed economy limit of the open economy model, we set the devaluation e_0 so that $\hat{\sigma}^{-1} e_0$ exactly equals the initial consumption response $\int_0^\infty \alpha_s^c g_{t+s} ds$ of the closed economy model, ie,

$$e_0 = \int_0^\infty \kappa(1-\xi)e^{-\bar{\nu}s}\left(\frac{e^{(\bar{\nu}-\nu)s}-1}{\bar{\nu}-\nu}\right)g_s ds, \tag{11}$$

then we find exactly the same response for consumption and inflation as in the closed economy model. This means that if we combined the government spending shock with an initial devaluation given by (11), then the multipliers of the closed economy limit of the open economy model would coincide with those of the closed economy model.[m]

This analysis shows that the policy analysis conducted for our closed economy model implicitly combines a shock to government spending with a devaluation.[n] In contrast, our open economy analysis assumes fixed exchange rates, ruling out such devaluations. The positive response of consumption in the closed economy model relies entirely on this one-time devaluation. Thus, the key difference between the two models is in monetary policy, not whether the economy is modeled as open or closed. Indeed, we have taken the closed-economy limit $\alpha \to 0$, but the results hold more generally: the degree of openness α matters only indirectly through its impact on $\hat{\sigma}$, ν and $\bar{\nu}$ and in the Cole–Obstfeld case, α actually does not even affect these parameters.

7. LIQUIDITY CONSTRAINTS AND NON-RICARDIAN EFFECTS

In this section, we explore non-Ricardian effects of fiscal policy in a closed and open economy setting. To do so, we follow Campbell and Mankiw (1989), Mankiw (2000), and Gali et al. (2007) and introduce hand-to-mouth consumers, a tractable

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[m] Note that the size of this devaluation is endogenous and grows without bound as prices become more flexible, ie, as κ increases. This explains why large multipliers are possible with high values of κ in the closed economy model: they are associated with large devaluations.

[n] To see what this implies, suppose the spending shock has a finite life so that $g_t = 0$ for $t \geq T$ for some T and that monetary policy targets inflation for $t \geq T$. In the closed economy model, inflation is always positive and the price level does not return to its previous level. In contrast, in the open economy model with a fixed exchange rate (no devaluation) inflation is initially positive but eventually negative and the price level returns to its initial steady state value. Indeed, if $g_t > 0$ for $t < T$ and $g_t = 0$ for $t \geq T$ for some T, then inflation is strictly negative for $t \geq T$ and the price level falls towards its long run value asymptotically.

way of modeling liquidity constraints. The latter paper studied the effects of government spending under a Taylor rule in a closed economy. Instead, our focus here is on liquidity traps and currency unions.

7.1 Hand-to-Mouth in a Liquidity Trap

The model is modified as follows. A fraction $1 - \chi$ of agents are optimizers, and a fraction χ are hand-to-mouth. Optimizers are exactly as before. Hand–to–mouth agents cannot save or borrow, and instead simply consume their labor income in every period, net of lump–sum taxes. These lump–sum taxes are allowed to differ between optimizers (T_t^o) and hand-to-mouth agents (T_t^r). We define

$$t_t^o = \frac{T_t^o - T^o}{Y} \qquad t_t^r = \frac{T_t^r - T^r}{Y},$$

where T^o and T^r are the per-capita steady state values of T_t^o and T_t^r.

We log-linearize around a steady state where optimizers and hand-to-mouth consumers have the same consumption and supply the same labor. In the appendix, we show that the model can be summarized by the following two equations

$$\dot{c}_t = \widetilde{\sigma}^{-1}(i_t - \bar{r}_t - \pi_t) + \widetilde{\Theta}_n \dot{g}_t - \widetilde{\Theta}_\tau \dot{t}_t^r,$$

$$\dot{\pi}_t = \rho\pi_t - \kappa[c_t + (1 - \xi)g_t],$$

where $\widetilde{\sigma}$, $\widetilde{\Theta}_n$ and $\widetilde{\Theta}_\tau$ are positive constants defined in the appendix, which are increasing in χ and satisfy $\widetilde{\Theta}_n = \widetilde{\Theta}_\tau = 0$ and $\widetilde{\sigma} = \hat{\sigma}$ when $\chi = 0$. The presence of hand-to-mouth consumers introduces two new terms in the Euler equation, one involving government spending and the other one involving taxes—both direct determinants of the consumption of hand-to-mouth agents. These terms drop out without hand-to-mouth consumers, since $\chi = 0$ implies $\widetilde{\Theta}_n = \widetilde{\Theta}_\tau = 0$ and $\widetilde{\sigma} = \hat{\sigma}$.

As before we define

$$\widetilde{\nu} = \frac{\rho - \sqrt{\rho^2 + 4\kappa\widetilde{\sigma}^{-1}}}{2} \qquad \widetilde{\overline{\nu}} = \frac{\rho + \sqrt{\rho^2 + 4\kappa\widetilde{\sigma}^{-1}}}{2}.$$

We write the corresponding multipliers with a HM superscript to denote "hand-to-mouth."

Proposition 6 (Closed Economy Multipliers, Hand–to–Mouth) *With hand-to-mouth consumers, we have*

$$c_t = \widetilde{c}_t + \widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r + \int_0^\infty \alpha_s^{c,HM} g_{t+s} ds - \int_0^\infty \gamma_s^{c,HM} t_{t+s}^r ds,$$

where

$$\alpha_s^{c,HM} = \left(1 + \frac{\widetilde{\Theta}_n}{1-\xi}\right)\widetilde{\alpha}_s^{c,HM} \qquad \gamma_s^{c,HM} = \frac{\widetilde{\Theta}_\tau}{1-\xi}\widetilde{\alpha}_s^{c,HM}.$$

$$\widetilde{\alpha}_s^{c,HM} = \widetilde{\sigma}^{-1}\kappa(1-\xi)e^{-\widetilde{\nu}s}\left(\frac{e^{(\widetilde{\nu}-\widetilde{\nu})s} - 1}{\widetilde{\nu} - \widetilde{\nu}}\right).$$

In these expressions, g_t and t_t^r can be set independently of each other because the government can always raise the necessary taxes on optimizing agents by adjusting t_t^o, so that total taxes $t_t = \chi t_t^r + (1-\chi)t_t^o$ are sufficient to balance the government budget over time

$$0 = \int_0^\infty (t_t - g_t)e^{-\rho t}dt.$$

If there are additional constraints on the tax system, then g_t and t_t^r become linked. For example, imagine that tax changes on optimizing and hand-to-mouth have to be identical so that $t_t^o = t_t^r = t_t$. In this case, taxes on hand-to-mouth agents satisfy

$$0 = \int_0^\infty (t_t^r - g_t)e^{-\rho t}dt.$$

Imagine in addition that the government must run a balanced budget, then we must have $t_t^o = t_t^r = t_t = g_t$. In this case, taxes on hand-to-mouth agents satisfy

$$t_t^r = g_t.$$

The presence of hand-to-mouth consumers affects the closed-form solution by modifying the coefficients on spending and adding new terms. The terms fall under two categories: the terms $\widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r$ capturing the concurrent effects of spending and the integral terms $\int_0^\infty \alpha_s^{c,HM}g_{t+s}ds - \int_0^\infty \gamma_s^{c,HM}t_{t+s}^r ds$ capturing the effects of future government spending and future taxes.

The concurrent terms appear because, with hand-to-mouth consumers, current fiscal policy has a direct and contemporaneous impact on spending. They represent traditional Keynesian effects, which are independent of the degree of price flexibility κ. The integral terms capture the effects of future fiscal policy through inflation. They represent New Keynesian terms, which scale with the degree of price flexibility κ, and disappear when prices are perfectly rigid $\kappa = 0$.

Let us start by discussing the concurrent terms $\widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r$. First, the term $-\widetilde{\Theta}_\tau t_t^r$ captures the fact that a reduction in current taxes on hand-to-mouth consumers increases their total consumption directly by redistributing income towards them, away from

either unconstrained consumers, who have a lower marginal propensity to consume, or from future hand-to-mouth consumers. Second, the term $\widetilde{\Theta}_n g_t$ captures the fact that higher current government spending increases labor income and hence consumption of hand-to-mouth consumers, who have a higher marginal propensity to consume than optimizers. Even when government spending is balanced so that $g_t = \chi t_t^o + (1 - \chi) t_t^r$ and taxes are levied equally on optimizers and hand-to-mouth agents so that $t_t^r = g_t$, the sum of the concurrent terms is not exactly zero because of the different effects of government spending and taxes on real wages.

In this case, since $\widetilde{\Theta}_\tau = \widetilde{\Theta}_n \dfrac{\mu}{1 + \phi}$, the sum of the concurrent terms $\widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r = \left(1 - \dfrac{\mu}{1 + \phi}\right) \widetilde{\Theta}_n g_t$ is likely to be positive in typical calibrations where steady state markups $\mu - 1$ are small compared to ϕ. This is because with sticky prices and flexible wages, real wages increase following increases in government spending, which reduces profit. With heterogeneous marginal propensities to consume, the incidence of this loss across agents matters for private spending, and hence for multipliers, and as we shall see below, these effects can be very large. We refer the reader to the appendix for a complete characterization of fiscal multipliers when these profit effects are taken out (profit offset).

We now turn to the integral terms $\int_0^\infty \alpha_s^{c,HM} g_{t+s} ds - \int_0^\infty \gamma_s^{c,HM} t_{t+s}^r ds$, lower taxes on hand-to-mouth consumers in the future, or higher government spending in the future, stimulates total future consumption.[o] This increases inflation, reducing the real interest rate which increases the current consumption of optimizing agents. This, in turn, stimulates spending by hand-to-mouth consumers. These indirect effects all work through inflation.

Going back to the example where tax changes on hand-to-mouth agents and optimizers discussed above $t_t^o = t_t^r = t_t$, our formulas reveal that the timing of deficits matters. Front-loading fiscal surpluses reduces multipliers through the New Keynesian effects, but increases multipliers early on (and lowers them eventually) through the Keynesian effects.

It is important to understand how these results depend on fixed interest rates, due, say, to a binding zero lower bound. Away from this bound, monetary policy could be chosen

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[o] Note that there are conflicting effects of the fraction of hand-to-mouth consumers χ on

$$\alpha_s^{c,HM} = \left(1 + \frac{\widetilde{\Theta}_n}{1 - \xi}\right) \widetilde{\alpha}_s^{c,HM} \text{ with } \widetilde{\alpha}_s^{c,HM} = \widetilde{\sigma}^{-1} \kappa (1 - \xi) e^{-\widetilde{\nu} s} \left(\frac{e^{(\widetilde{\nu} - \widetilde{\nu})s} - 1}{\widetilde{\nu} - \widetilde{\nu}}\right).$$ On the one hand, future spend-

ing increases future output and hence current inflation more when χ is higher, as captured by the multi-

plicative term $1 + \frac{\widetilde{\Theta}_n}{1-\xi}$ which increases with χ. On the other hand, a given amount of inflation leads to less intertemporal substitution when χ is higher, because hand-to-mouth consumers do not substitute intertemporally, as captured by the term $\widetilde{\sigma}^{-1}$ which decreases with χ. Overall, for plausible simulations, we find that the former effect tends to be stronger, and potentially much stronger, than the latter. Similar comments apply to the term $\gamma_s^{c,HM}$, which is always positive for $\chi > 0$ but is zero for $\chi = 0$.

to replicate the flexible price allocation with zero inflation. The required nominal interest rate is impacted by the presence of hand–to–mouth consumer

$$i_t = \tilde{\sigma} \left[(1 - \xi) + \tilde{\Theta}_n \right] \dot{g}_t + \tilde{\sigma} \tilde{\Theta}_\tau t_t^\tau,$$

but consumption is not

$$c_t = -(1 - \xi) g_t.$$

Hence away from the zero bound, we get the neoclassical multiplier, which is determined completely statically and does not depend on the presence of hand–to–mouth consumers.[P] In contrast, whenever monetary policy does not or cannot replicate the flexible price allocation, then hand–to–mouth consumers do make a difference for fiscal multipliers. Gali et al. (2007) consider a Taylor rule which falls short of replicating the flexible price allocation. Here, we have focused on fixed interest rates, motivated by liquidity traps.

7.2 Hand-to-Mouth in a Currency Union

We now turn to the open economy version with hand–to–mouth agents.

7.2.1 Complete Markets

We start with the case of complete markets for optimizers. In the appendix, we show that the system becomes

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \tilde{\kappa}(c_t + (1 - \tilde{\xi}) g_t) - (1 - \mathcal{G}) \lambda \tilde{\sigma} \tilde{\alpha} \tilde{\omega} \theta - \tilde{\kappa} \tilde{\tilde{\Theta}}_\tau t_t^\tau,$$

$$\dot{c}_t = -\tilde{\sigma}^{-1} \pi_{H,t} + \tilde{\Theta}_n \dot{g}_t - \tilde{\Theta}_\tau t_t^\tau,$$

with initial condition

$$c_0 = \tilde{\Theta}_n g_0 - \tilde{\Theta}_\tau t_0^\tau,$$

for some constants $\tilde{\kappa}, \tilde{\alpha}, \tilde{\omega}, \tilde{\sigma}, \tilde{\Theta}_n, \tilde{\Theta}_\tau$ and $\tilde{\tilde{\Theta}}_\tau$ defined in the appendix. Importantly $\tilde{\sigma}, \tilde{\Theta}_n,$ $\tilde{\Theta}_\tau$ are increasing in χ and $\tilde{\Theta}_n$ and $\tilde{\Theta}_\tau$ are decreasing in α. When $\chi = 0$ we have $\tilde{\kappa} = \kappa,$ $\tilde{\alpha} = \alpha, \tilde{\omega} = \omega, \tilde{\sigma} = \hat{\sigma}, \tilde{\Theta}_n = 0, \tilde{\Theta}_\tau = 0$ and $\tilde{\tilde{\Theta}}_\tau = 0$. As usual, we define

$$\tilde{\nu} = \frac{\rho - \sqrt{\rho^2 + 4 \tilde{\kappa} \tilde{\sigma}^{-1}}}{2} \qquad \tilde{\bar{\nu}} = \frac{\rho + \sqrt{\rho^2 + 4 \tilde{\kappa} \tilde{\sigma}^{-1}}}{2}.$$

[P] Note, however, that hand-to-mouth agents might change the associated allocation of optimizers. They just don't matter for the aggregate allocation.

Proposition 7 (Open Economy Multipliers, Hand–to–Mouth, Complete Markets) *With hand-to-mouth agents and complete markets for optimizers, we have*

$$c_t = \widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r + \int_{-t}^{\infty} \alpha_s^{c,t,HM,CM} g_{t+s} ds - \int_{-t}^{\infty} \gamma_s^{c,t,HM,CM} t_{t+s}^r ds,$$

where

$$\alpha_s^{c,t,HM,CM} = \left(1 + \frac{\widetilde{\Theta}_n}{1-\widetilde{\xi}}\right)\widetilde{\alpha}_s^{c,t,HM,CM}, \qquad \gamma_s^{c,t,HM,CM} = \frac{\widetilde{\Theta}_\tau - \widetilde{\widetilde{\Theta}}_\tau}{1-\widetilde{\xi}}\widetilde{\alpha}_s^{c,t,HM,CM},$$

$$\widetilde{\alpha}_s^{c,t,HM,CM} = \begin{cases} -\widetilde{\sigma}^{-1}\widetilde{\kappa}(1-\widetilde{\xi})e^{-\widetilde{\nu}s}\dfrac{1-e^{(\widetilde{\nu}-\widetilde{\nu})(t+s)}}{\widetilde{\nu}-\widetilde{\nu}} & s<0, \\[3mm] -\widetilde{\sigma}^{-1}\widetilde{\kappa}(1-\widetilde{\xi})e^{-\widetilde{\nu}s}\dfrac{1-e^{-(\widetilde{\nu}-\widetilde{\nu})t}}{\widetilde{\nu}-\widetilde{\nu}} & s\geq 0. \end{cases}$$

Just as in the closed economy case, hand-to-mouth consumers introduce additional Keynesian effects and New Keynesian effects through cumulated inflation, where the former are independent of price flexibility κ while the latter scale with price flexibility κ and disappear when prices are perfectly rigid so that $\kappa = 0$. Just as in the closed economy case, the Keynesian effects increase consumption in response to contemporaneous positive government spending shocks and decrease consumption in response to contemporaneous increases in taxes on hand-to-mouth agents. The difference with the closed economy case is that the New Keynesian effects tend to depress consumption in response to positive government spending shocks. A pure illustration of the Keynesian effect is initial consumption c_0 (for which New Keynesian effects are 0), which is not 0 anymore, but instead $c_0 = \widetilde{\Theta}_n g_0 - \widetilde{\Theta}_\tau t_0^r$. Importantly $\widetilde{\Theta}_n$ and $\widetilde{\Theta}_\tau$ are decreasing with the degree of openness α, simply because higher values of α reduce the marginal propensity to consume on domestic goods of hand-to-mouth agents, capturing the "leakage abroad" of fiscal policy.

7.2.2 Incomplete Markets

We now treat the case of incomplete markets for optimizers. We refer the reader to the appendix for the definitions of the constants $\widetilde{\Omega}_n$, $\widetilde{\Omega}_c$, Σ.

Proposition 8 (Open Economy Multipliers, Hand–to–Mouth, Incomplete Markets) *With hand-to-mouth agents and incomplete markets for optimizers, we have*

$$c_t = \widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r + \int_{-t}^{\infty} \alpha_s^{c,t,HM,IM} g_{t+s} ds - \int_{-t}^{\infty} \gamma_s^{c,t,HM,IM} t_{t+s}^r ds,$$

where

$$\alpha_s^{c,t,HM,IM} = \alpha_s^{c,t,HM,CM} + \delta_s^{c,t,HM,IM},$$

$$\gamma_s^{c,t,HM,IM} = \gamma_s^{c,t,HM,CM} + \epsilon_s^{c,t,HM,IM},$$

with

$$\delta_s^{c,t,HM,IM} = \rho \left[\frac{1-\widetilde{\alpha}}{\widetilde{\alpha}} e^{\widetilde{\nu}t} - (1-\mathcal{G})\lambda\widetilde{\sigma\kappa}^{-1}\widetilde{\omega}\left(1-e^{\widetilde{\nu}t}\right) \right]$$

$$\times \frac{\widetilde{\alpha}}{1-\Sigma\widetilde{\Omega}_c} \left[e^{-\rho(t+s)}\frac{(1-\mathcal{G})\widetilde{\Omega}_n}{\rho} + e^{-\rho(t+s)}\frac{(1-\mathcal{G})\widetilde{\Omega}_c}{\rho}\widetilde{\Theta}_n \right.$$

$$\left. + \frac{(1-\mathcal{G})\widetilde{\Omega}_c}{\rho}\left(1-\widetilde{\xi}\right)\left(1+\frac{\widetilde{\Theta}_n}{1-\widetilde{\xi}}\right)e^{-\rho(t+s)}\left(1-e^{\widetilde{\nu}(t+s)}\right) \right],$$

$$\epsilon_s^{c,t,HM,IM} = -\rho\left[\frac{1-\widetilde{\alpha}}{\widetilde{\alpha}}e^{\widetilde{\nu}t} - \lambda\widetilde{\sigma\kappa}^{-1}\widetilde{\omega}\left(1-e^{\widetilde{\nu}t}\right) \right]$$

$$\times \frac{\widetilde{\alpha}}{1-\Sigma\widetilde{\Omega}_c}\left[e^{-\rho(t+s)}\frac{(1-\mathcal{G})\widetilde{\Omega}_\tau}{\rho} - e^{-\rho(t+s)}\frac{(1-\mathcal{G})\widetilde{\Omega}_c}{\rho}\widetilde{\Theta}_\tau \right.$$

$$\left. + \frac{(1-\mathcal{G})\widetilde{\Omega}_c}{\rho}\left(1-\widetilde{\xi}\right)\frac{\widetilde{\widetilde{\Theta}}_\tau - \widetilde{\Theta}_\tau}{1-\widetilde{\xi}}e^{-\rho(t+s)}\left(1-e^{\widetilde{\nu}(t+s)}\right) \right].$$

The difference between the complete and incomplete market solution $\delta_s^{c,t,HM,IM}$ and $\epsilon_s^{c,t,HM,IM}$ are generally nonzero, can be understood along the same lines as in Section 6 in the absence of hand-to-mouth agents, generally switch signs with t and s, but do not substantively overturn the forces identified in the case of complete markets.

8. OUTSIDE-FINANCED FISCAL MULTIPLIERS

Up to this point, in our open economy analysis of currency unions, we have assumed that each country pays for its own government spending. Actually, with complete markets it does not matter who is described as paying for the government spending, since regions will insure against this expense. In effect, any transfers across regions arranged by governments are undone by the market. With incomplete markets, however, who pays matters. Transfers between regions cannot be undone and affect the equilibrium. Thus, for the rest of this section we assume incomplete markets.

We first examine what happens when the domestic country doesn't pay for the increase in domestic government spending. We show that this can make an important difference and lead to larger multipliers. This is likely to be important in practice: indeed,

a large part of the "local multiplier" literature considers experiments where government spending is not paid by the economic region under consideration.

8.1 Outside-Financed Fiscal Multipliers with No Hand-to-Mouth

We first start with the case where there are no hand-to-mouth agents. The only difference with the results with incomplete markets from Section 6.1 is that we now have

$$\theta = (1-\mathcal{G})\int_0^{+\infty} e^{-\rho s}\rho\frac{(\omega-\sigma)}{\omega+(1-\alpha)(1-\sigma)}c_s ds + (1-\mathcal{G})\frac{1-\alpha+\alpha\omega}{\omega+(1-\alpha)(1-\sigma)}\frac{\rho}{\alpha}\mathrm{nfa}_0,$$

where

$$\mathrm{nfa}_0 = \int_0^\infty e^{-\rho t}g_t dt$$

is the transfer from foreign to home that pays for the increase in government spending. In the Cole–Obstfeld case $\sigma = \eta = \gamma = \Omega = 1$.

We denote the consumption multipliers with a superscript PF, which stands for "paid for" by foreigners.

Proposition 9 (Outside-Financed Open Economy Multipliers) *When domestic government spending is outside-financed, the fiscal multipliers are given by the same expressions as in Proposition 5 with the difference that*

$$\alpha_s^{c,t,PF} = \alpha_s^{c,t,IM} + \delta_s^{c,t,PF},$$

where $\alpha_s^{c,t,IM}$ is the incomplete markets consumption multiplier characterized in Proposition 5 and

$$\delta_s^{c,t,PF} = \rho\left[\frac{1-\alpha}{\alpha}e^{\nu t} - \lambda\hat{\sigma}\omega\kappa^{-1}(1-e^{\nu t})\right]$$

$$\times \frac{1}{1-\hat{\Sigma}\dfrac{1}{1-\mathcal{G}}\rho\dfrac{\omega-\sigma}{\omega+(1-\alpha)(1-\sigma)}}\frac{1-\alpha+\alpha\omega}{\omega+(1-\alpha)(1-\sigma)}e^{-\rho(t+s)}.$$

The sign of $\delta_s^{c,t,PF}$ is the same as that of $(\hat{t}-t)$ and $\lim_{s\to\infty}\delta_s^{c,t,PF} = 0$.

In the Cole–Obstfeld case $\sigma = \eta = \gamma = 1$, the expression simplifies to

$$\delta_s^{c,t,PF} = \left[e^{\nu t}\frac{1-\alpha}{\alpha} - (1-e^{\nu t})\frac{1}{1-\mathcal{G}\frac{1}{1-\mathcal{G}}+\phi}\right]\rho e^{-\rho(t+s)}.$$

The intuition is most easily grasped by considering the Cole–Obstfeld case, which we focus on for now. When government spending is outside-financed, there is an associated transfer to domestic agents. Because agents are permanent-income consumers, only the net present value of the per-period transfer matters, which in turn depends on the

persistence of the shock to government spending. The effects of this transfer is captured by the term $\delta_s^{c,t,PF}$, which is higher, the higher the degree of home bias (the lower α). Indeed, more generally, we can compute net-present-value transfer multipliers for pure transfers nfa_0 unrelated to government spending[q]:

$$c_t = \beta^{c,t}\text{nfa}_0$$

with

$$\beta^{c,t} = \left[e^{\nu t}\frac{1-\alpha}{\alpha} - (1-e^{\nu t})\frac{1}{1-\mathcal{G}\frac{1}{1-\mathcal{G}}}\frac{1}{+\phi}\right]\rho.$$

We can also compute the effects of net-present-value transfers on inflation $\beta^{\pi,t} = -\nu e^{\nu t}\left[\rho\frac{1-\alpha}{\alpha} + \rho\frac{1}{\frac{1}{1-\mathcal{G}}+\phi}\right]$ and on the terms of trade $\beta^{s,t} = -[1-e^{\nu t}]\left[\rho\frac{1-\alpha}{\alpha} + \rho\frac{1}{\frac{1}{1-\mathcal{G}}+\phi}\right]$ (note that the terms of trade gap equals accumulated inflation $s_t = -\int_0^t \pi_{H,s}ds$). The presence of the discount factor ρ in all these expressions is natural because what matters is the annuity value ρnfa_0 of the transfer.

Net-present-value transfers have opposite effects on output in the short and long run. In the short run, when prices are rigid, there is a Keynesian effect due to the fact that transfers stimulate the demand for home goods: $\beta^{c,0} = \rho\frac{1-\alpha}{\alpha}$. In the long run, when prices adjust, the neoclassical wealth effect on labor supply lowers output: $\lim_{t\to\infty}\beta^{c,t} = -\rho\frac{1}{\frac{1}{1-\mathcal{G}}+\phi}$. In the medium run, the speed of adjustment, from the Keynesian short-run response to the neoclassical long-run response, is controlled by the degree of price flexibility κ, which affects ν.[r]

Note that the determinants of the Keynesian and neoclassical wealth effects are very different. The strength of the Keynesian effect hinges on the relative expenditure share of home goods $\frac{1-\alpha}{\alpha}$: the more closed the economy, the larger the Keynesian effect. The strength of the neoclassical wealth effect depends on the elasticity of labor supply $\frac{1}{\phi}$: the more elastic labor supply, the larger the neoclassical wealth effect.

Positive net-present-value transfers also increase home inflation. The long-run cumulated response in the price of home produced goods equals $\rho\frac{1-\alpha}{\alpha} + \rho\frac{1}{\frac{1}{1-\mathcal{G}}+\phi}$.

[q] In the particular case that we study here, transfers occur concurrently with an increase in government spending and exactly pay for the increase in government spending $\text{nfa}_0 = \int_0^\infty e^{-\rho t}g_t dt$.
[r] Note that ν is decreasing in κ, with $\nu = 0$ when prices are rigid ($\kappa = 0$), and $\nu = -\infty$ when prices are flexible ($\kappa = \infty$).

The first term $\rho\dfrac{1-\alpha}{\alpha}$ comes from the fact that transfers increase the demand for home goods, due to home bias. The second term $\rho\dfrac{1}{\frac{1}{1-\mathcal{G}}+\phi}$ is due to a neoclassical wealth effect that reduces labor supply, raising the wage. How fast this increase in the price of home goods occurs depends positively on the flexibility of prices through its effect on ν.[s]

These effects echo the celebrated Transfer Problem controversy of Keynes (1929) and Ohlin (1929). With home bias, a transfer generates a boom when prices are sticky, and a real appreciation of the terms of trade when prices are flexible. The neoclassical wealth effect associated with a transfer comes into play when prices are flexible, and generates an output contraction and a further real appreciation.

In the closed economy limit we have $\lim_{\alpha\to0}\beta^{c,t}=\infty$. In the fully open economy limit we have $\lim_{\alpha\to0}\beta^{c,t}=0$. The intuition is that the Keynesian effect of transfers is commensurate with the relative expenditure share on home goods $\dfrac{1-\alpha}{\alpha}$. This proposition underscores that transfers are much more stimulative than government spending, the more so, the more closed the economy. This robust negative dependence of transfer multipliers $\beta^{c,t}$ on openness α should be contrasted with the lack of clear dependence on openness of government spending multipliers $\alpha_s^{c,t,CM}$ noted above (indeed in the Cole–Obstfeld case, $\alpha_s^{c,t,CM}$ is independent of α).

Example 6 (Outside-Financed Spending, Cole–Obstfeld, AR(1)) Suppose that $g_t=ge^{-\rho_g t}$ and that domestic government spending is outside-financed. In the Cole–Obstfeld case $\sigma=\eta=\gamma=1$, we have

$$c_t=g\left[e^{\nu t}\frac{1-\alpha}{\alpha}-(1-e^{\nu t})\frac{1}{1-\mathcal{G}\frac{1}{1-\mathcal{G}}+\phi}\right]\frac{\rho}{\rho+\rho_g}$$
$$-ge^{\nu t}\left(\frac{1-e^{-(\nu+\rho_g)t}}{\nu+\rho_g}\right)\kappa(1-\xi)\frac{1-\mathcal{G}}{\bar{\nu}+\rho_g}.$$

Moreover we have $c_0=g\dfrac{1-\alpha}{\alpha}\dfrac{\rho}{\rho+\rho_g}$ and $\lim_{t\to\infty}c_t=-g\dfrac{1}{1-\mathcal{G}\frac{1}{1-\mathcal{G}}+\phi}\dfrac{\rho}{\rho+\rho_g}$.

Note that the second term on the right-hand side of the expression for c_t in Example 6 is simply the term identified in Example 5 in the complete markets case. The first term arises precisely because government spending is now paid for by foreign.

It is particularly useful to look at the predictions of this proposition for $t=0$ and $t\to\infty$. In the case of a stimulus $g>0$, we have $c_0>0>\lim_{t\to\infty}c_t$. Following a positive stimulus shock, we can get $c_0>0$ and actually $c_t>0$ for some time (because $\theta>0$) and eventually $c_t<0$. The conclusion would be that an unpaid for fiscal stimulus at

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[s] Recall that ν is decreasing in the degree of price flexibility κ.

home has a larger consumption multiplier in the short run and smaller in the long run. This is true as long as there is home bias $\alpha < 1$. The reason is that the associated transfer redistributes wealth from foreign to home consumers. This increases the demand for home goods because of home bias. In the neoclassical model with flexible prices, there would be an appreciation of the terms of trade and a reduction in the output of home goods because of a neoclassical wealth effect. With sticky prices, prices cannot adjust in the short term, and so this appreciation cannot take place right away, and so the output of home goods increases. In the long run, prices adjust and we get the neoclassical effect.

The lesson of this section is that we can partly overturn the conclusion of Proposition 3 when government spending is outside-financed. When the degree of home bias $1 - \alpha$, is high, or when increases in government spending are very persistent, then local multipliers estimates that involve increases in government spending that are not self-financed are potentially substantially inflated compared to the counterfactual of self-financed increases in government spending.

8.2 Outside-Financed Fiscal Multipliers with Hand-to-Mouth

We now turn to the case where there are hand–to–mouth agents.

Proposition 10 (Outside-Financed Open Economy Multipliers, Incomplete Markets, Hand–to–Mouth) *With hand-to-mouth agents, when domestic government spending is outside-financed, the fiscal multipliers are given by the same expressions as in Proposition 8 with the difference that*

$$\alpha_s^{c,t,HM,PF} = \alpha_s^{c,t,HM,IM} + \delta_s^{c,t,PF},$$

where

$$\delta_s^{c,t,HM,PF} = \rho \left[\frac{1 - \widetilde{\alpha}}{\widetilde{\alpha}} \widetilde{e^{\nu t}} - \lambda \widetilde{\sigma \kappa}^{-1} \widetilde{\omega} \left(1 - \widetilde{e^{\nu t}} \right) \right] \frac{1}{1 - \Sigma \widetilde{\Omega}_c} \frac{\widetilde{\alpha}(1 - \mathcal{G}) \widetilde{\Omega}_f}{\rho} e^{-\rho(t + s)}.$$

When domestic government spending is outside-financed, the question of the incidence of the accompanying transfer across domestic optimizers and hand-to-mouth agents naturally arises. These distributive effects are entirely captured by the adjustment in the taxes t_t^r paid by hand-to-mouth agents.

From now on, we focus on the benchmark case where taxes and the accompanying per-period transfer are distributed equally on optimizers and hand-to-mouth agents and where the domestic government runs a balanced budget, because this case is the most relevant to think about most of the estimates in the local multipliers literature where regions correspond to states with limited de jure or de facto ability to borrow.

When domestic government spending is self-financed, we have $t_t^o = t_t^r = g_t$, and instead when government spending is outside-financed, we have $t_t^o = t_t^r = 0$. Comparing fiscal multipliers when government spending is self-financed vs outside-financed, the effect of reduced taxes on optimizers in the latter case is captured by the corrective term

$\delta_s^{c,t,PF}$, while the effect of reduced taxes on hand-to-mouth agents is captured by the reduction in t_t^r from g_t to zero. In particular, in the short run before prices can fully adjust, both effects increase fiscal multipliers, the first effect for reasons already discussed in the case without hand-to-mouth agents in Section 8.1, the second effect because hand-to-mouth agents have a higher marginal propensity to consume than optimizers.

The presence of hand-to-mouth agents magnifies the difference between self-financed and outside-financed fiscal multipliers for temporary government spending shocks, simply because hand-to-mouth agents spend more of the temporary implicit transfer from foreigners that separate these two experiments in the short run, the more so, the more temporary the government spending shock.

Overall, this analysis shows that when the average marginal propensity to consume on domestic goods, as captured by the fraction of hand-to-mouth agents χ and by the degree of home bias $1 - \alpha$, is high, or when increases in government spending are very persistent, then local multipliers estimates that involve increases in government spending that are not self-financed are potentially substantially inflated compared to the counterfactual of self-financed increases in government spending.

9. TAKING STOCK: SOME SUMMARY MULTIPLIER NUMBERS

In this section, we provide numerical illustrations for the forces that we have identified in the chapter. We report summary multipliers $M^\gamma = 1 + M^c$ in liquidity traps and currency unions, computed as the ratio of the average response of output over the 2 years following the increase in spending to the average increase in government spending over the same period. Our baseline calibration features $\chi = 0$, $\sigma = 1$, $\epsilon = 6$, $\phi = 3$, and $\mathcal{G} = 0.3$ for liquidity traps and $\chi = 0$, $\sigma = 1$, $\eta = \gamma = 1$, $\epsilon = 6$, $\phi = 3$, $\mathcal{G} = 0.3$, and $\alpha = 0.4$ for currency unions. We take the government spending shock to be constant for $\tau_g = 1.25$ years (5 quarters) and zero afterward.[t] We then explore variations with higher values of χ. In all these experiments, we maintain the assumption that taxes fall equally on hand-to-mouth agents and on optimizers, and that markets are incomplete. In the deficit financed experiments, taxes are increased (discretely) only after three years, and are then constant for 1.25 years before reverting to zero. The first part of Table 1 corresponds to the case of perfectly rigid prices $\lambda = 0$ (infinite price duration), the second part to $\lambda = 0.12$ (price duration of 2.9 years), and $\lambda = 1.37$ (price duration of 0.9 year).

We start with the case of perfectly rigid prices in the first part of Table 1. This table presents summary multipliers in liquidity traps and currency unions, depending on whether

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[t] This shock has the same duration $\dfrac{\tau_g}{2} = \dfrac{1}{\rho_g}$ as an AR(1) with a coefficient with $\rho_g = 1.6$ (corresponding to a quarterly mean-reversion coefficient of 0.7), but dies off completely in finite time (after 1.6 years), leading to more reasonable values for liquidity trap multipliers when prices are somewhat flexible (the tail of the shock matters a great deal in this case because α_s^c and $\tilde{\alpha}_s^{c,HM}$ increase exponentially with the horizon s).

Table 1 Summary output multipliers

| | Liquidity trap | | | | | | Currency union | | | | | | | | |
	Tax-financed			Deficit-financed			Tax-financed			Deficit-financed			Foreign-financed		
Rigid prices ($\lambda=0$)	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$
$\chi=0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.1160	1.1160	1.1160
$\chi=0.25$	4.5000	1.4804	1.0000	6.0000	1.8922	1.2386	1.6459	1.1956	1.0000	1.9474	1.3786	1.1314	2.0387	1.4823	1.2446
$\chi=0.5$	*	*	1.0000	*	*	1.7159	*	2.2835	1.0000	*	3.4861	1.3361	*	3.5041	1.4514
$\chi=0.75$	*	*	1.0000	*	*	3.1477	*	*	1.0000	*	*	1.7385	*	*	2.4971
Sticky prices ($\lambda=0.12$)	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$
$\chi=0$	1.0542	1.0542	1.0542	1.0542	1.0542	1.0542	0.8968	0.8968	0.8968	0.8968	0.8968	0.8968	0.9550	0.9550	0.9550
$\chi=0.25$	6.9420	1.6437	1.0542	−191.4702	−1.0069	0.5347	1.2856	1.0321	0.8984	1.5476	1.2020	1.0241	1.5819	1.2410	1.0611
$\chi=0.5$	*	*	1.0542	−*	−*	−0.5044	*	1.5451	0.9009	*	2.5252	1.2233	*	2.3385	1.2302
$\chi=0.75$	*	*	1.0542	−*	−*	−3.6218	*	*	0.9083	*	*	1.5770	*	*	1.6241
Sticky prices ($\lambda\simeq1.37$)	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$	$o=0$	$o=0.5$	$o=1$
$\chi=0$	1.8315	1.8315	1.8315	1.8315	1.8315	1.8315	0.6529	0.6529	0.6529	0.6529	0.6529	0.6529	0.6638	0.6638	0.6638
$\chi=0.25$	168.2368	4.5741	1.8315	−3.4965e8	−5153.3064	−242.9734	0.8142	0.7101	0.6542	0.9127	0.7795	0.7096	0.9266	0.7883	0.7141
$\chi=0.5$	*	*	1.8315	−*	−*	−732.5833	*	0.9238	0.6563	*	1.2767	0.7999	*	1.2515	0.7941
$\chi=0.75$	*	*	1.8315	−*	−*	−2201.4125	*	*	0.6612	*	*	0.9670	*	*	0.9559

or not they are tax-financed (taxes equal to government spending in every period), deficit-financed (taxes are raised only 3 years after the increase in spending, and then mean-revert at the same rate as spending), or outside-financed (no change in taxes). For all these cases, we also report multipliers for different values of the profit-offset coefficient o: 0, 0.5, and 1. This profit-offset coefficient is equal to the share of marginal profits per agent which is transferred to each hand-to-mouth agent: when it is equal to 0, hand-to-mouth agents are completely shielded from the impact of government spending on profits, and when it is equal to 1, they are impacted exactly like optimizers. This is important because with sticky prices and flexible wages, real wages increase following increases in government spending, so that profits increase less than proportionately with output, while labor income increases more than proportionately. With heterogeneous marginal propensities to consume, the incidence of this loss across agents matters for private spending, and hence for multipliers, and as we shall see below, these effects can be very large. While our analysis in the main text of the paper is confined to the case $o = 0$, the appendix gives a full treatment of the arbitrary o case. We also vary the fraction of hand-to-mouth agents χ between 0 and 0.75.

The results are as follows. We start with our baseline calibration. The multiplier is always 1 in a liquidity trap, independently of whether government spending is tax- or debt-financed. In a currency union, the multiplier is 1 independently of whether government spending is tax- or debt-financed, but it increases to 1.1 when it is outside-financed.

We then depart from the baseline increasing the fraction of hand-to-mouth agents χ from 0 to 0.25, 0.5, and 0.75. We start with the case of full profit offset $o = 1$ and explain the role of profit offset later. In a liquidity trap, the tax-financed multiplier remains at 1 irrespective of χ. The deficit-financed multiplier increases with χ to 1.2 ($\chi = 0.25$), 1.7 ($\chi = 0.5$), or 3.1 ($\chi = 0.75$). Turning to currency unions, the tax-financed multiplier is 1 irrespective of χ. The deficit-financed multiplier increases with χ to 1.1 ($\chi = 0.25$), 1.3 ($\chi = 0.5$), or 1.8 ($\chi = 0.75$). Finally the outside-financed multiplier increases with χ to 1.3 ($\chi = 0.25$), 1.5 ($\chi = 0.5$), or 2.7 ($\chi = 0.75$). Importantly, the difference between outside- and self-financed multipliers is now larger than in our baseline, and the deficit-financed multiplier is in between these two multipliers.

In general, lower values of the profit offset coefficient o lead to higher multipliers. This is because with no profit offset, the contemporaneous reduction in profits resulting from the increase in government spending acts like a redistribution from low marginal propensity to consume optimizers toward high marginal propensity to consume hand-to-mouth agents, which increases output (and vice versa for the increase in taxes). This effect, which can be very large, disappears with full profit offset. The * in Table 1 indicates that the feedback loop between output and the distributive effects of profits on agents with different marginal propensities to consume is so powerful that it "blows up". When it occurs, our formulas cease to apply and the correct interpretation is that multipliers are positive infinite.

We continue with the case of sticky but not perfectly rigid prices in the second and third parts of Table 1, where we run through the exact same experiments as in the first part of Table 1. The key differences are as follows. First, in the case of liquidity traps, tax-financed multipliers are a lot higher than with rigid prices, illustrating the power of the positive feedback loop between inflation and output. Deficit-financed multipliers can be a lot lower than with rigid prices and can actually be negative when there are enough hand-to-mouth agents because the positive feedback loop for front-loaded government spending is weaker than the more back-loaded negative one for taxes (in this case, lower profit offset reduces multipliers, potentially leading to negative infinite values indicated by $-*$). Second, in the case of currency unions, multipliers are lower than with rigid prices, but the difference is not as large as in the case of liquidity traps. This is because in this case, there is no feedback loop between output and inflation since inflation lowers spending instead of increasing it, because of its accumulated effect appreciates the terms of trade and rebalances spending away from home goods toward foreign goods.

Although this is not illustrated in the table, we briefly comment on the role of the persistence of shocks and of the openness of the economy. In liquidity traps, more persistent government spending shocks tend to increase tax-financed multipliers because of the feedback loop between output and inflation (in fact tax-financed multipliers can become infinite when prices are not entirely rigid, even without hand-to-mouth agents). They increase deficit-financed multipliers with no hand-to-mouth agents but can decrease them with enough hand-to-mouth agents and somewhat flexible prices because the feedback loop between output and inflation is more potent for back-loaded taxes than for front-loaded government spending. In currency unions, more persistent government spending shocks tend to decrease tax-financed and deficit-financed multipliers, but to increase outside-financed multipliers when prices are rigid enough. In currency unions, multipliers tend to increase when the economy is more closed (α is lower) when government spending is outside-financed and prices are not too flexible or when it is deficit-financed and larger than one (less leakage abroad).

Our simulations are illustrative and do not attempt to explore a wide range of possible parameters. For example, we have kept the fraction of hand-to-mouth agents at a modest level. Likewise, we only explore a relatively open economy. Overall, even within this limited range, our results show that fiscal multipliers are somewhat sensitive to various primitive parameters, as well as the nature of the fiscal experiment. Differences were found comparing completely rigid prices to standard degrees of price stickiness, especially for the liquidity trap case. The presence of hand-to-mouth agents also affects the responses significantly. Perhaps most surprisingly, distributional impacts appear to be crucial. First, there is the difference between tax-financed, deficit-financed, and outside-financed spending. Second, there is the difference in the responses obtained depending on the way profits are redistributed. As explained earlier, this effect relies on the model prediction that profits relative to labor earnings are countercyclical. Thus, this

effect could be mitigated if wages, which are flexible in our standard New Keynesian model, were also assumed to be sticky.

Theoretically, in currency unions, outside-financed multipliers can be much larger than deficit-financed multipliers, especially when the economy is relatively closed and government spending shocks are relatively persistent. However, in our simulations with relatively open economies and relatively transitory government spending shocks (which capture the characteristics of many local multiplier studies), these differences are not very large. Since deficit-financed multipliers tend to be larger in liquidity traps than in currency unions (because there is less "leakage" abroad) with rigid enough prices, it would appear that outside-financed multipliers in currency unions (as estimated in the local multipliers literature) may provide a rough lower bound for national multipliers deficit-financed in liquidity traps with rigid enough prices. When prices are more flexible, the comparison is more delicate and the rough lower bound need not apply.

10. COUNTRY SIZE, AGGREGATION, AND FOREIGN GOVERNMENT SPENDING

So far, we have focused on the case where the country undertaking the fiscal stimulus is a small (infinitesimal) part of the currency union—this is implied by our modeling of countries as a continuum. Here, we relax this assumption. To capture country size, we interpret i as indexing regions and we imagine that countries $i \in [0, x]$ are part of a single country. They undertake the same fiscal stimulus g_t^i. We denote with a $- i \in (x, 1]$ the index of a typical region that is not undertaking fiscal stimulus so that $g_t^{-i} = 0$. We consider two situations: (1) monetary policy i_t^* at the union level achieves perfect inflation targeting (2) monetary policy at the union level is passive because the union is in a liquidity trap where interest rates i_t^* are at the zero lower bound. For simplicity, we focus on the Cole–Obstfeld case throughout.

10.1 Inflation Targeting at the Union Level
The aggregates variables satisfy

$$g_t^* = \int_0^1 g_t^i di = x g_t^i,$$

$$c_t^* = \int_0^1 c_t^i di = x c_t^i + (1 - x) c_t^{-i},$$

$$\pi_t^* = \int_0^1 \pi_t^i di = x \pi_t^i + (1 - x)] \pi_t^{-i}.$$

As long as the zero lower bound is not binding, monetary policy at the union level can be set to target zero inflation $\pi_t^* = 0$. The required interest rate i_t^* is

$$i_t^* - \rho = -\hat{\sigma}(1 - \xi)x\dot{g}_t^i,$$

and the corresponding value of c_t^* is

$$c_t^* = -(1 - \xi)xg_t^i.$$

The allocation for regions in the country undertaking the stimulus solves

$$\dot{\pi}_t^i = \rho\pi_t^i - \kappa(c_t^i + (1 - \xi)g_t^i),$$
$$\dot{c}_t^i = -(1 - \xi)x\dot{g}_t^i - \hat{\sigma}^{-1}\pi_t^i,$$

$$c_0^i = -(1 - \xi)xg_0^i.$$

Similarly the allocation for regions not undertaking the stimulus solves

$$\dot{\pi}_t^{-i} = \rho\pi_t^{-i} - \kappa c_t^{-i},$$
$$\dot{c}_t^{-i} = -(1 - \xi)x\dot{g}_t^i - \hat{\sigma}^{-1}\pi_t^{-i},$$

$$c_0^{-i} = -(1 - \xi)xg_0^i.$$

In the Cole–Obstfeld case, we define

$$\alpha_s^{c,t,CM*} = \begin{cases} \hat{\sigma}^{-1}\kappa(1 - \xi)e^{-\nu s}\dfrac{1 - e^{(\nu - \bar{\nu})(t + s)}}{\bar{\nu} - \nu} & s < 0, \\[4mm] \hat{\sigma}^{-1}\kappa(1 - \xi)e^{-\bar{\nu} s}\dfrac{1 - e^{(\nu - \bar{\nu})t}}{\bar{\nu} - \nu} & s \geq 0. \end{cases}$$

Proposition 11 (Large Countries, Union-Wide Inflation Targeting) *Suppose that the zero bound is not binding at the union level and that monetary policy targets union-wide inflation $\pi_t^* = 0$. Then in the Cole–Obstfeld case, we have*

$$c_t^i = -x(1 - \xi)g_t^i + (1 - x)\int_{-t}^{\infty} \alpha_s^{c,t,CM}g_{t+s}^i ds,$$

$$c_t^{-i} = -(1 - \xi)xg_t^i + x\int_{-t}^{\infty} \alpha_s^{c,t,CM*}g_t^i ds.$$

Let us first focus on the regions in the country undertaking the spending. This proposition shows that for regions in the country undertaking the stimulus, the effects on private spending on domestic goods are simply a weighted average of the effect $-(1 - \xi)g_t^i$ that would arise if the country undertaking the stimulus could set monetary policy to target their own domestic inflation $\pi_t^i = 0$, and the effect that arises if the country is a small (infinitesimal) part of a currency union, with weights given by x and $1 - x$, where x is the relative size of the country undertaking the stimulus.

Let us now turn to the regions in countries not undertaking the spending. There are both direct effects and indirect effects. The indirect effects work through inflation, which

affect the terms of trade and, hence, the demand for the goods produced by these regions. To isolate the direct effects set $\kappa = 0$, so that there is no inflation and $\alpha_s^{c,t,CM*} = 0$. The demand for home goods is then equal to $c_t^{-i} = -(1-\xi)g_t^* = -(1-\xi)xg_t^i$. When spending rises in regions $i \in [0, x]$, it depresses private spending by agents of these regions, lowering the demand for output in regions $-i \in (x, 1]$. When $\kappa > 0$, the indirect effect works through inflation. The lower demand for goods in regions $-i \in (x, 1]$ creates deflation in these regions, which makes these economies more competitive. The lower prices then increase the demand for the goods produced by these regions.

Example 7 (Union-Wide Inflation Targeting, AR(1)) Suppose that $g_t^i = g^i e^{-\rho_g t}$, then we have

$$c_t^{-i} = -e^{\nu t}(1-\xi)xg^i\left[1 - \frac{1-e^{-(\nu+\rho_g)t}}{\rho_g + \nu}\frac{\rho_g(\rho+\rho_g)}{\rho_g + \bar{\nu}}\right].$$

This implies that c_0^{-i} is negative if g^i is positive. If $\rho_g + \nu < 0$ then c_t^{-i} will remain negative. If instead $\rho_g + \nu > 0$ then c_t^{-i} starts out negative, but eventually switches signs.

This results suggests that a temporary increase in government spending abroad accompanied by monetary tightening to ensure no union-wide inflation induces a recession at home. This fits a common narrative regarding the post German reunification in the early 90s. The fiscal expansion was combined with a monetary contraction in Germany, so as to avoid inflation. The quasi-fixed exchange rate arrangements of the EMS forced other countries to follow suit and tighten monetary policy, negatively affecting their economic performance.

10.2 Zero Bound at the Union Level

If the zero bound binds at the union level, then c_t^* is given by

$$c_t^* = x\int_0^\infty \alpha_s^c g_{t+s}^i ds.$$

The allocation for regions in the country undertaking the stimulus solves

$$\dot{\pi}_t^i = \rho\pi_t^i - \kappa(c_t^i + (1-\xi)g_t^i),$$
$$\dot{c}_t^i = -\hat{\sigma}^{-1}\pi_t^i,$$
$$c_0^i = x\int_0^\infty \alpha_s^c g_{t+s}^i ds.$$

Similarly the allocation for regions not undertaking the stimulus solves

$$\dot{\pi}_t^{-i} = \rho\pi_t^{-i} - \kappa c_t^{-i},$$
$$\dot{c}_t^{-i} = -\hat{\sigma}^{-1}\pi_t^{-i},$$

$$c_0^{-i} = x \int_0^\infty \alpha_s^c g_{t+s}^i ds.$$

Proposition 12 (Large Countries, Union–Wide Zero Bound) *Suppose that the zero bound is binding at the union level, then in the Cole–Obstfeld case, we have*

$$c_t^i = x \int_0^\infty \alpha_s^c g_{t+s}^i ds + (1-x) \int_{-t}^\infty \alpha_s^{c,t,CM} g_{t+s}^i ds,$$

$$c_t^{-i} = x e^{\nu t} \int_0^\infty \alpha_s^c g_s^i ds.$$

Similarly to Proposition 11, this proposition shows that for the country undertaking the stimulus, the effects on private spending on domestic goods are simply a weighted average of the effect $\int_0^\infty \alpha_s^c g_{t+s}^i ds$ that would arise if the country undertaking the stimulus were a closed economy at the zero lower bound, and the effect that arises if the country were a small (infinitesimal) part of a currency union, with weights given by x and $1 - x$, where x is the relative size of the country undertaking the stimulus.

In contrast to the inflation targeting case, when the zero lower bound binds, an increase in government spending by regions $i \in [0, x]$ increases the demand for the goods of regions $-i \in (x, 1]$. This is natural since we now have a general expansion in private demand because inflation reduces real interest rates.[u]

11. CONCLUSION

We have explored the economic response to changes in government spending in a few benchmark models. Relative to the existing literature, our contribution is to characterize the dynamics of these responses analytically in some detail, rather than summarizing the effects in a single "summary multiplier." We have done so by defining the multipliers to be the partial derivative of private spending at any point in time, to public spending at any other date. We have also attempted to be relatively exhaustive in incorporating various elements that are important, but sometimes missing in standard analyses. In particular, we considered both closed and open economies and incorporated hand-to-mouth agents in both these frameworks. Most importantly, our analysis is the first to emphasize different forms of financing for the government spending shock, including tax-financed, deficit-financed, and outside-financed. It is our hope that our approach and analysis will prove useful in interpreting and unifying the large theoretical and empirical research on fiscal multipliers.

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[u] These findings on the spillover effects of fiscal policy complement the results in Cook and Devereux (2011) who focus on different configurations than us: they show that the spillover effects of fiscal policy at home on foreign when home is in a liquidity trap are negative with flexible exchange rates, but positive with fixed exchange rates. In this section, we focus on fixed exchange rates in a currency union and show how these spillover effects switch signs depending on whether the union is in a liquidity trap or targets inflation.

APPENDICES

Appendix A

This appendix derives the linear systems of equations to be solved for in order to derive fiscal multipliers in the following cases: liquidity trap; currency union with either complete markets (CM), incomplete markets (IM), and outside-financed government spending (PF). Appendix B then solves these systems equations to derive fiscal multipliers.

In both appendices, the general case with an arbitrary fraction χ of hand-to-mouth agents and with arbitrary profit offset o is derived first, followed by two special cases: no hand-to-mouth agents $\chi = 0$ (as in Sections 1–6) and no profit offset $o = 0$ (as in Sections 1–8).

Compared to the main text, the environment is generalized by allowing hand-to-mouth agents to receive a profit offset which redistributes a share of profits $o \in [0, 1]$ to hand-to-mouth agents:

$$P_t C_t^r = W_t N_t^r + \frac{o}{\chi}\Pi_t - P_t \underline{T}_t^r,$$

with

$$P_t \underline{T}_t^r = P_t T_t^r - \frac{o}{\chi}\Pi_t,$$
$$\Pi_t = P_{H,t} Y_t - w_t N_t.$$

A.1 Liquidity Trap

Assume that $c_t^* = 0, i_t^* = \bar{r}_t$ for all $t \geq 0$. The log-linearized equations are

$$\dot{c}_t^o = (1 - \mathcal{G})\sigma^{-1}(i_t - \bar{r}_t - \pi_t),$$
$$c_t^r = \frac{WN^r}{Y}(w_t + n_t^r) - \underline{t}_t^r,$$
$$w_t = \frac{\sigma}{1 - \mathcal{G}}c_t^r + \phi n_t^r,$$
$$w_t = \frac{\sigma}{1 - \mathcal{G}}c_t + \phi n_t,$$
$$c_t = \chi c_t^r + (1 - \chi)c_t^o,$$
$$n_t = \chi n_t^r + (1 - \chi)n_t^o,$$
$$\dot{\pi}_t = \rho\pi_t - \kappa[c_t + (1 - \xi)g_t],$$
$$\underline{t}_t^r = t_t^r - o\left[\left(1 - \frac{1}{\mu}\right)n_t - \frac{1}{\mu}w_t\right],$$

where w_t denotes real wages and μ is the steady state markup, with $\lambda = \rho_\delta(\rho + \rho_\delta), \kappa = \lambda(\hat{\sigma} + \phi), \xi = \dfrac{\hat{\sigma}}{\hat{\sigma} + \phi}.$

Combining and rearranging, we get

$$n_t^r = \phi^{-1}\left(w_t - \frac{\sigma}{1-\mathcal{G}}c_t^r\right),$$

$$c_t^r = \frac{WN^r}{Y}\left[(1+\phi^{-1})\left(\frac{\sigma}{1-\mathcal{G}}c_t + \phi n_t\right) - \phi^{-1}\frac{\sigma}{1-\mathcal{G}}c_t^r\right] - \underline{t}_t^r,$$

$$c_t^r = \frac{\frac{WN^r}{Y}(1+\phi^{-1})\left(\frac{\sigma}{1-\mathcal{G}}c_t + \phi n_t\right) - \underline{t}_t^r}{1+\phi^{-1}\frac{\sigma}{1-\mathcal{G}}\frac{WN^r}{Y}},$$

$$c_t\left[\frac{1-\chi\frac{WN^r}{Y}\frac{\sigma}{1-\mathcal{G}} + (1-\chi)\phi^{-1}\frac{\sigma}{1-\mathcal{G}}\frac{WN^r}{Y}}{1+\phi^{-1}\frac{\sigma}{1-\mathcal{G}}\frac{WN^r}{Y}}\right] = \chi\frac{\frac{WN^r}{Y}(1+\phi^{-1})\phi n_t - \underline{t}_t^r}{1+\phi^{-1}\frac{\sigma}{1-\mathcal{G}}\frac{WN^r}{Y}} + (1-\chi)c_t^o,$$

$$c_t = \chi\frac{\frac{WN^r}{Y}(1+\phi^{-1})\phi n_t - \underline{t}_t^r}{1-\chi\frac{WN^r}{Y}\frac{\sigma}{1-\mathcal{G}} + (1-\chi)\phi^{-1}\frac{\sigma}{1-\mathcal{G}}\frac{WN^r}{Y}} + (1-\chi)\frac{1+\phi^{-1}\frac{\sigma}{1-\mathcal{G}}\frac{WN^r}{Y}}{1-\chi\frac{WN^r}{Y}\frac{\sigma}{1-\mathcal{G}} + (1-\chi)\phi^{-1}\frac{\sigma}{1-\mathcal{G}}\frac{WN^r}{Y}}c_t^o,$$

$$c_t = \chi\frac{\phi(1+\phi)n_t - \frac{Y}{WN^r}\phi\underline{t}_t^r}{\frac{Y}{WN^r}\phi - \chi\frac{\sigma}{1-\mathcal{G}}\phi + (1-\chi)\frac{\sigma}{1-\mathcal{G}}} + (1-\chi)\frac{\frac{Y}{WN^r}\phi + \frac{\sigma}{1-\mathcal{G}}}{\frac{Y}{WN^r}\phi - \chi\frac{\sigma}{1-\mathcal{G}}\phi + (1-\chi)\frac{\sigma}{1-\mathcal{G}}}c_t^o,$$

$$c_t = \chi(1-\mathcal{G})\frac{\phi(1+\phi)n_t - \mu\phi\underline{t}_t^r}{(1-\mathcal{G})\mu\phi + \sigma - \chi\sigma(1+\phi)} + (1-\chi)\frac{(1-\mathcal{G})\mu\phi + \sigma}{(1-\mathcal{G})\mu\phi + \sigma - \chi\sigma(1+\phi)}c_t^o,$$

and finally

$$c_t = \Theta_n n_t - \Theta_\tau \underline{t}_t^r + \bar{\sigma}^{-1}\sigma\frac{1}{1-\mathcal{G}}c_t^o,$$

where

$$\bar{\sigma}^{-1} = \sigma^{-1}(1-\chi)(1-\mathcal{G})\frac{(1-\mathcal{G})\mu\phi + \sigma}{\phi(1-\mathcal{G})\mu + \sigma - \chi\sigma(1+\phi)},$$

$$\Theta_n = \chi(1-\mathcal{G})\frac{(1+\phi)\phi}{\phi(1-\mathcal{G})\mu + \sigma - \chi\sigma(1+\phi)},$$

$$\Theta_\tau = \chi(1-\mathcal{G})\frac{\mu\phi}{\phi(1-\mathcal{G})\mu + \sigma - \chi\sigma(1+\phi)},$$

Differentiating, we get

$$\dot{c}_t = \Theta_n\dot{n}_t - \Theta_\tau\underline{\dot{t}}_t^r + \bar{\sigma}^{-1}(i_t - \bar{r}_t - \pi_t),$$

and using $\dot{n}_t = \dot{c}_t + \dot{g}_t$, we find the Euler equation

$$\dot{c}_t = \widetilde{\sigma}^{-1}(i_t - \bar{r}_t - \pi_t) + \widetilde{\Theta}_n \dot{g}_t - \widetilde{\Theta}_\tau \underline{t}_t^r,$$

where

$$\widetilde{\sigma}^{-1} = \frac{\bar{\sigma}^{-1}}{1 - \Theta_n},$$

$$\widetilde{\Theta}_n = \frac{\Theta_n}{1 - \Theta_n},$$

$$\widetilde{\Theta}_\tau = \frac{\Theta_\tau}{1 - \Theta_n}.$$

By definition of t_t^r and using the expression for the wage,

$$\underline{t}_t^r = t_t^r - \frac{o}{\chi}\left[\left(1 - \frac{1}{\mu}\right)(c_t + g_t) - \frac{1}{\mu}\left[\frac{\sigma}{1 - \mathcal{G}}c_t + \phi(c_t + g_t)\right]\right].$$

Thus,

$$\underline{t}_t^r = t_t^r + \Psi_c c_t + \Psi_n g_t,$$

where

$$\Psi_c = -\frac{o}{\chi}\left[1 - \frac{1}{\mu}\left(\frac{\sigma}{1 - \mathcal{G}} + (1 + \phi)\right)\right],$$

$$\Psi_n = -\frac{o}{\chi}\left[1 - \frac{1}{\mu}(1 + \phi)\right].$$

Using the Euler equation and the expression for t_t^r, we get

$$[1 - \Theta_n + \Theta_\tau \Psi_c]\dot{c}_t = -\bar{\sigma}^{-1}\pi_t + [\Theta_n - \Theta_\tau \Psi_n]\dot{g}_t - \Theta_\tau \dot{t}_t^r.$$

Thus,

$$\dot{c}_t = -\underline{\widetilde{\sigma}}^{-1}\pi_t + \underline{\widetilde{\Theta}}_n \dot{g}_t - \underline{\widetilde{\Theta}}_\tau \dot{t}_t^r,$$

where

$$\underline{\widetilde{\sigma}}^{-1} = \frac{1}{\underline{\widetilde{\Theta}}_c}\bar{\sigma}^{-1},$$

$$\underline{\widetilde{\Theta}}_n = \frac{1}{\underline{\widetilde{\Theta}}_c}[\Theta_n - \Theta_\tau \Psi_n],$$

$$\underline{\widetilde{\Theta}}_\tau = \frac{1}{\underline{\widetilde{\Theta}}_c}\Theta_\tau,$$

$$\underline{\widetilde{\Theta}}_c = 1 - \Theta_n + \Theta_\tau \Psi_c.$$

Special case: no hand-to-mouth agents $\chi = 0$
The log-linear system is

$$\dot{c}_t = -\hat{\sigma}^{-1}\pi_t,$$
$$\dot{\pi}_t = \rho\pi_t - \kappa[c_t + (1-\xi)g_t],$$

for all $t \geq 0$.

Special case: no profit offset $o = 0$
The log-linear system is

$$\dot{c}_t = -\widetilde{\sigma}^{-1}\pi_t + \widetilde{\Theta}_n\dot{g}_t - \widetilde{\Theta}_\tau \dot{t}^r_t,$$
$$\dot{\pi}_t = \rho\pi_t - \kappa[c_t + (1-\xi)g_t].$$

for all $t \geq 0$.

A.2 Currency Union

Assume that $c^*_t = 0, i^*_t = \bar{r}_t$ for all $t \geq 0$. The log-linearized equations are

$$c^o_t = (1-\mathcal{G})\theta + \frac{(1-\alpha)(1-\mathcal{G})}{\sigma}s_t,$$

$$y_t = (1-\alpha)\hat{c}_t + (1-\mathcal{G})\alpha\left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma}\right]s_t + g_t,$$

$$y_t = n_t,$$
$$\dot{c}^o_t = -(1-\mathcal{G})\sigma^{-1}(\pi_{H,t} + \alpha\dot{s}_t),$$
$$c^r_t = \frac{1}{\mu}\left(w_t + n^r_t\right) - \underline{t}^r_t,$$

$$w_t = \frac{\sigma}{1-\mathcal{G}}c^r_t + \phi n^r_t,$$

$$w_t = \frac{\sigma}{1-\mathcal{G}}\hat{c}_t + \phi n_t,$$

$$\hat{c}_t = \chi c^r_t + (1-\chi)c^o_t,$$
$$n_t = \chi n^r_t + (1-\chi)n^o_t,$$
$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \lambda(w_t + \alpha s_t),$$

$$\int_0^{+\infty} e^{-\rho t}nx_t dt = -\mathrm{nfa}_0,$$

$$\underline{t}^r_t = t^r_t - \frac{o}{\chi}\left[\left(1 - \frac{1}{\mu}\right)n_t + \alpha p_{H,t} - \frac{1}{\mu}w_t\right],$$

with $\mathrm{nfa}_0 = 0$ in the IM case and $\mathrm{nfa}_0 = \int_0^{+\infty} e^{-\rho t}g_t dt$ in the PF case, where $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. Note that we have denoted total consumption of home agents by $\hat{c}_t$ to avoid a confusion with c_t, the total consumption of home goods by private agents (both home and foreign).

Using the expressions for the wage, aggregate consumption, and labor,

$$\hat{c}_t = \Theta_n n_t - \Theta_\tau \underline{t}_t^r + \bar{\sigma}^{-1} \sigma \frac{1}{1-\mathcal{G}} c_t^o,$$

where Θ_n, Θ_τ, and $\bar{\sigma}$ have been defined above. Differentiating the Backus–Smith condition, we get (we could have gotten this equation directly from the definition of s_t)

$$\dot{s}_t = -\pi_{H,t}.$$

Now we can get to an equation involving total (home + foreign) consumption of the domestic good $c_t = y_t - g_t$ which yields

$$c_t = (1-\alpha)\hat{c}_t + (1-\mathcal{G})\alpha \left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma} \right] s_t.$$

Differentiating, we get

$$\dot{c}_t = (1-\alpha)\dot{\hat{c}}_t + (1-\mathcal{G})\alpha \left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma} \right] \dot{s}_t,$$

then combining with the equation for $\hat{c}_t$,

$$\dot{c}_t = (1-\alpha) \left[\Theta_n \dot{n}_t - \Theta_\tau \underline{t}_t^r + \bar{\sigma}^{-1} \sigma \frac{1}{1-\mathcal{G}} \dot{c}_t^o \right] + (1-\mathcal{G})\alpha \left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma} \right] \dot{s}_t,$$

and replacing $n_t = c_t + g_t$,

$$\dot{c}_t = (1-\alpha) \left[\Theta_n (\dot{c}_t + \dot{g}_t) - \Theta_\tau \underline{t}_t^r + \bar{\sigma}^{-1} \sigma \frac{1}{1-\mathcal{G}} \dot{c}_t^o \right] + (1-\mathcal{G})\alpha \left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma} \right] \dot{s}_t,$$

and rearranging

$$\dot{c}_t = \widetilde{\Theta}_n \dot{g}_t - \widetilde{\Theta}_\tau \underline{t}_t^r + \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n} \sigma \frac{1}{1-\mathcal{G}} \dot{c}_t^o + \frac{1}{1-(1-\alpha)\Theta_n} \frac{\alpha(1-\mathcal{G})(\omega+1-\alpha)}{\sigma} \dot{s}_t,$$

where

$$\widetilde{\Theta}_n = \frac{(1-\alpha)\Theta_n}{1-(1-\alpha)\Theta_n},$$

$$\widetilde{\Theta}_\tau = \frac{(1-\alpha)\Theta_\tau}{1-(1-\alpha)\Theta_n},$$

then using the Euler equation for optimizers

$$\dot{c}_t = \widetilde{\Theta}_n \dot{g}_t - \widetilde{\Theta}_\tau \underline{t}_t^r - \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n} [\pi_{H,t} + \alpha \dot{s}_t] + \frac{1}{1-(1-\alpha)\Theta_n} \frac{\alpha(1-\mathcal{G})(\omega+1-\alpha)}{\sigma} \dot{s}_t,$$

and finally combining with the expression for $\dot{s}_t = -\pi_{H,t}$

$$\dot{c}_t = \widetilde{\Theta}_n \dot{g}_t - \widetilde{\Theta}_\tau \dot{\underline{t}}_t^r - \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}[(1-\alpha)\pi_{H,t}]$$

$$-\frac{1}{1-(1-\alpha)\Theta_n}\frac{\alpha(1-\mathcal{G})(\omega+1-\alpha)}{\sigma}\dot{\pi}_{H,t},$$

which we can rewrite as

$$\dot{c}_t = \widetilde{\Theta}_n \dot{g}_t - \widetilde{\Theta}_\tau \dot{\underline{t}}_t^r - \frac{\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\left[(1-\alpha)^2 + \alpha\frac{\bar{\sigma}}{\sigma}(1-\mathcal{G})(\omega+1-\alpha)\right]\dot{\pi}_{H,t},$$

$$\dot{c}_t = \widetilde{\Theta}_n \dot{g}_t - \widetilde{\Theta}_\tau \dot{\underline{t}}_t^r - \widetilde{\sigma}^{-1}\pi_{H,t},$$

where

$$\widetilde{\sigma}^{-1} = \frac{\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\left[(1-\alpha)^2 + \alpha\frac{\bar{\sigma}}{\sigma}(1-\mathcal{G})(\omega+1-\alpha)\right].$$

This is our Euler equation.[v]

To derive an initial condition, we use

$$c_t = (1-\alpha)\hat{c}_t + (1-\mathcal{G})\alpha\left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma}\right]s_t,$$

$$\hat{c}_t = \Theta_n n_t - \Theta_\tau \underline{t}_t^r + \bar{\sigma}^{-1}\sigma\frac{1}{1-\mathcal{G}}c_t^o,$$

$$c_t^o = (1-\mathcal{G})\theta + \frac{(1-\alpha)(1-\mathcal{G})}{\sigma}s_t,$$

and

$$n_t = c_t + g_t,$$

to get

$$c_t = \widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau \underline{t}_t^r + \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\frac{1}{1-\mathcal{G}}\left((1-\mathcal{G})\theta + \frac{(1-\alpha)(1-\mathcal{G})}{\sigma}s_t\right)$$

$$+ \frac{(1-\mathcal{G})\alpha\left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}s_t,$$

and apply it at $t=0$ with $s_0=0$ to get

[v] We can check that when there are no hand-to-mouth consumers, this boils down to

$$\dot{c}_t = -\sigma^{-1}(1-\mathcal{G})[1+\alpha(\omega-1)]\pi_t,$$

which is exactly the expression that we found.

$$c_0 = \widetilde{\Theta}_n g_0 - \widetilde{\Theta}_\tau \underline{t}_0^r + \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n} \sigma \frac{1}{1-\mathcal{G}}(1-\mathcal{G})\theta.$$

Hence with complete markets, this boils down to the simple condition

$$c_0 = \widetilde{\Theta}_n g_0 - \widetilde{\Theta}_\tau \underline{t}_0^r.$$

Finally we need to compute

$$mc_t = w_t + p_t - p_{H,t} = w_t + \alpha s_t,$$

We have

$$w_t = \frac{\sigma}{1-\mathcal{G}}\hat{c}_t + \phi n_t,$$

$$w_t = \frac{\sigma}{1-\mathcal{G}}\hat{c}_t + \phi(c_t + g_t),$$

which using

$$\hat{c}_t = \Theta_n n_t - \Theta_\tau \underline{t}_t^r + \bar{\sigma}^{-1}\sigma\frac{1}{1-\mathcal{G}}c_t^o,$$

we can rewrite as

$$w_t = \frac{\sigma}{1-\mathcal{G}}\left(\Theta_n(c_t + g_t) - \Theta_\tau \underline{t}_t^r + \bar{\sigma}^{-1}\sigma\frac{1}{1-\mathcal{G}}c_t^o\right) + \phi(c_t + g_t),$$

$$w_t = \frac{\sigma}{1-\mathcal{G}}\left[\Theta_n(c_t + g_t) - \Theta_\tau \underline{t}_t^r + \bar{\sigma}^{-1}\sigma\frac{1}{1-\mathcal{G}}\left((1-\mathcal{G})\theta + \frac{(1-\alpha)(1-\mathcal{G})}{\sigma}s_t\right)\right] + \phi(c_t + g_t),$$

so that

$$w_t + \alpha s_t = \left(\frac{\sigma\Theta_n}{1-\mathcal{G}} + \phi\right)(c_t + g_t) - \frac{\sigma}{1-\mathcal{G}}\Theta_\tau \underline{t}_t^r + \left(\frac{\sigma}{1-\mathcal{G}}\right)^2 \bar{\sigma}^{-1}(1-\mathcal{G})\theta$$
$$+ \left[\alpha + \left(\frac{\sigma}{1-\mathcal{G}}\right)^2 \bar{\sigma}^{-1}\frac{(1-\alpha)(1-\mathcal{G})}{\sigma}\right]s_t,$$

which using

$$c_t = \widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau \underline{t}_t^r + \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\frac{1}{1-\mathcal{G}}(1-\mathcal{G})\theta$$
$$+ \left[\frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\frac{1}{1-\mathcal{G}}\frac{(1-\alpha)(1-\mathcal{G})}{\sigma} + \frac{(1-\mathcal{G})\alpha\left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}\right]s_t,$$

i.e.

$$s_t = \frac{c_t - \widetilde{\Theta}_n g_t + \widetilde{\Theta}_\tau \underline{t}_t^r - \dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\dfrac{1}{1-\mathcal{G}}(1-\mathcal{G})\theta}{\dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\dfrac{1}{1-\mathcal{G}}\dfrac{(1-\alpha)(1-\mathcal{G})}{\sigma} + \dfrac{(1-\mathcal{G})\alpha\left[\dfrac{\omega}{\sigma}+\dfrac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}},$$

we can rewrite as

$$w_t + \alpha s_t = \left(\frac{\sigma\Theta_n}{1-\mathcal{G}} + \phi\right)(c_t + g_t) - \frac{\sigma}{1-\mathcal{G}}\Theta_\tau \underline{t}_t^r + \left(\frac{\sigma}{1-\mathcal{G}}\right)^2\bar{\sigma}^{-1}(1-\mathcal{G})\theta$$

$$+ \frac{\alpha + \dfrac{\sigma}{1-\mathcal{G}}\bar{\sigma}^{-1}(1-\alpha)}{\dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}(1-\alpha) + \dfrac{(1-\mathcal{G})\alpha\left[\dfrac{\omega}{\sigma}+\dfrac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}}$$

$$\times \left[c_t - \widetilde{\Theta}_n g_t + \widetilde{\Theta}_\tau \underline{t}_t^r - \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\frac{1}{1-\mathcal{G}}(1-\mathcal{G})\theta\right],$$

$$w_t + \alpha s_t = \left(\frac{\sigma\Theta_n}{1-\mathcal{G}} + \phi\right)(c_t + g_t) - \frac{\sigma}{1-\mathcal{G}}\Theta_\tau \underline{t}_t^r + \left(\frac{\sigma}{1-\mathcal{G}}\right)^2\bar{\sigma}^{-1}(1-\mathcal{G})\theta$$

$$+ \frac{\alpha + \dfrac{\sigma}{1-\mathcal{G}}\bar{\sigma}^{-1}(1-\alpha)}{\dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}(1-\alpha) + \dfrac{(1-\mathcal{G})\alpha\left[\dfrac{\omega}{\sigma}+\dfrac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}}$$

$$\times \left[c_t - \widetilde{\Theta}_n g_t + \widetilde{\Theta}_\tau \underline{t}_t^r - \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\frac{1}{1-\mathcal{G}}(1-\mathcal{G})\theta\right].$$

We can then replace this expression in to get the New Keynesian Phillips Curve

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \lambda(w_t + \alpha s_t).$$

The system is summarized by

$$\dot{c}_t = \widetilde{\Theta}_n \dot{g}_t - \widetilde{\Theta}_\tau \underline{t}_t^r - \widetilde{\sigma}^{-1}\pi_{H,t},$$
$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \lambda(w_t + \alpha s_t),$$
$$c_0 = \widetilde{\Theta}_n g_0 - \widetilde{\Theta}_\tau \underline{t}_0^r + \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\frac{1}{1-\mathcal{G}}(1-\mathcal{G})\theta,$$

and the nfa condition, where

$$w_t + \alpha s_t = \left(\frac{\sigma\Theta_n}{1-\mathcal{G}} + \phi\right)(c_t + g_t) - \frac{\sigma}{1-\mathcal{G}}\Theta_\tau \underline{t}_t^r + \left(\frac{\sigma}{1-\mathcal{G}}\right)^2 \bar{\sigma}^{-1}(1-\mathcal{G})\theta$$

$$+ \frac{\alpha + \dfrac{\sigma}{1-\mathcal{G}}\bar{\sigma}^{-1}(1-\alpha)}{\dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}(1-\alpha) + \dfrac{(1-\mathcal{G})\alpha\left[\dfrac{\omega}{\sigma} + \dfrac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}}$$

$$\times \left[c_t - \widetilde{\Theta}_n g_t + \widetilde{\Theta}_\tau \underline{t}_t^r - \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\frac{1}{1-\mathcal{G}}(1-\mathcal{G})\theta\right].$$

Define $\widetilde{\kappa}$ by

$$\widetilde{\kappa} = \lambda\left[\frac{\sigma\Theta_n}{1-\mathcal{G}} + \phi + \frac{\alpha + \dfrac{\sigma}{1-\mathcal{G}}\bar{\sigma}^{-1}(1-\alpha)}{\dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}(1-\alpha) + \dfrac{(1-\mathcal{G})\alpha\left[\dfrac{\omega}{\sigma} + \dfrac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}}\right].$$

Define $\widetilde{\xi}$ by

$$\widetilde{\kappa}\left(1 - \widetilde{\xi}\right) = \lambda\left[\frac{\sigma\Theta_n}{1-\mathcal{G}} + \phi - \frac{\alpha + \dfrac{\sigma}{1-\mathcal{G}}\bar{\sigma}^{-1}(1-\alpha)}{\dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}(1-\alpha) + \dfrac{(1-\mathcal{G})\alpha\left[\dfrac{\omega}{\sigma} + \dfrac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}}\widetilde{\Theta}_n\right].$$

Define $\widetilde{\alpha}$ by

$$\widetilde{\alpha} = 1 - \frac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\frac{1}{1-\mathcal{G}}.$$

Define $\widetilde{\omega}$ by

$$\widetilde{\omega} = \frac{1}{(1-\mathcal{G})\widetilde{\sigma}\widetilde{\alpha}}$$

$$\times \left[\left(\frac{\sigma}{1-\mathcal{G}}\right)^2\bar{\sigma}^{-1}(1-\mathcal{G}) - \frac{\alpha + \dfrac{\sigma}{1-\mathcal{G}}\bar{\sigma}^{-1}(1-\alpha)}{\dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}(1-\alpha) + \dfrac{(1-\mathcal{G})\alpha\left(\dfrac{\omega}{\sigma} + \dfrac{1-\alpha}{\sigma}\right)}{1-(1-\alpha)\Theta_n}}\frac{(1-\alpha)\bar{\sigma}^{-1}\sigma}{1-(1-\alpha)\Theta_n}\right].$$

Define $\widetilde{\widetilde{\Theta}}_\tau$ by

$$\widetilde{\widetilde{\Theta}}_\tau = \frac{\lambda}{\widetilde{\kappa}}\left[-\frac{\sigma}{1-\mathcal{G}}\Theta_\tau + \frac{\alpha + \frac{\sigma}{1-\mathcal{G}}\bar\sigma^{-1}(1-\alpha)}{\frac{(1-\alpha)\bar\sigma^{-1}}{1-(1-\alpha)\Theta_n}(1-\alpha) + \frac{(1-\mathcal{G})\alpha\left[\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}}\widetilde{\Theta}_\tau\right].$$

Define Γ_1 by

$$\Gamma_1 = (1-\alpha)^2\bar\sigma^{-1} + (1-\mathcal{G})\alpha\left(\frac{\omega}{\sigma} + \frac{1-\alpha}{\sigma}\right),$$

Then we can rewrite the system as

$$\dot\pi_{H,t} = \rho\pi_{H,t} - \widetilde{\kappa}\left(c_t + \left(1-\widetilde{\xi}\right)g_t\right) - (1-\mathcal{G})\lambda\widetilde{\sigma}\widetilde{\alpha}\widetilde{\omega}\theta - \widetilde{\kappa}\widetilde{\widetilde{\Theta}}_\tau\underline{t}_t^r,$$

$$\dot c_t = -\widetilde\sigma^{-1}\pi_{H,t} + \widetilde{\Theta}_n\dot g_t - \widetilde{\Theta}_\tau\underline{t}_t^r,$$

with an initial condition

$$c_0 = (1-\mathcal{G})(1-\widetilde\alpha)\theta + \widetilde{\Theta}_n g_0 - \widetilde{\Theta}_\tau\underline{t}_0^r,$$

and the nfa condition.

For net exports we get

$$nx_t = -(1-\mathcal{G})\alpha s_t + y_t - \hat c_t - g_t,$$

$$nx_t = (1-\mathcal{G})\left[\alpha\frac{\omega}{\sigma} + \alpha\frac{1-\alpha}{\sigma} - \alpha\right]s_t - \alpha\hat c_t,$$

$$nx_t = (1-\mathcal{G})\left[\alpha\frac{\omega}{\sigma} + \alpha\frac{1-\alpha}{\sigma} - \alpha\right]s_t$$
$$-\alpha\left[\Theta_n(c_t + g_t) - \Theta_\tau\underline{t}_t^r + \bar\sigma^{-1}\sigma\frac{1}{1-\mathcal{G}}c_t^o\right],$$

and finally

$$nx_t = (1-\mathcal{G})\left[\alpha\frac{\omega}{\sigma} + \alpha\frac{1-\alpha}{\sigma} - \alpha\right]s_t$$
$$-\alpha\left[\Theta_n(c_t + g_t) - \Theta_\tau\underline{t}_t^r + \bar\sigma^{-1}\sigma\frac{1}{1-\mathcal{G}}\left((1-\mathcal{G})\theta + \frac{(1-\alpha)(1-\mathcal{G})}{\sigma}s_t\right)\right],$$

where

$$s_t = \frac{c_t - \widetilde{\Theta}_n g_t + \widetilde{\Theta}_\tau \underline{t}_t^r - \dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\dfrac{1}{1-\mathcal{G}}(1-\mathcal{G})\theta}{\dfrac{(1-\alpha)\bar{\sigma}^{-1}}{1-(1-\alpha)\Theta_n}\sigma\dfrac{1}{1-\mathcal{G}}\dfrac{(1-\alpha)(1-\mathcal{G})}{\sigma} + \dfrac{(1-\mathcal{G})\alpha\left[\dfrac{\omega}{\sigma}+\dfrac{1-\alpha}{\sigma}\right]}{1-(1-\alpha)\Theta_n}}.$$

Using the Euler equation,

$$p_{H,t} = -s_0 - \frac{1}{\Gamma_1}[1-(1-\alpha)\Theta_n](c_t - c_0) + \frac{1}{\Gamma_1}(1-\alpha)\Theta_n(g_t - g_0) - (1-\alpha)\frac{1}{\Gamma_1}\Theta_\tau\left(\underline{t}_t^r - \underline{t}_0^r\right).$$

Using the initial condition for consumption,

$$p_{H,t} = -\frac{1}{\Gamma_1}[1-(1-\alpha)\Theta_n]c_t + \frac{1}{\Gamma_1}(1-\alpha)\Theta_n g_t - (1-\alpha)\frac{1}{\Gamma_1}\Theta_\tau \underline{t}_t^r + (1-\alpha)\frac{1}{\Gamma_1}\bar{\sigma}^{-1}\sigma\theta,$$

since $s_0 = 0$.

By definition of $\underline{t}_t^r$ and using the expressions for output, for prices and for the real wage,

$$\underline{t}_t^r = t_t^r - \frac{o}{\chi}\left[\left(1-\frac{1}{\mu}\right)(c_t + g_t)\right.$$

$$+ \alpha\left[-\frac{1}{\Gamma_1}[1-(1-\alpha)\Theta_n]c_t + \frac{1}{\Gamma_1}(1-\alpha)\Theta_n g_t - (1-\alpha)\frac{1}{\Gamma_1}\Theta_\tau \underline{t}_t^r + (1-\alpha)\frac{1}{\Gamma_1}\bar{\sigma}^{-1}\sigma\theta\right]$$

$$\left.-\frac{1}{\mu}\left[\frac{\sigma}{1-\mathcal{G}}\frac{1}{1-\alpha}\left[c_t - (1-\mathcal{G})\alpha\left(\frac{\omega}{\sigma}+\frac{1-\alpha}{\sigma}\right)s_t\right] + \phi n_t\right]\right].$$

Using the expression for the terms-of-trade,

$$\underline{t}_t^r = t_t^r - \frac{o}{\chi}\left[\left(1-\frac{1}{\mu}\right)(c_t + g_t)\right.$$

$$+ \alpha\left[-\frac{1}{\Gamma_1}[1-(1-\alpha)\Theta_n]c_t + \frac{1}{\Gamma_1}(1-\alpha)\Theta_n g_t - (1-\alpha)\frac{1}{\Gamma_1}\Theta_\tau \underline{t}_t^r + (1-\alpha)\bar{\sigma}^{-1}\sigma\theta\right]$$

$$-\frac{1}{\mu}\frac{\sigma}{1-\mathcal{G}}\frac{1}{1-\alpha}c_t$$

$$+\frac{1}{\mu}\sigma\frac{\alpha}{1-\alpha}\left(\frac{\omega}{\sigma}-\frac{1-\alpha}{\sigma}\right)\frac{1}{\Gamma_1}\left[[1-(1-\alpha)\Theta_n]c_t - (1-\alpha)\Theta_n g_t + (1-\alpha)\Theta_\tau \underline{t}_t^r - (1-\alpha)\bar{\sigma}^{-1}\sigma\theta\right]$$

$$\left.-\frac{1}{\mu}\phi n_t\right].$$

Thus,

$$\underline{t}_t^r = \psi_\tau t_t^r + \psi_c c_t + \psi_n g_t + \psi_\theta \theta,$$

where

$$\psi_c = -\frac{o}{\chi}\frac{1}{\hat{\psi}_\tau}\left[\left(1-\frac{1}{\mu}\right) - \alpha\frac{1}{\Gamma_1}[1-(1-\alpha)\Theta_n] - \frac{1}{\mu}\left[\frac{\sigma}{1-\alpha}\left[\frac{1}{1-\mathcal{G}} - \alpha\left(\frac{\omega}{\sigma}+\frac{1-\alpha}{\sigma}\right)\frac{1}{\Gamma_1}[1-(1-\alpha)\Theta_n]\right] + \phi\right]\right],$$

$$\psi_n = -\frac{o}{\chi}\frac{1}{\hat{\psi}_\tau}\left[\left(1-\frac{1}{\mu}\right) + \alpha\frac{1}{\Gamma_1}(1-\alpha)\Theta_n - \frac{1}{\mu}\left[\sigma\alpha\left(\frac{\omega}{\sigma}+\frac{1-\alpha}{\sigma}\right)\frac{1}{\Gamma_1}\Theta_n + \phi\right]\right],$$

$$\psi_\theta = -\frac{o}{\chi}\frac{1}{\hat{\psi}_\tau}\left[\alpha\frac{1}{\Gamma_1}(1-\alpha)\bar{\sigma}^{-1}\sigma - \frac{1}{\mu}\sigma\alpha\left(\frac{\omega}{\sigma}+\frac{1-\alpha}{\sigma}\right)\frac{1}{\Gamma_1}\bar{\sigma}^{-1}\sigma\right],$$

$$\psi_\tau = \frac{1}{\hat{\psi}_\tau},$$

$$\hat{\psi}_\tau = 1 - \frac{o}{\chi}\left[\alpha\frac{1}{\Gamma_1}(1-\alpha)\Theta_\tau - \frac{1}{\mu}\sigma\alpha\left(\frac{\omega}{\sigma}+\frac{1-\alpha}{\sigma}\right)\frac{1}{\Gamma_1}\Theta_\tau\right].$$

Using the Euler equation and the expression for $\underline{t}_t^r$,

$$[1-(1-\alpha)\Theta_n+(1-\alpha)\Theta_\tau\psi_c]\dot{c}_t = \Gamma_1\pi_{H,t}+(1-\alpha)[\Theta_n-\Theta_\tau\psi_n]\dot{g}_t - (1-\alpha)\Theta_\tau\psi_\tau \dot{t}_t^r.$$

Thus,

$$\dot{c}_t = -\underline{\tilde{\sigma}}^{-1}\pi_{H,t}+\underline{\tilde{\Theta}}_n\dot{g}_t - \underline{\tilde{\Theta}}_\tau \dot{t}_t^r,$$

where

$$\underline{\tilde{\sigma}}^{-1} = \frac{1}{\underline{\tilde{\Theta}}_c}\Gamma_1,$$

$$\underline{\tilde{\Theta}}_n = (1-\alpha)\frac{1}{\underline{\tilde{\Theta}}_c}[\Theta_n-\Theta_\tau\psi_n],$$

$$\underline{\tilde{\Theta}}_\tau = (1-\alpha)\frac{1}{\underline{\tilde{\Theta}}_c}\Theta_\tau\psi_\tau,$$

$$\underline{\tilde{\Theta}}_c = 1-(1-\alpha)\Theta_n+(1-\alpha)\Theta_\tau\psi_c.$$

Using the New Keynesian Phillips Curve and the expression for $\underline{t}_t^r$,

$$\dot{\pi}_{H,t} = \rho\pi_{H,t}-\tilde{\kappa}\left[c_t+\left(1-\tilde{\xi}\right)g_t\right] - (1-\mathcal{G})\lambda\tilde{\sigma}\tilde{\alpha}\tilde{\omega}\theta - \tilde{\kappa}\tilde{\Theta}_\tau\left[\psi_\tau t_t^r + \psi_c c_t + \psi_n g_t + \psi_\theta \theta\right].$$

Thus,

$$\dot{\pi}_{H,t} = \rho\pi_{H,t}-\underline{\tilde{\kappa}}_c c_t - \underline{\tilde{\kappa}}_n g_t - \underline{\tilde{\kappa}}_\theta \theta - \underline{\tilde{\kappa}}_\tau t_t^r,$$

where

$$\widetilde{\underline{\kappa}}_c = \widetilde{\kappa}\left(1 + \widetilde{\widetilde{\Theta}}_\tau \psi_c\right),$$

$$\widetilde{\underline{\kappa}}_n = \widetilde{\kappa}\left(1 - \widetilde{\xi} + \widetilde{\widetilde{\Theta}}_\tau \psi_n\right),$$

$$\widetilde{\underline{\kappa}}_\theta = (1 - \mathcal{G})\lambda\widetilde{\sigma}\widetilde{\alpha}\widetilde{\omega} + \widetilde{\kappa}\widetilde{\widetilde{\Theta}}_\tau \psi_\theta,$$

$$\widetilde{\underline{\kappa}}_\tau = \widetilde{\kappa}\widetilde{\widetilde{\Theta}}_\tau \psi_\tau.$$

Using the initial condition for consumption and the expression for $\underline{t}_t^r$,

$$c_0 = \Upsilon\theta + \widetilde{\underline{\Theta}}_n g_0 - \widetilde{\underline{\Theta}}_\tau t_0^r,$$

where

$$\Upsilon = \frac{1}{\widetilde{\underline{\Theta}}_c}[(1 - \mathcal{G})(1 - \widetilde{\alpha})[1 - (1 - \alpha)\Theta_n] - (1 - \alpha)\Theta_\tau \psi_\theta].$$

Using the expressions for net exports and for $\underline{t}_t^r$,

$$nx_t = \alpha(1 - \mathcal{G})\left[\frac{1}{1 - \alpha}\left(\frac{\omega}{\sigma} + \frac{1 - \alpha}{\sigma}\right) - 1\right]s_t - \frac{\alpha}{1 - \alpha}c_t.$$

Using the expression for the terms-of-trade,

$$nx_t = \alpha(1 - \mathcal{G})\left[\frac{1}{1 - \alpha}\left(\frac{\omega}{\sigma} + \frac{1 - \alpha}{\sigma}\right) - 1\right]$$
$$\times \frac{1}{\Gamma_1}\left[[1 - (1 - \alpha)\Theta_n]c_t - (1 - \alpha)\Theta_n g_t + (1 - \alpha)\Theta_\tau \underline{t}_t^r - (1 - \alpha)\bar{\sigma}^{-1}\sigma\theta\right]$$
$$- \frac{\alpha}{1 - \alpha}c_t.$$

Thus,

$$nx_t = \Omega_c c_t - (1 - \mathcal{G})\frac{\Gamma_2}{\Gamma_1}$$
$$\times \left[[(1 - \alpha)\Theta_n - (1 - \alpha)\Theta_\tau \psi_n]g_t - (1 - \alpha)\Theta_\tau \psi_\tau t_t^r + \left[(1 - \alpha)\bar{\sigma}^{-1}\sigma - (1 - \alpha)\Theta_\tau \psi_\theta\right]\theta\right],$$

where

$$\Omega_c = (1 - \mathcal{G})\frac{\Gamma_2}{\Gamma_1}[1 - (1 - \alpha)\Theta_n + (1 - \alpha)\Theta_\tau \psi_c] - \frac{\alpha}{1 - \alpha},$$

$$\Gamma_2 = \alpha\left[\frac{1}{1 - \alpha}\left(\frac{\omega}{\sigma} + \frac{1 - \alpha}{\sigma}\right) - 1\right].$$

Using the expressions for the Pareto weight and for net exports,

$$\theta = \int_0^{+\infty} e^{-\rho s}\left[\widetilde{\Omega}_c c_s + \widetilde{\Omega}_n g_s + \widetilde{\Omega}_\tau t_s^r\right]ds + \widetilde{\Omega}_f \mathrm{nfa}_0,$$

where

$$\widetilde{\Omega}_c = \rho\frac{\Gamma_1}{\Gamma_2}\frac{\Omega_c}{1-\mathcal{G}}\frac{1}{(1-\alpha)\bar{\sigma}^{-1}\sigma - \Theta_\tau\Psi_\theta},$$

$$\widetilde{\Omega}_n = -\rho\frac{(1-\alpha)\Theta_n - (1-\alpha)\Theta_\tau\Psi_n}{(1-\alpha)\bar{\sigma}^{-1}\sigma - \Theta_\tau\Psi_\theta},$$

$$\widetilde{\Omega}_\tau = \rho\frac{(1-\alpha)\Theta_\tau\Psi_\tau}{(1-\alpha)\bar{\sigma}^{-1}\sigma - \Theta_\tau\Psi_\theta},$$

$$\widetilde{\Omega}_f = \rho\frac{\Gamma_1}{\Gamma_2}\frac{1}{1-\mathcal{G}}\frac{1}{(1-\alpha)\bar{\sigma}^{-1}\sigma - \Theta_\tau\Psi_\theta}.$$

Special case: no hand-to-mouth agents $\chi = 0$

The log-linear system is

$$\dot{c}_t = -\hat{\sigma}^{-1}\pi_{H,t},$$

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \kappa[c_t + (1-\xi)g_t] - (1-\mathcal{G})\lambda\hat{\sigma}\alpha\omega\theta,$$

for all $t \geq 0$, with

$$c_0 = (1-\mathcal{G})(1-\alpha)\theta$$

And

$$\theta = \int_0^{+\infty} e^{-\rho s}\rho(1-\mathcal{G})\frac{\omega - \sigma}{\omega + (1-\alpha)(1-\sigma)}c_s ds + \rho\frac{1}{\alpha\omega}\frac{\alpha\omega + 1 - \alpha}{\alpha\omega + (1-\alpha)(1-\sigma)}\frac{1}{1-\mathcal{G}}\mathrm{nfa}_0,$$

where $\kappa = \lambda[\phi + \hat{\sigma}]$, $\xi = \dfrac{\hat{\sigma}}{\phi + \hat{\sigma}}$, $\hat{\sigma} = \dfrac{\sigma}{1-\mathcal{G}(1-\alpha) + \alpha\omega}$.

Special case: no profit offset $o = 0$

The log-linear system is

$$\dot{c}_t = -\widetilde{\sigma}^{-1}\pi_{H,t} + \widetilde{\Theta}_n\dot{g}_t - \widetilde{\Theta}_\tau t_t^r,$$

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \widetilde{\kappa}\left[c_t + \left(1 - \widetilde{\xi}\right)g_t\right] - (1-\mathcal{G})\lambda\widetilde{\sigma}\widetilde{\alpha}\widetilde{\omega}\theta - \widetilde{\kappa}\widetilde{\Theta}_\tau t_t^r,$$

for all $t \geq 0$, with

$$c_0 = \frac{1}{1-\Theta_n}\left[(1-\mathcal{G})(1-\widetilde{\alpha})[1 - (1-\alpha)\Theta_n]]\theta + \widetilde{\Theta}_n g_0 - \widetilde{\Theta}_\tau t_0^r\right.$$

and

$$\theta = \int_0^{+\infty} e^{-\rho s}\left[\rho\frac{\Gamma_1}{\Gamma_2}\frac{(1-\mathcal{G})\frac{\Gamma_2}{\Gamma_1}[1-(1-\alpha)\Theta_n]-\frac{\alpha}{1-\alpha}}{1-\mathcal{G}}\frac{1}{(1-\alpha)\bar{\sigma}^{-1}\sigma}c_s - \rho\frac{\Theta_n}{\bar{\sigma}^{-1}\sigma}g_s + \rho\frac{\Theta_\tau}{\bar{\sigma}^{-1}\sigma}t_s^r\right]ds$$

$$+ \rho\frac{\Gamma_1}{\Gamma_2}\frac{1}{1-\mathcal{G}}\frac{1}{(1-\alpha)\bar{\sigma}^{-1}\sigma}\mathrm{nf}a_0.$$

Appendix B

This appendix derives the solutions to the linear systems obtained in Appendix A. The same special cases are considered.

B.1 Liquidity Trap

Define

$$\widetilde{\nu} = \frac{\rho - \sqrt{\rho^2 + 4\kappa\underline{\widetilde{\sigma}}^{-1}}}{2}, \quad \widetilde{\overline{\nu}} = \frac{\rho + \sqrt{\rho^2 + 4\kappa\underline{\widetilde{\sigma}}^{-1}}}{2}.$$

The equilibrium is completely characterized by the following:

$$\dot{X}_t = AX_t + B_t,$$

where

$$X_t = [\pi_t, c_t]^t, \quad A = \begin{bmatrix} \rho & -\kappa \\ -\underline{\widetilde{\sigma}}^{-1} & 0 \end{bmatrix}, \quad B_t = -\kappa(1-\xi)g_t E_1 + \left[\widetilde{\Theta}_n\dot{g}_t - \widetilde{\Theta}_\tau\dot{t}_t^r\right]E_2,$$

for all $t \geq 0$.

The (unique) solution that satisfies saddle-path stability writes:

$$X_t = \int_t^{+\infty}\kappa(1-\xi)g_s e^{-A(s-t)}E_1 ds - \int_t^{+\infty}\left(\widetilde{\Theta}_n\dot{g}_s - \widetilde{\Theta}_\tau\dot{t}_s^r\right)e^{-A(s-t)}E_2 ds.$$

Equivalently, integrating the relevant objects by part,

$$X_t = \int_t^{+\infty}\kappa(1-\xi)g_s e^{-A(s-t)}E_1 ds + \left(\widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r\right)E_2$$
$$- \int_t^{+\infty}\left(\widetilde{\Theta}_n g_s - \widetilde{\Theta}_\tau t_s^r\right)Ae^{-A(s-t)}E_2 ds.$$

Thus,

$$c_t = \int_t^{+\infty}\kappa(1-\xi)E_2^t e^{-A(s-t)}E_1 ds + \left(\widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r\right) - \int_t^{+\infty}\left(\widetilde{\Theta}_n g_s - \widetilde{\Theta}_\tau t_s^r\right)E_2^t Ae^{-A(s-t)}E_2 ds.$$

Note that

$$E_2^t e^{-At} E_1 = \widetilde{\underline{\sigma}}^{-1} \frac{e^{-\tilde{\nu}t} - e^{-\bar{\tilde{\nu}}t}}{\bar{\tilde{\nu}} - \tilde{\nu}} \; , \quad E_2^t A e^{-At} E_2 = -\kappa(1-\xi)\widetilde{\underline{\sigma}}^{-1} \frac{e^{-\tilde{\nu}t} - e^{-\bar{\tilde{\nu}}t}}{\bar{\tilde{\nu}} - \tilde{\nu}},$$

for all $t \geq 0$.

Thus,

$$c_t = \widetilde{\underline{\Theta}}_n g_t - \widetilde{\underline{\Theta}}_\tau t_t^r$$

$$+ \kappa\widetilde{\underline{\sigma}}^{-1}\left(1 - \xi + \widetilde{\underline{\Theta}}_n\right)\int_t^{+\infty} \frac{e^{-\tilde{\nu}(s-t)} - e^{-\bar{\tilde{\nu}}(s-t)}}{\bar{\tilde{\nu}} - \tilde{\nu}} g_s ds$$

$$- \kappa(1-\xi)\widetilde{\underline{\sigma}}^{-1}\widetilde{\underline{\Theta}}_\tau \int_t^{+\infty} \frac{e^{-\tilde{\nu}(s-t)} - e^{-\bar{\tilde{\nu}}(s-t)}}{\bar{\tilde{\nu}} - \tilde{\nu}} t_s^r ds.$$

Therefore,

$$c_t = \widetilde{\underline{\Theta}}_n g_t - \widetilde{\underline{\Theta}}_\tau t_t^r + \int_0^{+\infty} \alpha_s^{c,HM} g_{t+s} ds - \int_0^{+\infty} \gamma_s^{c,HM} t_{t+s}^r ds,$$

where

$$\alpha_s^{c,HM} = \left(1 + \frac{\widetilde{\underline{\Theta}}_n}{1-\xi}\right)\widetilde{\alpha}_s^{c,HM} \; , \quad \gamma_s^{c,HM} = \frac{\widetilde{\underline{\Theta}}_\tau}{1-\xi}\widetilde{\alpha}_s^{c,HM},$$

$$\widetilde{\alpha}_s^{c,HM} = \kappa\widetilde{\underline{\sigma}}^{-1}(1-\xi)e^{-\tilde{\nu}s}\frac{e^{(\bar{\tilde{\nu}}-\tilde{\nu})s} - 1}{\bar{\tilde{\nu}} - \tilde{\nu}}.$$

Special case: no hand-to-mouth agents $\chi = 0$
Define

$$\nu = \frac{\rho - \sqrt{\rho^2 + 4\kappa\hat{\sigma}^{-1}}}{2} \; , \quad \bar{\nu} = \frac{\rho + \sqrt{\rho^2 + 4\kappa\hat{\sigma}^{-1}}}{2}.$$

We have

$$c_t = \int_0^{+\infty} \alpha_s^{c,HM} g_{t+s} ds,$$

where

$$\alpha_s^{c,HM} = \kappa\hat{\sigma}^{-1}(1-\xi)e^{-\bar{\nu}s}\frac{e^{(\bar{\nu}-\nu)s} - 1}{\bar{\nu} - \nu}.$$

Special case: no profit offset $o = 0$

We have

$$c_t = \widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r + \int_0^{+\infty} \alpha_s^{c,HM} g_{t+s} ds - \int_0^{+\infty} \gamma_s^{c,HM} t_{t+s}^r ds,$$

where

$$\alpha_s^{c,HM} = \left(1 + \frac{\widetilde{\Theta}_n}{1-\xi}\right) \widetilde{\alpha}_s^{c,HM}, \quad \gamma_s^{c,HM} = \frac{\widetilde{\Theta}_\tau}{1-\xi} \widetilde{\alpha}_s^{c,HM},$$

$$\widetilde{\alpha}_s^{c,HM} = \kappa \widetilde{\sigma}^{-1} (1-\xi) e^{-\widetilde{\nu} s} \frac{e^{(\widetilde{\nu}-\widetilde{\nu})s} - 1}{\widetilde{\nu} - \widetilde{\nu}}.$$

B.2 Currency Union

The IM and PF cases are considered here. The results for the CM case are obtained by direct analogy.

Define

$$\widetilde{\nu} = \frac{\rho - \sqrt{\rho^2 + 4\widetilde{\kappa}_c \underline{\widetilde{\sigma}}^{-1}}}{2}, \quad \widetilde{\nu} = \frac{\rho + \sqrt{\rho^2 + 4\widetilde{\kappa}_c \underline{\widetilde{\sigma}}^{-1}}}{2}.$$

The equilibrium is completely characterized by the following:

$$\dot{X}_t = AX_t + B_t,$$

With

$$E_2^t X_0 = \Upsilon \theta + \underline{\widetilde{\Theta}}_n g_0 - \underline{\widetilde{\Theta}}_\tau t_0^r,$$

$$\theta = \int_0^{+\infty} e^{-\rho s} \left[\widetilde{\Omega}_c c_t + \widetilde{\Omega}_n g_t + \widetilde{\Omega}_\tau t_t^r\right] ds + \widetilde{\Omega}_f \mathrm{nfa}_0,$$

where

$$X_t = [\pi_t, c_t]^t, \quad A = \begin{bmatrix} \rho & -\widetilde{\kappa}_c \\ -\underline{\widetilde{\sigma}}^{-1} & 0 \end{bmatrix}, \quad B_t = -\left(\underline{\widetilde{\kappa}}_n g_t + \underline{\widetilde{\kappa}}_\theta \theta + \underline{\widetilde{\kappa}}_\tau t_t^r\right) E_1 + \left[\underline{\widetilde{\Theta}}_n \dot{g}_t - \underline{\widetilde{\Theta}}_\tau \dot{t}_t^r\right] E_2,$$

for all $t \geq 0$.

The (unique) solution that satisfies saddle-path stability writes:

$$X_t = \alpha_{\widetilde{\nu}} e^{\widetilde{\nu} t} X_{\widetilde{\nu}} + \int_t^{+\infty} \left(\underline{\widetilde{\kappa}}_n g_s + \underline{\widetilde{\kappa}}_\theta \theta + \underline{\widetilde{\kappa}}_\tau t_s^r\right) e^{-A(s-t)} E_1 ds - \int_t^{+\infty} \left(\underline{\widetilde{\Theta}}_n \dot{g}_s - \underline{\widetilde{\Theta}}_\tau \dot{t}_s^r\right) e^{-A(s-t)} E_2 ds,$$

with

$$E_2^t X_0 = \Upsilon\theta + \underline{\widetilde{\Theta}}_n g_0 - \underline{\widetilde{\Theta}}_\tau t_0^r,$$

$$\theta = \int_0^{+\infty} e^{-\rho s}\left[\widetilde{\Omega}_c c_t + \widetilde{\Omega}_n g_t + \widetilde{\Omega}_\tau t_t^r\right]ds + \widetilde{\Omega}_f \mathrm{nf}a_0,$$

where $\alpha_{\tilde{\nu}} \in \mathbb{R}$.

Equivalently, integrating the relevant objects by part,

$$X_t = \alpha_{\tilde{\nu}} e^{\tilde{\nu}t} X_{\tilde{\nu}} + \int_t^{+\infty}\left(\underline{\widetilde{\kappa}}_n g_s + \underline{\widetilde{\kappa}}_\theta \theta + \underline{\widetilde{\kappa}}_\tau t_s^r\right)e^{-A(s-t)}E_1 ds + \left(\underline{\widetilde{\Theta}}_n g_t - \underline{\widetilde{\Theta}}_\tau t_t^r\right)E_2$$
$$- \int_t^{+\infty}\left(\underline{\widetilde{\Theta}}_n g_s - \underline{\widetilde{\Theta}}_\tau t_s^r\right)Ae^{-A(s-t)}E_2 ds,$$

with

$$E_2^t X_0 = \Upsilon\theta + \underline{\widetilde{\Theta}}_n g_0 - \underline{\widetilde{\Theta}}_\tau t_0^r,$$

$$\theta = \int_0^{+\infty} e^{-\rho s}\left[\widetilde{\Omega}_c c_t + \widetilde{\Omega}_n g_t + \widetilde{\Omega}_\tau t_t^r\right]ds + \widetilde{\Omega}_f \mathrm{nf}a_0.$$

Thus,

$$\Upsilon\theta - \int_0^{+\infty}\left(\underline{\widetilde{\kappa}}_n g_s + \underline{\widetilde{\kappa}}_\theta \theta + \underline{\widetilde{\kappa}}_\tau t_s^r\right)E_2^t e^{-As}E_1 ds$$
$$+ \int_0^{+\infty}\left(\underline{\widetilde{\Theta}}_n g_s - \underline{\widetilde{\Theta}}_\tau t_s^r\right)AE_2^t e^{-As}E_2 ds = \alpha_{\tilde{\nu}}.$$

Therefore,

$$c_t = \left[\Upsilon\theta - \int_0^{+\infty}\left(\underline{\widetilde{\kappa}}_n g_s + \underline{\widetilde{\kappa}}_\theta \theta + \underline{\widetilde{\kappa}}_\tau t_s^r\right)E_2^t e^{-As}E_1 ds + \int_0^{+\infty}\left(\underline{\widetilde{\Theta}}_n g_s - \underline{\widetilde{\Theta}}_\tau t_s^r\right)E_2^t Ae^{-As}E_2 ds\right]e^{\tilde{\nu}t}$$
$$+ \int_t^{+\infty}\left(\underline{\widetilde{\kappa}}_n g_s + \underline{\widetilde{\kappa}}_\theta \theta + \underline{\widetilde{\kappa}}_\tau t_s^r\right)E_2^t e^{-A(s-t)}E_1 ds + \left(\underline{\widetilde{\Theta}}_n g_t - \underline{\widetilde{\Theta}}_\tau t_t^r\right)$$
$$- \int_t^{+\infty}\left(\underline{\widetilde{\Theta}}_n g_s - \underline{\widetilde{\Theta}}_\tau t_s^r\right)E_2^t Ae^{-A(s-t)}E_2 ds.$$

Equivalently,

$$c_t = \left[\Upsilon e^{\tilde{\nu}t} - \underline{\widetilde{\kappa}}_\theta\left[e^{\tilde{\nu}t}\int_0^{+\infty}E_2^t e^{-As}E_1 ds - \int_t^{+\infty}E_2^t e^{-A(s-t)}E_1 ds\right]\right]\theta + \underline{\widetilde{\Theta}}_n g_t - \underline{\widetilde{\Theta}}_\tau t_t^r$$
$$- \underline{\widetilde{\kappa}}_n\left[e^{\tilde{\nu}t}\int_0^{+\infty}E_2^t e^{-As}E_1 g_s ds - \int_t^{+\infty}E_2^t e^{-A(s-t)}E_1 g_s ds\right]$$
$$+ \underline{\widetilde{\Theta}}_n\left[e^{\tilde{\nu}t}\int_0^{+\infty}E_2^t Ae^{-As}E_2 g_s ds - \int_t^{+\infty}E_2^t Ae^{-A(s-t)}E_2 g_s ds\right]$$
$$- \underline{\widetilde{\kappa}}_\tau\left[e^{\tilde{\nu}t}\int_0^{+\infty}E_2^t e^{-As}E_1 t_s^r ds - \int_t^{+\infty}E_2^t e^{-A(s-t)}E_1 t_s^r ds\right]$$
$$- \underline{\widetilde{\Theta}}_\tau\left[e^{\tilde{\nu}t}\int_0^{+\infty}E_2^t e^{-As}AE_2 t_s^r ds - \int_t^{+\infty}E_2^t Ae^{-A(s-t)}E_2 t_s^r ds\right].$$

Note that

$$E_2^t e^{-At} E_1 = \underline{\widetilde{\sigma}}^{-1} \frac{e^{-\tilde{\nu}t} - e^{-\bar{\tilde{\nu}}t}}{\bar{\tilde{\nu}} - \tilde{\nu}} \ , \quad E_2^t A e^{-At} E_2 = -\widetilde{\kappa}_c \underline{\widetilde{\sigma}}^{-1} \frac{e^{-\tilde{\nu}t} - e^{-\bar{\tilde{\nu}}t}}{\bar{\tilde{\nu}} - \tilde{\nu}},$$

for all $t \geq 0$.

Thus,

$$c_t = \left[\frac{\Upsilon e^{\tilde{\nu}t} - \underline{\widetilde{\kappa}}_\theta \underline{\widetilde{\sigma}}^{-1} \left(e^{\tilde{\nu}t} - 1 \right) \int_0^{+\infty} e^{-\tilde{\nu}s} - e^{-\bar{\tilde{\nu}}s}}{\bar{\tilde{\nu}} - \tilde{\nu}ds} \right] \theta + \underline{\widetilde{\Theta}}_n g_t - \underline{\widetilde{\Theta}}_\tau t_t^r$$

$$- \left(\widetilde{\underline{\kappa}}_n + \widetilde{\underline{\kappa}}_c \underline{\widetilde{\Theta}}_n \right) \underline{\widetilde{\sigma}}^{-1} \left[e^{\tilde{\nu}t} \int_0^{+\infty} \frac{e^{-\tilde{\nu}s} - e^{-\bar{\tilde{\nu}}s}}{\bar{\tilde{\nu}} - \tilde{\nu}} g_s ds - \int_t^{+\infty} \frac{e^{-\tilde{\nu}(s-t)} - e^{-\bar{\tilde{\nu}}(s-t)}}{\bar{\tilde{\nu}} - \tilde{\nu}} g_s ds \right]$$

$$- \left(\widetilde{\underline{\kappa}}_\tau - \widetilde{\underline{\kappa}}_c \underline{\widetilde{\Theta}}_\tau \right) \underline{\widetilde{\sigma}}^{-1} \left[e^{\tilde{\nu}t} \int_0^{+\infty} \frac{e^{-\tilde{\nu}s} - e^{-\bar{\tilde{\nu}}s}}{\bar{\tilde{\nu}} - \tilde{\nu}} t_s^r ds - \int_t^{+\infty} \frac{e^{-\tilde{\nu}(s-t)} - e^{-\bar{\tilde{\nu}}(s-t)}}{\bar{\tilde{\nu}} - \tilde{\nu}} t_s^r ds \right].$$

Using the expression for the Pareto weight θ,

$$c_t = \left[\Upsilon e^{\tilde{\nu}t} - \underline{\widetilde{\kappa}}_\theta \underline{\widetilde{\sigma}}^{-1} \left(e^{\tilde{\nu}t} - 1 \right) \frac{\tilde{\nu}^{-1} - \bar{\tilde{\nu}}^{-1}}{\bar{\tilde{\nu}} - \tilde{\nu}} \right] \left(\int_0^{+\infty} e^{-\rho s} \left[\widetilde{\Omega}_c c_s + \widetilde{\Omega}_n g_s + \widetilde{\Omega}_\tau t_s^r \right] ds + \widetilde{\Omega}_f \mathrm{nfa}_0 \right)$$

$$+ \underline{\widetilde{\Theta}}_n g_t - \underline{\widetilde{\Theta}}_\tau t_t^r$$

$$- \left(\widetilde{\underline{\kappa}}_n + \widetilde{\underline{\kappa}}_c \underline{\widetilde{\Theta}}_n \right) \underline{\widetilde{\sigma}}^{-1} \left[e^{\tilde{\nu}t} \int_0^{+\infty} \frac{e^{-\tilde{\nu}s} - e^{-\bar{\tilde{\nu}}s}}{\bar{\tilde{\nu}} - \tilde{\nu}} g_s ds - \int_t^{+\infty} \frac{e^{-\tilde{\nu}(s-t)} - e^{-\bar{\tilde{\nu}}(s-t)}}{\bar{\tilde{\nu}} - \tilde{\nu}} g_s ds \right]$$

$$- \left(\widetilde{\underline{\kappa}}_\tau - \widetilde{\underline{\kappa}}_c \underline{\widetilde{\Theta}}_\tau \right) \underline{\widetilde{\sigma}}^{-1} \left[e^{\tilde{\nu}t} \int_0^{+\infty} \frac{e^{-\tilde{\nu}s} - e^{-\bar{\tilde{\nu}}s}}{\bar{\tilde{\nu}} - \tilde{\nu}} t_s^r ds - \int_t^{+\infty} \frac{e^{-\tilde{\nu}(s-t)} - e^{-\bar{\tilde{\nu}}(s-t)}}{\bar{\tilde{\nu}} - \tilde{\nu}} t_s^r ds \right].$$

From Fubini's Theorem, assuming that the integrals are finite,

$$\int_0^{+\infty} e^{-\rho t} \int_t^{+\infty} \frac{e^{-\tilde{\nu}(s-t)} - e^{-\bar{\tilde{\nu}}(s-t)}}{\bar{\tilde{\nu}} - \tilde{\nu}} x_s ds dt = \int_0^{+\infty} e^{-\rho s} \int_0^s \frac{e^{(\rho - \tilde{\nu})(s-t)} - e^{(\rho - \bar{\tilde{\nu}})(s-t)}}{\bar{\tilde{\nu}} - \tilde{\nu}} dt x_s ds$$

$$= -\frac{1}{\bar{\tilde{\nu}}} - \tilde{\nu} \int_0^{+\infty} e^{-\rho s} \left[(\rho - \tilde{\nu})^{-1} \left(1 - e^{(\rho - \tilde{\nu})s} \right) - (\rho - \bar{\tilde{\nu}})^{-1} \left(1 - e^{(\rho - \bar{\tilde{\nu}})s} \right) \right] x_s ds,$$

for each $x \in \{g, t^r\}$.

Note that $\frac{\tilde{\nu}^{-1} - \bar{\tilde{\nu}}^{-1}}{\bar{\tilde{\nu}} - \tilde{\nu}} = -\widetilde{\underline{\kappa}}_c^{-1} \underline{\widetilde{\sigma}}$ by definition of $\tilde{\nu}, \bar{\tilde{\nu}}$. Thus,

$$\int_0^{+\infty} e^{-\rho t} c_t dt = \frac{1}{1 - \Sigma \widetilde{\Omega}_c} \left(\int_0^{+\infty} \varsigma_n^t g_s ds + \int_0^{+\infty} \varsigma_\tau^t t_s^r ds + \varsigma_f \mathrm{nfa}_0 \right),$$

where

$$\varsigma_n^t = \Sigma e^{-\rho t}\widetilde{\Omega}_n + e^{-\rho t}\underline{\widetilde{\Theta}}_n - \left(\underline{\widetilde{\kappa}}_n + \underline{\widetilde{\kappa}}_c\underline{\widetilde{\Theta}}_n\right)\underline{\widetilde{\sigma}}^{-1}\left[\frac{1}{\rho - \widetilde{\nu}}\frac{e^{-\widetilde{\nu}t} - e^{-\bar{\widetilde{\nu}}t}}{\bar{\widetilde{\nu}} - \widetilde{\nu}}\right.$$

$$\left. + \frac{1}{\bar{\widetilde{\nu}} - \widetilde{\nu}}e^{-\rho t}\left[(\rho - \widetilde{\nu})^{-1}\left(1 - e^{(\rho - \widetilde{\nu})t}\right) - (\rho - \bar{\widetilde{\nu}})^{-1}\left(1 - e^{(\rho - \bar{\widetilde{\nu}})t}\right)\right]\right],$$

$$\varsigma_\tau^t = \Sigma e^{-\rho t}\widetilde{\Omega}_\tau - e^{-\rho t}\underline{\widetilde{\Theta}}_\tau - \left(\underline{\widetilde{\kappa}}_\tau - \underline{\widetilde{\kappa}}_c\underline{\widetilde{\Theta}}_\tau\right)\underline{\widetilde{\sigma}}^{-1}\left[\frac{1}{\rho - \widetilde{\nu}}\frac{e^{-\widetilde{\nu}t} - e^{-\bar{\widetilde{\nu}}t}}{\bar{\widetilde{\nu}} - \widetilde{\nu}}\right.$$

$$\left. + \frac{1}{\bar{\widetilde{\nu}} - \widetilde{\nu}}e^{-\rho t}\left[(\rho - \widetilde{\nu})^{-1}\left(1 - e^{-(\rho - \widetilde{\nu})t}\right) - (\rho - \bar{\widetilde{\nu}})^{-1}\left(1 - e^{(\rho - \bar{\widetilde{\nu}})t}\right)\right]\right],$$

$$\varsigma_f = \Sigma\widetilde{\Omega}_f,$$

$$\Sigma = \Upsilon\frac{1}{\rho - \widetilde{\nu}} - \underline{\widetilde{\kappa}}_\theta\underline{\widetilde{\kappa}}_c^{-1}\left(\frac{1}{\rho} - \frac{1}{\rho - \widetilde{\nu}}\right).$$

Therefore,

$$c_t = \underline{\widetilde{\Theta}}_n g_t - \underline{\widetilde{\Theta}}_\tau t_t^r + \int_{-t}^{+\infty}\alpha_s^{c,t,HM,IM}g_{t+s}ds - \int_{-t}^{+\infty}\gamma_s^{c,t,HM,IM}t_{t+s}^r ds,$$

where

$$\alpha_s^{c,t,HM,IM} = \alpha_s^{c,t,HM,CM} + \delta_s^{c,t,HM,IM} + \delta_s^{c,t,HM,PF},$$
$$\gamma_s^{c,t,HM,IM} = \gamma_{s0}^{c,t,HM,CM} + \epsilon_s^{c,t,HM,IM},$$

with

$$\alpha_s^{c,t,HM,CM} = -\left(\underline{\widetilde{\kappa}}_n + \underline{\widetilde{\kappa}}_c\underline{\widetilde{\Theta}}_n\right)\underline{\widetilde{\sigma}}^{-1}\left[\widetilde{\nu}^t\frac{e^{-\widetilde{\nu}(t+s)} - e^{-\bar{\widetilde{\nu}}(t+s)}}{\bar{\widetilde{\nu}} - \widetilde{\nu}} - \mathbb{1}_{s\geq 0}\frac{e^{-\widetilde{\nu}s} - e^{-\bar{\widetilde{\nu}}s}}{\bar{\widetilde{\nu}} - \widetilde{\nu}}\right],$$

$$\gamma_s^{c,t,HM,CM} = \left(\underline{\widetilde{\kappa}}_\tau - \underline{\widetilde{\kappa}}_c\underline{\widetilde{\Theta}}_\tau\right)\underline{\widetilde{\sigma}}^{-1}\left[e^{\widetilde{\nu}t}\frac{e^{-\widetilde{\nu}(t+s)} - e^{-\bar{\widetilde{\nu}}(t+s)}}{\bar{\widetilde{\nu}} - \widetilde{\nu}} - \mathbb{1}_{s\geq 0}\frac{e^{-\widetilde{\nu}s} - e^{-\bar{\widetilde{\nu}}s}}{\bar{\widetilde{\nu}} - \widetilde{\nu}}\right],$$

$$\delta_s^{c,t,HM,IM} = \left(\frac{1}{1 - \Sigma\widetilde{\Omega}_c}\widetilde{\Omega}_c\varsigma_n^{t+s} + e^{-\rho(t+s)}\widetilde{\Omega}_n\right)\left[\Upsilon e^{\widetilde{\nu}t} - \underline{\widetilde{\kappa}}_\theta\underline{\widetilde{\kappa}}_c^{-1}\left(1 - e^{\widetilde{\nu}t}\right)\right],$$

$$\epsilon_s^{c,t,HM,IM} = -\left(\frac{1}{1 - \Sigma\widetilde{\Omega}_c}\widetilde{\Omega}_c\varsigma_\tau^{t+s} + e^{-\rho(t+s)}\widetilde{\Omega}_\tau\right)\left[\Upsilon e^{\widetilde{\nu}t} - \underline{\widetilde{\kappa}}_\theta\underline{\widetilde{\kappa}}_c^{-1}\left(1 - e^{\widetilde{\nu}t}\right)\right],$$

and $\delta_s^{c,t,HM,PF} = 0$ in IM case, and

$$\delta_s^{c,t,HM,PF} = \left(\frac{1}{1 - \Sigma\widetilde{\Omega}_c}\widetilde{\Omega}_c\varsigma_f + \widetilde{\Omega}_f\right)\left[\Upsilon e^{\widetilde{\nu}t} - \underline{\widetilde{\kappa}}_\theta\underline{\widetilde{\kappa}}_c^{-1}\left(1 - e^{\widetilde{\nu}t}\right)\right]e^{-\rho(t+s)}$$

in PF case.

We can reexpress these as

$$\alpha_s^{c,t,HM,CM} = -\left(\underline{\widetilde{\kappa}}_n + \underline{\widetilde{\kappa}}_c\underline{\widetilde{\Theta}}_n\right)\underline{\widetilde{\sigma}}^{-1}\left[e^{\check{\nu}t}\frac{e^{-\check{\nu}(t+s)} - e^{-\bar{\check{\nu}}(t+s)}}{\bar{\check{\nu}} - \check{\nu}} - \mathbb{1}_{s\geq 0}\frac{e^{-\check{\nu}s} - e^{-\bar{\check{\nu}}s}}{\bar{\check{\nu}} - \check{\nu}}\right],$$

$$\gamma_s^{c,t,HM,CM} = \left(\underline{\widetilde{\kappa}}_\tau - \underline{\widetilde{\kappa}}_c\underline{\widetilde{\Theta}}_\tau\right)\underline{\widetilde{\sigma}}^{-1}\left[e^{\check{\nu}t}\frac{e^{-\check{\nu}(t+s)} - e^{-\bar{\check{\nu}}(t+s)}}{\bar{\check{\nu}} - \check{\nu}} - \mathbb{1}_{s\geq 0}\frac{e^{-\check{\nu}s} - e^{-\bar{\check{\nu}}s}}{\bar{\check{\nu}} - \check{\nu}}\right],$$

$$\delta_s^{c,t,HM,IM} = \left[\Upsilon e^{\check{\nu}t} - \underline{\widetilde{\kappa}}_\theta\underline{\widetilde{\kappa}}_c^{-1}\left(1 - e^{\check{\nu}t}\right)\right] \times$$

$$\frac{1}{1 - \Sigma\widetilde{\Omega}_c}\left[e^{-\rho(t+s)}\widetilde{\Omega}_n + e^{-\rho(t+s)}\widetilde{\Omega}_c\widetilde{\Theta}_n\right.$$

$$\left. +\widetilde{\Omega}_c\left(\underline{\widetilde{\kappa}}_n + \underline{\widetilde{\kappa}}_c\underline{\widetilde{\Theta}}_n\right)\underline{\widetilde{\sigma}}^{-1}\frac{1}{\widetilde{\kappa}_c\underline{\widetilde{\sigma}}^{-1}}e^{-\rho(t+s)}\left(1 - e^{\check{\nu}(t+s)}\right)\right],$$

$$\epsilon_s^{c,t,HM,IM} = -\left[\Upsilon e^{\check{\nu}t} - \underline{\widetilde{\kappa}}_\theta\underline{\widetilde{\kappa}}_c^{-1}\left(1 - e^{\check{\nu}t}\right)\right] \times$$

$$\frac{1}{1 - \Sigma\widetilde{\Omega}_c}\left[e^{-\rho(t+s)}\widetilde{\Omega}_\tau - e^{-\rho(t+s)}\widetilde{\Omega}_c\widetilde{\Theta}_\tau\right.$$

$$\left. +\widetilde{\Omega}_c\left(\underline{\widetilde{\kappa}}_\tau - \underline{\widetilde{\kappa}}_c\underline{\widetilde{\Theta}}_\tau\right)\underline{\widetilde{\sigma}}^{-1}\frac{1}{\widetilde{\kappa}_c\underline{\widetilde{\sigma}}^{-1}}e^{-\rho(t+s)}\left(1 - e^{\check{\nu}(t+s)}\right)\right],$$

and $\delta_s^{c,t,HM,PF} = 0$ in the IM case, and

$$\delta_s^{c,t,HM,PF} = \left[\Upsilon e^{\check{\nu}t} - \underline{\widetilde{\kappa}}_\theta\underline{\widetilde{\kappa}}_c^{-1}\left(1 - e^{\widetilde{\nu}t}\right)\right]\frac{1}{1 - \Sigma\widetilde{\Omega}_c}\widetilde{\Omega}_f e^{-\rho(t+s)}$$

in the PF case.

By direct analogy,

$$c_t = \underline{\widetilde{\Theta}}_n g_t - \underline{\widetilde{\Theta}}_\tau t_t^r + \int_{-t}^{+\infty}\alpha_s^{c,t,HM,CM}g_{t+s}ds - \int_0^{+\infty}\gamma_s^{c,t,HM,CM}t_{t+s}^r ds$$

in CM case.

Special case: no hand-to-mouth agents $\chi = 0$
Define

$$\nu = \frac{\rho - \sqrt{\rho^2 + 4\kappa\hat{\sigma}^{-1}}}{2} \quad , \quad \bar{\nu} = \frac{\rho + \sqrt{\rho^2 + 4\kappa\hat{\sigma}^{-1}}}{2},$$

and

$$\hat{\Sigma} = (1 - \mathcal{G})(1 - \alpha)\frac{1}{\bar{\widetilde{\nu}}} + (1 - \mathcal{G})\frac{\hat{\sigma}}{\phi + \hat{\sigma}}\alpha\omega\frac{1}{\bar{\widetilde{\nu}}}\frac{\widetilde{\nu}}{\rho}.$$

Define

$$\alpha_s^{c,t,HM,CM} = -\kappa(1-\xi)\hat{\sigma}^{-1}\left[e^{\nu t}\frac{e^{-\nu(t+s)} - e^{-\bar{\nu}(t+s)}}{\bar{\nu} - \nu} - \mathbb{1}_{s\geq 0}\frac{e^{-\nu s} - e^{-\bar{\nu} s}}{\bar{\nu} - \nu}\right].$$

We have

$$c_t = \int_{-t}^{+\infty} \alpha_s^{c,t,HM,IM} g_{t+s}\,ds,$$

where

$$\alpha_s^{c,t,HM,IM} = \alpha_s^{c,t,HM,CM} + \delta_s^{c,t,HM,IM} + \delta_s^{c,t,HM,PF},$$

with

$$\delta_s^{c,t,HM,IM} = \rho\left[\frac{1-\alpha}{\alpha}e^{\tilde{\nu}t} - \frac{\hat{\sigma}}{\phi+\hat{\sigma}}\omega\left(1-e^{\tilde{\nu}t}\right)\right]$$
$$\times \frac{\alpha\dfrac{\omega-\sigma}{\omega+(1-\alpha)(1-\sigma)}}{1-\hat{\Sigma}\rho\dfrac{\omega-\sigma}{\omega+(1-\alpha)(1-\sigma)}\dfrac{1}{1-\mathcal{G}}}(1-\xi)e^{-\rho(t+s)}\left(1-e^{\tilde{\nu}(t+s)}\right),$$

and $\delta_s^{c,t,HM,PF} = 0$ in the IM case, and

$$\delta_s^{c,t,HM,PF} = \rho\left[\frac{1-\alpha}{\alpha}e^{\tilde{\nu}t} - \frac{\hat{\sigma}}{\phi+\hat{\sigma}}\omega\left(1-e^{\tilde{\nu}t}\right)\right]\frac{\dfrac{\alpha\omega+1-\alpha}{\omega+(1-\alpha)(1-\sigma)}}{1-\hat{\Sigma}\rho\dfrac{\omega-\sigma}{\omega+(1-\alpha)(1-\sigma)}\dfrac{1}{1-\mathcal{G}}}e^{-\rho(t+s)}$$

in the PF case.

Special case: no profit offset $o = 0$
In that case we have

$$\Sigma = (1-\mathcal{G})(1-\widetilde{\alpha})\frac{1}{\widetilde{\nu}} + (1-\mathcal{G})\lambda\widetilde{\sigma}\widetilde{\alpha}\widetilde{\omega}\widetilde{\kappa}^{-1}\frac{1}{\rho}\frac{\widetilde{\nu}}{\widetilde{\nu}}.$$

Define

$$\widetilde{\alpha}_s^{c,t,HM,CM} = -\widetilde{\kappa}\left(1-\widetilde{\xi}\right)\widetilde{\sigma}^{-1}\left[e^{\tilde{\nu}t}\frac{e^{-\tilde{\nu}(t+s)} - e^{-\bar{\tilde{\nu}}(t+s)}}{\bar{\tilde{\nu}} - \tilde{\nu}} - \mathbb{1}_{s\geq 0}\frac{e^{-\tilde{\nu}s} - e^{-\bar{\tilde{\nu}}s}}{\bar{\tilde{\nu}} - \tilde{\nu}}\right].$$

We have

$$c_t = \widetilde{\Theta}_n g_t - \widetilde{\Theta}_\tau t_t^r + \int_{-t}^{+\infty} \alpha_s^{c,t,HM,IM} g_{t+s}\,ds - \int_{-t}^{+\infty} \gamma_s^{c,t,HM,IM} t_{t+s}^r\,ds,$$

where

$$
\alpha_s^{c,t,HM,IM} = \alpha_s^{c,t,HM,CM} + \delta_s^{c,t,HM,IM} + \delta_s^{c,t,HM,PF},
$$
$$
\gamma_s^{c,t,HM,IM} = \gamma_s^{c,t,HM,CM} + \epsilon_s^{c,t,HM,IM},
$$

with

$$
\alpha_s^{c,t,HM,CM} = -\left(1 + \frac{\widetilde{\Theta}_n}{1-\widetilde{\xi}}\right)\widetilde{\kappa}\left(1-\widetilde{\xi}\right)\widetilde{\sigma}^{-1}\left[e^{\tilde{\nu}t}\frac{e^{-\tilde{\nu}(t+s)} - e^{-\bar{\bar{\nu}}(t+s)}}{\bar{\bar{\nu}} - \tilde{\nu}} - \mathbb{1}_{s\geq 0}\frac{e^{-\tilde{\nu}s} - e^{-\bar{\bar{\nu}}s}}{\bar{\bar{\nu}} - \tilde{\nu}}\right],
$$

$$
\gamma_s^{c,t,HM,CM} = -\frac{\widetilde{\Theta}_\tau - \widetilde{\widetilde{\Theta}}_\tau}{1-\widetilde{\xi}}\widetilde{\kappa}\left(1-\widetilde{\xi}\right)\widetilde{\sigma}^{-1}\left[e^{\tilde{\nu}t}\frac{e^{-\tilde{\nu}(t+s)} - e^{-\bar{\bar{\nu}}(t+s)}}{\bar{\bar{\nu}} - \tilde{\nu}} - \mathbb{1}_{s\geq 0}\frac{e^{-\tilde{\nu}s} - e^{-\bar{\bar{\nu}}s}}{\bar{\bar{\nu}} - \tilde{\nu}}\right],
$$

$$
\delta_s^{c,t,HM,IM} = \rho\left[\frac{1-\widetilde{\alpha}}{\widetilde{\alpha}}e^{\tilde{\nu}t} - \lambda\widetilde{\sigma}\widetilde{\kappa}^{-1}\widetilde{\omega}\left(1 - e^{\tilde{\nu}t}\right)\right]
$$

$$
\times \frac{\widetilde{\alpha}}{1-\Sigma\widetilde{\Omega}_c}\left[e^{-\rho(t+s)}\frac{(1-\mathcal{G})\widetilde{\Omega}_n}{\rho} + e^{-\rho(t+s)}\frac{(1-\mathcal{G})\widetilde{\Omega}_c}{\rho}\widetilde{\Theta}_n.
$$

$$
+ \frac{(1-\mathcal{G})\widetilde{\Omega}_c}{\rho}\left(1-\widetilde{\xi}\right)\left(1 + \frac{\widetilde{\Theta}_n}{1-\widetilde{\xi}}\right)e^{-\rho(t+s)}\left(1 - e^{\tilde{\nu}(t+s)}\right)\right],
$$

$$
\epsilon_s^{c,t,HM,IM} = -\rho\left[\frac{1-\widetilde{\alpha}}{\widetilde{\alpha}}e^{\tilde{\nu}t} - \lambda\widetilde{\sigma}\widetilde{\kappa}^{-1}\widetilde{\omega}\left(1 - e^{\tilde{\nu}t}\right)\right]
$$

$$
\times \frac{\widetilde{\alpha}}{1-\Sigma\widetilde{\Omega}_c}\left[e^{-\rho(t+s)}\frac{(1-\mathcal{G})\widetilde{\Omega}_\tau}{\rho} - e^{-\rho(t+s)}\frac{(1-\mathcal{G})\widetilde{\Omega}_c}{\rho}\widetilde{\Theta}_\tau.
$$

$$
+ \widetilde{\Omega}_c\left(1-\widetilde{\xi}\right)\frac{\widetilde{\widetilde{\Theta}}_\tau - \widetilde{\Theta}_\tau}{1-\widetilde{\xi}}e^{-\rho(t+s)}\left(1 - e^{\tilde{\nu}(t+s)}\right)\right],
$$

and $\delta_s^{c,t,HM,PF} = 0$ in the IM case, and

$$
\delta_s^{c,t,HM,PF} = \rho\left[\frac{1-\widetilde{\alpha}}{\widetilde{\alpha}}e^{\tilde{\nu}t} - \lambda\widetilde{\sigma}\widetilde{\kappa}^{-1}\widetilde{\omega}\left(1 - e^{\tilde{\nu}t}\right)\right]\frac{1}{1-\Sigma\widetilde{\Omega}_c}\frac{\widetilde{\alpha}(1-\mathcal{G})\widetilde{\Omega}_f}{\rho}e^{-\rho(t+s)}
$$

in the PF case.

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