

Global Imbalances and Policy Wars at the Zero Lower Bound

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Abstract

This paper explores the consequences of extremely low real interest rates in a world with integrated but heterogenous capital markets and nominal rigidities. We establish four main results: (i) Liquidity traps spread to the rest of the world through the current account, which we illustrate with a new *Metzler diagram in quantities*; (ii) Beggar-thy-neighbor currency and trade wars provide stimulus to the undertaking country at the expense of other countries; (iii) (Safe) public debt issuances and increases in government spending *anywhere* are expansionary *everywhere*; (iv) At the ZLB, net issuers of safe assets experience a disproportionate share of the global stagnation.

JEL Codes: E0, F3, F4, G1,

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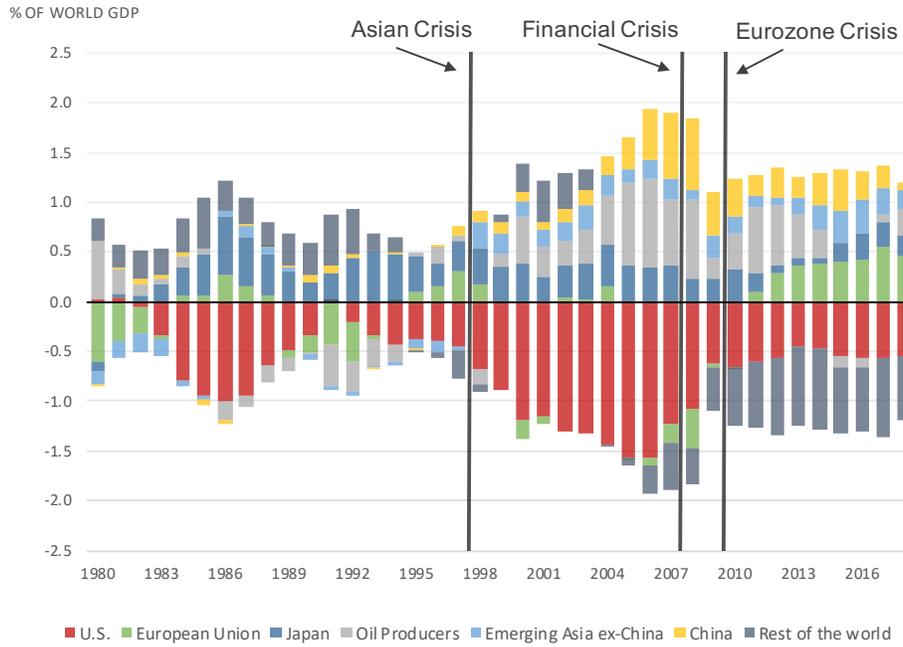
1 Introduction

In Caballero, Farhi and Gourinchas (2008a,b) we modeled global imbalances as the result of global differences in the capacity to produce assets, and the decline in potential growth in the developed world. The steady decline in interest rates was a natural outcome of this process. Figs. 1 and 2 illustrate these trends. The world has changed since then: interest rates have reached extremely low levels and there is limited space for further downward adjustment. We denote this downward rigidity in policy rates the ‘Zero Lower Bound’ (ZLB). How are global imbalances resolved in this ‘ZLB’ context? How do policies in one country spill over to others in this environment? And how do local policymakers’ incentives change at the ZLB?

We build a stylized model to address these questions. Our basic framework is a two-country perpetual-youth model with nominal rigidities, designed to highlight the heterogeneous relative demand for, and supply of, financial assets across countries. In the body of the paper, we study a stationary world in which all countries share the same preferences for domestic and foreign goods (i.e., there is no home bias) and financial markets are fully integrated. This is an *all-or-none* world: Either all countries experience a *permanent* ‘liquidity trap’—characterized by an inefficiently low level of aggregate economic activity—or none do. Within this model, we show that: (i) the current account plays a key role in spreading liquidity traps; (ii) local governments have an incentive to engage in zero-sum currency and trade wars; and (iii) fiscal deficits and public debt issuance generate positive global spillovers. We then expand the model to consider heterogeneity in the demand for, and supply of, *safe* assets across countries. In our setting, the overall scarcity of safe assets tips the global economy into a global ‘safety trap,’ and safe-asset issuers experience a disproportionate share of the adjustment burden.

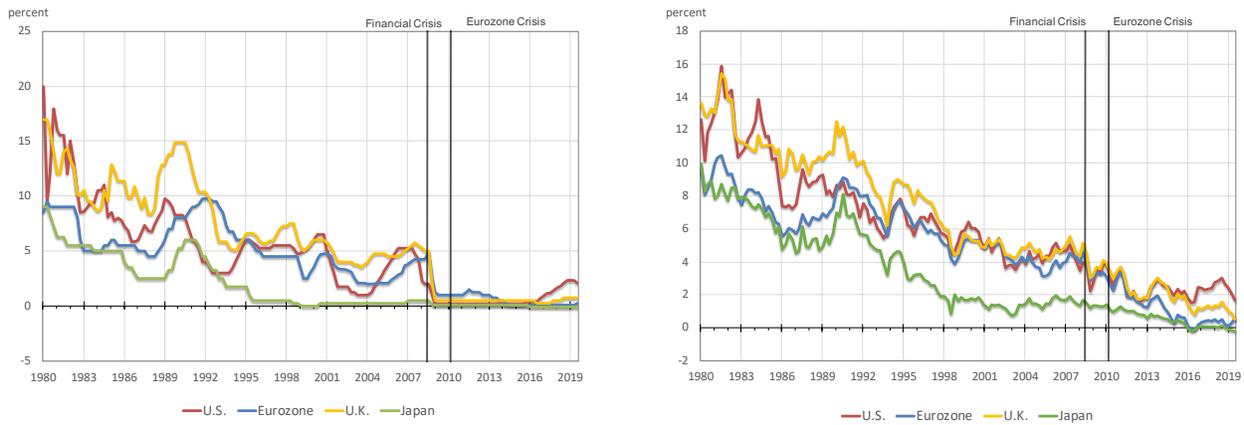
The ZLB emerges as a natural tipping point. Away from the ZLB, real interest rates clear global asset markets: A shock that creates an *asset shortage* (excess demand for assets) at the prevailing real interest rate results in an endogenous reduction in global real interest rates that restores equilibrium in global asset markets. At the ZLB, real interest rates cannot play their equilibrating role and global output must adjust to clear asset markets: Global output endogenously declines, reducing income and therefore net global asset demand and restoring equilibrium in global asset markets. Moreover, the role of capital flows changes at the ZLB. Away from the ZLB, current account surpluses propagate low interest rates from the origin country to the rest of the world. At the ZLB, current account surpluses propagate recessions.

We characterize global imbalances at the ZLB with a *Metzler diagram in quantities* that connects the size of the global recession and net foreign asset positions (and current accounts) to the recessions that would prevail in each country under financial autarky. This is analogous to the case away from the ZLB, where the world equilibrium real interest rate and net foreign asset (and current account) positions are connected



Note: The graph shows current account balances as a fraction of world GDP from 1980 to 2018. We observe the build-up of global imbalances in the early 2000s, until the financial crisis of 2008. Since then, global imbalances have receded but not disappeared. Deficits subsided in the U.S., and surpluses emerged in Europe. Source: World Economic Outlook (Oct. 2018), and Authors' calculations. Oil Producers: Bahrain, Canada, Iran, Iraq, Kuwait, Libya, Mexico, Nigeria, Norway, Oman, Russia, Saudi Arabia, United Arab Emirates, Venezuela; Emerging Asia ex-China: India, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, Thailand, Vietnam.

Figure 1: Global Imbalances, 1980-2018



(a) Policy Rates

(b) Long Rates

Note: Panel (a) reports policy rates while panel (b) reports nominal yields on 10-years government securities. We use Germany's 10-year yield as a proxy for the Eurozone 10-year yield and the Bundesbank Lombard rate prior to 1999 as a proxy for the Eurozone policy rate. Both panels show the large decline in global interest rates. Following the financial crisis, the developed world has remained at or near the Zero Lower Bound. Source: Global Financial Database and FRED.

Figure 2: Short and Long Nominal Interest Rates, 1980-2019

to the equilibrium real interest rate that would prevail in each country under autarky. Our analysis shows that, other things equal, when a country's autarky recession is more (less) severe than the global recession, that country is also a net creditor (debtor) and runs current account surpluses (deficits) in the financially integrated environment, effectively exporting its recession abroad. In turn, a country experiences a more (less) severe autarky recession than the average recession when its autarky asset shortage is more (less) severe than the global asset shortage. In this environment, a large country with a severe autarky liquidity trap recession can pull the world economy into a global liquidity trap recession.

But other things need not be equal. In particular, our benchmark model has a critical degree of indeterminacy at the ZLB. This indeterminacy is related to the seminal result by [Kareken and Wallace \(1981\)](#) that the nominal exchange rate is indeterminate in a world with pure interest rate targets. This is *de facto* the case when the economy is in a persistent global liquidity trap at the ZLB. However, in our framework and in contrast to the environments envisioned by [Kareken and Wallace \(1981\)](#), this indeterminacy has substantive real implications. In the presence of nominal rigidities, different values of the nominal exchange rate correspond to different values of the real exchange rate, and therefore to different output levels and current account balances across countries. In a global liquidity trap, global output needs to decline, but the exchange rate affects the *distribution* of recessions across countries. This creates fertile grounds for *zero-sum* beggar-thy-neighbor devaluations achieved by direct interventions in exchange rate markets, stimulating output and improving the current account in one country at the expense of others. These sorts of beggar-thy-neighbor policies can lead to “currency wars” when countries are at the ZLB. By the same token, this indeterminacy implies that countries have an incentive to engage in “trade wars”: Countries may hike tariffs to divert global demand away from foreign goods and toward domestic goods.

In sharp contrast, policies that alleviate asset scarcity have positive spillovers. In particular, fiscal expansions by countries with sound fiscal accounts have powerful positive spillovers. A balanced budget expansion reduces the net demand for (safe) assets, while an unbalanced expansion has the additional virtue of directly expanding asset supply. Moreover, as the global liquidity trap becomes more persistent, fiscal capacity constraints become less and less relevant. The upshot is that public debt issuance and increases in government spending *anywhere* are expansionary *everywhere*.

Our benchmark model considers a general scarcity of stores of value. In practice, the distinction between a general scarcity of stores of value versus a scarcity of *safe* assets matters. To address this issue, we relax our risk neutrality assumption and introduce *safe* and *risky* assets, along the lines of [Caballero and Farhi \(2017\)](#). This enriched model delivers five additional results: First, macroeconomic outcomes depend on whether there is a scarcity of *safe* assets, not on whether there is an overall scarcity of stores of value. When the return on safe assets reaches the ZLB, the economy enters a ‘safety trap’. Second, the scarcity of

safe assets depresses the return on safe assets relative to the expected return on risky assets: risk premia increase. Third, our model uncovers a ‘reserve currency paradox’: a country issuing a reserve currency, i.e. a currency expected to appreciate in bad times, faces lower safe real rates and will enter a safety trap earlier, or experience a larger recession in a global safety trap. Fourth, as before, the financial account plays a key role in transmitting economic shocks at the ZLB. However, the most important dimension of the financial account is the net flow of safe assets. At the ZLB, countries that are net issuers of safe assets experience a worse recession, and vice-versa. Fifth, net issuers of safe assets experience an ‘exorbitant privilege’, i.e. a high return on their (riskier) external assets relative to their (safer) external liabilities.

We present several important extensions in the appendix. There, we introduce home bias, allow for milder nominal rigidities, relax some elasticity assumptions, and consider a model with heterogeneity in the propensity to save both within and across countries.

Related literature. Our paper is related to several strands of literature. Most closely related is the literature that identifies the shortage of assets, and especially the shortage of safe assets, as a key macroeconomic driver of global interest rates and capital flows (see e.g. [Bernanke \(2005\)](#), [Caballero \(2006, 2010\)](#); [Caballero et al. \(2008a,b\)](#), [Caballero and Krishnamurthy \(2009\)](#), [Mendoza, Quadrini and Ríos-Rull \(2009\)](#), [Bernanke, Bertaut, DeMarco and Kamin \(2011\)](#), [Gourinchas, Rey and Govillot \(2010\)](#), [Maggiore \(2012\)](#) and [Coeurdacier, Guibaud and Jin \(2015\)](#)). In particular, [Caballero et al. \(2008a\)](#) develops the idea that global imbalances originated in the superior development of financial markets in developed economies (as well as in the decline in potential growth of Europe and Japan relative to the U.S.). This paper analyzes the implications of asset scarcity when the world economy experiences ultra-low natural real interest rates and is constrained by the Zero Lower Bound: The adjustment now occurs through quantities (output) rather than prices (interest rates), and exchange rates play an important role in allocating a global slump across countries.

Another strand of the literature emphasizes that public debt is safe because it is insensitive to information, mitigating the role of information asymmetries and discouraging investors from acquiring information (see for example [Gorton \(2010\)](#), [Stein \(2012\)](#), [Moreira and Savov \(2014\)](#), [Gorton and Ordóñez \(2013, 2014\)](#), [Dang, Gorton and Holmström \(2015\)](#) and [Greenwood, Hanson and Stein \(2015\)](#)). A recent literature also considers relative degrees of safety and what makes some assets ‘safe’ in equilibrium when there are coordination problems (see for example [He, Krishnamurthy and Milbradt \(2015\)](#)). Our model offers a different interpretation, where the “specialness” of public debt and close substitutes arises from their safety in bad aggregate states (see also [Gennaioli, Shleifer and Vishny \(2012\)](#), [Barro and Møllerus \(2014\)](#), and [Caballero and Farhi \(2017\)](#)).

There is an extensive literature on liquidity traps (see e.g. [Keynes \(1936\)](#), [Krugman \(1998\)](#), [Eggertsson and Woodford \(2003\)](#), [Christiano, Eichenbaum and Rebelo \(2011\)](#), [Guerrieri and Lorenzoni \(2011\)](#), [Eggertsson and Krugman \(2012\)](#), [Werning \(2012\)](#), and [Correia, Farhi, Nicolini and Teles \(2013\)](#)). This literature emphasizes that the binding Zero Lower Bound on nominal interest rates presents an important challenge for macroeconomic stabilization. A subset of this literature considers the implications of a liquidity trap in the open economy (see e.g. [Svensson \(2003\)](#), [Jeanne \(2009\)](#), [Farhi and Werning \(2012\)](#), [Cook and Devereux \(2013a,b, 2014\)](#), [Devereux and Yetman \(2014\)](#), [Benigno and Romei \(2014\)](#), [Erceg and Lindé \(2014\)](#), and [Fornaro and Romei \(2019\)](#)). While many of these papers share similar themes, our paper makes three distinct contributions. First, we use our *Metzler diagram in quantities* to elucidate the link between the global recession and net foreign asset positions. We also allow for permanent liquidity traps and capital flows (global imbalances). Finally, we make a distinction between risky and safe assets.

Our paper is also related to the recent literature on secular stagnation (see e.g. [Kocherlakota \(2014\)](#), [Eggertsson and Mehrotra \(2014\)](#), [Caballero and Farhi \(2017\)](#)). Like us, these papers use an OLG structure with a zero lower bound and nominal rigidities, but in a closed economy. Our contribution is to explore the open economy dimension of the secular stagnation hypothesis. We study the propagation of liquidity traps from one country to another and the role of global imbalances and policy spillovers at the ZLB.

From that perspective, the paper closest to ours is [Eggertsson, Mehrotra, Singh and Summers \(2015\)](#) which finds, like us, that exchange rates have powerful effects when the economy is in a global liquidity trap. Complementary to ours, their paper explores the role of market integration and capital controls. Our paper emphasizes other methodological and substantive dimensions, such as the Metzler diagram in quantities, the “reserve currency paradox”, the spillovers of safe public debt issuance, the role of capital flows in spreading liquidity traps and macroeconomic policies, and the role of safe vs. risky assets.

2 A Model of the Diffusion of Liquidity Traps

This section introduces our baseline model of a global economy with structurally low real interest rates. We first lay out the assumptions of the model and characterize the world equilibrium. Throughout the paper, we focus on steady state balanced growth paths. We start under financial autarky, i.e. when trade is balanced, then move to the integrated equilibrium. In each case, we characterize the equilibrium and discuss the relevant economic mechanisms both at and away from the Zero Lower Bound (ZLB).

2.1 Assumptions and Competitive Equilibrium

Time is continuous. There are two countries, *Home* and *Foreign*. Foreign variables are denoted with stars. We first describe *Home*, and then move on to *Foreign*.

Demographics. Population is constant and normalized to one. Agents are born and die at a constant hazard rate θ , independent across agents. Each dying agent is instantaneously replaced by a newborn. Therefore, in an interval dt , θdt agents die and θdt agents are born, leaving total population unchanged.

Preferences. Agents have a single opportunity to consume, c_t , at the time of death. Until they die, agents save and reinvest all their income.¹ Formally, we let τ_θ denote the stopping time for the idiosyncratic death process. Agents value home and foreign goods according to a Cobb-Douglas aggregate with an expenditure share on the home good $\gamma \in [0, 1]$, are risk neutral over short time intervals, and do not discount the future. For a given stochastic consumption process of home and foreign goods $\{c_{H,t}, c_{F,t}\}$, which is measurable with respect to the information available at date t , we define the utility U_t of an agent alive at that date with the following stochastic differential equation:

$$\begin{aligned} U_t &= 1_{\{t-dt \leq \tau_\theta < t\}} c_t + 1_{\{t \leq \tau_\theta\}} \mathbb{E}_t[U_{t+dt}], \\ c_t &= c_{H,t}^\gamma c_{F,t}^{1-\gamma}, \end{aligned} \tag{1}$$

where we use the notation $\mathbb{E}_t[U_{t+dt}]$ to denote the expectation of U_{t+dt} conditional on the information available at date t .²

Nominal rigidities, potential output and actual output. In an interval dt , potential output of the home good is $\bar{Y}_t dt$, where \bar{Y}_t grows at the exogenous rate g . Because of nominal rigidities, actual output Y_t is demand-determined and can be lower than potential output, \bar{Y}_t . We define $\xi_t = Y_t/\bar{Y}_t \in [0, 1]$, the ratio of output to potential output and, slightly abusing terminology, we refer to ξ_t as the *output gap*, with $\xi < 1$ when the economy is in a recession.

We assume that nominal rigidities take an extreme form: the prices of home goods are fully and permanently rigid in the home currency.³ We normalize home prices to one, $P_{H,t} = 1$, and assume that

¹This assumption allows us to focus on the store-of-value scarcity we wish to highlight. It also simplifies the algebra by removing non-central intertemporal substitution considerations, while delivering an aggregate consumption function with the same economic properties as if agents had log-preferences. See [Gourinchas and Rey \(2014\)](#) for details.

²Note that the information at date t contains the information about the realization of the idiosyncratic shocks up to t , implying that $1_{\{t-dt \leq \tau_\theta < t\}}$ and $c_{H,t}$ and $c_{F,t}$ are known at date t . Similarly, the conditional expectation \mathbb{E}_t is an expectation over idiosyncratic death shocks.

³This provides us with a sharp characterization of the equilibrium. Appendix [A.2](#) relaxes this assumption.

the Law of One Price holds so that the price of home goods in the foreign currency is $P_{H,t}/E_t$ where E_t is the nominal exchange rate, defined as the home price of the foreign currency. With this definition, an increase in E_t represents a depreciation of the home currency. Home's consumer price index (CPI) satisfies $P_t = (1/\gamma)^\gamma (E_t/(1-\gamma))^{1-\gamma}$, and Home CPI inflation is $\pi_t = (1-\gamma)\dot{E}_t/E_t$. Appendix A.1 provides a micro-foundation in the New Keynesian tradition with monopolistic competition and rigid prices à la Blanchard and Kiyotaki (1987).

Private incomes, assets, and financial development. Domestic income has two components: the income of newborns and financial income. In the interval dt , newly born agents receive income $(1-\delta)\xi_t\bar{Y}_t dt$. The remainder of income, $\delta\xi_t\bar{Y}_t dt$, is distributed as financial income. Specifically, we assume there is a mass \bar{Y}_t of Lucas trees, each producing a claim to a dividend of $\delta\xi_t$ units of output in the interval dt . With independent and instantaneous probability ρ each tree dies and the corresponding stream of dividends is transferred to a new tree. The stock of trees grows at rate g to accommodate growth in potential output. All new trees are bestowed to newborns.

Financial development is controlled by two key parameters: ρ and δ . The assumption that trees die ($\rho > 0$) can be interpreted either as a consequence of creative-destruction, or as a form of weak property rights. Either way, this assumption reduces the share of future output that is capitalized into assets that are traded today: a higher ρ reduces the aggregate supply of assets.

The assumption that only a fraction of output can be capitalized into traded financial claims ($\delta < 1$) captures many factors behind the limited pledgeability of income, as in Caballero et al. (2008a). At the most basic level, one can interpret δ as the share of income paid to capital in production. But in reality only a fraction of this share can be committed to asset holders, as the government, managers, and other insiders can dilute and divert part of the profits. For this reason, we refer to δ as an index of how well-defined and tradable rights over earnings are in the home country's financial markets. A lower δ reduces asset supply and simultaneously increases asset demand, since newborns receive a higher share of total income, which they save.

Public debt and the provision of public liquidity. In addition to private assets, we assume that a home government issues short-term public debt D_t , which it services by levying taxes τ_t on the income $(1-\delta)\xi_t\bar{Y}_t$ of newborns. We let $d_t = D_t/\bar{Y}_t$ denote the ratio of home public debt to potential output and assume that the tax rate is adjusted to maintain the desired ratio of debt to potential output, d_t .

Public debt plays a critical role in our model. Since the environment is non-Ricardian, public debt does

not fully crowd out private financial assets.⁴ An increase in the ratio of public debt to potential output (d_t) increases the total supply of assets, while the concomitant increase in taxes decreases the demand for these assets, since it reduces the disposable income of newborns. Public debt therefore provides ‘public liquidity’ in the sense of [Holmström and Tirole \(1998\)](#). By taxing the income of future (unborn) generations, the government capitalizes part of the economy’s non-financial income into public debt. Moreover, by adjusting the tax rate, it can fix the supply of *public* assets $d\bar{Y}_t$ independently of the state of the aggregate economy, ξ_t . By contrast, along a balanced-growth path with interest rate r , the supply of *private* financial assets is proportional to the level of aggregate activity, ξ_t : $V_t = \delta\xi_t\bar{Y}_t/(r + \rho)$. While our baseline set-up does not feature aggregate risk, this provision of public liquidity makes public debt ‘*safer*’ than private financial assets, in the sense that the value of public debt does not vary across possible realizations of aggregate demand ξ_t .

Monetary policy and ZLB. Home monetary policy follows a truncated Taylor rule that can be summarized as

$$i_t = \max\{r_t^n + \pi_t, 0\} \text{ with } i_t = 0 \text{ whenever } \xi_t < 1, \quad (2)$$

where i_t is the home nominal interest rate and r_t^n is the relevant *natural real interest rate* at Home, defined as the real interest rate that clears markets ignoring the ZLB constraint. This monetary policy rule defines two regimes: when $r_t^n + \pi_t \geq 0$, monetary policy is not constrained by the ZLB and can achieve potential output, $\xi_t = 1$. When instead $r_t^n + \pi_t < 0$, monetary policy is constrained at the ZLB, $i_t = 0$ and $\xi_t < 1$.

Foreign. Foreign differs from Home along five dimensions. First, potential output of the foreign good is given by \bar{Y}_t^* , which also grows at rate g , and the output gap is denoted $\xi_t^* = Y_t^*/\bar{Y}_t^*$. Second, we allow financial development to differ, with the financial capacity of the foreign country given by δ^* . Third, public debt in the foreign country is given by D_t^* , the debt to output ratio by d_t^* , and taxes by τ_t^* . Fourth, Foreign has its own currency and the prices of foreign goods are sticky in this currency. We normalize the price of the Foreign good to one in the foreign currency: $P_{F,t}^* = 1$. Fifth, foreign monetary policy follows a truncated Taylor rule similar to Home’s:

$$i_t^* = \max\{r_t^{n*} + \pi_t^*, 0\} \text{ with } i_t^* = 0 \text{ whenever } \xi_t^* < 1, \quad (3)$$

⁴Our framework becomes Ricardian if all future financial income is capitalized into existing financial assets (i.e. if $\rho = g = 0$ so that there are no new trees) and if taxes fall entirely on financial income. This would occur despite the overlapping generations: public debt would crowd out private assets one-for-one. See [Caballero et al. \(2008a\)](#) and [Gourinchas and Rey \(2014\)](#) for a detailed discussion of the Non-Ricardian features of this type of model.

where r_t^{n*} is the relevant natural real rate in the foreign country and $\pi_t^* = -\gamma \dot{E}_t/E_t$ is Foreign's CPI inflation rate.

We assume that there is no home bias and that in both countries the share γ of home consumption is equal to the share of potential output of home goods in total output: $\gamma = y$, where $y \equiv \bar{Y}_t/(\bar{Y}_t + \bar{Y}_t^*)$.

Competitive equilibrium. We denote by W_t and W_t^* the total wealth of home and foreign households in their respective currencies. V_t and V_t^* are the total value of home and foreign private assets in their respective currencies.

We consider two environments: *financial autarky* and *financial integration*. Under financial autarky, agents are free to trade goods across countries, but they cannot trade financial claims. Under financial integration, agents can also trade claims to the Lucas trees and public debt across borders.

We now write the domestic and foreign wealth dynamics, asset pricing conditions, government constraints, and market clearing conditions, then define and characterize a competitive equilibrium of our economy in each environment.

First, at each instant aggregate nominal consumption expenditure satisfies $P_t c_t = \theta W_t$ and $P_t^* c_t^* = \theta W_t^*$, since a fraction θ of the population in each country dies every instant and consumes all its wealth.

Second, the evolution of Home and Foreign aggregate wealth follow:

$$\dot{W}_t = (1 - \tau_t)(1 - \delta)\xi_t \bar{Y}_t - \theta W_t + i_t W_t + (\rho + g)V_t, \quad (4a)$$

$$\dot{W}_t^* = (1 - \tau_t^*)(1 - \delta^*)\xi_t^* \bar{Y}_t^* - \theta W_t^* + i_t^* W_t^* + (\rho + g)V_t^*. \quad (4b)$$

The change in home aggregate wealth has three components: (i) the newborn's net of-tax-income $(1 - \tau_t)(1 - \delta)\xi_t \bar{Y}_t$ is earned and consumption expenditure θW_t from dying agents are subtracted; (ii) home wealth earns a return equal to the home nominal risk-free rate, i_t , given that there is no aggregate uncertainty; (iii) new trees with aggregate value $(\rho + g)V_t$, accounting both for creative destruction and growth of potential output, are endowed to newborns. Foreign wealth follows similar dynamics with a return equal to the foreign nominal risk-free rate, i_t^* .

Third, since there is no aggregate risk, the return to private assets equals the nominal risk free rate in each country:

$$i_t V_t = \delta \xi_t \bar{Y}_t - \rho V_t + \dot{V}_t - g V_t, \quad (5a)$$

$$i_t^* V_t^* = \delta^* \xi_t^* \bar{Y}_t^* - \rho V_t^* + \dot{V}_t^* - g V_t^*. \quad (5b)$$

This return consists of three terms. First a dividend payment of $\delta\xi_t\bar{Y}_t$; second a capital loss equal to the fraction of trees that die, $-\rho V_t$; third a capital gain $\dot{V}_t - gV_t$ for trees that survive.⁵

In addition, under financial integration Uncovered Interest Parity (UIP) holds between Home and Foreign since agents are risk neutral:

$$i_t = i_t^* + \frac{\dot{E}_t}{E_t}. \quad (6)$$

Combined with the expression for domestic and foreign CPI inflation rates, UIP ensures that real returns are equalized under financial integration: $r_t = r_t^*$ where $r_t = i_t - \pi_t$ and $r_t^* = i_t^* - \pi_t^*$.

Fourth, government debt dynamics can be expressed as

$$\dot{D}_t = i_t D_t - \tau_t (1 - \delta) \xi_t \bar{Y}_t, \quad (7a)$$

$$\dot{D}_t^* = i_t^* D_t^* - \tau_t^* (1 - \delta^*) \xi_t^* \bar{Y}_t^*, \quad (7b)$$

where the first term represents interest payments (at the risk free local interest rate) and the second term represents tax revenues on local non-financial income.

Fifth, market clearing conditions for home and foreign goods require

$$c_{H,t} + c_{H,t}^* = \gamma \theta (W_t + E_t W_t^*) = \xi_t \bar{Y}_t, \quad (8a)$$

$$E_t (c_{F,t} + c_{F,t}^*) = (1 - \gamma) \theta (W_t + E_t W_t^*) = E_t \xi_t^* \bar{Y}_t^*. \quad (8b)$$

To understand the first expression, observe that home consumption expenditure on the home good (in home currency), $c_{H,t}$, represents a fraction γ of total home consumption expenditure θW_t , while foreign consumption expenditure on the home good (in foreign currency), $c_{H,t}^*/E_t$, represent the same fraction γ of total foreign consumption expenditure θW_t^* since there is no home bias in consumption. The second expression is derived in a similar way.

Finally, under financial integration, asset market clearing requires that total asset demand equals total asset supply:

$$(V_t + D_t) + E_t (V_t^* + D_t^*) = W_t + W_t^*, \quad (9)$$

⁵The term $-gV_t$ is a correction for the fact that the number of trees is growing with potential output. To obtain this expression, observe that the value of a single home Lucas tree, v_t , defined as a claim to $\delta\xi_t$ units of output, satisfies $i_t v_t = \delta\xi_t - \rho v_t + \dot{v}_t$. The value of all home trees is $V_t = v_t \bar{Y}_t$.

while under financial autarky, asset demand must equal asset supply in each country:

$$V_t + D_t = W_t, \quad (10a)$$

$$V_t^* + D_t^* = W_t^*. \quad (10b)$$

We can now define a competitive equilibrium, both under financial integration, when home and foreign agents are free to trade financial claims, and under financial autarky, when they are restricted to trade financial assets within their country.

Definition 1. (*Competitive Equilibrium under Financial Integration and Financial Autarky*)

Given paths for the ratio of public debt to potential output, d_t and d_t^* , a competitive equilibrium consists of sequences for output gaps ξ_t and ξ_t^* , natural real rates r_t^n and r_t^{n*} , household wealth W_t and W_t^* , private financial assets V_t and V_t^* , taxes τ_t and τ_t^* , consumptions c_t and c_t^* , consumer prices P_t and P_t^* , policy rates i_t and i_t^* , and the nominal exchange rate E_t , such that (i) household consumption, wealth and private assets satisfy Eqs. (4) and (5); (ii) debt dynamics follow Eq. (7) with $D_t = d_t \bar{Y}_t$ and $D_t^* = d_t^* \bar{Y}_t^*$; (iii) policy rates are set according to Eqs. (2) and (3); and (iv) goods markets clear Eq. (8). Moreover:

- Under financial integration, global asset markets clear (Eq. (9)) and UIP holds (Eq. (6));
- Under financial autarky, asset markets clear only locally (Eq. (10)).

We now specialize the model by focusing on steady state Balanced Growth Paths (BGP) where both economies grow at rate g and the ratio of debt to potential output in both countries, d and d^* , are constant. With some abuse of notation we drop the time subscript. Along a BGP, the exchange rate E , prices P , P^* , output gaps ξ , ξ^* , policy rates i , i^* and taxes τ , τ^* are constant, while wealth W , W^* , private assets V , V^* , public debt D , D^* and consumption c , c^* grow at rate g .

First, we characterize the financial autarky equilibrium both at and away from the ZLB, then we move to the case of financial integration.

2.2 Financial Autarky

It is useful to introduce the concepts of *financial autarky natural rates*, $r^{a,n}$ and $r^{a,n*}$ and *financial autarky natural output gaps* $\xi^{a,n}$ and $\xi^{a,n*}$:

$$r^{a,n} \equiv -\rho + \frac{\delta\theta}{1-\theta d} \quad ; \quad r^{a,n*} \equiv -\rho + \frac{\delta^*\theta}{1-\theta d^*} \quad (11a)$$

$$\xi^{a,n} \equiv \frac{\theta d}{1-\delta\theta/\rho} \quad , \quad \xi^{a,n*} \equiv \frac{\theta d^*}{1-\delta^*\theta/\rho}. \quad (11b)$$

The financial autarky natural rate is the real interest rate consistent with potential output under financial autarky when we ignore the ZLB constraint. The financial autarky natural output gap is the level of output that obtains when the interest rate is set at the ZLB under financial autarky. We make the following assumptions on the parameters.

Assumption 1.

$$0 < \delta, \delta^* < \rho/\theta \quad ; \quad 0 < d, d^* < 1/\theta$$

We will discuss the role of [Assumption 1](#) in detail after we state our first proposition, which characterizes the economy under financial autarky, both away from the ZLB and at the ZLB.

Proposition 1 (Financial Autarky Away from and At the Zero Lower Bound). *Under financial autarky and [Assumption 1](#), the competitive equilibrium is as follows:*

- *The home economy satisfies $i^a = r^a = \max\{r^{a,n}, 0\}$ and $\xi^a = \min\{\xi^{a,n}, 1\}$.*
 - *If $r^{a,n} \geq 0$, then $\xi^{a,n} \geq 1$ and the home economy is away from the Zero Lower Bound. There is a unique balanced growth path equilibrium with a positive interest rate, $i^a = r^a = r^{a,n}$, and output at its potential level, $\xi^a = 1$.*
 - *If $r^{a,n} < 0$, then $\xi^{a,n} < 1$ and the home economy is at the Zero Lower Bound. There is a unique balanced growth path equilibrium with $i^a = r^a = 0 > r^{a,n}$, and home output is below its potential level, with $\xi^a = \xi^{a,n} < 1$.*
- *Similarly, the foreign economy satisfies $i^{a*} = r^{a*} = \max\{r^{a,n*}, 0\}$ and $\xi^{a*} = \min\{\xi^{a,n*}, 1\}$.*
- *The autarky exchange rate satisfies:*

$$E^a = \frac{\xi^a}{\xi^{a*}}. \tag{12}$$

Proof. See text. □

To understand the economics behind this proposition, observe first that along a BGP the nominal exchange rate is constant, so all prices are constant and there is no inflation: $\pi^a = \pi^{a*} = 0$. It follows that nominal and real interest rates coincide, $i^a = r^a$ and $i^{a*} = r^{a*}$. From the goods market conditions [Eqs. \(8a\)](#) and [\(8b\)](#), the autarky exchange rate obtains immediately as the ratio of the output gaps, $E^a = \xi^a/\xi^{a*}$, which establishes the last part of the proposition.

Consider now a BGP financial autarky equilibrium with home output gap ξ and home real interest rate r . From [Eq. \(5a\)](#), total home asset supply along a BGP is given by

$$V + D = \frac{\delta}{r + \rho} \xi \bar{Y} + d \bar{Y}, \quad (13)$$

which is decreasing with the interest rate. From [Eq. \(4a\)](#), home asset demand along the BGP satisfies

$$W = \frac{\xi}{\theta} \bar{Y}, \quad (14)$$

which is invariant to the interest rate. The financial autarky natural rate $r^{a,n}$ given by [Eq. \(11a\)](#) equates asset demand with asset supply ($V + D = W$) when output is at its potential level, $\xi = 1$. This is only possible if $r^{a,n} \geq 0$. Next, observe that we can rewrite $\xi^{a,n} = 1 + r^{a,n}(1 - \theta d)/(\rho - \delta\theta)$, so that under [Assumption 1](#), $\xi^{a,n} \geq 1$ if and only if $r^{a,n} \geq 0$. This establishes the first part of the proposition.

Suppose now that $r^{a,n} < 0$. Inspecting [Eq. \(11a\)](#), this occurs when δ is low or ρ is high (i.e. a low supply of private assets), when d is low (i.e. a low supply of public assets) or when θ is low (i.e. a high demand for stores of value). In this case, the ZLB constraint imposes $i^a = r^a = 0$. A ZLB equilibrium arises when there is *a shortage of private or public assets that cannot be resolved by a decline in equilibrium real interest rates*.

Instead, an alternative (perverse) equilibrating mechanism endogenously arises in the form of a recession with $\xi^a < 1$. Under [Assumption 1](#), at a fixed zero interest rate, the recession reduces asset demand ([Eq. \(14\)](#)) more than asset supply ([Eq. \(13\)](#)), which helps restore equilibrium in the global asset market. The size of the required home recession is given by [Eq. \(11b\)](#). This establishes the second part of the proposition. The last part of the proposition obtains by symmetry.

We can now understand the role of [Assumption 1](#). The conditions $\delta\theta - \rho < 0$ and $d > 0$ ensure that asset demand decreases faster than asset supply as ξ^a declines, and that the intersection satisfies $0 < \xi^{a,n} < 1$. The condition $\theta d < 1$ ensures that the supply of public assets is not large enough to satisfy asset demand (in which case the equilibrium would require an infinite interest rate). The restrictions imposed by [Assumption 1](#) are exceedingly mild. For instance, if we assume $\rho = 3\%$ and $\theta = 5\%$, [Assumption 1](#) implies that $\delta, \delta^* < 3/5$ and $d < 20$. A reasonable estimate of δ is likely smaller than the capital share, often estimated around $1/3$, while realistic debt-output ratios are significantly lower than 20. The condition $d > 0$ is economically important: it ensures that part of the asset supply is *not* affected by the recession. In that sense, public debt is a ‘safe asset’.⁶

⁶We develop this notion in more detail in [Section 5](#) where we introduces ‘safe assets’ explicitly.

An equivalent interpretation of the ZLB equilibrium comes from the goods market. At every instant, the demand for goods arises from old households who die. At the ZLB, the aggregate purchasing power of these households in local currency (aggregate demand) is given by $\theta(V + D) = \theta(\xi^a \theta \delta / \rho + d) \bar{Y}$. The market value of domestic goods brought to the market in local currency (aggregate supply) is $\xi^a \bar{Y}$. Since trade is balanced under financial autarky, the two must be equal. When $r^{a,n} < 0$, aggregate supply exceeds aggregate demand at $\xi^a = 1$. In other words, old agents don't have enough purchasing power to buy all the goods supplied by the young. With nominally rigid prices, output is demand-determined, as in standard New Keynesian models. The recession simultaneously reduces aggregate supply and aggregate demand, but supply falls more than demand under the conditions of Assumption 1, helping to restore equilibrium in the goods market.

Even though agents from each country consume goods from both countries, ZLB recessions stay in their own economy. According to Proposition 1, whether a country is in a ZLB equilibrium depends only on its own financial autarky natural rate $r^{a,n}$. Under autarky, the nominal exchange rate adjusts to reflect the relative scarcity of goods according to Eq. (12). Countries with a more severe ZLB recession (a lower ξ^a) have a stronger currency (a lower E^a) that sustains their purchasing power for the foreign good and prevents the ZLB recession from spilling over to the other country. In other words, under financial autarky domestic financial conditions determine the level of domestic output, while the exchange rate simply adjusts to make sure that the corresponding equilibrium is consistent with an integrated goods market.

2.3 Financial Integration

We now consider the case of financial integration. As in the case of financial autarky, the exchange rate is constant along a BGP. From Eq. (6), it follows that $i = i^* = i^w = r = r^* = r^w$, where i^w and r^w denote the world nominal and real interest rates. This implies that either no country is trapped at the ZLB, $i^w = r^w > 0$, or all countries are, $i^w = r^w = 0$.

By analogy with the case of financial autarky, we define the *world natural interest rate* $r^{w,n}$ as

$$r^{w,n} = -\rho + \frac{\bar{\delta}\theta}{1 - \theta\bar{d}}, \quad (15)$$

where $\bar{\delta} = y\delta + (1-y)\delta^*$ is the world's financial capacity and $\bar{d} = yd + (1-y)d^*$ is the world's public debt to potential output ratio evaluated at $E = 1$. The world natural interest rate is the real rate consistent with global potential output when we ignore the ZLB constraint. It is similar to the autarky natural interest

rate, Eq. (11a), but for the world as a whole. We further define two bounds on the nominal exchange rate:

$$\bar{E} = 1 - \frac{(1 - \theta\bar{d})r^{w,n}}{(1 - y)d^*\theta\rho} \quad ; \quad \underline{E} = \left(1 - \frac{(1 - \theta\bar{d})r^{w,n}}{yd\theta\rho}\right)^{-1}. \quad (16)$$

The next proposition characterizes the BGP equilibrium away from the ZLB and at the ZLB.

Proposition 2 (Financial Integration). *Under financial integration and Assumption 1, competitive equilibria along a BGP are as follows:*

- If $r^{w,n} \geq 0$, then the global economy is away from the Zero Lower Bound. There is a unique balanced growth path with a positive interest rate, $i^w = r^w = r^{w,n}$, output is at its potential level, $\xi = \xi^* = 1$, and $E = 1$.
- If $r^{w,n} < 0$, then the global economy is at the ZLB. There is a continuum of balanced growth path equilibria with $i^w = r^w = 0$, indexed by $E \in [\underline{E}, \bar{E}]$, where ξ and ξ^* satisfy

$$\xi = \frac{\theta\bar{d}(E)}{1 - \bar{\delta}\theta/\rho} \quad ; \quad \xi^* = \frac{\theta\bar{d}(E)/E}{1 - \bar{\delta}\theta/\rho}, \quad (17)$$

and $\bar{d}(E) = yd + (1 - y)d^*E$ is the exchange-rate-adjusted ratio of global public debt to potential output.

- In all cases, the exchange rate satisfies

$$E = \frac{\xi}{\xi^*}. \quad (18)$$

Proof. See text. □

As before, the last part of the proposition obtains immediately from the goods market equilibrium conditions Eq. (8). Consider a BGP under financial integration away from the ZLB, $\xi = \xi^* = 1$. It follows from Eq. (18) that the exchange rate is $E = 1$. Define world potential output $\bar{Y}^w = \bar{Y} + E\bar{Y}^*$, the world supply of private assets $V^w = V + EV^*$, the world supply of public assets $D^w = D + ED^*$, and world wealth $W^w = W + EW^*$, all in Home's currency. From Eqs. (5) and (7), total asset supply $V^w + D^w$ along the BGP is given by

$$V^w + D^w = \left(\frac{\bar{\delta}}{r^w + \rho} + \bar{d}\right)\bar{Y}^w, \quad (19)$$

which is decreasing in the global real rate r^w , while total asset demand along the BGP satisfies

$$W^w = \frac{\bar{Y}^w}{\theta}, \quad (20)$$

which is invariant to the interest rate. The natural rate $r^{w,n}$ given by Eq. (15) equates global asset demand and global asset supply, $W^w = V^w + D^w$. This is only possible if $r^{w,n} \geq 0$, which establishes the first part of the proposition.

We can express the world natural rate $r^{w,n}$ as a weighted average of home and foreign financial autarky real rates:

$$r^{w,n} = y \frac{1 - \theta d}{1 - \theta \bar{d}} r^{a,n} + (1 - y) \frac{1 - \theta d^*}{1 - \theta \bar{d}^*} r^{a,n^*}. \quad (21)$$

In this expression, the weights represent the relative supply of *private* assets under autarky, V/V^w and V^*/V^w .⁷ Under Assumption 1, the weights are positive and sum to one. It follows that the world natural rate always lies between the home and foreign financial autarky rates. Hence, the global economy may escape the ZLB ($r^{w,n} \geq 0$) even if Home (but not Foreign) finds itself at the ZLB under financial autarky, i.e. when $r^{a,n} < 0 \leq r^{w,n} < r^{a,n^*}$. This occurs when the scarcity of assets in Home is offset by an abundance of assets in Foreign. In this case, financial integration pulls Home away from the ZLB.

Suppose now that $r^{w,n} < 0$. Inspecting Eq. (15), this occurs when $\bar{\delta}$ is low or ρ is high (i.e. a low global supply of private assets), when \bar{d} is low (i.e. a low global supply of public assets) or when θ is low (i.e. a high global demand for stores of value). The ZLB constraint imposes $i^w = r^w = 0$. In other words, under financial integration a ZLB equilibrium arises when there is a *global* shortage of private or public assets that cannot be resolved by a decline in the world real rate.

A global ZLB can only arise if at least one country (e.g. Home) is at the ZLB under financial autarky. However, the global economy may be pushed against the ZLB ($r^{w,n} < 0$) even if Foreign would have remained away from the ZLB under financial autarky, i.e. when $r^{a,n} < r^{w,n} < 0 \leq r^{a,n^*}$. This occurs when the scarcity of assets in Home is too large to be offset by asset supply in Foreign. Financial integration drags Foreign into a ZLB trap it would avoid under autarky.

Along a balanced growth path at the ZLB, total asset supply can be expressed as

$$V^w + D^w = \frac{\delta \xi \bar{Y} + \delta^* E \xi^* \bar{Y}^*}{\rho} + d \bar{Y} + E d^* \bar{Y}^*, \quad (22)$$

and global asset demand satisfies

$$W^w = \frac{\xi \bar{Y} + E \xi^* \bar{Y}^*}{\theta}. \quad (23)$$

In addition, both the asset market Eq. (9) and the goods market Eq. (18) need to clear.

This is a system of *four* equations Eqs. (9), (18), (22) and (23), in *five* unknowns V^w , W^w , ξ , ξ^* , and E .

⁷Under autarky $W = 1/\theta \bar{Y}$ while $D = d \bar{Y}$ so $V = W - D = (1/\theta - d)y \bar{Y}^w$ while $V^w = W^w - D^w = (1/\theta - \bar{d}) \bar{Y}^w$. Taking the ratio yields $V/V^w = y(1 - \theta d)/(1 - \theta \bar{d})$. Similar expressions apply to Foreign.

That is, there is a *degree of indeterminacy*. This indeterminacy is related to the seminal result by [Kareken and Wallace \(1981\)](#) that the exchange rate is indeterminate with pure interest rate targets, which is *de facto* the case when both countries are at the ZLB. Unlike in [Kareken and Wallace \(1981\)](#), money is *not* neutral in our model: different exchange rates correspond to different levels of output at Home and in Foreign, as prescribed by [Eq. \(18\)](#). In other words, while global output needs to decline to restore equilibrium in asset markets, different combinations of domestic and foreign output—corresponding to different values of the exchange rate—are possible.

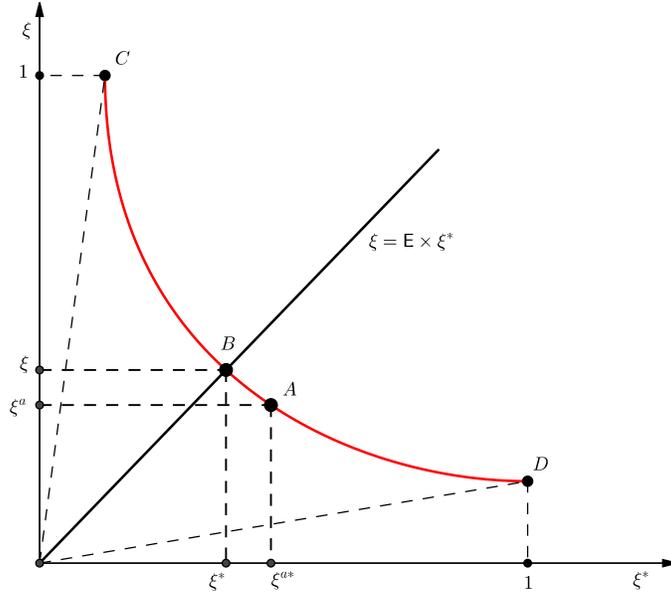
From a technical point of view, indeterminacy arises from the assumption that the liquidity trap is perceived as permanent. In [Online Appendix B.2](#), we extend our model to consider the possibility of exit from the ZLB at some future stochastic time τ . Post-exit, the exchange rate is determinate. By the usual arbitrage arguments and backward induction, this pins down the exchange rate path pre-exit as well, removing the indeterminacy. There are, however, important reasons to be skeptical of the rational expectations backward-induction logic that pins the exchange rate today to its value after the economy exits the trap, especially when the trap may be very persistent. A natural practical interpretation is that the longer the liquidity trap is expected to last, the less anchored to fundamentals is the exchange rate today. Our model considers the limit case where exchange rate expectations are not anchored by long run outcomes or when long run outcomes themselves are constrained by the ZLB.

Indexing these different solutions by the exchange rate, we can substitute $\xi^*E = \xi$ and equate world asset demand and world asset supply:

$$\frac{\xi}{\theta} = \frac{\bar{\delta}}{\rho}\xi + \bar{d}(E). \tag{24}$$

Solving for the home output gap yields [Eq. \(17\)](#). A similar derivation holds for Foreign. Any value of the exchange rate is possible as long as *both* countries are at the ZLB, i.e. $\xi \leq 1$ and $\xi^* \leq 1$. This determines a range $[\underline{E}, \bar{E}]$ with $\xi = 1$ for $E = \bar{E}$ and $\xi^* = 1$ for $E = \underline{E}$, where \underline{E} and \bar{E} are defined in [Eq. \(16\)](#). This establishes the second part of [Proposition 2](#).

As in the case of financial autarky, under [Assumption 1](#) a recession at Home or in Foreign reduces total asset demand (the left hand side of [Eq. \(24\)](#)) faster than it reduces total asset supply (the right hand side of [Eq. \(24\)](#)). The novelty under financial integration is that a weaker currency (a higher E) increases the total supply of public assets in local currency $\bar{d}(E)$, while simultaneously reducing the total supply of public assets in foreign currency $\bar{d}(E)/E$. This results in a smaller recession at Home (higher ξ) and a larger recession in Foreign (lower ξ^*). As before, an equivalent interpretation of the ZLB equilibrium comes from the goods market: a cheaper currency acts as a positive aggregate demand shifter at Home and a negative aggregate demand shifter in Foreign.



This figure illustrates home (ξ) and foreign (ξ^*) output gaps at the global ZLB for different values of the exchange rate $E \in [\underline{E}, \bar{E}]$ when $\xi^a < 1$ and $\xi^{a*} < 1$. Point A denotes the autarky equilibrium ($E = E^a = \xi^a / \xi^{a*}$). When $E > E^a$, $\xi > \xi^a$ and $\xi^* < \xi^{a*}$ (point B). When $E = \bar{E}$, $\xi = 1$, Home escapes the ZLB and Foreign absorbs all the output loss (point C). The red segment $[CD]$ plots the output frontier at the ZLB.

Figure 3: Output Determination in the Global ZLB

To summarize our findings, under financial integration global financial conditions (reflected in the determinants of the world natural rate $r^{w,n}$) determine whether the global economy is at the ZLB. Unlike under financial autarky, however, the exchange rate is not anchored by goods market fundamentals. Different values of the exchange rate affect local financial conditions by changing the relative supply of public assets. This affects relative demand and the allocation of output across countries.

We can illustrate the indeterminacy by considering the special case where both countries experience a liquidity trap under financial autarky (that is, when $r^{a,n} < 0$ and $r^{a,n*} < 0$). The equilibrium autarky exchange rate simplifies to

$$E^a = \frac{d}{d^*} \frac{\rho - \delta^* \theta}{\rho - \delta \theta}.$$

The country with worse asset scarcity (lower d or lower δ) has lower output and a stronger currency under financial autarky. Under financial integration, if $E = E^a$ the financial integration equilibrium coincides with the financial autarky equilibrium: $\xi = \xi^a$ and $\xi^* = \xi^{a*}$. For $E > E^a$, Eq. (17) implies that we have $\xi > \xi^a$ and $\xi^* < \xi^{a*}$, and vice-versa for $E < E^a$. Fig. 3 summarizes this relationship and maps home and foreign output when the exchange rate varies between \underline{E} and \bar{E} . The segment $[CD]$ in red reports the possible combinations of the Home and Foreign output gaps ξ, ξ^* that satisfy Eq. (17).

2.4 Net Foreign Assets, Current Accounts and the Metzler Diagram

We briefly characterize Net Foreign Asset positions and Current Accounts under financial integration, both away from the ZLB and at the ZLB. Consider the case where the global economy is away from the ZLB ($\xi = \xi^* = 1$).

Proposition 3 (Net Foreign Assets and Current Accounts Away from the ZLB). *Under Assumption 1, if $r^{w,n} > 0$ then along a Balanced Growth Path:*

- The world interest rate is a weighted average of the home and foreign autarky rates $r^{a,n}$ and $r^{a,n*}$, as in Eq. (21).
- Home is a net creditor and runs a current account surplus if and only if the world interest rate is higher than the autarky interest rate: $r^{a,n} < r^{w,n} < r^{a,n*}$.
- Home's Net Foreign Asset position (NFA) and Current Account (CA) are given by

$$\frac{NFA}{\bar{Y}} = \frac{(1 - \theta d)(r^w - r^{a,n})}{(g + \theta - r^w)(\rho + r^w)} \quad , \quad \frac{CA}{\bar{Y}} = g \frac{NFA}{\bar{Y}}. \quad (25)$$

Proof. See text. □

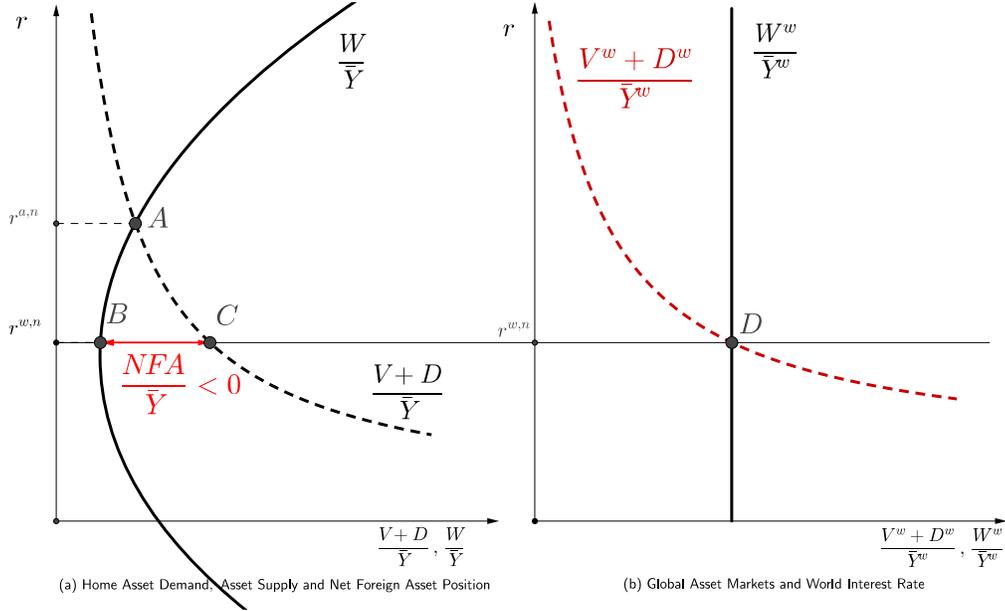
We have already established that away from the ZLB, the world interest rate is a weighted average of the financial autarky rates in both countries. Next, note that along a BGP and for a given world interest rate r^w , we can express home wealth accumulation (Eq. (4a)), the home asset pricing equation Eq. (5a), and the home government budget constraint Eq. (7a) as

$$V = \frac{\delta}{r^w + \rho} \bar{Y}, \quad (26a)$$

$$W = \frac{(1 - \delta) - (r^w - g)d + (\rho + g) \frac{\delta}{r^w + \rho}}{g + \theta - r^w} \bar{Y}. \quad (26b)$$

The net foreign asset position is defined as $NFA = W - (V + D)$, and the current account is the change in the net foreign asset position: $CA = \dot{NFA} = gNFA$ along the BGP. Substituting, we obtain Eq. (25), which tells us that the home Net Foreign Asset position increases with global interest rates r^w .

Similar equations hold for Foreign, which together with equilibrium in the world asset market allow us to characterize the world interest rate r^w in a conventional Metzler diagram (Fig. 4).



Panel (a) reports asset demand W/\bar{Y} (solid line) and asset supply $(V + D)/\bar{Y}$ (dashed line) in Home, scaled by Home potential output. The two lines intersect at the autarky natural interest rate $r^{a,n}$ (point A). Panel (b) reports world asset demand W^w/\bar{Y}^w (solid line) and world asset supply $(V^w + D^w)/\bar{Y}^w$ (dashed red line). The two lines intersect at the world natural interest rate $r^{w,n}$ (point D). When the world interest rate is below the autarky rate ($0 < r^{w,n} < r^{a,n}$) the country is a net debtor and runs a current account deficit.

Figure 4: World Interest Rates and Net Foreign Asset Positions: the Metzler Diagram

Panel (a) of Fig. 4 reports home asset supply $V + D$ (dashed line) and home asset demand W (solid line), scaled by Home potential output \bar{Y} , as functions of the world interest rate r^w .⁸ The two curves intersect at the financial autarky natural interest rate $r^{a,n}$ —assumed positive—where the country is neither a debtor nor a creditor (point A). For lower values of the world interest rate, Home is a net debtor: $NFA/\bar{Y} < 0$. For higher values, it is a net creditor. Panel (b) reports global asset supply $V^w + D^w$ (red dashed line) and global asset demand W^w (solid line), scaled by global potential output \bar{Y}^w , as a function of the global interest rate r^w (Eqs. (19) and (20)). Global asset supply decreases with the world interest rate, while global asset demand is constant. The two curves intersect at the world natural interest rate $r^{w,n}$, assumed positive. The figure assumes $r^{a,n*} < r^{a,n}$, hence $r^{w,n} < r^{a,n}$ and Home runs a current account deficit.

Away from the ZLB, *foreign's Current Account surplus helps propagate its asset shortage*, increasing the foreign interest rate above its autarky level ($r^{a,n*} < r^{w,n}$), while reducing the home interest rate below autarky ($r^{w,n} < r^{a,n}$).

⁸Asset supply $(V + D)/\bar{Y}$ is monotonically decreasing in the world interest rate r^w . Asset demand W/\bar{Y} is non-monotonic because of two competing effects. First, higher interest rates imply that wealth accumulates faster. But higher interest rates also reduce the value of the new trees endowed to the newborns and increase the tax burden required to pay the higher interest on public debt. For high levels of the interest rate and low levels of debt, the first effect dominates and asset demand increases with r^w . For low levels of the interest rate, the second effect dominates and asset demand decreases with r^w . Regardless of the shape of W/\bar{Y} , NFA/\bar{Y} is always increasing in the interest rate.

Consider the case where the global economy is at the ZLB ($r^{w,n} < 0$, $\xi \leq 1$ and $\xi^* \leq 1$) described in [Proposition 2](#). The next proposition characterizes global imbalances.

Proposition 4 (Net Foreign Assets and Current Accounts at the ZLB). *Under Assumption 1, if $r^{w,n} < 0$, then given an exchange rate $E \in [\underline{E}, \bar{E}]$:*

- Domestic output ξ is a weighted average of home and exchange-rate-weighted foreign financial autarky outputs, $\xi^{a,n}$ and $E\xi^{a,n*}$, according to

$$\xi = y \frac{1 - \frac{\delta\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} \xi^{a,n} + (1 - y) \frac{1 - \frac{\delta^*\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} E \xi^{a,n*}. \quad (27)$$

- Home is a net creditor and runs a current account surplus if and only if home output ξ exceeds its financial autarky level: $\xi^{a,n} < \xi < E\xi^{a,n*}$. Along the BGP, Home's Net Foreign Asset Position and Current Account are given by

$$\frac{NFA}{\bar{Y}} = \frac{(1 - \frac{\delta\theta}{\rho})(\xi - \xi^{a,n})}{g + \theta}, \quad \frac{CA}{\bar{Y}} = g \frac{NFA}{\bar{Y}}. \quad (28)$$

Proof. See text. □

The first part of the proposition obtains directly by manipulating [Eq. \(24\)](#), using the definition of $\xi^{a,n}$ and $\xi^{a,n*}$ in [Proposition 2](#). In this expression, the weights represent the relative supply of *public* assets under autarky, D/D^w and $E^a D^*/D^w$.⁹ Under [Assumption 1](#), the weights are positive and sum to one. This implies that $\xi^{a,n} \leq \xi \leq E\xi^{a,n*}$.

Assume that $r^{w,n} < 0$ and fix a nominal exchange rate $E \in [\underline{E}, \bar{E}]$. We can rewrite wealth accumulation [Eq. \(5a\)](#) and home asset pricing [Eq. \(4a\)](#) along the BGP as a function of the domestic output level ξ :

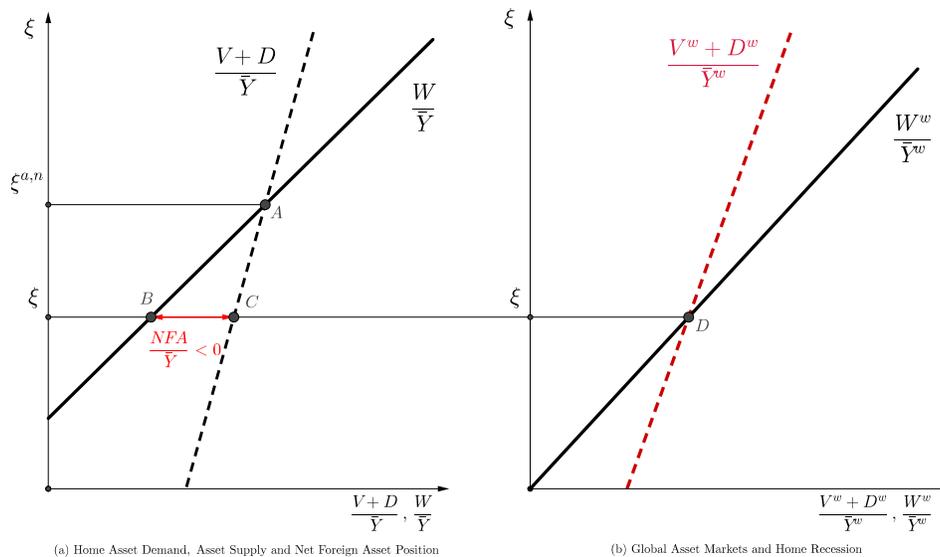
$$V = \frac{\delta\xi}{\rho} \bar{Y}, \quad (29a)$$

$$W = \frac{\xi + gd + g \frac{\delta\xi}{\rho}}{g + \theta} \bar{Y}, \quad (29b)$$

which immediately implies [Eq. \(28\)](#). This establishes the last part of the proposition.

Since $\xi \leq 1$, Home always runs a Current Account deficit when $\xi^{a,n} > 1$, i.e. when Home would escape the liquidity trap under financial autarky. A similar equation holds for Foreign, which together

⁹Under autarky at the ZLB, $W = \xi^a/\theta\bar{Y}^w$ while $V = \delta\xi^a/\rho$ so that $D = W - V = (1/\theta - \delta/\rho)\xi^a y \bar{Y}^w$ while $D^w = W^w - D^w = (1/\theta - \bar{\delta}/\rho)\xi^a \bar{Y}^w$. Taking the ratio yields $D/D^w = y(1 - \delta\theta/\rho)/(1 - \bar{\delta}\theta/\rho)$. Similar expressions apply to Foreign.



Panel (a) reports Home asset demand W/\bar{Y} (solid line) and asset supply $(V + D)/\bar{Y}$ (dashed line) as functions of home output ξ . The two lines intersect at the autarky level of output $\xi^{a,n}$ (point A). Panel (b) reports global asset demand W^w/\bar{Y}^w (solid line) and asset supply $V^w + D^w/\bar{Y}^w$ (red dashed line) scaled by world potential output \bar{Y}^w as a function of home output ξ , for a given exchange rate $E < E^a$, when $\xi^{a,n} < 1$ and $\xi^{a,n*} < 1$. The two lines intersect at the home level of output ξ (point D). Home experiences a worse recession, $\xi < \xi^a$, when it is a net debtor, $NFA/\bar{Y} < 0$ and runs a Current Account deficit ($CA/\bar{Y} < 0$).

Figure 5: Recessions and Net Foreign Asset Positions in a Global Liquidity Trap: the Metzler Diagram in Quantities

with equilibrium in the world asset market allows us to characterize the equilibrium Home recession ξ as a function of the exchange rate E in a *modified Metzler diagram in quantities* (Fig. 5).

Panel (a) of Fig. 5 reports home asset supply $V + D$ (dashed line) and home asset demand W (solid line) scaled by home potential output \bar{Y} , as functions of domestic output ξ , for a given exchange rate E (Eqs. (29a) and (29b)). Both asset demand and asset supply are increasing in output, but supply increases faster than demand. The two curves intersect at the financial autarky output $\xi^{a,n}$ (point A). For lower values of output, Home is a net debtor: $NFA/\bar{Y} < 0$. For higher values, it is a net creditor: $NFA/\bar{Y} > 0$. Panel (b) reports global asset supply $V^w + D^w$ (red dashed line) and global asset demand W^w (solid line) scaled by global potential output \bar{Y}^w , as a function of the home recession ξ (Eqs. (22) and (23)). Both global asset demand and supply are increasing in output, but supply increases faster than demand. The two curves intersect at the equilibrium level of home output $0 < \xi < 1$. The figure assumes $E\xi^{a,n*} < \xi^{a,n}$ or equivalently $E < E^a$.

Replacing ξ and $\xi^{a,n}$ from Eq. (17) and Eq. (11b) respectively, we can rewrite the home Net Foreign

Asset position and Current Account in Eq. (28) as

$$\frac{NFA}{\bar{Y}} = \frac{(1 - \frac{\delta\theta}{\rho})}{g + \theta} \left[\frac{\theta\bar{d}(E)}{1 - \frac{\delta\theta}{\rho}} - \frac{\theta d}{1 - \frac{\delta\theta}{\rho}} \right], \quad \frac{CA}{\bar{Y}} = g \frac{NFA}{\bar{Y}}. \quad (30)$$

A cheaper home currency implies a larger home Net Foreign Asset position and hence a larger Current Account, allowing Home to export more of its recession abroad. Depending on the value of the exchange rate E , Home can be a surplus country or a deficit country.¹⁰

When both countries are in a liquidity trap under financial autarky, we can express the Net Foreign Asset position and Current Account directly as a function of the exchange rate E , relative to the autarky exchange rate E^a . Substituting the expression for the exchange rate-adjusted financial capacity, we obtain

$$\frac{NFA}{\bar{Y}} = \frac{1 - \frac{\delta\theta}{\rho}}{1 - \frac{\delta\theta}{\rho}} \frac{(1 - y)\theta d^*(E - E^a)}{g + \theta}, \quad \frac{CA}{\bar{Y}} = g \frac{NFA}{\bar{Y}}. \quad (31)$$

We can now connect our results to the case of financial autarky. Under autarky, the exchange rate is determinate precisely because the capital account is closed. If both countries are in a liquidity trap under financial autarky, $\xi^a = \xi^{a,n} < 1$, $\xi^{a*} = \xi^{a,n*} < 1$ and $E^a = \xi^{a,n}/\xi^{a,n*}$. Then for $E = E^a$, the financial integration equilibrium coincides with the financial autarky equilibrium and there are no current account imbalances. For $E > E^a$, we have $\xi > \xi^a$, $\xi^* < \xi^{a*}$ and $NFA/\bar{Y} > 0$, and vice versa for $E < E^a$. By depreciating its exchange rate and running a Current Account surplus, Home can reduce the size of its recession.

In the ZLB equilibrium, *Home's current account surplus helps propagate recessions*, increasing Home's output and reducing Foreign's output. The ZLB is a *'tipping point'* for global imbalances, where the economy transitions from *benign* (current account surpluses propagating low interest rates) to *malign* (current account surpluses propagating recessions).

3 Negative Policy Spillovers: Currency and Trade Wars

The adverse impact of current account surpluses on foreign output in the ZLB equilibrium is a symptom of a more general increased policy interdependence. At the global ZLB, some policies have large positive spillovers; others have large negative spillovers. This section focuses on the negative spillovers. In particular, we consider currency wars and trade wars. Each of these policies affects the global equilibrium by reallocating demand towards the home country and away from the foreign country, without addressing the

¹⁰In a global liquidity trap, there can be global imbalances even though the two countries are identical, which could never happen outside of a global liquidity trap.

underlying cause of global stagnation.

3.1 Currency Wars

Consider the role of exchange rate policy away from the ZLB and at the ZLB. Our model provides a way of thinking about “currency wars”, i.e. the incentives for one country to manipulate its currency at the expense of its trading partners.

Outside the global liquidity trap, the exchange rate is pinned down ($E = 1$), output in each country is at its potential level ($\xi = \xi^* = 1$) and the real interest rate is equal to its Wicksellian natural counterpart ($r = r^* = r^{w,n}$). Countries have nothing to gain from manipulating their exchange rates. This result accords with the theoretical literature on the gains from monetary policy coordination in models with nominal rigidities, concluding that the gains are at best modest (Corsetti and Pesenti, 2001; Obstfeld and Rogoff, 2002).

In the global liquidity trap, the global asset shortage cannot be offset by lower world interest rates and the world enters a recession. The distribution of this global recession across countries is mediated by the exchange rate and global imbalances. Even though the exchange rate is indeterminate in this global liquidity trap regime, it is in principle possible for the home monetary authority to peg the exchange rate at any level E in the indeterminacy region $[\underline{E}, \bar{E}]$, by simply standing ready to buy and sell the home currency for the foreign currency at the exchange rate E .

By choosing a sufficiently depreciated exchange rate, Home is able to partly export its recession abroad by running a Current Account surplus (Proposition 2 and Proposition 4). That is, once interest rates are at the ZLB, our model indicates that exchange rate policies generate powerful beggar-thy-neighbor effects. This zero-sum logic resonates with concerns regarding “currency wars”: in the global stagnation equilibrium, attempts to depreciate one’s currency affect relative output one-for-one, according to Eq. (18).

Of course, if both countries attempt to simultaneously depreciate their currencies, these efforts cancel out, and the exchange rate remains a pure matter of coordination. Moreover, if agents coordinate on an equilibrium where the home exchange rate is appreciated, as could be the case if the home currency were perceived to be a “reserve currency,” then this would worsen the recession at Home. In other words, while the reserve currency status may be beneficial outside a liquidity trap, it exacerbates the domestic recession in a global liquidity trap. This “*paradox of the reserve currency*” captures a dimension of the appreciation struggles of countries like Switzerland during the recent European turmoil in 2015, and of Japan before the implementation of Abenomics in 2012.

We can develop these insights further by extending our baseline model. For tractability the baseline

model of [Section 2](#) assumed a unitary elasticity of substitution between home and foreign goods. Under this assumption, while a depreciation can stimulate output, the value of home vs. foreign goods $\xi/(E\xi^*)$ remains invariant to the exchange rate because income and substitution effects perfectly cancel each other. In order to analyze currency wars, we move away from the assumption of a unitary elasticity. [Appendix A.4](#) presents this extension, allowing for an arbitrary elasticity of substitution σ between home and foreign goods.

The analysis under financial autarky, or outside the ZLB under financial integration, is identical to the case $\sigma = 1$ except for the value of the financial autarky exchange rate. In particular, $\xi = \xi^* = 1$ and $E = 1$ when the global natural rate is positive, $r^{w,n} \geq 0$, under [Proposition 2](#). In the case of a global liquidity trap under financial integration, [Appendix A.4](#) shows that domestic and foreign output satisfy

$$\frac{\xi}{E\xi^*} = E^{\sigma-1}, \quad (32a)$$

$$\xi = \frac{\theta \bar{d}(E)}{(1 - \frac{\theta \hat{\delta}(E)}{\rho}) P^{1-\sigma}}, \quad (32b)$$

where $\hat{\delta}(E) = (\delta y + \delta^*(1-y)E^{1-\sigma})/P^{1-\sigma}$ is a weighted average of δ in Home and Foreign using the relative price of H and F goods as weights, and $P = (y + (1-y)E^{1-\sigma})^{1/(1-\sigma)}$ is the consumer price index at Home. The first equation indicates that the value of home vs. foreign goods $\xi/(E\xi^*)$ increases with the exchange rate when $\sigma > 1$. The second equation illustrates that the exchange rate affects domestic output via the supply of public assets, $\bar{d}(E)$, as in the case $\sigma = 1$, but also via the supply of private assets, $\hat{\delta}(E)$, and via the price level P that affects both asset demand and asset supply. When $\sigma > 1$, one can check that the net effect of a depreciation is expansionary at Home and contractionary in Foreign.

Along a BGP at the Zero Lower Bound under financial integration, domestic wealth W is increasing in ξ according to [Eq. \(29b\)](#). It follows that a depreciation of the exchange rate E has two effects on real consumption $c = \theta W/P$: it stimulates output ξ , which increases wealth and consumption, but also leads to an increase in the price level P , which reduces real consumption. The analysis of the general case, although conceptually straightforward, leads to a nonlinear system of equations which is not amenable to a closed form solution. Things simplify in the limit $\sigma \rightarrow \infty$ where the goods become perfect substitutes.

In that limit, $E = 1$ and $P = 1$, yet there is still a degree of indeterminacy indexed by a re-normalization of the exchange rate $\hat{E} \equiv E^\sigma$. We show in the appendix that, when $\sigma \rightarrow \infty$, home and foreign output

satisfy

$$\xi = \frac{\theta \bar{d}}{y(1 - \frac{\delta\theta}{\rho}) + \frac{1}{\hat{E}}(1 - y)(1 - \frac{\delta^*\theta}{\rho})}, \quad (33a)$$

$$\xi^* = \frac{\theta \bar{d}}{y(1 - \frac{\delta\theta}{\rho})\hat{E} + (1 - y)(1 - \frac{\delta^*\theta}{\rho})}. \quad (33b)$$

Since $P = 1$ in that limit, we can ignore the effect of the (renormalized) exchange rate on the price index: real consumption $\theta W/P$ is proportional to output ξ . By choosing a more depreciated \hat{E} , Home can stimulate domestic output and consumption at the expense of Foreign.¹¹

To develop this idea further, assume that the central bank at Home can take some ‘non-conventional’ action $a \geq 1$, while the central bank in Foreign can take an action $a^* \geq 1$. These actions can be interpreted as non-conventional monetary policies such as large-scale asset purchases, foreign exchange interventions or any other (costly) communication by central banks. We rule out policies with a ‘fiscal dimension’, for instance non-conventional monetary policies that expand the supply of public or quasi-public debt, since these would have positive spillovers (see [Section 4](#)).

These actions come at a *non-pecuniary* cost $\mathcal{C}(a) \geq 0$ and $\mathcal{C}(a^*) \geq 0$ per unit of output, which can be interpreted as the political-economy cost for the central bank of deviating from a narrow interest rate policy. We assume that the function $\mathcal{C}(a)$ is twice continuously differentiable and convex in a , with $\mathcal{C}(1) = \mathcal{C}'(1) = 0$ and let $\eta_c = a\mathcal{C}''(a)/\mathcal{C}'(a) > 0$ denote the elasticity of the marginal cost. We assume further that these actions can potentially impact the renormalized exchange rate, with $\hat{E} = \mathcal{E}(a, a^*) \equiv (a/a^*)^n$ denoting how the exchange rate responds to the actions of both central banks, and $0 < n < 1$. A stronger action by the Home (resp. Foreign) central bank depreciates (resp. appreciates) the currency, at a decreasing rate.

We do not explicitly spell out the mechanism by which central banks can affect the exchange rate. One possibility is that this is just a communication game, where the central bank announcement a is expected to affect the exchange rate according to $\mathcal{E}(a, a^*)$. Another possibility is that non-conventional monetary policy affects the exchange rate via its effect on relative output both away from the ZLB and at the ZLB.

If the economy were outside the global liquidity trap, there would be no incentive to manipulate the exchange rate: $a = a^* = 1 = \hat{E} = 1$ since output would already be at its potential level ($\xi = 1$). Any stimulative non-conventional policy at Home would trigger a countervailing monetary tightening according to the Home Taylor rule [Eq. \(2\)](#).

Consider now what happens in the global liquidity trap when the central bank aims to maximize domestic consumption c , net of the non-pecuniary cost $\mathcal{C}(a)$, given foreign action a^* . Using [Eqs. \(29b\)](#)

¹¹While the exchange rate E remains equal to 1, we can interpret changes in \hat{E} as infinitesimal attempts to manipulate the exchange rate, with an effect on output described by [Eq. \(33\)](#).

and (33a), Home's optimal non-conventional action a satisfies

$$\xi \frac{\frac{1}{E}(1-y)(1-\frac{\delta^*\theta}{\rho})}{y(1-\frac{\delta\theta}{\rho}) + \frac{1}{E}(1-y)(1-\frac{\delta^*\theta}{\rho})} \frac{\mathbf{n}}{a} = \mathcal{C}'(a), \quad (34)$$

which generates a best-response function $a = \mathcal{A}(a^*)$. By symmetry, the foreign central bank aims to maximize foreign consumption c^* , net of the non-pecuniary cost $\mathcal{C}(a^*)$, given home action a . Using Eq. (33b), Foreign's optimal action a^* satisfies

$$\xi^* \frac{y(1-\frac{\delta\theta}{\rho})}{y(1-\frac{\delta\theta}{\rho}) + \frac{1}{E}(1-y)(1-\frac{\delta^*\theta}{\rho})} \frac{\mathbf{n}}{a^*} = \mathcal{C}'(a^*), \quad (35)$$

which defines a best-response function $a^* = \mathcal{A}^*(a)$. A Nash equilibrium of the Currency War game obtains when $a = \mathcal{A}(a^*)$ and $a^* = \mathcal{A}^*(a)$ hold simultaneously. Under some restrictions on the parameters (described in the appendix), a Nash equilibrium exists, is unique and is asymptotically stable. This equilibrium features $a > 1$ and $a^* > 1$: both countries have an incentive to depreciate their currency. This is generically inefficient since the efforts of each country are undone by the other, while each country bears the full cost of its action, $\mathcal{C}(a)$ and $\mathcal{C}(a^*)$.

Furthermore, we show in appendix A.4 that each country's optimal action is increasing in the amount of public debt \bar{d} : $\partial a / \partial \bar{d} > 0$. It follows that, if *one* country issues more public debt, *all* countries attempt to depreciate their currency, in a largely futile effort.

We summarize these results in the following proposition.

Proposition 5 (Currency Wars). *Under financial integration, in the limit of $\sigma \rightarrow \infty$ and under the parameter restriction described in appendix A.4, the Nash equilibria of the Currency War game where the central banks tries to maximize consumption c, c^* by choosing actions (a^N, a^{N*}) , are as follows:*

- If $r^{w,n} \geq 0$, then the global economy is away from the Zero Lower Bound. There is a unique balanced growth path Nash equilibrium with positive interest rate $i^w = r^w = r^{w,n}$, output is at its potential, $\xi = \xi^* = 1$, and there is no incentive to manipulate the exchange rate: $E = 1, a^N = a^{N*} = 1$.
- If $r^{w,n} < 0$, then the global economy is at the ZLB: $i^w = r^w = 0$. There is a unique asymptotically stable balanced growth path Nash equilibrium with $a^N > 1, a^{N*} > 1$ characterized by Eqs. (34) and (35); the normalized exchange rate satisfies $\hat{E}^N = \mathcal{E}(a^N, a^{N*})$ and $\xi^N \leq 1, \xi^{N*} \leq 1$ satisfy Eq. (33).
- At the ZLB Nash equilibrium, the more public debt a country issues, the more each country tries to depreciate its currency: $\partial a^N / \partial \bar{d} > 0, \partial a^{N*} / \partial \bar{d} > 0$.

Proof. See text and appendix A.4. □

3.2 Trade Wars

Trade wars share the mechanisms and negative spillovers of currency wars at the ZLB. To explore this issue, we introduce the possibility of asymmetric tariffs into the baseline model of Section 2. This provides a way to think about “trade wars,” i.e. the incentives for one country to erect trade barriers at the expense of its trading partners.

The setup is identical to Section 2, except that we now allow Home to impose an ad-valorem tariff λ on imports from Foreign, and conversely allow Foreign to impose an ad-valorem tariff λ^* on imports from Home. Under the law of one price, households in Home now face import prices $P_{F,t} = E_t P_{F,t}^* (1 + \lambda)$, while households in Foreign now face import prices $P_{H,t}^* = P_{H,t} (1 + \lambda^*) / E_t$. As before, we assume that prices are fully rigid in their home market and normalize: $P_{H,t} = P_{F,t}^* = 1$.

In addition, we assume that each country instantaneously rebates tariff revenues to the consuming households. With Cobb-Douglas preferences, aggregate expenditure shares are

$$c_H = \frac{\gamma(1 + \lambda)}{1 + \gamma\lambda} \theta W \quad , \quad c_F = \frac{(1 - \gamma) \theta W}{1 + \gamma\lambda} \frac{1}{E} \quad (36a)$$

$$c_H^* = \frac{\gamma}{1 + \lambda^*(1 - \gamma)} \theta E W^* \quad , \quad c_F^* = \frac{(1 - \gamma)(1 + \lambda^*)}{1 + \lambda^*(1 - \gamma)} \theta W^*. \quad (36b)$$

Everything else equal, tariffs shift households’ expenditure shares towards domestic goods: as Home increases its tariffs on Foreign goods, demand for Foreign goods by Home households decreases by a factor $(1 + \gamma\lambda)^{-1} < 1$. Further, since tariff revenues are rebated lump sum to consumers, demand for Home goods by Home households increases by a factor $(1 + \lambda)/(1 + \gamma\lambda) > 1$. The same effect holds for the tariffs imposed by Foreign.

Substituting Eq. (36) into the goods market clearing conditions, Eq. (8) becomes

$$\theta \left(\frac{y(1 + \lambda)}{1 + y\lambda} W + \frac{y}{1 + \lambda^*(1 - y)} E W^* \right) = \xi \bar{Y}, \quad (37a)$$

$$\theta \left(\frac{1 - y}{1 + y\lambda} W + \frac{(1 - y)(1 + \lambda^*)}{1 + \lambda^*(1 - y)} E W^* \right) = E \xi^* \bar{Y}^*. \quad (37b)$$

Because tariff revenues are rebated lump sum to households, all remaining equilibrium conditions are unchanged: wealth accumulation Eq. (4), asset pricing Eq. (5) and government debt dynamics Eq. (7).

Manipulating the equilibrium conditions, under financial autarky the natural rate $r^{a,n}$, the natural output gap $\xi^{a,n}$ and the equilibrium allocations are the same as in Proposition 1, regardless of the tariffs

λ and λ^* : $r^a = \max\{r^{a,n}, 0\}$ and $\xi^a = \min\{\xi^{a,n}, 1\}$. The only effect of the tariffs is to force an adjustment in the autarky exchange rate, now equal to

$$E^a = \frac{\xi^a}{\xi^{a,*}} \frac{1 + \lambda^*(1 - y)}{1 + \lambda y}. \quad (38)$$

Under autarky, the natural rate $r^{a,n}$ is entirely determined in asset markets. Since asset market conditions are not changed by the tariffs, the natural rate is unchanged: whether the economy is away from or at the ZLB is unaffected by the tariffs. It follows that wealth (in domestic currency) is also independent of the tariffs. Consequently, the exchange rate must adjust to counteract the shift in relative demand induced by the tariffs in Eq. (36). Since the root of the ZLB equilibrium lies in the financial sphere, reallocating demand between Home and Foreign goods cannot resolve this problem: an increase in tariffs in Home simply appreciates the currency, leaving the domestic economy just as depressed.

Things are different under financial integration. We can distinguish between two cases. First, away from the ZLB, output is at its potential level ($\xi = \xi^* = 1$) in both countries. In that case, a change in tariffs requires an adjustment in exchange rates. As the exchange rate varies, so does the global supply of assets relative to global asset demand, hence global interest rates need to adjust as well. This can be illustrated most directly by combining the asset supply and asset demand conditions Eqs. (4), (5) and (7) with the fact that global wealth spent must equal global output, $\theta W^w = \bar{Y} + E\bar{Y}^*$. This yields an expression for the world risk free rate as a weighted average of the home and foreign natural autarky rates, where the weights are a function of the exchange rate:

$$r^w = \frac{y(1 - \theta d)}{y(1 - \theta d) + E(1 - x)(1 - \theta d^*)} r^{a,n} + \frac{(1 - y)E(1 - \theta d^*)}{y(1 - \theta d) + E(1 - x)(1 - \theta d^*)} r^{a,n*}. \quad (39)$$

An appreciation of the exchange rate shifts the global interest rate towards the Home country's autarky natural rate as it increases Home asset supply relative to Foreign.

Given a global interest rate r^w , Home and Foreign asset demands (Eq. (4) along the BGP) satisfy Eq. (26). Substituting this into the goods market equilibrium conditions Eq. (36) yields an expression for the exchange rate needed to clear the goods markets, given a global interest rate r^w :

$$(g + \theta - r^w)(\rho + r^w) = \frac{\theta y(1 + \lambda)}{1 + y\lambda} [\rho + g(\delta + d) + r^w(1 - \delta - d)] \quad (40a)$$

$$+ E \frac{\theta(1 - y)}{1 + \lambda^*(1 - y)} [\rho + g(\delta^* + d^*) + r^w(1 - \delta^* - d^*)]. \quad (40b)$$

As before, for a given world interest rate (and therefore asset demands), an increase in domestic tariffs

requires an appreciation of the domestic currency to clear the goods markets. But this movement in the exchange rate now affects world interest rates according to [Eq. \(39\)](#).

As long as this system admits a solution $r^{w,n}$ with $r^{w,n} > 0$, the economy escapes the ZLB. Output in each country is unaffected by tariffs, whose effect is absorbed by a combination of exchange rate and global interest rate adjustments.

While tariffs leave output unchanged, they do affect global imbalances: Home's net foreign asset position along the BGP is still given by [Eq. \(25\)](#) from [Proposition 3](#), reproduced here:

$$\frac{NFA}{\bar{Y}} = \frac{(1 - \theta d)(r^w - r^{a,n})}{(g + \theta - r^w)(\rho + r^w)} \quad , \quad \frac{CA}{\bar{Y}} = g \frac{NFA}{\bar{Y}}.$$

An increase in tariffs at Home, which appreciates the currency, *reduces* global imbalances at Home (relative to its output) as it reduces the gap between the world interest rate and Home's autarky rate. This is true regardless of whether the country is a creditor or a debtor: following an increase in its tariffs, a creditor country runs a smaller surplus; a debtor country runs a smaller deficit.¹² This illustrates that global imbalances are not driven by the expenditure switching effect due to the tariffs or to the exchange rate appreciation, for otherwise the current account would always either improve or deteriorate regardless of its initial position. Instead, global imbalances are determined in global financial markets and reflect the tension between the local and global supply and demand of assets.

Consider what happens when the natural rate $r^{w,n}$ becomes negative and the economy experiences a global liquidity trap. As in [Proposition 2](#), given tariff policies λ and λ^* , $r^w = 0$ and there is a continuum of balanced growth path equilibria indexed by the exchange rate E within a range $[\underline{E}, \bar{E}]$. Combining asset demand, supply and goods market conditions [Eqs. \(4\), \(5\), \(7\)](#) and [\(36\)](#) for a given exchange rate E , the output gaps satisfy the following system:

$$\frac{\theta + g}{\theta} \xi = \frac{y(1 + \lambda)}{1 + \lambda y} \left[gd + \left(1 + \frac{g\delta}{\rho} \right) \xi \right] + \frac{1 - y}{1 + \lambda^*(1 - y)} E \left[gd^* + \left(1 + \frac{g\delta^*}{\rho} \right) \xi^* \right] \quad (41a)$$

$$\frac{\theta + g}{\theta} E \xi^* = \frac{y}{1 + \lambda y} \left[gd + \left(1 + \frac{g\delta}{\rho} \right) \xi \right] + \frac{(1 - y)(1 + \lambda^*)}{1 + \lambda^*(1 - y)} E \left[gd^* + \left(1 + \frac{g\delta^*}{\rho} \right) \xi^* \right]. \quad (41b)$$

This system boils down to [Eq. \(17\)](#) in the absence of tariffs, $\lambda = \lambda^* = 0$.¹³ Conditional on an exchange rate E , an increase in tariffs in Home increases Home's output, i.e. $\partial \xi / \partial \lambda > 0$, while decreasing Foreign's output: $\partial \xi^* / \partial \lambda^* < 0$. The intuition is simple: at the global ZLB asset prices and the exchange rate are

¹²The same expression implies that Foreign's external imbalances, as a fraction of foreign output, must become larger when Home tariffs increase. This is consistent with $NFA + E NFA^* = 0$ since the exchange rate appreciates.

¹³The range of indeterminacy $[\underline{E}, \bar{E}]$ is also constrained by tariff policy. \bar{E} is defined such that $\xi = 1$ in [Eq. \(41a\)](#), while \underline{E} is defined such that $\xi^* = 1$. As tariffs increase, this range shrinks and converges to E^a .

fixed. Hence Home tariffs, which tilt global demand towards the Home good, must reduce home slack at the expense of foreign slack. In that sense, trade wars, like currency wars, simply reallocate a global deficiency of aggregate demand without addressing the underlying cause, which lies in global financial markets.

Next, observe that global imbalances still satisfy [Eq. \(28\)](#) from [Proposition 4](#):

$$\frac{NFA}{\bar{Y}} = \frac{(1 - \frac{\delta\theta}{\rho})(\xi - \xi^{a,n})}{g + \theta}.$$

Consequently, at the ZLB tariffs always increase a country's net foreign asset position and current account, regardless of its autarky position, and deteriorate the net foreign position and current account of the rest of the world.

The following proposition extends [Proposition 2](#) and characterizes the BGP equilibrium away from the ZLB and at the ZLB in the presence of asymmetric tariffs. This is intuitive, since at the ZLB the effect of tariffs on aggregate output operates entirely via the reallocation of demand, with no effect on the underlying global financial conditions.

The monotonicity of the output gap with respect to tariffs suggests that countries face strong (local) incentives to increase their tariffs. If we identify a country's objective with minimizing the size of its own recession, a country's "best response" to any fixed tariff level by their neighbors is to increase their own tariffs, so long as they are not at potential output. In other words, countries have strong incentives to engage in 'trade wars' to mitigate their own recessions at the expense of their neighbors. This is a direct parallel to the currency wars result presented in [Section 3.1](#). The next proposition summarizes these results and characterizes what happens when the trade war logic is taken to an extreme.

Proposition 6 (Financial Integration with Asymmetric Tariffs). *Under financial integration and [Assumption 1](#), competitive equilibria along a BGP in the presence of asymmetric tariffs are as follows:*

- *If $r^{w,n} \geq 0$, then the global economy is away from the Zero Lower Bound, where $r^{w,n}$ and E jointly solve [Eqs. \(39\) and \(40a\)](#). There is a unique balanced growth path with positive interest rate $i^w = r^w = r^{w,n}$ and output at its potential level, $\xi = \xi^* = 1$. Tariffs have no effect on output but reduce global imbalances: $\partial\xi/\partial\lambda = 0$; $\partial|NFA|/\partial\lambda < 0$.*
- *If $r^{w,n} < 0$, then the global economy is at the ZLB. There is a continuum of balanced growth path equilibria with $i^w = r^w = 0$, indexed by $E \in [\underline{E}, \bar{E}]$, where ξ and ξ^* satisfy [Eq. \(41a\)](#). Tariffs reduce the output gap: $\partial\xi/\partial\lambda > 0$, and increase the net foreign asset position, $\partial NFA/\partial\lambda > 0$, at the expense of the rest of the world: $\partial\xi^*/\partial\lambda < 0$, $\partial NFA^*/\partial\lambda < 0$.*

- Suppose the world starts in a global liquidity trap at a fixed $E \in [\underline{E}, \bar{E}]$. Then, as $\lambda \rightarrow \infty$ and $\lambda^* \rightarrow \infty$ jointly, the world converges to a no-trade equilibrium where $\xi \rightarrow \xi^{a,n}$, $\xi^* \rightarrow \xi^{*a,n}$ and $E \rightarrow E^a$. Consequently, $NFA \rightarrow 0$ and $NFA^* \rightarrow 0$.

Proof. See text. □

4 Positive Policy Spillovers: Public Debt (Deficits) and Balanced-Budget Fiscal Expansions

Currency depreciations and tariffs are zero-sum (at best) because they do not address the key shortage of (safe) assets to store value, which lies behind the global liquidity trap. In contrast, public debt issuances (deficits) and balanced-budget fiscal expansions have the potential to generate positive spillovers. These two methods of expansionary fiscal policy reduce the net supply of safe assets via distinct channels: public debt issuances increase the supply of (safe) assets and balanced-budget fiscal expansion reduce the net demand for (safe) assets. Both have the potential to stimulate the economy by alleviating the global excess demand for financial assets and the corresponding global excess supply of goods in the ZLB equilibrium.

4.1 Public Debt (Deficits): The Net Asset Creation Channel

We first focus on public debt (deficits), assuming that there is no change in government spending. At the ZLB, public debt issuances can be financed without levying any extra taxes. Public debt is essentially a rational bubble. Public debt issuances increase the global (safe) asset supply, and stimulate global output, thereby generating positive spillovers.

In the interest of space, we only consider the case of a global liquidity trap. From [Proposition 2](#), [Eq. \(17\)](#), it is immediate that for a given exchange rate E , an increase in public debt at Home (D) or in Foreign (D^*) increases world net asset supply and reduces the world asset shortage. As a result, home and foreign outputs ξ and ξ^* increase proportionately. It does not matter whether the increase in public debt originates at Home or at Foreign: an increase in public debt anywhere is expansionary everywhere. From [Eq. \(30\)](#) an increase in debt at Home (D) decreases the Home Net Foreign Asset position and pushes the Home Current Account toward a deficit.

It is important to note that in a liquidity trap we have $r^w \leq g$ since $r^w = 0$ and $g \geq 0$. This implies that the government does not need to levy taxes to sustain debt, and in fact can afford to rebate some tax revenues to households. Fiscal capacity is therefore not a constraint on the use of debt as an instrument

to stimulate the economy. This stark conclusion rests on the assumption that the trap is permanent.¹⁴

Note also that at the ZLB, public debt and money are (at the margin) perfect substitute zero interest rate government liabilities. As a result, issuing government bonds and issuing money as a helicopter drop are equivalent at the ZLB. Hence all the results regarding the issuance of public debt at the ZLB apply *identically* to the issuance of money.¹⁵ Through the lens of the model, helicopter drops anywhere are expansionary everywhere.

4.2 Balanced-Budget Fiscal Expansions: the Net Asset Demand Channel

We now focus on a specific case of fiscal policy: balanced-budget increases in government spending. Because of this assumption, we shut down the (safe) asset creation mechanism of public debt (deficit) increases. Budget-balanced government spending has two components: contemporaneous government consumption spending and taxes on private income (which would have been partly saved). Thus, on net budget-balanced government spending reduces desired global savings and global (safe) asset demand. At the ZLB, this reduction in net asset demand stimulates output in all countries, thereby generating positive spillovers. This can also be seen in the goods market, where the reduction in net asset demand directly increases the demand for the goods that are consumed by the government, and indirectly increases the demand for all goods by increasing income via a Keynesian multiplier.

At Home, government spending on domestic goods $\gamma_G \bar{Y}$ is financed by increasing the tax τ on the income of newborns, $(1 - \delta) \xi \bar{Y}$, for a constant level of public debt D/\bar{Y} . The same applies to Foreign where government spending $\gamma_G^* \bar{Y}^*$ is financed by increasing the tax τ^* on the income of newborns, $(1 - \delta^*) \xi^* \bar{Y}^*$, for a constant level of public debt D^*/\bar{Y}^* . In the interest of space, we only consider the case of a global

¹⁴If the trap were only temporary, as in the model with exit in Online Appendix B.2, the results could be different, depending on whether the post-exit economy is dynamically efficient (i.e. $r^w < g$) or not. If the post-exit economy is dynamically inefficient, then the conclusion holds. But if the post-exit economy is dynamically efficient, then fiscal capacity is eventually required to service debt, constraining the use of debt issuance as a stimulus tool during the liquidity trap.

¹⁵In both cases, if the liquidity trap were temporary, and if the economy were dynamically efficient after exiting the trap, fiscal capacity would be needed either to soak up the extra money that was issued at the ZLB, or to service the government bonds that were issued at the ZLB.

liquidity trap. Following the same steps as in the baseline model, the BGP equilibrium satisfies:

$$\begin{aligned}
E &= \frac{\xi - \gamma_G}{\xi - \gamma_G^*}, \\
\xi &= \gamma_G + \frac{\theta \bar{d}(E) + \frac{\overline{\delta\gamma_G}(E)\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}}, \quad \xi = \gamma_G^* + \frac{1}{E} \frac{\theta \bar{d}(E) + \frac{\overline{\delta\gamma_G}(E)\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}}, \\
\frac{NFA}{\bar{Y}} &= \frac{\xi(1 - \frac{\delta\theta}{\rho}) - \gamma_G - \theta d}{g + \theta} = \frac{(1 - \frac{\delta\theta}{\rho})}{g + \theta} \left[\frac{\theta \bar{d}(E) + \frac{\overline{\delta\gamma_G}(E)\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} - \frac{\frac{\delta\gamma_G\theta}{\rho} + \theta d}{1 - \frac{\delta\theta}{\rho}} \right], \\
\frac{CA}{\bar{Y}} &= g \frac{NFA}{\bar{Y}},
\end{aligned}$$

where $\overline{\delta\gamma_G}(E) \equiv y\delta\gamma_G + (1 - y)\delta^*\gamma_G^*E$. These equations show that, given the exchange rate E , home government spending stimulates home output more than one-for-one, i.e. with a Keynesian government spending multiplier:

$$\frac{\partial \xi}{\partial \gamma_G} = 1 + \frac{y\delta\theta}{\rho - \bar{\delta}\theta} > 1,$$

while it stimulates foreign output but less so, with a Keynesian government spending multiplier of

$$\frac{\partial \xi^*}{\partial \gamma_G} = \frac{1}{E} \frac{y\delta\theta}{\rho - \bar{\delta}\theta} > 0.$$

These two effects are intuitive given that government spending not only increases the demand for home goods and reduces the asset demand arising from Home households, but also indirectly increases asset supply by stimulating home output. This explains why the domestic government spending multiplier is greater than one, and why the effect on foreign output is positive. Moreover, and for the same reason, home government spending reduces the home Net Foreign Asset position and pushes the home Current Account toward a deficit. Similar effects apply for foreign government spending.

All in all, fiscal policy—be it in the form of public debt issuances, helicopter drops of money, or budget-balanced increases in government spending—is a *positive-sum* remedy to a ZLB environment.

4.3 Public Debt and Currency Wars

Unfortunately, the incentives to expand public debt are further depressed by the possibility of currency wars. Indeed, [Proposition 5](#) shows that both a and a^* increase in \bar{d} : public debt issuances increase the efforts of each country to engage in a currency war to depreciate its currency. This in turn discourages each country from issuing more public debt.

That is, the possibility of a currency war reduces the domestic benefits from issuing public debt. Recall

that—at a constant exchange rate—issuing more public debt in one country raises output in all countries. This creates room for some coordination, as countries will not take into account the impact of their debt issuance on foreign output. However, there is a more pernicious effect: issuing more public debt in one country increases the incentives for foreign exchange intervention in the other country. The resulting appreciation of the Home currency reduces the expansion in home output, in favor of foreign. Hence, the possibility of a future currency war dilutes the domestic benefits from an expansion of public debt.¹⁶

5 A Model of the Diffusion of Safety Traps

So far, all assets have been safe in terms of their individual payoffs. Yet, from a *macroeconomic* perspective, public debt was safer than private assets since its payoff did not change with the size of the recession at the ZLB. We now extend our discussion by distinguishing between safe and risky private assets, and between public and private provision of safe assets. This extension is conceptually important for five reasons. First, what matters in our extended environment is not the overall scarcity of stores of value, but whether there is a *scarcity of safe assets*. Second, this scarcity of safe assets depresses the return on safe assets, relative to the expected return on risky assets: the risk premium increases. Third, our model uncovers a new form of the ‘reserve currency paradox’ described earlier: a country issuing a reserve currency, i.e. a currency expected to appreciate in bad times, faces lower safe real rates and can enter a safety trap earlier, or experience a larger recession in a global safety trap. Fourth, as before, the financial account plays a key role in transmitting economic shocks at the ZLB. However, the dimension of the financial account that matters is the net flow of safe assets. At the ZLB, countries that are net issuers of safe assets experience a worse recession, and vice-versa. Fifth, very naturally in our framework, net issuers of safe assets experience an ‘exorbitant privilege’ i.e. a high return on their (riskier) gross external assets relative to their (safer) gross external liabilities.

We present a simple extension of the model, building on [Caballero and Farhi \(2017\)](#), that carries most of the relevant intuition and establishes the results described above. In this extension, risk arises from the (vanishingly) small possibility of a disaster shock, modeled as a permanent drop in output in both countries. After the realization of this Poisson shock, all uncertainty is resolved: risk disappears and the

¹⁶In the model, countries benefit from a depreciation of their currency because prices are set in the producer’s currency. A large body of evidence indicates that international prices are instead set in a few dominant currencies ([Gopinath, Boz, Casas, Diez, Gourinchas and Plagborg-Møller, 2018](#)). In that case, a depreciation of the dominant currency may be expansionary in the dominant currency’s country and in the rest of the world. [Mukhin \(2017\)](#) develops this point in a New Keynesian model. His result holds in our model in the limit where all trade is invoiced in a single dominant currency. In practice, while dollar currency pricing is prevalent among emerging market economies, it is less common for developed economies. The potential for currency wars among developed countries is therefore stronger than the potential for currency wars between advanced and emerging countries.

economy behaves as in the benchmark model of [Section 2](#).

We focus on the case where the post-disaster economy stays away from the ZLB. Formally, this means that the post-shock world natural interest rate $r^{w,n}$ from [Eq. \(15\)](#) remains positive. This assumption allows us to isolate the role of the demand for, and supply of, *safe* assets in generating a stagnation equilibrium.

Prior to the shock, a fraction of (locally) infinitely risk-averse investors only hold safe assets. These assets consist of (default and risk-free) public debt—unaffected by the realization of the Poisson shocks—and private assets that are created by tranching and securitizing private risky assets. All remaining assets are held by risk neutral investors. These infinitely risk-averse investors capture a central notion of “fear,” i.e. extreme uncertainty that translates into an extreme reluctance to hold any risk. When the supply of safe assets, private or public, is insufficient to meet the demand from risk averse investors, the market segments and safe assets earn a safety premium relative to risky assets. As the demand for safe assets grows, or as the capacity of the global economy to produce them shrinks, this safety premium can become so large that it pushes the global economy into what [Caballero and Farhi \(2017\)](#) call a ‘safety trap.’

The safety trap of this section and the liquidity trap of [Section 2](#) are intimately related in the sense that both arise when interest rates cannot fall far enough to restore equilibrium in asset markets. There are, however, two important differences. First, exiting a *safety* trap requires an increase in the supply of, or a reduction in the demand for, *safe* assets, regardless of the overall supply of, or demand for, other assets. By contrast, exiting a liquidity trap only requires an increase in the net supply of assets, regardless of their risk characteristics. Second, safety traps can exist and persist even in environments with long-dated assets, as long as these assets remain risky: while the yield on safe assets decreases, risk-premia increase, which bounds the value of long-lived assets and reduces the associated wealth effects.

5.1 Assumptions and Constrained Symmetric Competitive Equilibrium

We modify the model of [Section 2](#) along the following lines.

Risk. A Poisson shock occurs with instantaneous probability λ . When the Poisson shock occurs, all uncertainty is resolved and output in both countries drops instantaneously and permanently by a factor $\mu < 1$. We assume that the natural interest rate after the Poisson shock remains positive: $r^{w,n} = -\rho + \theta\bar{\delta}/(1 - \theta\bar{d}/\mu) > 0$. However, the possibility of an adverse future shock depresses the world natural interest rate before the Poisson shock, which might be negative or positive. It follows that the economy may be at the ZLB *before* the Poisson shock, but never *after* it.¹⁷ For simplicity, we study the limit $\lambda \rightarrow 0$. Since some

¹⁷We have also built a version of the model where liquidity traps are possible after the Poisson shock, for example because of a deleveraging shock. This version of the model endogenizes μ as a recession ξ^μ brought about by the deleveraging liquidity trap after the Poisson shock. See [Section 5.5](#) for a detailed discussion. We chose not to include this version of the model

agents are risk averse, the Poisson shock matters even in the limit where its intensity becomes vanishingly small.

Heterogeneity in Risk Appetite: Neutrals and Knightians. Following the closed economy analysis in Caballero and Farhi (2017), we allow for a fraction α of savers in each country to be ‘Knightians,’ i.e. infinitely locally risk-averse agents. The remaining fraction $1 - \alpha$ of savers are ‘Neutrals,’ i.e. risk-neutral agents as in the benchmark model. We assume that Knightians have full home bias: they only consume the goods of their own country. This implies that domestic Knightians only value financial assets whose payoffs are constant in the home good numeraire. By contrast, Neutrals have no home bias. Formally, the preferences of Home Knightians and Neutrals are given by the following stochastic differential equations:

$$\begin{aligned} U_t^K &= 1_{\{t-dt \leq \tau_\theta < t\}} c_{H,t} + 1_{\{t \leq \tau_\theta\}} \min_t \{U_{t+dt}^K\}, \\ U_t^N &= 1_{\{t-dt \leq \tau_\theta < t\}} c_{H,t}^\gamma c_{F,t}^{1-\gamma} + 1_{\{t \leq \tau_\theta\}} \mathbb{E}_t[U_{t+dt}^N]. \end{aligned}$$

Home Neutral and Knightian savers receive the same income $(1 - \delta) \xi_t \bar{Y}_t$ at birth, and save it until the time of death by investing in different portfolios. We make two further assumptions that simplify the analysis but do not matter for our substantive results: Neutral newborns pay all taxes and receive all new trees. The foreign country has a similar setup.

Private Safe Assets: Tranching and securitization. At any point in time, Knightian savers will only invest their wealth in domestic *safe assets*. These safe assets come in two varieties: public debt, which we assume is default free, and private safe assets. We now describe the supply of these private safe assets.

First, we simplify the analysis by assuming that $\delta = \delta^*$, so that there are no differences across countries in the ability to pledge future output into current assets. Instead, the difference will come from their ability to manufacture safe assets from risky ones. At any point in time in each country, a fraction of the existing Lucas trees can be arbitrarily tranced into Arrow-Debreu securities that can then be arbitrarily traded and recombined. These Arrow-Debreu securities cannot be sold short. The remaining fraction of the trees can only be traded as a whole. Countries differ in their ability to generate tranced trees, and we denote $0 \leq \phi \leq 1$ (resp. $0 \leq \phi^* \leq 1$) the fraction of tranced trees at Home (resp. in Foreign). As we shall see, private safe assets can be synthesized using the right mix of Arrow-Debreu securities. Equilibrium in the asset markets requires that Neutral savers hold all remaining assets: all the untranced trees as well as the remaining public debt and Arrow-Debreu securities originating from tranced private assets and not held

because it is more complex and less connected to the baseline model.

by Knightians.

It is convenient to classify combinations of Arrow-Debreu securities into two categories. We use the term “macro puts” to denote combinations of Arrow-Debreu securities that pay zero dividends until the Poisson shock realizes, and positive dividends after. Similarly, we use the term “macro calls” to denote Arrow-Debreu securities that only pay positive dividends before the Poisson shock realizes, but zero dividends after. It is immediate that the right combination of macro puts and macro calls on local trees will deliver a riskless payoff in terms of the local good, regardless of the occurrence of the Poisson shock.

Monetary policy and ZLB. By analogy with [Section 2](#), Home monetary policy follows a truncated Taylor rule that can be summarized as

$$i_t = \max\{r_t^{K,n} + \pi_t, 0\} \text{ with } i_t = 0 \text{ whenever } \xi_t < 1, \quad (42)$$

where i_t is the home nominal interest rate and $r_t^{K,n}$ is the relevant *natural real risk-free interest rate* at Home, defined as the risk-free real interest rate that clears markets when ignoring the ZLB constraint. The remaining assumptions are the same as in [Section 2](#).

Constrained Symmetric Competitive Equilibrium under Financial Integration. We focus throughout on constrained symmetric stochastic steady states under financial integration. We denote by r_t^K and r_t^{K*} the risk-free interest rates in the home and foreign numeraires; by r_t^w the risky rate of return, which is the same in the home and foreign numeraires since we are working in the limit $\lambda \rightarrow 0$; and by E_t and E_τ the value of the exchange rate before and immediately after the Poisson shock. We further denote by W_t^K and W_t^N the wealth of Home Knightians and Neutrals in the Home currency; by V_t and V_t^S the value of Home private assets prior to and after the Poisson shock in the Home currency; and by \hat{V}_t^S the value of Home macro puts in the Home currency. Similar definitions hold for Foreign.

First, along a BGP prior to the Poisson shock, the evolution of wealth for the two groups of Home savers satisfies (with similar equations for Foreign):

$$gW_t^K = \dot{W}_t^K = -\theta W_t^K + \alpha(1 - \delta)\xi_t \bar{Y}_t + r_t^K W_t^K, \quad (43a)$$

$$gW_t^N = \dot{W}_t^N = -\theta W_t^N + (1 - \tau)\alpha(1 - \delta)\xi_t \bar{Y}_t + r_t^w W_t^N + (\rho + g)V_t. \quad (43b)$$

Second, home private asset value satisfies (with a similar equation for Foreign)

$$V_t = \hat{V}_t^S + \frac{\delta \xi_t}{r_t^w + \rho} \bar{Y}_t. \quad (44)$$

To understand Eq. (44) observe that home private assets V_t are composed of home macro puts worth \hat{V}_t^S , home macro calls, and untranched home trees. Macro calls constitute claims to a future stream of dividends $\phi\delta\bar{Y}_t e^{-\rho(s-t)} ds$ in an interval ds until the Poisson shock realizes, and 0 afterwards. In the limit $\lambda \rightarrow 0$, they are worth $\phi\delta\bar{Y}_t/(r_t^w + \rho)$. By the same logic, untranched home trees constitute claims to a future stream of dividends $(1 - \phi)\delta\bar{Y}_t e^{-\rho(s-t)} ds$ in an interval ds until the Poisson shock realizes, and $(1 - \phi)\delta\mu\bar{Y}_t e^{-\rho(s-t)} ds$ afterwards. They are worth $(1 - \phi)\delta\bar{Y}_t/(r_t^w + \rho)$.¹⁸ Equation (44) follows.

Third, the value of Home macro puts \hat{V}_t^S satisfies

$$\hat{V}_t^S = \frac{r_t^w - r_t^K}{r_t^w} \phi V_t^S. \quad (45)$$

To understand Eq. (45) observe that Home macro puts can be combined with macro calls to create safe assets from the perspective of Home Knightians worth ϕV_t^S —i.e. a fraction ϕ of the value of home assets after the Poisson shock. The required macro calls represent a constant future stream of dividends $r_t^K \phi V_t^S ds$ in an interval ds until the Poisson shock realizes. They are worth $(r_t^K/r_t^w)\phi V_t^S$. The value of home macro puts is the residual $\phi V_t^S - (r_t^K/r_t^w)\phi V_t^S$. Eq. (45) follows.

Moreover, home macro puts can be combined with macro calls to create safe assets from the perspective of *foreign* Knightians worth $(E/E_\tau)\phi V_t^S$. The required macro calls are worth $(r_t^{K^*}/r_t^w)(E_t/E_\tau)\phi V_t^S$. The value of home macro puts is also given by $\hat{V}_t^S = \phi V_t^S(1 - r_t^{K^*}/r_t^w)(E_t/E_\tau)$. Equating the two expressions for the value of Home macro puts implies the following ‘modified UIP’ equation for the exchange rate:

$$\frac{r_t^w - r_t^K}{r_t^w - r_t^{K^*}} = \frac{E_t}{E_\tau}. \quad (46)$$

Eq. (46) indicates that the exchange rate of the country with the *lowest safe interest rate* r_t^K , or the *highest risk premium* $r_t^w - r_t^K$, *appreciates* following the realization of the Poisson shock. To gain some intuition for this ‘modified UIP’ relationship, suppose that the Home currency appreciates upon the realization of the Poisson shock ($E_t > E_\tau$). It follows that Home safe assets in the Home numeraire are also safe from the perspective of Foreign Knightians. Conversely, Foreign safe assets in Foreign’s numeraire are not safe from the perspective of Home Knightians (they depreciate). This tilts the demand for safe assets towards Home safe assets and away from Foreign safe assets: in equilibrium, Home safe assets must offer a lower return $r_t^K < r_t^{K^*}$, as indicated by Eq. (46).

Another way to interpret the ‘modified UIP’ condition is in terms of an *endogenous foreign exchange risk premium* ψ_t in the UIP equation Eq. (6) : $r_t^K = r_t^{K^*} + \psi_t + \dot{E}_t/E_t$. Prior to the Poisson shock along

¹⁸Because these trees cannot be tranched, they are held by Neutrals, who do not value the associated dividends after the Poisson shock realizes because the intensity of the Poisson process λ is vanishingly small.

the BGP, $\dot{E}_t/E_t = 0$. Substituting into Eq. (46), we obtain an expression for ψ_t ,

$$\psi_t = \left(\frac{E_\tau}{E_t} - 1 \right) (r_t^w - r_t^K), \quad (47)$$

that links the foreign exchange risk premium ψ_t and the risk premium $(r_t^w - r_t^K)$.

In general, we can have a safety trap in one country but not in the other. For example, Home can be in a safety trap with $r^K = 0$ and $\xi < 1$ while Foreign is not: $r^{K*} > 0$ and $\xi^* = 1$. Going back to our modified UIP equation Eq. (46), this requires that $E_t > E_\tau$, i.e. that the currency in Home appreciates when the Poisson shock occurs, with corresponding negative foreign exchange risk premium $\psi_t < 0$. This offers another interpretation of the “reserve currency paradox” discussed earlier. If the home currency is a reserve currency, i.e. a currency expected to appreciate in bad times, then the home risk free rate r^K is lower, and Home can be in a safety trap even if Foreign is not.

If the economy escapes the ZLB after the shock, $E_\tau = 1$ as prescribed by Proposition 2. We focus on *symmetric* equilibria both inside and outside the global safety trap such that $E_t = 1$ and $\xi_t = \xi_t^*$ prior to the shock. It follows from Eq. (46) that $r_t^K = r_t^{K*} = r_t^{K,w}$. Equilibrium in asset markets after the Poisson shock then imposes that $V_t^S = (1 - \theta\bar{d}/\mu)\mu\bar{Y}_t/\theta$.¹⁹

Fourth, along the BGP government debt dynamics can be expressed as

$$(r_t^K - g) d = \tau_t (1 - \alpha) (1 - \delta) \xi_t, \quad (48a)$$

$$(r_t^{K*} - g) d^* = \tau_t^* (1 - \alpha) (1 - \delta) \xi_t^*. \quad (48b)$$

Fifth, market clearing conditions for home and foreign goods require

$$W_t^K + y (W_t^N + E_t W_t^{N*}) = \xi_t \frac{\bar{Y}_t}{\theta}, \quad (49a)$$

$$E_t W_t^{K*} + (1 - y) (W_t^N + E_t W_t^{N*}) = E_t \xi_t^* \frac{\bar{Y}_t^*}{\theta}. \quad (49b)$$

To understand the first expression, recall that Home Knightians have full home bias, while Home and Foreign Neutrals spend a share y of expenditures on the Home good.

¹⁹To see this, observe that global asset demand after the shock is $\mu\bar{Y}^w/\theta$ while global asset supply is $V_t^{S,w} + \bar{d}\bar{Y}^w$. By symmetry of the equilibrium, $V_t^S/\bar{Y}_t = V_t^{S*}/\bar{Y}_t^*$. The result follows.

Finally, asset market clearing requires

$$W_t^w = W_t^K + W_t^N + W_t^{K*} + W_t^{N*} = V_t + V_t^* + D_t + D_t^* = V_t^w + D_t^w, \quad (50a)$$

$$W_t^K + W_t^{K*} \leq \phi V_t^S + \phi^* V_t^{S*} + d\bar{Y}_t + d^*\bar{Y}_t^*. \quad (50b)$$

The first equation states that global asset demand equals global asset supply. The second equation states that Knightians' wealth must be smaller than the payoff from safe public and private assets in the event of a Poisson shock. As discussed in [Caballero and Farhi \(2017\)](#), there are different regimes depending on whether [Eq. \(50b\)](#) holds as a strict inequality or as an equality. If [Eq. \(50b\)](#) holds as a strict inequality, the marginal holder of safe assets is a Neutral investor and there are no risk premia: $r_t^K = r_t^w$ (unconstrained regime). In this regime, there is no scarcity of safe assets. In the second case, the marginal holder of safe assets is a Knightian, [Eq. \(50b\)](#) holds with equality and there is a risk premium: $r_t^w > r_t^K$ (constrained regime). We assume throughout that we are in the constrained regime, which occurs when α is large enough (so that the demand for safe assets is high enough), or when μ , ϕ or \bar{d} are small enough (so that the supply of safe assets is small). We define a constrained symmetric competitive equilibrium under financial integration.

Definition 2. *Constrained Symmetric Competitive Equilibrium under Financial Integration*

Given paths for the ratio of public debt to potential output, d_t and d_t^ , a constrained competitive symmetric equilibrium under financial integration prior to the Poisson process consists of sequences for output gaps ξ_t and ξ_t^* , natural risk-free rates $r_t^{K,n}$ and $r_t^{K,n*}$, household wealth for Knightians and Neutrals W_t^K , W_t^N , W_t^{K*} and W_t^{N*} , private financial assets V_t and V_t^* , macro puts V_t^S and V_t^{S*} , taxes τ_t and τ_t^* , policy rates i_t and i_t^* , risky real return r_t^w , and nominal exchange rate E_t , such that (i) household wealth and private assets satisfy equations [Eq. \(43\)](#) [Eq. \(44\)](#), and [Eq. \(45\)](#); (ii) debt dynamics follow [Eq. \(48\)](#); (iii) policy rates are set according to [Eq. \(42\)](#); (iv) goods markets clear [Eq. \(49\)](#); (v) global asset markets clear [Eq. \(50a\)](#); (vi) the equilibrium is constrained, so that [Eq. \(50b\)](#) holds as an equality; and (vii) the equilibrium is symmetric, so that $E_t = 1$, $\xi_t = \xi_t^*$, $r_t^K = r_t^{K*}$.*

5.2 The Diffusion of Safety Traps

We can now characterize the constrained symmetric competitive equilibria. First, along a symmetric BGP the exchange rate is constant $E = 1$, so real and nominal risk-free rates are constant and equated across countries: $r^K = r^{K*} = r^{K,w}$ while $\xi = \xi^*$ at the ZLB. We make the following assumption on the parameters of the problem.

Assumption 2.

$$\mu > \theta \bar{d} \quad ; \quad \alpha > (\mu - \theta \bar{d}) \bar{\phi} + \theta \bar{d},$$

where $\bar{\phi} = y\phi + (1 - y)\phi^*$ is the global tranching capacity.

The first part of [Assumption 2](#) ensures that the supply of public assets remains limited after the Poisson shock. It is necessary for $r^{w,n} > 0$ after the Poisson shock, as we assumed. The second part ensures that the competitive equilibrium is constrained, i.e. that the demand for safe assets (controlled by α) exceeds the supply (controlled by μ , $\bar{\phi}$ and \bar{d}).

Safety Trap Equilibria. From the constrained safe asset market condition [Eq. \(50b\)](#), we obtain

$$W^{K,w} = [(\mu - \theta \bar{d}) \bar{\phi} + \theta \bar{d}] \frac{\bar{Y}^w}{\theta}. \quad (51)$$

This equation indicates how the market value of safe assets varies with the supply of private safe assets (controlled by $\bar{\phi}$ and μ) and the supply of public safe assets (controlled by \bar{d}).

Next, from the wealth accumulation equation for Knightians [Eq. \(43a\)](#), we obtain

$$W^{K,w} = \frac{\alpha(1 - \delta) \xi \bar{Y}^w}{g + \theta - r^{K,w}}. \quad (52)$$

Combining these two expressions, we obtain an expression for the natural risk free rate $r^{K,w,n}$ and the natural output gap $\xi^{w,n}$:

$$r^{K,w,n} = g + \delta\theta - (1 - \delta)\theta \frac{\alpha - ((\mu - \theta \bar{d}) \bar{\phi} + \theta \bar{d})}{(\mu - \theta \bar{d}) \bar{\phi} + \theta \bar{d}}, \quad (53a)$$

$$\xi^{w,n} = \frac{g + \theta}{(1 - \delta)\theta} \frac{(\mu - \theta \bar{d}) \bar{\phi} + \theta \bar{d}}{\alpha}. \quad (53b)$$

When $r^{K,w,n} \geq 0$, the economy avoids the safety trap and $\xi = 1$. When instead the natural risk free rate becomes negative, the global economy experiences a recession, $\xi = \xi^{w,n}$. [Eq. \(53a\)](#) reveals that whether a safety trap occurs is determined entirely by the demand for and supply of *safe assets*. The tighter the scarcity of safe assets (as measured by the gap between α and $(\mu - \theta \bar{d}) \bar{\phi} + \theta \bar{d}$), the lower the risk-free rate. A safety trap obtains when this scarcity is so acute that the risk free rate reaches the ZLB. This may occur even if the overall supply of assets is sufficient to avoid a liquidity trap when the regime is unconstrained.

Risk Premium. Next, we solve for the risky return r^w . We first use the market clearing conditions [Eq. \(49\)](#) and the global asset market equilibrium [Eq. \(50a\)](#) to obtain total asset demand W^w and private

asset supply V^w :

$$W^w = \xi \frac{\bar{Y}^w}{\theta} = V^w + \bar{d}\bar{Y}^w. \quad (54)$$

Replacing V^w in Eq. (54) using Eq. (44) and Eq. (45), we obtain

$$\xi \left(1 - \frac{\delta\theta}{r^w + \rho} \right) = \frac{r^w - r^K}{r^w} \bar{\phi} (\mu - \theta\bar{d}) + \theta\bar{d}. \quad (55)$$

This implicitly defines r^w . Since $r^w \geq r^K$ and $\xi \leq 1$, it is easy to check that $r^w > -\rho + \delta\theta/(1 - \theta\bar{d})$, i.e. the risky return is higher than the autarky natural rate in the benchmark model of Section 2. The scarcity of safe assets increases the risk premium $r^w - r^{K,w}$. Everything else equal, r^w is decreasing in ξ so that a deeper safety trap is associated with higher risk premia.

Net Foreign Assets, Current Accounts and ‘Exorbitant Privilege’. We can express Home’s Net Foreign Asset position independent of whether the global economy is in a safety trap equilibrium. After some simple manipulations, the Net Foreign Asset position can be expressed as

$$\frac{NFA}{\bar{Y}} = -\frac{\theta - (r^w - r^{K,w})}{\theta + g - r^w} (d - \bar{d}) - \frac{\theta - r^w - \rho}{\theta + g - r^w} \frac{r^w - r^{K,w}}{\theta r^w} [\phi(\mu - \theta d) - \bar{\phi}(\mu - \theta\bar{d})], \quad (56a)$$

$$\frac{CA}{\bar{Y}} = g \frac{NFA}{\bar{Y}}. \quad (56b)$$

Suppose that Home has more securitization capacity than Foreign ($\phi > \phi^*$) while $d = d^* = \bar{d}$. In this case, Home runs a negative Net Foreign Asset position and a Current Account deficit as long as $r^w + \rho - \theta < 0$, which is automatically verified in equilibrium.²⁰ Home’s larger securitization capacity increases the value of its assets: it is able to tranche out more “expensive” safe assets (high price, low rate of return) from its risky assets than Foreign. Therefore, Home experiences a version of the “exorbitant privilege” documented by Gourinchas and Rey (2007): It is able to run a permanent negative Net Foreign Asset position and Current Account deficit because it pays a lower interest rate on its liabilities than on its assets. A similar effect arises if Home issues more debt than Foreign ($d > \bar{d}$).

Gross Capital Flows and Metzler Diagram in Safe Assets. Both outside of a safety trap (when $r^{K,w,n} > 0$) and in a symmetric safety trap (when $r^{K,w,n} < 0$), we can represent the equilibrium determi-

²⁰There are two opposing effects of a larger securitization capacity. First, it increases the value of home assets. Second, the wealth of home agents accumulates faster because of the larger value of new securitized trees. The first effect worsens the Net Foreign Asset position and Current Account, while the second effect improves them. Since the strength of the second effect depends negatively on the propensity to consume θ and positively on the rate of depreciation ρ , the condition $r^w < \theta - \rho$ essentially bounds the strength of the second effect and guarantees that the first effect dominates. This condition is automatically verified in equilibrium.

nation of the safe interest rate $r^{K,w}$ and of the recession ξ through a *Metzler diagram in safe assets*. The key is to focus on the safe asset component CA^K of the Current Account and the corresponding safe asset Component of the Net Foreign Asset position NFA^K :

$$\begin{aligned}\frac{NFA^K}{\bar{Y}} &= \frac{\alpha(1-\delta)\xi^w}{\theta + g - r^{K,w}} - \frac{\phi(\mu - \theta d) + \theta d}{\theta}, \\ \frac{CA^K}{\bar{Y}} &= g \frac{NFA^K}{\bar{Y}}.\end{aligned}$$

The Metzler diagram in safe assets implies that in the global equilibrium, countries that are net suppliers of safe assets experience a larger recession than under financial autarky.

We summarize these results in the following proposition.

Proposition 7 (Constrained Symmetric Competitive Equilibrium, Safety Traps, Risk Premia, Net Foreign Assets and Current Accounts under Financial Integration). *Under Assumption 2, the constrained symmetric competitive equilibrium is such that:*

- *The global economy satisfies $r^{K,w} = \max(r^{K,w,n}, 0)$, $\xi = \min(\xi^{w,n}, 1)$ and $E = 1$.*
 - *If $r^{K,w,n} \geq 0$ then $\xi^{w,n} \geq 1$ and the global economy is outside the Safety Trap. There is a unique constrained symmetric balanced growth path equilibrium with a positive risk free interest rate and output at its potential, $\xi = 1$;*
 - *If $r^{K,w,n} < 0$ then $\xi^{w,n} < 1$ and the global economy is in a Safety Trap, there is a unique constrained symmetric balanced growth path equilibrium with $r^{K,w} = 0$ and $\xi = \xi^{w,n} < 1$.*
- *Outside or inside the Safety Trap, there is a positive risk premium $r^w - r^{K,w}$, decreasing in the size of the global recession ξ .*
- *If Home has a larger capacity to produce public ($d > \bar{d}$) or private ($\phi > \bar{\phi}$) safe assets, it is a net creditor, runs a current account surplus, and enjoys a version of the ‘exorbitant privilege’, paying lower interest rates on its (safe) liabilities than on its (riskier) assets.*
- *In the global equilibrium, countries that are net suppliers of safe assets experience a larger recession than under financial autarky.*

Proof. See text. □

5.3 Policies to escape Safety Traps

We now briefly consider two policies to address a safety trap: debt issuance and quantitative easing. We focus on these policies because their mechanisms differ in safety traps and in liquidity traps. By contrast, trade wars work in similar ways in both cases.²¹

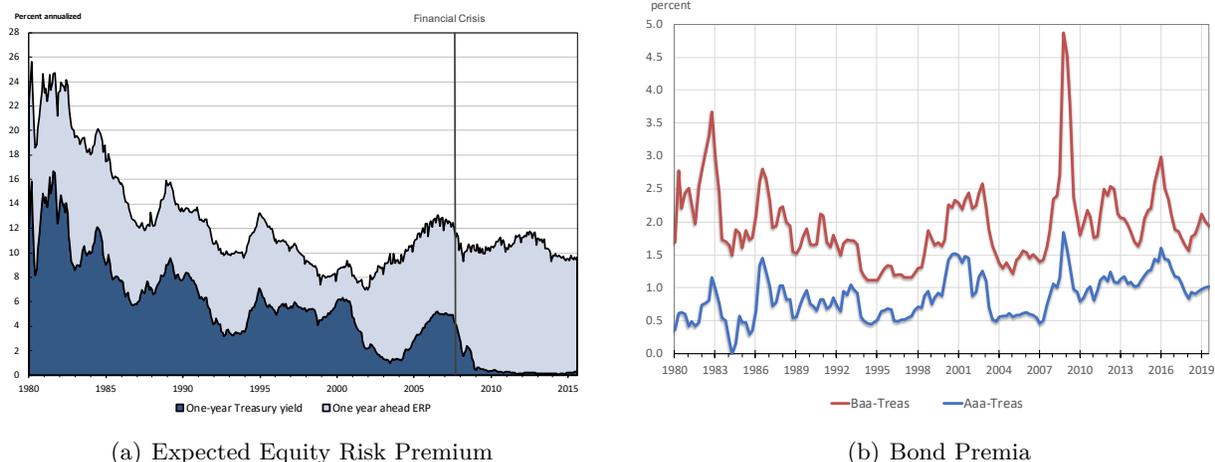
Debt Issuance and Fiscal Capacity. Assume that the global economy is experiencing a global safety trap with $r^{K,w,n} < 0$. Before the Poisson shock, fiscal capacity is irrelevant since $r^{K,w} - g = -g < 0$. It might therefore be tempting to increase public debt d in order to escape the safety trap, as in [Section 4.1](#). However, fiscal capacity still matters after the Poisson shock if $r^{K,w} = r^w > g$ and taxes are needed to stabilize the debt. Imagine an initial situation where the foreign country is at its fiscal capacity after the Poisson shock, defined as the maximum level of debt achievable at the highest sustainable tax rate. Suppose that the foreign country nonetheless decides to increase its debt before the Poisson shock. Any additional increase in debt is effectively risky: it does not increase its payoff after the Poisson shock. In turn, Knightians will not be willing to hold the increase in public debt: debt issuance would have no effect on ξ prior to the Poisson shock. The payoff to an increase in foreign debt after the Poisson shock could shrink if the increase in debt causes the foreign country to default on its debt, as could happen with a fixed cost of defaulting. This would reduce ξ . It is the fiscal capacity of the country *after* it exits the safety trap that matters for the issuance of public debt during a safety trap.

Quantitative Easing. Consider a government purchase of risky assets (whether macro calls or untranching trees) funded by issuing public debt. The effect is the same as issuing debt, with one additional advantage: the assets on the government balance sheet can be sold after the Poisson shock to help pay down the debt, if fiscal capacity is stretched. Government purchases of safe assets (Poisson puts) are ineffective since they swap private safe assets for public safe assets.

5.4 Risk Premia and Safe Asset Imbalances

Our safety trap extension makes two important predictions ([Proposition 7](#)): risk premia $r^w - r^{K,w}$ increase at the ZLB with the size of the recession (a lower ξ), and net suppliers of safe assets experience a disproportionate share of global stagnation. We briefly review the empirical evidence on these two predictions. Panel (a) of [Fig. 6](#) shows the [Duarte and Rosa \(2015\)](#) estimate of the expected return to U.S. equities along

²¹Currency wars also work in the same way, but they are easier to analyze in the version of the model with a liquidity trap after the Poisson shock discussed in [Section 5.5](#), which recovers the exchange rate indeterminacy of the baseline model. In the absence of liquidity traps after the Poisson shock, the exchange rate is determinate, and currency wars would have to involve forms of forward guidance involving commitments to low rates after the Poisson shock.



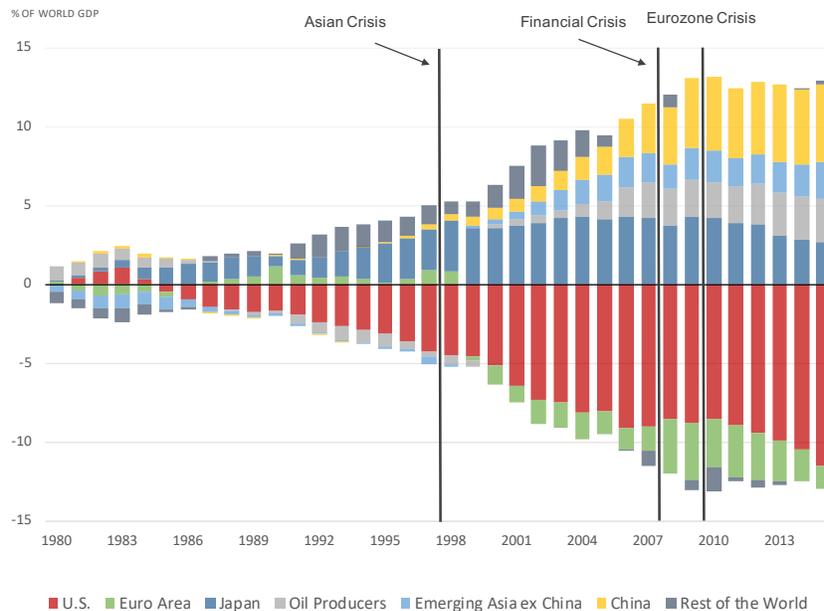
Note: Panel (a) shows the one-year US Treasury yield (dark area) and the one-year expected risk premium (ERP) (grey area), calculated as the first principal component of 20 models of the one-year-ahead equity risk premium. The figure shows that the equity risk premium has increased, especially since the Global Financial Crisis. Source: One-year Treasury yield: Federal Reserve H.15; ERP: Duarte and Rosa (2015). Panel (b) shows the spread between Moody's Aaa and Baa seasoned corporate bond yields and the 20-year constant maturity Treasury. The Baa and Aaa spreads have increased since the 2008 financial crisis. Source: Fred.

Figure 6: Bond and Equity Premia, 1980-2019

with the yield on one-year Treasuries. The difference between the two lines (light blue area) represents an estimate of the one-year expected equity risk premium (ERP). The figure illustrates how the decline in safe interest rates has not been matched by a decline in expected equity return, i.e. the risk premium has increased dramatically. Panel (b) reports two estimates of corporate bond risk premia: the Baa and Aaa spread over a 20-year Treasury yield. This figure also indicates a gradual increase in bond risk premia, especially after the financial crisis of 2008, a point first made by Negro, Giannone, Giannoni and Tambalotti (2017). Krishnamurthy (2019) further observes that bond risk premia are even more elevated when one takes into account the decline in volatility and default risk: because volatility has substantially decreased, default risk within a rating class, for example Baa, has decreased; a given credit spread for a given rating class is therefore indicative of a greater price of risk now than in the past. Fixing the riskiness of a bond instead of its rating would therefore result in more rapidly increasing spreads and risk premia over time.

Finally, Fig. 7 reports a simple estimate of the Net Foreign Asset position in safe assets, NFA^K , relative to world GDP from 1980 to 2015. The Net Foreign Asset position in safe assets is constructed from Lane and Milesi-Ferretti (2018)'s update to their External Wealth of Nations dataset as the sum of Official Reserves (minus Gold holdings), Portfolio Debt and Other Assets, minus Portfolio Debt and Other Liabilities.²² The figure shows that the net supply of safe assets originates largely with the U.S. and—to

²²This is a crude estimate of net safe asset positions since neither portfolio debt assets and liabilities nor other assets and liabilities (mostly cross border bank loans) need be safe. Nevertheless, these holdings can be considered *safer* than portfolio



Note: The graph shows Net Safe positions as a fraction of world GDP. Net Safe positions are defined as the sum of Official Reserves (minus Gold), Portfolio Debt and Other Assets, minus Portfolio Debt and Other Liabilities. The net supply of safe assets originates largely with the US and—to a smaller extent—the Eurozone. Source: Lane and Milesi-Ferretti (2018) update of the External Wealth of Nations. Oil Producers: Bahrain, Canada, Iran, Iraq, Kuwait, Libya, Mexico, Nigeria, Norway, Oman, Russia, Saudi Arabia, United Arab Emirates, Venezuela; Emerging Asia ex-China: India, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, Thailand, Vietnam.

Figure 7: Net Safe Asset Imbalances, 1980-2015

a smaller extent—the Eurozone. In 2015, the U.S. net supply of safe assets accounted for 11.5% of world GDP, up from 5% in 2000, while the Eurozone net supply accounted for 1.5% of world GDP. On the net demand side, we observe a large increase from China, mostly in the form of Official Reserves, from 0.7% of world GDP in 2000 to 4.9% in 2015; a large increase from oil producers, from 0.24% in 2000 to 2.70% in 2015; and a continued large absorption from Japan (around 2.7% of world GDP). This figure differs substantially from Fig. 1. It indicates that the U.S. external safe asset imbalances have been increasing over time, unlike global imbalances which have stabilized.²³

5.5 Liquidity Trap after the Poisson Shock

Finally, we offer some remarks for the case where the global economy experiences a liquidity trap after the Poisson shock. This occurs if $r^{w,n} < 0$. Because we want the ZLB equilibrium prior to the Poisson shock to be driven by safe asset scarcity—and not by a general lack of stores of value—we consider a slightly equity and direct investment.

²³Figure Fig. 1 reports flows (current accounts), while Fig. 7 reports stocks (net safe asset holdings). This is mostly because it is easier to construct estimates of the Net Foreign Asset position in safe assets NFA^K than the corresponding Current Account CA^K .

different version of the model where the Poisson shock results in a decline in the marginal propensity to consume from θ to $\theta^\mu < \theta$.²⁴ In the interest of brevity, we qualitatively describe the main results. First, because the economy remains in a liquidity trap, the exchange rate post Poisson E^μ is indeterminate within some range. In a symmetric equilibrium the exchange rate before the Poisson shock will remain constant: $E = E^\mu$. Next, the intensity of the scarcity of safe assets determines if the economy is in a Safety Trap. When it is, as in the benchmark model, the exchange rate determines the relative output levels in Home and Foreign. A cheaper exchange rate translates into higher output and a larger Net Foreign Asset Position. A depreciation of the currency increases the supply of (foreign) safe assets in Home, lifting its output. As in the benchmark model, we recover the currency and trade wars result. As in that model, debt issuances are not an issue since $r^K - g$ is always negative: the government can always issue more safe assets and does not face any fiscal capacity constraint.

6 Final Remarks

World interest rates and global imbalances go hand in hand: Countries with large safe asset shortages run Current Account surpluses and push the world interest rate down. At the ZLB, the global asset market remains in disequilibrium when output is at its potential: the resulting global (safe) asset shortage cannot be resolved by lower world interest rates. It is instead alleviated by a world recession, which is propagated by global imbalances: Current Account surplus countries push world output down, exerting a negative effect on the world economy. Economic policy becomes more interconnected across the world, with either negative or positive spillovers depending on the policy instrument. Exchange rate policy becomes a zero-sum game of currency wars where each country can depreciate its exchange rate to stimulate its economy, at the expense of other countries. The same logic and incentives hold for tariffs on foreign goods. In contrast, safe public debt issuances, increases in government spending, and support for private securitization are positive-sum and stimulate output in all countries. Our Metzler Diagram in Quantities is a powerful new tool for illustrating the economics of global imbalances and economic wars at the ZLB. Safe asset shortages also push the global economy into a global downturn, a global safety trap. This depresses the return on safe assets relative to risky ones as risk premia increase. Reserve currency countries can be pushed into a safety trap earlier, or experience a larger recession. Finally, recessions propagate through the safe component of the financial account. Net Safe Asset issuers experience a disproportionate share of the global recession.

²⁴In our earlier model, it is possible to generate a liquidity trap after the shock if $r^{w,n} = -\rho + \theta\delta/(1 - \theta\bar{d}/\mu) < 0$. However, in that case it is immediate that the unconstrained regime also experiences a liquidity trap before the Poisson shock, i.e. $-\rho + \theta\delta/(1 - \theta\bar{d}) < 0$. To isolate the effect of safe asset scarcity, we consider instead an environment where $-\rho + \theta\delta/(1 - \theta\bar{d}) > 0$ while $-\rho + \theta^\mu\delta/(1 - \theta^\mu\bar{d}) < 0$.

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A Appendix

A.1 New-Keynesian Microfoundations for Section 2.1

We provide one possible exact microfoundation for demand determined output in the presence of nominal rigidities in the model in Section 2.1. The microfoundation is in the New Keynesian tradition. We focus on the case of Home (the case of Foreign is identical). In a nutshell, domestic monopolistic firms produce imperfectly substitutable varieties of home intermediate goods and compete in prices. The firms' posted prices are rigid in the home currency, and that they accommodate demand at the posted price. The different varieties of the home intermediate goods are combined into a home final good by a competitive sector according to a Dixit Stiglitz aggregator. These assumptions are standard in the New Keynesian literature starting with [Blanchard and Kiyotaki \(1987\)](#).

Between t and $t + dt$, there is an endowment $\bar{Y}_t dt$ of each differentiated variety $i \in [0, 1]$ of non-traded input. Each variety i of non-traded input can be transformed into one unit of variety i of home intermediate good using a one-to-one linear technology by a monopolistic firm indexed by i which is owned and operated by the agents supplying variety i of the non-traded input, in proportion to their holdings of non-traded inputs.

The differentiated varieties of final home intermediate goods are then combined together into a home final good by a competitive sector according to a standard Dixit-Stiglitz aggregator

$$Y_t = \left(\int_0^1 Y_{H,i,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} dt,$$

where $Y_{H,i,t} dt$ is the quantity of variety i of the final good. The price of the final home good is

$$P_{H,t} = \left(\int_0^1 p_{H,i,t}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}},$$

where $p_{H,i,t}$ is the home currency price posted by monopolistic firm i for variety i of the home intermediate good. The resulting individual demand for each variety is given by

$$Y_{H,i,t} dt = \left(\frac{p_{H,i,t}}{P_{H,t}} \right)^{-\sigma} Y_t dt.$$

Prices set by monopolistic firms are perfectly rigid in the home currency, equal to each other. We normalize these prices in the home currency to one

$$p_{H,i,t} = P_{H,t} = 1.$$

All the varieties of intermediate goods are then produced in the same amount

$$Y_{H,i,t} dt = Y_t dt = \xi_t \bar{Y}_t dt.$$

Between t and $t + dt$, the varieties of non-traded inputs indexed by $i \in [\delta, 1]$, are distributed equally to the different agents who are born during that interval of time. The varieties of non-traded inputs indexed by $i \in [0, \delta]$ accrue equally as dividends on the different Lucas trees.

Real income (equal to real output) $Y_t dt$ is divided into an endowment $(1 - \delta)Y_t dt$ distributed equally to agents who are born during that interval of time, and the dividend $\delta Y_t dt$ of the Lucas trees. This provides an exact microfoundation for the model presented in Section 2.1.

A.2 Inflation

So far, we have assumed that prices are fully rigid. In this section, we relax this assumption and allow for *some* price adjustment through a Phillips curve.

This extension gives us the opportunity to reiterate some well-known insights about the economics of liquidity traps, and to obtain some new ones. The former are that credibly higher inflation targets reduce the severity of a liquidity trap, that more (downward) price flexibility can exacerbate the severity of the trap as the economy may fall into a deflationary spiral. The less known one is that in a *global* liquidity trap, it is the *more rigid* country that experiences the worst trap (note the contrast between this relative rigidity and the aggregate rigidity implication).

Moreover, it is now possible for some regions of the world to escape the liquidity trap if their inflation expectations are sufficiently high.

A.2.1 Extending the Model

Phillips curve. We wish to capture the idea that wages, or prices, are rigid downwards, but not upwards. We follow the literature and assume that prices and wages cannot fall faster than a certain limit pace, perhaps determined by a “social norm” and that this limit pace is faster if there is more slack in the economy:²⁵

$$\begin{aligned}\pi_{H,t} &\geq -\kappa_0 - \kappa_1(1 - \xi_t), \\ \pi_{F,t}^* &\geq -\kappa_0^* - \kappa_1^*(1 - \xi_t^*),\end{aligned}$$

where $\pi_{H,t} = \dot{P}_{H,t}/P_{H,t}$ (resp. $\pi_{F,t}^* = \dot{P}_{F,t}^*/P_{F,t}^*$) denotes the *domestic* (resp. foreign) inflation rate, and where $\kappa_1 \geq 0$ and $\kappa_1^* \geq 0$. Moreover, we assume that if there is slack in the economy, prices or wages fall as fast as they can: $\xi_t < 1$ implies that $\pi_{H,t} = -\kappa_0 - \kappa_1(1 - \xi_t)$ and $\xi_t^* < 1$ implies that $\pi_{F,t}^* = -\kappa_0^* - \kappa_1^*(1 - \xi_t^*)$. We capture this requirement with the complementary slackness conditions $[\pi_{H,t} + \kappa_0 + \kappa_1(1 - \xi_t)](1 - \xi_t) = 0$ and $[\pi_{F,t}^* + \kappa_0^* + \kappa_1^*(1 - \xi_t^*)](1 - \xi_t^*) = 0$.

To summarize, there are two Phillips curves, one for Home and one for Foreign. The home Phillips curve traces out an increasing curve in the $(\pi_{H,t}, \xi_t)$ space, which becomes vertical at $\xi_t = 1$. The foreign Phillips curve is similar.

Monetary policy. We assume that monetary policy is conducted according to simple truncated Taylor rules, where the nominal interest rate responds to domestic inflation:

$$\begin{aligned}i_t &= \max\{r_t^n + \bar{\pi} + \psi_\pi(\pi_{H,t} - \bar{\pi}), 0\}, \\ i_t^* &= \max\{r_t^{n*} + \bar{\pi}^* + \psi_\pi^*(\pi_{F,t}^* - \bar{\pi}^*), 0\}.\end{aligned}$$

In these equations r_t^n and r_t^{n*} are the relevant natural interest rates at Home and in Foreign, which depend on whether we analyze the financial integration equilibrium or the financial autarky equilibrium. We denote by $\bar{\pi} \geq \max\{-\kappa_0, 0\}$ and $\bar{\pi}^* \geq \max\{-\kappa_0^*, 0\}$ the home and foreign inflation targets, and $\psi_\pi > 1$ and $\psi_\pi^* > 1$ are Taylor rule coefficients.

For simplicity, we take the limit of large Taylor rule coefficients $\psi_\pi \rightarrow \infty$ and $\psi_\pi^* \rightarrow \infty$. This specification of monetary policy implies that inflation in any given country is equal to its target and that there is no recession as long as the country’s interest rate is positive. For example, for Home, either $\pi_{H,t} = \bar{\pi}$, $\xi_t = 1$, and $i_t = r_t^n + \bar{\pi} \geq 0$ or $\pi_{H,t} \leq -\kappa_0 \leq \bar{\pi}$, $\xi_t \leq 1$, and $i_t = 0$. The same holds for Foreign.

A.2.2 Equilibria

We assume that the world natural interest rate $r^{w,n} = -\rho + \bar{\delta}\theta/(1 - \bar{d}\theta) < 0$ is low enough that $r^{w,n} < \min\{\kappa_0, \kappa_0^*\}$, which yields the existence of a *global* liquidity trap equilibrium. We show that in this case, there are *several* possible equilibrium configurations once inflation considerations are added. First, there can be equilibria with no liquidity traps either at Home or in Foreign. Second, there can be equilibria with a symmetric global liquidity trap both at Home and in Foreign. Third, there can be asymmetric liquidity trap equilibria with a liquidity trap only in one country. We treat each in turn.

No liquidity trap equilibrium. We solve for the no-liquidity trap case. This equilibrium is such that $\xi = 1$, $\xi^* = 1$, $\pi_H = \bar{\pi}$, $\pi_F^* = \bar{\pi}^*$, $i = r^{w,n} + \bar{\pi}$, and $i^* = r^{w,n} + \bar{\pi}^*$.

It is straightforward to show that the terms of trade $S_t = E_t P_{F,t}^*/P_{H,t}$ is constant at $S_t = 1$ which implies that $\dot{E}_t/E_t = \pi_H - \pi_F^* = \bar{\pi} - \bar{\pi}^*$. The condition for this equilibrium to exist is that $i \geq 0$ and $i^* \geq 0$, i.e. $\min\{r^{w,n} + \bar{\pi}, r^{w,n} + \bar{\pi}^*\} \geq 0$. This condition shows that *the no-liquidity trap equilibrium exists if and only if the inflation targets $\bar{\pi}$ and $\bar{\pi}^*$ in both countries are high enough.*

Note, however, that this is an existence, not a uniqueness result. In fact, as we shall see next, other equilibria exist even if inflation targets are high enough to make the no-liquidity trap equilibrium feasible.

²⁵The introduction of this kind of Phillips curves borrows heavily from Eggertsson and Mehrotra (2014) and Caballero and Farhi (2017).

Symmetric global liquidity trap equilibrium. Let us now focus on the other extreme and solve for the symmetric global liquidity trap case.

Observe that in a stationary equilibrium the terms of trade $S_t = E_t P_{F,t}^* / P_{H,t}$ must be constant at $S_t = \xi / \xi^*$, so that $\dot{E}_t / E_t = \pi_H - \pi_F^*$. Uncovered Interest Parity then requires that $i = i^* + \dot{E}_t / E_t$, which combined with $i = i^* = 0$ implies that $\dot{E}_t / E_t = 0$ and hence $\pi_F^* = \pi_H = \pi^w$. That is, *in a global liquidity trap, inflation rates are equal across countries*, hence real interest rates are equalized, $r = r^* = -\pi^w$.

In Appendix ??, we provide a detailed exposition of the equilibrium equations. We can represent the equilibrium as an Aggregate Demand (AD)-Aggregate Supply (AS) diagram which constitutes a system of four equations in four unknowns π_H , π_F^* , ξ , and ξ^* . The home and foreign AD curves are given by:

$$\xi = \frac{\frac{1 - \frac{\pi_H}{\rho}}{1 - \frac{\delta\theta}{\rho} - \frac{\pi_H}{\rho}} y \theta d}{1 - \frac{1 - \frac{\pi_H}{\rho}}{1 - \frac{\delta\theta}{\rho} - \frac{\pi_H}{\rho}} \frac{1-y}{\xi^*} \theta d^*} ; \quad \xi^* = \frac{\frac{1 - \frac{\pi_F^*}{\rho}}{1 - \frac{\delta\theta}{\rho} - \frac{\pi_F^*}{\rho}} (1-y) \theta d^*}{1 - \frac{1 - \frac{\pi_F^*}{\rho}}{1 - \frac{\delta\theta}{\rho} - \frac{\pi_F^*}{\rho}} \frac{y}{\xi} \theta d}$$

The home and foreign AS curves are given by:

$$\pi_H = -\kappa_0 - \kappa_1(1 - \xi) \quad ; \quad \pi_F^* = -\kappa_0^* - \kappa_1^*(1 - \xi^*)$$

as long as $\xi < 1$ and $\xi^* < 1$, and become vertical at $\xi = 1$ and $\xi^* = 1$.

It can be verified that the home and foreign AD equations imply $\pi_H = \pi_F^* = \pi^w$. If $\kappa_0 = \kappa_0^*$, this implies that

$$\frac{1 - \xi^*}{1 - \xi} = \frac{\kappa_1}{\kappa_1^*},$$

so that *Home has a smaller recession than Foreign*, $\xi > \xi^*$, *if and only if home prices or wages are more flexible than foreign prices or wages*: $\kappa_1 > \kappa_1^*$. More (downward) price or wage flexibility reduces the size of the recession at Home relative to Foreign because it depreciates the domestic terms of trade. In a stationary equilibrium, deflation rates are equalized across countries so relatively more wage flexibility implies a relatively smaller recession.

The rest of the equilibrium simplifies greatly when the Phillips curves are identical in both countries so that $\kappa_0^* = \kappa_0$ and $\kappa_1^* = \kappa_1$. Indeed, this requires that the recession is identical at Home and in Foreign: $\xi = \xi^* = \xi^w$, and $S = 1$. Moreover, in this case, we have the following simpler *global* AD-AS representation:

$$\xi^w = \frac{1 - \frac{\pi^w}{\rho}}{1 - \frac{\delta\theta}{\rho} - \frac{\pi^w}{\rho}} \theta \bar{d},$$

$$\pi^w = -\kappa_0 - \kappa_1(1 - \xi^w).$$

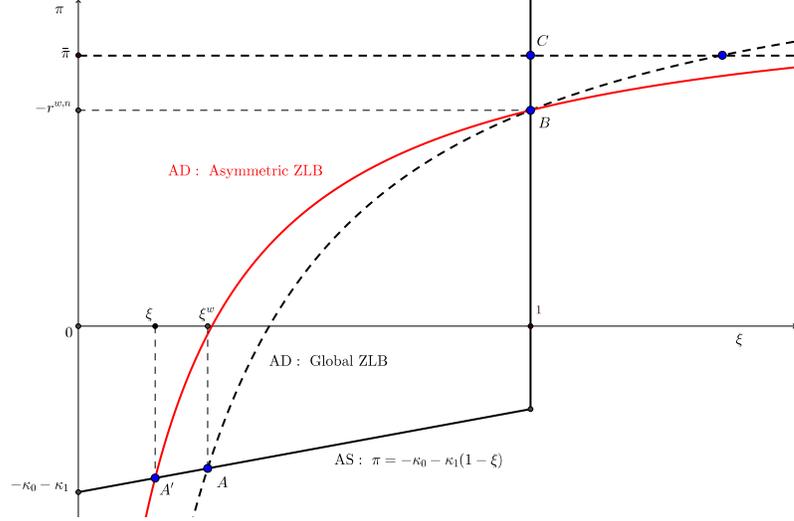
This representation makes clear that compared with the case with no inflation, *there is now a negative feedback loop between the global recession and inflation*. A larger recession reduces inflation, which in turn raises the real interest rate, causing a further recession etc. *ad infinitum*. This *feedback loop is stronger, the more flexible prices and wages are*, as captured by the slope of the Phillips curve κ_1 . That is, wage flexibility plays out differently across countries and at the global level: Countries with more price flexibility bear a smaller share of the global recession than countries with less wage flexibility; but at the global level, more wage flexibility exacerbates the global recession.

The equilibrium is guaranteed to exist under some technical conditions on the Phillips curves parameters κ_0 and κ_1 , which ensure that the feedback loop is not so powerful to lead to a total collapse of the economy.²⁶

Figure 8 reports the global AD-AS diagram and displays both the no liquidity trap equilibrium (if it exists) and the symmetric liquidity trap equilibrium. For simplicity the figure is drawn in the case where Phillips curves and inflation targets are identical in both countries so that $\kappa_0^* = \kappa_0$ and $\kappa_1^* = \kappa_1$, $\bar{\pi}^* = \bar{\pi}$.

We focus on the no liquidity trap equilibrium and the symmetric liquidity trap equilibrium for now. The AS curve (black solid line) slopes upwards, then becomes vertical at $\xi = \xi^w = 1$: A smaller recession is associated with less deflation, until full employment is achieved. At the ZLB, the global AD curve (black dashed line) also slopes upwards since an increase in inflation reduces the real interest rate, which increases output. Away from the ZLB, the

²⁶For $\xi^w = 1^-$, the AD curve has $\pi^w = -r^{w,n}$, while the AS curve has $\pi^w = -\kappa_0 < -r^{w,n}$. For $\xi^w = 0$, the AD curve has $\pi^w = \rho$, while the AS curve has $\pi^w = -(\kappa_0 + \kappa_1)$. A sufficient condition for a unique intersection is that $\kappa_0 + \kappa_1 \leq -\rho$.



The figure reports Aggregate Supply (solid black line) and Aggregate Demand (dashed black line) in a symmetric liquidity trap equilibrium (point A) and in a no liquidity trap equilibrium (point C), when $\kappa_0^* = \kappa_0$, $\kappa_1^* = \kappa_1$, and $\bar{\pi}^* = \bar{\pi}$. The red solid line represents the home AD curve in the asymmetric equilibrium where Foreign is out of the liquidity trap (point A').

Figure 8: Aggregate Demand and Aggregate Supply in a symmetric and asymmetric liquidity trap equilibria.

AD curve becomes horizontal at $\bar{\pi}$. We always assume that the upward sloping part of the AD curve is steeper than the non-vertical part of the AS curve and that they intersect at one point, A. The AD and AS schedules intersect at either exactly point A, or at three points, A, B, and C. Point A is the symmetric liquidity trap equilibrium: $i = 0$, $\pi^w = -\kappa_0 - \kappa_1(1 - \xi^w) < \bar{\pi}$, and $\xi^w < 1$. Point C, if it exists, corresponds to the no liquidity trap equilibrium with $i = i^* = \bar{\pi} > 0$, $\pi^w = \bar{\pi}$, and $\xi = \xi^w = 1$. Point B, if it exists, is unstable, and so we ignore it.

Asymmetric liquidity trap equilibria. Can we have an asymmetric equilibrium where one country is in a liquidity trap but not the other? As we shall see, it is *always possible*. These asymmetric liquidity trap equilibria are associated with different values of the real exchange rate, and are a manifestation of the same fundamental indeterminacy that we identified in the case with no inflation.

Suppose that one country is in a liquidity trap (say Home) but not the other (say Foreign). Then because the terms of trade must be constant at $S_t = \xi$, we must have $i = 0$, $i^* = i - \dot{E}_t/E_t = \pi_F^* - \pi_H > 0$, $\xi < 1$, $\xi^* = 1$, $\pi_F^* = \bar{\pi}^*$, and $\pi_H + \kappa_0 + \kappa_1(1 - \xi) = 0$. In Appendix B.1, we provide a detailed exposition of the equilibrium equations. We find:

$$\xi = \frac{y \frac{1 - \frac{\pi_H}{\rho}}{1 - \frac{\delta\theta}{\rho} - \frac{\pi_H}{\rho}} \theta d}{1 - (1 - y) \frac{1 - \frac{\pi_H}{\rho}}{1 - \frac{\delta\theta}{\rho} - \frac{\pi_H}{\rho}} \theta d^*},$$

$$\pi_H = -\kappa_0 - \kappa_1(1 - \xi).$$

The equilibrium is guaranteed to exist under the same technical conditions on Phillips curves that the ones derived above.

It is easy to see that *the home recession is larger and home inflation is lower in this asymmetric liquidity trap equilibrium where only Home is in a liquidity trap, than in the symmetric equilibrium where both Home and Foreign are in a liquidity trap*. In Figure 8, the red solid line reports the Home AD curve in the asymmetric equilibrium when Foreign is not in a liquidity trap. Point A' is the corresponding equilibrium. We can verify immediately that $\xi < \xi^w$, that is: The recession is more severe for the country that remains in the trap.

Inflation, exchange rates, and the structure of equilibria. Let us take stock and summarize the structure of equilibria when $r^{w,n} < \min\{\kappa_0, \kappa_0^*\}$. There may exist an equilibrium with no liquidity trap, which occurs if and only if $\min\{r^{w,n} + \bar{\pi}, r^{w,n} + \bar{\pi}^*\} \geq 0$. But there always exists a symmetric global liquidity trap equilibrium, as well as two asymmetric liquidity trap equilibria where only one country is in a liquidity trap. These symmetric and asymmetric liquidity trap equilibria are associated with different values of the real exchange rate and this multiplicity is a manifestation of the same fundamental indeterminacy that we identified in the case with no inflation. Indeed, it is immediate to see that terms of trade S are the most depreciated in the asymmetric liquidity trap equilibrium where Home is not in a liquidity trap but Foreign is, the most appreciated in the asymmetric liquidity trap equilibrium where Home is in a liquidity trap but Foreign is not, and intermediate between these two values in the symmetric liquidity trap equilibrium where both Home and Foreign are in a liquidity trap. The severity of the recession at Home is directly commensurate with the degree of appreciation of the terms of trade S .

“Currency wars”. Suppose that $\min\{r^{w,n} + \bar{\pi}, r^{w,n} + \bar{\pi}^*\} < 0$. A country (say Home) can target its exchange rate by standing ready to exchange unlimited quantities of home currency for foreign currency at a given crawling exchange rate. Doing so can in effect rule out both the symmetric liquidity trap equilibrium and the asymmetric liquidity trap equilibrium where it is in a liquidity trap. Home can therefore always guarantee that it will not be in a liquidity trap, and avoid a recession by shifting it entirely to Foreign, which then experiences a deeper recessions.²⁷

A.3 Home Bias

We assume that the spending share on home goods of home agents is $y + (1 - y)\beta$, and similarly that the spending share on foreign goods of foreign agents is $y^* + (1 - y^*)\beta$, where $y^* = 1 - y$ and $\beta \in [0, 1]$ indexes the degree of home bias. Full home bias corresponds to $\beta = 1$. The case of no home bias analyzed previously corresponds to $\beta = 0$.

With home bias in preferences, the good market clearing conditions become:

$$[y + (1 - y)\beta]\theta W + (1 - y^*)(1 - \beta)\theta W^* E = \xi \bar{Y}, \quad (\text{A.1})$$

$$(1 - y)(1 - \beta)\theta W + [y^* + (1 - y^*)\beta]\theta W^* E = E \xi^* \bar{Y}^*. \quad (\text{A.2})$$

This applies both under financial integration and under financial autarky, whether Home is in a liquidity trap ($\xi < 1$) or not ($\xi = 1$), and similarly whether Foreign is in a liquidity trap ($\xi^* < 1$) or not ($\xi^* = 1$).

For conciseness, we only consider the case where there is a global liquidity trap under financial integration. In that case, just like in the case of no home bias, there is a degree of indeterminacy indexed by the exchange rate E .

The asset and wealth dynamic equations Eqs. (4) and (5) are unchanged. After simple manipulations, we can express all equilibrium variables as a function of the nominal exchange rate:

$$\begin{aligned} \xi &= \frac{\theta \bar{d}^\beta(E)}{1 - \frac{\delta \beta \theta}{\rho}}, \quad (\text{A.3}) \\ \frac{NFA}{\bar{Y}} &= \frac{(1 - \frac{\delta \theta}{\rho}) \left[\frac{\theta \bar{d}^\beta(E)}{1 - \frac{\delta \beta \theta}{\rho}} - \frac{\theta d}{1 - \frac{\delta \theta}{\rho}} \right]}{g + \theta} = \frac{(1 - \frac{\delta \theta}{\rho})(\xi - \xi^{a,l})}{g + \theta}, \\ \frac{CA}{\bar{Y}} &= g \frac{NFA}{\bar{Y}}, \end{aligned}$$

where we have defined the averages modified by home bias β as

$$\bar{d}^\beta(E) = \frac{[\beta + (1 - \beta)y \left(1 + \frac{\frac{\delta^* \theta + \theta}{\rho}}{1 - \frac{\delta^* \theta}{\rho}} \right)] \theta d + (1 - y)(1 - \beta) \left(1 + \frac{\frac{\delta^* \theta + \theta}{\rho}}{1 - \frac{\delta^* \theta}{\rho}} \right) E \theta d^*}{[\beta + (1 - \beta)y \left(1 + \frac{\frac{\delta^* \theta + \theta}{\rho}}{1 - \frac{\delta^* \theta}{\rho}} \right)] + (1 - y)(1 - \beta) \left(1 + \frac{\frac{\delta^* \theta + \theta}{\rho}}{1 - \frac{\delta^* \theta}{\rho}} \right)},$$

²⁷In Appendix B.1, we characterize Net Foreign Asset positions and Current Accounts in all the equilibria described above. In the no liquidity trap equilibrium, these quantities are given by exactly the same formula as in the case with no inflation and can be represented in a Metzler diagram. In a symmetric global liquidity trap equilibrium instead, they are given by a Metzler diagram in quantities augmented with a global AS curve. The qualitative effects are essentially similar to the no inflation case.

$$\bar{\delta}^\beta = \frac{[\beta + (1 - \beta)y \left(1 + \frac{\frac{\delta^* \theta + \theta}{\rho}}{1 - \frac{\delta^* \theta}{\rho}}\right)] \left(1 - \frac{\delta \theta}{\rho}\right) + (1 - y)(1 - \beta) \frac{1 + \frac{\theta}{\rho}}{1 - \frac{\delta^* \theta}{\rho}} \left(1 - \frac{\delta^* \theta}{\rho}\right)}{[\beta + (1 - \beta)x \left(1 + \frac{\frac{\delta^* \theta + \theta}{\rho}}{1 - \frac{\delta^* \theta}{\rho}}\right)] + (1 - y)(1 - \beta) \left(1 + \frac{\frac{\delta^* \theta + \theta}{\rho}}{1 - \frac{\delta^* \theta}{\rho}}\right)}.$$

and where $\xi^{a,l}$ is defined exactly as in the case with no home bias, and given by the same formula. Equations (A.3) and its equivalent for the foreign country show that, as before, home output ξ is increasing in the exchange rate E while foreign output ξ^* is decreasing in E . Finally, as before, the home Net Foreign Asset Position and Current Account are increasing in the gap between the domestic recession and the home financial autarky recession $\xi^{a,l}$. *The key difference introduced by home bias β is that the home and foreign outputs ξ and ξ^* become less responsive to the exchange rate E .* This can be seen directly by examining (A.3) in the case of home bias ($\beta > 0$) and comparing to Eq. (17) in the case with no home bias ($\beta = 0$). This effect is seen most transparently in the limit with full home bias ($\beta \rightarrow 1$) in which case the outputs ξ and ξ^* become completely insensitive to the exchange rate E .

Assume further that both countries are in a liquidity trap under financial autarky. Then, just like in the case with no home bias, the integrated equilibrium coincides with financial autarky when $E = E^a$. For $E > E^a$, we have $\xi > \xi^a$ and $\xi^* < \xi^a$ and vice versa for $E < E^a$.²⁸

A.4 Trade Elasticities

We now assume away home bias and investigate instead the role of the elasticity of substitution σ between home and foreign goods. The main difference in the system of equilibrium equations is once again the goods market clearing conditions, which become

$$\begin{aligned} \frac{y}{y + E^{1-\sigma}(1-y)}(W + EW^*) &= \frac{\xi \bar{Y}}{\theta}, \\ \frac{(1-y)E^{1-\sigma}}{y + E^{1-\sigma}(1-y)}(W + EW^*) &= E \frac{\xi^* \bar{Y}^*}{\theta}. \end{aligned}$$

This applies both under financial integration and under financial autarky, whether Home is in a liquidity trap ($\xi < 1$) or not ($\xi = 1$), and similarly whether Foreign is in a liquidity trap ($\xi^* < 1$) or not ($\xi^* = 1$). This implies that we now have

$$E = \hat{E}^{\frac{1}{\sigma}}$$

where \hat{E} is a renormalized exchange rate given by

$$\hat{E} = \frac{\xi}{\xi^*}.$$

The analysis under financial autarky is identical to the case $\sigma = 1$ except for the value of the financial autarky exchange rate. Under financial integration and outside the liquidity trap, the analysis is also unchanged compared to the case $\sigma = 1$: from Section A.4, $\xi = \xi^* = 1$ implies $E = 1$. Since the asset demands and asset supplies are unchanged, so is the equilibrium. This obtains as long as $r^{w,n} \geq 0$ where $r^{w,n}$ is defined in Eq. (15).

Consider now the case of a global liquidity trap ($r^{w,n} < 0$). In that case, as before, we can index the solution by the exchange rate E (or equivalently \hat{E}). Using Eqs. (5) and (7), we can express the world supply of assets as:

$$\begin{aligned} V^w + D^w &= \frac{\delta y \xi + \delta^*(1-y)E\xi^*}{\rho} \bar{Y}^w + \bar{d}(E) \bar{Y}^w \\ &= \left[\frac{\hat{\delta}(E)}{\rho} \xi P^{1-\sigma} + \bar{d}(E) \right] \bar{Y}^w. \end{aligned}$$

where $\hat{\delta}(E) = (\delta y + \delta^*(1-y)E^{1-\sigma})/P^{1-\sigma}$ is a weighted average of δ using the relative price of H and F goods as weights, and $P = (y + (1-y)E^{1-\sigma})^{1/(1-\sigma)}$ is the consumption price index at Home. From Eq. (4) we can write asset

²⁸One can readily check that the range $[\underline{E}, \bar{E}]$ increases with β , so that the model with home bias admits a larger range of indeterminacy. In the limit of full home bias, any value of the exchange rate is admissible.

demand along a BGP as:

$$W^w = \xi P^{1-\sigma} \frac{\bar{Y}^w}{\theta}.$$

Putting the two together, we can solve for the Home recession ξ :

$$\xi = \frac{\theta \bar{d}(E)}{\left(1 - \frac{\theta \hat{\delta}(E)}{\rho}\right) P^{1-\sigma}}.$$

Compared to the case of $\sigma = 1$, a change in the exchange rate now has three effects on the recession at Home ξ . First, as before, it increases the supply safe ‘public’ assets $\bar{d}(E)$. This increases output. In addition, an increase in E changes the value of private assets, captured by $\hat{\delta}(E)$. Finally, it increases the price level P , which affects both private asset supply and private asset demand. Formally, we can write:

$$\frac{\partial \xi}{\partial E} = \frac{\theta(1-y)}{\left(1 - \frac{\theta \hat{\delta}(E)}{\rho}\right) P^{1-\sigma}} \left[d^* - \frac{\bar{d}(E)}{\left(1 - \frac{\theta \hat{\delta}(E)}{\rho}\right) P^{1-\sigma}} (1-\sigma) E^{-\sigma} \left(1 - \frac{\delta^* \theta}{\rho}\right) \right].$$

The first term in bracket corresponds to the expansion of public assets. The second term is the net effect of the expansion of private assets and the decline in asset demands. When $\sigma > 1$, this second effect is also positive: output becomes more responsive to the exchange rate as goods become more substitutable.

Things simplify in the limit $\sigma \rightarrow \infty$, where the goods become perfect substitutes, to which we now turn. For conciseness, we only treat the case where there is a global liquidity trap under financial integration. As we take the limit $\sigma \rightarrow \infty$, we have $E = P = 1$, but there is still a degree of indeterminacy indexed by the renormalized exchange rate \hat{E} . Indeed, we can compute all the equilibrium variables as a function of \hat{E} :

$$\begin{aligned} \xi &= \frac{\theta \bar{d}}{y(1 - \frac{\delta \theta}{\rho}) + \frac{1}{\hat{E}}(1-y)(1 - \frac{\delta^* \theta}{\rho})}, \\ \xi^* &= \frac{\theta \bar{d}}{y(1 - \frac{\delta \theta}{\rho}) \hat{E} + (1-y)(1 - \frac{\delta^* \theta}{\rho})}, \\ \frac{NFA}{\bar{Y}} &= \frac{(1 - \frac{\delta \theta}{\rho}) \xi - \theta d}{g + \theta} = \frac{(1 - \frac{\delta \theta}{\rho})(\xi - \xi^{a,l})}{g + \theta}, \\ \frac{CA}{\bar{Y}} &= g \frac{NFA}{\bar{Y}}, \end{aligned}$$

where $r^{w,n}$ and $\xi^{a,l}$ are defined exactly as in the unitary elasticity case, and are given by the same formulas. Home output ξ is increasing in the renormalized exchange rate \hat{E} , and foreign output is decreasing in the renormalized exchange rate \hat{E} . Finally, the home Net Foreign Asset Position is increasing in the gap between home output and home financial autarky output $\xi^{a,l}$ under zero home nominal interest rates. *The key difference introduced by $\sigma > 1$ over $\sigma = 1$ is that home and foreign outputs ξ and ξ^* become more responsive to the exchange rate E .* Indeed in the limit $\sigma \rightarrow \infty$, ξ and ξ^* become infinitely sensitive to the exchange rate E . In other words, larger trade elasticities magnify the stimulative effect of an exchange rate depreciation on the home recession.

Assume further that both countries are in a liquidity trap under financial autarky. Then, just like in the case with $\sigma = 1$, the financially integrated equilibrium coincides with financial autarky when $\hat{E} = \hat{E}^a$. For $\hat{E} > \hat{E}^a$, we have $\xi > \xi^a$ and $\xi^* < \xi^a$ and vice versa for $\hat{E} < \hat{E}^a$.

Let’s now consider the Nash equilibrium where each central bank maximizes consumption $\theta W/P$ net of the cost

$\mathcal{C}(a)$. The first-order conditions are:

$$\xi \frac{\frac{1}{\hat{E}}(1-y)(1-\frac{\delta^*\theta}{\rho})}{y(1-\frac{\delta\theta}{\rho}) + \frac{1}{\hat{E}}(1-y)(1-\frac{\delta^*\theta}{\rho})} \frac{\mathbf{n}}{a} = \mathcal{C}'(a),$$

$$\xi^* \frac{y(1-\frac{\delta\theta}{\rho})}{y(1-\frac{\delta\theta}{\rho}) + \frac{1}{\hat{E}}(1-y)(1-\frac{\delta^*\theta}{\rho})} \frac{\mathbf{n}}{a^*} = \mathcal{C}'(a^*).$$

The second order conditions are satisfied as long as $\mathbf{n} < 1$. From the first order conditions, it is immediate that the Nash equilibrium satisfies $a, a^* > 1$ and we can also easily check that $\partial a / \partial d > 0$ and $\partial a^* / \partial d > 0$ so that higher public debt in either country increases the incentives to intervene to depreciate the currency.

Further, define $x = y(1 - \theta\delta/\rho)$ and $x^* = (1 - y)(1 - \theta\delta^*/\rho)/\hat{E}$. The slope of the Home best-response is given by:

$$[(x + x^*)(1 + \eta_c) + \mathbf{n}(x - x^*)] \frac{da}{a} = \mathbf{n}(x - x^*) \frac{da^*}{a^*}$$

while the slope of the foreign best response is given by

$$(x^* - x)\mathbf{n} \frac{da}{a} = [(x + x^*)(1 + \eta_c) + \mathbf{n}(x^* - x)] \frac{da^*}{a^*}$$

When $\hat{E} < \hat{E}^o \equiv ((1 - y)(1 - \theta\delta^*/\rho))/(y(1 - \theta\delta/\rho))$, the actions are strategic substitutes, while when $\hat{E} > \hat{E}^o$, they are strategic complements. Either way, the equilibrium is asymptotically stable in the sense of Fudenberg and Tirole when

$$\left| \frac{da}{da^*} \right|_A < \left| \frac{da}{da^*} \right|_{A^*}$$

This condition boils down to

$$\mathbf{n} < \frac{1 + \eta_c}{\sqrt{2}} \frac{\hat{E} + \hat{E}^o}{\hat{E}^o - \hat{E}} \equiv \mathcal{H}(\hat{E})$$

It is easy to check that the function $\mathcal{H}(\hat{E})$ is increasing, so its lowest value will be reached at the lower bound $\underline{\hat{E}}$ such that $\xi^* = 1$:

$$\begin{aligned} \underline{\hat{E}} &= \frac{\theta\bar{d}}{y(1-\frac{\theta\delta}{\rho})} - \hat{E}^o \\ &= \frac{\theta\bar{d} - (1-y)(1-\frac{\theta\delta^*}{\rho})}{y(1-\frac{\theta\delta}{\rho})} \end{aligned}$$

and the condition for an asymptotically stable Nash equilibrium is always satisfied if

$$\mathbf{n} < \mathcal{H}(\underline{\hat{E}}) = \frac{1 + \eta_c}{\sqrt{2}} \frac{\theta\bar{d}}{2(1-y)(1-\frac{\theta\delta^*}{\rho}) - \theta\bar{d}}$$

A sufficient condition is

$$2(1-y)(1-\frac{\theta\delta^*}{\rho}) > \theta\bar{d} > \frac{2(1-y)(1-\frac{\theta\delta^*}{\rho})}{1 + \frac{1+\eta_c}{\sqrt{2}}}$$

We assume that this condition is satisfied.

A.5 Within Country Heterogeneity: Borrowers and Savers

Here we work out a version of our model incorporating within country heterogeneity between borrowers and savers. For simplicity, we abstract away from public debt by setting $D/\bar{Y} = D^*/\bar{Y}^* = 0$.

We add a mass of borrowing constrained impatient borrowers (B) agents. The rest of the agents are savers (S) and are modeled as before. Borrowers consume as much as possible when they are born, and the rest when they die. They only get an endowment when they die, and they can only pledge a part of it. They must therefore borrow in order to consume when born. They then roll over their debt until they die, at which point they use their income to repay their debt and consume the remainder. In a small interval dt , a part $\eta\xi_t\bar{Y}_t dt$ of total income accrues to dying borrowers in the form of labor income. Because of the borrowing constraint, borrowers born in the interval dt can only consume $\chi\bar{Y}_t dt$, where we imagine that χ is small compared to η .²⁹ We assume that the new trees accrue to savers.

Note that there is now a distinction between financial wealth and human wealth for borrowers. Indeed a borrower receives income when he dies. This future income is a form a human wealth and is not part of his financial wealth. When the borrower dies, this human wealth allows him to repay the debt that he has incurred to borrow when he was born and rolled over until his death (his financial wealth), and to consume the residual.

The evolution equations for the financial wealth of borrowers and savers are given by:^{30,31}

$$\begin{aligned} gW^B &= -\theta W^B - \chi X + r^w W^B, \\ gW^S &= -\theta W^S + (1 - \delta - \eta)\xi\bar{Y} + r^w W^S + (\rho + g)V, \\ gW^{B*} &= -\theta W^{B*} - \chi^*\bar{Y}^* + r^w W^{B*}, \\ gW^{S*} &= -\theta W^{S*} + (1 - \delta - \eta)\xi^*\bar{Y}^* + r^w W^{S*} + (\rho + g)V^*. \end{aligned}$$

We continue to denote by $W = W^B + W^S$ total home wealth and by $W^* = W^{B*} + W^{S*}$ total foreign wealth and obtain the evolution equations for total wealth by aggregating the evolution equations for wealth by borrowers and savers in both countries:

$$\begin{aligned} gW &= -\theta W + (1 - \delta - \eta)\xi\bar{Y} - \chi\bar{Y} + r^w W + (\rho + g)V, \\ gW^* &= -\theta W^* + (1 - \delta^* - \eta^*)\xi^*\bar{Y}^* - \chi^*\bar{Y}^* + r^w W^* + (\rho + g)V^*. \end{aligned}$$

The good market clearing conditions are now given by:³²

$$\begin{aligned} \xi\bar{Y} &= y(\theta(W + EW^*) + \eta\xi\bar{Y} + E\eta^*\xi^*\bar{Y}^* + \chi\bar{Y} + E\chi^*\bar{Y}^*) \\ E\xi^*\bar{Y}^* &= (1 - y)(\theta(W + EW^*) + \eta\xi\bar{Y} + E\eta^*\xi^*\bar{Y}^* + \chi\bar{Y} + E\chi^*\bar{Y}^*). \end{aligned}$$

²⁹Note that, as in [Guerrieri and Lorenzoni \(2011\)](#) and [Eggertsson and Krugman \(2012\)](#), the borrowing limit χX does not depend on whether the economy is in recession. This assumption is crucial to generate a liquidity trap, as it implies that the debt issued by borrowers does not scale with output, so asset demand declines faster than asset supply in the recession. While the assumption that the credit constraint is invariant to the recession is perhaps extreme, all that is needed for our result to go through is that the borrowing limit does not scale one for one with output.

³⁰Let us for example explain in details the wealth evolution equation for borrowers at Home. The wealth of borrowers is negative $W^B < 0$, it represents their debt. In an interval dt , the wealth of borrowers W^B changes because of because of dying borrowers repaying their debt ($-\theta W^B dt$), because of newborn borrowers taking on new debt ($-\chi\bar{Y} dt$), and because of the accumulation of interest ($r^w W^B$). In a steady state, the wealth of borrowers W^B must also change by $gW^B dt$. This gives the wealth evolution equation for borrowers.

³¹Note that the wealth of borrowers does not take into account the income of borrowers when they die, because it is not part of their financial wealth. But of course, the income of borrowers influences their consumption when they die. It therefore appears in the goods market clearing conditions. This explains the terms $\eta\xi\bar{Y}$ and $E\eta^*\xi^*\bar{Y}^*$ in the market clearing conditions at Home and in Foreign.

³²For example, the home market clearing condition can be understood as follows. The demand arising from dying savers is given by $y\theta(W - W^B + EW^* - EW^{B*})$. The demand arising from newborn borrowers is given by $y(\chi X + E\chi^*\bar{Y}^*)$. And the demand arising from dying borrowers is given by $y(\eta\xi\bar{Y} + \theta W^B + E\eta^*\xi^*\bar{Y}^* + E\theta W^{B*})$.

The asset pricing equations are unchanged:

$$\begin{aligned} r^w V &= -\rho V + \delta \xi \bar{Y}, \\ r^w V^* &= -\rho V^* + \delta^* \xi^* \bar{Y}^*. \end{aligned}$$

And we must still impose $r^w \geq 0$, $0 \leq \xi \leq 1$, $0 \leq \xi^* \leq 1$, and the complementary slackness conditions $r^w(1 - \xi) = 0$ and $r^w(1 - \xi^*) = 0$.

In the interest of space, we only treat the liquidity trap case. We get:

$$\begin{aligned} E &= \frac{\xi}{\xi^*}, \\ \xi &= \frac{\bar{\chi}(E)}{1 - \bar{\eta} - \frac{\delta \theta}{\rho}}, \\ \frac{NFA}{\bar{Y}} &= \frac{\frac{1 - \eta - \frac{\delta \theta}{\rho}}{1 - \bar{\eta} - \frac{\delta \theta}{\rho}} \bar{\chi}(E) - \chi}{g + \theta}, \\ \frac{CA}{\bar{Y}} &= g \frac{NFA}{\bar{Y}}. \end{aligned}$$

where for any variable z , we use the notation $\bar{z}(E) = xz(E) + (1 - x)z^*(E)$.

The variable χ (χ^*) increases with home (foreign) financial development, and decreases with a home (foreign) deleveraging shock. Identifying the borrowers with the young and the savers with the middle-aged and the old, proportional decreases in the variables η and χ (η^* and χ^*) capture home (foreign) population aging.

These equations indicate that a deleveraging shock at Home (a decrease in χ) or in Foreign (a decrease in χ^*) can push the global economy into a liquidity trap. For a given exchange rate E , the larger the world deleveraging shock, the larger the recession in any given country. For a given exchange rate E and world deleveraging shock $\bar{\chi}(E)$, a larger home deleveraging shock (a lower χ) pushes the home Current Account towards a surplus.

Similarly, aging at Home (a proportional decrease in χ and η) or in Foreign (a proportional decrease in χ^* and η^*) can push the global economy in a liquidity trap. For a given exchange rate E , the larger the shock, the larger the recession in any given country. For a given exchange rate E , more aging at Home pushes the Home Current Account towards a surplus.

This analysis also shows how countries with tighter credit constraints or lower fraction of income accruing to borrowers act as if they had a larger asset demand (lower θ).

B Online Appendix: Not For Publication

B.1 Derivations for the Model with Inflation in Appendix A.2

Global liquidity trap equilibrium equations. In a global liquidity trap equilibrium, the equilibrium values of $V^w = V + SV^*$, $W^w = W + SW^*$ (expressed in terms of the home good numeraire) and π_H , π_F^* , S , ξ , and ξ^* solve the following system of equations

$$\begin{aligned} S &= \frac{\xi}{\xi^*}, \\ \theta W^w &= \xi X + S\xi^* X^* \\ -\pi_H V^w &= -\rho V^w + \delta \xi \bar{Y} + \delta^* S \xi^* \bar{Y}^*, \\ g W^w &= -\theta W^w + (1 - \delta) \xi \bar{Y} + (1 - \delta^*) S \xi^* \bar{Y}^* + g D^w - \pi_H W^w + (\rho + g) V^w, \end{aligned}$$

$$\pi_H = -\kappa_0 - \kappa_1(1 - \xi), \tag{B.1}$$

$$\pi_F^* = -\kappa_0^* - \kappa_1^*(1 - \xi^*) \tag{B.2}$$

$$\pi_F^* = \pi_H, \tag{B.3}$$

where $D^w = D + SD^*$. The first equation is the equation for the terms of trade. The second equation is the equation for total world wealth. Both result directly from combining the home and foreign goods market clearing conditions. The third equation is the asset pricing equation for world private assets. The fourth equation is the accumulation equation for world wealth, where we have used the government budget constraints to replace taxes as a function of public debt $\tau(1 - \delta)\xi\bar{Y} = -gD$ and $\tau^*(1 - \delta)\xi^*\bar{Y}^* = -gD^*$. The fifth and sixth equations are the home and foreign Phillips curves. The seventh equation represents the requirement derived above that the terms of trade be constant.

Asymmetric liquidity trap equilibrium equations. In an asymmetric liquidity trap equilibrium where one country (say Home) is in a liquidity trap but not the other (say Foreign), the equilibrium equations are instead given by:

$$\begin{aligned} S &= \xi, \\ \theta W^w &= \xi \bar{Y} + S \bar{Y}^* \\ -\pi_H V^w &= -\rho V^w + \delta \xi \bar{Y} + \delta^* S \bar{Y}^*, \\ g W^w &= -\theta W^w + (1 - \delta) \xi \bar{Y} + (1 - \delta^*) S \bar{Y}^* + g D^w - \pi_H W^w + (\rho + g) V^w, \\ \pi_H &= -\kappa_0 - \kappa_1(1 - \xi), \end{aligned}$$

and we have $i = 0$, $i^* = \bar{\pi}^* - \pi_H = i - \dot{E}_t/E_t > 0$.

Net Foreign Assets, Current Accounts, and Metzler Diagram in quantities. In this section, we characterize Net Foreign Asset positions and Current Accounts in the model with inflation of Appendix A.2. We express these quantities in real terms in the home good numeraire. In the no liquidity trap equilibrium, these quantities are given by exactly the same formula as in the case with no inflation. In a symmetric global liquidity trap equilibrium, or in an asymmetric liquidity trap equilibrium, we have

$$\begin{aligned} \frac{NFA}{\bar{Y}} &= \frac{W - (V + D)}{\bar{Y}} = \frac{\xi(1 - \frac{\delta\theta}{r+\rho}) - \theta d}{g + \theta - r}, \\ \frac{CA}{\bar{Y}} &= g \frac{NFA}{\bar{Y}}, \end{aligned}$$

where $\xi < 1$ and $r = -\pi_H$ if Home is in a liquidity trap and $\xi = 1$ and $r = -\pi_F^*$ if Home is not in a liquidity trap (but Foreign is).

The same forces that we identified in the model with no inflation are at play. For example, in a symmetric global

liquidity trap equilibrium when the Phillips curves are identical across countries (so that $\kappa_0^* = \kappa_0$ and $\kappa_1^* = \kappa_1$),

$$\begin{aligned}\frac{NFA}{\bar{Y}} &= \frac{W - (V + D)}{\bar{Y}} = \frac{(1 - \frac{\delta\theta}{\rho} - \frac{\pi^w}{\rho})[\frac{\theta\bar{d}}{1 - \frac{\delta\theta}{\rho} - \pi^w} - \frac{\theta d}{1 - \frac{\delta\theta}{\rho} - \frac{\pi^w}{\rho}}]}{g + \theta + \pi^w}, \\ \frac{CA}{\bar{Y}} &= g \frac{NFA}{\bar{Y}}.\end{aligned}$$

Hence to the extent that Home has a higher financial capacity than Foreign $\delta > \delta^*$, or a higher public debt ratio than Foreign $d > d^*$, then Home runs a negative Net Foreign Asset position and a Current Account deficit. We can also represent the equilibrium with a Metzler diagram in quantities augmented with a global AS curve. Indeed we have

$$\begin{aligned}\frac{NFA}{\bar{Y}} &= \frac{\xi^w(1 - \frac{\frac{\delta\theta}{\rho}}{1 - \frac{\pi^w}{\rho}}) - \theta d}{g + \theta + \pi^w}, \\ S \frac{NFA^*}{\bar{Y}^*} &= \frac{\xi^w(1 - \frac{\frac{\delta^*\theta}{\rho}}{1 - \frac{\pi^w}{\rho}}) - \theta d^*}{g + \theta + \pi^w},\end{aligned}$$

and we must have

$$\begin{aligned}y \frac{NFA}{\bar{Y}} + (1 - y) S \frac{NFA^*}{\bar{Y}^*} &= 0, \\ \pi^w + \kappa_0 + \kappa_1(1 - \xi^w) &= 0,\end{aligned}$$

with $S = 1$.

B.2 Recovery: Exchange Rates Movements and Interest Rates Differentials

We start with the model with permanently rigid prices, and consumption home bias of section A.3. For simplicity, we assume that there is no public debt so that $D/\bar{Y} = D^*/\bar{Y}^* = 0$.

We then assume that a Poisson shock occurs with instantaneous probability $\lambda > 0$. When the Poisson shock occurs, the fraction of output δ that accrues in the form of dividends jumps instantaneously and permanently by a factor $\nu > 1$ in both countries. This alleviates the asset shortage and increases the world natural interest rate. We assume that ν is large enough that upon the realization of the Poisson shock the world natural interest rate rises above zero: $-\rho + \nu\delta\theta > 0$. This implies that the economy may experience a liquidity trap *before* the Poisson shock, but never *after* it.

The steady state of the post-Poisson shock economy is uniquely determined, and so are its dynamics from any initial position.³³ By backward induction, this means that the exchange rate during the liquidity trap phase is also pinned down, conditional on the exchange rate E_τ that occurs at the time τ of the realization of the Poisson shock. This removes the indeterminacy in the nominal exchange rate à la [Kareken and Wallace \(1981\)](#) that we found in our baseline model.³⁴

But another form of indeterminacy appears which we can index by the exchange rate E_τ . This is because, in our model, agents are risk neutral, so that international portfolios are indeterminate.³⁵ Yet, a given portfolio allocation will determine relative wealths immediately after the Poisson shock. In the presence of home bias in consumption, this pins down relative demands for Home and Foreign goods and therefore the nominal exchange rate E_τ . Conversely, for a given value E_τ , one can construct international portfolios that are consistent with this value of the exchange

³³One can verify that the dynamics of the economy are saddle-path stable.

³⁴This is because we have assumed that the economy is not in a global liquidity trap after the Poisson shock. If we assume instead that the economy is in a liquidity trap after the Poisson shock (so that the recovery is only a partial recovery which doesn't lift the economy out of the ZLB), then even without home bias, the exchange rate indeterminacy à la [Kareken and Wallace \(1981\)](#) is reinstated. Indeed, in this case, the exchange rate E_τ after the Poisson shock is indeterminate. The exchange rate E before the Poisson shock, which depends on its value E_τ after the Poisson shock, inherits this indeterminacy. In the interest of space, we do not develop this model formally.

³⁵This other form of indeterminacy hinges on our assumption that some (here all) agents are risk neutral. If all agents were somewhat risk averse, then portfolios would be pinned down and this other form of indeterminacy would disappear.

rate at the time of the Poisson shock. We summarize by writing domestic and foreign wealth and asset values at the time of the shock as $W_\tau = w_\tau \bar{Y}/\theta$, $W_\tau^* = w_\tau^* \bar{Y}^*/\theta$, $V_\tau = v_\tau \bar{Y}/\theta$, and $V_\tau^* = v_\tau^* \bar{Y}^*/\theta$ where it is understood that the coefficients w_τ, w_τ^*, v_τ and v_τ^* are functions of the exchange rate E_τ at the time of the shock.

We focus on the stochastic steady state before the Poisson shock. Because of the jump in the exchange rate at the time of the Poisson shock, Home and Foreign typically experience different real interest rates. This is in contrast to our baseline model where real interest rates are always equalized across countries. To see this most clearly, note that financial integration imposes that in the stochastic steady state prior to the Poisson shock, we have the following UIP equation:

$$r = r^* + \lambda \left(\frac{E_\tau}{E} - 1 \right). \quad (\text{B.4})$$

This implies that the home interest rate $r < r^*$ if the home currency is expected to appreciate after the Poisson shock $E_\tau/E < 1$.³⁶

The asset pricing equations include news terms accounting for capital gains and losses triggered by the realization of the Poisson shock:

$$rV = -\rho V + \delta \xi \bar{Y} + \lambda (V_\tau - V),$$

$$r^*V^* = -\rho V^* + \delta^* \xi^* \bar{Y}^* + \lambda (V_\tau^* - V^*).$$

For example, a higher value of home assets V_τ after the Poisson shock increases the value of home assets V in the stochastic steady state before the Poisson shock.

The wealth accumulation equations include new terms accounting for the risk and return of each country's portfolio:

$$gW = -\theta W + (1 - \delta) \xi \bar{Y} + rW + \lambda (W - W_\tau) + (g + \rho) V,$$

$$gW^* = -\theta W^* + (1 - \delta) \xi^* \bar{Y}^* + r^*W^* + \lambda (W^* - W_\tau^*) + (g + \rho) V^*.$$

For example, a lower value of home wealth W_τ after the Poisson shock means that home agents have a riskier portfolio, and therefore collect higher returns as long as the Poisson shock does not materialize. This in turn increases home wealth W in the stochastic steady state before the Poisson shock.

The goods market clearing equations (A.1) and (A.2) are unchanged, and we must still impose $r \geq 0$, $r^* \geq 0$, $0 \leq \xi \leq 1$, $0 \leq \xi^* \leq 1$, and the complementary slackness conditions $r(1 - \xi) = 0$ and $r^*(1 - \xi^*) = 0$.

The jump in the exchange rate at the time of the Poisson shock opens the door to the possibility that Home and Foreign may not experience a liquidity trap simultaneously prior to the shock. Real interest rates can differ across countries, resulting in the possibility of more strongly asymmetric liquidity trap equilibria than those we have encountered so far, where one country has zero nominal interest rates, zero real interest rates and a recession, while the other country has positive nominal interest rates, positive real interest rates, and no recession.

Going back to the UIP equation (B.4), we see that for Home to be the only country in a liquidity trap, we need $r = 0$, $\xi < 1$, $r^* > 0$, $\xi^* = 1$ and $\lambda (E_\tau/E - 1) = -r^* < 0$. This requires that the home exchange rate appreciate at the time of the shock, $E_\tau < E$. We focus on this configuration from here onwards.

Home output ξ is then given by

$$\xi = \frac{\beta \frac{g-\lambda}{g-\lambda+\theta} \frac{\lambda v_\tau - \frac{(\rho+\lambda)\lambda}{g-\lambda} (w_\tau - v_\tau)}{\rho+\lambda} + y^* (1 - \beta) E}{\left[1 - \frac{\beta \frac{g-\lambda}{g-\lambda+\theta}}{\beta \frac{g-\lambda}{g-\lambda+\theta} + y^* (1 - \beta)} \frac{\delta \theta}{\rho+\lambda} \right] \left[\beta \frac{g-\lambda}{g-\lambda+\theta} + y^* (1 - \beta) \right]}. \quad (\text{B.5})$$

This equation shows that everything else equal, as long as there is home bias $\beta > 0$, a higher value v_τ/θ of the home asset after the Poisson shock, and a lower value of the home Net Foreign Asset position after the Poisson shock $(w_\tau^* - v_\tau^*)/\theta$, contribute to a lower home output. Both increase the value of home wealth before the Poisson shock

³⁶We can also have equilibria with different values of $E = E_\tau$, with similar implications in terms of relative outputs and "currency wars" as in the main text—lower values of $E = E_\tau$ are associated with higher values of ξ and lower values of ξ^* . Interestingly, But here, this logic can be more extreme in that we can also have equilibria with asymmetric liquidity traps where there is a liquidity trap in one country but not in the other. For example, Home can be in a liquidity trap with $r = 0$ and $\xi < 1$ while Foreign is not: $r^* > 0$ and $\xi^* = 1$. In this case, going back to the UIP equation, the exchange rate appreciates when the Poisson shock occurs $E > E_\tau$.

and, because of home bias, of the demand for home goods. The reason is that a higher value of v_τ/θ increases the value of new trees, and that a lower value of $(w_\tau^* - v_\tau^*)/\theta$ indicates that home agents take more risk, and are hence rewarded by a higher return before the Poisson shock.

The foreign interest rate r^* and the exchange rate are then given by the following system of nonlinear equations

$$0 = r^* + \lambda \left(\frac{E_\tau}{E} - 1 \right),$$

$$1 - \left(\frac{\xi}{E} - 1 \right) g \frac{1 - \beta}{\beta} = \frac{\theta + \frac{g - \lambda - r^*}{\rho + \lambda + r^*} (\delta^* \theta + \lambda v_\tau^*) - \lambda (w_\tau^* - v_\tau^*)}{g - \lambda + \theta - r^*},$$

where we use the equation above to express ξ as a function of E . This is an equilibrium as long as $\xi < 1$ and $r^* \geq 0$.

We can also compute

$$\frac{NFA}{\bar{Y}} = \frac{\xi \left(1 - \frac{\delta \theta}{\rho + \lambda} \right) - \frac{\lambda v_\tau}{\rho + \lambda} - \lambda \frac{\theta}{g - \lambda} (w_\tau - v_\tau)}{g - \lambda + \theta} = \frac{1 - \frac{\delta \theta}{\rho + \lambda}}{g - \lambda + \theta} (\xi - \hat{\xi}^{a,l}),$$

$$\frac{CA}{\bar{Y}} = g \frac{NFA}{\bar{Y}}.$$

where $\hat{\xi}^{a,l}$ is home output in the equilibrium where Home is in financial autarky before the Poisson shock, but not after the Poisson shock (and where the equilibrium coincides with that under consideration after the Poisson shock).³⁷

Financial integration before the Poisson shock increases output $\xi \geq \hat{\xi}^{a,l}$ if and only if the exchange rate is more depreciated $E \geq \bar{E}^{a,l}$.³⁸

³⁷By financial autarky we mean that Net Foreign Asset positions at Home and in Foreign are equal to 0. We allow countries to trade actuarially fair insurance contracts on the realization of the Poisson shock. These contracts have zero ex-ante value for both home and foreign agents.

³⁸The analysis simplifies drastically in the limit of full home bias ($\beta \rightarrow 1$). In this case, we have $\xi^a = \xi^{a,l}$, $w_\tau = v_\tau = w_\tau^* = v_\tau^* = 1$. This implies that ξ and r^* are given by their financial autarky values $\xi = \xi^{a,l}$, $r^* = r^{*a,n} = \delta^* \theta - \rho$, and Net Foreign Asset Positions and Current Accounts are zero $\frac{NFA}{\bar{Y}} = \frac{CA}{\bar{Y}} = 0$. This is an equilibrium if and only if $r^{*a,n} \geq 0$ and $r^{a,n} \leq 0$ (which is equivalent to $\xi^{a,l} \leq 1$).