RARE DISASTERS AND EXCHANGE RATES*

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Abstract

We propose a new model of exchange rates, based on the hypothesis that the possibility of rare but extreme disasters is an important determinant of risk premia in asset markets. The probability of world disasters as well as each country's exposure to these events is time-varying. This creates joint fluctuations in exchange rates, interest rates, options, and stock markets. The model accounts for a series of major puzzles in exchange rates: excess volatility and exchange rate disconnect, forward premium puzzle and large excess returns of the carry trade, and comovements between stocks and exchange rates. It also makes empirically successful signature predictions regarding the link between exchange rates and telltale signs of disaster risk in currency options. *JEL* codes: G12, G15.

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I. Introduction

We propose a new model of exchange rates, based on the hypothesis of Rietz (1988) and Barro (2006) that the possibility of rare but extreme disasters is an important determinant of risk premia in asset markets. The model accounts for a series of major puzzles in exchange rates. It also makes signature predictions about the link between exchange rates and currency options, which are broadly supported empirically. Overall, the model explains classic exchange rate puzzles and more novel links between options, exchange rates and stock market movements.

In the model, at any point in time, a world disaster might occur. Disasters correspond to bad times — they therefore matter disproportionately for asset prices despite the fact that they occur with a low probability. Countries differ by their riskiness, that is by how much their exchange rate would depreciate if a world disaster were to occur (something that we endogenize in the paper, relating it to the productivity of the export sector). Because the exchange rate is an asset price whose future risk affects its current value, relatively riskier countries have more depreciated exchange rates.

The probability of a world disaster as well as each country's exposure to these events is time-varying. This creates large fluctuations in exchange rates, which rationalize their apparent "excess volatility". To the extent that perceptions of disaster risk are not perfectly correlated with conventional macroeconomic fundamentals, our disaster economy exhibits an "exchange rate disconnect" (Meese and Rogoff 1983).

Relatively risky countries also feature high interest rates, because investors need to be compensated for the risk of an exchange rate depreciation in a potential world disaster. This allows the model to account for the forward premium puzzle. This is true both in samples with no disasters and in full samples with a representative number of disasters, but the intuition is easier to grasp in the case of samples with no disasters.[1] Indeed, suppose that a country is temporarily risky: it has high interest rates, and its exchange rate is depreciated. As its riskiness reverts to the mean, its exchange rate appreciates. Therefore, the currencies of high

[1] According to the uncovered interest rate parity (UIP) equation, the expected depreciation of a currency should be equal to the interest rate differential between that country and the reference region. A regression of exchange rate changes on interest rate differentials should yield a coefficient of 1. However, empirical studies starting with Tryon (1979), Hansen and Hodrick (1980), Fama (1984), and those surveyed by Lewis (2011) consistently produce a regression coefficient that is less than 1, and often negative. This invalidation of UIP has been termed the *forward premium puzzle*: currencies with high interest rates tend to appreciate. In other words, currencies with high interest rates feature positive predictable excess returns.

interest rate countries appreciate on average, conditional on no disaster occurring. In the paper, we also offer a detailed intuition in terms of time-varying disaster risk premia for the case of full samples with a representative number of disasters.

The disaster hypothesis also makes specific predictions about option prices. This paper works them out, and finds that those signature predictions are reasonably well borne out in the data. We view this as encouraging support for the disaster hypothesis.

The starting point is that, in our theory, the exchange rate of a risky country commands high put premia in option markets – as measured by high "risk reversals" (a risk reversal is the difference in implied volatility between an out-of-the-money put and a symmetric out-of-the-money call). Indeed, investors are willing to pay a high premium to insure themselves against the risk that the exchange rate depreciates in the event of a world disaster. A country's risk reversal is therefore a reflection of its riskiness.

Accordingly, the model makes four signature predictions regarding these put premia ("risk reversals"). First, investing in countries with high risk reversals should have high returns on average. Second, countries with high risk reversals should have high interest rates. Third, when the risk reversal of a country goes up, its currency contemporaneously depreciates. These predictions, and a fourth one detailed below, are broadly consistent with the data (see p. 24).

The model is very tractable, and we obtain simple and intuitive closed form expressions for the major objects of interest, such as exchange rates, interest rates, carry trade returns, yield curves, forward premium puzzle coefficients, option prices, and stocks. To achieve this, we build on the closed-economy model with a stochastic intensity of disasters proposed in Gabaix (2012) (Rietz 1988 and Barro 2006 assume a constant intensity of disasters), and use the "linearity-generating" processes developed in Gabaix (2009). Our framework is also very flexible. We show that it is easy to extend the basic model to incorporate several factors and inflation.

We calibrate a version of the model and obtain quantitatively realistic values for the quantities of interest, such as the volatility of the exchange rate, the interest rate, the forward premium, the return of the carry trade, as well as the size and volatility of risk reversals and their link with exchange rate movements and interest rates. The underlying disaster numbers largely rely on Barro and Ursua (2008)'s empirical numbers which imply that rare disasters matter five times as much as they would if agents were risk neutral. As a result, changes in beliefs about disasters translate into meaningful volatility. This is why the model yields substantial volatility which is difficult to obtain with more traditional models (e.g. Obstfeld and Rogoff 1995).

In addition, our calibration matches the somewhat puzzling link between stock market and exchange rate returns. Empirically, there is no correlation between movements in the stock market and the currency of a country. However, the most risky currencies have a positive correlation with world stock market returns, while the least risky currencies have a negative correlation. Our calibration replicates these facts.

Finally, recent research (Lustig, Roussanov and Verdelhan (2011)) has documented a one-factor structure of currency returns (they call this new factor HML_{FX}). Our proposed calibration matches this pattern. In addition, our model delivers the new prediction that risk reversals of the most risky countries (respectively least risky) should covary negatively (respectively positively) with this common factor. This prediction holds empirically.

To sum up, our model delivers the following patterns.

Classic puzzles

1. Excess volatility of exchange rates.

2. Failure of uncovered interest rate parity. The coefficient in the Fama regression is less than 1, and sometimes negative.

   Link between options and exchange rates

3. High interest rate countries have high put premia (as measured by "risk reversals").

4. Investing in countries with high (respectively low) risk reversals delivers high (respectively low) returns.

5. When the risk reversal of a country's exchange rate increases (which indicates that the currency becomes riskier), the exchange rate contemporaneously depreciates.

   Link between stock markets and exchange rates

6. On average, the correlation between a country's exchange rate returns and stock market returns is zero.

7. However, high (respectively low) interest rate countries have a positive (respectively negative) correlation of their currency with the world stock market: their currency appreciates (respectively depreciates) when world stock markets have high returns.

   Comovement structure in exchange rates

8. There is a broad 1-factor structure in the excess currency returns (the HML_{FX} factor of Lustig, Roussanov and Verdelhan 2011): high interest rate currencies tend to comove, and comove negatively with low interest rate currencies.

9. There is a broad 1-factor structure of stock market returns: stock market returns tend to be positively correlated across countries.

10. There is a positive covariance between the above two factors.

    At the same time, we match potentially challenging domestic moments, e.g.

11. High equity premium.

12. Excess volatility of stocks.

Hence, we obtain a parsimonious model of exchange rates, interest rates, options, and stocks that matches the main features of the data. It delivers novel predictions borne out in the data, notably the link between movements in option prices ("risk reversals"), currency returns and stock returns.

Relation to the literature. Our paper is part of a broader research movement using modern asset pricing models to understand exchange rates, especially the aforementioned puzzles.

In the closed-economy literature, there are three main paradigms for representative agent rational expectations models to explain both the level and the volatility of risk premia (something that the plain consumption CAPM with low risk aversion fails to generate):[2] habits (Abel 1990, Campbell and Cochrane 1999), long run risks (Epstein and Zin 1989, Bansal and Yaron

[2]Pavlova and Rigobon (2007, 2008) provide an elegant and tractable framework for analyzing the joint behavior of bonds, stocks, and exchange rates which succeeds in accounting for comovements among international assets. However, their model is based on a traditional consumption CAPM, and therefore generates low risk premia and small departures from UIP.

2004) and rare disasters (Rietz 1988, Barro 2006, and for time-varying disasters, Gabaix 2012, Gourio 2012 and Wachter 2013).

Economists have extended these closed-economy paradigms to open-economy setups to understand exchange rates. Habit models were used by Verdelhan (2010), Heyerdahl-Larsen (2014), and Stathopoulos (2012) to generate risk premia in currency markets. Long run risks models were applied by Colacito and Croce (2011, 2013) and Bansal and Shaliastovich (2013), using a two-country setting. Given our model features some form of long-lasting shocks, it would be interesting to extend our framework to an Epstein-Zin setting, in particular with a more usual production function and capital accumulation, e.g. along the lines of Colacito et al. (2014). One specific feature of the disaster approach is that it allows to think naturally about the risk reversals, which makes the four signature predictions outlined above. In addition, the present disaster model is particularly tractable, so that closed forms obtain and we can think about an arbitrary number of countries rather than just two.

To the best of our knowledge, we are the first to adapt the disaster paradigm to exchange rates. After the present paper was circulated, Gourio, Siemer, and Verdelhan (2013) and Guo (2010) studied related and complementary models numerically in an RBC and a monetary context, respectively, while Du (2013) explores quantitatively a related model, with a different focus. His results are mostly numerical (as they apply to a more complex Epstein-Zin world, where closed forms are hard to obtain), apply to two countries (which makes it impossible to address an inherently multicountry set of issues, like Lustig, Roussanov and Verdelhan (2011)'s HML_{FX}) and do not touch upon the cross-moments between stocks and exchange rates, and between stocks and currency options. Martin (2013) presents a two-country model with i.i.d. shocks and characterizes the impact of deviations from lognormality using cumulants, and generates a Fama coefficient equal to zero; however he does not investigate a number of issues that we explore, such as currency options and stocks. Because of his conceptual focus he only offers a limited numerical illustration rather than a full-blown quantitative analysis.[3]

On the empirical front, several recent papers investigate the hypothesis that disaster risk accounts for the forward premium puzzle. This debate is active and ongoing. Brunnermeier,

[3]Another strand of the literature departs from the assumption of frictionless markets. Alvarez, Atkeson, and Kehoe (2002) rely on a model with endogenously segmented markets to qualitatively generate the forward premium anomaly. Pavlova and Rigobon (2012) study the importance of incomplete markets for external adjustment. Gabaix and Maggiori (2015) present a model of exchange rate determination and carry trade based on limited risk bearing capacity of the financial sector.

Nagel and Pedersen (2009) find evidence of a strong link between currency carry trade premium and currency crash risk. Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) compare hedged and unhedged carry trade returns, and conclude that the carry trade premium can be explained by rare events, reflecting of high values of the stochastic discount factor and negative carry trade returns – consistent with the rare disaster hypothesis. Using a similar methodology, Jurek (2014) reaches a more skeptical conclusion about the disaster hypothesis, and argues that it accounts for at most one third of the carry trade returns. Using a different methodology that makes direct use of option prices at various degrees of moneyness, Farhi et al. (2015) find that global disaster risk accounts for a large fraction of the carry trade risk premium in advanced countries in the 1996 to 2014 sample, and that global disaster risk is an important factor in the cross-sectional and time-series variation of exchange rates, interest rates, and equity tail risk. These and our papers are also related to an older literature on so-called "peso problems"(Lewis 2011). Under the "pure peso" view, there are no risk premia and the forward premium puzzle is simply due to a small sample bias. By contrast, under the "rare disasters"view there are risk premia, and the forward premium puzzle also holds in full samples.

Outline. The rest of the paper is organized as follows. In Section II, we set up the basic model and in Section III derive its implications for the major puzzles. Section IV contains extensions to options, stocks and the nominal yield curve. Section V shows the calibration of the model. Section VI concludes. Most proofs are in the Appendix.

II. Model Setup

II.A. Macroeconomic Environment

We consider a stochastic infinite horizon open economy model. There are n countries indexed by $i = 1, 2, ..., n$. There are 2 goods in each country i: a traded good, called T, and a nontraded good, called NT. The traded good is common to all countries, the non-traded good is country-specific.

Preferences. In country i, agents value consumption of traded good C_{it}^{T} and nontraded good C_{it}^{NT} according to

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\exp(-\rho t)u\left(C_{it}^{T},C_{it}^{NT},\theta_{it}\right)\right], \tag{1}$$

where θ_{it} is a preference shock. We choose the following specification of utility

$$u\left(C_{it}^{T},C_{it}^{NT},\theta_{it}\right)=\frac{\zeta_{it}\left[\left(C_{it}^{T}\right)^{\frac{\sigma-1}{\sigma}}+\frac{1}{\xi_{it}}\left(C_{it}^{NT}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}(1-\gamma)}}{1-\gamma}, \tag{2}$$

where γ is the coefficient of relative risk aversion, σ is the elasticity of substitution between tradables and non-tradables, and the preference shock is a vector $\theta_{it}=(\zeta_{it},\xi_{it})$ with a shifter ζ_{it} for the marginal utility of wealth and a shifter ξ_{it} for the relative expenditure on traded versus non-traded goods.[4] Most of our theoretical results do not in fact require this specific structure for preferences. We adopt it only for concreteness, and because we make use of it in our calibration.

Endowments and Technology. At each date t, each country receives a random endowment of the tradable good, η_{it}^{T} and of the non-tradable good, η_{it}^{NT}. The endowment of the non-tradable good can be used in one of two ways. It can be either consumed (C_{it}^{NT}) or invested in the production of the tradable good (ι_{it}^{T}). Investing one unit of the non-traded good at time t yields $\exp(-\lambda s)\omega_{i,t+s}$ units of the traded good in all future periods $t+s\geq t$. Here λ is the depreciation rate of the initial investment, and ω_{it} is the yield of the export technology. This gives the following feasibility constraints

$$\eta_{it}^{NT}=\iota_{it}^{T}+C_{it}^{NT}, \tag{3}$$

$$Y_{it}^{T}=\eta_{it}^{T}+\sum_{s=0}^{\infty}\exp(-\lambda s)\omega_{it}\iota_{i,t-s}^{T}. \tag{4}$$

where Y_{it}^{T} is the total output of tradable goods in country i in period t.

The consumption of tradable goods is subject to the world-wide feasibility constraint:

$$\sum_i Y_{it}^{T}=\sum_i C_{it}^{T}. \tag{5}$$

[4] The shifters are not essential. Their only role is to help capture the relative movements in the consumption of traded goods, non-traded goods, and the exchange rate.

Complete markets. Markets are complete: there exists a complete set of state and date contingent securities. As a result, the welfare theorems apply and we can study the competitive equilibrium as the solution of a planning problem. The planner chooses a sequence of ι_{it}^T, C_{it}^T and C_{it}^{NT} to maximize a weighted sum of welfare (1) across countries with Pareto weights μ_i subject to (3)-(5). The Lagrangian is:

$$\begin{aligned} L = & \sum_i \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \mu_i \exp(-\rho t) u \left(C_{it}^T, C_{it}^{NT}, \theta_{it} \right) \right] \\ & + \sum_i \sum_{t=0}^{\infty} \mathbb{E}_0 \left[M_t^* \left(\sum_{s=0}^{\infty} \exp(-\lambda s) \omega_{it} \left(\eta_{i,t-s}^{NT} - C_{i,t-s}^{NT} \right) + \eta_{it}^T - C_{it}^T \right) \right] \end{aligned} \tag{6}$$

where M_t^* is the Lagrange multiplier on the economy-wide resource constraint for tradable goods in period t. The first order conditions $\frac{\partial L}{\partial C_{it}^T} = 0$ and $\frac{\partial L}{\partial C_{it}^{NT}} = 0$ deliver respectively:

$$\mu_i \exp(-\rho t)\, u_{C_{it}^T} - M_t^* = 0,$$
$$\mu_i \exp(-\rho t)\, u_{C_{it}^{NT}} - \mathbb{E}_t \sum_{s=0}^{\infty} M_{t+s}^* \exp(-\lambda s) \omega_{i,t+s} = 0.$$

The relative price $e_{it} = \frac{u_{C_{it}^{NT}}}{u_{C_{it}^T}}$ of non-tradable to tradable goods within each country is obtained as

$$e_{it} = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \exp(-\lambda s) \omega_{i,t+s} \frac{M_{t+s}^*}{M_t^*} \right].$$

World numéraire and pricing kernel. We choose the traded good as the world numéraire. As a result, M_t^* is the pricing kernel in the world numéraire. The price at time t of an asset with a stochastic stream of cash flows $(D_{t+s})_{s \geq 0}$ (expressed in units of the world numéraire) is given by $\mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t+s}^* D_{t+s} \right] / M_t^*$.

Exchange rate. Recall that e_{it} is the relative price of non-tradables to tradables in country i. Since the tradable good is the world numéraire, e_{it} is the exchange rate of country i vis-à-vis the world numéraire if the domestic numéraire in country i is the non-tradable good. We sometimes refer to it as the "absolute" exchange rate of country i. Our convention is such that when e_{it} increases, the exchange rate appreciates. We also define the bilateral exchange rate between country i and country j to be $\frac{e_{it}}{e_{jt}}$: an exchange rate appreciation of i with respect to j corresponds to an increase of $\frac{e_{it}}{e_{jt}}$.

If the domestic numéraire is a basket of non-tradable and tradable goods, then e_{it} and $\frac{e_{it}}{e_{jt}}$ correspond only approximately to the corresponding traditional notions of real exchange rates. This correspondence becomes exact in the limit where non-tradables represent a large fraction of the consumption basket ($\xi_{it} \to 0$). Section VII.A quantitatively discusses the issues further.

We collect these results in a proposition.

Proposition 1 (Value of the exchange rate). *The bilateral exchange rate between country i and country j is $\frac{e_{it}}{e_{jt}}$, where the absolute exchange rate e_{it} of country i is the present value of its future export productivity:*

$$e_{it} = \mathbb{E}_t \left[\sum_{s=0}^{\infty} M^*_{t+s} \exp(-\lambda s) \omega_{i,t+s} \right] / M^*_t, \tag{7}$$

with the convention that an increase in e_{it} means an appreciation of country i's currency.

Equation (7) expresses the exchange rate directly as the net present value of future fundamentals. The non-tradable good is an asset that produces dividends $D_{i,t+s} = \exp(-\lambda s)\omega_{i,t+s}$, and is priced accordingly.

Existence of equilibrium. To fully specify the model, we find it convenient to proceed in the following way. Take a process for productivity ω_{it}. We will posit a specific world pricing kernel M^*_t. Our specification will be chosen to be realistic yet deliver closed-form solutions for exchange rates and interest rates. The following Lemma shows that endowment processes η^T_{it} and η^{NT}_{it} can always be found to rationalize this pricing kernel. In addition, this procedure allows us to match any process for net exports $nx_{it} = Y^T_{it} - C^T_{it}$.[5]

It would be desirable to uncover empirical evidence for the endowment and preference shocks processes postulated in the Lemma.

Lemma 1 (Existence of equilibrium). *(i) Take as given a process for the world pricing kernel M^*_t, productivity ω_{it}, preference shocks θ_{it}, and net exports nx_{it} with the restriction that $\sum_i nx_{it} = 0$ for all t. There exist endowment processes for traded goods η^T_{it} and non-traded*

[5] We could have introduced Epstein-Zin preferences. Given processes for M^*_t and ω_{it}, this would only change the implied processes for C^T_{it} and C^{NT}_{it}, as well as the endowment processes η^T_{it} and η^{NT}_{it}. But it would not change our main results, namely the characterization of exchange rates, interest rates, stocks and options.

goods η_{it}^{NT} as well as Pareto weights μ_i for each country i such that the equilibrium world pricing kernel and net exports are indeed given by M_t^* and nx_{it}, respectively. (ii) Take as given a process for the world pricing kernel M_t^*, productivity ω_{it}, consumptions C_{it}^T and C_{it}^{NT}, and net exports nx_{it} with the restriction that $\sum_i nx_{it} = 0$ for all t. There exist endowment processes for traded goods η_{it}^T and non traded goods η_{it}^{NT}, preference shocks θ_{it} as well as Pareto weights μ_i for each country i such that the equilibrium world pricing kernel, consumptions and net exports are indeed given by M_t^*, C_{it}^T and C_{it}^{NT}, and nx_{it}, respectively.

II.B. Disaster Risk

We now specialize the model to incorporate variable disaster risk. We study equilibria where the world consumption of the tradable good M_t^* follows the following stochastic process. In line with Rietz (1988) and Barro (2006), we assume that in each period $t+1$ a disaster may happen with probability p_t. If no disaster happens, $\frac{M_{t+1}^*}{M_t^*} = \exp(-R)$ for some R. If a disaster happens, then $\frac{M_{t+1}^*}{M_t^*} = \exp(-R)B_{t+1}^{-\gamma}$, with $B_{t+1} > 0$. To sum up, the pricing kernel M_t^* evolves according to[6]

$$\frac{M_{t+1}^*}{M_t^*} = \exp(-R) \times \begin{cases} 1 & \text{if there is no disaster at } t+1, \\ B_{t+1}^{-\gamma} & \text{if there is a disaster at } t+1. \end{cases} \tag{8}$$

In the calibration, we relate R to the discount factor ρ and the growth rate g_c of the world endowment of tradable goods. We use the Barro-Ursua (2008) numbers to discipline the choice of B_{t+1} by relating it to the recovery rate of consumption in disasters. Using the first-order conditions of the planning problem, we have:

$$C_{it}^{NT} = C_{it}^T \left(\xi_{it} e_{it}\right)^{-\sigma} \tag{9}$$

and

$$C_{it}^T = \left(\frac{M_t^*}{\mu_i e^{-\rho t}\zeta_{it}}\right)^{-\frac{1}{\gamma}} \left[1 + \xi_{it}^{-\sigma} e_{it}^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}\left(\frac{1}{\gamma}-1\right)-\frac{1}{\gamma}}. \tag{10}$$

[IGOR please proofread next 2 paragraphs] If there was just one good, the fact that the

[6] In a more complex variant, disasters could be followed by partial recoveries (e.g. Gourio 2008, Nakamura et al. 2013). For a given γ, that lowers the risk premia coming from disaster risk. However, a slight increase in γ could counteract that effect. All in all, we find it simpler and more transparent to keep the simplest disaster formulation, at fairly little cost to the economics.

pricing kernel M_t^* is multiplied by $B_{t+1}^{-\gamma}$ in a disaster would mean that consumption is multiplied by B_{t+1} in that disaster. In our situation with multiple goods, the economics is broadly similar, though a bit more complex in its details.

Consider first the case where e_{it}, ξ_{it}, and ζ_{it} do not jump if a disaster occurs at $t+1$. Then our assumption (8) implies that both C_{it}^T and C_{it}^{NT} are multiplied by B_{t+1} between t and $t+1$, and so is the consumption basket $\left[\left(C_{it}^T\right)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\xi_{it}}\left(C_{it}^{NT}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$.

Of course we will be interested in cases where e_{it}, ξ_{it}, and ζ_{it} do jump if a disaster occurs at $t+1$, potentially leading to jumps in the relative consumption of traded versus nontraded goods in the event of a disaster. This is easiest to see in the case where $\sigma\gamma = 1$. Consider for example the situation where the exchange e_{it} is multiplied by a factor $F_{i,t+1}$ but ξ_{it}, and ζ_{it} do not jump if a disaster occurs at $t+1$. Then, (10) reduces to $C_{it}^T = \left(\frac{M_t^*}{\mu_i e^{-\rho t}\zeta_{it}}\right)^{-\frac{1}{\gamma}}$ and is independent of e_{it} so that C_{it}^T is multiplied by B_{t+1} if a disaster occurs at $t+1$ while (9) indicates C_{it}^{NT} is multiplied by $B_{t+1}F_{i,t+1}^{-\sigma}$.

Productivity. We assume that productivity of country i follows:

$$\frac{\omega_{i,t+1}}{\omega_{it}} = \exp(g_{\omega_i}) \times \begin{cases} 1 & \text{if there is no disaster at } t+1, \\ F_{i,t+1} & \text{if there is a disaster at } t+1. \end{cases} \tag{11}$$

During a disaster, the relative productivity of the nontraded good is multiplied by $F_{i,t+1}$. For instance, if productivity falls by 20%, then $F_{i,t+1} = 0.8$.

In the model, a sufficient statistic for many quantities of interest is the "resilience" of a country i, defined as:

$$H_{it} = p_t \mathbb{E}_t^D \left[B_{t+1}^{-\gamma} F_{i,t+1} - 1\right], \tag{12}$$

where $\mathbb{E}_t^D$ (resp. $\mathbb{E}_t^{ND}$) is the expected value conditional on a *d*isaster happening at $t+1$ (resp. conditional on *n*o *d*isaster happening). A relatively safe country has a high resilience H_{it}, as it has a high recovery rate $F_{i,t+1}$. Conversely, a relatively risky country has low resilience. In equation (12), the probability p_t and the world intensity of disasters B_{t+1} are common to all countries, but the recovery rate $F_{i,t+1}$ is country-specific. The changes in prospective recovery rates could be correlated across countries.

Rather than separately specifying laws of motion for its components (p_t, B_{t+1}, and $F_{i,t+1}$), we directly model the law of motion for H_{it} and we assume that it takes a particular, convenient functional form. This will allow us to compute exchange rates, interest rates and options in closed form.

We decompose

$$H_{it} = H_{i*} + \widehat{H}_{it}, \tag{13}$$

where H_{i*} and $\widehat{H}_{it}$ are the constant and variable parts of resilience, respectively. For tractability, we posit that the law of motion for $\widehat{H}_{it}$ follows a linearity-generating process:

$$\widehat{H}_{i,t+1} = \frac{1 + H_{i*}}{1 + H_{it}} \exp(-\phi_{H_i}) \widehat{H}_{it} + \varepsilon^H_{i,t+1}, \tag{14}$$

where ϕ_{H_i} denotes the speed of mean reversion of resilience and the innovations $\varepsilon^H_{i,t+1}$ have mean zero, both unconditionally and conditional on a disaster ($\mathbb{E}_t\left[\varepsilon^H_{i,t+1}\right] = \mathbb{E}^D_t\left[\varepsilon^H_{i,t+1}\right] = 0$).

The economic meaning of equation (14) is that $\widehat{H}_{it}$ mean-reverts towards zero, but is subject to shocks. Because H_{it} hovers around H_{i*}, $\frac{1+H_{i*}}{1+H_{it}}$ is close to one and the process behaves like a regular AR(1) up to second-order terms in $\widehat{H}_{it}$: $\widehat{H}_{i,t+1} \simeq \exp(-\phi_{H_i})\widehat{H}_{it} + \varepsilon^H_{i,t+1}$. The "twist" term $\frac{1+H_{i*}}{1+H_{it}}$ is innocuous from an economic perspective but provides analytical tractability (see the technical appendix in Gabaix 2009 for a discussion). Linearity-generating processes allow the derivation of the equilibrium exchange rate in closed form.[7]

Resilience as a sufficient statistic. As we shall see shortly, resilience H_{it} is a sufficient statistic for the impact of p_t, B_{t+1}, and $F_{i,t+1}$ on exchange rates, interest rates, as well as consumption. We leverage this insight and do not independently specify the processes for p_t, B_{t+1}, and $F_{i,t+1}$. We simply assume that they lead to the process for resilience H_{it} that we have posited above.

[7] For the process to be well-behaved, we need to impose that $\widehat{H}_{it}$ is always above a negative lower bound $\underline{\widehat{H}}_i = (1 + H_{i*})(\exp(-\phi_{H_i}) - 1)$. To ensure this, the variance of the innovations $\varepsilon^H_{i,t+1}$ goes to zero when $\widehat{H}_{it}$ is close to $\underline{\widehat{H}}_i$. The online appendix details this.

III. Main Results

III.A. Exchange Rates and Interest Rates

Exchange rate. We start by deriving the value of the exchange rate. We define $h_{i*} := \ln(1 + H_{i*})$ and

$$r_{ei} := R + \lambda - g_{\omega_i} - h_{i*}. \tag{15}$$

As we shall see below, $r_{ei} - \lambda$ is the interest rate when the temporary component of resilience, $\widehat{H}_{it}$, is zero.

Proposition 2 (Level of the exchange rate). *The bilateral exchange rate between country i and country j is $\frac{e_{it}}{e_{jt}}$, where e_{it} is the exchange rate of country i in terms of the world numéraire and is equal to*

$$e_{it} = \frac{\omega_{it}}{r_{ei}} \left(1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}} \right) \tag{16}$$

in the limit of small time intervals.

Equation (16) implies that the exchange rate e_{it} increases (appreciates) with h_{i*} and $\widehat{H}_{it}$: risky (i.e., low resilience) countries have a low (depreciated) exchange rate. Safer (i.e., high resilience) currencies have a high (appreciated) exchange rate. Risky countries are those whose currency value (and, more primitively, whose relative price of non-tradables) is expected to drop during disasters.

The exchange rate fluctuates with the resilience $\widehat{H}_{it}$. As we shall see in the calibration, these fluctuations are plausibly large, and can therefore generate "excess volatility" of the exchange rate (see the online appendix for a generalization with several factors).

To the extent that they are imperfectly correlated with traditional macroeconomic fundamentals, the fluctuations in resilience can generate an "exchange rate disconnect".

Derivation of Proposition 2. The appendix provides a rigorous proof. Here we guess and verify a solution of the form

$$\frac{e_{it}}{\omega_{it}} = a + b\widehat{H}_{it} \tag{17}$$

with coefficients a and b to be determined. Equation (7) gives:

$$\frac{e_{it}}{\omega_{it}} = 1 + e^{-\lambda}\mathbb{E}_t\left[\frac{M^*_{t+1}}{M^*_t}\frac{\omega_{i,t+1}}{\omega_{it}}\frac{e_{i,t+1}}{\omega_{i,t+1}}\right].$$

Using the process for M^*_t, ω_{it} (equations (8) and (11)) we have:

$$\mathbb{E}_t\left[\frac{M^*_{t+1}}{M^*_t}\frac{\omega_{i,t+1}}{\omega_{it}}\right] = e^{-R+g_{\omega_i}}\left((1-p_t) + p_t\mathbb{E}^D_t\left[B^{-\gamma}_{t+1}F_{i,t+1}\right]\right)$$

$$\mathbb{E}_t\left[\frac{M^*_{t+1}}{M^*_t}\frac{\omega_{i,t+1}}{\omega_{it}}\right] = e^{-R+g_{\omega_i}}\left(1+H_{it}\right) \tag{18}$$

which shows the origin of the definition of resilience. Using the process for $\widehat{H}_{i,t+1}$ in (14), and the fact that innovation $\varepsilon^H_{i,t+1}$ has mean 0 independently of the realization of the disaster (i.e. $\mathbb{E}^D_t\left[\varepsilon^H_{i,t+1}\right] = \mathbb{E}^{ND}_t\left[\varepsilon^H_{i,t+1}\right] = 0$), we get:

$$\begin{aligned}
\frac{e_{it}}{\omega_{it}} - 1 &= e^{-\lambda}\mathbb{E}_t\left[\frac{M^*_{t+1}}{M^*_t}\frac{\omega_{i,t+1}}{\omega_{it}}\right]\mathbb{E}_t\left[\frac{e_{i,t+1}}{\omega_{i,t+1}}\right] \\
&= e^{-\lambda-R+g_{\omega_i}}\left(1+H_{it}\right)\mathbb{E}_t\left[\frac{e_{i,t+1}}{\omega_{i,t+1}}\right] \\
&= e^{-\lambda-R+g_{\omega_i}}\left(1+H_{it}\right)\mathbb{E}_t\left[a + b\widehat{H}_{i,t+1}\right], \text{ using (17)} \\
&= e^{-\lambda-R+g_{\omega_i}}\left(a\left(1+H_{it}\right) + b\left(1+H_{it}\right)\mathbb{E}_t\widehat{H}_{i,t+1}\right)
\end{aligned}$$

Using (13) and (14),

$$\begin{aligned}
\frac{e_{it}}{\omega_{it}} - 1 &= e^{-\lambda-R+g_{\omega_i}}\left(a\left(1+H_{i*}+\widehat{H}_{it}\right) + b\left(1+H_{i*}\right)\exp(-\phi_{H_i})\widehat{H}_{it}\right) \\
&= e^{-\lambda-R+g_{\omega_i}}e^{h_{i*}}\left(a + \left(ae^{-h_{i*}} + b\exp(-\phi_{H_i})\right)\widehat{H}_{it}\right), \text{ using } 1+H_{i*} = e^{h_{i*}} \\
&= e^{-r_{ei}}\left(a + \left(ae^{-h_{i*}} + b\exp(-\phi_{H_i})\right)\widehat{H}_{it}\right).
\end{aligned}$$

By (17) we have $\frac{e_{it}}{\omega_{it}} - 1 = a - 1 + b\widehat{H}_{it}$, hence:

$$\begin{aligned}
a - 1 &= e^{-r_{ei}}a, \\
b &= ae^{-r_{ei}-h_{i*}} + be^{-r_{ei}-\phi_{H_i}}.
\end{aligned}$$

Solving for a and b gives:

$$e_{it} = \frac{\omega_{it}}{1-\exp\left(-r_{ei}\right)}\left(1+\frac{\exp\left(-r_{ei}-h_{i*}\right)}{1-\exp\left(-r_{ei}-\phi_{H_i}\right)}\widehat{H}_{it}\right). \tag{19}$$

Taking the limit of small time intervals (i.e. small r_{ei} and ϕ_{H_i}) gives expression (16). $\square$

Domestic pricing kernel. We can use this proposition to derive an expression for the domestic pricing kernel, $M_{it} = \mu_i e^{-\rho t} u_{i,C_{it}^{NT}}$. We get

$$M_{it} = M_t^* e_{it} = M_t^* \frac{\omega_{it}}{r_{ei}}\left(1+\frac{\widehat{H}_{it}}{r_{ei}+\phi_{H_i}}\right) \tag{20}$$

If there is a world disaster on the tradable good (so that M_t^* is multiplied by $B_{t+1}^{-\gamma}$), there is also a disaster on the domestic, non-tradable good (so that so that M_{it} is multiplied by $B_{t+1}^{-\gamma}F_{i,t+1}$, as ω_{it} is multiplied by $F_{i,t+1}$). Disasters affect marginal utility with respect to both tradable and non-tradable goods.

The decomposition also shows how the domestic pricing kernel M_{it} is subject to both common disaster shocks with country-specific loadings ($B_{t+1}^{-\gamma}F_{i,t+1}$) that command an international risk premium and non-disaster shocks that do not command an international risk premium: the innovations to $\widehat{H}_{it}$ (and also the innovations to ω_{it} that are orthogonal to disasters, if we had introduced them), which can be correlated across countries, but still do not command a disaster premium.

Interest rate. We first derive the interest rate in terms of the pricing kernel, before giving its concrete value. The argument is standard, but for completeness we spell it out here. One unit of the domestic currency is worth e_{it} at time t. One can invest it in the domestic bond and get $(1+r_{it})$ in the domestic currency at time $t+1$, hence $(1+r_{it})\,e_{i,t+1}$ of the world numéraire at time $t+1$. The value of the payoff is $\mathbb{E}_t\left[\frac{M_{t+1}^*}{M_t^*}\left(1+r_{it}\right)e_{i,t+1}\right]$ at time t. Hence

$$e_{it} = \mathbb{E}_t\left[\frac{M_{t+1}^*}{M_t^*}\left(1+r_{it}\right)e_{i,t+1}\right],$$

so that the interest rate is given by:

$$\frac{1}{1+r_{it}} = \mathbb{E}_t \left[\frac{M^*_{t+1} e_{i,t+1}}{M^*_t e_{it}} \right], \tag{21}$$

The following proposition calculates its value in the disaster model. Recall that $r_{ei} := R + \lambda - g_{\omega_i} - h_{i*}$.

Proposition 3 (Interest rate). *The value of the interest rate in country i is*

$$r_{it} = r_{ei} - \lambda - \frac{r_{ei} \widehat{H}_{it}}{r_{ei} + \phi_{H_i} + \widehat{H}_{it}} \tag{22}$$

in the limit of small time intervals.

Proof of Proposition 3. In this proof, it is useful to define $x_{it} = e^{-h_{i*}} \widehat{H}_{it}$. Then, (18) gives:

$$\mathbb{E}_t \left[\frac{M^*_{t+1} \omega_{i,t+1}}{M^*_t \omega_{it}} \right] = \exp\left(-R + g_{\omega_i}\right)\left(1 + H_{it}\right) = \exp\left(-r_{ei} + \lambda\right)\left(1 + x_{it}\right).$$

Equation (14) implies $\mathbb{E}_t\left[x_{i,t+1}\right] = \exp\left(-\phi_{H_i}\right) \frac{x_{it}}{1+x_{it}}$, and equation (19) yields $e_{it} = \omega_{it} A \left(1 + B x_{it}\right)$ with $A = \frac{1}{1-\exp(-r_{ei})}$, $B = \frac{\exp(-r_{ei})}{1-\exp\left(-r_{ei}-\phi_{H_i}\right)}$. Thus:

$$\begin{aligned}
1 + r_{it} &= \frac{e_{it}}{\mathbb{E}_t\left[\frac{M^*_{t+1}}{M^*_t} e_{i,t+1}\right]} = \frac{\omega_{it} A \left(1 + B x_{it}\right)}{\mathbb{E}_t\left[\frac{M^*_{t+1}}{M^*_t} \omega_{i,t+1} A \left(1 + B x_{i,t+1}\right)\right]} = \frac{1 + B x_{it}}{\mathbb{E}_t\left[\frac{M^*_{t+1}\omega_{i,t+1}}{M^*_t \omega_{it}}\right] \mathbb{E}_t\left[1 + B x_{i,t+1}\right]} \\
&= \frac{1 + B x_{it}}{\exp\left(-r_{ei} + \lambda\right)\left(1 + x_{it}\right)\left(1 + B \frac{\exp\left(-\phi_{H_i}\right) x_{it}}{1 + x_{it}}\right)} = \exp\left(r_{ei} - \lambda\right) \frac{1 + B x_{it}}{1 + x_{it}\left(1 + B \exp\left(-\phi_{H_i}\right)\right)} \\
&= \exp\left(r_{ei} - \lambda\right) \frac{1 + \frac{\exp(-r_{ei})}{1-\exp\left(-r_{ei}-\phi_{H_i}\right)} x_{it}}{1 + \frac{1}{1-\exp\left(-r_{ei}-\phi_{H_i}\right)} x_{it}} \\
&= \exp\left(r_{ei} - \lambda\right)\left[1 - \frac{\left(1 - \exp\left(-r_{ei}\right)\right) \exp\left(-h_{i*}\right) \widehat{H}_{it}}{1 - \exp\left(-r_{ei} - \phi_{H_i}\right) + \exp\left(-h_{i*}\right) \widehat{H}_{it}}\right].
\end{aligned}$$

Hence,

$$r_{it} = \exp\left(r_{ei} - \lambda\right)\left[1 - \frac{\left(1 - \exp\left(-r_{ei}\right)\right) \exp\left(-h_{i*}\right) \widehat{H}_{it}}{1 - \exp\left(-r_{ei} - \phi_{H_i}\right) + \exp\left(-h_{i*}\right) \widehat{H}_{it}}\right] - 1. \tag{23}$$

Taking the limit of small time intervals gives (22). □

When a country is "risky" (low h_{i*} or $\widehat{H}_{it}$), its interest rate is high according to (22) because its currency has a high risk of depreciating in bad states of the world. Note that this risk is a real risk of depreciation, not a default risk. Safe countries can borrow at a lower interest rate, which may explain why historically the dollar or Swiss franc interest rates were low (Gourinchas and Rey 2007).

Safe haven countries can borrow at low interest rates and have an appreciated currency. Consider two countries, one with low risk / high average resilience h_{i*} ("Switzerland", the safe haven), and one with high risk / lower average resilience h_{i*} ("Brazil"). Equations (15) - (22) imply that, on average (i.e., when $\widehat{H}_{it} = 0$), Switzerland has low interest rates (equation (22) with $\widehat{H}_{it} = 0$), while Brazil has high interest rates. This is a compensation for disaster risk, not default: investors are willing to lend to Switzerland at low interest rates because the Swiss exchange rate will appreciate relative to Brazil's in a disaster.

At the same time, the exchange rates are $e_{it} = \frac{\omega_{it}}{r_{ei}}$ when resiliences are at their central value (equation (16) with $\widehat{H}_{it} = 0$). Hence, the Swiss exchange rate is on average appreciated ("strong") compared to the Brazilian exchange rate.

Switzerland (the safe haven) therefore benefits from the "exorbitant privilege" of borrowing at low interest rates. This underlying mechanism is different from those of Gourinchas, Govillot and Rey (2010), who emphasize differences in risk aversion, and Maggiori (2013), who emphasizes differences in financial development. A distinctive feature of our model is that the exchange rates of safe haven countries appreciate in the times of crisis. Colacito et al. (2015) generate similar facts using a model with heterogeneous exposures to a long run risk, rather than a disaster, factor.

III.B. Forward Premium Puzzle and Carry Trade

Carry trade. Consider the following carry trade from the perspective of an international investor. The trade borrows one unit of international numéraire in currency j at interest rate r_{jt} and invests it in currency i at interest rate r_{it} for one period. The return on this carry trade is

$$R_{ij,t+1} = \frac{e_{i,t+1}}{e_{it}} (1 + r_{it}) - \frac{e_{j,t+1}}{e_{jt}} (1 + r_{jt}).$$

It is important to characterize the expected return on this carry trade. Because disasters are rare events, it is useful to compute this expected return both in samples with no disasters and in full samples with representative disasters. Formally, the former corresponds to the expectation conditional on no disasters $\mathbb{E}_t^{ND}[R_{ij,t+1}]$ while the latter corresponds to the unconditional expectation $\mathbb{E}_t[R_{ij,t+1}]$.

Proposition 4 (Carry trade return). *Consider two countries i and j with $r_{ek} = r_e$, $H_{k*} = H_*$, $\phi_{H_k} = \phi_H$ for $k = i, j$, and consider the limit of small time intervals. The expected returns of the carry trade (unconditionally, and conditionally on no disasters in the sample) are equal to:*

$$\mathbb{E}_t[R_{ij,t+1}] = \left(1 - \overline{B}^{\gamma}\right)\left(\widehat{H}_{jt} - \widehat{H}_{it}\right) = \left(1 - \overline{B}^{\gamma}\right)\left(1 + \frac{\phi_H}{r_e}\right)(r_{it} - r_{jt}). \tag{24}$$

$$\mathbb{E}_t^{ND}[R_{ij,t+1}] = \widehat{H}_{jt} - \widehat{H}_{it} = \left(1 + \frac{\phi_H}{r_e}\right)(r_{it} - r_{jt}). \tag{25}$$

The first expression requires the additional assumption that B_{t+1} is constant with value $\overline{B}$. The two right-hand sides expressed with $r_{it} - r_{jt}$ hold up to second order terms in $\widehat{H}_{it}$ and $\widehat{H}_{jt}$.

The intuition for expression (24) is as follows. First, if there are no disasters ($\overline{B} = 1$), the expected carry trade return is zero, $\mathbb{E}_t[R_{ij,t+1}] = 0$. Second, if there are disasters ($\overline{B} < 1$), then the expected carry trade return is positive if the investment currency i has lower resilience than the funding currency j ($\widehat{H}_{jt} - \widehat{H}_{it} > 0$), i.e. if currency i is riskier than currency j, so that this carry trade will have highly negative returns in a disaster. At the same time, we have $\widehat{H}_{jt} - \widehat{H}_{it} = -\left(1 + \frac{\phi_H}{r_e}\right)(r_{jt} - r_{it})$: if currency i is riskier than currency j, its (real) interest rate r_{it} is greater than country j's interest rate r_{jt}. In other words, investing in currencies with high interest rates yields positive expected returns, which is simply a compensation for bearing disaster risk.

A related intuition can be provided for expression (25). Suppose again that currency i is riskier than currency j ($\widehat{H}_{jt} - \widehat{H}_{it} > 0$, $r_{it} - r_{jt} > 0$). If no disaster occurs, the investor pockets the interest differential $r_{it} - r_{jt}$. Moreover, because $\widehat{H}_{j,t+1} - \widehat{H}_{i,t+1}$ mean-reverts towards zero, the exchange rate of country i appreciates in expectation (conditional on no disasters) against that of country j. As a result, the investor also collects an expected capital gain, and the expected return of the carry trade (conditional on no disasters) $\left(1 + \frac{\phi_H}{r_e}\right)(r_{it} - r_{jt})$ is greater than $r_{it} - r_{jt}$.[8]

Fama regression. These results can be understood using the related language of Fama (1984) regressions:

$$\text{Fama regression: } \frac{e_{i,t+1}}{e_{it}} - \frac{e_{j,t+1}}{e_{jt}} = \alpha + \beta(r_{jt} - r_{it}) + \varepsilon_{ij,t+1}, \tag{26}$$

where $\varepsilon_{ij,t+1}$ is a random variable with mean zero. As for the expected return on the carry trade, it is useful to evaluate this regression both in samples with no disasters (conditional on no disasters) and in full samples with representative disasters (unconditionally). We denote the corresponding coefficients by β^{ND} and β^{Full}.

We have $\frac{e_{i,t+1}}{e_{it}} - \frac{e_{j,t+1}}{e_{jt}} = R_{ij,t} - (r_{it} - r_{jt})$ in the limit of small time intervals, so that the coefficients in the Fama regressions are simply $\beta^{Full} = 1 - b^{Full}$ and $\beta^{ND} = 1 - b^{ND}$, where $b^{Full} = \left(1 - \overline{B}^{\gamma}\right)\left(1 + \frac{\phi_H}{r_e}\right)$ and $b^{ND} = 1 + \frac{\phi_H}{r_e}$ are the coefficients on interest rate differentials in Proposition 4. Hence, we obtain the following Proposition.

PROPOSITION 5 (Fama coefficients). *Consider two countries i and j with $r_{ek} = r_e$, $\phi_{H_k} = \phi_H$ for $k = i, j$, and consider the limit of small time intervals as well as small $\widehat{H}_{it}$ and $\widehat{H}_{jt}$. In the Fama regression (26), if in addition B_t is constant with value $\overline{B}$, then in a full sample the coefficient β^{Full} is:*

$$\beta^{Full} = -\frac{\phi_H}{r_e} + \left(1 + \frac{\phi_H}{r_e}\right)\overline{B}^{\gamma} \leq 1, \tag{27}$$

while in a sample with no disasters the coefficient β is:

$$\beta^{ND} = -\frac{\phi_H}{r_e}. \tag{28}$$

The UIP condition can be restated as requiring $\beta = 1$. The "forward premium puzzle" is the observation that empirically, the coefficient in the Fama regression is $\beta < 1$. Our results can be seen as a rationalization of the forward premium puzzle.

When there are no disasters ($\overline{B} = 1$) we have $\beta^{Full} = 1$. But when there are disasters

[8] We can derive related expressions that are valid whether or not there are differences in steady state resiliences ($H_{i*} \neq H_{j*}$) or not. Indeed, we have $\mathbb{E}_t\left[R_{ij,t+1}\right] = \left(1 - \overline{B}^{\gamma}\right)(H_{jt} - H_{it})$ and $\mathbb{E}_t^{ND}\left[R_{ij,t+1}\right] = H_{jt} - H_{it}$. This applies in particular with constant resiliences $H_{jt} - H_{it} = H_{j*} - H_{i*} = r_{i*} - r_{j*}$ (assuming $g_{\omega_i} = g_{\omega_j}$). In that case, there is a positive return to the carry trade equal to the constant interest rate differential (as argued by Hassan and Mano (2015)).

($\overline{B} < 1$), we have $\beta^{Full} < 1$. The intuition is similar to the one given for the expected return on the carry trade and can be traced back to the existence of a time varying disaster risk premium. Note that we can even have $\beta^{Full} < 0$.

In addition, we always have $\beta^{ND} < 0$. There again, the intuition is similar to the one given for the expected return on the carry trade (conditional on no disasters), and derives from the mean-reversion in relative resilience $\widehat{H}_{j,t+1} - \widehat{H}_{i,t+1}$. In this simplest model with one factor (resilience), β^{ND} is always negative; in richer models with more factors (resilience and inflation, see below), β^{ND} can have both signs depending on the relative importance of the different factors.

IV. Extensions

IV.A. Options and Risk Reversals

Disaster risk is inherently hard to measure, but options offer a powerful way to assess its importance. Here, we characterize the way disasters are incorporated into option prices. We discuss the empirical validity of the model's predictions.

Consider two countries i and j. Call j the domestic currency and i the foreign currency. A put gives the investor the right at date 1 to receive K units of currency j in exchange for $\frac{e_{j0}}{e_{i0}}$ units of currency i, where K is the strike. As those $\frac{e_{j0}}{e_{i0}}$ units of currency i have a market value of $\frac{e_{i1}}{e_{j1}}\frac{e_{j0}}{e_{i0}}$ units of currency j at time 1, the put pays off $x_1 = \left(K - \frac{e_{i1}}{e_{j1}}\frac{e_{j0}}{e_{i0}}\right)^+$ in units of currency j at date 1. This corresponds to $e_{j1}x_1$ units of the international numéraire, so the date-0 price of the put is $\mathbb{E}_0\left[\frac{M_1^*}{M_0^*}e_{j1}x_1\right]$ in the international numéraire, and $V^P(K) = \frac{1}{e_{j0}}\mathbb{E}_0\left[\frac{M_1^*}{M_0^*}e_{j1}x_1\right]$ in currency j. Hence, the put price in currency j is:

$$V^P(K) = \mathbb{E}_0\left[\frac{M_1^*}{M_0^*}\left(\frac{e_{j1}}{e_{j0}}K - \frac{e_{i1}}{e_{i0}}\right)^+\right]. \tag{29}$$

Likewise, the currency j price at date 0 of a call with strike K is $V^C(K) = \mathbb{E}_0\left[\frac{M_1^*}{M_0^*}\left(\frac{e_{i1}}{e_{i0}} - \frac{e_{j1}}{e_{j0}}K\right)^+\right]$.

Option prices without disasters. The Black-Scholes formula for equity options was adapted by Garman-Kohlhagen (1983) to currency options. We call $V_{BS}^P(S, \kappa, \sigma, r, r^\star, T)$ and $V_{BS}^C(S, \kappa, \sigma, r, r^\star, T)$ the Black-Scholes prices for a put and a call, respectively, when the exchange rate is S, the strike is κ, the exchange rate volatility is σ, the home interest rate is r, the foreign interest rate is $r^{\star}$, and the time to maturity is T. The pricing formulas in that case are well-known.[9]

Option prices in the model. We use the index k for a statement applying to both countries i, j. For tractability, we make two simplifying assumptions as in Gabaix (2012). First, we assume that if a disaster occurs in period 1, ε_{k1}^{H} is equal to zero. Second, we assume that the distribution of e_{k1} conditional on date 0 information and no disaster occurring in period 1 is lognormal with drift μ_k and volatility σ_k, where k indexes countries: $\frac{e_{k1}}{e_{k0}} = \exp\left(\mu_k + \varepsilon_k - \sigma_k^2/2\right)$, where $\varepsilon_k \sim N(0, \sigma_k^2)$, and $\mu_k := R - \ln\left(\left(1 + r_k\right)\left(1 + H_k\right)\right)$ is the expected exchange rate appreciation conditional on no disasters.[10] This enables us to derive option prices in closed form. The standard deviation of the bilateral log exchange rate (conditional on no disaster) is $\sigma_{i|j} := \left(\sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j\right)^{1/2}$, where ρ_{ij} is the correlation between ε_i and ε_j.

Proposition 6 (Option prices). *The price of a put with strike K and maturity 1 is:*

$$V^{P}(K) = V^{P,ND}(K) + V^{P,D}(K), \tag{30}$$

where $V^{P,ND}(K)$ and $V^{P,D}(K)$ are the part of the price corresponding respectively to the no-disaster and disaster states:

$$V^{P,ND}(K) = \exp\left(-R + \mu_i\right)\left(1 - p_0\right) V_{BS}^{P}\left(K \exp\left(\mu_j - \mu_i\right), \sigma_{i|j}\right), \tag{31}$$

$$V^{P,D}(K) = \exp\left(-R + \mu_i\right) p_0 \mathbb{E}_0^{D}\left[B_1^{-\gamma}\left(K \exp\left(\mu_j - \mu_i\right) F_{j1} - F_{i1}\right)^{+}\right], \tag{32}$$

where $V_{BS}^{P}(K, \sigma) := V_{BS}^{P}(1, K, \sigma, 0, 0, 1)$ is the Black-Scholes put value when the strike is K, the volatility σ, the interest rates 0, the maturity 1, the spot price 1.

[9] Calling Φ the Gaussian cumulative distribution function, we have:

$$V_{BS}^{C}(S, \kappa, \sigma, r, r^{\star}, T) = Se^{-r^{\star}T}\Phi(d_1) - \kappa e^{-rT}\Phi(d_2),$$
$$V_{BS}^{P}(S, \kappa, \sigma, r, r^{\star}, T) = \kappa e^{-rT}\Phi(-d_2) - Se^{-r^{\star}T}\Phi(-d_1),$$
$$d_1 = \left[\ln(S/\kappa) + (r - r^{\star} + \sigma^2/2)T\right]/\sigma\sqrt{T}, \qquad d_2 = d_1 - \sigma\sqrt{T}.$$

[10] This can be ensured as in Gabaix (2012). We assume that if there is no disaster, then $\omega_{t+1}/\omega_t = e^{g}\left(1 + \varepsilon_{t+1}^{\omega}\right)$, with $\mathbb{E}_t\left[\varepsilon_{t+1}^{\omega}\right] = \mathbb{E}_t\left[\varepsilon_{t+1}^{\omega}\varepsilon_{t+1}^{H}\right] = 0$. This does not change any of the formulas for the exchange rate and the interest rate. The disturbance term $\varepsilon_{t+1}^{\omega}$ can be designed to ensure that e_{i1}^{ND}/e_{i0} has the lognormal noise described above. The exact expression for μ_i comes from the Euler equation $1 = \mathbb{E}\left[\left(1 + r_i\right)e^{-R+\mu_i}\left(1 + H_i\right)\right]$.

The price of a call is given by the put-call parity equation:

$$V^C(K) = V^P(K) + \frac{1}{1+r_{i0}} - \frac{K}{1+r_{j0}}. \tag{33}$$

The option price (30) is the sum of two terms. The first one is a familiar Black-Scholes term. The second is a pure disaster term.

If the foreign currency i is riskier than the home currency j, then out-of-the-money put prices on the currency pair (home, foreign) should be higher than out-of-the-money call prices as the price of protection against a devaluation of the foreign currency should be high. We next present a simple metric – risk reversals – to measure the gap between out-of-the-money puts and out-of-the-money calls.

Implied volatility smirk and risk reversals. We start by surveying well-known notions in option theory. Given a call option with strike K and price v, the implied volatility of the option is the volatility $\widehat{\sigma}(K)$ that needs to be assumed in the Black-Scholes formula to match the price: $V_{BS}^C(K, \widehat{\sigma}(K)) = v$. Implied volatilities on puts are defined similarly. For instance, if a currency has a lot of disaster risk, its put price will be high (Proposition 6) and its implied volatility will be high.

In particular, consider the implied volatility curve (i.e., the graph of the implied volatility $\widehat{\sigma}(K)$ as a function of the strike K) of a pair of currencies: a risky currency and a safe currency. Out-of-the-money puts protect against the crash of the exchange rate of i versus j and out-of-the-money calls protect against the crash of the exchange rate of j versus i. Imagine that i is riskier than j. Then the implied volatility of deep out-of-the-money puts is higher than the implied volatility of out-of-the-money calls— a pattern referred to as a “smirk”.

A popular way to quantify the smirk is the “risk reversal” (RR). Intuitively, it is the difference in implied volatility of an out-of-the-money put and a symmetric out-of-the-money call. Hence, a very risky currency will have a high RR.

To formulate a more precise definition of the RR, we need to define the delta of an option. It is the derivative with respect to the time-0 currency price in the Black-Scholes formula. Formally, if the call price (in the Black-Scholes / Garman-Kohlhagen model) is $V_{BS}^C(S, \kappa, r, r^*, \sigma, T)$, the delta is $\Delta := \frac{\partial V_{BS}^C}{\partial S}$). The delta of a call option decreases monotonically from 1 to 0 as κ/S increases. Symmetrically, the delta of a put option decreases monotonically from 0 to -1 as κ/S

increases. Given a value $\Delta \in (0,1)$, we define the Δ risk reversal to be the difference in implied volatilities between an out-of-the-money put and an out-of-the-money call with the following properties. The strike of the put is chosen such that the Black-Scholes / Garman-Kohlhagen delta is $-\Delta$. Symmetrically, the strike of the call is chosen such that the Black-Scholes / Garman-Kohlhagen delta is Δ. In practice we will work with $\Delta = 0.25$, corresponding to a "25 delta" risk reversal.

We state a Lemma to better understand the risk reversal. It is drawn from Farhi et al. (2015, Proposition 5), and the online appendix gives a self-contained proof.[11]

LEMMA 2 *In the limit of small time intervals, the risk reversal can be expressed as:*

$$RR_{ijt} = k_\Delta \left(H_{jt} - H_{it} \right). \tag{34}$$

where $k_\Delta := \frac{1-2\Delta}{\phi(\Phi^{-1}(\Delta))}$ *is a numerical constant, with* ϕ *and* Φ *the density and cumulative distribution functions of a standard Gaussian.*

Hence, if country i has more disaster risk than country j ($H_{jt} - H_{it} > 0$), then the risk reversal is positive: put prices on currency i are very expensive and have a high implied volatility (compared to symmetric call prices).

Four signature predictions of disasters. The model makes four broad predictions regarding option prices. The first three were seen above, and the fourth one will be detailed in section V.C.

1. Countries with high risk reversals have high interest rates.
2. Investing in countries with high risk reversals generates high expected returns.
3. When risk reversals go up, the exchange rate contemporaneously depreciates.
4. The risk reversal of risky (i.e., high risk reversal, high interest rate) countries should covary negatively with HML_{FX} (which is the payoff of a portfolio going long high interest rate

[11] Formula (34) holds for a "one-period" option, and H_{it} is expressed per period. Suppose that "one period" is τ years (e.g. $\tau = \frac{1}{12}$ if a period is one month), and implied volatilities are expressed in annual units, and the maturity of the option is T years. Then, formula (34) becomes $\overline{RR}_{ijt} = \frac{1-2\Delta}{\phi(\Phi^{-1}(\Delta))} \left(\overline{H}_{jt} - \overline{H}_{it} \right) \sqrt{T}$, where $\overline{RR}_{ijt} = RR_{ijt}/\sqrt{\tau}$ and $\overline{H}_{it} = H_{it}/\tau$ are the RR and the resilience expressed in annual units. In addition, for 25 delta options ($\Delta = 0.25$), $\frac{1-2\Delta}{\phi(\Phi^{-1}(\Delta))} \simeq 1.57$.

currencies and short low interest rate currencies), while the risk reversal of less risky countries should covary positively with it.

Empirical support for these predictions can be found in various papers: Carr and Wu (2007, prediction 3), Brunnermeier, Nagel, and Pedersen (2009, predictions 1-3), Du (2013, prediction 1), Farhi et al. (2015, predictions 1-2). Section V.C finds support for prediction 4.

These four signature predictions of the disaster hypothesis are therefore qualitatively borne out in the data. The calibration will show that the correspondence between empirics and theory can be made quantitative as well.

Illustration: impact of a change in the world disaster probability. An important object is the probability of a world disaster, p_t. Its movements have a number of signature effects that we now study. Consider two countries, again one safe (high F_{jt}), "Switzerland", and one risky (low $F_{it} < F_{jt}$), "Brazil". The difference in their resiliences is

$$H_{jt} - H_{it} = p_t \mathbb{E}_t^D \left[B_{t+1}^{-\gamma} \left(F_{j,t+1} - F_{i,t+1} \right) \right].$$

Suppose that p_t increases, while $\mathbb{E}_t^D \left[B_{t+1}^{-\gamma} \left(F_{j,t+1} - F_{i,t+1} \right) \right]$ remains the same. Then, $H_{jt} - H_{it}$ increases: Switzerland becomes relatively more resilient than Brazil. As a result, Switzerland's currency appreciates relative to Brazil's.

IV.B. Stocks

Our model allows us to think in a tractable way about the joint determination of exchange rate and equity values. We consider the case of a stock of a generic firm in country i, that produces the traded good.[12]. Its dividend is D_{it} in units of the traded good (i.e. the international numéraire), and $d_{it} = \frac{D_{it}}{e_{it}}$ when expressed in the domestic currency. It follows the following process

$$\frac{D_{i,t+1}}{D_{it}} = \exp\left(g_D\right)\left(1 + \varepsilon_{i,t+1}^D\right) \times \begin{cases} 1 & \text{if there is no disaster,} \\ F_{Di,t+1} & \text{if there is a disaster,} \end{cases}$$

where $\varepsilon_{i,t+1}^D$ is an idiosyncratic shock uncorrelated with the pricing kernel.

[12] The dividend of this firm may not be equal to total exports, as only a segment of firms are listed in the stock market. The NBER working paper version of this paper also develops the case of a firm producing the nontraded good.

We define the resilience H_{Dit} of the dividend D_{it} of stock i as

$$H_{Dit} = p_t \mathbb{E}_t^D \left[B_{t+1}^{-\gamma} F_{Di,t+1} - 1 \right] = H_{Di*} + \widehat{H}_{Dit}.$$

As before, we posit that the law of motion for $\widehat{H}_{Dit}$ is a LG process:

$$\widehat{H}_{Di,t+1} = \frac{1 + H_{Di*}}{1 + H_{Dit}} \exp\left(-\phi_{H_{Di}}\right) \widehat{H}_{Dit} + \varepsilon_{i,t+1}^{H_D}, \tag{35}$$

where $\phi_{H_{Di}}$ is the speed of mean reversion of the resilience of the stock. We also define $h_{Di*} := \ln\left(1 + H_{Di*}\right)$,

Proposition 7 (Price of stocks). *The domestic price of the stock P_{Dit} is, in the continuous time limit*

$$P_{Dit} = d_{it} \frac{1 + \frac{\widehat{H}_{Dit}}{r_{Di} + \phi_{H_{Di}}}}{r_{Di}}, \tag{36}$$

where d_{it} is the dividend expressed in the domestic currency and $r_{Di} := R - g_D - h_{Di*}$.

A more resilient stock (high $\widehat{H}_{Dit}$) has a higher price-dividend ratio and lower future returns. The next Lemma states the equity premium.

Lemma 3 (Equity premium). *The expected excess return of stocks (in the domestic currency, over the domestic risk-free rate) is, in the limit of small time intervals, and neglecting second-order Ito terms: $p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} \left(F_{i,t+1} - F_{Di,t+1} \right) \right]$.*

IV.C. Yield Curve, Forward Rates, and Nominal Exchange Rates

Until recently, forward real interest rates were not available. Only their nominal counterparts were actively traded. Even today, most bonds are nominal bonds. To model nominal bonds, we build on the real model developed above. Let $P_{it} = P_{i0} / \prod_{s=0}^{t-1} \left(1 - \pi_{is}\right)$ be the price level in country i, where π_{it} is inflation at time t (this formulation will prove tractable). The nominal exchange rate is:

$$\widetilde{e}_{it} = e_{it} / P_{it}, \tag{37}$$

where we denote nominal variables with a tilde. The nominal interest rate $\widetilde{r}_t$ satisfies $1 = \mathbb{E}_t \left[\frac{M^*_{t+1} \widetilde{e}_{i,t+1}}{M^*_t \widetilde{e}_{it}} \left(1 + \widetilde{r}_{it}\right) \right]$, so that in the continuous-time limit

$$\widetilde{r}_{it} = r_{it} + \pi_{it}, \tag{38}$$

i.e. the nominal interest rate is the real interest rate plus inflation. As there is no burst of inflation during disasters, Fisher neutrality applies. With a risk of a burst of inflation, even short-term bonds would command a risk premium.

We posit that inflation hovers around π_{i*}, roughly according to an AR(1) process. More specifically, to ensure tractability of the model, we posit the linearity-generating process:

$$\pi_{i,t+1} = \pi_{i*} + \frac{1 - \pi_{i*}}{1 - \pi_{it}} \exp\left(-\phi_{\pi i}\right) \left(\pi_{it} - \pi_{i*}\right) + \varepsilon^{\pi}_{i,t+1}, \tag{39}$$

where $\varepsilon^{\pi}_{i,t+1}$ has mean zero and is uncorrelated with innovations in M^*_{t+1}, in particular with disasters. This means, to the leading order, that $\pi_{i,t+1} - \pi_{i*} \simeq \exp\left(-\phi_{\pi i}\right) \left(\pi_{it} - \pi_{i*}\right) + \varepsilon^{\pi}_{i,t+1}$, i.e. the process is a (twisted) AR (1). One could allow for non-zero correlation, but the analysis would become a bit more complicated.

Proposition 8 (Forward rates). *In the continuous-time limit, the domestic nominal forward rate is, up to second-order terms in $\widehat{H}_{it}$ and $\pi_{it} - \pi_{i*}$:*

$$f_{it}\left(T\right) = r_{ei} - \lambda - \frac{r_{ei}}{r_{ei} + \phi_{H_i}} \exp\left(-\phi_{H_i} T\right) \widehat{H}_{it} + \pi_{i*} + \exp\left(-\phi_{\pi i} T\right) \left(\pi_{it} - \pi_{i*}\right). \tag{40}$$

The nominal forward rate in (40) depends on real and nominal factors. The real factor is the resilience of the economy $\widehat{H}_{it}$. The nominal factor is inflation π_{it}.

Each term is multiplied by a term of the type $\exp\left(-\phi_{H_i} T\right)$. For small speeds of mean reversion ϕ, the forward curve is fairly flat.

We can derive the implications of our model for a Fama regression in nominal terms:

$$\frac{\tilde{e}_{i,t+1}}{\tilde{e}_{it}} - \frac{\tilde{e}_{j,t+1}}{\tilde{e}_{jt}} = \tilde{\alpha} + \tilde{\beta}(\tilde{r}_{jt} - \tilde{r}_{it}) + \tilde{\varepsilon}_{ij,t+1}, \tag{41}$$

where $\tilde{r}_{it}$ and $\tilde{r}_{jt}$ are the nominal interest rates in countries i and j. Our model's prediction is in the next proposition.

Proposition 9 (Value of the coefficient in the Fama regression in nominal terms). *In the nominal Fama regression (41) with forward rates, the coefficients are:*

$$\tilde{\beta}^{ND} = \widetilde{\nu}\beta^{ND} + 1 - \widetilde{\nu}, \qquad \tilde{\beta}^{Full} = \widetilde{\nu}\beta^{Full} + 1 - \widetilde{\nu}, \tag{42}$$

where β^{ND} and β^{Full} are the coefficients in the Fama regression defined in Proposition 5 and

$$\widetilde{\nu} = \frac{1}{1 + \frac{\mathrm{Var}(\pi_{it} - \pi_{jt})}{\left(\frac{r_{ei}}{r_{ei} + \phi_{H_i}}\right)^2 \mathrm{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right)}} \tag{43}$$

is the share of variance in the forward rate due to $\widehat{H}_t$.

In this simple model, the higher the variance of inflation, the closer $\tilde{\beta}^{ND}$ is to 1. Hence, countries with very variable inflation (typically countries with high average inflation) approximately satisfy the uncovered interest rate parity condition. Bansal and Dahlquist (2000) provide empirical support for this phenomenon. When disaster risks are very variable – and the real exchange rate is very variable – then $\tilde{\beta}^{ND}$ is smaller and can turn negative.

V. Quantitative Analysis

V.A. Calibration

We present a simple calibration of the model, to work out its basic quantitative properties. We leave a full-blown estimation of the model for future work. As our data is nominal, use the extension to a nominal setup presented in Section IV.C.[13] We use monthly data from JP Morgan presented in Farhi et al. (2015) for the January 2009-May 2013 period.[14] We take the same parameters for all countries.

Key and ancillary parameters. Table I summarizes the main inputs of the calibration, which are sufficient statistics for the output of the model, summarized in Tables III-VI. These sufficient statistics can in turn be rationalized in terms of "ancillary" parameters shown in

[13] The countries we use are: Australia, Canada, Euro area, Japan, Norway, New Zealand, Sweden, Switzerland, U.K., U.S.

[14] There was a large crash of carry trade in the Fall 2008. The post-2008 sample is more homogeneous economically than the full sample.

Table II. We call these parameters "ancillary" because they matter only via their impact on the aforementioned sufficient statistics listed in Table I. For instance, take $r_{ei} = R + \lambda - g_{\omega_i} - h_{i*}$ (see equation (15)). Table I calibrates it to be 6%. At the same time, many combinations of $R, \lambda, -g_{\omega_i}$, and $-h_{i*}$ can add up to $r_{ei} = 6\%$. We present one such combination of ancillary parameters in Table II.

Similarly, we directly calibrate the process (14) for resilience $H_{it} = p_t \mathbb{E}_t^D \left[B_{t+1}^{-\gamma} F_{i,t+1} - 1 \right]$. In the ancillary Table II, we show one example of underlying processes of the disaster probability p_t, severity B_{t+1} and country-specific factors $F_{i,t+1}$, that generate the sufficient statistic H_{it}. This allows for a more parsimonious calibration that focuses on the key drivers of exchange rates.

Calibrating the key parameters. *Cross-country correlations.* In the baseline calibration, we take the innovations to $\varepsilon_{it}^H, \varepsilon_{it}^{H_D}$ and ε_{it}^D to be uncorrelated across countries. This is just for clarity and simplicity. We introduce cross-country correlations later in Section V.C. The online appendix (Section XII) details the exact specification that we adopt for the mean-zero innovations $\varepsilon_{it}^H, \varepsilon_{it}^{H_D}$ and ε_{it}^D.

Exchange rate and interest rate. We call Δ the time-difference operator, $\Delta x_t = x_t - x_{t-1}$, and $\sigma_x = stdev\left(\Delta x_t\right)$ the volatility of a variable x_t. For two countries, we define the volatility of the bilateral exchange rate as $\sigma_e^{bil} = stdev\left(\Delta \ln \frac{e_{it}}{e_{jt}}\right)$ and the volatility of the difference in interest rates $\sigma_r^{bil} = stdev\left(\Delta\left(r_{it} - r_{jt}\right)\right)$. Equations (16) and (22) give $\sigma_r^{bil} = r_{ei}\sigma_e^{bil}$. The above equation guides our calibration. We set $r_{ei} = 6\%$. This implies $\sigma_e^{bil} \simeq \sigma_{\tilde{e}}^{bil} \simeq 11\%$ and a reasonable value of the bilateral real interest rate volatility, $\sigma_r^{bil} = 0.7\%$.

For the speed of mean-reversion of resilience, we take $\phi_{H_i} = 18\%$, which gives a half-life of $\ln 2/\phi_{H_i} = 3.8$ years, in line with estimates from the exchange rate predictability literature (Rogoff (1996)). The standard deviation of the innovations to resilience H_{it} are chosen to roughly match the level and volatility of the risk reversals, as well as the volatility of the bilateral exchange rate, as reported in Table III. This leads us to set $\sigma_{H_i} = 1.87\%$.

Inflation. Data (e.g., on currency options) are nominal, and the essence of our model is real. Inflation contains a substantial high-frequency and transitory component, which is in part due to measurement error. Call $\widetilde{\pi}_{it} = \pi_{it} + \eta_{it}$ the measured inflation (which can be thought of as

TABLE I: Key Parameter Inputs.

Exchange rate discount rate	$r_{ei} = 6\%$
Resilience: volatility and speed of mean-reversion	$\sigma_{H_i} = 1.87\%, \phi_{H_i} = 18\%$
Inflation: volatility and speed of mean-reversion	$\sigma_{\pi_i} = 0.6\%, \phi_{\pi_i} = 23\%$
Stocks: discount rate, volatility and growth of dividends	$r_{Di} = 4.4\%, \sigma_{D_i} = 8.4\%, g_{D_i} = 2.5\%$
Stocks resilience: volatility and speed of mean-reversion	$\sigma_{H_{Di}} = 3.1\%, \phi_{H_{Di}} = 13\%$

Notes: This table reports the coefficients used in the model. σ_X is the average volatility, and ϕ_X is the speed of mean-reversion. The time unit is a year (the model is simulated at a monthly frequency, but for readability the numbers reported above are all annualized)

TABLE II: Ancillary Parameters.

Coefficient of risk aversion, IES	$\gamma = 4, \sigma = 1/\gamma$
Rate of time preference	$\rho = 5.9\%$
Probability and severity of disasters	$\overline{p} = 3.6\%$, $\overline{B} = 0.66$
Recovery rate of export productivity	$\overline{F} = 1$, $\sigma_{F_i} = 9.8\%$
Growth rate of consumption, export productivity	$g_{c_i} = 2.5\%, g_{\omega_i} = 0$
Depreciation rate	$\lambda = 5.5\%$
Recovery rate of stock dividend	$\overline{F}_{D_i} = \overline{B}$

Notes: This table reports the coefficients used in the model. σ_X is the average volatility, and ϕ_X is the speed of mean-reversion. The time unit is a year (the model is simulated at a monthly frequency, but for readability the numbers reported above are all annualized). The above parameters also generate: $H_{i*} = 15.37\%, R = 15.9\%, \sigma_{F_{Di}} = 1.5\%$.

trend inflation plus mean-zero noise), while π_{it} is the trend inflation. We estimated inflation using the Kalman filter, with $\pi_{i,t+1} = C_1 + C_2\pi_{i,t} + \varepsilon'_{i,t+1}$for the trend inflation and $\widetilde{\pi}_{it} = \pi_{it} + \eta_{it}$ for the noisy measurement of inflation, in the sample 1985-2013 (so that we capture the post-Volcker disinflation regime). This led to a volatility of inflation of $\sigma_{\pi_i} = 0.6\%$ (by the way the estimate is quite unstable and sample-dependent) and speed of mean-reversion of $\phi_{\pi_i} = 23\%$.

Carry trade returns. A simple and important statistic that captures risk premia in exchange rates is the average carry trade return. We operationalize this strategy in the following manner (see e.g. Farhi et al. (2015)). To better capture disaster risk, we sort on risk reversals rather than interest rates. We divide countries into two equally-sized bins of resilience: the risky countries are those in the bottom half of resilience and the less risky countries are in the top half. We define the carry trade as going long \$1 in the equally-weighted portfolio of risky countries (low $\widehat{H}_{it}$) and going short \$1 in the equally-weighted portfolio of safe countries (high

$\widehat{H}_{it}$).

Stocks. Empirically, domestic stock returns and the exchange rate are uncorrelated on average (see Table IV). We specify the correlation between innovations to H_{Dit} and H_{it} to match this (the procedure is detailed in the online appendix). We match the volatility of dividends d_{it} of 11%, as in Campbell and Cochrane (1999); this leads to $\sigma_{D_i} = 8.4\%$ (remember that D_{it} is in the world numéraire, while d_{it} is expressed in the domestic currency). We use the stock calibration in Gabaix (2012). We take the speed of mean-reversion of dividend resilience to be $\phi_{H_{Di}} = 13\%$, which mimics the speed of mean-reversion of the stock's price-dividend ratio and $r_{Di} = 4.4\%$, to match the average price-dividend ratio. We choose $\sigma_{H_{Di}}$ to match the volatility of stock returns.

Interpreting key parameters in terms of specific ancillary parameters. Resiliences are sufficient statistics for the calibration. To be more precise, our calibration only requires specifying the joint laws of motion for the resiliences $H_{it} = p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} F_{i,t+1} - 1\right]$. The results of the calibration do not depend on whether the shocks that drive the changes in resiliences come from movements in p_t, $B_{t+1}^{-\gamma}$ or $F_{i,t+1}$. However, we discuss now how their variations are related to deeper disaster parameters.

One simple specification to generate the volatility of resilience in Table I is to assume that resilience innovations come entirely from idiosyncratic movements of F_{it}. We keep p_t and B_{t+1} constant at $\overline{p} = 3.6\%$ and $\overline{B} = 0.66$, which as recommended by Barro and Ursua (2008), is based on a certainty equivalent of historical disasters ($\overline{B} := \mathbb{E}\left[B^{-\gamma}\right]^{-1/\gamma} = 0.66$, averaging over historical episodes). We use a coefficient of relative risk aversion $\gamma = 4$.

Then, we calibrate $\sigma_{F_i} = \frac{\sigma_{H_i}}{\overline{p}\overline{B}^{-\gamma}} = 9.8\%$.[15] This generates a dispersion in prospective recovery rates of $var\left(F_{it}\right)^{1/2} \simeq \frac{\sigma_{F_i}}{\sqrt{2\phi_{H_i}}}$, so that the dispersion in expected bilateral recovery rates is $var\left(\ln F_{it} - \ln F_{jt}\right)^{1/2} \simeq \frac{\sigma_{F_i}}{\sqrt{\phi_{H_i}}} = 23\%$. Hence, if the theory is correct, a weak consequence is that the realized dispersion of currency jumps during disasters should be at least 23%.[16] Em-

[15] An interesting alternative is as follows. Suppose that innovations in differential resilience come entirely from movements in p_t (keeping $\mathbb{E}_t\left[B_{t+1}^{-\gamma}\left(F_{i,t+1} - F_{j,t+1}\right)\right]$ constant). With fixed values of F_{it}, e.g. $\left|F_{i,t+1} - F_{j,t+1}\right| = 0.23$ (similar to the numbers above) we have $\sigma_{p_t} = 1.5\%$. This is of the same order of magnitude as the calibration in Wachter (2013), which uses $\sigma_{p_t} \simeq 1.1\%$.

[16] Indeed, consider the very mild enrichment of the model, where $\frac{\omega_{i,t+1}}{\omega_{it}}$ is the process assumed in the paper (equation (11)), but multiplied by $\left(1 + \varepsilon_{i,t+1}^{\omega}\right)$ where $\varepsilon_{i,t+1}^{\omega}$ is 0 if there is no disaster, and is a mean-0 random variable is there is a disaster. Then, none of the pricing expressions in the paper change (including the value of the exchange rate and interest rate). If there is a disaster at t, $\Delta \ln \frac{e_{it}}{e_{jt}} = \Delta \ln \frac{F_{it}}{F_{jt}} + \Delta \ln \frac{\varepsilon_{it}^{\omega}}{\varepsilon_{jt}^{\omega}}$, so $var^D \left(\Delta \ln \frac{e_{it}}{e_{jt}}\right) \geq$

pirically, the dispersion of actual currency movements is indeed larger, at 53% (the interquartile range of 41%) during World War I and II.[17] Hence, by that metric, the volatility in resilience is reasonable.

Exports and productivity. In the calibration, we take $\sigma = 1/\gamma$ for parsimony (so that the utility becomes separable) and also assume that ζ_{it} and ξ_{it} do not jump in a disaster. In a disaster, M_{t+1}^* is multiplied by $B_{t+1}^{-\gamma}$. So, since $C_{it}^T = \left(\frac{M_t^*}{\mu_i e^{-\rho t}\zeta_{it}}\right)^{-\frac{1}{\gamma}}$ (from equation (10)), we have that C_{it}^T is multiplied by B_{t+1} in a disaster. Given $C_{it}^{NT} = C_{it}^T\left(\xi_{it}e_{it}\right)^{-\frac{1}{\gamma}}$ (from equation (9)), C_{it}^{NT} is multiplied by $B_{t+1}F_{i,t+1}^{-\frac{1}{\gamma}}$ in a disaster. For simplicity in the choice of ancillary parameters, we will take the case where the typical value of $F_{i,t+1}$ is $\overline{F} = 1$. This way, in a typical disaster, the ratio of nontradable to tradable consumption remains unchanged. In other words, both types of consumption collapse by the same factor, B_{t+1}.

This leads us to take $H_{i*} = \overline{p}\left(\overline{B}^{-\gamma}\overline{F} - 1\right) = 15.3\%$. We categorize H_{i*} as ancillary because its value (given the existing key parameters of Table I) does not affect (up to second order terms) the exchange rate and interest rate moments that we report in Tables III and IV.

The growth rate of productivity g_{ω_i} is irrelevant in practice, but for completeness we propose a specific value. Though results are not sensitive to the choice of this parameter, we set $g_{\omega_i} = 0$; in an economy where endowments all grow at a rate g_{c_i}, the consumption of tradables and non-tradables then both grow at g_{c_i}, as there is no differential advantage to the production of tradables. We take $\lambda = 5.5\%$ which leads to an average real interest rate of $r_{ei} - \lambda = 0.5\%$.

Given $R = r_{ei} - \lambda + g_{\omega_i} + h_{i*}$, we obtain $R = 15.9\%$. Given our above assumptions, we have $R = \rho + \gamma g_{c_i}$, where g_{c_i} is the growth rate of tradable consumption. We choose the growth rates so that in normal times consumption of non-tradables grows at a rate $g_{c_i} = 2.5\%$. This in turn pins down the rate of time preference, $\rho = R - \gamma g_{c_i} = 5.9\%$.

Stocks. We take the recovery of the stock dividend to be that of the economy $\overline{F}_{D_i} = \overline{B}$. By the same reasoning as above, we have $\sigma_{F_{Di}} = \frac{\sigma_{H_{Di}}}{\overline{p}\overline{B}^{-\gamma}} = 1.5\%$.

$var^D\left(\Delta \ln \frac{F_{it}}{F_{jt}}\right)$.

[17] The data comes from Global Financial Data, and comprises Australia, Austria, Belgium, Brazil, Canada, Denmark, Finland (only WWII), France, Germany, India, Ireland (only WWII), Italy, Japan, Netherlands, New Zealand (only WWII), Norway, Portugal (only WWII), Russia (only WWII), South Africa, Spain, Sweden, Switzerland, and UK.

TABLE III: Moments: Empirical and in the Model

Moments	Data	Calibration
Std Dev($\Delta \ln \tilde{e}_{ijt}$)	12.35%	11.9%
Carry Trade Return	3.44%	4.9%
Fama Regression Coefficient	-1.29	-0.61
Mean($\lvert RR_{ijt} \rvert$)	1.31%	1.25%
Std Dev(RR_{ijt})	1.24%	1.28%
Std Dev(ΔRR_{ijt})	2.60%	1.10%
Std Dev($\tilde{r}_{it}$)	1.38%	1.28%
Std Dev($\Delta(\tilde{r}_{it} - \tilde{r}_{jt})$)	0.71%	1.18%
Corr($\Delta \ln \tilde{e}_{ijt}, \Delta RR_{ijt}$)	-0.58	-0.90
Corr($\ln \tilde{e}_{ij,t+1}, \ln \tilde{e}_{ijt}$)	0.88	0.98
Corr($\Delta \ln \tilde{e}_{ij,t+1}, \Delta \ln \tilde{e}_{ijt}$)	-0.13	0.00
Corr($\tilde{r}_{it} - \tilde{r}_{jt}, RR_{ijt}$)	0.55	0.65

Notes: The table reports the moments generated by the model, using the inputs from Table I. The risk reversal RR_{ijt} is defined as the implied volatility of an out-of-the-money put minus the implied volatility of an out-of-the-money call, all at 25-delta. A high RR_{ijt} means that the price of protection from depreciation of currency i (against country j) is high. $\tilde{r}_{it}$ is the nominal interest rate. We define $\widetilde{e}_{ijt} := \frac{\widetilde{e}_{it}}{\widetilde{e}_{jt}}$, the nominal bilateral exchange rate between countries i and j: a high $\widetilde{e}_{ijt}$ means that currency i appreciates. Carry trade returns are the returns from a long-short portfolio going \$1 long (resp. short) an equally-sized portfolio of high (resp. low) RR countries. The time unit is a year (the model is estimated and simulated at a monthly frequency, but for readability the numbers reported above are all annualized).

V.B. Main Quantitative Findings

Tables III and IV present results from the calibration.[18]

As Table III shows, the model hits the volatility of the bilateral exchange rate, i.e. generates the right amount of "excess volatility" in exchange rates. The model also roughly matches (and slightly undershoots) the size of disaster risk as measured by the average size of risk reversals. At the same time, the model generates a moderate volatility of the interest rate, as in the data.

We showed earlier that in the model countries with high risk reversals have high interest rates, and that increases in risk reversals are associated with depreciations of the exchange rate. The calibration confirms this. Table III reports the calibrated values of Cov $(\widetilde{r}_{it} - \widetilde{r}_{jt}, RR_t)$, which broadly matches up with its empirical counterpart. Corr$(\Delta \ln \tilde{e}_{ijt}, \Delta RR_{ijt})$ is a bit far

[18] The online appendix presents a Table VI, which verifies that our calibration of stocks also matches the pattern of predictability of the stock market with the price-dividend ratio, with a slope and R^2 rising with the horizon (at least for a while).

TABLE IV: Moments related to Stocks: Empirical and in the Model

Moments	Data	Calibration
Equity premium: Mean($r_{it}^{\text{Stock}} - r_{it}$)	6%	7.5%
Volatility of stock returns: Std Dev(r_{it}^{Stock})	18%	18%
Mean P_{it}/D_{it}	23	18.7
Standard deviation of $\ln P_{it}/D_{it}$	0.33	0.37
Volatility of stock returns in foreign currency: Std Dev($r_{it}^{\text{Stock,\$}}$)	20%	21%
$\text{Corr}(\Delta \ln \tilde{e}_{ijt}, r_{it}^{\text{Stock}} - r_{jt}^{\text{Stock}})$	0.00	0.03
$\text{Corr}(\Delta \ln \tilde{e}_{ijt}, r_{it}^{\text{Stock,\$}} - r_{jt}^{\text{Stock,\$}})$	0.75	0.55
$\text{Corr}(\Delta RR_{ijt}, r_{it}^{\text{Stock}} - r_{jt}^{\text{Stock}})$	-0.05	-0.01
$\text{Corr}(\Delta RR_{ijt}, r_{it}^{\text{Stock,\$}} - r_{jt}^{\text{Stock,\$}})$	-0.46	-0.49

Notes. The table reports the moments generated by the model, using inputs from Table I. r_{it}^{stock} (resp. $r_{it}^{stock,\$}$) is the stock return of country i expressed in country i's domestic currency (resp. in US dollars). The risk reversal RR_{ijt} is defined as the implied volatility of an out-of-the-money put minus the implied volatility of an out-of-the-money call, all at 25-delta. A high RR_{ijt} means that the price of protection from depreciation of currency i (against country j) is high. We define $\widetilde{e}_{ijt} := \frac{\widetilde{e}_{it}}{\widetilde{e}_{jt}}$, the nominal bilateral exchange rate between countries i and j: an increase in $\widetilde{e}_{ijt}$ means that currency i appreciates. Here we take country j to be the US. The time unit is a year (the model is estimated and simulated at a monthly frequency, but for readability the numbers reported above are all annualized).

from its empirical value. The reason is that we have a one-factor model, which (essentially) tends to generate a -1 correlation. If we have a two-factor model (as in the online appendix, Section XIII), then the fit becomes quite good.

The carry trade generated by the model gives average returns in line with the empirical evidence (see Farhi et al. 2015 for more variants of the carry trade). Investing in countries with high risk reversals generates high expected returns. Indeed, the expected return of the carry trade (given positive RR) is about 4.9% per annum. Finally, the model generates Fama coefficients $\beta^{ND} = -0.61$ in line with estimates of the literature cited above. In the table, we report Verdelhan (2010)'s estimate, which is representative.

Stocks. Table IV shows the moments related to stocks. We call $r_{it}^{\text{Stock}} = \frac{P_{it}+D_{it}}{P_{i,t-1}} - 1$ the return of the stock in country i, in the domestic currency. We also call $r_{it}^{\text{Stock,\$}} = \frac{e_{ijt}}{e_{ij,t-1}} \frac{P_{it}+D_{it}}{P_{i,t-1}} - 1 \simeq r_{it}^{\text{Stock}} + \Delta \ln e_{ijt}$ the return in a foreign currency, which we will take to be the dollar, and call $\widetilde{e}_{ijt} = \frac{\widetilde{e}_{it}}{\widetilde{e}_{jt}}$ the bilateral exchange rate between countries i and j. The equity premium is about 6%, in line with the usual empirical estimates. We report the correlations between changes in the exchange rate of two countries and changes in the relative stock returns:

$corr\left(\Delta \ln \widetilde{e}_{ijt}, r_{it}^{\text{Stock}} - r_{jt}^{\text{Stock}}\right)$. Empirically, this correlation is close to 0; movements in the stock market and the exchange rate are uncorrelated on average. The model reproduces that fact. Likewise, changes in risk reversals and relative stock returns are essentially uncorrelated in the data and in the model ($corr\left(\Delta RR_{ijt}, r_{it}^{\text{Stock}} - r_{jt}^{\text{Stock}}\right)$ is close to 0).

We also study the correlation between stock returns in a common currency and the change in the exchange rate: $corr\left(\Delta \ln \widetilde{e}_{ijt}, r_{it}^{\text{Stock},\$} - r_{jt}^{\text{Stock},\$}\right)$. Empirically, this correlation is high, about 0.75. The model is roughly in line with this. Likewise, there is a negative correlation between changes in risk reversals and relative stock returns: $corr\left(\Delta RR_{ijt}, r_{it}^{\text{Stock},\$} - r_{jt}^{\text{Stock},\$}\right) < 0$ in the data and the model. Economically, when country i becomes less risky, the risk-reversal RR_{ijt} goes down, the exchange rate $\widetilde{e}_{ijt}$ appreciates and its realized stock return differential $r_{it}^{\text{Stock},\$} - r_{jt}^{\text{Stock},\$}$ is higher.

We conclude that the disaster model can be made quantitatively broadly congruent with the salient empirical facts.

V.C. Additional Findings: Covariance Structure in Currencies and HML_{FX}

In their influential work, Lustig, Roussanov and Verdelhan (2011) find a one-factor structure in currency returns. Namely, they form a portfolio, HML_{FX}, going long high interest rate currencies and short low interest rate currencies. They also find that currency excess returns are accounted for by the HML_{FX} factor. They find, in essence, that regressing currency return $r_{it}^{Currency} = \alpha_i + \beta_i HML_{FX_t} + \varepsilon_{it}$ yields $\alpha_i = 0$. Here, $r_{it}^{Currency}$ is the currency return (capital gains plus interest rate) when going long a basket i (e.g., high interest rate currencies) and short a diversified basket of currencies.

In this subsection, we keep the previous calibration, but give it the additional structure of a one-factor model in currencies and stocks, respectively. We find that we can match the salient facts related to HML_{FX} and make a new, successful prediction linking it to risk reversals.

Factor structure. We define the normalized resiliences of stocks and exchange rates as:

$$h_{Dit} := \frac{1 + \frac{\widehat{H}_{Dit}}{r_{Di} + \phi_{H_{Di}}}}{1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}}} - 1, \qquad h_{eit} := \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}}.$$

Using this notation, we can re-express equations (16) and (36) as follows:

$$e_{it} = \frac{\omega_{it}}{r_{ei}} \left(1 + h_{eit}\right), \qquad P_{Dit} = \frac{r_{ei}}{r_{Di}} \frac{D_{it}}{\omega_{it}} \left(1 + h_{Dit}\right).$$

The innovation to the exchange rate and the stock price (and return) are captured by h_{eit} and h_{Dit}, respectively. We call ε_{X_t} the innovation to a random variable X_t ($\varepsilon_{X_t} = X_t - \mathbb{E}_{t-1}[X_t]$).

We posit that the innovation to normalized resilience follows a one-factor structure: $\varepsilon_{h_{eit}} = \beta_{ei,t-1} f_{et} + \eta_{eit}$ where f_{et} and η_{eit} are mean-0 innovations. We also specify $\beta_{eit} = b\left(H_{at} - H_{it}\right)$ with $b > 0$, and H_{at} is the average of H_{it} over all other countries. This means that when f_{et} is positive, the spread in the resilience between risky and less risky countries shrinks, i.e. risky currencies appreciate over less risky currencies. Hence, f_{et} is proportional to $HML_{FX,t}$. When f_{et} is positive, $HML_{FX,t}$ is positive and the risk reversal of risky countries goes down while their exchange rate appreciates. The key free parameter is b, which we set to 0.114.

Empirically, international stock markets tend to covary. This naturally suggests a one-factor structure of stock resilience: $\varepsilon_{h_{Dit}} = \beta_{Di,t-1} f_{Dt} + \eta_{Dit}$. We set $\beta_{Dit} = 1$ and $corr\left(f_{et}, f_{Dt}\right) = 0.65$, which allows us to match the empirical correlation between the HML_{FX} factor and the average of international stock market returns (the online appendix details the process, including linearity-generating terms.) The factors η_{eit} and η_{Dit} are uncorrelated with other variables.

TABLE V: Moments related to HML_{FX}: Empirical and in the Model

Moments	Data	Calibration
$\text{Corr}(HML_{FX,t}, r_{at}^{\text{Stock}})$	0.39	0.33
Test of the one factor-structure: α_i	$0^{(*)}$	$0^{(*)}$
in $r_{it}^{Currency} = \alpha_i + \beta_{i,HML} HML_{FX_t} + \varepsilon_{it}$		
	(for H,M,L portfolio)	(for H,M,L portfolio)
$\text{Corr}(HML_{FX,t}, \Delta \ln \frac{\widetilde{e}_{it}}{\widetilde{e}_{at}})$	$(0.60, -0.01, -0.51)$	$(0.50, 0.04, -0.43)$
$\text{Corr}(HML_{FX,t}, \Delta RR_{it})$	$(-0.43, -0.05, 0.36)$	$(-0.47, -0.02, 0.43)$

Notes. Here $HML_{FX,t}$ is the return of a portfolio going long high interest rate currencies and short low interest rate currencies. r_{at}^{Stock} is the average of stock market returns across countries. $r_{it}^{Currency}$ is the currency return (capital gains plus interest rate) when going long a basket i (high / medium / low interest rate currencies), and short an equally-weighted basket of all currencies. $0^{(*)}$ means that the value is not statistically different from 0. $\frac{\widetilde{e}_{it}}{\widetilde{e}_{at}}$ is the nominal exchange rate of country i vis-à-vis an equally-weighted basket of all currencies ($\widetilde{e}_{at}$ is the average exchange rate across countries, $\ln \widetilde{e}_{at} = \frac{1}{n} \sum_{j=1}^{n} \ln \widetilde{e}_{jt}$). Countries are sorted by interest rates, and are divided into three groups of High, Medium and Low interest rates (H,M,L).

Results. Table V shows the results.[19] As expected, if we sort countries by interest rates (High, Medium and Low interest rates: H,M,L - recall that in our model, risky countries have high real interest rates), we observe that risky countries have a positive correlation with HML_{FX} and the least risky countries have a negative correlation with it (see the row $\mathrm{Corr}(HML_{FX,t}, \Delta \ln \frac{\widetilde{e}_{it}}{\widetilde{e}_{at}}))$. The model produces a good quantitative fit with this fact.

We also verify that the one-factor structure shown in Lustig, Roussanov and Verdelhan (2011) is replicated in our model (see the row on $r_{it}^{Currency} = \alpha_i + \beta_{i,HML} HML_{FX_t} + \varepsilon_{it}$).[20] In addition, the model replicates the positive correlation between HML_{FX} returns and average stock market returns.

The new prediction of the model is that risk reversals should covary with HML_{FX}: the risk reversal of risky countries should covary negatively with HML_{FX}, while the risk reversal of less risky countries should covary positively with it. This is indeed the case in the data, as indicated in Table V. We view this as an additional comforting, previously undocumented, disaster-like feature of the data.

In conclusion, a parsimonious calibration of the model can replicate the major moments of the link between currencies, interest rates, stocks and options, including the factor structure documented in stocks and currencies.

VI. Conclusion

We have proposed a disaster-based tractable framework for exchange rates. Our framework accounts qualitatively and quantitatively for both classic exchange rate puzzles (e.g. excess volatility of exchange rates, forward premium puzzle, excess return of the carry trade) and links between currency options, exchange rates and interest rates – signature predictions of the disaster hypothesis.

The model is fully solved in closed form. It can readily be extended in several ways. The online appendix of this paper works out various extensions, including a detailed model of the term structure and the incorporation of business cycle movements.[21]

[19] This Table is computed over the whole sample, to maximize representativeness. The numbers are broadly the same when restricting to the post-2009 sample, except $\mathrm{Corr}(HML_{FX,t}, r_{at}^{\mathrm{Stock}})$, which is smaller in that sample. We suspect that this number is not representative of typical samples.

[20] If we computed the returns of portfolios short a given currency (say the dollar), then we would need to add a second factor, namely the return of that currency.

[21] These extensions rationalize additional empirical facts uncovered by Boudoukh, Richardson and Whitelaw

The model offers a unified, tractable and calibrated treatment of the major assets and their links: exchange rates, bonds, stocks and options. Hence, we hope it may be a useful point of departure to think about issues in international macro-finance. In particular, studying more specifically the dynamics of production and consumption in the disaster environment seems like a fruitful direction for research.[22] We speculate that this might involve modelling adjustment costs in investment, imperfect risk-sharing, and price setting imperfections leading to pricing to market and incomplete cost-to-price pass-through. Pursuing this direction could lead to a unified international macro model to think jointly about prices and quantities.

(2012) on forward rates and deviations from UIP, and Lustig, Stathopoulos and Verdelhan (2014) on the term premium in the bond and currency riskiness. Boudoukh, Richardson and Whitelaw (2012) find that the carry trades based on long-dated forward rates exhibit small deviations from UIP: in our model this is because at a long horizon, other factors (e.g. business cycle or inflation) are more important, and they tend to generate no deviation from UIP. Lustig, Stathopoulos and Verdelhan (2014) find that "risky" currencies have a lower term premium. In our model this holds even if all countries have the same inflation dynamics, where inflation goes up in disasters (as in the historical experience on average), which creates a term premium. Because risky currencies will depreciate in a disaster, a portfolio long their short-term bond and short their long-term bond will have little value in a disaster, whereas the equivalent portfolio for safe countries will have a positive value: hence, the term premium is high in safe countries and small or zero in risky countries.

[22] To keep a tractable model of production with disasters, the "disasterization" procedure may be useful. It was proposed in Gabaix (2011), and later used in Gourio (2012).

VII. Appendix: Complements and Proofs

VII.A. *Different Notions of the Exchange Rate*

In the paper, we define the "absolute" exchange rate e_{it} to be the price of the non-traded good in country i in terms of the world numéraire. The more traditional definition would be $\mathcal{E}_{it}$, the price of the consumption basket in country i in terms of the world numéraire.[23] Using the usual algebra of CES price indices, the link between the two is:

$$\mathcal{E}_{it} = \left(\xi_{it}^{\sigma} + {e_{it}}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{44}$$

The share of traded goods in consumption is $\xi_{it}^{\sigma} e_{it}^{1-\sigma}\left(\xi_{it}^{\sigma} + {e_{it}}^{1-\sigma}\right)^{-1}$. In the data, this share is small, so that ξ_{it} is close to 0. The consequence is that $\mathcal{E}_{it} \simeq e_{it}$ and $\frac{\mathcal{E}_{it}}{\mathcal{E}_{jt}} \simeq \frac{e_{it}}{e_{jt}}$, so that the two notions are quantitatively close. This approximation is exact up to a term $O\left(\xi_{it}^{\sigma}\right)$. It is analytically simpler to characterize the behavior of e_{it}. In any case, it is possible to go back and forth between the two notions using equation (44).

VII.B. *Proofs*

We present here the proofs of the main results. Additional proofs are in the online appendix. For simplicity, we drop the country index i in most proofs.

The limit of small time intervals. We often take the limit of small time intervals. We formalize this procedure here. Take for instance the interest rate "per period" r_{it}. Let us call Δt the physical length of a time period (e.g. $\Delta t = \frac{1}{12}$ of a year if the time period is one month), and $\boldsymbol{r}_{it}$ the interest rate in continuous time notation. Then, $r_{it} = \boldsymbol{r}_{it}\Delta t$. Likewise,

$$\widehat{H}_{it} = \widehat{\boldsymbol{H}}_{it}\Delta t, \qquad p_t = \boldsymbol{p}_t\Delta t, \qquad \rho = \boldsymbol{\rho}\Delta t, \qquad \lambda = \boldsymbol{\lambda}\Delta t \tag{45}$$

However, the exchange rate is not in "per period units", so that $e_{it} = \boldsymbol{e}_{it}$. Likewise, $B_t = \boldsymbol{B}_t$, $F_{it} = \boldsymbol{F}_{it}$. On the other hand, $t = \boldsymbol{t}/\left(\Delta t\right)$, where $\boldsymbol{t}$ is the physical time (in years). The limit of small time intervals corresponds to $\Delta t \to 0$.

[23] The bilateral exchange rate between country i and country j is then $\frac{\mathcal{E}_{it}}{\mathcal{E}_{jt}}$.

As an example, let us detail how (19) becomes (16) in the limit of small time intervals.

$$
\begin{aligned}
e_{it} &= \frac{\omega_{it}}{1-\exp\left(-r_{ei}\right)}\left(1+\frac{\exp\left(-r_{ei}-h_{i*}\right)}{1-\exp\left(-r_{ei}-\phi_{H_i}\right)}\widehat{H}_{it}\right) \\
&= \frac{\boldsymbol{\omega}_{it}\Delta t}{1-\exp\left(-\boldsymbol{r}_{ei}\Delta t\right)}\left(1+\frac{\exp\left(-\boldsymbol{r}_{ei}\Delta t-\boldsymbol{h}_{i*}\Delta t\right)}{1-\exp\left(-\boldsymbol{r}_{ei}\Delta t-\boldsymbol{\phi}_{H_i}\Delta t\right)}\widehat{\boldsymbol{H}}_{it}\Delta t\right) \\
&= \frac{\boldsymbol{\omega}_{it}\Delta t}{\boldsymbol{r}_{ei}\Delta t+O\left((\Delta t)^2\right)}\left(1+\frac{1+O\left(\Delta t\right)}{\boldsymbol{r}_{ei}\Delta t+\boldsymbol{\phi}_{H_i}\Delta t+O\left((\Delta t)^2\right)}\widehat{\boldsymbol{H}}_{it}\Delta t\right) \\
&= \frac{\boldsymbol{\omega}_{it}}{\boldsymbol{r}_{ei}+O\left(\Delta t\right)}\left(1+\frac{1+O\left(\Delta t\right)}{\boldsymbol{r}_{ei}+\boldsymbol{\phi}_{H_i}+O\left(\Delta t\right)}\widehat{\boldsymbol{H}}_{it}\right) \\
&= \frac{\boldsymbol{\omega}_{it}}{\boldsymbol{r}_{ei}}\left(1+\frac{\widehat{\boldsymbol{H}}_{it}}{\boldsymbol{r}_{ei}+\boldsymbol{\phi}_{H_i}}\right)+O\left(\Delta t\right) = \frac{\omega_{it}}{r_{ei}}\left(1+\frac{\widehat{H}_{it}}{r_{ei}+\phi_{H_i}}\right)+O\left(\Delta t\right).
\end{aligned}
$$

Complement to the Proof of Proposition 2. We now present a rigorous proof. Let $D_t = \exp(-\lambda t)\,\omega_t$ and $X_t = \widehat{H}_t$ (for simplicity, we drop the subscript i in this proof). By Proposition 1, we have

$$
e_0 = \mathbb{E}_0\left[\sum_{t=0}^{\infty} M_t^* D_t\right]/M_0^*. \tag{46}
$$

We calculate the moments:

$$
\begin{aligned}
\mathbb{E}_t\left[\frac{M_{t+1}^* D_{t+1}}{M_t^* D_t}\right] &= \exp\left(-R-\lambda+g_\omega\right)\left\{(1-p_t)+p_t\mathbb{E}_t^D\left[B_{t+t}^{-\gamma}F_{t+1}\right]\right\} \\
&= \exp\left(-R-\lambda+g_\omega\right)\left(1+H_t\right) = \exp\left(-R-\lambda+g_\omega\right)\left(1+H_*\right)+\exp\left(-R-\lambda+g_\omega\right)\widehat{H}_t \\
&= \exp\left(-R-\lambda+g_\omega\right)\left(1+H_*\right)+\exp\left(-R-\lambda+g_\omega\right)X_t \\
&= \exp\left(-r_e\right)+\exp(-r_e-h_*)X_t,
\end{aligned}
$$

using $r_e = R+\lambda-g_\omega-h_*$. Also:

$$
\begin{aligned}
\mathbb{E}_t\left[\frac{M_{t+1}^* D_{t+1}}{M_t^* D_t}X_{t+1}\right] &= \mathbb{E}_t\left[\frac{M_{t+1}^* D_{t+1}}{M_t^* D_t}\right]\mathbb{E}_t\left[X_{t+1}\right] = e^{-R-\lambda+g_\omega}\left(1+H_t\right)\frac{1+H_*}{1+H_t}e^{-\phi_H}\widehat{H}_t \\
&= e^{-R-\lambda+g_\omega-\phi_H}\left(1+H_*\right)\widehat{H}_t = e^{-r_e-\phi_H}X_t.
\end{aligned}
$$

There are two ways to conclude. *The first way* uses the notation of Gabaix (2009)[24]: the above two moment calculations show that $Y_t = M_t^* D_t\left(1, X_t\right)$ is an LG process with generator

[24] See also the compact version, on Gabaix's web page: "Précis of Results on Linearity-Generating Processes".

Ω:

$$\Omega = \begin{pmatrix} \exp(-r_e) & \exp(-r_e - h_*) \\ 0 & \exp(-r_e - \phi_H) \end{pmatrix}.$$

Using Theorem 2 in Gabaix (2009), or equation 3 of the "Précis", we find $e_{i0} = \omega_{i0} (1, 0) (I - \Omega)^{-1} (1, X_t)'$,

$$e_{i0} = \frac{\omega_{i0}}{1 - \exp(-r_{ei})} \left(1 + \frac{\exp(-r_{ei} - h_{i*})}{1 - \exp(-r_{ei} - \phi_{H_i})} \widehat{H}_{i0}\right). \tag{47}$$

More generally, where ω_{it} is the current productivity of the country, we get (19). *The second way* is the more heuristic proof in the main text.

Proof of Proposition 4. *Derivation of (25).* Using (16), we calculate:

$$\begin{aligned}
\mathbb{E}_t^{ND}\left[\frac{e_{i,t+1}}{e_{it}} - 1\right] &= \mathbb{E}_t^{ND}\left[\frac{\omega_{i,t+1}}{\omega_{it}} \frac{1 + \frac{\widehat{H}_{i,t+1}}{r_e + \phi_H}}{1 + \frac{\widehat{H}_{it}}{r_e + \phi_H}} - 1\right] = \mathbb{E}_t^{ND}\left[\exp(g_{\omega_i}) \frac{1 + \frac{\widehat{H}_{i,t+1}}{r_e + \phi_H}}{1 + \frac{\widehat{H}_{it}}{r_e + \phi_H}} - 1\right] \\
&= g_{\omega_i} + \mathbb{E}_t^{ND}\left[\frac{1 + \frac{\widehat{H}_{i,t+1}}{r_e + \phi_H}}{1 + \frac{\widehat{H}_{it}}{r_e + \phi_H}} - 1\right] + o(\Delta t) = g_{\omega_i} + \mathbb{E}_t^{ND}\left[\frac{\widehat{H}_{i,t+1} - \widehat{H}_{it}}{r_e + \phi_H + \widehat{H}_{it}}\right] + o(\Delta t) \\
&= g_{\omega_i} - \frac{\phi_H \widehat{H}_{it} + \widehat{H}_{it}^2}{r_e + \phi_H + \widehat{H}_{it}} + o(\Delta t),
\end{aligned}$$

where we use the fact that (14) becomes, in the limit of small time intervals, $\widehat{H}_{i,t+1} - \widehat{H}_{it} = -\left(\phi_H + \widehat{H}_{it}\right) \widehat{H}_{it} + \varepsilon_{i,t+1}^H + o\left((\Delta t)^2\right)$.

Using (22), $r_{it} = r_e - \lambda - \frac{r_e \widehat{H}_{it}}{r_e + \phi_H + \widehat{H}_{it}}$ and $r_e = R + \lambda - g_{\omega_i} - h_{i*}$,

$$\begin{aligned}
\mathbb{E}_t^{ND}\left[\frac{e_{i,t+1}}{e_{it}} (1 + r_{it})\right] - 1 &= \mathbb{E}_t^{ND}\left[\frac{e_{i,t+1}}{e_{it}} - 1\right] + r_{it} + o(\Delta t) \\
&= g_{\omega_i} - \frac{\phi_H \widehat{H}_{it} + \widehat{H}_{it}^2}{r_e + \phi_H + \widehat{H}_{it}} + r_e - \lambda - \frac{r_e \widehat{H}_{it}}{r_e + \phi_H + \widehat{H}_{it}} + o(\Delta t) \\
&= R - h_{i*} - \frac{\left(r_e + \phi_H + \widehat{H}_{it}\right) \widehat{H}_{it}}{r_e + \phi_H + \widehat{H}_{it}} + o(\Delta t) \\
&= R - H_{i*} - \widehat{H}_{it} + o(\Delta t) = R - H_{it} + o(\Delta t).
\end{aligned}$$

This implies:

$$\begin{aligned}\mathbb{E}_t^{ND}\left[R_{ij,t+1}\right] &= \mathbb{E}_t^{ND}\left[\frac{e_{i,t+1}}{e_{it}}\left(1+r_{it}\right)\right] - \mathbb{E}_t^{ND}\left[\frac{e_{j,t+1}}{e_{jt}}\left(1+r_{jt}\right)\right] \\ &= H_{jt} - H_{it} + o\left(\Delta t\right) = \widehat{H}_{jt} - \widehat{H}_{it} + o\left(\Delta t\right)\end{aligned}$$

Next, we turn to proving expression (24).

$$\begin{aligned}\mathbb{E}_t\left[\frac{e_{i,t+1}}{e_{it}}\left(1+r_{it}\right)\right] - 1 &= \left(1-p_t\right)\mathbb{E}_t^{ND}\left[\frac{e_{i,t+1}}{e_{it}}\left(1+r_{it}\right)-1\right] + p_t\mathbb{E}_t^{D}\left[\frac{e_{i,t+1}}{e_{it}}\left(1+r_{it}\right)-1\right] \\ &= \left(1-p_t\right)\left(R-H_{it}\right) + p_t\mathbb{E}_t^{D}\left[F_{i,t+1}-1\right] + o\left(\Delta t\right)\end{aligned}$$

The last step is verified as follows:

$$\begin{aligned}p_t\mathbb{E}_t^{D}\left[\frac{e_{i,t+1}}{e_{it}}\left(1+r_{it}\right)-1\right] &= \boldsymbol{p}_t\Delta t\mathbb{E}_t^{D}\left[F_{i,t+1}\left(1+\boldsymbol{r}_{it}\Delta t\right)-1\right] = \boldsymbol{p}_t\Delta t\left(\mathbb{E}_t^{D}\left[F_{i,t+1}-1\right]+O\left(\Delta t\right)\right) \\ &= p_t\mathbb{E}_t^{D}\left[F_{i,t+1}-1\right] + o\left(\Delta t\right)\end{aligned}$$

Given $H_{it} = p_t\mathbb{E}_t^{D}\left[\overline{B}^{-\gamma}F_{i,t+1}-1\right]$ (equation (12)), as we assumed that $B_{t+1} = \overline{B}$ is deterministic,

$$\begin{aligned}\mathbb{E}_t\left[R_{ij,t+1}\right] &= \mathbb{E}_t\left[\frac{e_{i,t+1}}{e_{it}}\left(1+r_{it}\right)\right] - \mathbb{E}_t\left[\frac{e_{j,t+1}}{e_{jt}}\left(1+r_{jt}\right)\right] \\ &= \left(1-p_t\right)\left(H_{jt}-H_{it}\right) + p_t\mathbb{E}_t^{D}\left[F_{i,t+1}-F_{j,t+1}\right] + o\left(\Delta t\right) \\ &= \left(1-p_t\right)\left(H_{jt}-H_{it}\right) + \overline{B}^{\gamma}\left(H_{it}-H_{jt}\right) + o\left(\Delta t\right) \\ &= \left(1-\overline{B}^{\gamma}\right)\left(H_{jt}-H_{it}\right) + o\left(\Delta t\right).\end{aligned}$$

The last step uses (45), which gives $p_t = O\left(\Delta t\right)$ and $H_{jt} - H_{it} = O\left(\Delta t\right)$, so that $p_t\left(H_{jt}-H_{it}\right) = O\left(\left(\Delta t\right)^2\right) = o\left(\Delta t\right)$.

Finally, we use (22), which gives $r_{it} - r_{jt} = -\frac{r_e\widehat{H}_{it}}{r_e+\phi_H} + O\left(\widehat{H}_{it}^2\right) + \frac{r_e\widehat{H}_{jt}}{r_e+\phi_H} + O\left(\widehat{H}_{jt}^2\right)$, hence $\widehat{H}_{jt} - \widehat{H}_{it} = \left(1+\frac{\phi_H}{r_e}\right)\left(r_{it}-r_{jt}\right) + O\left(\widehat{H}_{jt}^2 + \widehat{H}_{it}^2\right)$.

Proof of Proposition 5. We use (25), which gives (in the limit of small time intervals and resiliences):

$$E_t^{ND}\left[\frac{e_{i,t+1}}{e_{it}}-\frac{e_{j,t+1}}{e_{jt}}\right]+(r_{it}-r_{jt}) = E_t^{ND}\left[\frac{e_{i,t+1}}{e_{it}}(1+r_{it})-\frac{e_{j,t+1}}{e_{jt}}(1+r_{jt})\right]+o(\Delta t)$$
$$=\left(1+\frac{\phi_H}{r_e}\right)(r_{it}-r_{jt})+o(\Delta t)$$

i.e.

$$E_t^{ND}\left[\frac{e_{i,t+1}}{e_{it}}-\frac{e_{j,t+1}}{e_{jt}}\right]=\frac{\phi_H}{r_e}(r_{it}-r_{jt})+o(\Delta t)$$

Hence, an econometrician running the Fama regression (26) will find: $\beta^{ND}=-\frac{\phi_H}{r_e}$.

Likewise,

$$E_t\left[\frac{e_{i,t+1}}{e_{it}}-\frac{e_{j,t+1}}{e_{jt}}\right]=\left[\left(1-\overline{B}^{\gamma}\right)\left(1+\frac{\phi_H}{r_e}\right)-1\right](r_{it}-r_{jt})+o(\Delta t)$$
$$=\left[-\frac{\phi_H}{r_e}+\left(1+\frac{\phi_H}{r_e}\right)\overline{B}^{\gamma}\right](r_{jt}-r_{it})+o(\Delta t)$$

so that $\beta^{Full}=-\frac{\phi_H}{r_e}+\left(1+\frac{\phi_H}{r_e}\right)\overline{B}^{\gamma}$.

Proof of Proposition 6. *Put price.* We start with the put price:

$$V^P(K)=\mathbb{E}_0\left[\frac{M_1^*}{M_0^*}\left(K\frac{e_{j1}}{e_{j0}}-\frac{e_{i1}}{e_{i0}}\right)^+\right]$$
$$=(1-p_0)\,\mathbb{E}_0^{ND}\left[\frac{M_1^*}{M_0^*}\left(K\frac{e_{j1}}{e_{j0}}-\frac{e_{i1}}{e_{i0}}\right)^+\right]+p_0\mathbb{E}_0^{D}\left[\frac{M_1^*}{M_0^*}\left(K\frac{e_{j1}}{e_{j0}}-\frac{e_{i1}}{e_{i0}}\right)^+\right]$$
$$=(1-p_0)\,e^{-R}\mathbb{E}_0^{ND}\left[\left(K\frac{e_{j1}}{e_{j0}}-\frac{e_{i1}}{e_{i0}}\right)^+\right]+p_0e^{-R}\mathbb{E}_0^{D}\left[B_1^{-\gamma}\left(K\frac{e_{j1}}{e_{j0}}-\frac{e_{i1}}{e_{i0}}\right)^+\right],$$

where ND and D superscripts denote expectation conditional on no disasters and a disaster, respectively. The next calculation uses the following lemma, which is standard.[25]

[25] To verify it, we calculate that the characteristic function of y is the characteristic function of distribution (48):

$$\mathbb{E}^Q\left[e^{ky}\right]=\mathbb{E}\left[e^{x-\mathbb{E}[x]-\sigma_x^2/2}e^{ky}\right]=\exp\left(k\mathbb{E}[y]+\frac{k^2\sigma_y^2}{2}+k\,\mathrm{Cov}(x,y)\right)=\exp\left[k\left(\mathbb{E}[y]+\mathrm{Cov}(x,y)\right)+\frac{k^2\sigma_y^2}{2}\right].$$

Lemma 4 *(Discrete-time Girsanov). Suppose that (x, y) are jointly Gaussian under P. Consider the measure Q defined by $dQ/dP = \exp(x - \mathbb{E}[x] - \operatorname{Var}(x)/2)$. Then, under Q, y is Gaussian, with distribution*

$$y \sim^Q \mathcal{N}(\mathbb{E}[y] + \operatorname{Cov}(x, y), \operatorname{Var}(y)), \tag{48}$$

where $\mathbb{E}[y]$, $\operatorname{Cov}(x, y)$, *and* $\operatorname{Var}(y)$ *are calculated under P.*

To perform the calculation, write for the ND case

$$\frac{e_{i1}}{e_{i0}} = \exp\left(\mu_i + \varepsilon_i - \sigma_i^2/2\right) \tag{49}$$

and the analogue for j. We call $\eta = \varepsilon_i - \varepsilon_j$, and calculate:

$$\begin{aligned} V_1 &= \mathbb{E}_0^{ND}\left[\left(K\frac{e_{j1}}{e_{j0}} - \frac{e_{i1}}{e_{i0}}\right)^+\right] = \mathbb{E}_0^{ND}\left[\left(K\exp\left(\mu_j + \varepsilon_j - \sigma_j^2/2\right) - \exp\left(\mu_i + \varepsilon_i - \sigma_i^2/2\right)\right)^+\right] \\ &= \exp(\mu_i)\,\mathbb{E}_0^{ND}\left[\exp\left(\varepsilon_j - \sigma_j^2/2\right)\left(K\exp(\mu_j - \mu_i) - \exp\left(\varepsilon_i - \varepsilon_j + \sigma_j^2/2 - \sigma_i^2/2\right)\right)^+\right] \end{aligned}$$

We define $dQ/dP = \exp\left(\varepsilon_j - \sigma_j^2/2\right)$, and use Lemma 4. Under Q, with $\eta = \varepsilon_i - \varepsilon_j$, $y = \eta + \sigma_j^2/2 - \sigma_i^2/2$ is a Gaussian variable with variance σ_η^2 and mean:

$$\begin{aligned} \mathbb{E}^Q[y] &= \sigma_j^2/2 - \sigma_i^2/2 + \operatorname{Cov}(\varepsilon_i - \varepsilon_j, \varepsilon_j) \\ &= -\sigma_j^2/2 - \sigma_i^2/2 + \sigma_{i,j} = -\operatorname{Var}(\eta)/2. \end{aligned}$$

Hence,

$$\begin{aligned} V_1 &= \exp(\mu_i)\,\mathbb{E}^Q\left[(K\exp(\mu_j - \mu_i) - e^y)^+\right] \\ &= \exp(\mu_i)\,V_{BS}^P\left(K\exp(\mu_j - \mu_i), \sigma_{i|j}\right), \end{aligned}$$

where $\sigma_{i|j} = (\operatorname{Var}(\varepsilon_j - \varepsilon_i))^{1/2}$ and $V_{BS}^P(K, \sigma) = \mathbb{E}\left[(K - \exp(\sigma u - \sigma^2/2))^+\right]$ (with u a standard Gaussian) is the Black-Scholes put value when the interest rate is 0, the maturity 1, the strike K, the spot price 1, and the volatility σ.

Next, we observe that in disasters, $\frac{e_{i1}}{e_{i0}} = \exp(\mu_i) F_{i1}$. This implies:

$$\mathbb{E}_0^D \left[B_1^{-\gamma} \left(K \frac{e_{j1}}{e_{j0}} - \frac{e_{i1}}{e_{i0}} \right)^+ \right] = \mathbb{E}_0^D \left[B_1^{-\gamma} \left(\exp(\mu_j) K F_{j1} - \exp(\mu_i) F_{i1} \right)^+ \right].$$

We conclude that the value of the put is (30).

Call price. We use the put-call parity. Using the identity $x^+ = x + (-x)^+$ and the fact that $\mathbb{E}_0 \left[\frac{M_1^*}{M_0^*} \frac{e_{i1}}{e_{i0}} \right] = \frac{1}{1+r_{i0}}$, we have:

$$\begin{aligned} V^C(K) &= \mathbb{E}_0 \left[\frac{M_1^*}{M_0^*} \left(\frac{e_{i1}}{e_{i0}} - \frac{e_{j1}}{e_{j0}} K \right)^+ \right] = \mathbb{E}_0 \left[\frac{M_1^*}{M_0^*} \left(\frac{e_{i1}}{e_{i0}} - \frac{e_{j1}}{e_{j0}} K \right) \right] + \mathbb{E}_0 \left[\frac{M_1^*}{M_0^*} \left(\frac{e_{j1}}{e_{j0}} K - \frac{e_{i1}}{e_{i0}} \right)^+ \right] \\ &= \frac{1}{1+r_{i0}} - \frac{K}{1+r_{j0}} + V^P(K) \end{aligned}$$

Proof of Proposition 7. Using the same proof as in Gabaix (2012, Theorem 1), the price of the stock in the international numéraire (the traded good) is:

$$\overline{P}_{Dit} = D_{it} \frac{1 + \frac{\widehat{H}_{Dit}}{r_{Di} + \phi_{H_{Di}}}}{r_{Di}}.$$

Hence, expressed in the domestic currency, the price is: $P_{Dit} = \frac{\overline{P}_{Dit}}{e_{it}} = d_{it} \frac{1 + \frac{\widehat{H}_{Dit}}{r_{Di} + \phi_{H_{Di}}}}{r_{Di}}$. Away from the continuous time limit, the price of the stock is:

$$P_{Dit} = d_{it} \frac{1 + \frac{\exp(-r_{Di} - h_{Di*})}{1 - \exp\left(-r_{Di} - \phi_{H_{Di}}\right)} \widehat{H}_{Dit}}{1 - \exp(-r_{Di})}.$$

The model with nominal prices. The inflation process is as in Gabaix (2012), so we can take results from that paper. Let $Q_{it} = Q_{i0} \prod_{s=0}^{t-1} (1 - \pi_{is})$ be the value of money (the inverse of the price level) in country i. Using the LG results, the expected value of one unit of currency

T periods later is:

$$\mathbb{E}_t\left[\frac{Q_{i,t+T}}{Q_{it}}\right] = (1-\pi_{i*})^T\left(1-\frac{1-\exp\left(-\phi_{\pi i}T\right)}{1-\exp\left(-\phi_{\pi i}\right)}\frac{\pi_{it}-\pi_{i*}}{1-\pi_{i*}}\right), \tag{50}$$

or $\mathbb{E}_t\left[\frac{Q_{i,t+T}}{Q_{it}}\right] = \exp\left(-\pi_{i*}T\right)\left(1-\frac{1-\exp(-\phi_{\pi i}T)}{\phi_{\pi i}}\left(\pi_{it}-\pi_{i*}\right)\right)$ in the continuous-time limit.

The time-t price of a nominal bond yielding one unit of currency at time $t+T$ is $\widetilde{Z}_t(T) = \mathbb{E}_t\left[\frac{M^*_{t+T}e_{t+T}Q_{i,t+T}}{M^*_t e_t Q_{it}}\right]$. Because we assume that shocks to inflation are uncorrelated with disasters, the present value of one nominal unit of the currency is:

$$\widetilde{Z}_t(T) = \mathbb{E}_t\left[\frac{M^*_{t+T}e_{t+T}}{M^*_t e_t}\right]\mathbb{E}_t\left[\frac{Q_{i,t+T}}{Q_{it}}\right]. \tag{51}$$

Proof of Proposition 8. The derivation of the forward rate is as in Gabaix (2012), Theorem 2 and Lemma 2, using (51).

Proof of Proposition 9. We start with the case of the regression in a sample that does not contain disasters. So, up to second-order terms in $\widehat{H}_t$ and π_{it}, using the fact that (up to $o(\Delta t)$ terms), the change in the nominal exchange rate is the sum of the change in the real rate plus inflation differential,

$$\begin{aligned}\mathbb{E}^{ND}_t\left[\frac{\widetilde{e}_{i,t+1}-\widetilde{e}_{it}}{\widetilde{e}_{it}}-\frac{\widetilde{e}_{j,t+1}-\widetilde{e}_{jt}}{\widetilde{e}_{jt}}\right] &= \frac{-\phi_H}{r_e+\phi_H}\left(\widehat{H}_{it}-\widehat{H}_{jt}\right)-\left(\pi_{it}-\pi_{jt}\right)+o\left(\Delta t\right)\\ &:= a\left(\widehat{H}_{it}-\widehat{H}_{jt}\right)+b\left(\pi_{it}-\pi_{jt}\right)+c+o\left(\Delta t\right)\end{aligned}$$

$$\begin{aligned}\widetilde{r}_{it}-\widetilde{r}_{jt} &= -\frac{r_e}{r_e+\phi_H}\left(\widehat{H}_{it}-\widehat{H}_{jt}\right)+\left(\pi_{it}-\pi_{jt}\right)+o\left(\Delta t\right)\\ &:= A\left(\widehat{H}_{it}-\widehat{H}_{jt}\right)+B\left(\pi_{it}-\pi_{jt}\right)+C+o\left(\Delta t\right),\end{aligned}$$

hence

$$\begin{aligned}\widetilde{\beta}^{ND} &= -\frac{\mathrm{Cov}\left(\mathbb{E}^{ND}_t\left[\frac{\widetilde{e}_{i,t+1}-\widetilde{e}_{it}}{\widetilde{e}_{it}}-\frac{\widetilde{e}_{j,t+1}-\widetilde{e}_{jt}}{\widetilde{e}_{jt}}\right],\widetilde{r}_{it}-\widetilde{r}_{jt}\right)}{\mathrm{Var}\left(\widetilde{r}_{it}-\widetilde{r}_{jt}\right)} = -\frac{aA\,\mathrm{Var}\left(\widehat{H}_{it}-\widehat{H}_{jt}\right)+bB\,\mathrm{Var}\left(\pi_{it}-\pi_{jt}\right)}{A^2\,\mathrm{Var}\left(\widehat{H}_{it}-\widehat{H}_{jt}\right)+B^2\,\mathrm{Var}\left(\pi_{it}-\pi_{jt}\right)}\\ &= -\widetilde{\nu}\frac{a}{A}-\left(1-\widetilde{\nu}\right)\frac{b}{B} = \widetilde{\nu}\beta^{ND}+1-\widetilde{\nu},\end{aligned}$$

where $\widetilde{\nu} := \frac{1}{1+\frac{B^2 \operatorname{Var}\left(\pi_{it}-\pi_{jt}\right)}{A^2 \operatorname{Var}\left(\widehat{H}_{it}-\widehat{H}_{jt}\right)}}$.

The case of the full sample regression is proved similarly.

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