The Microeconomic Foundations of Aggregate Production Functions

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Aggregate Production Functions

- Aggregate production functions pervasive in modern macro.
- Reduced-form macro attributes treated as structural objects: marginal products, factor demands, elasticities of substitution, bias of technical change.
- Microfoundations needed:
  - reduced-form macro attributes from structural micro parameters (easier to estimate, more stable),
  - macro impact of micro phenomena,
  - micro consequences of aggregate phenomena.
- More and more pressing given newly available data sets with high levels of granularity and sources of variation.
- No satisfactory general theory to date.

Aggregate Consumption Functions

- Contrast with tremendous progress on microfoundations of aggregate consumption functions in last 20 years:
  - heterogeneous agents models,
  - nonlinear aggregation over and dynamics of distributions.
- Different theory needed for aggregate production functions:
  - multiple sectors and intermediate goods (gross vs. value added),
  - nonlinear networks and input-output linkages.

Goal

- General microfoundations for aggregate production functions.
- Arbitrary number of sectors and factors.
- Arbitrary network or input-output linkages.
- Arbitrary pattern of micro-elasticities.
- Arbitrary CRS and DRS (and some IRS).
- Arbitrary set of frictions or distortions.
- Further generalizations (see Conclusion).

Research Agenda

- Baqaee-Farhi (17): “The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten’s Theorem”.
- Baqaee-Farhi (18a): “Productivity and Misallocation in General Equilibrium”.
- Baqaee-Farhi (18b): “Macroeconomics with Heterogenous Agents and Input Output Linkages”.
- Baqaee-Farhi (19): “Networks, Barriers, and Trade”.
- ...

Related Literature

- Literature related to Cambridge-Cambridge controversy (see references in Cohen-Harcourt 03).
- Literature deriving Cobb-Douglas aggregate production functions from Pareto distribution of techniques at micro or aggregate level: Houthakker (55), Jones (05), Boehm-Oberfield (18)...
- Literature on nested CES models: Kremer (93), Jones (11,13), Oberfield-Raval (14), Rognlie (15)...
- Literature on production networks: Long-Plosser (83), Jovanovic (87), Drulauf (93), Scheninkman-Woodford (94), Horvath (98), Dupor (99), Carvalho (10), Gabaix (11), Forrester et al. (11), Acemoglu et al. (12), Di Giovanni et al. (14), Atalay (16), Bigio-Lao (16), Baqaee (16a,b), Grassi (17), Carvalho-Grassi (17), Baqaee-Farhi (17a,17b), ...

Outline

Cambridge-Cambridge Controversy

[T]he production function has been a powerful instrument of miseducation. The student of economic theory is taught to write $Y = F(K, L)$ where L is a quantity of labour, K a quantity of capital and Y a rate of output of commodities. He is instructed to [...] measure L in man-hours of labour; he is told something about the index-number problem involved in choosing a unit of output ; and then he is hurried on [...], in the hope that he will forget to ask in what units K is measured. Before ever he does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next.

— Joan Robinson (1953)

- Solow, Samuelson, Hahn, Bliss vs. Robinson, Sraffa, Pasinetti,...
- Many aspects: theoretical, methodological, ideological.

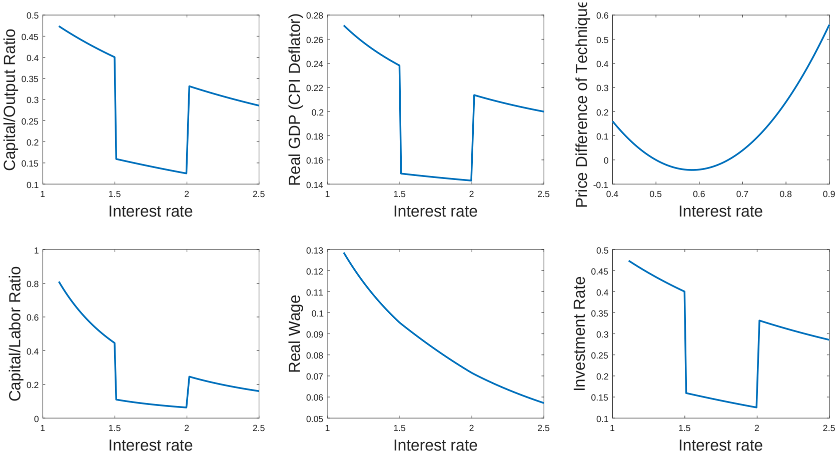
Samuelson's Three Key Parables of Capital Theory

- Aggregate production function with two factors $Y = F(K, L)$.
- Samuelson's three key "parables" of neoclassical writings:
  1. rate of interest determined by technical property $r = F_K$,
  2. diminishing returns to capital $(K/Y)(\underset{-}{r})$,
  3. distribution of income via relative scarcity of factors $(r/w)(K\underset{-}{/}L)$.
- Dependent on interpretation of capital as physical quantity, breaks down with heterogeneous capital goods (cannot be aggregated in physical units, must be aggregated in valuations units).

Re-switching and Capital-Reversing

- Samuelson (1966): re-switching / capital-reversing example.
- Use "Austrian" circulating-capital model (Hayek, Böhm-Bawerk).
- Two techniques to produce at t:
  1. Invest two units of labor in $t-2$, combine with six units at t,
  2. Invest seven units of labor in $t-1$.
- (2) dominates with high r since delays expensive.
- (2) dominates with low r: lower total labor requirement (7 vs. 8).
- (1) dominates with intermediate r.
- Reswitching leads to violation of parables (capital reversing).

Re-switching and Capital-Reversing



Aftermath

- “Pathology illuminates healthy physiology [...] If this causes headaches for those nostalgic for the old time parables of neoclassical writing, we must remind ourselves that scholars are not born to live an easy existence.”
  — Samuelson (1966).

- Solow’s “High-brow”, “middle-brow”, “low-brow” answers.

- “Solow, in the interest of empirical measurements and approximation, has been willing occasionally to drop his rigorous insistence upon a complex-heterogeneous capital model; instead, by heroic abstraction, has [...] estimated a single production function for society and has had a tremendous influence [...]. One might almost say that there are two Solows: the orthodox priest of the MIT school and the busman on a holiday who operates brilliantly and without inhibitions in the rough-and-ready realm of empirical heuristics.” — Samuelson (1962)

Aftermath

- Aggregate production functions not well founded in theory.
- Disagreement on curiosity vs. deep problem.
- Keeps being used for empirics.
- After short detour of “general equilibrium” approach (Bliss, Hahn), RBC revolution leads to quasi-universal adoption of aggregate production functions and neglect of controversies.
- Focus of macro professsion shifts from “heterogeneity and aggregation” to “dynamics and expectations”.

Our Starting Point

- Pick up quest abandoned after Cambridge-Cambridge controversy.
- Move away from question of factor aggregation by allowing for many disaggregated factors.
- General characterization of aggregate production functions.

Outline

General Setup

- General nested CES economy.
- Arbitrary number of sectors, factors, and input-output linkages.
- Arbitrary pattern of elasticities.
- Assume CRS (can handle DRS and some IRS with fixed factors).
- Can handle joint production.
- Shocks to sectoral productivity and factor supply or prices.

Networks and Input-Output

- "Relabel" each CES nest to be a new sector with elasticity ε_i.
- Input-output matrix
$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}.$$
- Leontief inverse
$$\Psi = (I - \Omega)^{-1} = \sum_{n=0}^{\infty} \Omega^n.$$
- Ω_{ij} and Ψ_{ij} direct and total reliance of i on j.
- Domar weights $\lambda_i = b' \Psi_{(i)}$. Write Λ_f instead of λ_f for factor.

Aggregate Production Function

- Aggregate production function

$$Y(A_1, \ldots, A_n, L_1, \ldots, L_Q)$$

  defined from maximization planning problem of choosing allocation of factors, intermediate and final goods, to maximize quantity of output bundle, subject to resource constraints.

- Characterization:
  - first order: marginal products of factors, aggregate TFP,
  - second order: elasticities of substitution, bias of technical change, nonlinearities.

- Most (but not only) useful for economies with inelastic factors.

Aggregate Cost Function

- Aggregate cost function

$$C(A_1, \ldots, A_n, r_1, \ldots, r_Q)$$

  defined from minimization planning problem of choosing allocation of factors, intermediate and final goods, to minimize cost of a unit of output bundle, subject to resource constraints.

- Characterization:
  - first order: factor demands, aggregate TFP,
  - second order: elasticities of substitution, bias of technical change, nonlinearities.
- Most (but not only) useful for economies with elastic factors.

Outline

First Order: Aggregate Production and Cost Functions

- Hulten (1978)'s theorem (macro envelope condition):

$$\frac{d\log Y}{d\log L_f} = \Lambda_f \quad , \frac{d\log Y}{d\log A_i} = \lambda_i,$$

$$\frac{d\log C}{d\log r_f} = \Lambda_f \quad , \frac{d\log C}{d\log A_i} = -\lambda_i.$$

- Marginal products, factor demands, macro impact of micro shocks.

- Shares as sufficient statistics.

- Irrelevance of: network, returns to scale, micro-elasticities,...

Outline

Second Order: Roadmap

- Propagation equations for shocks to factors or sectors.
- Apply to get Hessians of aggregate production and cost functions.
- Macro impact of micro shocks.
- Macro elasticities of substitution.
- Maco bias of technical change.
- Start with aggregate cost function (easier).

Outline

Propagation Equations with Elastic Factors

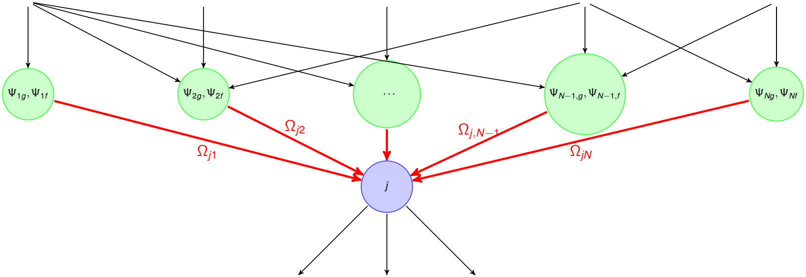
$$\mathrm{d}\Lambda_f = \sum_j (\theta_j - 1)\lambda_j Cov_{\Omega^{(j)}}\Big(\sum_k \Psi_{(k)}\,\mathrm{d}\log A_k - \sum_g \Psi_{(g)}\,\mathrm{d}\log r_g, \Psi_{(f)}\Big),$$

$$\mathrm{d}\lambda_i = \sum_j (\theta_j - 1)\lambda_j Cov_{\Omega^{(j)}}\Big(\sum_k \Psi_{(k)}\,\mathrm{d}\log A_k - \sum_g \Psi_{(g)}\,\mathrm{d}\log r_g, \Psi_{(i)}\Big).$$

- Shocks propagate downstream.

Explaining Covariance Operator: Ex. Shock $\mathrm{d}\log r_g > 0$

$$\mathrm{d}\Lambda_f = -\sum_j (\theta_j - 1)\lambda_j \underbrace{Cov_{\Omega^{(j)}}\left(\Psi_{(g)}\,\mathrm{d}\log r_g, \Psi_{(f)}\right)}.$$



- Ψ_{ig}: exposure of i to g.
- Ψ_{if}: exposure of i to f.

Hessian of Aggregate Cost Function

- Characterization of Hessian of aggregate cost function:

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log r_f \, \mathrm{d} \log r_g} = \frac{\mathrm{d} \Lambda_f}{\mathrm{d} \log r_g},$$

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log A_i \, \mathrm{d} \log A_j} = -\frac{\mathrm{d} \lambda_i}{\mathrm{d} \log A_j},$$

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log r_f \, \mathrm{d} \log A_i} = \frac{\mathrm{d} \Lambda_f}{\mathrm{d} \log A_i}.$$

- Summarizes same information as propagation equations.
- Sufficient statistics: network, returns to scale, micro-elasticities.

Macro Impact of Micro Shocks

- First- and second-order macro impact of micro shocks:

$$\frac{\partial \log C}{\partial \log A_i} = -\lambda_i,$$

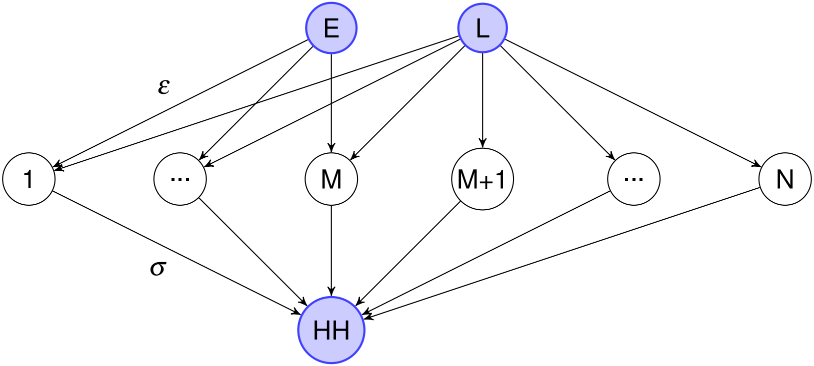
  and

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log A_i \, \mathrm{d} \log A_j} = -\frac{\mathrm{d}\lambda_i}{\mathrm{d} \log A_j}.$$

- Nonlinearities.
- Comovement.
- Macro moments: standard deviation, skewness, kurtosis.

“Universal” Energy Example

- Two factors: electricity and labor.
- Downstream sectors use electricity and labor with elasticity $\varepsilon < 1$.
- Final demand uses downstreams sectors with elasticity $\sigma \gg \varepsilon$



$$\frac{\mathrm{d}^2 \log C}{\mathrm{d}\log r_E^2} = -\frac{\mathrm{d}\Lambda_E}{\mathrm{d}\log r_E} = -\Lambda_E\left[(\varepsilon-1)(1-\frac{N}{M}\Lambda_E)+(\sigma-1)\Lambda_E(\frac{N}{M}-1)\right].$$

Macro Elasticities of Substitution

- Definition of macro Morishima Elasticity of Substitution (MES):

$$\sigma_{fg}^{C} = -\frac{\mathrm{d}\log(L_f/L_g)}{\mathrm{d}\log(r_f/r_g)},$$

  where $L_f = \mathrm{d}\,C/\mathrm{d}\,r_f$.

- More convenient to compute as:

$$1 - \sigma_{fg}^{C} = \frac{\mathrm{d}\log(\Lambda_f/\Lambda_g)}{\mathrm{d}\log(r_f/r_g)}.$$

Advantage of MES vs. Other Notions

- Measure of curvature of isoquants or ease of substitution.
- Sufficient statistic for effect on relative factor shares of changes in relative factor prices.
- Log derivative of quantity ratio w.r.t. price ratio.
- “Works” for CES: constant MES if and only if CES.

Characterization of Macro Elasticities of Substitution

$$\sigma^C_{fg} = \sum_j \theta_j \lambda_j Cov_{\Omega^{(j)}}(\Psi_{(g)}, \Psi_{(g)}/\Lambda_g - \Psi_{(f)}/\Lambda_f),$$

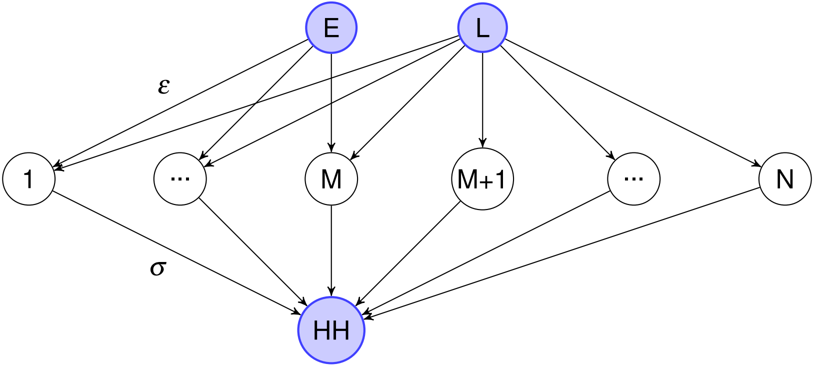
where

$$\sum_j \lambda_j Cov_{\Omega^{(j)}}(\Psi_{(g)}, \Psi_{(g)}/\Lambda_g - \Psi_{(f)}/\Lambda_f) = 1.$$

- General properties:
  - non-symmetry $\sigma^C_{fg} \neq \sigma^C_{gf}$ in general,
  - σ^C_{fg} weighted average of all θ_j, weights functions of network,
  - $\sigma^C_{fg} = \theta$ if uniform micro-elasticities $\theta_j = \theta$ (network irrelevance).

“Universal” Energy Example (Hicksian)

- Two factors: electricity and labor.
- Downstream sectors use electricity and labor with elasticity $\varepsilon < 1$.
- Final demand uses downstreams sectors with elasticity $\sigma \gg \varepsilon$



$$\sigma^C_{EL} = 1 - \frac{d\log(\Lambda_E/\Lambda_L)}{d\log(r_E/r_L)} = \sigma \frac{\left(\frac{N}{M} - 1\right)\Lambda_E}{1 - \Lambda_E} + \varepsilon \frac{1 - \frac{N}{M}\Lambda_E}{1 - \Lambda_E}.$$

Macro Bias of Technical Change

- Definition:

$$B^C_{fgk} = \frac{d \log(\Lambda_f/\Lambda_g)}{d \log A_k} = \frac{d \log(L_f/L_g)}{d \log A_k}.$$

- Characterization:

$$B^C_{fgk} = \sum_j (\theta_j - 1)\lambda_j Cov_{\Omega^{(j)}}(\Psi_{(k)}, \Psi_{(f)}/\Lambda_f - \Psi_{(g)}/\Lambda_g).$$

- No network-irrelevance result.

Capital-Biased Technical Change in a Task-Based Model

- Inspired by Acemoglu-Restrepo (17).
- CES aggregator over set of tasks with elasticity θ_D.
- Task i is CES of capital and labor with shares ω_{iK} and ω_{iL} and elasticity θ_{KL}.
- Micro capital-biased technical change in k.
- Macro capital bias:

$$B^C_{KLkK} = (\theta_{KL} - 1)\lambda_k \frac{\omega_{kK}}{\Lambda_K}\frac{\omega_{kL}}{\Lambda_L} + (\theta_D - 1)\lambda_k \frac{\omega_{kK}}{\Lambda_K}(1 - \frac{\omega_{kL}}{\Lambda_L}).$$

Outline

Propagation Equations with Inelastic Factors

$$
\begin{aligned}
\mathrm{d}\Lambda_f = & \sum_j (\theta_j - 1)\lambda_j Cov_{\Omega^{(j)}}\Big(\sum_k \Psi_{(k)} \,\mathrm{d}\log A_k + \sum_g \Psi_g \,\mathrm{d}\log L_g, \Psi_{(f)}\Big) \\
& - \sum_j (\theta_j - 1)\lambda_j Cov_{\Omega^{(j)}}\Big(\sum_g \Psi_{(g)} \frac{1}{\Lambda_g} \,\mathrm{d}\Lambda_g, \Psi_{(f)}\Big),
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{d}\lambda_i = & \sum_j (\theta_j - 1)\lambda_j Cov_{\Omega^{(j)}}\Big(\sum_k \Psi_{(k)} \,\mathrm{d}\log A_k + \sum_g \Psi_g \,\mathrm{d}\log L_g, \Psi_{(i)}\Big) \\
& - \sum_j (\theta_j - 1)\lambda_j Cov_{\Omega^{(j)}}\Big(\sum_g \Psi_{(g)} \frac{1}{\Lambda_g} \,\mathrm{d}\Lambda_g, \Psi_{(i)}\Big).
\end{aligned}
$$

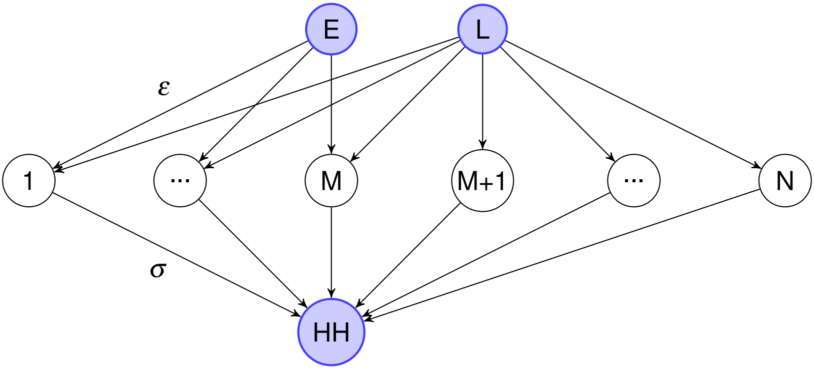
- Shocks propagate downstream and upstream.
- Need to solve a linear system, i.e. invert a matrix.

Extending Definitions and Characterizations

- Hessian.
- Macro impact of micro shocks.
- Macro elasticities of substitution between factors (MES).
- Macro bias of technical change.

"Universal" Energy Example

- Two factors: electricity and labor.
- Downstream sectors use electricity and labor with elasticity $\varepsilon < 1$.
- Final demand uses downstreams sectors with elasticity $\sigma \gg \varepsilon$.



$$\frac{d^2 \log Y}{d \log E^2} = \frac{d\Lambda_E}{d \log E^2} = \Lambda_E \frac{(\varepsilon - 1)(1 - \frac{N}{M}\Lambda_E) + (\sigma - 1)\Lambda_E(\frac{N}{M} - 1)}{1 + (\sigma - 1)\frac{(\frac{N}{M} - 1)\Lambda_E}{1 - \Lambda_E} + (\varepsilon - 1)\frac{(1 - \frac{N}{M}\Lambda_E)}{1 - \Lambda_E}}$$

"Universal" Energy Example (Hicksian)

- Two factors: electricity and labor.
- Downstream sectors use electricity and labor with elasticity $\varepsilon < 1$.
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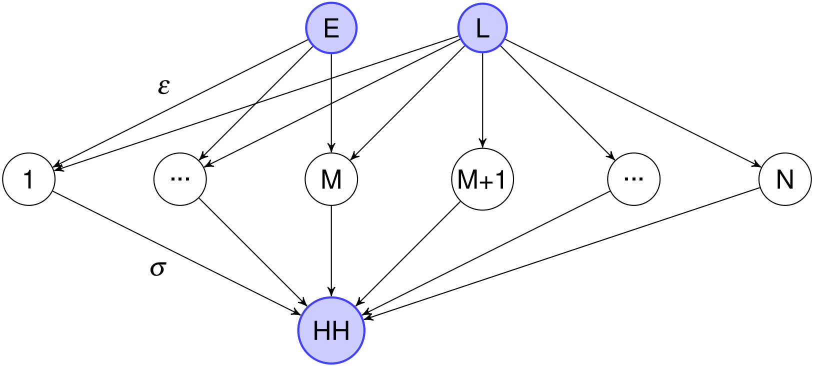


$$\sigma_{EL}^{F} = \frac{1}{1 - \frac{\mathrm{d}\log(\Lambda_E/\Lambda_L)}{\mathrm{d}\log(E/L)}} = \sigma \frac{\left(\frac{N}{M} - 1\right)\Lambda_E}{1 - \Lambda_E} + \varepsilon \frac{1 - \frac{N}{M}\Lambda_E}{1 - \Lambda_E}.$$

Capital-Biased Technical Change in a Task-Based Model

- Inspired by Acemoglu-Restrepo (17).
- Cobb-Douglas output aggregator over set of tasks.
- Task i is CES of capital and labor with shares ω_{iK} and ω_{iL} and elasticity θ_{KL}.
- Effects of capital-biased technical change in task k:

$$\frac{d\log w_L}{d\log A_{kK}} = \lambda_k \omega_{kK} \frac{1 + (\theta_{KL} - 1)\sum_i \lambda_i \left(\frac{\omega_{iL}}{\Lambda_L} - \frac{\omega_{kL}}{\Lambda_L} \right) \frac{\omega_{iK}}{\Lambda_K}}{1 + (\theta_{KL} - 1)\sum_i \lambda_i \frac{\omega_{iL}}{\Lambda_L} \frac{\omega_{iK}}{\Lambda_K}}.$$

- Can get decrease in labor share *and* decrease in real wage from capital-biased technical change.
- Impossible with usual aggregate production function approach.

Back to Cambridge-Cambridge Controversy

- Use formalism to capture Samuelson's reswitching example.

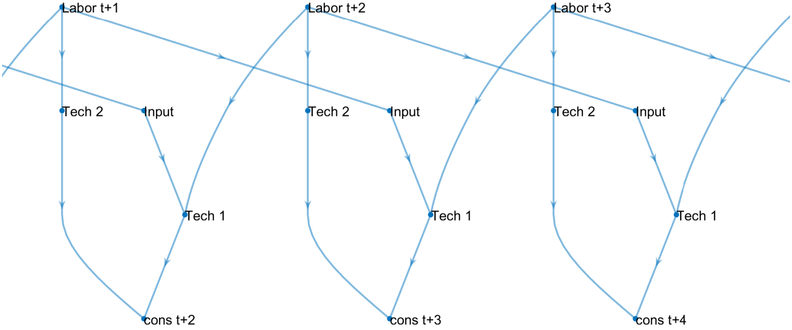


Figure: Reswitching Economy as a Network

Back to Cambridge-Cambridge Controversy

- Use formalism to capture Samuelson's reswitching example.
- Controversy as debate on relevance of network.

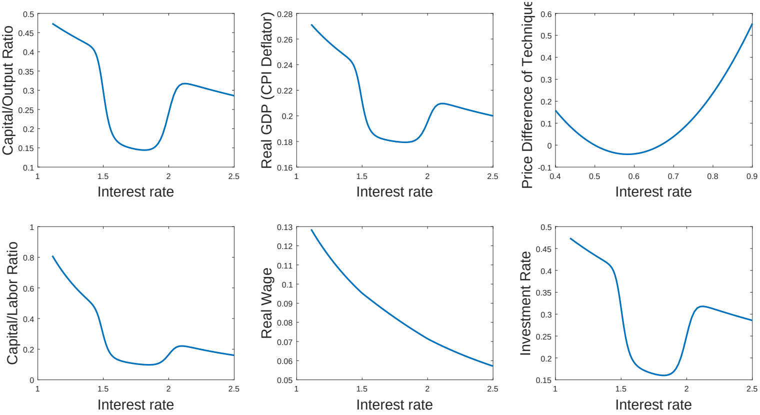


Figure: Smoothed Version of Samuelson Economy

Outline

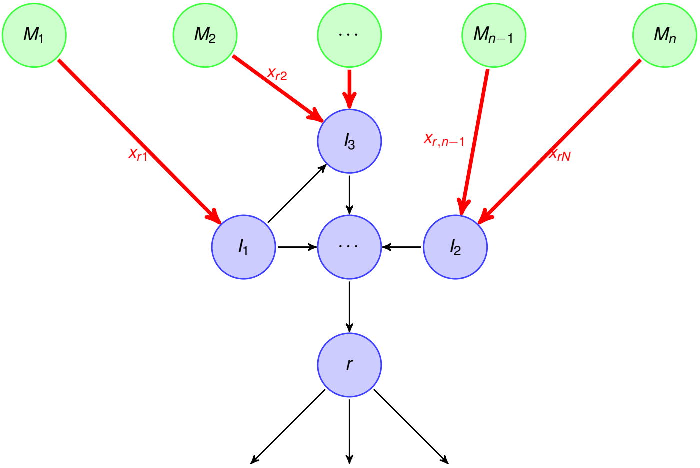
Beyond CES

- Can extend results to general non-CES functional forms.
- Input-output substitution operator generalizes input-output covariance operator.

Separating Production from Final Demand

- Can separate production from final demand via ”aggregate distance function”.
- Allows to combine analysis with heterogeneous agents or non-homothetic final demand.

Sub-Aggregate Production Functions



- Can get sub-aggregate production functions for ”islands”.
- More generally, can get sub-aggregate distance function.

Frictions and Distortions

- Characterizing aggregate production and cost functions harder:
  - shares no longer sufficient statistics for aggregate production and cost functions to the first order,
  - need shares, input-output matrix, micro elasticities, and distortions.
- Aggregate production and cost functions less useful:
  - first derivatives divorced from shares,
  - second derivatives divorced from elasticities of shares to shocks.
- Propagation equations robust instead to characterize elasticities of shares and get: marginal products, factor demands, macro impact of micro shocks, macro elasticities of substitution, macro bias of technical change.

Outline

Capital Skill Complementarity and Skill Premium

- Krusell et al. (00) study impact of declining price of equipment investment goods on skill premium in presence of capital-skill complementarities.
- Nested CES aggregate production function with elasticities motivated in part by micro-evidence:
  - 0.67 between skilled labor and equipment capital (inner nest),
  - 1.67 between unskilled labor and aggregate of skilled labor and equipment capital (outer nest).
- Revisit in calibrated disaggregated model with 66 sectors and input-output linkages:
  - same value added micro-elasticities between factors as above.
  - 0.5 elasticity between value added and intermediates,
  - 0.1 elasticity across intermediates,
  - 0.9 elasticity across sectors in consumption.

Macro Elasticities of Substitution

	Production Function			Cost Function		
	Capital	Non-college	College	Capital	Non-college	College
Capital	–	1.67	0.67	–	1.67	0.67
Non-college	1.04	–	0.89	1.26	–	1.09
College	0.67	1.67	–	0.67	1.67	–

Table: MESs between factors in the aggregate production function and in the aggregate cost function for the aggregate model.

	Production Function			Cost Function		
	Capital	Non-college	College	Capital	Non-college	College
Capital	–	1.43	0.69	–	1.47	0.72
Non-college	0.94	–	0.94	1.09	–	1.09
College	0.66	1.59	–	0.64	1.54	–

Table: MESs between factors in the aggregate production function and in the aggregate cost function for the disaggregated model.

Equipment Capital Shock and Skill Premium

	Capital	Non-college	College
Aggregate model	0.05	0.07	-0.13
Disaggregated Model	0.06	0.03	-0.12

Table: The (log point) change in factor income shares in response to the shock $\mathrm{d}\log K = -0.37$ in the aggregated and disaggregated model, reversing the effects of the equipment capital shock.

Change in skill premium

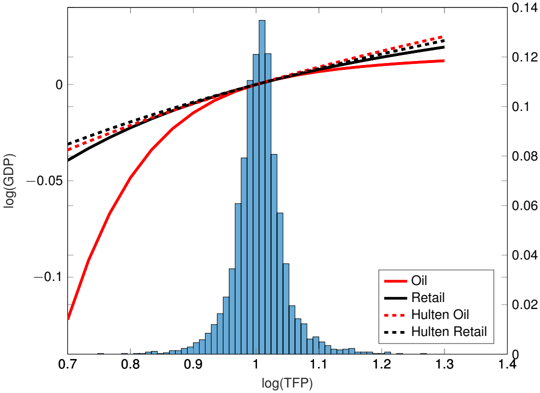
$$\left(\frac{1}{\sigma_{HK}} - \frac{1}{\sigma_{LK}}\right) \times \mathrm{d}\log K = \left(\frac{\mathrm{d}\log \Lambda_H}{\mathrm{d}\log K} - \frac{\mathrm{d}\log \Lambda_L}{\mathrm{d}\log K}\right) \times \mathrm{d}\log K,$$

is -0.20 log points in aggregate model vs. -0.16 log points in disaggregated model (20% lower).

Outline

Macro Impact of Oil vs. Retail Shocks

- Macro impact of micro shocks in calibrated disaggregated model with 66 sectors and input-output linkages.



Nonlinear Impact of Oil Shocks

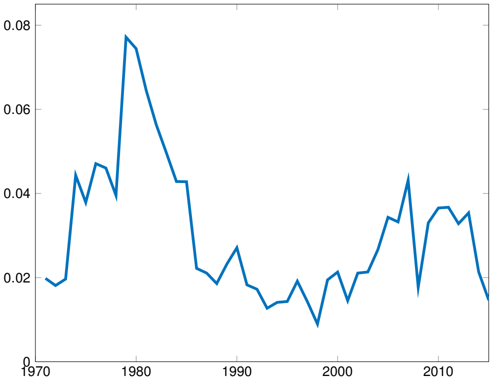


Figure: Global expenditures on crude oil as a fraction of world GDP.

- First-order effect: $1.8\% \times -13\% \approx -0.2\%$.
- Second-order effect: $\frac{1}{2}(1.8\% + 7.6\%) \times -13\% \approx -0.6\%$.

Outline

Baumol's Cost Disease and U.S. TFP Growth

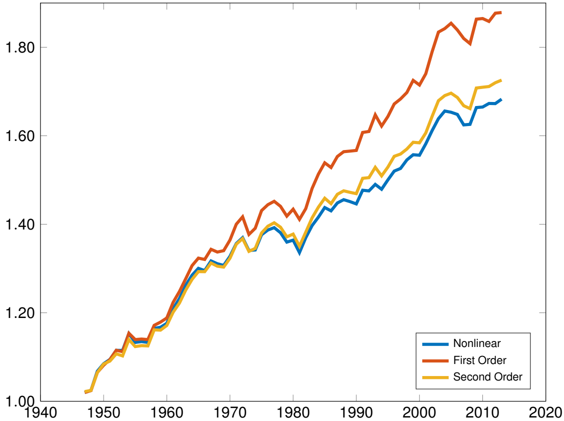


Figure: Cumulative change in *TFP*: nonlinear (actual), first-order approximation, and second-order approximation.

Baumol's cost disease reduced U.S. aggregate TFP growth by 20 percentage points.

Conclusion

- Pick up where Cambridge-Cambridge controversy left.
- General microfoundations for aggregate production functions.
- Many applications.
- Research agenda:
  - Baqaee-Farhi (17,18a,b,19),
  - ongoing...IRS, entry, exit,
  - ongoing...explicit IO models of market structure,
  - active and exciting area!