Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis

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Abstract

This paper studies the effects of supply and demand shocks in a general disaggregated model with multiple sectors, factors, and input-output linkages, as well as downward nominal wage rigidities and a zero lower bound constraint. We use the model to try to understand how the Covid-19 crisis, an omnibus of various supply and demand shocks, affects output, unemployment, and inflation. Throughout our analysis, we focus on how the structure of the production network and the elasticities of substitution matter for determining outcomes. We use some quantitative thought experiments, grounded in data, to gauge the importance of the forces that we identify. We find that with only negative labor supply shocks, Keynesian effects are larger for more heterogeneous shocks. Furthermore, Keynesian unemployment is increasing and Keynesian output effects are hump-shaped in the degree of complementarities, meaning that Keynesian forces matter more for unemployment than for output. Finally, negative supply shocks are necessarily stagflationary. On the other hand, negative aggregate demand shocks are deflationary, and once they are large relative to the negative supply shocks, they amplify Keynesian unemployment and output effects. We illustrate the intuition for these results using a nonlinear AS-AD representation of our model.

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1 Introduction

The outbreak of Covid-19 is an unusual macroeconomic shock. It cannot be easily categorized as a supply shock or as a demand shock, and it does not affect every part of the economy in the same way. While some sectors of the economy, like food retail and consumer manufacturing are running into supply constraints and struggling to keep up with demand, other sectors like commercial manufacturing, restaurants, and many service industries are laying off workers to reduce excess capacity.[1]

To analyze this divergent situation, we use a general disaggregated model and aggregate up from the micro level to the macro level. We allow for an arbitrary number of sectors and factors as well as unrestricted input-output linkages and elasticities of substitution. We incorporate downward nominal wage rigidities and a zero lower bound constraint.

We model the outbreak of the pandemic as an exogenous shock to the quantity of factors supplied, to the productivity of producers, and to the overall level and composition of final demand by consumers. On the one hand, the epidemic reduces the quantity of factors available, as workers withdraw from the labor force due to lock-downs or a reduced willingness to work. On the other hand, the epidemic reduces productivity of firms by forcing them to change their production plans. Finally, the epidemic changes final demand across and within periods, as households reduce spending and change the way they consume at given prices.

We characterize the responses of aggregate output, inflation, unemployment to these supply and demand shocks. We also characterize the responses of disaggregated variables like individual sales, prices, and quantities of the different sectors, as well as individual income shares, wages, and Keynesian unemployment of the different factors.[2] The determination of which factor markets become slack and which factor markets become tight is endogenous and highly nonlinear. One of the contributions of the paper is to investigate these forms of nonlinearities.

[1] See Gopinath (2020) for a discussion of why the Covid-19 crisis should be thought as a combination of supply and demand shocks.

[2] Keynesian unemployment measures the amount of slack in a given factor market. When the demand for the factor goes down, and when the wage of the factor cannot fall enough because of downward nominal wage rigidities, some of the available supply of the factor, which may itself have been reduced by a negative supply shock, is not utilized in equilibrium. This happens, for example, if in a labor market, wages cannot fall enough and so some workers would like to work at the going wage but cannot find a job. Measured unemployment in the data reflects not only Keynesian unemployment but also frictional unemployment and classical unemployment, as well as workers who are simply locked down.

We show that to a first order, changes in aggregate output depend only on sales-shares-weighted changes in productivities and factors supplied in equilibrium. Changes in the quantities of factors supplied in turn depend on the structure of the network and on the elasticities of substitution in production and in final demand. This result, which extends Hulten's theorem (1978) to environments with nominal rigidities, is useful to obtain local comparative statics and also as an intermediate step to derive global comparative statics.

We discuss the impact of each of the shocks in turn, starting with the case where there are only negative factor supply shocks. We show that downward nominal wage rigidities always weakly *magnify* the impact on output of negative factor supply shocks: in equilibrium, the shocks can endogenously reduce demand more than supply in some factor markets, push them against their downward nominal wage rigidity constraint, and lead to Keynesian unemployment.

With shocks to factor supply only, we also prove a *network-irrelevance* result: as long as all elasticities of substitution are the same, then conditional on the initial factor income shares, the structure of the input-output network is *globally* irrelevant. In other words, even though there are many sectors and potentially complex and nonlinear supply chains, this information only matters through the initial implied factor income shares as long as the elasticities of substitution are uniform. However, the model's behavior remains very different from that of an aggregate model because the different factor markets endogenously experience different cyclical conditions.

For this network-irrelevant case, we provide global comparative statics. We show that as long as the uniform elasticity of substitution is less than one, so that there are *complementarities*, the set of Keynesian equilibria is a *lattice*. This implies that there is a unique best (worst) equilibrium with the minimal (maximal) number of slack factor markets and the minimal (maximal) amount of Keynesian unemployment in each factor market. In the best and worst equilibria, a reduction in the supply of a factor lowers spending on the other factors. Therefore, a negative factor supply shock in one market depresses the other factor markets. Similarly, a binding downward nominal wage rigidity constraint in one factor market pushes other factor markets towards their constraint.[3] For this case with complementarities, we can use a simple algorithm similar to the one used by Vives (1990) or Elliott et al. (2014) to compute the best and worst equilibria.

[3]The behavior of the model with factor supply shocks is related to Guerrieri et al. (2020), who show that under certain configurations of elasticities of substitution, negative factor supply shocks can cause negative demand spillovers for other factors. However, the economic mechanism in this paper is different, since we focus on complementarities while they focus on substitutability.

We illustrate graphically how the equilibrium changes in response to shocks with an aggregate supply (AS) and aggregate demand (AD) diagram. A novelty of our model is that supply shocks do not simply shift the AS curve, but they also change its shape, resulting in apparent instability of the AS relationship. The unstable shape of the AS curve reflects the nonlinearities arising from the interaction of complementarities and occasionally-binding downward nominal wage rigidities.

Although the model with complementarities is capable of generating negative Keynesian spillovers across factor markets, the effect of these spillovers is mitigated in two ways. First, the factor markets that become slack are precisely those factor markets whose income shares have fallen. Since their income shares have fallen, these factors have become less central to aggregate output. As a result, Keynesian unemployment in those factor markets has a smaller effects on output than would otherwise be expected. Therefore, although negative supply shocks to one factor can cause large amounts of Keynesian unemployment in the others, the effect of this unemployment on output is mitigated. In fact, in the limit of perfect complements, a negative supply shock to one factor leads to mass Keynesian unemployment in the other factor markets, but this unemployment has no effect on output since the marginal product of these other factors goes down to zero.

Second, if there are flexible factor markets that do not experience any negative supply shocks (such as, for example, some capital markets), then these factor markets can act like shock absorbers. Intuitively, the flexible factors' share of income decreases, and this reduces the amount of slack in rigid factor markets.

We also provide local comparative statics outside of the network-neutral case, though global comparative statics are no longer available. We show how elasticities of substitution interact with the input-output network to redirect demand away from some factors and towards others, causing Keynesian unemployment in factor markets where demand goes down more than supply, and further reducing output.

When there are shocks to the composition of demand within the period, say because households demand more groceries and fewer cruises, then there is no network-irrelevance result and the network structure always matters. In this case, changes in household spending across goods changes nominal expenditures in different factor markets, as final demand is distributed to factors through the input-output network. If households shift their spending away from goods that are intensive, directly and indirectly through their supply chains, on potentially-rigid factors, then this results in Keynesian unemployment and reduces output.

Finally, households' intertemporal preferences may change, so that households may choose to spend more in the future and less today. For example, households may be afraid of consuming certain goods in the midst of a pandemic and might, at given prices, prefer to delay spending until the pandemic has abated when consuming these goods become safe again.[4] This acts like a falling tide, lowering nominal expenditures on all factors simultaneously, potentially causing all factor markets to become slack. We call this a negative aggregate demand shock. Like negative supply shocks and shocks to the composition of demand, negative aggregate demand shocks can cause Keynesian unemployment and reduce output. However, unlike those two shocks which are stagflationary, aggregate demand shocks are deflationary.

We use a quantitative input-output model of the US economy to gauge the importance of the various forces that we identify. We find that when labor supply shocks are large and sufficiently homogeneous, the model behaves similarly to a neoclassical one. Although negative supply shocks can cause large amounts of Keynesian unemployment, the impact of this unemployment on output remains moderate (at least in comparison to the neoclassical effect of the negative supply shocks). Similarly the additional inflation resulting from the fact that wages cannot fall remains moderate. However, when labor supply shocks are very heterogeneous, Keynesian effects on output and inflation start to matter more. In addition, if negative supply shocks coincide with a negative but smaller aggregate demand shock, then the aggregate demand shock acts more to mitigate inflation than to further reduce output. Once the negative aggregate demand shock becomes large relative to the negative supply shocks, it reduces both inflation and output.

We extend our basic framework to allow for endogenous supply and demand shocks. In the first extension, we show that occasionally-binding credit constraints on households reduce nominal expenditures overall, acting like negative demand shocks. In the second extension, we show that occasionally-binding credit constraints on firms, by causing firm failures, reduce productivity, acting like negative supply shocks. We show how these endogenous supply and demand shocks can then propagate through the production network and affect output, unemployment, and inflation.

Finally, we analyze policy responses to the Covid-19 shock. We study the effect of fiscal, monetary, and tax incentives such as payroll tax cuts. We show that indiscriminate fiscal stimulus is wasteful and should instead be targeted towards the sectors that use more intensively, directly and indirectly through the network, the factors that are depressed. We

[4] Eichenbaum et al. (2020a) provide a model where such changes in behavior are endogenously motivated.

also show that complementarities in production reduce the effectiveness of these different policies. For example, monetary stimulus (such as, for example, forward guidance) increases the relative price of flexible factors relative to rigid ones, and, if there are complementarities, the stimulus is mitigated because expenditures endogenously shift away from rigid factors and toward flexible ones.

The outline of the paper is as follows. In Section 2, we set up the model, define the equilibrium and notation, and discuss the shocks. In Section 3, we establish the basic comparative statics of the model and illustrate them with a simple example. In Section 4, we establish a network-neutrality result and prove some global comparative statics. In Section 5, we conduct a quantitative exercise to understand the importance of the various mechanisms we have emphasized for the Covid-19 crisis using a series of thought experiments. In Section 6, we include some additional examples which go beyond our quantitative exercise and which illustrate how the network can matter. The rest of the paper contains extensions: in Section 7, we extend the model to include occassionally-binding credit constraints on households; in Section 8, we extend the model to include occassionally-binding credit constraints on firms, causing firm failures; and in Section 9 we investigate monetary, fiscal, and tax policy responses. We conclude in Section 10.

Related Literature

The paper is part of the literature on economic effects of the Covid-19 crisis. Below is a brief description of the most closely related papers. In future versions of the paper, we hope to expand our discussion of other related literature.

Guerrieri et al. (2020) show that under certain configurations of the elasticities of substitution, negative labor supply shocks can cause negative demand spillovers. They focus on substitutabilities whereas we focus on complementarities and the associated nonlinearities from occasionally-binding downward wage rigidity.[5]

Bigio et al. (2020) study optimal policies in response to Covid-19 crisis in a two-sector Keynesian model with a directly affected sector and a sector hit by spillovers. We differ

[5]The economics of these two cases are different because with substituabilities, a negative labor supply shock reduces the share of that labor and reduces aggregate nominal expenditure through intertemporal substitution, if intertemporal substitution outpaces intratemporal substitution, this reduces demand for the other labors despite the increase in their shares. With complementarities instead, the propagation of the shock is not driven by intertemporal substitution but by the fact that a negative labor supply shock increases the share of that labor, which in turn reduces the demand for the other labors; the same logic applies to the endogenous reductions in labor induced by the initial shock, which further reduce the demand for the other labors, etc.

in both focus and framework, since we are not focused on optimal policy and instead try to understand the importance of the production structure.[6]

Fornaro and Wolf (2020) study Covid-19 in a New-Keynesian model where the pandemic is assumed to have persistent effects on productive capacity in the future by lowering aggregate productivity growth. The expected loss in future income reduces aggregate demand. They show that a feedback loop can arise between aggregate supply and aggregate demand if productivity growth in turn depends on the level of economic activity.[7] We differ in that we focus on the effects of current supply disruptions.

Barrot et al. (2020) study the effect of Covid-19 using a quantitative production network with complementarities and detailed administrative data from France. Bodenstein et al. (2020) analyze optimal shutdown policies in a two-sector model with complementarities and minimum-scale requirements. Our approach differs due from these papers due our focus on nominal rigidities and Keynesian effects.

Other economics papers studying the effects of Covid-19 include Eichenbaum et al. (2020a,b), Dingel and Neiman (2020), Berger et al. (2020), Alvarez et al. (2020), Atkeson (2020a,b), Bethune and Korinek (2020), Caballero and Simsek (2020), Faria-e Castro (2020), Gourinchas (2020), Jones et al. (2020), Jorda et al. (2020), Kaplan et al. (2020), Krueger et al. (2020), Jorda et al. (2020), Bodenstein et al. (2020), Barro et al. (2020), Fernández-Villaverde and Jones (2020), Hall et al. (2020), Glover et al. (2020), Bonadio et al. (2020), and Acemoglu et al. (2020).

This paper is also related to previous work by the authors, including Baqaee (2015), studying the effect of shocks to the composition of demand in a production network with downward wage rigidity, Baqaee (2018) and Baqaee and Farhi (2020a), studying the effects of entry and exit in production networks, Baqaee and Farhi (2019), studying the effects of nonlinearities in neoclassical production networks, and most relatedly, Baqaee and Farhi (2020b), a companion paper to this which studies the nonlinear effects of negative supply and demand shocks associated with the Covid-19 crisis using a neoclassical model.

Our analysis is also related to production network models with nominal rigidities, like Pasten et al. (2017) and Pasten et al. (2019) who study propagation of monetary and TFP shocks in models with sticky prices, Ozdagli and Weber (2017) who study the interaction of monetary policy, production networks, and asset prices, and Rubbo (2020) and La'O

[6]Bigio et al. (2020) study a fully dynamic model specified in continuous time, which allows them to analyze how the effects unfold over time.

[7]This could be because of reduced investment in research and development due to a reduced size of the market à la Benigno and Fornaro (2018)

and Tahbaz-Salehi (2020) who study optimal monetary policy with sticky prices.

2 Setup

In this section, we set up the basic model. We break the description of the model in two. First, we discuss the intertemporal problem of how the representative household chooses to spend its income across periods. Second, we discuss the intratemporal problem of how a given amount of expenditures is spent across different goods within a period. We then define the equilibrium notion and discuss the shocks that we will be studying.

2.1 Environment and Equilibrium

There are two periods, the present denoted without stars, and the future denoted with stars, and there is no investment.[8] We take the equilibrium in the future as given. As in Krugman (1998) and Eggertsson and Krugman (2012), this is isomorphic to an infinite-horizon model where after an initial unexpected shock in period 1, the economy returns to a long-run equilibrium with market clearing and full employment. We denote the supply of the future composite final-consumption good by $\bar{Y}_*$, its price by $\bar{p}^Y_*$, and future final income and expenditure by $\bar{I}_* = \bar{E}_* = \bar{p}^Y_* \bar{Y}_*$, which are all taken to be exogenous.

We focus on the present, where there are a set of producers $\mathcal{N}$ and a set of factors $\mathcal{G}$ with supply functions L_f. We denote by $\mathcal{N} + \mathcal{G}$ the union of these sets. We abuse notation and also denote by $\mathcal{N}$ and $\mathcal{G}$ the cardinalities of these sets.

Consumers. The representative consumer maximizes intertemporal utility

$$(1-\beta)\frac{Y^{1-1/\rho}-1}{1-1/\rho} + \beta\frac{Y_*^{1-1/\rho}-1}{1-1/\rho},$$

where ρ is the intertemporal elasticity of substitution (IES) and $\beta \in [0,1]$ captures households' time-preferences. The intertemporal budget constraint is

$$\sum_{i\in\mathcal{N}} p_i c_i + \frac{\bar{p}^Y_* Y_*}{1+i} = \sum_{f\in\mathcal{G}} w_f L_f + \sum_{i\in\mathcal{N}} \pi_i + \frac{\bar{I}_*}{1+i},$$

[8] We abstract from investment in the main body of the paper in order to keep the exposition manageable. However, our approach generalizes to environments with investment. See Appendix A.

where p_i and c_i are the price and final consumption of good i, the nominal interest rate is $(1+i)$, the wage and quantity of factor f are w_f and L_f, and π_i is the profit of producer i. The consumption bundle in the present period is given by

$$Y = \mathcal{D}(c_1, \ldots, c_{\mathcal{N}}; \omega_{\mathcal{D}}),$$

a homothetic final-demand aggregator of the final consumptions c_i of the different goods i. The parameter $\omega_{\mathcal{D}}$ is a preference shifter capturing changes in the composition of final demand.

For future reference, we define the price p^Y of the consumption bundle Y by

$$p^Y = \mathcal{P}(p_1, \ldots, p_{\mathcal{N}}; \omega_{\mathcal{D}}).$$

where $\mathcal{P}$ is the dual price index of the quantity index $\mathcal{D}$. We also denote by

$$E = p^Y Y$$

the present final expenditure. In the rest of the paper, we will refer to Y as *output*.[9]

Producers. Producer i maximizes profits

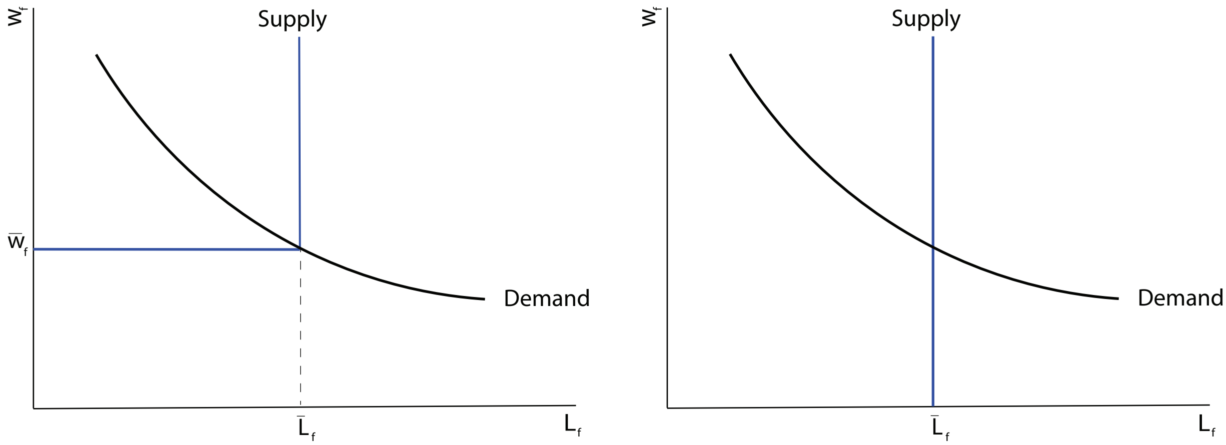
$$\pi_i = \max_{\{y_i\},\{x_{ij}\},\{L_{if}\}} p_i y_i - \sum_{j\in\mathcal{N}} p_j x_{ij} - \sum_{f\in\mathcal{G}} w_f L_{if}$$

using a production function

$$y_i = A_i F_i\left(\left\{x_{ij}\right\}_{j\in\mathcal{N}}, \left\{L_{if}\right\}_{f\in\mathcal{G}}\right),$$

where A_i is a Hicks-neutral productivity shifter, y_i is total output, and x_{ij} and L_{if} are intermediate and factor inputs used by i.

[9]Output here corresponds to welfare. Changes in output and changes in welfare always coincide to the first order, but they do not always coincide to higher orders. Without changes in the preference shifter $\omega_{\mathcal{D}}$, changes in welfare coincide with changes in real GDP at any order. With changes in the preference shifter, changes in changes in welfare do not coincide with changes in real GDP at the second order. See Baqaee and Farhi (2020b) for more information.



(a) Factors in $\mathcal{L}$ (b) Factors in $\mathcal{K}$

Figure 2.1: Equilibrium in the factor markets.

Market equilibrium. Market equilibrium for goods is standard. The market for i is in equilibrium if

$$c_i + \sum_{j \in \mathcal{N}} x_{ji} = y_i.$$

Market equilibrium for factors is non-standard. We assume that the wages of the different factors cannot fall below some exogenous lower bound.[10] We say that factor market f is in equilibrium if the following there conditions hold:

$$(w_f - \bar{w}_f)(L_f - \bar{L}_f) = 0, \quad \bar{w}_f \leq w_{f,} \quad L_f \leq \bar{L}_f,$$

where

$$L_f = \sum_{i \in \mathcal{N}} L_{if}$$

is the total demand for factor f. The parameters $\bar{w}_f$ and $\bar{L}_f$ are exogenous minimum nominal wage and endowment of the factor.

In words, there are two possibilities. One possibility is $w_f \geq \bar{w}_f$ and full employment of the factor with $L_f = \bar{L}_f$. In this case, we say that the wage is flexible and that the market clears. The other possibility is that $w_f = \bar{w}_f$ and less than full employment of the factor with $L_f \leq \bar{L}_f$. We then say that the market is rigid or slack and that it does not clear.

We only consider two cases: the case where $\bar{w}_f$ is equal to its pre-shock market-clearing value, denoted by the set $\mathcal{L} \subseteq G$; and the case where $\bar{w}_f = -\infty$, making the wage of f

[10]In Appendix B, we extend the model to allow for some downward wage flexibility.

flexible and ensuring the market for f always clears, denoted by the subset $\mathcal{K} \subseteq \mathcal{G}$. For concreteness, we call $\mathcal{K}$ the capital factors and $\mathcal{L}$ the labor factors. For future reference, we denote by $\mathcal{F} \subseteq \mathcal{G}$ denote the endogenous equilibrium set of factor markets that clear. In other words, $f \in \mathcal{F}$ if, and only if, $L_f = \bar{L}_f$. We also denote by $\mathcal{R} \subseteq \mathcal{G}$ the endogenous equilibrium set of factor markets that do not clear (rigid factors) so that $f \in \mathcal{R}$ if, and only if, $L_f < \bar{L}_f$. Of course, $\mathcal{K} \subseteq \mathcal{F}$, and $\mathcal{R} \subseteq \mathcal{L}$. Figure 2.1 illustrates the supply and demand curves in the factor markets.

Equilibrium. Given a nominal interest rate $(1 + i)$, factor supplies $\bar{L}_f$, productivities A_i, and demand shifters $\omega_{\mathcal{D}}$, an equilibrium is a set of prices p_i, factor wages w_f, intermediate input choices x_{ij}, factor input choices L_{if}, outputs y_i, and final demands c_i, such that: each producer maximizes its profits subject to its technological constraint; consumers maximize their utility; and the markets for all goods and factors are in equilibrium. Without loss of generality, we normalize $\bar{Y} = \bar{Y} = 1$ so that the expenditure shares on the present and on the future are given by $1 - \beta$ and β.

2.2 Comparative Statics w.r.t. Shocks

We will provide comparative statics with respect to shocks, starting at an initial equilibrium with full employment of all factors. A natural disaster, like the Covid epidemic, can be captured as a combination of negative supply and demand shocks.

Supply shocks. On the one hand, the pandemic causes a reduction in the available endowment of labor $\bar{L}_f$, driven either by government-mandated shutdowns, deaths, or reduced willingness to work.

Similarly, the epidemic might reduced the productivity A_i of the different producers by changing the way firms can operate, for instance by reducing person-to-person interactions.

Demand shocks. Similarly, the pandemic can also change the composition of final demand today, since at given prices, households may shift expenditure away from some goods like restaurants and cruises, and towards other goods like groceries and healthcare. We model this as a change in the preference shifter $\omega_{\mathcal{D}}$.

On the other hand the pandemic can reduce households' willingness to consume today relative to tomorrow: at given prices, households may choose to consume less during the epidemic and more afterwards. We model this as an increase in the discount factor $\beta/(1-\beta)$.

Finally, in an attempt to offset a negative shock, the monetary authority might lower the nominal interest rate $1+i$. It might be restricted in its ability to do so by the binding of the zero lower bound (ZLB). We also allow the monetary authority to increase future prices $\bar{p}_*^Y$. Other policies might also raise future output $\bar{Y}_*$, and other developments might instead lower $\bar{Y}_*$.

2.3 Input-Output Definitions

To analyze the model, we define some input-output objects such as input-output matrices, Leontief inverse matrices, and Domar weights associated with any equilibrium. To make the exposition more intuitive, we slightly abuse notation by treating factors with the same notation as goods. For each factor f, we interchangeably use the notation L_{if} or $x_{i(\mathcal{N}+f)}$ to denote its use by producer i, the notation L_f or y_f to denote total factor supply, and p_f or w_f to refer to its price or wage. This allows us to add factor supply and demand into the input-output matrix along with the supply and demand for goods. Furthermore, we define final demand as an additional good produced by producer 0 according to the final demand aggregator. We interchangeably use the notation c_i or x_{0i} to denote final consumption of good i. We write $1+\mathcal{N}$ for the union of the sets $\{0\}$ and $\mathcal{N}$, and $1+\mathcal{N}+\mathcal{G}$ for the union of the sets $\{0\}$, $\mathcal{N}$, and $\mathcal{G}$. With this abuse of notation, we can stack every market in the economy into a single input-output matrix.

Input-output matrix. We define the input-output matrix to be the $(1+\mathcal{N}+\mathcal{G})\times(1+\mathcal{N}+\mathcal{G})$ matrix Ω whose ijth element is equal to i's expenditures on inputs from j as a share of its total revenues

$$\Omega_{ij} \equiv \frac{p_j x_{ij}}{p_i y_i} = \frac{p_j x_{ij}}{\sum_{k\in\mathcal{N}+\mathcal{G}} p_k x_{ik}}.$$

The input-output matrix Ω records the *direct* exposures of one producer to another, forward from upstream to downstream in costs, and backward from downstream to upstream in demand.

Leontief inverse matrix. We define the Leontief inverse matrix as

$$\Psi \equiv (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \ldots.$$

The Leontief inverse matrix Ψ records instead the *direct and indirect* exposures through the supply chains in the production network. This can be seen from the fact that $(\Omega^n)_{ij}$ measures the weighted sums of all paths of length n from producer i to producer j.

Nominal expenditure and Domar weights. Recall that nominal expenditure is the total sum of all final expenditures

$$E = \sum_{i \in N} p_i c_i = \sum_{i \in N} p_i x_{0i}.$$

We define the Domar weight λ_i of producer i to be its sales share as a fraction of GDP

$$\lambda_i \equiv \frac{p_i y_i}{E}.$$

Note that $\sum_{i \in N} \lambda_i > 1$ in general since some sales are not final sales but intermediate sales. Note that the Domar weight λ_f of factor f is simply its total income share.

The accounting identity $p_i y_i = p_i x_{0i} + \sum_{j \in \mathcal{N}} p_i x_{ji} = \Omega_{0i} E + \sum_{j \in \mathcal{N}} \Omega_{ji} \lambda_j E$ links the Domar weights to the Leontief inverse via

$$\lambda_i = \Psi_{0i} = \sum_{j \in \mathcal{N}} \Omega_{0j} \Psi_{ji},$$

where $\Omega_{0j} = (p_j x_{0j}) / (\sum_{k \in \mathcal{N} + \mathcal{G}} p_k x_{0k}) = (p_j c_j)/E$ is the share of good j in final expenditure.

2.4 Nested-CES Economies

For simplicity, we restrict attention to nested-CES economies. That is, we assume every production function and the final demand function can be written as nested-CES functions (albeit with an arbitrary set of nests). Any nested-CES economy can be written in *standard form*, defined by a tuple $(\bar{\omega}, \theta, F)$. The $(1 + \mathcal{N} + \mathcal{G}) \times (1 + \mathcal{N} + \mathcal{G})$ matrix $\bar{\omega}$ is a matrix of input-output parameters. The $(1 + \mathcal{N}) \times 1$ vector θ is a vector of microeconomic elasticities of substitution. Each good $i \in \mathcal{N}$ is produced with the production function

$$\frac{y_i}{\overline{y}_i} = \frac{A_i}{\bar{A}_i} \left(\sum_{j \in \mathcal{N}+\mathcal{G}} \bar{\omega}_{ij} \left(\frac{x_{ij}}{\overline{x}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}},$$

where x_{ij} are intermediate inputs from j used by i. We represent final demand as the purchase of good 0 from producer 0 producing the final good

$$\frac{y_0}{\overline{y}_0} = \left(\sum_{j \in \mathcal{N}+\mathcal{G}} \bar{\omega}_{0j} \frac{\omega_{0j}}{\bar{\omega}_{0j}} \left(\frac{x_{0j}}{\overline{x}_{0j}} \right)^{\frac{\theta_0 - 1}{\theta_0}} \right)^{\frac{\theta_0}{\theta_0 - 1}},$$

where ω_{0j} is a demand shifter. In these equations, variables with over-lines are normalizing constants equal to the values at some initial competitive equilibrium and we then have $\bar{\omega} = \bar{\Omega}$.[11] To simplify the notation below, we think of ω_0 as a $1 \times (1 + \mathcal{N} + \mathcal{G})$ vector with k-th element ω_{0k}.

Through a relabelling, this structure can represent any nested-CES economy with an arbitrary pattern of nests and elasticities. Intuitively, by relabelling each CES aggregator to be a new producer, we can have as many nests as desired.

3 Benchmark Comparative Statics

In this section, we describe the comparative statics of the basic model and provide some examples. Our results here are general but local (first-order) comparative statics. Below in Section 4, we provide global comparative statics in important special cases.

Because of downward wage-rigidity, variables like aggregate output and inflation are not differentiable everywhere. Therefore, our local comparative statics should be understood as holding almost-everywhere. Furthermore, there are potentially multiple equilibria, in which case, local comparative statics should be understood as perturbations of a given locally-isolated equilibrium.

We write $d \log X$ for the differential of an endogenous variable X, which can also be

[11]Note that when mapping the original economy to the re-labeled economy, the different nests in final demand are mapped intro different producers j. To simplify the exposition, we have imposed that there are only demand shocks in the outermost nest mapped to producer 0. It is easy to generalize the results to allow for demand shocks in all the nests corresponding to final demand.

understood as the (infinitesimal) change in an endogenous variable X in response to (infinitesimal) shocks. For example, the supply shocks are $d\log A_i$ and $d\log \bar{L}_f$, and the shocks to the composition of demand are $d\log \omega_{0j}$. We sometimes write them in vector notation as $d\log A$, $d\log \bar{L}$, and $d\log \omega_0$.

3.1 Euler Equation for Output

Log-linearizing the Euler equation gives changes in output $d\log Y$ as a function of changes in the price index $d\log p^Y$ and the shocks

$$d\log Y = -\rho d\log p^Y + d\log \zeta, \tag{3.1}$$

where

$$d\log \zeta = -\rho\Big(d\log(1+i) + \frac{1}{1-\beta} d\log \beta - d\log \bar{p}_*^Y\Big) + d\log \bar{Y}_*$$

is an *aggregate demand* shock. A positive aggregate demand shock can come about from a reduction in the nominal interest rate or the discount factor, or an increase in future prices or output (a proxy for forward guidance).

Equation (3.1) implies that unless there are negative aggregate demand shocks, supply shocks move output and prices in opposite directions. Negative supply shocks are hence *stagflationary*. Negative aggregate demand shocks are necessary in order to produce a reduction in both output and prices.[12] These remarks for supply shocks also apply to shocks to the intratemporal composition of demand.

3.2 Euler Equation for Spending

Changes in nominal expenditure $d\log E$ are similarly given by

$$d\log E = d\log(p^Y Y) = (1-\rho) d\log p^Y + d\log \zeta. \tag{3.2}$$

Recall that ρ is the intertemporal elasticity of substitution (IES). When $\rho > 1$, increases in prices $d\log p^Y > 0$ reduce nominal expenditure as consumers substitute towards the future. Conversely, when $\rho < 1$, increases in prices $d\log p^Y > 0$ increase nominal expenditure as consumers substitute towards the present. When $\rho = 1$, changes in nominal

[12] In section 7, we extend the basic model to include some credit-constrained or hand-to-mouth households, and this provides an endogenous mechanism for delivering negative aggregate demand shocks.

expenditure are exogenously given by the shocks $d\log E = d\log\zeta$. Although our propositions allow for arbitrary values of ρ, we will focus primarily on the case where $\rho = 1$, abstracting from intertemporal substitution.

3.3 Output

We can express changes in output as a function of changes in nominal expenditure and changes in factor shares.

Proposition 1. *Changes in output are given by*

$$\begin{aligned} d\log Y &= \sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log L_f, \\ &= \underbrace{\sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log \bar{L}_f}_{\text{neoclassical effect}} + \underbrace{\sum_{f\in\mathcal{L}} \lambda_f \min\left\{d\log\lambda_f + d\log E - d\log\bar{L}_f, 0\right\}}_{\text{Keynesian effect}}. \end{aligned}$$

The first expression for $d\log Y$ shows that a version of Hulten's theorem holds for this economy. In particular, to a first-order, changes in output can only be driven by changes in the productivities $d\log A_i$ weighted by their producer's sales share λ_i, or by changes in the quantities of factors $d\log L_f$ weighted by their income shares λ_f.[13]

The second expression uses the fact that while changes in capitals $f \in \mathcal{K}$ are exogenous with $d\log L_f = d\log\bar{L}_f$, changes in labors $f \in \mathcal{L}$ are endogenous with $d\log L_f = d\log\bar{L}_f + \min\left\{d\log\lambda_f + d\log E - d\log\bar{L}_f, 0\right\} \leq d\log\bar{L}_f$. Therefore, in the Keynesian model with downward nominal wage rigidities, negative shocks can be amplified relative to a neoclassical economy with flexible wages where all factor markets clear at full employment. To a first-order, the response of output in the Keynesian model is the same as the neoclassical model, plus a negative adjustment for Keynesian unemployment which depends on endogenous changes in nominal expenditure $d\log E$ and factor income shares $d\log\lambda_f$. It is only through the determination of these endogenous sufficient statistics that the structure of the network matters.

[13]This expression also shows that changes in the composition of demand within the period $d\log\omega_0$, or changes in aggregate demand $d\log\zeta$, can only change output through changes in the quantities of factors.

3.4 Shares, Prices, and Factor Employment

We make use of the following notation. For a matrix M, we denote by $M_{(i)}$ its i-th row by $M^{(j)}$ its j-th column. We write $Cov_{\Omega^{(j)}}(\cdot,\cdot)$ to denote the covariance of two vectors of size $1+\mathcal{N}+\mathcal{G}$ using the j-the row of the input-ouput matrix $\Omega^{(j)}$ as a probability distribution.

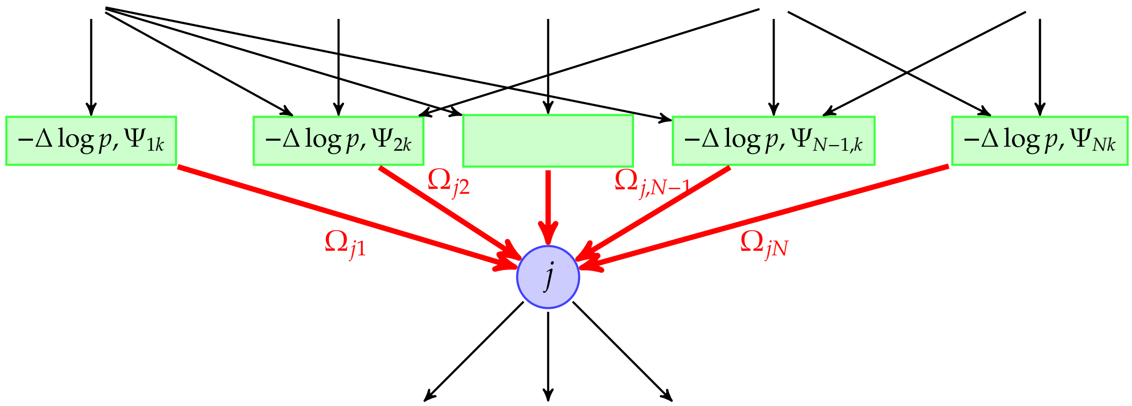


Figure 3.1: Graphical illustration of the IO covariance operator.

Proposition 2. *Changes in sales and factor shares are given by*

$$d\log\lambda_k = \theta_0 Cov_{\Omega^{(0)}}\left(d\log\omega_0, \frac{\Psi_{(k)}}{\lambda_k}\right) + \sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j-1)Cov_{\Omega^{(j)}}\left(\sum_{i\in\mathcal{N}}\Psi_{(i)}d\log A_i - \sum_{f\in\mathcal{G}}\Psi_{(f)}\left(d\log\lambda_f - d\log L_f\right), \frac{\Psi_{(k)}}{\lambda_k}\right)$$

almost everywhere, where changes in factor employments are given by

$$d\log L_f = \begin{cases} d\log\bar{L}_f, & \textit{for} \quad f\in\mathcal{K}, \\ \min\left\{d\log\lambda_f + d\log E, d\log\bar{L}_f\right\}, & \textit{for} \quad f\in\mathcal{L}. \end{cases}$$

We can break down these equations into forward and backward propagation equations. Forward propagation equations describe changes in prices:

$$d\log p_k = -\sum_{i\in\mathcal{N}}\Psi_{ki}d\log A_i + \sum_{f\in\mathcal{G}}\Psi_{kf}\left(d\log\lambda_f + d\log E - d\log L_f\right).$$

Changes in prices propagate downstream (forward) through costs. A negative productivity shock $\Delta\log A_i$ to a producer i upstream from k increases the price of k in proportion to how much k buys from i directly and indirectly as measured by Ψ_{ki}. Similarly an increase $d\log w_f = d\log\lambda_f - d\log L_f + d\log E$ in the wage of factor f increases the price of k in proportion to the direct and indirect exposure of k to f.

Backward propagation equations describe changes in sales or factor shares:

$$d\log\lambda_k = \theta_0 Cov_{\Omega^{(0)}}\left(d\log\omega_0, \Psi_{(k)/\lambda_k}\right) + \sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j - 1)Cov_{\Omega^{(j)}}\left(-d\log p, \Psi_{(k)}/\lambda_k\right).$$

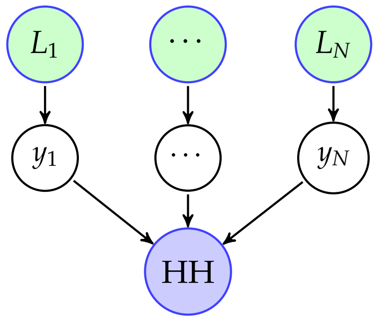
Changes in sales propagate upstream (backward) through demand. The first term on the right-hand side $\theta_0 Cov_{\Omega^{(0)}}(d\log\omega_0, \Psi_{(k)}/\lambda_k)$ on the right-hand side is the direct effect of shocks to the composition of final demand on the sales of k. These shocks directly increase the share of k if they redirect demand towards goods j that have high direct and indirect exposures to k relative to the rest of the economy as measured by Ψ_{jk}/λ_k to k .

The second term $\sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j - 1)Cov_{\Omega^{(j)}}(-d\log p, \Psi_{(k)}/\lambda_k)$ on the right-hand side captures the changes in the sales of i from substitutions by producers j downstream from k. This is depicted in Figure 3.1. If producer j has an elasticity of substitution θ_j below one so that its inputs are complements, then it shifts its expenditure towards those inputs l with higher price increases $d\log p_l$, and this increases the demand for k if those goods l buy a lot from k directly and indirectly relative to the rest of the economy as measured by Ψ_{lk}/λ_k. These expenditure-switching patterns are reversed when θ_j is above one (the inputs of j are substitutes). When θ_j is equal to one (the inputs of j are Cobb-Douglas) these terms disappear.

Note that once a factor market f becomes slack, the change in its income share $d\log\lambda_f$ becomes irrelevant for changes in all the other sales and factor shares since they then translate one for one into changes in employment of the factor $d\log L_f$ and leave its wage unchanged with $d\log w_f = 0$.

3.5 A (Somewhat) Universal Example

In section 6 we work through some illustrative examples of supply and demand shocks, showing how supply and demand shocks propagate up and down supply chains. However, for now, we instead focus on a simpler example which will nevertheless prove to contain an element of universality. For this example, we assume that the intertemporal elasticity of substitution is $\rho = 1$, so that changes in nominal expenditures are equal to negative aggregate demand shocks $d\log E = d\log\zeta$.



$$Y/\bar{Y} = \left(\sum_i \bar{\lambda}_i (y_i/\bar{y}_i)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$
$$y_i/\bar{y}_i = L_i/\bar{L}_i,$$
$$L_i = \min\{\bar{L}_i, \lambda_i E/\bar{w}_i\}$$
$$w_i = \max\{\lambda_i E/\bar{L}_i, \bar{w}_i\}.$$

Figure 3.2: Horizontal Economy. The arrows represent the flow of resources for production. Each sector has its own factor market.

Consider the horizontal economy depicted in Figure 3.2. We call it horizontal because there are no intermediate inputs. Each sector produces linearly with its own labor and sells directly to the household who substitutes across them with an elasticity of substitution $\theta < 1$. Labor cannot be reallocated across sectors, and so there are as many labor markets as there are sectors. These different labor markets are all potentially rigid ($\mathcal{L} = \mathcal{G}$ and $\mathcal{K} = \varnothing$). We introduce negative labor supply shocks $d \log \bar{L}_f \leq 0$ in the different sectors. To start with, suppose that there are neither shocks to the composition of demand ($d \log \omega_{0j} = 0$) nor aggregate demand shocks ($d \log \zeta = 0$).

Recall that $\mathcal{F}$ and $\mathcal{R}$ are the equilibrium sets of flexible and rigid factors. We give comparative statics conditional on the post-shock equilibrium being in the interior of the set of equilibria that have the same sets of flexible and rigid factors. We then give conditions for these sets of flexible and rigid factors to indeed arise in equilibrium.

We define the average negative labor supply shock to the flexible factors

$$d \log \bar{L}_{\mathcal{F}} = \sum_{f \in \mathcal{F}} \frac{\lambda_f}{\lambda_{\mathcal{F}}} d \log \bar{L}_f,$$

where $\lambda_{\mathcal{F}} = \sum_{f \in \mathcal{F}} \lambda_f$, as well as the average employment change in the rigid factors

$$d \log L_{\mathcal{R}} = \sum_{f \in \mathcal{R}} \frac{\lambda_f}{\lambda_{\mathcal{R}}} d \log L_f < \sum_{f \in \mathcal{R}} \frac{\lambda_f}{\lambda_{\mathcal{R}}} d \log \bar{L}_f = d \log \bar{L}_{\mathcal{R}},$$

where $\lambda_{\mathcal{R}} = \sum_{f \in \mathcal{R}} \lambda_f$. Keynesian unemployment is given by $d \log L_{\mathcal{R}} - d \log \bar{L}_{\mathcal{R}}$.

By Proposition 2, the change in the share of a factor f is given by

$$d\log\lambda_f = (\theta-1)\Big(\sum_{g\in\mathcal{F}}\lambda_g\left(d\log\lambda_g - d\log\bar{L}_g\right) - \left(d\log\lambda_f - L_f\right)\Big).$$

Summing across all flexible factors and solving the resulting linear equation gives changes in total spending on flexible factors

$$\lambda_{\mathcal{F}}d\log\lambda_{\mathcal{F}} = -\frac{(1-\theta)(1-\lambda_{\mathcal{F}})\lambda_{\mathcal{F}}d\log\bar{L}_{\mathcal{F}}}{1-(1-\theta)\,(1-\lambda_{\mathcal{F}})}.$$

This can be used to deduce average changes in employment in the rigid factors

$$\lambda_{\mathcal{R}}d\log L_{\mathcal{R}} = \sum_{f\in\mathcal{R}}\lambda_f d\log L_f = \sum_{f\in\mathcal{R}}\lambda_f d\log\lambda_f = -\sum_{f\in\mathcal{F}}\lambda_f d\log\lambda_f = -\lambda_{\mathcal{F}}d\log\lambda_{\mathcal{F}}.$$

A negative effective supply shock $d\log\bar{L}_{\mathcal{F}} < 0$ increases the shares of the flexible factors. The shock increases the wages of the flexible factors, which redirects expenditure towards their sectors because of complementarities, which further increases the wages of the flexible factors, etc. ad infinitum. Of course, if spending on flexible sectors increases, then spending on rigid sectors decreases, and this reduces employment in those sectors because wages cannot fall.

Using Proposition 1, we can see that Keynesian channels amplify the output effect of the negative supply shocks to the flexible factors since

$$d\log Y = \lambda_{\mathcal{F}}d\log\bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}}d\log L_{\mathcal{R}} = \frac{\lambda_{\mathcal{F}}d\log\bar{L}_{\mathcal{F}}}{1-(1-\theta)\,(1-\lambda_{\mathcal{F}})}. \tag{3.3}$$

The direct impact on output of the negative supply shock to the flexible factors is given by $\lambda_{\mathcal{F}}d\log\bar{L}_{\mathcal{F}}$, and the amplification of this shock through Keynesian channels is given by the multiplier $1/[1-(1-\theta)(1-\lambda_{\mathcal{F}})]$. Naturally, amplification is stronger, the lower is the elasticity of substitution $\theta < 1$. Amplification is also stronger when the share of the flexible factors $\lambda_{\mathcal{F}}$ is low.

We now go back and check that our conjectured set of flexible factors is indeed the equilibrium set of flexible factors. A given factor f, which is potentially rigid, is actually rigid in equilibrium if, and only if,

$$d\log \bar{L}_f > \frac{(1-\theta)\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}.$$

That is, as long as the negative shock to factor f is sufficiently small in magnitude compared to the average shock affecting the flexible part of the economy. This condition is harder to satisfy the smaller is the set of flexible factors $\lambda_{\mathcal{F}}$ and the higher is the elasticity of substitution θ. In particular, if we had assumed that sectors were substitutes $\theta \geq 1$ instead of being complements with $\theta < 1$, then this condition could not be satisfied and all factors would be flexible.

This condition also shows that Keynesian channels amplify the shock compared to the neoclassical economy with flexible wages since

$$d\log Y = \underbrace{\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d\log \bar{L}_{\mathcal{R}}}_{\text{neoclassical effect}} + \underbrace{\sum_{f\in\mathcal{R}} \lambda_f \Big(\frac{(1-\theta)\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} - d\log \bar{L}_f\Big)}_{\text{Keynesian effect}},$$

where the Keynesian effect is always negative. Note that here, as in Proposition 1, Keynesian amplification is defined relative to the output reduction that would take place in response to the negative factor supply shocks *to all the factors* in a neoclassical economy. This notion is related to but different from the Keynesian amplification of the negative factor supply shocks to the flexible factors, that we used to discuss equation 3.3. This different notion is defined relative to the output reduction that would take place in response to the negative factor supply shocks *to the flexible factors* in a neoclassical economy, and it is also informative since the factor supply shocks to the rigid factors have no impact on the equilibrium because these markets are slack.

If in addition to the negative labor supply shocks, there were also shocks to the composition of demand $d\log \omega^0$ and to aggregate demand $d\log \zeta < 0$, then the response of output would become

$$d\log Y = \frac{\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} - \frac{\theta\lambda_{\mathcal{F}} d\log \omega_{0\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} + \left(1 - \frac{(1-\theta)\lambda_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}\right)(1-\lambda_{\mathcal{F}}) d\log \zeta,$$

where $d\log \omega_{0\mathcal{F}} = \sum_{f\in\mathcal{F}} (\lambda_f/\lambda_{\mathcal{F}}) d\log \bar{\omega}_{0f}$. The second term on the right-hand side captures the fact that if consumers redirect expenditure towards flexible factors and away from rigid factors, then this exacerbates Keynesian unemployment in rigid factors and further reduces output. The third term is the effect of the negative aggregate demand shock. The direct effect of the negative aggregate demand shock, captured by $(1 - \lambda_{\mathcal{F}}) d \log \zeta$, is to lower employment in rigid factor markets and to reduce output. This direct effect is mitigated because the shock lowers the prices of flexible factors, bringing them closer to those of rigid factors, and triggering expenditure switching away from flexible factors and towards rigid ones, as captured by $-[(1-\theta)\lambda_{\mathcal{F}}/(1-(1-\theta)(1-\lambda_{\mathcal{F}}))](1-\lambda_{\mathcal{F}}) d \log \zeta$.

4 Network Irrelevance

Proposition 2 shows that information about the whole input-output network is required to compute counterfactuals. However, in some cases, there are very simple sufficient statistics that summarize this information. In this section, we focus on a useful special case of the model where the input-output structure is summarized by the initial factor shares. The disaggregated nature of the model remains critical because the different factor markets endogenously experience different cyclical conditions.

This special case is useful for several reasons. First, it shows that many of the intuitions that hold in the horizontal economy above can be applied to economies with a network structure. So, there is a sense in which the horizontal economy example is universal. Second, network-neutrality clarifies exactly what ingredients are necessary for the production network to matter beyond sales and factor shares. Third, we are able to obtain not only local but also global comparative statics.

We assume that the intertemporal elasticity of substitution is $\rho = 1$, that all the elasticities in production and in final demand are the same with $\theta_j = \theta$ for all $j \in 1 + \mathcal{N}$, and that there are no productivity shocks $\Delta \log A = 0$ and no shocks to the composition of demand $\Delta \log \omega_0 = 0$. We can then write changes in output $\Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega})$ as a function of discrete labor supply shocks $\Delta \log \bar{L}$, aggregate demand shocks $\Delta \log \zeta$, and the initial input-output matrix $\bar{\Omega}$. We use Δ to denote discrete global changes to distinguish them from infinitesimal local changes which we denote with d.

4.1 A Network-Irrelevance Result

The next proposition shows that Y depends on the input-output network $\bar{\Omega}$ *only* through the initial factor shares $\bar{\lambda}_f$ for $f \in \mathcal{G}$.

Proposition 3. *Suppose that the intertemporal elasticity of substitution is $\rho = 1$ and that the elasticities of substitution in production and in final demand are all the same with $\theta_j = \theta$ for every $j \in 1 + \mathcal{N}$. Suppose that there are only factor supply shocks $\Delta \log \bar{L}$ and aggregate demand shocks $\Delta \log \zeta$ but no productivity shocks and no shocks to the composition of demand. Then*

$$\Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}) = \Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}')$$

for every $\bar{\Omega}$ and $\bar{\Omega}'$ such that $\bar{\lambda}_f = \bar{\Psi}_{0f} = \bar{\Psi}'_{0f} = \bar{\lambda}'_f$ for every $f \in \mathcal{G}$.

An implication is that the local comparative statics for the horizontal economy in Section 3.5 actually apply much more generally. In particular, they apply to *any* production network as long as the elasticities of substitution in production and final demand are uniform.

Another implication of this proposition is that the network can only matter globally beyond the initial factor shares if: the elasticities of substitution are different or if there are shocks to the composition of demand or to productivities.

4.2 Lattice Structure and Global Comparative Statics

In general, the equilibrium of the Keynesian model is not unique. However, for the network-neutral case, we can prove there are simple-to-compute unique "best" and "worst" equilibria as long as there are complementarities ($\theta < 1$). We can also provide global comparative statics for these equilibria.

To state our result, we endow $\mathbb{R}^{\mathcal{G}}$ with the partial ordering $x \leq y$ if and only if $x_f \leq y_f$ for all $f \in \mathcal{G}$. Formally, we show that set of equilibrium values of the changes in factor quantities $\Delta \log L$ is a complete lattice under the partial ordering $\leq$ defined by $\Delta \log L' \leq \Delta \log L$ if and only if $\Delta \log L'_f \leq \Delta \log L_f$ for all $f \in \mathcal{G}$.

Proposition 4. *Under the assumptions of Proposition 3, and assuming in addition that $\theta < 1$, there is a unique best and worst equilibrium: for any other equilibrium, $\Delta \log Y$ and $\Delta \log L$ are lower than at the best and higher than at the worst. Furthermore, both in the best and in the worst equilibrium, $\Delta \log Y$ and $\Delta \log L$ are increasing in $\Delta \log \bar{L}$ and in $\Delta \log \zeta$.*

The global comparative static result in the proposition generalize the insight of the horizontal economy in Section 3.5. In particular, negative labor supply shocks in some factor markets create Keynesian unemployment in other factor markets. This would not happen with substitutes ($\theta \geq 1$) where negative labor supply shocks would not lead to any Keynesian unemployment in any factor market.

Proposition 4 also provides a straightforward way to compute this best equilibrium using a greedy algorithm along the lines of Vives (1990) or, more recently, Elliott et al. (2014). We can find the best equilibrium as follows. Solve the model assuming all factor markets are flexible. If one of the wages is below the minimum, call this market rigid and set its wage equal to its lower bound. Recompute the equilibrium assuming that these factor markets are rigid. Continue in this manner until the wage in every candidate flexible market is above its lower bound. The worst equilibrium can be found in the same way but starting from the assumption that all markets are rigid, and checking at every step if a priori rigid markets have employments below their endowments.

4.3 AS-AD Representation

We can represent the best equilibrium as the point at which an aggregate supply and aggregate demand curve intersect. The AD curve, which is a decreasing log-linear relationship, is given by the Euler equation, and relates aggregate output to the price level today. Deriving the AS curve is less straightforward. To do so, fix some level of output Y. There is a price level $p^Y(Y)$ such that: given the implied level of expenditure $E(Y) = p^Y(Y)Y$, the wage of every factor is consistent with the amount of expenditures on that factor; and these wages give rise to prices that are consistent with $p^Y(Y)$.

An example is plotted in Figure 4.1 at the initial equilibrium in the absence any exogenous shock. The downward slope of the left-side of the AS curve depends on the downward flexibility of factor prices. If the set of flexible factors is empty ($\mathcal{K} = \emptyset$), then the AS curve is horizontal to the left. If the set of rigid factors is empty ($\mathcal{L} = \emptyset$), then the AS curve is vertical to the left. Of course, in the case when there are no potentially-rigid factor markets, we recover the neoclassical model.

Since the AD curve is just the Euler equation, aggregate demand shocks $d \log \zeta$ shift the AD curve in the usual way, and it is easy to see from this figure that a negative aggregate demand shock reduces present prices and output. The AS curve, on the other hand, does not have a simple closed-form representation, and supply shocks transform the shape of the AS curve in non-obvious ways. In the next few subsections, we consider how different negative shocks affect output and inflation, and we illustrate our findings using nonlinear AS-AD diagrams.

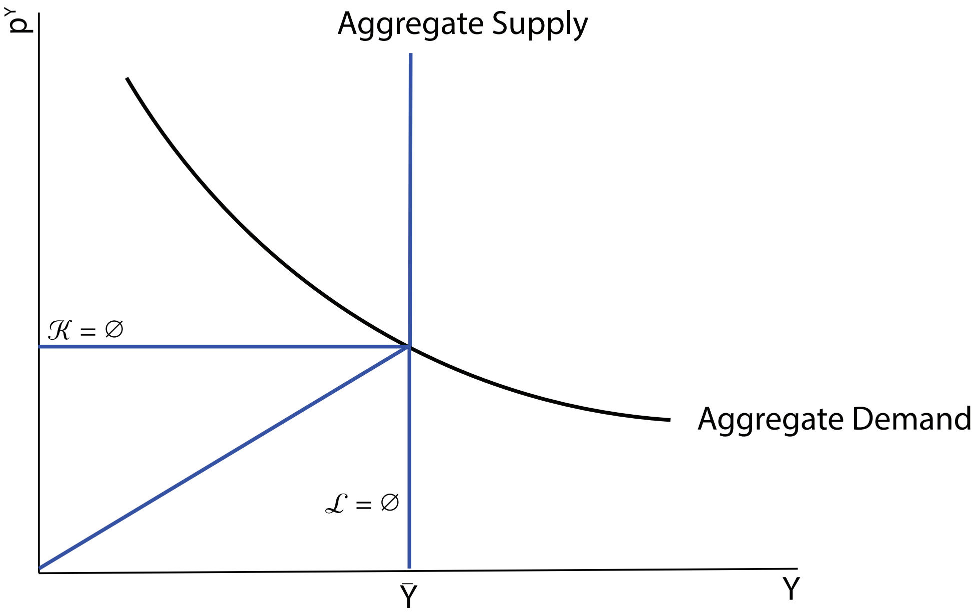


Figure 4.1: AS-AD representation of the equilibrium without shocks. The $\mathcal{K} = \varnothing$ case is when all factors are potentially rigid, and $\mathcal{L} = \varnothing$ case is when all factors are always flexible.

4.4 Hump-Shaped Effect of Complementarities

As discussed earlier, complementarities across producers can transmit negative supply shocks in one factor market as negative demand shocks to other factor markets. This negative spillover is larger, the stronger are the complementarities. In other words, the amount of Keynesian unemployment in the rigid factor markets is decreasing as a function of the elasticity of substitution θ.

In Figure 4.2, we plot an example for a uniform-elasticity economy with two equally-sized factor markets. Both factors are labors and are therefore potentially rigid. There are no capitals ($\mathcal{K} = \varnothing$). We feed a 20% negative shock the supply of one of the factors. When there are complementarities ($\theta < 1$), the negative supply shock in one factor market causes the downward nominal wage rigidity to bind and triggers Keynesian unemployment in the other factor market. By contrast, with substitutability ($\theta \geq 1$), the downward nominal wage rigidity constraint does not bind in any of the two factor markets and the model behaves exactly like the neoclassical one.

However, the strength of this effect on output is hump-shaped in the elasticity of substitution. In Figure 4.3, we plot the change in output in the Keynesian model with downward wage rigidity against the response of the neoclassical model with flexible

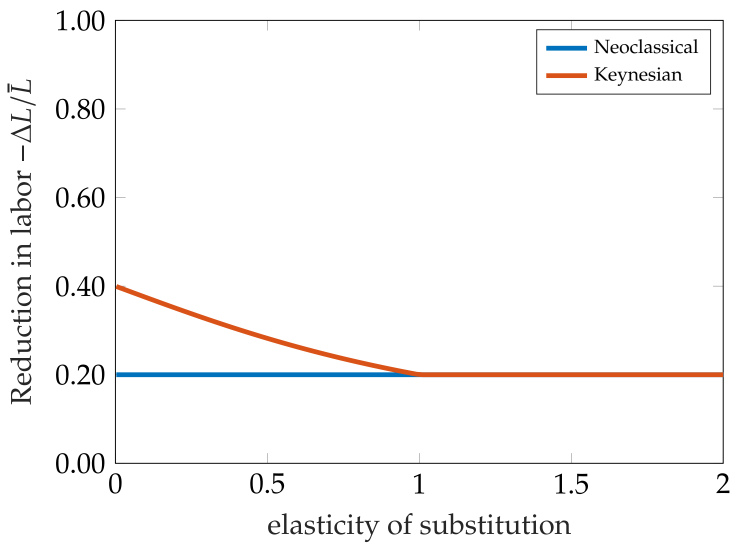


Figure 4.2: The change in the quantity of labor supplied in the neoclassical (flexible wage) and Keynesian example as a function of the elasticity of substitution.

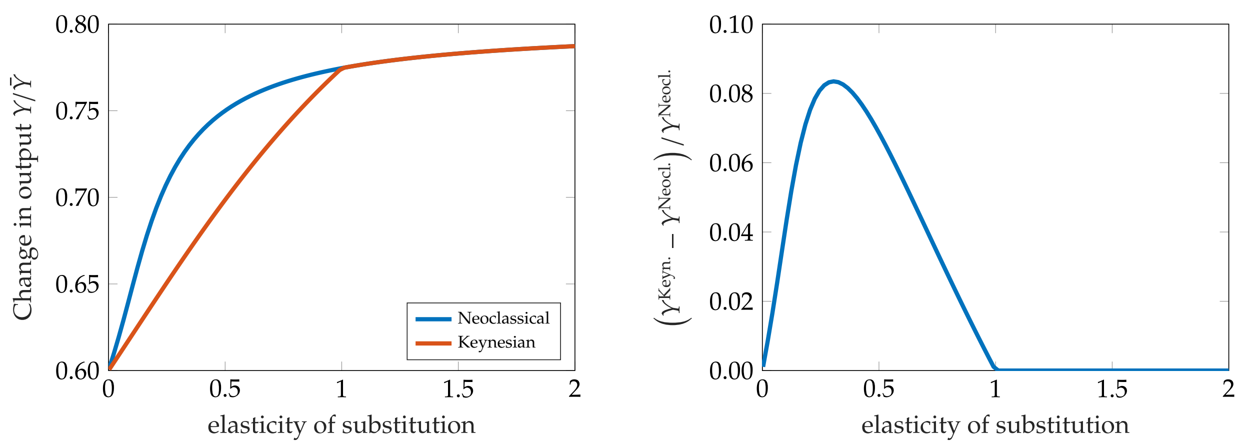


Figure 4.3: The panel on the left shows the change in output, in a neoclassical (flexible wage) and Keynesian example, in response to a reduction in one sector's labor as a function of the elasticity of substitution. The panel on the right shows the percentage difference between the neoclassical and Keynesian models.

wages. As we already discussed, the behavior of output in the two models coincides when $\theta \geq 1$ but diverges as soon as $\theta < 1$. However, the behavior of the two models coincides again as θ approaches zero.

Intuitively, as complementarities become stronger, the marginal product of the rigid factor falls more. Output is more and more determined by the productive capacity of the negatively shocked flexible factor. In other words, as complementarities become stronger,

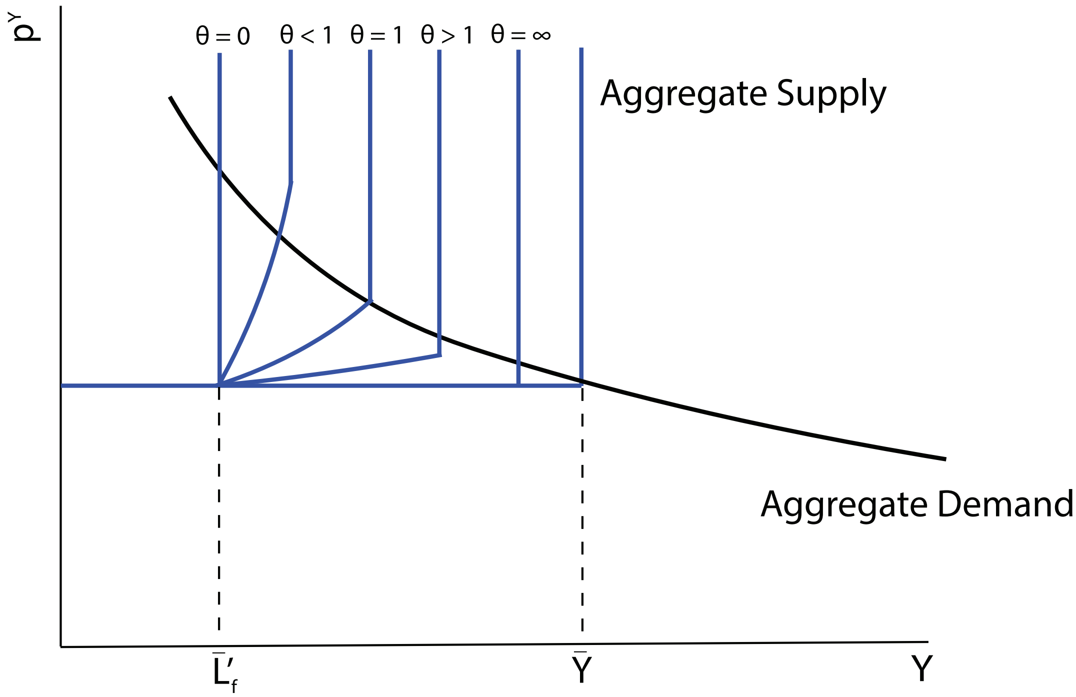


Figure 4.4: The effect of the same negative supply shock to a factor for different values of the elasticity of substitution θ.

the income share of the non-shocked rigid factor falls more in response to the negative shock to the flexible factor, and, as a result, the rigid factor becomes less critical and its Keynesian unemployment matters less for output. In particular, in the perfect complement limit, the unemployed workers in the rigid factor market have a marginal product of zero, and so their loss is irrelevant for output.[14]

Figure 4.4 represents this negative supply shock using an AS-AD diagram. The AS curve is horizontal to the left since there are no capitals ($\mathcal{K} = \varnothing$). The initial level of output is given by $\bar{Y}$ and the new level of labor available in the shocked sector is given by $\bar{L}'_f$. The negative supply shock shifts the AS curve to the left.

In the figure, we draw the new AS curve for different values of the elasticity of substitution θ. Unlike standard models, in this model, the shape of the AS curve itself changes in response to supply shocks. In particular, the negative labor supply shock introduces two kinks into the AS curve. The first kink is the point at which the AS curve becomes

[14] The non-montonic pattern in Figure 4.3 does not show in a linear approximation, and so does not appear in equation (3.3). Intuitively, as the negative supply shock gets smaller, the hump in Figure 4.3 moves towards the left and is pressed up against the axis, and so the amplification of output reductions is increasing in the degree of complementarities over a bigger range. In the limit of infinitesimal shocks, this range becomes complete.

horizontal, and the second kink is the point at which the AS curve becomes vertical. The first kink always occurs at the point where $Y = \bar{L}'_f$. Intuitively, this is the level of aggregate output that would cause the shocked sector itself to become rigid The second kink, on the other hand, moves as we vary the elasticity of substitution.

As we lower the elasticity of substitution θ, the kink point at which the AS curve becomes vertical shifts north-westwards. As long as $\theta > 1$, the second kink is below the AD curve, and so the equilibrium is the same as the neoclassical one, because the AS and AD intersect along the neoclassical portion of the AS curve. Intuitively, when $\theta > 1$, no factor market becomes rigid and so downward nominal wage rigidity is never triggered. Once the elasticity of substitution has been lowered to $\theta = 1$, the Cobb-Douglas case, the second kink exactly intersects the AD curve. As θ goes below one, the second kink moves above the AD curve, downward nominal wage rigidities are triggered, and the equilibrium has lower output and higher inflation than the neoclassical model. Finally, as θ goes to zero and we approach the Leontief case, the second kink point moves directly above the first kink point, and so the reduction in output in the neoclassical model and Keynesian model become the same again.

4.5 Keynesian Amplification of Heterogeneous Shocks

Next, we consider how heterogeneity in the size of the shock affects the equilibrium. In Figure 4.5, we consider the same example as in Section 4.4, but we now allow for negative supply shocks in both factor markets.

First consider the case where there is only a negative supply shock to one of the factors. The shock shifts the AS curve back and introduces two kinks. It results in Keynesian unemployment and a reduction in output over and above that which takes place in the neoclassical model represented by $\bar{Y}'$.

Now, consider the case where there is also a negative supply shock of the same magnitude to the other factor market, so that the negative supply shock is now uniform across the two factor markets. The kink disappears, output falls to its neoclassical level $\bar{Y}''$, and there are no longer any Keynesian forces in the model: downward nominal wage rigidities do not bind in any factor market, there is no Keynesian unemployment, and there is no Keynesian amplification of output reductions. Once again, this is because the first and second kink are now directly on top of each other.

The lesson is that we should expect Keynesian forces from negative factor supply

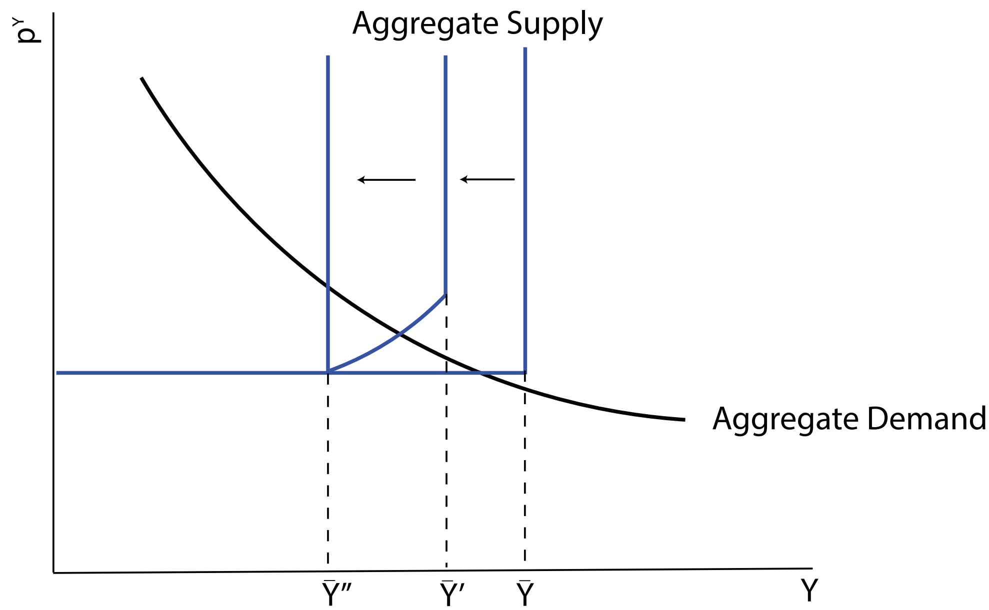


Figure 4.5: Negative labor supply shocks in a two-sector model, $\bar{Y}$ is output without any shocks, $\bar{Y}'$ is output with shocks to only one sector, and $\bar{Y}''$ is output with shocks to both sectors.

shocks to be stronger when the shocks are more heterogeneous. If the negative supply shocks are more homogeneous, then it is less likely that supply outstrips demand in any factor market. Indeed, when the shock uniformly affects all factor markets together, then relative factor prices do not change, all factor prices increase, and the nominal rigidities are never triggered.

Covid-19 plausibly caused a heterogeneous shock to labor supply, since it affected labor supply in some sectors much more severely than in others. Whereas many white-collar jobs can be done at home, most blue-collar work require workers to work in close proximity to each other and to their clients.[15] This means that lock-downs disproportionately affect some sectors, and the more heterogeneous are these effects, the more likely they are to trigger Keynesian unemployment.

4.6 Interaction of Negative Supply and Demand Shocks

Next, we show how negative factor supply shocks and aggregate demand shocks interact with one another. In Figure 4.6, we show how the equilibrium responds to a negative labor supply shock together with a negative aggregate demand shock, assuming there

[15] See e.g. Mongey et al. (2020).

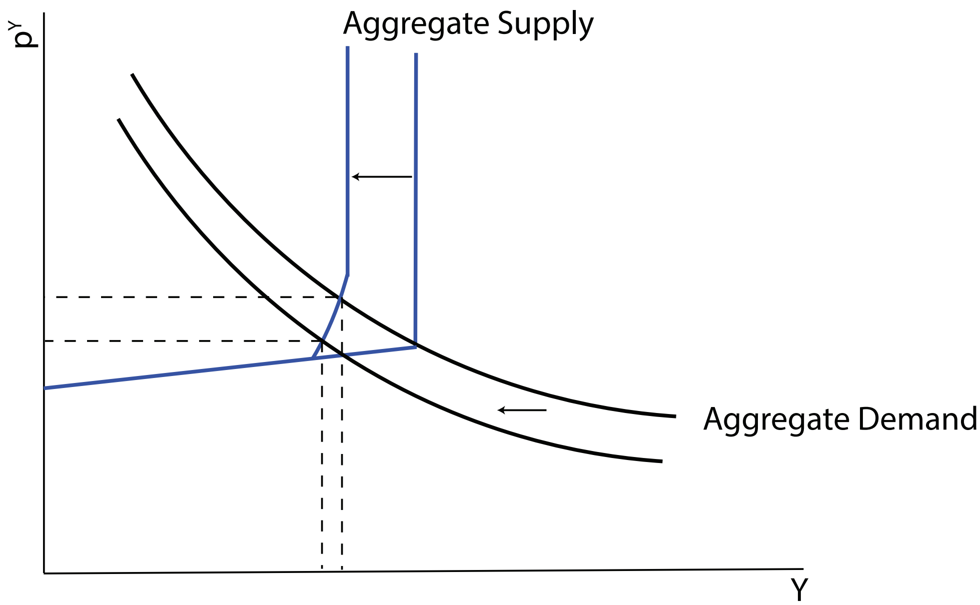


Figure 4.6: Negative labor supply shock coupled with a negative aggregate demand shock.

are complementarities. We deviate from the example of Section 4.4 by allowing for more than two factors, and by allowing for potentially-rigid labors and always flexibe capitals ($\mathcal{K} \neq \emptyset$).

As expected, the negative aggregate demand shock shifts the AD curve backwards. If there are no supply shocks, then aggregate demand shocks are potent, causing output to fall by a lot. If there are some capitals ($\mathcal{K} \neq \emptyset$), then the aggregate demand shock can also reduce prices a lot.

Now, consider what happens if the negative aggregate demand shock coincides with negative supply shocks. As usual, the negative labor supply shock introduces kinks into the AS curve and shifts the curve backwards. In equilibrium, the effect of the negative AD shock is now much less potent for output. In fact, in the extreme case where the first and second kink are on top of each, the negative aggregate demand shock has no effect on output unless it is very large. However, even though the negative supply shock blunts the importance of aggregate demand for output, aggregate demand shocks remain critical for the determination of prices. In particular, aggregate demand shocks reduce inflation, and without them, it is impossible to deliver a reduction in output without inflation.

4.7 Benefits of Wage Flexibility and of Factor Reallocation

We end this section with two propositions: that wage flexibility and factor reallocation are desirable. These two propositions may at first seem obvious, but they are by no means universally valid. Since the model with nominal rigidities is inefficient, the theory of the second best means that seemingly desirable attributes like flexibility and reallocation can actually turn out to be harmful in general. However, to the extent that the network-neutral case is likely to be realistic, then these propositions guarantee that neoclassical intuitions about flexibility and reallocation are still empirically relevant.

To show that wage flexibility is desirable, we take a factor $f \in \mathcal{L}$ and remove its downward wage rigidity constraint by moving it to $\mathcal{K}$. This amounts to creating a more flexible economy.

Corollary 5. *Under the assumptions of Proposition 3 at the best equilibria, $\Delta \log Y$ and $\Delta \log L$ are higher in the more flexible than the less flexible economy.*

In addition to the fact that flexibility is desirable, we can also prove that reallocation is desirable. We consider two factors h and h' that are paid the same wage at the initial equilibrium and that have the same minimum nominal wage. The idea is that these two factors are really the same underlying factor, but that frictions to reallocation prevent them from being flexibly reallocated one into the other. We then consider a reallocation economy where these reallocations are allowed to take place. This amounts to a renormalization of the input-output matrix and of the shocks.

Corollary 6. *Under the assumptions of Proposition 3, the best equilibrium of the no-reallocation economy has lower output $\Delta \log Y$ and employment $\Delta \log L$ than the best equilibrium of the reallocation economy.*

5 Quantitative Application

We now turn to the quantification of the model. Although the quantitative model does not satisfy network-neutrality, the model's quantitative behavior closely matches up with the intuitions we developed using the network-neutral case.

For our quantitative experiment, we use an input-output model to capture a stylized version of the U.S. economy. We hit this economy with shocks to labor supply and aggregate demand and analyze the response of output. We also consider the response of employment and inflation to these shocks.

5.1 Setup

We start by describing our calibration of the model and of the shocks.

Calibrating the economy. There are 66 sectors and sectoral production functions use labor, capital, and intermediates. The share parameters of the functions are calibrated so that at the initial pre-shock allocation, expenditure shares match those in the input-output tables provided by the BEA. We focus on the short run and assume that labor and capital cannot be reallocated across sectors. We consider two versions of the model: Keynesian and neoclassical. The Keynesian model assumes that every labor market has wages that are perfectly rigid downwards, whereas the neoclassical model assumes flexible wages and market clearing.

The nesting structure is the following. In each sector, labor and capital are combined via a value-added nest with an elasticity η, intermediates are combined in an intermediates nest with an elasticity θ, these two nests are then combined in a sector nest with an elasticity ϵ, and final consumptions are combined in a final-demand nest with an elasticity σ. We therefore allow for differences in the elasticities, but we do not allow them to vary by sector, because such disaggregated estimates are not available.

We construct the input-output matrix using the annual U.S. input-output data from the BEA, dropping the government, noncomparable imports, and second-hand scrap industries. The dataset contains industrial output and inputs for 66 industries.

Based on the empirical literature, we set the elasticity of substitution between labor and capital to be $\eta = 0.5$, between value-added and intermediate inputs to be $\epsilon = 0.5$, across intermediates to be $\theta = 0.001$. We set the elasticity of substitution across final uses to be 0.95. These numbers are broadly in line with Atalay (2017), Herrendorf et al. (2013), Oberfield (2013), and Boehm et al. (2019), and our numerical findings are fairly robust to variations in these particular numbers. In particular, if we set all elasticities of to be equal to 0.5, making the economy network-neutral, it would not alter the qualitative nature of our findings.

Calibrating the shocks. We consider two types of shocks, and argue that both are essential to understanding the Covid crisis. First, we consider exogenous negative labor supply shocks, representing the workers who are removed from the labor force due to the lock-down. Second, we consider reductions in aggregate demand, modelled by changes in the discount factor, representing changes in households' desire to spend during the crisis. In the future, we plan to also analyze shocks to the composition of demand across sectors within the period.

In the absence of data on the size of the shocks, we use quantitative thought experiments. For labor supply shocks, we ground our thought experiment in the following available data. We use the fraction of workers that can work from home in each sector. The particular measure that we use builds on a measure at the occupation level constructed by Mongey et al. (2020) and closely related to that of Dingel and Neiman (2020), mapping occupations to sectors to come up with a measure at the sectoral level.[16] We construct scenarios by varying the fraction $x \in [0, 1]$ of workers who cannot work from home that are removed in each sector. When $x = 1$, that means all workers who cannot work from home are removed from the labor force.

To model aggregate demand shocks, we consider $\Delta \log \zeta = \{-0.1, -0.2\}$. This is motivated by survey evidence, from for example McKinsey, that indicates US consumer spending has fallen across almost all categories of spending.[17]

For example, we could imagine that at the height of the lock-down, the negative labor supply shock is large with x close to one, and then as the economy reopens, x shrinks but the aggregate demand shock remains unchanged. In future versions of the paper, we also plan to investigate shocks to the composition of demand.

We solve the model nonlinearly for each shock and graph the results. Since the Keynesian version of the model is not everywhere differentiable, there is some numerical instability in the graphs, and the reader should not interpret small spikes as being meaningful. Future versions of the paper will not include these numerical imperfections.

5.2 Results

We now describe our results. Figure 5.1 plots the response of output as a function of the size of the shock to labor x for four different benchmark cases: 'Neocl. Supply' is the neoclassical model with flexible wages where only labor supply shocks matter and

[16]We start with the 1 year ACS from 2017. We convert the NAICS codes to BEA industry codes. We use a crosswalk to get the occupation codes from the ACS into the occupation codes in Dingel and Neiman, and Money and Weinberg. This does mean 24 occupations are not included in the data, since they are not in the ONET. We merge the occupation measure in Mongey et al. (2020). Finally, we calculate the number of people in each BEA code who could work from home, and then divide by all the people in that sector (employed an unemployed, but who report participating in the labor force and being in that occupation). The collapse the ACS data into BEA code bins is weighted by the probability weights from the ACS.

[17]See https://www.statista.com/statistics/1105623/coronavirus-expected-changes-to-consumer-spending-by-product-category-us/.

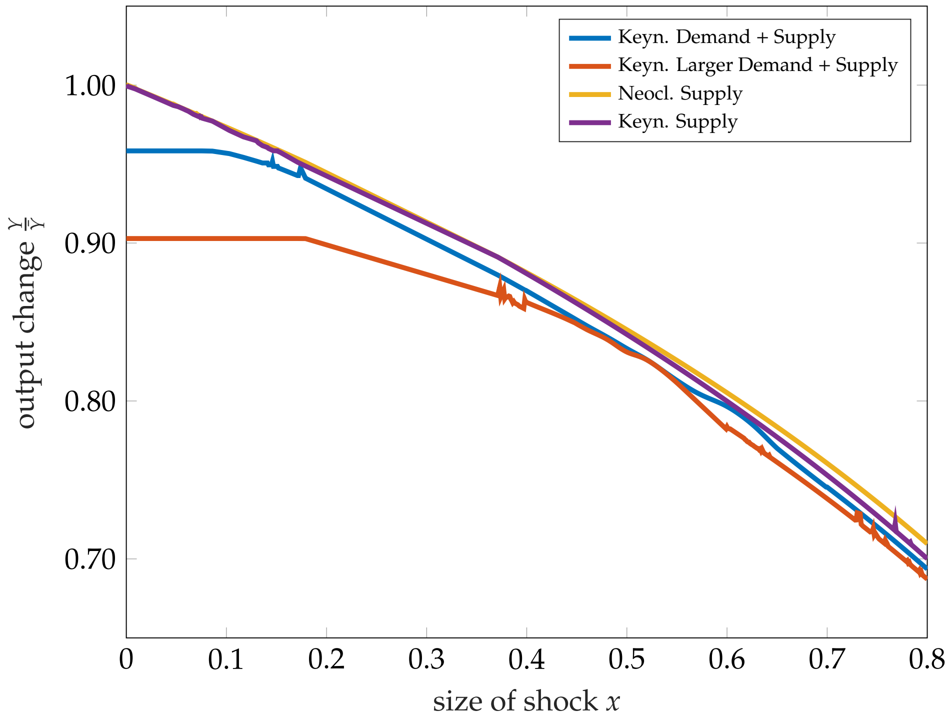


Figure 5.1: Reduction in output as a function of the size of the labor supply shock. 'Keyn. Demand + Supply' is the Keynesian model with an initial −10% aggregate demand shock, ''Keyn. Larger Demand + Supply' is the Keynesian model with an initial −20% aggregate demand shock, 'Keyn. Supply' is the Keynesian model without an initial aggregate demand shock, and 'Neocl. Supply' is the neoclassical model with flexible wages.

aggregate demand shocks do not; 'Keyn. Supply' is the Keynesian model without an initial aggregate demand shock; 'Keyn. Demand + Supply' is the Keynesian model with an initial −10% aggregate demand shock; and ''Keyn. Larger Demand + Supply' is the Keynesian model with an initial −20% aggregate demand shock. We discuss each line in turn.

The yellow line ('Neocl. Supply') shows the response of the neoclassical model with flexible wages. Output declines monotonically as the labor supply shock gets bigger. The reduction in output is fairly nonlinear, implying higher output losses than in a Cobb-Douglas model. This is because complementarities endogenously increase the importance (as measured by their income share) of the labors hit with the larger shocks. When 80% of workers who cannot work from home have been removed from the labor force, output has fallen by about 29%.

The purple line ('Keyn. Supply') is the response of output for Keynesian model with downward wage rigidity and only labor supply shocks. The model behaves remarkably similarly to the neoclassical model, only slightly dipping below the neoclassical model. When 80% of workers who cannot work from home have been removed from the labor force, output has fallen by 30%. Keynesian unemployment is then only about 2.5%.

Keynesian unemployment remains relatively small is because most labor markets do not become rigid in response to the shock. There are two reasons. First, the shock, which is quite heterogeneous, is not heterogeneous enough, and so in many labor markets, demand does not go down much more than supply. Second, the capitals of the different sectors are flexible factors which do not experience negative supply shocks, and so their income income shares fall as demand is redirected away from capital and towards labor.

However, even granting the fact that Keynesian unemployment is relatively mild, the output effect of this Keynesian unemployment still needs to be explained. Ordinarily, we would expect a 2.5% reduction in the labor force to shave approximately 1.6% off aggregate output: after all, the labor share is 2/3 at the initial equilibrium, and $2/3 \times 2.5\% \approx 1.6\%$. However, the sectors that experience Keynesian unemployment are precisely those sectors whose share of aggregate income has shrunk, or in other words, sectors that have become relatively less central to maintaining production. This is why the additional reduction in output in the Keynesian model is so small at around 1%.

The blue line and the red line combine negative labor supply shocks and negative aggregate demand shocks. One could imagine that at the height of the lock-down, the negative labor supply shock is large with x close to one, and then as the economy reopens, x shrinks but the large aggregate demand shock remains or intensifies.

The blue line ('Keyn. Demand + Supply') considers the same labor supply shocks, but begins the economy with a negative 10% aggregate demand shock. As long as the labor supply shock is sufficiently small ($x < 0.15$), the negative labor supply shocks have no effect. This is because the negative aggregate demand shock has already put all labor markets into a state of Keynesian unemployment, and so small reductions in excess supply have no effect on output. However, at some point, the labor supply shocks becomes large enough to overtake the reduction in aggregate demand. At this point, output begins to fall and output quickly behaves as if there were no negative aggregate demand shock. When 80% of workers who cannot work from home are removed, output has fallen by 31% and unemployment has increased to about 3.2%. Interestingly, for this model, unemployment is non-monotone in the shock x. Initially, the aggregate demand shock causes around 4%

Keynesian unemployment. As the negative supply shock gets larger, this unemployment number falls, as the reduction in supply causes some factor markets to become flexible. Eventually, once the shock gets big enough, strong complementarities across goods cause Keynesian unemployment to start going up again. These patterns mirror shifts in the AS and AD pictures in Figure 4.6.

The red line (''Keyn. Larger Demand + Supply'), is similar to the blue line except the economy starts with a negative 20% aggregate demand shock. As with the other example, initially, employment and inflation fall, and as long as the labor supply shocks are sufficiently small ($x < 0.25$), they has no effect on output. Once the supply shock gets large enough, then the same pattern as for the blue line emerge. By the time the supply shock has removed 80% of workers who cannot work from home, output has fallen by 32%.

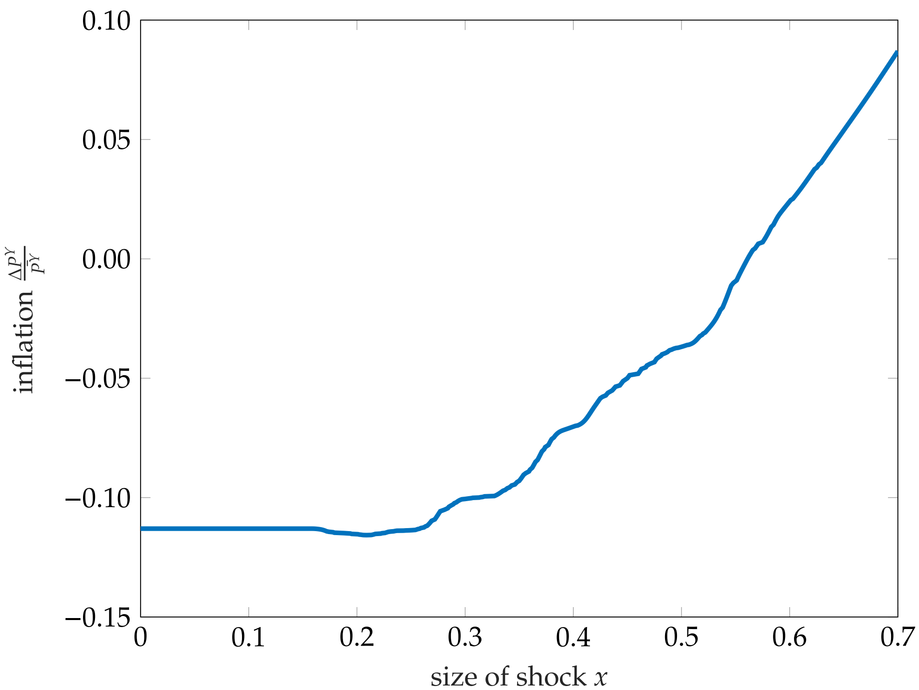


Figure 5.2: The change in inflation in the Keynesian model with a −20% aggregate demand shock as a function of the size of the negative supply shock.

Two lessons seem to emerge from Figure 5.1: (1) as long as the negative supply shocks are large enough, negative aggregate demand shocks are irrelevant for understanding the behavior of output; (2) as long as the negative labor supply shocks are large and homogeneous enough, Keynesian forces are not quantitatively important for understanding the behavior of output. While both are true for Figure 5.1, one must be careful not to over-interpret either finding. We discuss each of these issues in turn.

Starting with (1), we point out that although aggregate demand shocks do not matter for the behavior of output once the negative supply shocks become very large, they are crucial for the behavior of inflation. Figure 5.2 displays inflation for the Keynesian model with a negative −20% aggregate demand shock. For small values of the negative supply shock, inflation is negative. However, as the labor supply shock becomes large enough, inflation eventually becomes positive, but lower than it would be in the absence of the aggregate demand shock. So, whereas the negative aggregate demand shock exerts a deflationary influence throughout Figure 5.2, the negative aggregate demand shock affects aggregate output only if it is large relative to the negative supply shock, as shown in the AS-AD diagram in Figure 4.6.

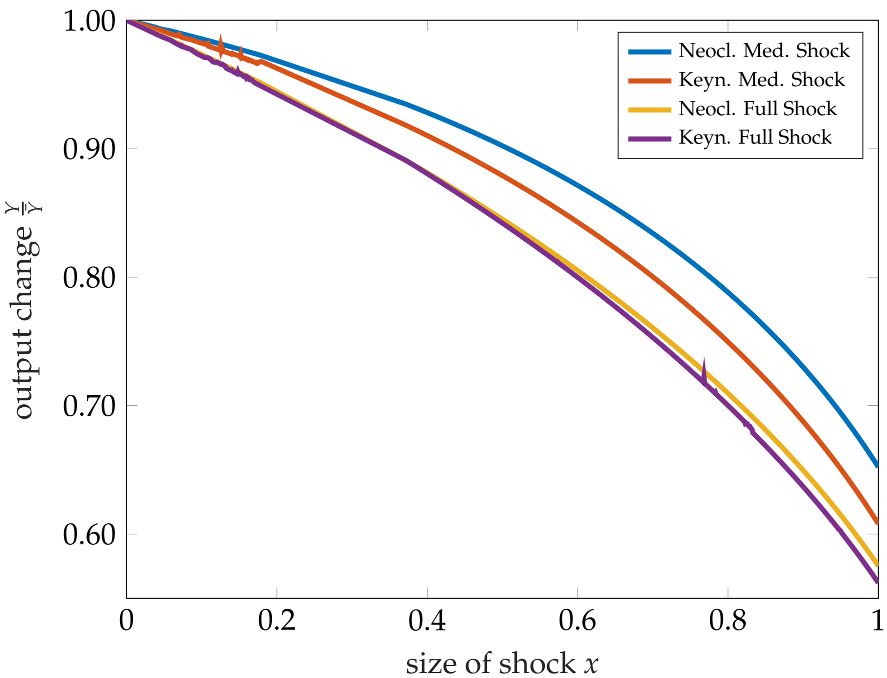


Figure 5.3: Reduction in output as a function of the size of the labor supply shock for the model without negative aggregate demand shocks.

Turning to (2), note that although Keynesian forces might not matter much for output with sufficiently homogeneous supply shocks in the absence of aggregate demand shocks, they can matter a lot if the supply shocks are sufficiently heterogeneous. To see this, in Figure 5.3, we again compare the neoclassical model to the Keynesian model, without any aggregate demand shocks, but with different negative supply shocks. The 'Full Shock' lines are the same as the ones in Figure 5.1. The 'Med Shock' lines suppose that only those sectors who are below the median in terms of their ability to work from home are subject to negative supply shocks. That means, some labor supplies are not directly hit with a negative supply shock. In this case, we see that the nominal rigidities are able to generate a substantially larger reduction in output. When 80% of the candidate workers have been removed from the labor force, output in the neoclassical model has fallen by 20%, whereas in the Keynesian model it has fallen by 25%. Intuitively, if some labor markets are not directly experiencing reductions in supply, then they are more likely to become slack. This is the intuition conveyed by the AS-AD diagram in Figure 4.5.[18]

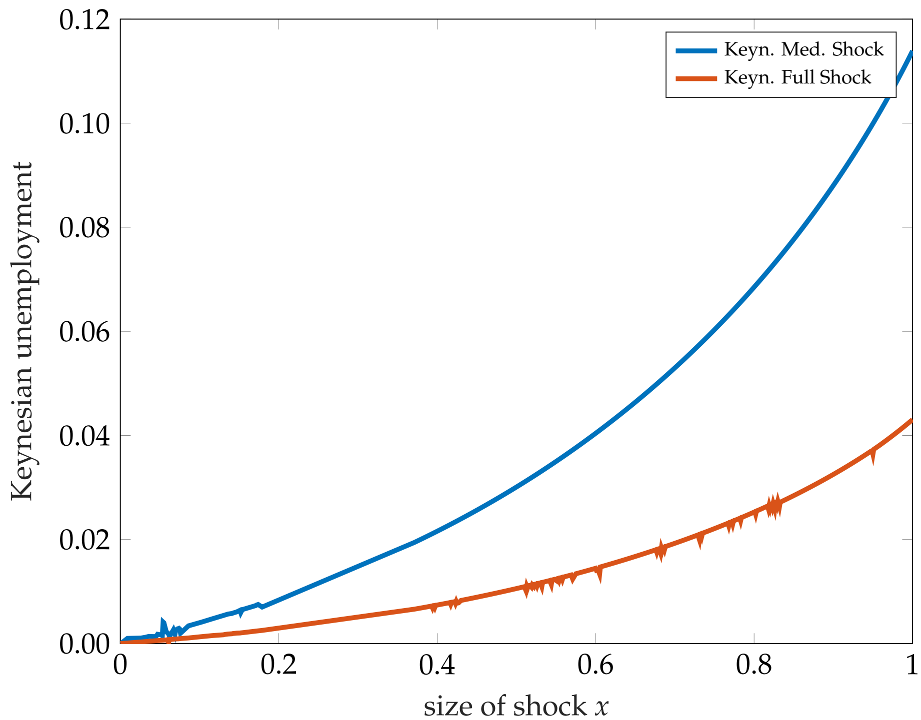


Figure 5.4: The change in involuntary Keynesian unemployment, without aggregate demand shocks, plotted as a function of the size of the negative labor supply shock.

In Figure 5.4, we plot unemployment in the Keynesian model with the 'Full Shock' and the 'Med. Shock.' The latter generates a much larger increase in involuntary unemployment than the former even though the supply shock in the former is much larger than in the latter. When 80% of the candidate workers have been removed from the labor

[18]Allowing for very elastic variable utilization of flexible factors, like capital, would increase output reduction in the neoclassical model and the amount of Keynesian output amplification.

force, the model with the smaller negative supply shock generates 8% Keynesian unemployment, whereas the 'Full Shock' specification only generates about 2.5% Keynesian unemployment.

Of course, if one observes only the equilibrium reduction in employment, and if one does not have information on wages, then one cannot identify whether those reductions are entirely driven by entirely by reductions in supply, or also partly by endogenous reductions in demand.

6 Examples of Network Relevance

Our discussion so far has focused on economies where the network is either irrelevant, or almost irrelevant, conditional on the initial factor income shares. Our quantitative application seems to suggest a small role for the network structure beyond its influence on initial factor income shares since making all the elasticities uniform does not substantively change our results. This is because the quantitative application almost satisfies network neutrality: all elasticities of substitution are below one and reasonably close to one another, and there are no shocks to the composition of demand.

In this section, we use some analytical examples to show how the network structure can matter. We show how shocks to the composition of demand and substitutability in supply chains can also act to reduce output. Throughout all these examples, we assume that the intertemporal elasticity of substitution is $\rho = 1$ so that nominal expenditure is exogenous $d\log E = d\log \zeta$.

6.1 Cobb-Douglas Economy

We first consider how demand shocks affect output and employment in a Cobb-Douglas production-network economy where all elasticities of substitution in production and in final demand are equal to one ($\theta_j = 1$ for all j). This example recalls findings in Baqaee (2015). We allow for shocks to productivities $d\log A_i$, labor supplies $d\log \bar{L}_f$, composition of demand $d\log \omega_{0i}$, and aggregate demand $d\log \zeta$.

Proposition 2 implies that factor shares change only due to changes in the composition of demand:

$$d\log \lambda_f = Cov_{\Omega^{(0)}}\left(d\log \omega_0, \frac{\Psi_{(f)}}{\lambda_f}\right) = \sum_j \Omega_{0j} d\log \omega_{0j} \frac{\Psi_{jf}}{\lambda_f}.$$

The parameter Ψ_{jf} is a network-adjusted measure of use factor f by producer j. The covariance captures the fact that a shock that redirects expenditure away from j reduces the share of factor f if j is more intensive in its use of factor f than the rest of the economy.

Plugging back into Proposition 1 yields response of output

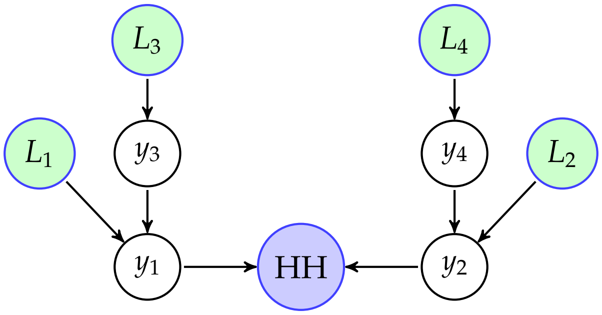
$$d\log Y = \underbrace{\sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log \bar{L}_f,}_{\text{neoclassical effect}}$$
$$+ \underbrace{\sum_{f\in\mathcal{L}} \lambda_f \min\left\{ Cov_{\Omega^{(0)}}\left(d\log\omega_0, \frac{\Psi_{(f)}}{\lambda_f}\right) + d\log\zeta - d\log\bar{L}_f, 0\right\}}_{\text{Keynesian effect}}.$$

The terms on the first line summarize the impact of the shock if the economy were neoclassical with no downward nominal wage rigidity. The terms on the second line are negative and capture the additional endogenous reduction in output through Keynesian channels: output is additionally reduced if the composition of demand shifts away from sectors whose network-adjusted use of labors with small shocks is high, or if there is a negative aggregate demand shock. Conditional on shares λ_f, the input-output network matters *only* in so far as it translates changes in household demand into changes in factor demands.

In the Cobb-Douglas example, demand shocks $d\log\omega_0$ and $d\log\zeta$ only propagate backward along supply chains to cause unemployment upstream. On the other hand, supply shocks $d\log\bar{L}_f$ and $d\log A_i$ only propagate forward along supply chains but do not cause any unemployment downstream. In fact, since these shocks do not change factor shares, supply shocks do not cause any unemployment in any of the factors, and so these shocks do not trigger the Keynesian channels. The next example shows how deviating from Cobb-Douglas changes these conclusions.

6.2 Substitutable Supply Chains

Our second example shows how production networks can feature Keynesian unemployment in response to negative supply shocks even without complementarities. However, doing so requires having non-uniform elasticities of substitution (otherwise network-irrelevance applies). In particular, once elasticities of substitution are non-uniform, labor supply shocks can create unemployment upstream and downstream. In contrast to Ex-



$$
\begin{aligned}
(Y/\bar{Y})^{\frac{\theta_0-1}{\theta_0}} &= \bar{\lambda}_1(y_1/\bar{y}_1)^{\frac{\theta_0-1}{\theta_0}} + \bar{\lambda}_2(y_2/\bar{y}_2)^{\frac{\theta_0-1}{\theta_0}}, \\
y_i/\bar{y}_i &= L_i/\bar{L}_i, \quad i \in \{3,4\}, \\
y_i/\bar{y}_i &= (L_i/\bar{L}_i)^{1-\omega}(y_{i+2}/\bar{y}_{i+2})^{\omega}, \quad i \in \{1,2\} \\
L_i &= \min\{\lambda_i E, \bar{L}_i\} \qquad , \\
w_i &= \max\{\lambda_i E/\bar{L}_i, \bar{w}_i\}.
\end{aligned}
$$

Figure 6.1: Horizontal Economy. The arrows represent the flow of resources for production. Each sector has its own factor market.

ample 3.5, where complementarities create unemployment in the non-shocked supply chains, in this example, substitutabilities create unemployment *within* the shocked supply chain. We assume away productivity shocks and demand shocks so that $d\log A_i = 0$ for all i, $d\log \omega_{0j} = 0$ for all j, and $d\log\zeta = 0$.

We consider the example in Figure 6.1, where the household consumes the output of sectors 1 and 2 with elasticity of substitution $\theta_0 > 1$. The initial expenditure shares are λ_1 and $\lambda_2 = 1 - \lambda_1$ for sectors 1 and 2 respectively. The two downstream sectors have Cobb-Douglas production functions combining sector-specific labor with an upstream input, with respective shares $1-\omega$ and ω. The upstream supplier of 1 is 3 and the one for 2 is 4. The two upstream suppliers produce using industry-specific labor. The sales shares of sector 3 and 4 are given by $\lambda_3 = \omega\lambda_1$, and $\lambda_4 = \omega\lambda_2$. The factor shares of labors in the different sectors are given by $(1-\omega)\lambda_1$, $(1-\omega)\lambda_2$, λ_3, and λ_4. We denote by p_i the price of i and by w_i the wage of workers in i.

We will only consider negative labor supply shocks $d\log\bar{L}_1 \leq 0$ and $d\log\bar{L}_3 \leq 0$ to 1 and 3, and we will maintain the assumption that $d\log\bar{L}_2 = d\log\bar{L}_4 = 0$. Hence the quantity of 1 will decrease, its relative price will increase, and because $\theta_0 > 1$, consumers will substitute expenditure towards good 2. This in turn implies that wages in 2 and 4 will increase. There will not be any unemployment in 2 and 4. However, there may be unemployment in 1 and/or 3 and we focus our attention on these sectors.

Preliminaries. To conduct the analysis, we rely on Proposition 2 which implies that changes in the sales share of sector 1 are given by

$$
d\log\lambda_1 = (\theta_0 - 1)(1-\lambda_1)(d\log p_2 - d\log p_1), \tag{6.1}
$$

where $d\log p_1 = (1-\omega)d\log w_1 + \omega d\log p_3$ and $d\log p_3 = d\log w_3$. Changes in the sales share of sector 2 are then given by

$$d\log\lambda_2 = -\frac{\lambda_1}{1-\lambda_1}d\log\lambda_1, \tag{6.2}$$

and since $d\log y_2 = 0$ and $d\log E = 0$, we also have $d\log p_2 = d\log\lambda_2$. Finally, we have $d\log\lambda_3 = d\log\lambda_1$ and $d\log\lambda_2 = d\log\lambda_4$.

Negative downstream labor supply shock. To start with, suppose that $d\log\bar{L}_1 < d\log\bar{L}_3 = 0$. That is, the downstream producer in supply chain 1 is negatively affected.

Then the only equilibrium features

$$d\log\lambda_3 - d\log\bar{L}_3 < d\log w_3 = 0 < d\log\lambda_1 - d\log\bar{L}_1 = d\log w_1.$$

The wage in sector 1 increases and the wage in sector 3 hits its downward rigidity constraint. There is full employment in sector 1 but there is unemployment in sector 3. w_1 increases but w_3 falls. This is because the negative labor supply shock in 1 causes the price of 1 to rise, which causes consumers to redirect expenditures away from 1 since $\theta_0 > 1$, which in turn reduces the demand for 1 and for 3.

This can be verified by substituting these expressions into equation (6.1) and (6.2) to get

$$d\log\lambda_1 = \frac{(\theta_0-1)(1-\lambda_1)(1-\omega)d\log\bar{L}_1}{1+(\theta_0-1)\left[(1-\lambda_1)(1-\omega)+\lambda_1\right]} > d\log\bar{L}_1$$

as needed.[19]

Using this expression for $d\log\lambda_1$ and plugging back into Proposition 1 gives

$$d\log Y = \underbrace{\lambda_1(1-\omega)d\log\bar{L}_1}_{\text{neoclassical effect}} + \underbrace{\frac{\omega(\theta_0-1)(1-\lambda_1)}{1+(\theta_0-1)\left[(1-\lambda_1)(1-\omega)+\lambda_1\right]}\lambda_1(1-\omega)d\log\bar{L}_1}_{\text{Keynesian effect}}.$$

Here the first term on the right-hand side coincides with the impact of the negative labor supply shock in the neoclassical model. The second term on the right-hand side is negative and captures the additional reduction in output through Keynesian channel via increases in unemployment in sector 3. Hence, the negative supply shock is transmitted upstream

[19]In fact, this would continue to be the case even if the upstream supplier was also negatively affected $d\log\bar{L}_3 < 0$, as long as this negative shock is not too large in magnitude.

as a negative demand shock. The shock has its greatest impact for intermediate values of ω, balancing the fact that a higher ω magnifies the negative demand effect but lowers the negative supply effect.

Overall this example shows that, once we deviate from Cobb-Douglas, then expenditure switching causes supply shocks to travel in either direction along the supply chain, reducing employment in other parts of the economy, and amplifying the effect of the original shock.

Negative downstream labor supply shock. Similarly, the shock can be transmitted in the opposite direction. To see this, suppose instead that $d\log \bar{L}_3 < d\log \bar{L}_1 = 0$.

The only equilibrium features

$$d\log \lambda_1 - d\log \bar{L}_1 < d\log w_1 = 0 < d\log \lambda_3 - d\log \bar{L}_3 = d\log w_3.$$

This time, it is the downstream sector that suffers the negative demand shock and experiences unemployment. This can be verified by substituting these expressions into equations (6.1) and (6.2) to get as needed

$$d\log \lambda_1 = \frac{(\theta_0 - 1)(1-\lambda_1)\omega d\log \bar{L}_3}{1 + (\theta_0 - 1)\left[(1-\lambda_1)\omega + \lambda_1\right]} > d\log \bar{L}_3.$$

Using this expression for $d\log \lambda_3 = d\log \lambda_1$ and plugging back into Proposition 1 gives

$$d\log Y = \underbrace{\lambda_3 d\log \bar{L}_3}_{\text{neoclassical effect}} + \underbrace{\frac{(\theta_0 - 1)(1-\lambda_1)(1-\omega)}{1 + (\theta_0 - 1)\left[(1-\lambda_1)\omega + \lambda_1\right]}\lambda_3 d\log \bar{L}_3}_{\text{Keynesian effect}}.$$

Once again, the first term on the right-hand side coincides with the impact of the negative labor supply shock in the neoclassical model. The second term on the right-hand side is negative and captures the additional reduction in output through Keynesian channel via increases in unemployment in sector 1. The negative supply shock is now transmitted downstream where it reduces demand.

7 Extension I: Constrained Consumers

We consider heterogeneous consumers who can save but cannot borrow. This means that if their income declines, then their consumption declines one-for-one with their income. However, if their income increases, holding fixed prices, these households would choose to save some of this income rather than spend all of it today. We assume that all agents have the same preferences which allows us to easily aggregate their consumptions.

7.1 Comparative Statics

Suppose there are two types of consumers: Ricardian and non-Ricardian. Index non-Ricardian consumers by $h \in \mathcal{H}$, and suppose that consumers of type h own a fraction χ_{hf} of factor $f \in \mathcal{G}$. The representative Ricardian consumer $r \notin \mathcal{H}$ behaves similarly to the representative household in Section 2. On the other hand, for the non-Ricardian households $h \in \mathcal{H}$ expenditures are determined by their contemporaneous earnings if those earnings fall more than what they would need to satisfy their Euler equation. We start with local comparative static results and end with a brief comment on global results.

Denote the endogenous set of households that are constrained in equilibrium by $\mathcal{H}^c \subseteq \mathcal{H}$. Hence $h \in \mathcal{H}^c$ if and only if

$$\sum_{f \in \mathcal{G}} \frac{\chi_{hf} \lambda_f}{\sum_{g \in \mathcal{G}} \chi_{hg} \lambda_g} \left(d \log \lambda_f + d \log E \right) < (1-\rho) d \log p^Y + d \log \zeta. \tag{7.1}$$

In words, household h is constrained if their nominal income today is less than how much they wish to consume, as given by their Euler equation.[20]

The key step in this extension is the determination of changes in aggregate nominal expenditure. In the interest of space, we omit the derivations and jump directly to the result, which applies almost everywhere given the set of constrained households $\mathcal{H}^c$:

$$d \log E = \frac{Cov_\lambda \left(\chi_{\mathcal{H}^c}, d \log \lambda \right)}{1 - \mathbb{E}_\lambda \left(\chi_{\mathcal{H}^c} \right)} + (1-\rho) d \log p^Y + d \log \zeta.$$

The covariance uses the factor shares λ_f for $f \in \mathcal{G}$ as the probability distribution and computes the covariance of the income share of constrained households for each factor

[20]Note that we assume that the reduction in the income of type h households is borne uniformly by all households of type h. That is, we do not allow some type-h households to get fired (becoming constrained) while others stay on the job.

$\chi_{\mathcal{H}_f^c} = \sum_{h \in \mathcal{H}^c} \chi^h$ with the changes in the factor shares $d \log \lambda_f$.[21]

Therefore, even in the absence of exogenous aggregate demand shocks $d \log \zeta$ such as changes in nominal interest rates or in the desire to save, there is now an endogenous aggregate demand shock. In particular, if the income shares of factors owned by constrained households shrink, then this imparts a negative aggregate demand shock that shrinks nominal expenditures today $d \log E < 0$. Since it is precisely those households whose income falls that become constrained, this endogenously reduces expenditures, and the reductions in expenditures exasperates unemployment.

The accompanying propagation equations determining changes in factor shares $d \log \lambda_f$ as a function of changes in aggregate nominal expenditure $d \log E$ are exactly the same as in Proposition 2. And the aggregation determining $d \log Y$ as a function of the $d \log \lambda_f$'s and $d \log E$ is exactly the same as in Proposition 1.

Furthermore, all the global results of Section 4 generalize to this case with constrained consumers: the set of equilibria has a lattice structure with a best and a worst equilibria, these equilibria are decreasing in $\Delta \log \bar{L}$ and $\Delta \log \zeta$, and there are benefits from wage flexibility.

7.2 Example with Constrained Households

Consider again the horizontal economy analyzed in Section 3.5. Suppose each labor factor $f \in \mathcal{L}$ is wholly owned by a type of consumer facing potentially binding borrowing constraints and whose only source of income is f. We assume that the intertemporal elasticity of substitution is $\rho = 1$. We consider shocks to the composition of demand $d \log \omega_{0j}$ and to factor supplies $d \log \bar{L}_f$ but we abstract from shocks to productivities and to aggregate demand.

Household h becomes constrained if it owns a factor that has become rigid because its wage has hit the downward nominal wage rigidity constraint. As a result, changes in nominal expenditure are given by

$$d \log E = \frac{\sum_{f \in \mathcal{R}} \lambda_f d \log \lambda_f}{\lambda_{\mathcal{F}}} = -d \log \lambda_{\mathcal{F}}.$$

Hence, an increase in the expenditure share of flexible sectors, by depriving credit-

[21] We could allow for the possibility that some households are exogenously hand-to-mouth. The equation for changes in aggregate nominal expenditure $d \log E$ would stay the same, but we would no longer have to verify that (7.1) holds for those households.

constrained workers of income, reduces nominal expenditures today one-for-one. This is the endogenous negative demand shock.

We then proceed as in Section 3.5 to get

$$\begin{aligned} d\log Y &= \lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d\log L_{\mathcal{R}} \\ &= \lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}} + (1-\lambda_{\mathcal{F}})(1-\theta) d\log \bar{L}_{\mathcal{F}} - \theta\lambda_{\mathcal{F}} d\log \omega_{0\mathcal{F}}, \end{aligned}$$

The first term on the second line is the direct effect of the negative shock to the supplies flexible factors. The second term captures the amplification from the fact that the shock redistributes demand away from the rigid sectors, causing unemployment. The final term captures the fact that a change in the composition of demand towards flexible sectors further reduces employment.

Surprisingly, when we compare the reduction in output to the one we obtained without credit-constrained consumers in Section 3.5, we see that adding endogenously credit-constrained households to the model attenuates the effect on output. Without credit constraints, in response to a negative supply shock $d\log \bar{L}_{\mathcal{F}} < 0$, expenditures on $\mathcal{F}$ increase, which further raises the price of factors in $\mathcal{F}$, which further increases expenditures due to complementarities, and so on. However, with credit constraints, this feedback loop short-circuits. In response to a negative supply shock $d\log \bar{L}_{\mathcal{F}} < 0$, expenditures on $\mathcal{F}$ increase, but this reduces the income of constrained households, reducing total income and spending.

This example also shows how the model can generate recessions without inflation, even in the absence of exogenous aggregate demand shocks. To see this, note that the change in today's price level is

$$d\log p^Y = d\log E - d\log Y = -\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}}.$$

Hence, changes in the composition of demand have no effect on the price level. This is because a change in the composition of demand causes output and nominal expenditures to decline at the same rate, leaving the overall price level unchanged. However, changes in the composition of demand can reduce output. If there are only shocks to the composition of demand then we can get $d\log p^Y = 0$ and $d\log Y < 0$. This is impossible in the absence of potentially credit-constrained consumers, because the Euler equation necessitates that output and the price level move in opposite directions. The difference here is that consumers who become credit-constrained reduce their spending, lowering nominal demand for all factors, including the flexibly priced ones.

The example above shows how the presence of credit-constrained households can generate recessions without any accompanying inflation, even in the absence of negative aggregate demand shocks. Of course, since wages are constrained to never fall, it is not surprising that the model cannot generate deflation. In Appendix B, we extend the model to allow for some downward wage flexibility and show how this can help to generate a recession and deflation at the same time, without any exogenous negative aggregate demand shocks.

8 Extension II: Firm Failures

Whereas credit-constraints can act as endogenous demand negative shocks, exits and firm-failures can act as endogenous supply shocks.

To capture firm failures, we modify the general Keynesian structure described in Section 2 as follows. We assume that output in sector $i \in \mathcal{N}$ is a CES aggregate of identical producers j each with constant returns production functions $y_{ik} = A_i f_i(x_{ij}^k)$, where x_{ij}^k is the quantity of industry j's output used by producer k in industry i. Assuming all firms within an industry use the same mix of inputs, sectoral output is

$$y_i = \left(\int y_{ik}^{\frac{\sigma_i - 1}{\sigma_i}} dk \right)^{\frac{\sigma_i}{\sigma_i - 1}} = M_i^{\frac{1}{\sigma_i - 1}} A_i f_i(x_{ij}),$$

where x_{ij} is the quantity of input j used by industry i, M_i is the mass of producers in industry i, $\sigma_i > 1$ is the elasticity of substitution across producers, and A_i is an exogenous productivity shifter. From this equation, we see that a change in the mass of operating firms acts like a productivity shock and changes the industry-level price. Therefore, if shocks outside sector i trigger a wave of exits, then this will set in motion endogenous negative productivity shock $(1/(\sigma_i - 1))\Delta \log M_i$ in sector i.

Suppose that each firm must maintain a minimum level of revenue in order to continue operation.[22, 23] We are focused on a short-run application, so we do not allow new entry,

[22]One possible micro-foundation is each producer must pay its inputs in advance by securing within-period loans and that these loans have indivisibilities: only loans of size greater than some minimum level can be secured. This minimum size is assumed to coincide with the initial costs $\bar{\lambda}_i \bar{E}/\bar{M}_i$ of the producer.

[23]Another possible micro-foundation is as follows. Producers within a sector charge a CES markup $\mu_i = \sigma_i/(\sigma_i - 1)$ over marginal cost. These markups are assumed to be offset by corresponding production subsidies. Producers have present nominal debt obligations corresponding to their initial profits (1 −

but of course, this would be important for long-run analyses.[24]

The mass of firms that operate in equilibrium is therefore given by

$$M_i = \min\left\{\frac{\lambda_i E}{\bar{\lambda}_i \bar{E}}\bar{M}_i, \bar{M}_i\right\},$$

where $\bar{M}_i$ is the exogenous initial mass of varieties, $\lambda_i E$ is nominal revenue earned by sector i and $\bar{\lambda}_i \bar{E}$ is the initial nominal revenue earned by i. If nominal revenues fall relative to the baseline, then the mass of producers declines to ensure that sales per producer remain constant. In order to capture government-mandated shutdowns of certain firms, we allow for shocks that reduce the exogenous initial mass of producers $\Delta \log \bar{M}_i \leq 0$.

8.1 Local Comparative Statics

We can generalize Propositions 1 and 2 to this context. The only difference is that we must replace $d\log A_i$ by $d\log A_i + (1/(\sigma_i - 1))d\log M_i$, where

$$d\log M_i = d\log \bar{M}_i + \min\{d\log\lambda_i + d\log E - d\log \bar{M}_i, 0\}.$$

This backs up the claim that the $d\log M_i$'s act like endogenous negative productivity shocks. They provide a mechanism whereby a negative demand shock, say in the composition of demand or in aggregate demand $d\log\zeta$, triggers exits which are isomorphic to negative supply shocks.

As in the other examples, the general lesson is that the output response, to a first-order, is again given by an application of Hulten's theorem along with an amplification effect which depends on how the network redistributes demand and triggers Keynesian unemployment in some factors and firm failures in some sectors.

8.2 Illustrative Example

Consider once again the horizontal economy analyzed in Section 3.5. We assume that there are no shocks to aggregate demand ($d\log\zeta = 0$). Since $\rho = 1$, this ensures that nominal expenditure is constant ($d\log E = 0$). We also assume that there are no exogenous

$1/\mu_i)\bar{\lambda}_i\bar{E}/\bar{M}_i$. The same is true in the future. If present profits $(1 - 1/\mu_i)\lambda_i E/M_i$ fall short of the required nominal debt payment $(1 - 1/\mu_i)\bar{\lambda}_i\bar{E}/\bar{M}_i$, then the firm goes bankrupt and exits. Alternatively, we can imagine that there is no future debt obligation but that firms cannot borrow.

[24] See Baqaee (2018) and Baqaee and Farhi (2020a) for production networks with both entry and exit.

shocks to productivities ($d\log A_i = 0$), no shocks to factor supplies ($d\log \bar{L}_f = 0$), and no shocks to the composition of demand ($d\log \omega_{0j} = 0$). Finally, we assume that all sectors have the same within-sector elasticity of substitution $\sigma_i = \sigma > 1$.

We focus on exogenous shocks $d\log \bar{M}_i \leq 0$ capturing government-mandated shutdowns. We show how endogenous failures can amplify these negative supply shocks. The insights are more general and also apply to labor supply shocks. Similarly, failures can be triggered by negative aggregate demand shocks, and the resulting endogenous negative supply shocks can result in stagflation with simultaneous reductions in output and increases in inflation.

Preliminaries. Changes in the sales of i are given by

$$d\log \lambda_i = (1-\theta_0)(1-\lambda_i)\Big(d\log p_i - \sum_{j\in\mathcal{N}} \lambda_j d\log p_j\Big), \tag{8.1}$$

where changes in the price of i depend on changes in the wage in i and on the endogenous reduction in the productivity of i driven by firm failures

$$d\log p_i = d\log w_i - \frac{1}{\sigma-1} d\log M_i, \tag{8.2}$$

where change in wages in i are given by

$$d\log w_i = \max\{d\log \lambda_i - d\log \bar{L}_i, 0\}, \tag{8.3}$$

and changes in the mass of producers in i are given by

$$d\log M_i = \min\{d\log \lambda_i, d\log \bar{M}_i\}. \tag{8.4}$$

We compare and contrast the effect of $d\log \bar{L}_i$ to $d\log \bar{M}_i$ in order, starting with the case where sectors are complements and then the case where they are substitutes.

Shut-down shock with complements. We assume that sector are complements ($\theta < 1$), and we consider the government-mandated shutdown of some firms in only one sector i. We can aggregate the non-shocked sectors into a single representative sector indexed by $-i$. We therefore have $d\log \bar{M}_i < 0 = d\log \bar{M}_{-i}$.

The closures of firm in i raise its price ($d\log p_i > 0$), which because of complementarities, increases its share ($d \log \lambda_i > 0$). It therefore does not trigger any further endogenous exit in this shocked sector ($d \log M_i = d \log \bar{M}_i$). In addition, the wages of its workers increases ($d \log w_i > 0$). The shock reduces expenditure on the other sectors ($d \log \lambda_{-i} < 0$), and this reduction in demand triggers endogenous exits ($d \log M_{-i} < 0$), pushes wages against their downward rigidity constraint ($d \log w_{-i} = 0$) and creates unemployment ($d \log L_{-i} < 0$), both of which endogenously amplify the reduction in output through failures and Keynesian effects.

Using equations (8.1), (8.2), (8.3), and (8.4), we find

$$d \log \lambda_i = -\frac{(1-\theta)(1-\lambda_i)}{1-(1-\theta)(1-\lambda_i)\left(1-\frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)}\frac{1}{\sigma-1} d \log \bar{M}_i > 0,$$

$$d \log M_{-i} = d \log L_{-i} = -\frac{\lambda_i}{1-\lambda_i} d \log \lambda_i < 0,$$

and finally

$$d \log Y = \lambda_i \frac{1}{\sigma-1} d \log \bar{M}_i + \frac{(1-\theta)(1-\lambda_i)\frac{\sigma}{\sigma-1}}{1-(1-\theta)(1-\lambda_i)\left(1-\frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)} \lambda_i \frac{1}{\sigma-1} d \log \bar{M}_i.$$

The first term on the right-hand side is the direct reduction in output from the shut-down in sector i. The second term capture the further indirect equilibrium reduction in output via firm failures and unemployment in the other sectors.

Shut-down shock with substitutes. Consider the same experiment as above but assume now that sectors are substitutes ($\theta > 1$). We conjecture an equilibrium where sales in sector i do not fall more quickly than the initial shock $d \log \lambda_i - d \log \bar{M}_i > 0$. Sector i loses demand following the exogenous shutdown of some of its firms, and this results in unemployment in in the sector ($d \log L_i < 0$) but no endogenous firm failures ($d \log M_i = d \log \bar{M}_i$) . On the other hand, sector $-i$ maintains full employment and experiences no failures.

To verify that this configuration is indeed an equilibrium, we compute

$$d \log \lambda_i = \frac{(\theta-1)(1-\lambda_i)}{1-(\theta-1)\lambda_i} \frac{1}{\sigma-1} d \log \bar{M}_i.$$

We must verify that

$$0 > d \log \lambda_i > d \log \bar{M}_i.$$

The first inequality is verified as long as $\theta > 1$ is not too large. The second inequality is verified if $\sigma > 1$ is large enough and $\theta > 1$ is not too large.

If these conditions are violated, then we can get a jump in the equilibrium outcome. Intuitively, in those cases, the shutdown triggers substitution away from i, and that substitution is so dramatic than it causes more firms to shutdown, and the process feeds on itself ad infinitum. Any level of $d \log L_i < 0$ and $d \log M_i < d \log \bar{M}_i$ can then be supported as equilibria. Although we do not focus on it, this possibility illustrates how allowing for firm failures can dramatically alter the model's behavior.

Assuming the regularity conditions above are satisfied, the response of output is given by

$$d \log Y = \lambda_i \frac{1}{\sigma - 1} d \log \bar{M}_i + \frac{(\theta_0 - 1)(1 - \lambda_i)}{1 - (\theta_0 - 1)\lambda_i} \lambda_i \frac{1}{\sigma - 1} d \log \bar{M}_i,$$

where the first term on the right-hand side is the direct effect of the shutdown and the second term is the amplification from the indirect effect of the shutdown which results in Keynesian unemployment in i.

9 Extension III: Policy

In this section, we briefly review how policy can affect outcomes. We analyze three different types of policy: monetary, fiscal, and tax policy.

9.1 Monetary Policy

A monetary expansion in this model comes in the form of positive aggregate demand shock

$$d \log \zeta = -\rho \left(d \log(1 + i) + \frac{d \log \beta}{1 - \beta} - d \log \bar{p}^Y \right) + d \log \bar{Y} > 0,$$

this can come about either via lower nominal interest rates, or failing that, forward guidance about the price level in the future. If nominal rates are stuck at the zero-lower bound, then an increase in future prices, by lowering the real interest rate, will stimulate spending today. A positive aggregate demand shock increases nominal expenditures since

$$d \log E = \frac{Cov_{\lambda_{\mathcal{G}}} \left(\chi_{\mathcal{H}^c}, d \log \lambda_{\mathcal{G}} \right)}{1 - \mathbb{E}_{\lambda_{\mathcal{G}}} \left(\chi_{\mathcal{H}^c} \right)} + (1 - \rho) d \log p^Y + d \log \zeta.$$

If nominal expenditures $d\log E$ are sufficiently high, then the economy can maintain full employment regardless of the shocks by guaranteeing that nominal wages do not have to fall in equilibrium

$$\min_{f\in\mathcal{G}}\{d\log\lambda_i + d\log E - d\log\bar{L}_f\} > 0.$$

This is obviously the optimal policy for the monetary authority to pursue, if it is feasible.

Setting aside full-employment policy, we can also consider how output responds to a given monetary stimulus $d\log\zeta > 0$. Since the model is non-linear, the effectiveness of monetary policy depends on what other shocks have hit the economy. A canonical example is if the monetary stimulus coincides with a set of negative supply shocks. In this case, complementarities in production act to reduce the effectiveness of monetary policy.

To see this, consider again the horizontal economy described in Example 3.5. We assume that sectors are complements with $\theta < 1$ and that the intertemporal elasticity of substitution is $\rho = 1$. For simplicity, suppose that there are no constrained households. We hit the economy with negative factor supply shocks $d\log\bar{L} < 0$,. Suppose that in addition, through forward-guidance, the monetary authority is able to raise $d\log\zeta > 0$.

Then, working through the same equations as in the original example, we find that the overall effect on output is

$$d\log Y = \frac{\lambda_{\mathcal{F}}d\log\bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} + \left(1 - \frac{(1-\theta)\lambda_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}\right)(1-\lambda_{\mathcal{F}})d\log\zeta.$$

The first term is the effect of the negative supply shock, amplified by the nominal rigidities, exactly as in Example 3.5. The second term is the effect of the monetary stimulus. The term $(1-\lambda_{\mathcal{F}})d\log\zeta$ is the direct effect of the stimulus on the employment of rigid factors. However, this direct effect is mitigated. This is because monetary stimulus raises the prices of flexible factors in absolute terms and relative to those of rigid factors, and since flexible and rigid factors are complements, this causes expenditures to switch towards flexible factors and away from rigid factors. This force attenuates the effectiveness of monetary policy, and it would not appear if not for the heterogeneity in cyclical conditions across factor markets.

9.2 Payroll Tax Cuts

In this section, we briefly consider the effect of payroll tax cuts used by many governments in the wake of Covid-19. If correctly targeted, payroll tax cuts can alleviate the demand short-fall in slack factor markets. By selectively cutting taxes (or subsidizing) unemployed sectors, a policymaker can actually implement the first-best outcome.[25]

Even without going all the way to the first best, these policies can be helpful. However, as with monetary policy, complementarities in production also reduce the effectiveness of a given intervention. To see this, consider once more the horizontal, economy of Example 3.5. We assume that sectors are complements with $\theta < 1$ and that the intertemporal elasticity of substitution is $\rho = 1$. For simplicity, we assume that there are no constrained households.[26] We hit the economy with negative factor supply shocks $d \log \bar{L}_f < 0$. In addition, we assume that the government institutes a gross payroll subsidy $d \log s_{\mathcal{R}}$ on the rigid factors financed by a tax on the resulting profits. In this case, the output response is

$$d \log Y = \frac{\lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}}}{1 - (1-\theta)(1-\lambda_{\mathcal{F}})} + \left(1 - \frac{(1-\theta)\lambda_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}\right)(1-\lambda_{\mathcal{F}}) d \log s_{\mathcal{R}}.$$

As usual, the first term is the effect of the negative supply shock to the flexible sectors, amplified by the nominal rigidities. The second term is the effect of the payroll subsidy. Naturally, a subsidy on rigid factors increases output, and the term $(1-\lambda_{\mathcal{F}}) d \log s_{\mathcal{R}}$ is the direct effect of the increase in employment. However, the subsidy on rigid factors also reduces the price of rigid sectors relative to flexible ones. Since factors are complements, this means that expenditures shift towards flexible factors and away from rigid ones, attenuating the effect of the payroll subsidy.

9.3 Fiscal Policy

Finally, we consider the effect of changes in size of and composition of government spending. We assume that $G = 0$ and denote by dG the change in government expenditure and by Ω_k^G the shares of the different sectors in government expenditure. We assume that government spending is deficit-financed, and that the debt is repaid with taxes in the future. We assume that only a fraction $\alpha_{\text{Ricardian}}$ of these future taxes falls on Ricardian households, and the rest falls either on non-Ricardian households or on future generations. Without loss of generality, we normalize $\bar{Y} = \bar{Y}_* = 1$. We denote the average

[25]See e.g. Correia et al. (2013); Farhi et al. (2014).

[26]With exogenously constrained households, payroll tax cuts can be helpful through a different channel if they increase their income by boosting their wages, thereby effectively redistributing away from households with low marginal propensities to consume and towards households with high marginal propensities to consume.

marginal propensity to consumer by $\overline{MPC} = \mathbb{E}_\lambda(1-\chi_{\mathcal{H}^c})MPC_{\text{Ricardian}} + \mathbb{E}_\lambda(\chi_{\mathcal{H}^c})$, where $MPC_{\text{Ricardian}} = 1-\beta$ is the marginal propensity to consumer of Ricardian households and 1 is the marginal propensity to consume of Ricardian households.

The only changes in the analysis concern the determination of changes in nominal expenditure and the propagation equations for sales and factor shares. Changes in nominal expenditure are given by

$$d\log E = \frac{Cov_\lambda\left(\chi_{\mathcal{H}^c}, d\log\lambda\right)}{1-\mathbb{E}_\lambda\left(\chi_{\mathcal{H}^c}\right)} + (1-\rho)d\log p^Y + d\log\zeta + \frac{1-\alpha_{\text{Ricardian}}MPC_{\text{Ricardian}}}{1-\overline{MPC}}dG.$$

Changes in sales and factor shares are given by

$$\begin{aligned}\lambda_f d\log\lambda_f &= \sum_{k\in\mathcal{N}}\Psi_{kf}\Omega_{0k}d\log\omega_{0k} + \sum_{k\in\mathcal{N}}\Psi_{kf}(\Omega^G_k - \Omega_{0k})\frac{dG}{E} \\ &+ \sum_{j\in 1+\mathcal{N}}\lambda_j(\theta_j-1)Cov_{\Omega^{(j)}}\left(\sum_{k\in\mathcal{N}}\Psi_{(k)}\left(d\log A_k\right) + \sum_{g\in\mathcal{G}}\Psi_{(g)}\left(d\log L_g - d\log\lambda_g\right), \Psi_{(f)}\right),\end{aligned}$$

where $d\log L_g = d\log\bar{L}_g$ for $f\in\mathcal{K}$ and $d\log L_g = \min\left\{d\log\lambda_g + d\log E, d\log\bar{L}_g\right\}$ for $f\in\mathcal{L}$. We can then combine these formulas with Proposition 1 to get the change in aggregate output exactly as before. This generalizes Baqaee (2015) beyond the Cobb-Douglas special case.

These results show how changes in government spending can stimulate output in two different ways. The first reason is the standard Keynesian-cross argument: an increase in government spending stimulates the incomes of constrained households, who then proceed to consume more. This boosts nominal expenditure by $dG(1-\alpha_{\text{Ricardian}}MPC_{\text{Ricardian}})/(1-\overline{MPC})$, which is higher, the higher is the average marginal propensity to consume $\overline{MPC}$ and the lower is the fraction $\alpha_{\text{Ricardian}}$ of future taxes that fall on Ricardian consumers.[27] Interestingly, in the context of the pandemic, the fiscal multiplier could be lower in a partial lock-down if Ricardian households have a low marginal propensity to consume because they want to postpone consumption until the lock-down is fully lifted.

The second reason is slightly more subtle. By choosing the composition of government spending wisely, the government can target its spending to boost the demand of sectors whose factor markets are depressed. This effect is captured by $\sum_{k\in\mathcal{N}}\Psi_{kf}(\Omega^G_k - \Omega_{0k})\frac{dG}{E}$. Intuitively, fiscal policy can move the AD curve by changing both the size and compo-

[27] See e.g. Farhi and Werning (2016) for a discussion.

sition of government expenditures. To the extent that the government cannot perfectly target depressed factor markets, some of the government expenditures will end up wastefully increasing the wages of flexible factors instead of stimulating employment, thereby lowering the fiscal multiplier. Furthermore, fiscal multipliers are further dampened in economies with complementarities since to some extent, government spending always ends up increasing the wages of some flexible factors, causing private expenditure to be redirected towards those factors and away from rigid factors.

10 Conclusion

This paper analytically characterizes and numerically quantifies the importance of occasionally-binding downward wage rigidity in the presence of a combination of supply and demand shocks. Since data on the effects of the Covid-19 crisis are as yet scarce, we use thought experiments to gauge the relevance of various forces for current events. In future versions of the paper, as more data becomes available, we hope to revise our quantitative analysis and to further investigate shocks to the composition of demand.

Our results suggest that with only negative labor supply shocks, Keynesian effects are more important for unemployment than for output. This is because the sectors that become rigid in response to negative labor supply shocks are precisely those sectors that are becoming relatively less important for aggregate output. Furthermore, these negative supply shocks are stagflationary. Negative aggregate demand shocks, on the other hand, are deflationary, and once they are large enough relative to the negative supply shocks, can cause large effects on output.

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Appendix A Investment

To model investment, we can simply add intertemporal production functions into the model. An investment function transforms goods and factors in the present period into goods that can be used in the future. In that case, instead of breaking the problem into an intertemporal and intratemporal problem, we must treat both problems at once. In this case, Proposition 1 still applies without change.

However, we can no longer use the Euler equation to pin down nominal expenditures today, since nominal GDP today includes investment expenditures and output tomorrow can no longer taken to be exogenous. Instead, to determine $d\log E$, we must use a version of Proposition 2.

In particular, let λ_i^I denote the intertemporal sales share — expenditures on quantity i as a share of the net present value of household income. Furthermore, let $\bar{\Omega}^I$ represent the intertemporal input-output matrix, which includes the capital accumulation equations. Then, letting intertemporal consumption be the zero-th good, and abstracting from shocks to the composition of demand for simplicity, we can write

$$d\log\lambda_k^I = +\sum_{j\in\mathcal{N}}\lambda_j^I(\theta_j-1)Cov_{\Omega^{I,(j)}}\left(\sum_{i\in\mathcal{N}}\Psi_{(i)}^I d\log A_i - \sum_{f\in\mathcal{G}}\Psi_{(f)}^I\left(d\log\lambda_f^I - d\log L_f\right), \frac{\Psi_{(k)}^I}{\lambda_k^I}\right)$$

almost everywhere, where changes in factor employments are given by

$$d\log L_f = \begin{cases} d\log\bar{L}_f, & \text{for} \quad f\in\mathcal{K}, \\ \min\left\{d\log\lambda_f + d\log E, d\log\bar{L}_f\right\}, & \text{for} \quad f\in\mathcal{L}. \end{cases}$$

Changes in nominal expenditures today are given by

$$d\log E = \sum_{f\in\mathcal{G}}\frac{\lambda_f^I}{\beta}d\log\lambda_f^I + d\log Y_*,$$

but now $d\log Y_*$ is endogenous. In particular, we have

$$d\log Y_* = \sum_{i\in\mathcal{N}_*}\lambda_{*,i}d\log A_{*,i} + \sum_{i\in\mathcal{G}_*}\lambda_{*,i}d\log L_{*,i},$$

where asterisks denote future variables. The set $\mathcal{G}_*$ is the set of factors from the perspective of the future period. This includes both factor endowments in the future as well as endogenously accumulated capital stocks.

To complete the characterization, we note that for each endogenously accumulated factor $f \in \mathcal{G}_*$, we have

$$d \log L_f = d \log \lambda_f^I + (1-\beta) d \log E + \beta d \log Y_* - d \log p_f,$$

and $d \log p_f$ given by the usual set of forward propagation equations

$$d \log p_f = \sum_{i \in \mathcal{N}} \Psi_{fi}^I d \log A_i + \sum_{j \in \mathcal{G}^I} \Psi_{fj}^I d \log p_j,$$

where $\mathcal{G}^I$ is the set of factor endowments across all periods. The price of these factor endowments are, in turn, given by

$$d \log p_j = \max\{d \log \lambda_j^I + (1-\beta) d \log E + \beta d \log Y_2 - d \log \bar{L}_f, 0\},$$

if j is a sticky factor and $d \log p_j = d \log \lambda_j^I + (1-\beta) d \log E + \beta d \log Y_2 - d \log \bar{L}_f$ otherwise.

Appendix B Some Downward Wage Flexibility

In practice, we might imagine that wages can fall albeit not by enough to clear the market. The possibility that wages may fall obviously has important implications for inflation. Indeed, we show that with shocks to the composition of demand, and even without shocks to aggregate demand, we can get simultaneous reductions in output *and* inflation.

For each factor $f \in \mathcal{L}$, suppose the following conditions hold

$$\frac{L_f}{\bar{L}_f} = \begin{cases} \left(\frac{w_f}{\bar{w}_f}\right)^{\phi_f}, & \text{if} \quad w_f \leq \bar{w}_f, \\ 1, & \text{if} \quad w_f > \bar{w}_f. \end{cases}$$

The parameter ϕ_f controls the degree of downward wage flexibility. If $\phi_f = \infty$, then the wage is perfectly rigid downwards. If $\phi_f = 0$, then the wage is fully flexible, and we recover the neoclassical case.

B.1 Generalizing the Results

The only change to Proposition 1 is that we now have

$$d\log Y = \sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log \bar{L}_f + \sum_{f\in\mathcal{L}} \frac{\phi_f}{1+\phi_f}\lambda_f \min\left\{d\log\lambda_f + d\log E - d\log\bar{L}_f, 0\right\},$$

and the only change to Proposition 2 is that we now have

$$d\log L_f = \begin{cases} \frac{\phi_f}{1+\phi_f}\left(d\log\lambda_f + d\log E\right) + \frac{1}{1+\phi_f} d\log\bar{L}_f & \text{if} \quad f\in\mathcal{R} \\ d\log\bar{L}_f & \text{if} \quad f\in\mathcal{F}. \end{cases} \tag{B.1}$$

B.2 Illustrative Example

We now construct an example showing how allowing for some degree of downward wage flexibility allows the model to generate a recession *and* deflation at the same time, without relying on aggregate demand shocks. We return to the example of Section 3.5. However, this time, suppose that wages have some degree of downward flexibility $0 < \phi < \infty$ common across all factor markets $f \in \mathcal{L}$.

We now get

$$d\log Y = \lambda_{\mathcal{F}} d\log\bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d\log L_{\mathcal{R}},$$

where $\lambda_{\mathcal{R}} = \sum_{f\in\mathcal{R}}\lambda_f = 1-\lambda_{\mathcal{F}}$ is the total share of the rigid factors and $d\log L_{\mathcal{R}}$ is the "representative" employment reduction in the rigid sectors

$$d\log L_{\mathcal{R}} = \sum_{f\in\mathcal{R}} \frac{\lambda_f}{\lambda_{\mathcal{R}}} d\log\bar{L}_f < \sum_{f\in\mathcal{R}} \frac{\lambda_f}{\lambda_{\mathcal{R}}} d\log\bar{L}_f = d\log\bar{L}_{\mathcal{R}}.$$

In turn, this employment reduction is given as a function of the change $d\log\lambda_{\mathcal{F}}$ in the share of the flexible sectors by

$$\lambda_{\mathcal{R}} d\log L_{\mathcal{R}} = -\frac{\phi}{1+\phi}\lambda_{\mathcal{F}} d\log\lambda_{\mathcal{F}} + \frac{1}{1+\phi}\lambda_{\mathcal{R}} d\log\bar{L}_{\mathcal{R}},$$

and the the change $d\log\lambda_{\mathcal{F}}$ in the share of the flexible sectors is given by

$$\lambda_{\mathcal{F}} d\log\lambda_{\mathcal{F}} = \frac{\lambda_{\mathcal{F}} d\log\omega_{0\mathcal{F}} - (1-\theta)\lambda_{\mathcal{F}}(1-\lambda_{\mathcal{F}})\left[d\log\bar{L}_{\mathcal{F}} - \frac{1}{1+\phi} d\log\bar{L}_{\mathcal{R}}\right]}{1-\frac{\phi}{1+\phi}(1-\theta)(1-\lambda_{\mathcal{F}})}.$$

Starting with the last equation, we see that once again, the share of flexible factors increases if the shock to the composition of demand redirects expenditure towards these factors or if the labor supply shocks for those factors is larger than the ones hitting the rigid factors. This reduces the shares of rigid factors, creates unemployment, and further reduces output through Keynesian effects. Indeed, putting everything together, we get

$$d\log Y = \lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}} + \frac{\frac{\phi}{1+\phi}(1-\theta)\lambda_{\mathcal{F}}(1-\lambda_{\mathcal{F}})d\log \bar{L}_{\mathcal{F}} + \left(1-\frac{1}{1+\phi}(1-\theta)\right)\lambda_{\mathcal{R}} d\log \bar{L}_{\mathcal{R}} - \frac{\phi}{1+\phi}\theta\lambda_{\mathcal{F}} d\log \omega_{0\mathcal{F}}}{1-\frac{\phi}{1+\phi}(1-\theta)(1-\lambda_{\mathcal{F}})}.$$

The difference between the case where wages have some downward flexibility ($\phi < \infty$) and the case where they do not ($\phi = \infty$) is that now the wages of the rigid factors falls, and this mitigates the increase in unemployment and the reduction in output. However, there is also a countervailing amplification effect: the labor supply shocks to the rigid factors now also over and above ensuring that they are rigid. This is because these shocks now reduce the wages of the rigid factors, which further redirects expenditure away from them because of complementarities, and further reduces employment of the rigid factors. Of course, allowing for some degree of wage flexibility can endogenously change the sets of flexible and rigid factors, and so we do no try to push the comparison any further.

Instead, we turn our attention to inflation. Using $d\log p^Y = d\log E - d\log Y$, the effect on inflation is

$$d\log p^Y = -\frac{1}{1+\phi} d\log \lambda_{\mathcal{F}} - \lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}} - \frac{1}{1+\phi}\lambda_{\mathcal{R}} d\log \bar{L}_{\mathcal{R}}.$$

The first term is negative, since the share of flexible factors expands in response to the negative demand shock, capturing the fact that as demand switches to flexible factors, the price of sticky sectors starts to decline, generating deflation. In the simple case where there are no negative supply shocks $d\log \bar{L} = 0$ but the composition of demand has shifted, we get that output *and* inflation both fall.