Compared to What?

Variation in the Impacts of Early Childhood Education by Alternative Care-Type Settings[1]

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Abstract

Early childhood education research often compares a group of children who receive the intervention of interest to a group of children who receive care in a range of different care settings. In this paper, we estimate differential impacts of an early childhood intervention by alternative care setting, using data from the Head Start Impact Study, a large-scale randomized evaluation. To do so, we utilize the principal stratification framework, a generalization of the instrumental variables approach, to estimate separate impacts for two types of Compliers: those children who would otherwise be in other center-based care when assigned to control and those who would otherwise be in home-based care. We find strong, positive short-term effects of Head Start on receptive vocabulary for those Compliers who would otherwise be in home-based care. By contrast, we find no meaningful impact of Head Start on vocabulary for those Compliers who would otherwise be in other center-based care. Our findings suggest that alternative care type is a potentially important source of variation in early childhood education interventions.

1 Introduction

Access to publicly funded prekindergarten in the United States has expanded substantially in recent years. In the last decade, the percentage of U.S. four-year-old children enrolled in public preschool has increased by one-third—from 31 to 40 percent—with some states now serving nearly 90 percent of all four-year-old children through publicly funded preschool programs (Barnett et al., 2014). Many cities, such as Boston, Los Angeles, New York, and Washington, D.C., have added to this expansion through locally-funded prekindergarten programs. The Obama Administration has called for additional funds to support even greater access to high-quality early childhood education across the country.

Those who support the expansion of publically funded preschool point to nearly 50 years of research indicating that participation in high-quality pre-school programs can yield individual and societal benefits in both the short and long term, often highlighting historically important interventions such as the Perry Preschool Project (e.g., Barnett, 1995; Heckman, 2006). Opponents argue that current public preschool programs, especially Head Start, the largest and most prominent public preschool program in the United States, have failed to replicate these initial successes at scale (e.g., Coulson, 2013; Whitehurst, 2013b). This belief stems in part from the results of the Head Start Impact Study (HSIS), a randomized evaluation that found that the opportunity to enroll in Head Start improved children's performance on short term measures of cognitive and social-emotional development but that, in general, these initial impacts were no longer apparent after children finished first grade (Puma et al., 2010a).

Researchers and policymakers have posited a wide range of explanations for differences between the Head Start results and those of early model programs like Perry preschool, including differences in program features, program intensity, and program targeting (Barnett, 2011; Bitler et al., 2014). We focus on one prominent explanation: that the care settings of control group children attenuated the reported effects for Head Start (e.g., National Forum on Early Childhood Policy and Programs, 2010). In the Perry Preschool Project, all control group children were cared for in their homes by a parent or other adult. By contrast, in the Head Start Impact Study, roughly one-third of children not in Head Start enrolled in other center-based care, with services similar to those provided by Head Start, while the remaining two-thirds were cared for in a home-based setting.

In this paper, we conduct a comprehensive analysis of the differential impact of enrolling in Head Start by the setting in which children would otherwise receive care. Our main result is that enrollment in Head Start yields strong, positive short-term effects on a measure of receptive vocabulary among those children who would enroll in Head Start when offered the opportunity to do so but who would otherwise be cared for by a parent or other caregiver at home or in a home-based setting. For this group of children, we estimate that, after one year, enrollment in Head Start improved children's performance by over 0.2 standard deviations, more than 50 percent larger than the corresponding intent-to-treat estimates reported in Puma et al. (2010a). By contrast, we find no meaningful impact of Head Start for those children who would otherwise enroll in non-Head

Start center-based care.[1]

Our analysis makes three main substantive contributions. First, we find meaningful impact variation by alternative care type that is masked by the topline results. This suggests that sweeping claims of Head Start's ineffectiveness (e.g., Whitehurst, 2013a) are misplaced, at least in terms of impact on receptive vocabulary. Moroever, we find no evidence that other center-based alternatives are more effective than Head Start on average, despite research arguing that this might be the case (Gormley et al., 2010). Second, this pattern of impact variation broadly holds across outcome quantiles (Bitler et al., 2014) and within key subgroups (Bloom and Weiland, 2014), although these estimates are imprecise. We find especially large impacts among Dual-Language Learner children who would otherwise be in home-based care. Third, consistent with the topline HSIS results (Puma et al., 2010a), we find that the impact of Head Start indeed declines over time. However, we find a gradual decline in effects, rather than the rapid attenuation identified by prior work (Gibbs et al., 2011), and also find modest evidence of positive impacts of Head Start through first grade.

Our paper also makes several methodological contributions. First, we set up an approach for identifying and estimating impacts in the presence of multiple counterfactual treatment options, which is common in early childhood education studies and in program evaluation more generally (e.g., Duncan and Magnuson, 2013). To do so, we use the *principal stratification* framework of Frangakis and Rubin (2002), which is a generalization of the usual instrumental variables (IV) approach for non-compliance in randomized experiments (Angrist et al., 1996). In the standard IV case, the goal is to estimate the impact for subgroups defined by each child's care setting under both experimental conditions: Compliers are children who would enroll in Head Start under treatment and would not enroll in Head Start under control. In our analysis, we are instead interested in two different types of Compliers: Center-based Compliers, children who would enroll in Head Start under treatment and would enroll in other center-based care under control, and Home-based Compliers, children who would enroll in Head Start under treatment and would otherwise enroll in home-based care. This approach yields two Local Average Treatment Effects (LATEs), rather than just one.

Identifying and estimating impacts for these subgroups is challenging. Extending results from the IV setting (Imbens and Rubin, 1997b; Abadie, 2003), we show that a range of quantities of interest are non-parametrically identified, including the relative sample shares of Center- and Home-based Compliers and the outcome distributions for these groups under control. The outcome distributions under treatment, however, are more difficult to identify. To obtain the two LATEs, we leverage results from the literature on finite mixtures and show semi-parametric identification under a broad class of models. Unlike some other identification results in the principal stratification literature (e.g., Jo, 2002; Ding et al., 2011; Mealli and Pacini, 2013), we demonstrate that there are alternatives to parametric identification that do not require conditional independence assumptions. This emphasis on the deep connections between principal stratification and mixture models suggests

[1]These results are corroborated in independent work by Kline and Walters (2014), who find the same general pattern using a structural model. We compare our approaches in Section 7.

many promising avenues for future research. Finally, we estimate our quantities of interest via a Bayesian hierarchical model, which allows us to naturally account for many of the real-world complications in the Head Start Impact Study, including missing data and a multilevel structure, with children nested within Head Start centers. We do so via an implementation of Hamiltonian Monte Carlo called Stan (Stan Development Team, 2014), which builds on recent advances in Bayesian computation.[2] To the best of our knowledge, this is the first implementation of a principal stratification model with site-level random effects.

We organize the paper as follows. Section 2 gives background on Head Start and the principal stratification approach. Section 3 describes the HSIS data. Sections 4 and 5 provide an overview of the analytic framework and give some descriptive information about the principal strata. Section 6 gives an overview of our identification and estimation approaches. Section 7 presents our results. We close with a discussion of the substantive implications for this work for early childhood policy as well as the methodological implications for policy research more broadly. We defer all detailed technical discussions and proofs to the appendix.

2 Background

2.1 Background on Head Start and the Head Start Impact Study

Originally launched in the summer of 1965 as a two-month intervention to help low-income children prepare for kindergarten, Head Start programs across the United States currently provide early childhood education and family support services to more than 900,000 low-income children and their families each year. Head Start services are administered by nearly 1,600 local grantee agencies that receive a total of $8 billion in annual state and federal funds (Administration for Children and Families, 2014). Today, Head Start programs must adhere to a set of performance standards that specify requirements for program services, curricula, teacher preparation and professional development. For example, current Head Start classes serving four or five-year-olds can have no more than 20 children, and those serving three-year olds can have no more than 17 children. Programs must screen all enrolled children for developmental, sensory, and behavioral disabilities and have a written curricula to support each child's cognitive and language development. Head Start programs are also required to engage in collaborative partnership-building with parents through processes that include structured home visits, parenting education classes, and assistance in accessing food, housing, clothing, and transportation.

Researchers and policy makers have debated the effectiveness of Head Start since the program's inception. In their summary of the initial research on Head Start from the 1960s, Zigler and Muenchow (1992) show that children enrolled in these early evaluations of Head Start exhibited large gains on measures of cognitive achievement between their initial enrollment and program completion. Excitement regarding these impressive findings was soon tempered, however, by additional

[2] All our software is open source. In addition to our publically available code, we also provide tutorials for fitting these models in Stan.

research indicating that the effects of Head Start participation were no longer apparent once children reached elementary school (Westinghouse Learning Corporation, 1969). Nevertheless, many of the quasi-experimental studies that followed over the next four decades indicated positive impacts of Head Start on a range of outcomes from short-term academic skill development to long-term outcomes measured in adulthood (e.g., Currie and Thomas, 1993; Garces et al., 2002; Ludwig and Miller, 2007; Deming, 2009; Carneiro and Ginja, 2014).

The mixed results of the randomized Head Start Impact Study did little to settle this debate (National Forum on Early Childhood Policy and Programs, 2010). Nonetheless, the rich HSIS data has led to a host of secondary analyses. Bloom and Weiland (2014) and Walters (2014), for example, examine impact variation across Head Start centers, finding substantial heterogeneity. Bitler et al. (2014) use quantile regression to examine impact variation across the entire outcome distribution, finding substantially larger effects for children with low scores. Bitler et al. (2014) and Bloom and Weiland (2014) also examine heterogeneity across important subgroups, with both studies highlighting significantly larger effects among Dual-Language Learners than among native English speaking students. Finally, other studies, such as Gelber and Isen (2013) and Miller et al. (2014), find that parents play an important role in the effects of Head Start.

2.2 Heterogeneity by Alternative Care Type

The goal of this paper is to explore a specific type of impact heterogeneity: whether or not the impact of Head Start varies by alternative care type. There is substantial evidence in the literature suggesting that this might be the case. First, a recent meta-analysis of 28 studies of Head Start conducted between the program's inception and 2007 found that much of the variation in the findings regarding Head Start's impact on child achievement and cognitive development could be explained by differences in the types of preschool services used by the control group (Shager et al., 2013). Although studies of Head Start programs yielded overall positive effects on short term indicators of children's cognitive skills and achievement (with average effect sizes of +0.27), those studies in which the children in the control group experienced other forms of center-based care yielded significantly smaller effects as compared to those studies of Head Start in which control group children received no additional services (see also Duncan and Magnuson, 2013, for a broader discussion of the counterfactual problem). Zhai et al. (2011) find a similar result using longitudinal data from the Fragile Families and Child Wellbeing Study, concluding that impacts of Head Start were largest relative to non-center-based care.

Second, a few authors have used HSIS data to address this question. Using a matching approach, Zhai et al. (2014) find significant effects of Head Start relative to parent care and relative/nonrelative care but find no meaningful differences in outcomes between Head Start and other center-based care.[3] Using variation across sites, Walters (2014) finds that impacts are smaller for

[3]As in other matching approaches, the critical assumption for this analysis in Zhai et al. (2014) is that, given the observed covariates, alternative care type is as good as random (i.e., principal stratum membership is ignorable). This is a strong assumption in the case of the Head Start Impact Study, since we cannot observe a range of key factors associated with child care decisions, such as the mother's employment status at the time of randomization.

Head Start centers that draw more children from other center-based programs rather than from home-based care. Finally, using a structural model, Kline and Walters (2014) also find that the effects of Head Start are larger relative to home-based care than relative to other center-based care. We discuss the relationship between our results and those of Kline and Walters (2014) in Section 7.

At the same time, some authors have argued against alternative care type as an important source of impact variation. Bitler et al. (2014), for example, find no relationship between observed impacts and the distribution of counterfactual care type across a range of subgroups in HSIS. Barnett (2011) points to the Abecedarian study, initially launched in 1972, which demonstrated large, sustained program impacts, even though roughly two-thirds of control group children attended high-quality center care.

2.3 Principal Stratification

There is a small but growing literature on the use of model-based principal stratification in social science applications. Schochet et al. (2014) provide a recent review. Some previous education examples include Barnard et al. (2003) on the effect a randomized lottery for private school voucher use in New York City with complex noncompliance patterns (see also, Jin and Rubin, 2009); Page (2012) on the relative importance of student exposure to the labor market in career academy high schools; and Schochet (2013) on student mobility in school-based randomized trials. Outside of education, several studies have used principal stratification to analyze the JobCorps evaluation (e.g., Zhang et al., 2009; Frumento et al., 2012) and JOBS II evaluation (Mattei et al., 2013). Finally, a separate series of papers use a selection on observables assumption, rather than model-based inference, to estimate similar quantities of interest. These include Hill et al. (2002), who analyze the Infant Home Development Program, Schochet and Burghardt (2007), who analyze the JobCorps data, Jo and Stuart (2009), who analyze the JOBS II data, and Scott-Clayton and Minaya (2014), who analyze student employment data.

3 Head Start Impact Study

3.1 Overview

Our primary source of data is the HSIS, which was conducted within oversubscribed Head Start centers throughout the U.S. In the HSIS, children randomized to treatment were offered enrollment in a Head Start program for the 2002-2003 school year, while children randomized to control were not offered enrollment. In total, 4,440 children, aged either three or four years old, were randomized to treatment or control across 351 Head Start centers.[4] The randomization itself was complex; treatment probabilities varied by the child's age, the date the child was first put on a Head Start

Moreover, the authors refer to this method as "principal score matching," which has the goal of estimating effects for principal strata like the kind defined in Section 4.3. However, even under the appropriate ignorability assumption, their estimation method does not yield estimates of the relevant principal strata.

[4]We exclude all children from Puerto Rico, because they are not available in the public use data set.

center wait list, and the distribution of eligible children across neighboring Head Start centers.[5] While it is infeasible to recreate the true randomization procedure using currently available data, we can approximately account for the complex structure of the randomization by analyzing the data as if randomization were conducted separately within each center. After excluding children from centers that did not have at least one child in each experimental condition, we obtain a data set with 4,385 children across 340 Head Start centers.[6] We refer to the first year of the study as the Head Start year.

3.2 Outcomes

The HSIS research team collected a wide array of outcomes on children in the sample. A key requirement of our analytic approach, however, is the ability to find a close parametric approximation to the underlying outcome distribution. Therefore, we currently cannot assess several important cognitive outcomes, such as the Woodcock-Johnson III Applied Problems test, and social-emotional outcomes, such as externalizing behavior, since they are poorly suited to typical parametric approximations.

We therefore restrict our analysis to the Peabody Picture Vocabulary Test (PPVT), a standardized measure of children's receptive vocabulary in which the evaluator shows the child a page containing three to four pictures and asks the child to identify the picture that best represents the meaning of a word presented orally by the assessor.[7] First, the PPVT, which is derived from an item response theory score, appears to be well approximated by a Normal distribution, as we show in Appendix B.[8] Second, the PPVT is a widely used assessment and is predictive of key skills later in life (Romano et al., 2010). Based on results from the pre-test, the average child at the beginning of the HSIS performed at roughly the 30th percentile of national PPVT performance, reflecting this group's relative disadvantage in pre-academic skills.

An important complication in the HSIS is the high proportion of missing outcomes. Overall, around 18 percent of PPVT scores are missing in the Head Start year, increasing to around 22 percent two years later. Twenty five percent of PPVT pre-test scores are missing. Furthermore, treatment group children are much more likely to have observed outcomes than control group children: in the Head Start year, 24 percent of control group children have missing PPVT scores, compared to just 13 percent of treatment group children. Around 40 percent of children are missing at least one PPVT score from the pre-test through the second year of follow-up; around 10 percent do not have an observed PPVT score for any of those four tests.

[5]The official HSIS report also uses a complex set of weights to extrapolate the experimental results to a "nationally representative" population of potentially eligible Head Start children (see Gibbs et al., 2011, for a discussion). We do not use those weights here, instead focusing on the results for the experimental sample.

[6]We exclude 55 children from centers of random assignment in which all children in the center were assigned to the same experimental condition.

[7]To reduce the time required to test individual children, the HSIS used a shortened version of the PPVT created using item response theory. See Puma et al. (2010b), section 3-10 for additional details.

[8]Ho and Yu (2014) provide additional discussion of the Normality assumption for IRT scores.

3.3 Covariates

Covariates play a particularly important role in principal stratification models (Feller et al., 2014). Thankfully, the HSIS data set includes a rich set of covariates on child and family characteristics. As part of a broader research effort, we also appended center-level characteristics and neighborhood-level variables for the area around each child's Head Start center of random assignment. Neighborhood-level information includes geocoded data from the 2000 Census, the 2002 Business Census, the Department of Education, and the FBI crime database.

Table 1 shows descriptive statistics for the HSIS covariates we use in our analysis. Overall, HSIS children had diverse background characteristics: around 30 percent identified as Black, 37 percent as Hispanic, 29 percent spoke a non-English language at home, roughly half lived with both biological parents, and one-fifth had a mother who was a recent immigrant. The children generally come from disadvantaged households: over 70 percent have a mother with at most a high school degree or GED, and 84 percent have an assessed family risk that is moderate to high.[9] As would be expected, the children's households are generally situated in disadvantaged neighborhoods. Based on the census data for the Head Start centers, nearly one-quarter of neighborhood households were in poverty, and over one-third of adult residents were high school dropouts. Further, while the national unemployment rate in the US was roughly four percent in 2000, the unemployment rate in these communities was nearly eleven percent, although there is substantial heterogeneity across neighborhoods (McCoy et al., 2014).

For each covariate, Table 1 also shows the normalized differences, a standardized measure of covariate balance across treatment conditions (Imbens and Rubin, 2014; Imai et al., 2008).[10] Overall, this table shows excellent covariate balance between treatment and control groups, with all normalized differences below 0.1 in absolute value.

3.4 Child Care Setting

Table 2 shows the distribution of observed child care settings for children in the HSIS treatment and control groups. Among treatment group children, 77 percent took up the offered slot and enrolled in Head Start in the treatment year.[11] Approximately eight percent of children assigned to treatment enrolled in a non-Head Start center, and nine percent were cared for by a parent or

[9] Family risk in HSIS is based on the sum of five variables: "(1) whether the household received food stamps or TANF in Fall 2002; (2) if neither parent was a high school graduate; (3) if neither parent is working; (4) if the mother was a teen mother; (5) and if the mother is a single mother" (Puma et al., 2010b).

[10] As Imbens and Rubin (2014) note, the normalized difference is a more sensible measure of covariate balance than the more traditional t-test. In particular, the goal is not to assess whether there is sufficient evidence to conclude that the covariate means differ between the two experimental conditions, but rather the degree of adjustment needed to account for any imbalance. As they discuss, the goal is "to assess whether the differences between the two distributions are so large that simple adjustment methods, such as linear covariance (i.e., regression) adjustment, are unlikely to be adequate to remove most biases in estimated treatment/control average differences associated with differences in covariates."

[11] Throughout, we focus on the setting in which the child was cared for in year one of the intervention, even for outcomes collected in subsequent years. We believe that this is the appropriate definition, as the randomization encourages participation in Head Start in the first year only. Nonetheless, other definitions are possible for later years. See Walters (2014).

other relative or enrolled in a home-based childcare program.[12] In principle, children randomized to the control group were free to take up any available early childhood program except for that provided by the Head Start center to which they had applied and had not been offered enrollment. In practice, 13 percent of control group children nevertheless enrolled in a Head Start center (most in the center in which they had lost the lottery), 31 percent enrolled in a non-Head Start center, and 56 percent were cared for by a parent, a relative, or within a home-based childcare program.[13] Note that the HSIS sample consists entirely of families who actively sought to enroll a child in Head Start. Thus, there was at least some initial indication of a preference for Head Start.

4 Analytic Framework

We next outline the technical aspects of our analytic framework. We begin with a general setup for the problem, review the case with binary treatment compliance—that is, Head Start vs. not Head Start—and then extend this setup to the more general multi-valued treatment setting. Additional technical details are deferred to Appendix A.

4.1 Setup

We observe N children, N_1 of whom are randomized to receive the opportunity to enroll in Head Start, with treatment indicator $Z_i = 1$ for child i, and N_0 of whom are not, with $Z_i = 0$. We analyze the HSIS data as a stratified randomized evaluation, with child-level randomization conducted separately within each Head Start center.

We set up the problem using the potential outcomes notation (Neyman, 1990; Rubin, 1974). To do so, we first make the standard Stable Unit Treatment Value Assumption (SUTVA; Rubin, 1980), which states that the treatment assignment of one child does not affect the outcome of another child. Next, we define the relevant potential outcomes. First, let $D_i^{\text{obs}} \in \mathcal{D}$ denote the observed care setting for child i, where $\mathcal{D}$ is the set of possible care settings, and $D_i(z)$ is the care setting for child i if that child had been assigned to treatment condition z.[14] Second, let $Y_i^{\text{obs}} \in \mathbb{R}$ denote the observed outcome of interest (e.g., PPVT), with corresponding potential outcomes, $Y_i(z)$.[15] With this setup, $Y_i^{\text{obs}} = Z_i Y_i(1) + (1 - Z_i)Y_i(0)$ and $D_i^{\text{obs}} = Z_i D_i(1) + (1 - Z_i)D_i(0)$. Finally, we formalize the assumption regarding the validity of randomization (Imbens and Rubin, 2014), which is sensible in the HSIS context:

[12] Standard practice in early childhood education research is to divide care settings into home-based versus center-based care (e.g., Gormley, 2007). We therefore categorize care settings into three main groups: Head Start, non-Head Start center care, and home care. Home care encompasses a variety of home-based settings including being cared for by a parent at home (73 percent), being cared for in a non-relative home-based child carecare setting (11 percent), being cared for by a relative in that relative's home (9 percent), and being cared for by a non-parent in the family's home (6 percent). Although it may be of minor substantive interest to separate out these different home-based settings, it was not feasible given the small sample sizes.

[13] Percentages are among children with observed care setting.

[14] Throughout, we refer to Z as treatment assigned and D as treatment received. An alternative labeling is to refer to Z as the instrument and D as the treatment.

[15] Often researchers double index as $Y_i(z, d)$. However, as z fully defines which outcome we observe, we use a single-indexed potential outcome, i.e., $Y_i(z) = Y_i(z, D_i(z))$.

Assumption R. *(Random assignment.)* Treatment assignment probabilities do not depend on the potential outcomes:

$$Z_i \perp\!\!\!\perp (Y_i(0), Y_i(1), D_i(0), D_i(1)) \, .$$

Finally, we define the Intention-to-Treat (ITT) estimand as $ITT = \mathbb{E}[Y_i(1) - Y_i(0)]$.[16]

4.2 IV: $D_i^* \in \{\textbf{Head Start}, \textbf{Not Head Start}\}$

We briefly walk through the assumptions necessary to identify the Local Average Treatment Effect. Let D_i^* be a binary indicator for whether or not child i participated in Head Start in the first year. Following Angrist et al. (1996), define child i's compliance type, S_i^*, via the joint values $(D_i^*(0), D_i^*(1))$, as shown in Table 3a. For continuity with the next section, we refer to these compliance types by the more general term, principal strata, taking values $S_i^* \in$ {Always Head Start, Never Head Start, Complier, Defier}. As usual, we define the $LATE$ as the impact of randomization on the Compliers, $LATE = \mathbb{E}\{Y_i(1) - Y_i(0) \mid S_i^* = \text{Complier}\}$. To identify this quantity, we then make two standard assumptions: (1) the "no defiers" assumption; and (2) the exclusion restrictions for Always Head Start and Never Head Start children.

Assumption IV-1. *(IV Monotonicity/No Defiers.)*

$$\mathbb{P}\{D_i^*(0) = 1, D_i^*(1) = 0\} \equiv \mathbb{P}\{S_i^* = \text{Defier}\} = 0$$

Assumption IV-2. *(IV Exclusion Restrictions.)* For $S_i^* \in \{\text{Always Head Start}, \text{Never Head Start}\}$, $Y_i(0) = Y_i(1)$.

The monotonicity assumption and the exclusion restriction for Never Head Start children are both innocuous in this application. However, as Gibbs et al. (2011) note, the exclusion restriction for Always Head Start children is more questionable. In particular, roughly half of Always Head Start children enroll in Head Start centers other than the center of random assignment. If these alternative centers systematically differ from centers of random assignment, then the exclusion restriction might not hold for this group. While we maintain the exclusion restriction in the end, we nonetheless note that there are a range of available model-based approaches for relaxing this assumption (e.g., Imbens and Rubin, 1997a; Hirano et al., 2000).

4.3 Principal Stratification: $D_i \in \{\textbf{Head Start}, \textbf{Other Center}, \textbf{Home}\}$

The IV approach allows us to estimate the impact of Head Start among Compliers. However, we wish to estimate differential impacts for children within this group. Our inferential goal is

[16] With some abuse of notation, we use the expectation and probability operators to denote the finite sample average for the finite sample of children actually randomized in the HSIS experiment. See Imbens and Rubin (2014) for further discussion.

therefore to divide the overall LATE into one LATE for those who would otherwise receive care in another non-Head Start center and a second LATE for those who would otherwise receive care in a home-based setting. To do so, we disaggregate the binary indicator, D_i^*, to three levels: $D_i \in \{\text{Head Start}, \text{Other Center}, \text{Home}\}$, where $D_i^* = 1$ if $D_i = \text{Head Start}$ and $D_i^* = 0$ otherwise. We also disaggregate the set of three standard compliance types, $\mathcal{S}^*$, into a more complete set of principal strata, $\mathcal{S}$.

Table 3b shows the nine possible combinations of care types under both treatment and control. Column headings correspond to the type of care each child would experience if assigned to the control condition; row headings correspond to the type of care each would experience if assigned to the treatment condition. Strata on the diagonal are the "always" strata, where the Always Center-based and Always Home-based strata make up the Never Head Start stratum in Table 3a. Finally, the Compliers are divided into Center Compliers and Home Compliers.

As in the standard IV case, we make two key types of assumptions: monotonicity assumptions and exclusion restrictions. The standard monotonicity assumption from the IV setting becomes a statement about four strata rather than just one. We break this statement into two parts.

Assumption PS-1a. *(PS Monotonicity/No Defiers.)*

$$\mathbb{P}\{D_i(0) = \text{HS}, D_i(1) = \text{Center}\} = 0$$
$$\mathbb{P}\{D_i(0) = \text{HS}, D_i(1) = \text{Home}\} = 0$$

Assumption PS-1a states that there are no children who would take up Head Start under assignment to control but not under assignment to treatment. Therefore, strata A and C in Table 3b do not exist. This is a natural extension of Assumption IV-1 to multi-valued D. And, as with Assumption IV-1, this seems to be a reasonable assumption in HSIS.

Assumption PS-1b. *(Irrelevant Alternatives.)*

$$\mathbb{P}\{D_i(0) = \text{Center}, D_i(1) = \text{Home}\} = 0$$
$$\mathbb{P}\{D_i(0) = \text{Home}, D_i(1) = \text{Center}\} = 0$$

Assumption PS-1b states that the Head Start offer does not change the care setting for families choosing between non-Head Start options. Therefore, strata B and D in Table 3b do not exist. Walters (2014) motivates this assumption with a revealed preference argument: since the availability of non-Head Start preschool is not affected by a Head Start offer, preferences among non-Head Start care options should not be affected either. While this is an unverifiable assumption, it is likely that, if such families do exist, they make up only a very small fraction of the overall population.

This yields five possible principal strata: Always Head Start (ahs), Always Center (ac), Always Home (ah), Center Complier (cc), and Home Complier (hc). As in the IV case, we can naturally make exclusion restrictions for principal strata unaffected by randomization. In particular we assume zero treatment effect for the Always Head Start, Always Center, and Always Home strata.

Assumption PS-2. *(PS Exclusion Restrictions.)* For $S_i \in$ {Always Head Start, Always Center, Always Home}, $Y_i(0) = Y_i(1)$.

As with the IV exclusion restriction for Never Head Start children, we believe that the exclusion restrictions for Always Center and Always Home children are innocuous. The exclusion restriction for Always Head Start children is identical to the IV case. The remaining strata are Center Compliers and Home Compliers. Our goal is to estimate the impacts of randomization for these groups, which are the effects of receiving Head Start versus receiving other center-based care and home-based care, respectively:

$$LATE_{\text{cc}} = \mathbb{E}\{Y_i(1) - Y_i(0) \mid S_i = \text{Center Complier}\}$$
$$LATE_{\text{hc}} = \mathbb{E}\{Y_i(1) - Y_i(0) \mid S_i = \text{Home Complier}\}.$$

As with the overall LATE, these are local effects since they are only defined for specific subgroups. In other words, we cannot interpret the difference between $LATE_{\text{cc}}$ and $LATE_{\text{hc}}$ as the causal effect of other center-based care versus home care—these two subgroups are not the same children. They differ across a range of unobserved and observed characteristics, such as child pre-test scores and family characteristics.

Finally, the overall LATE is a weighted average of these two estimands:

$$LATE = \frac{\pi_{\text{cc}}}{\pi_{\text{cc}} + \pi_{\text{hc}}} LATE_{\text{cc}} + \frac{\pi_{\text{hc}}}{\pi_{\text{cc}} + \pi_{\text{hc}}} LATE_{\text{hc}}$$

where π_s denotes the proportion of children in stratum s.

5 Describing Principal Strata

In most subgroup analyses, the groups themselves are known and fixed. For example, we can easily estimate the differential impact of Head Start for boys and girls: after collecting baseline data, each child's gender is known. While principal strata are well-defined subgroups, just like three and four year olds, we cannot directly observe subgroup membership for all children.

Fortunately, we can extend some results from the IV case to provide useful descriptions of the principal strata themselves. In particular, we non-parametrically identify the overall distribution of principal strata as well as the distribution of covariates within each stratum. Unsurprisingly, we find that these principal strata indeed differ across observed characteristics and that this variation is consistent with intuition and results in the early childhood literature.

The Appendix gives further details for the results we present below along with proofs of all the lemmas.

5.1 Overall Distribution of Principal Strata

Extending the standard results from the IV case (Angrist et al., 1996), we can estimate the overall size of each principal stratum.

Lemma 1 (Distribution of Principal Strata)**.** *Under Assumptions R, PS-1a, and PS-1b, the distribution of principal strata, $\pi_s \equiv \mathbb{P}\{S_i = s\}$, is non-parametrically identified for all s.*

For intution on Lemma 1, it is useful to see the analogue in the IV setting: we first estimate the proportion of Always Head Start children in the control group and Never Head Start children in the treatment group, and then subtract to estimate the proportion of Compliers. Table 4 shows point estimates for the distribution of principal strata in the sample. Roughly one-third of all children are non-compliers of various types; each non-complier stratum is around 10 percent of the overall sample. The remaining two-thirds are split between the two Complier groups; Home Compliers total around 70 percent of all Compliers.

5.2 Using Covariates to Predict Stratum Membership

Since HSIS is a randomized experiment, we can examine the distribution of principal strata for specific subgroups, such as for all boys in the sample. Following Hill et al. (2002), we define the *principal score* as $\pi_{s|\mathbf{x}} \equiv \mathbb{P}(S_i = s \mid \mathbf{X}_i = \mathbf{x})$, the probability that a child belongs to principal stratum s given that child's observed covariates (see also Abadie, 2003; Jo and Stuart, 2009). Note that this is a simple generalization of modeling the "first stage" in the standard IV setting as a function of the covariates (e.g., Angrist, 2004).[17]

For HSIS, we estimate the principal score using multinomial logistic regression and a simple data augmentation procedure.[18] Figure 1 shows the resulting logistic regression coefficients for select covariates that are predictive of being a Center-based vs. Home-based Complier. We discuss these results below.

5.3 Distribution of Covariates by Principal Stratum

We can also estimate the distribution of covariates for each principal stratum.

Lemma 2 (Distribution of Covariates by Principal Stratum)**.** *Under Assumptions R, PS-1a, and PS-1b, $\mathbb{P}\{\mathbf{X}_i = \mathbf{x} \mid S_i = s\}$ is non-parametrically identified for all s.*

This lemma is a simple extension of the comparable IV result in Abadie (2003), and allows us to make concrete observations about otherwise unobservable groups (see also Angrist and Pischke,

[17] Unlike the usual first stage model, $\mathbb{P}\{D^{*,\text{obs}} \mid X_i = \mathbf{x}\}$, the principal score is vector-valued, since S_i is discrete rather than binary.

[18] This approach improves on simpler versions of this model fit by Walters (2014) and Zhai et al. (2014). Walters (2014) effectively estimates the share of Center-based Compliers and Home-based Compliers for each Head Start center, doing so via two separate logistic regressions, rather than via multinomial logistic regression. Zhai et al. (2014) estimate a multinomial logistic regression using covariates to predict $D(0)$ rather than stratum membership, therefore conflating Always Center-based children and Center Compliers under control and conflating Always Home-based children and Home Compliers under control.

2008; Frumento et al., 2012). Table 6 shows the means for select covariates for each stratum; Figure 2 separately shows the means by pre-test score. There are key differences in observable characteristics across the latent groups. Columns 1–3 on Table 6 show variation in pre-treatment covariates across the different types of non-compliers. Overall, these results suggest that children who always enroll in a non-Head Start center-based setting outperform their counterparts who would always be in Head Start or in a home-based setting. For example, as shown in Figure 2, Always Center-based children strongly outperform Always Head Start and Always Home-based children on the PPVT pre-test. Further, consistent with prior research (Hirshberg et al., 2005), Always Center-based children generally have mothers with higher educational attainment.[19] Other covariates also sensibly predict differences among the non-complier types. For example, Always Center children are much more likely to live in a state that has state-funded preschool than Always Home children. In general, this ordering is consistent with the selection results from Deming (2009), who finds that families of children in non-Head Start preschools have higher income and maternal education than families of children in Head Start or in no preschool.

We can compare our two complier groups by examining columns 4 and 5 of Table 6, which are the complement to the logistic regression coefficients in Figure 1. Consistent with research that has found that parents typically prefer center-based care for four-year olds (e.g., Huston et al., 2002; Rose and Elicker, 2010), roughly 60 percent of Home Compliers are three years old, compared to only 45 percent of Center Compliers. We also find that Home Compliers enter the study with lower pre-academic skills. Home Compliers exhibit lower PPVT performance at the beginning of the study and are more likely to be in the bottom third of PPVT performance compared to Center Compliers. Home Compliers are additionally more likely to have a mother with less than a high school education. As above, Center Compliers are more likely to live in states that, during the time of the HSIS, provided state-funded pre-kindergarten. Note that we do not find meaningful differences between these two groups based on race or ethnicity or based on Dual Language Learner status.

Overall, these differences in covariate means by principal stratum underscore that children in different principal strata do, indeed, differ in terms of their baseline characteristics. Therefore, while estimates of causal effects *within* each principal stratum are valid, comparisons *between* principal strata are descriptive rather than causal, in the same way that comparing treatment effects for males and females is descriptive rather than causal. In other words, differential impacts across strata could also be due to differences in observed or unobserved characteristics other than care type.[20]

[19] Note that we do not have information on maternal employment at the time of randomization.

[20] See Gallop et al. (2009) for a discussion of using principal stratification for mediation analysis, which generally requires much stronger assumptions than those presented here.

6 Overview of Identification and Estimation

This section provides an overview of the identification and estimation strategies used in this paper. Interested readers can find greater detail in the Appendix, which gives an in-depth discussion of possible identification approaches, our hierarchical Bayesian estimation procedure, robustness to different parameteric assumptions, and other technical information. Conversely, readers can skip to Section 7 for a discussion of the results.

6.1 Overview of Identification Strategy

The identification strategy rests on the idea that we can identify the outcome distributions for each principal stratum. This builds on earlier work in the IV case from Imbens and Rubin (1997b) and Abadie (2003). We provide a brief sketch of the idea here. The Appendix provides additional discussion of identification in principal stratification models (see also Zhang et al., 2009).

To illustrate the identification approach, first consider a standard subgroup analysis, for example, estimating the impact of Head Start for the subgroup of boys. Formally, we can achieve this in two distinct steps. The first step is to identify the distribution of outcomes for boys in the treatment group, which we denote $g_{\text{boys}\,1}(y)$, and the corresponding distribution of outcomes for boys in the control group, which we denote $g_{\text{boys}\,0}(y)$. Since HSIS is a randomized experiment, and since we directly observe which children are boys, we can non-parametrically identify both $g_{\text{boys}\,0}(y)$ and $g_{\text{boys}\,1}(y)$ from the corresponding sample (e.g., via kernel density estimation). We can then obtain the average impact of Head Start on boys by comparing the means of the two distributions. While not necessarily practical, this is nonetheless a valid procedure for identifying an average treatment effect for a subgroup.

6.1.1 Instrumental Variables

While we directly observe gender, we do not directly observe compliance type for all children. We therefore must adopt a different approach for estimating the outcome distributions by compliance type. For illustration, we again begin with the standard IV set up for non-compliance, where we compare Head Start versus not Head Start:

- **Always Head Start and Never Head Start.** Under monotonicity, we know that any children in the control group who enroll in Head Start must be Always Head Start children. As a result, we can directly estimate the outcome distribution for the Always Head Start subgroup under control, $g_{\text{ahs}\,0}(y)$. Since we assume that there is no treatment effect for this group (i.e., that the exclusion restriction holds for Always Head Start children), then $g_{\text{ahs}\,1}(y) = g_{\text{ahs}\,0}(y) = g_{\text{ahs}}(y)$. We can repeat this approach for Never Head Start children in the treatment group, which yields $g_{\text{nhs}\,1}(y) = g_{\text{nhs}\,0}(y) = g_{\text{nhs}}(y)$.

- **Compliers.** We must take a different approach for the Compliers. First, we cannot directly observe which children are Compliers. Second, since we are interested in the LATE, we can no longer assume that Compliers have the same outcome distribution under treatment and control. The key insight is to focus on the relationship between the observed treatment and the unobserved compliance type; Table 5a shows these relationships for the IV case. For example, children in the control group who do not enroll in Head Start are either Compliers or Never Head Start children. In other words, the observed outcome distribution for these children is a mixture of $g_{\text{nhs}}(y)$ and $g_{\text{co}\,0}(y)$. Formally:

$$f_{00}(y) = \frac{\pi_{\text{nhs}}}{\pi_{\text{nhs}} + \pi_{\text{co}}} g_{\text{nhs}}(y) + \frac{\pi_{\text{co}}}{\pi_{\text{nhs}} + \pi_{\text{co}}} g_{\text{co}\,0}(y), \tag{1}$$

where $f_{zd}(y)$ is the observed outcome distribution for children with treatment assignment $Z_i = z$ and treatment received $D_i^* = d$. For example, $f_{00}(y)$ is the observed outcome distribution for children assigned to the control condition who do not experience Head Start. Since we can directly observe $f_{00}(y)$, π_{nhs}, π_{co}, and $g_{\text{nhs}}(y)$, we can re-arrange terms to identify $g_{\text{co}\,0}(y)$, the outcome distribution for Complier children in the control group. We can repeat this with the mixture of Always Head Start and Compliers under treatment to obtain $g_{\text{co}\,1}(y)$. Therefore, we can non-parametrically identify both $g_{\text{co}\,0}(y)$ and $g_{\text{co}\,1}(y)$, even though we cannot observe these distributions directly. See Imbens and Rubin (1997b) for additional discussion.

Once we have all the outcome distributions, we can immediately obtain the average outcomes by principal stratum, μ_{sz}, and finally obtain $LATE = \mu_{\text{co}\,1} - \mu_{\text{co}\,0}$. Also see Kling et al. (2007) for an example in which the Complier means are substantively meaningful in their own right. More generally, Abadie (2003) shows that we can use this approach to identify a broad range of features by compliance type, including covariate distributions.

6.1.2 Principal Stratification

We now extend the argument from the IV case to identify the outcome distributions for our principal strata of interest. We again have observed mixtures, as shown in Table 5b.

- **Always Head Start, Always Center-based, and Always Home-based.** Just as with the Always Head Start and Never Head Start groups, we directly observe the outcome distributions for the Always Head Start, Always Center-based, and Always Home-based strata. For example, we directly observe the Always Home-based children under treatment and can therefore non-parametrically identify $g_{\text{ah}\,1}(y)$. Since we assume that there is no impact of randomization on this group, $g_{\text{ah}\,1}(y) = g_{\text{ah}\,0}(y) = g_{\text{ah}}(y)$. We repeat this for the Always Head Start and Always Center-based strata, yielding non-parametric identification for $g_{\text{ahs}}(y)$, $g_{\text{ac}}(y)$, and $g_{\text{ah}}(y)$.

- **Center-based Compliers (control) and Home-based Compliers (control).** As in the IV case, we cannot directly observe the outcome distributions for Center-based Compliers and Home-based Compliers and must instead identify these distributions indirectly. We begin with the outcome distribution for Home-based Compliers under control, $g_{\text{hc}\,0}(y)$. Analogous to Equation (1), the outcome distribution for control group children in home-based care is a mixture of $g_{\text{ah}}(y)$ and $g_{\text{hc}\,0}(y)$:

$$f_{0\,\text{Home}}(y) = \frac{\pi_{\text{ah}}}{\pi_{\text{ah}} + \pi_{\text{hc}}}\; g_{\text{ah}}(y) + \frac{\pi_{\text{hc}}}{\pi_{\text{ah}} + \pi_{\text{hc}}}\; g_{\text{hc}\,0}(y). \tag{2}$$

where we have previously identified $g_{\text{ah}}(y)$. Similarly, we re-arrange terms to non-parametrically identify $g_{\text{hc}\,0}(y)$ and repeat this procedure for the Center-based Compliers under control, $g_{\text{cc}\,0}(y)$.

- **Center-based Compliers (treated) and Home-based Compliers (treated).** Identifying the corresponding Complier distributions under treatment requires additional effort and some additional assumptions on the outcome distributions. As in the IV case, we can reduce the problem to estimating a mixture of two types:

$$f^*_{1\,\text{HS}}(y) = \frac{\pi_{\text{cc}}}{\pi_{\text{cc}} + \pi_{\text{hc}}}\; g_{\text{cc}\,1}(y) + \frac{\pi_{\text{hc}}}{\pi_{\text{cc}} + \pi_{\text{hc}}}\; g_{\text{hc}\,1}(y). \tag{3}$$

where $f^*_{1\,\text{HS}}(y)$ is the observed outcome distribution after "backing out" the Always Head Start outcome distribution. Unlike in the previous case, however, neither mixture component is known, leading to a two-component finite mixture model.

The key idea in mixture modeling is that assumptions on the shape of the component densities, $g_{\text{cc}\,1}(y)$ and $g_{\text{hc}\,1}(y)$, can yield point identification for component-specific parameters, such as component means. In other words, we leverage the higher-order moments of the observed distribution, such as skewness and kurtosis, to separate out the mixture components. The earliest result of this kind is due to Pearson (1894), who showed that it is sufficient to assume that the components are Normal. See Frühwirth-Schnatter (2006) for a more modern overview. In Appendix A, we follow a more recent literature on semi-parametric identification (e.g., Bordes et al., 2006; Hunter et al., 2007) and show that symmetry of the component densities is one potentially powerful identifying assumption in this setting.

Once we have identified the outcome distributions for Center-based and Home-based Compliers, we can immediately obtain the relevant outcome means, μ_{sz}, for stratum $S_i = s$ and treatment $Z_i = z$. This yields $LATE_{\text{cc}} = \mu_{\text{cc}\,1} - \mu_{\text{cc}\,0}$ and $LATE_{\text{hc}} = \mu_{\text{hc}\,1} - \mu_{\text{hc}\,0}$.

6.2 Overview of Estimation Approach

Following a large literature (e.g., Imbens and Rubin, 1997a; Little and Yau, 1998; Hirano et al., 2000), we estimate our quantities of interest via a full Bayesian model. We use this approach, rather than utilizing non- or semi-parametric estimators, for two main reasons.

First, as Imbens and Rubin (1997b) discuss, parsimoniously parameterized models can often lead to better practical performance (in the sense of lower root-mean-squared-error) than corresponding nonparametric approaches.[21] Here, the Normal distribution seems to be an excellent approximation to the PPVT, suggesting that gains from a parsimonious parameterization would be substantial. Moreover, non- and semi-parametric estimators for finite mixture models can have extremely poor finite sample characteristics (Feller et al., 2014).

Second, we face a range of real-world complications in the HSIS example: missing data and study attrition; stratified randomization across many, small Head Start centers; and a mix of child- and center-level covariates. Addressing these issues is natural in a full Bayesian model but would be quite difficult with a non- or semi-parametric approach.

To develop intuition for the full Bayesian model, we briefly describe a data augmentation procedure for estimating the parameters of interest. The key idea is to alternate between (1) estimate the vector of model parameters, θ, given stratum membership, S, and (2) imputing each child's principal stratum membership, S, given θ. In practice, we begin with an initial guess of principal stratum membership for each child and then alternate between these steps thousands of times to generate posterior distributions for all quantities of interest.

- **Step 1: Given stratum membership, estimate model parameters.** We estimate model parameters via two sub-models.
  - *Step 1A: Outcome sub-model, $g_{sz|\mathbf{x}}(y)$.* First, we estimate the following regression with center-level random effects:

    $$Y_i^{obs} = \alpha_s + \beta_s' X_i + \tau_{\text{cc}} \cdot z_i \cdot I_i^{\text{cc}} + \tau_{\text{hc}} \cdot z_i \cdot I_i^{\text{hc}} + \varepsilon_i \tag{4}$$

    where α_s is the intercept for stratum s; β_s is a vector of baseline covariate coefficients, which vary across strata; I_i^{cc} and I_i^{hc} are indicators for Center-based and Home-based Compliers, respectively; and τ_{cc} and τ_{hc} are the average causal effects of assignment to Head Start in these two strata. Due to the exclusion restrictions, the other τ terms are assumed to be zero. We use these regression results to derive $g_{sz|\mathbf{x}}(y)$ for each stratum.
  - *Step 1B: Principal score sub-model, $\pi_{s|\mathbf{x}}$.* Second, we estimate a multinomial logistic regression predicting principal stratum membership as a function of observed covariates.

- **Step 2: Given model parameters, predict stratum membership.** Given the outcome model, $g_{sz|\mathbf{x}}(y)$, and principal score model, $\pi_{s|\mathbf{x}}$, we can estimate the probability of stratum membership via Bayes' Rule. For example, if we observe a child in the control group who is in home-based care, the child's probability of being a Home Complier is:

  $$\mathbb{P}\{S_i = \text{hc} \mid \text{data}, \theta\} = \frac{\pi_{\text{hc}|\mathbf{x}} \cdot g_{\text{hc}\,0|\mathbf{x}}(y)}{\pi_{\text{hc}|\mathbf{x}} \cdot g_{\text{hc}\,0|\mathbf{x}}(y) + \pi_{\text{ah}|\mathbf{x}} \cdot g_{\text{ah}|\mathbf{x}}(y)}.$$

[21] In the context of the IV model, the authors argue that "the role of the parametric model is solely to provide a good fit to the [...] underlying outcome distributions. The estimand of interest [...] is well defined irrespective of the specific parametric model used" (Imbens and Rubin, 1997b, p. 568). In our case, the parametric assumption plays a slightly more important role; nonetheless, we believe that the advantages of the Bayesian approach remain.

We then flip a weighted coin to predict S_i for that child. By contrast, if we observe a child in the treatment group who is in home-based care, the child must be in the Always Home-based stratum. So $\mathbb{P}\{S_i = \text{ah} \mid \text{data}, \theta\} = 1$.

These steps highlight the important role that covariates play in estimation. Even if they are not formally required for identification, covariates reduce residual variation and are predictive of principal stratum membership, $\pi_{s|\mathbf{x}}$. As an extreme case, as the predicted probabilities move closer to 0 and 1, the overall estimation procedure moves closer to standard subgroup analysis. Such covariates can be especially important for improving model performance with small samples and weak instruments (Feller et al., 2014).

Computationally, we estimate this model via Stan, a Bayesian programming language that implements a variant of Hamiltonian Monte Carlo (HMC; Stan Development Team, 2014; Hoffman and Gelman, 2014).[22] HMC differs from off-the-shelf Metropolis-Hastings algorithms in that the proposal distribution is based on the idea that parameters are fictional particles, with potential energy determined by the negative log probability function. These fictional particles then move around the parameter space the same way that real particles would move around physical space. Note that, while we utilize a Bayesian implementation, we could also implement this principal stratification model via EM (see, for example, Frumento et al., 2012).

7 Results

We now summarize results for the Intent-to-Treat, Instrumental Variable, and Principal Stratification models, beginning with impacts in the Head Start year. We then briefly explore impacts after the first year as well as additional impact heterogeneity, including distributional treatment effects. Appendix B includes extensive sensitivity and robustness checks, such as Bayesian model checking via posterior predictive p-values, assessments of the Normality assumption, and estimation results under complete case and under a t distribution.

7.1 Impacts in the Head Start Year

The first row of Table 7 shows the ITT estimate, the impact of opportunity to enroll in Head Start, on PPVT in effect size units (i.e., SD of the control group). Consistent with the original Head Start results (Puma et al., 2010a), we find that the overall impact of randomization to treatment is +0.14 in the Head Start year (posterior median). There is strong evidence that this impact is greater than zero.[23]

[22]Note that Stan directly evaluates the observed data posterior and does not rely on data augmentation. Nonetheless, describing the data augmentation procedures helps to build intuition for the estimation procedure.

[23]In general, the results we present here give much stronger statistical evidence that the impacts are positive than the evidence presented in Puma et al. (2010a). Multiple factors contribute to these differences. First, unlike Puma et al. (2010a), we pool the three- and four-year-old cohorts, which roughly doubles the sample size. Second, unlike Puma et al. (2010a), we control for Head Start center of random assignment in the outcome model, which improves precision. Finally, we do not use the HSIS weights, which greatly inflate the resulting standard errors. See Bloom and Weiland (2014) for additional discussion.

The second row of Table 7 shows the corresponding LATE estimate from the IV model. Among Compliers, the impact of enrolling in Head Start on PPVT is +0.18.[24] As with the ITT, there is strong evidence that this impact is positive.[25] This effect is comparable to the average effects of early childhood education programs reported in a recent meta-analysis (effect size of +0.21; Duncan and Magnuson, 2013) and represents approximately one-quarter of the Black-White test score gap at the end of kindergarten (Fryer and Levitt, 2004).

The last three rows of Table 7 show the principal stratification results from the full model. For Home Compliers, we find a treatment effect of +0.23 on PPVT, with strong evidence that these impacts are greater than zero. This is much larger than the corresponding ITT effect. For Center Compliers, however, we find an effect of zero. Because we jointly estimate $LATE_{\text{hc}}$ and $LATE_{\text{cc}}$, we can calculate that $\mathbb{P}\{LATE_{\text{hc}} > LATE_{\text{cc}}\} = 0.99$. As we discussed above, this is a descriptive comparison—like claiming that the treatment effect is larger for boys than girls—but it nonetheless shows that impacts for these two latent groups are meaningfully different.[26]

It is useful to compare our results to those of Kline and Walters (2014), who use a structural model to estimate a range of different treatment effects for the HSIS data, including $LATE_{\text{hc}}$ and $LATE_{\text{cc}}$. In particular, Kline and Walters (2014) focus on a parametric choice model, combining a multinomial Probit with some additional restrictions on the outcome model. In other words, the authors impose Normality on a latent distribution while we impose Normality on the observed outcome distribution itself.[27] Reassuringly, Kline and Walters (2014) also find the same overall pattern of effects, with positive and significant impacts for Home Compliers and negligible impacts for Center Compliers. While their point estimate for $LATE_{\text{hc}}$ is somewhat larger than ours (0.35 vs. 0.23), it appears as though this discrepancy is largely due to a different choice of outcome; Kline and Walters (2014) estimate impacts on an index of outcomes while we focus on PPVT alone.[28] As the analytic strategy in Kline and Walters (2014) differs starkly from our own, it is rather remarkable that the results are so close.

7.2 Impacts after the Head Start Year

A key feature of the HSIS design is that children in the three-year-old cohort control group were given access to the Head Start program in the second year of the study. In practice, nearly half of the control group took up the opportunity to enroll, with another 34 percent enrolling in other,

[24]This estimate is nearly identical to that of Bloom and Weiland (2014), who conduct a similar analysis.

[25]Note that this estimate differs from the usual Wald estimator for IV, $\frac{ITT}{\pi_c} = \frac{0.14}{0.7} = 0.20$. This is primarily due to the multi-site randomization and differences in compliance rates across Head Start centers. See Raudenbush et al. (2012) and Reardon and Raudenbush (2013) for further discussion of this issue.

[26]Note that we separately estimate the IV and Principal Stratification models and do not constrain the weighted average of $LATE_{\text{cc}}$ and $LATE_{\text{hc}}$ to be equal to the overall $LATE$. Nonetheless, the implied $LATE$ from the principal stratification model is +0.16, which is quite close to the IV model estimate of +0.18.

[27]In our case, we can empirically assess the Normality assumption in five of the seven outcome distributions of interest. See Mealli and Pacini (2008) for a more detailed comparison of principal stratification and selection models.

[28]We can assess the influence of the outcome choice with a simple back-of-the-envelope calculation. Their estimate of the overall $LATE$, which is non-parametrically identified, is roughly 40 percent larger than ours (0.25 vs. 0.18); their estimate of $LATE_{\text{hc}}$ is roughly 50 percent larger than ours (0.35 vs. 0.23).

non-Head Start center care during that year. Enrollment was similarly high for treatment group children: 64 percent enrolled in Head Start, with another 24 percent enrolling in other center care.[29] Therefore, by the second year of HSIS, the randomization only increased the probability of enrolling in Head Start by 16 percentage points and only increased the probability of enrolling in any center-based care setting by 6 percentage points.

As a result, simply pooling cohorts after the Head Start year does not necessarily estimate a substantively meaningful quantity. Moreover, re-defining the principal strata based on care setting in the second year is challenging, both because so few children were in home-based care in that period and because the monotonicity and exclusion restriction assumptions of Section 4.3 are less likely to hold for the second year. Following Puma et al. (2010a), we therefore analyze the results separately by cohort to assess impacts after the Head Start year.

Unfortunately, further dividing Center and Home Compliers into separate three- and four-year-old subgroups leads to a range of estimation challenges. In particular, Feller et al. (2014) show that finite mixture models can yield unstable estimates in small samples or in settings with weakly predictive covariates. Moreover, even if stratum membership were known, sample sizes would still be relatively small. Finally, outcome missingness increases substantially over the course of the study, with roughly a quarter of all outcomes missing by the third year. Therefore, the cohort and subgroup results presented below should all be considered exploratory.

With this caveat in mind, Figure 3 shows the treatment effect on PPVT by cohort by assessment year for all Compliers, for Center Compliers, and Home Compliers.[30] Consistent with the official HSIS results, we find a decline in the treatment effect as children age. Nonetheless, unlike in the official HSIS results, we find impacts that are positive and meaningfully different from zero by the time children are in 1st grade, with $LATE$ estimates of 0.09 and 0.14 for the three- and four-year-old cohorts, respectively. The effects for Home Compliers follow the same decline as for the overall Compliers, albeit with slightly larger point estimates and with less precision. By contrast, the impacts among Center Compliers are best described as noise around zero, though this null result could be due to the limited sample size.

While we regard these results as exploratory, they nonetheless suggest that the impact of Head Start might indeed persist into early elementary school, even if the magnitudes are modest. In particular, Gibbs et al. (2011) argue that a key puzzle of the HSIS results is not that they decrease over time, but that they attenuate to zero as soon as children leave the program, much more rapidly than estimates based on quasi-experimental methods (e.g., Currie and Thomas, 1993; Deming, 2009). The results in Figure 3 show that the decline in treatment effects is not nearly as rapid as in the reported topline results.

[29]For the three year old cohort, 22 percent of control group children and 14 percent of treatment group children do not have an observed care setting in the second year of the study. Reported percentages are among children with observed care type.

[30]These are the normative grades for a given cohort. Children who began the study as three-year-olds were able to gain access to Head Start in year 2 and then enrolled in kindergarten in year 3. The four-year-olds transitioned to kindergarten and then first grade in the second and third years of the study. Therefore, by year 3, all children, if following a standard educational trajectory, were in elementary school.

7.3 Subgroup and Quantile Treatment Effects

Several recent papers have explored variation in Head Start's impact across observed subgroups and across quantiles of the outcome distribution. Since Center and Home Compliers differ across a range of observed and unobserved characteristics, an important question is therefore the extent to which these differences explain the different impacts for the two Complier groups. Again, these estimates should be considered exploratory.

First, we turn to variation across subgroups defined by pre-treatment characteristics. Following Bloom and Weiland (2014) and Bitler et al. (2014), we focus on variation by (1) whether a child is in the bottom third of pre-test score by cohort; and (2) whether a child is a Dual-Language Learner (DLL). Table 8 shows the corresponding principal stratification estimates during the Head Start year. First, across all four subgroups, we observe the same pattern of positive, significant effects for Home Compliers and negligible effects for Center Compliers. While the smaller sample sizes limit statistical power, this consistency nonetheless bolsters the overall findings. Second, as in Bloom and Weiland (2014), we find larger Home Complier effects for children in the bottom third by pre-test score and for DLL students. The effect for DLL students is especially striking, with an effect size of around +0.35 SD in the Head Start year, more than double the point estimate for non-DLL students. This suggests that, at least in terms of vocabulary development, there is substantial impact of Head Start relative to a home-based setting in which English is not spoken. See Bloom and Weiland (2014) for additional discussion.

Another likely source of impact variation is heterogeneity across the outcome distribution (Bitler et al., 2003). In a recent paper, Bitler et al. (2014) estimate distributional effects for Head Start via quantile treatment effects, $G^{-1}_{\text{co}\,1}(q) - G^{-1}_{\text{co}\,0}(q)$, the difference between the qth quantiles of the outcome distributions for Compliers under treatment and control, respectively. The authors find that the impacts of Head Start on PPVT and other measures are largest at the bottom of the outcome distribution, both overall and among Compliers. As we discuss in Appendix B, we can leverage our framework both to replicate and to extend their results. Figure 4 shows the quantile treatment effect estimates for all Compliers, Center Compliers, and Home Compliers during the Head Start year. As expected, our estimates for all Compliers are very close to those of Bitler et al. (2014), showing large, positive effects at the bottom of the distribution of between +0.4 and +0.6 SD. The effects for Home Compliers are also positive and significant throughout, with larger effects at the bottom of the distribution. By contrast, the quantile treatment effects for Center Compliers are essentially zero across the entire distribution.

8 Discussion

Our primary contribution is to develop a framework for estimating impact variation by alternative care setting and to apply this framework to the Head Start Impact Study. In particular, we find positive and meaningful impacts on key outcomes among Home-based Compliers, those children who would enroll in Head Start under treatment and who would otherwise be in home-based care.

By contrast, we find no meaningful effects among Center-based Compliers, those children who would otherwise receive non-Head Start center care.

In doing so, we present a much more nuanced view of Head Start's impact than the topline experimental results indicate. We also refute sweeping generalizations made about Head Start, such as "Head Start does not improve the school readiness of children from low-income families" (Whitehurst, 2013a). In addition, we do not find any evidence that available center-based alternatives are more effective than Head Start on average (e.g., Gormley et al., 2010; Barnett and Haskins, 2010). In the HSIS sample, around half of the control group children who enrolled in some other form of center-based care did so in either a state-funded prekindergarten program or a prekindergarten program based in the public schools.[31] While statistical power is limited, the null finding for Center Compliers suggests that concerns over Head Start's comparative effectiveness may be misplaced.

In addition to showing larger impacts in the Head Start year, we also find that the fade out in treatment effects over time is gradual, not rapid (Gibbs et al., 2011). This pattern closely resembles the observed fade out in other early childhood education studies (Magnuson et al., 2007; Leak et al., 2010). In addition, while our estimates are imprecise, we find impacts between 0.10 to 0.15 for Home Complier children in first grade. These point estimates are very close to those in Deming (2009), who estimates Head Start impacts of 0.15 for children aged 5 to 6 and 0.13 for children aged 7 to 10.[32] Importantly, Deming (2009) observes outcomes for these same children in young adulthood, showing large long-term impacts. It is therefore possible that future follow up from the Head Start Impact Study will also find meaningful long-term impacts despite treatment effect fade out on short-term outcomes.

More generally, our analysis highlights the critical role that variation in counterfactual care type plays in early childhood education evaluations. Duncan and Magnuson (2013) argue that improving counterfactual conditions are a primary reason for a sharp decline in reported impacts of early childhood education interventions over the last half-century. We not only provide evidence consistent with this claim, but also outline a framework for re-analyzing other early childhood education studies to create comparable estimates. Of course, the issue of variation in counterfactual treatments is common in program evaluation settings, including for alternative schools (Bloom and Unterman, 2014) and job training programs (Heckman et al., 2000; Schochet et al., 2008). Our approach could easily be extended to these settings as well.

At the same time, the proposed method is constrained by the need for a good parametric approximation of the outcome distribution. In our analysis, for example, we are unable to examine a number of key non-cognitive outcomes because they are, for example, measured on a Likert scale. Even if we were to impose a parametric model on these outcomes, such as an ordered

[31] Like Head Start, these publicly funded programs typically feature minimum standards for important structural aspects of program quality such as teacher preparation, teacher-child ratio and curricula. This result is also consistent with a recent study in Tulsa that found that Head Start and a publicly funded prekindergarten program led to comparable school readiness (Jenkins et al., 2014) and with the larger literature comparing quality for publicly funded versus private preschool programs (Kagan, 1991; Morris and Helburn, 2000).

[32] The outcome in Deming (2009) combines PPVT with the Peabody Individual Achiement Tests (PIAT) for math and reading.

probit, the resulting inference would be especially sensitive to the latent Normality assumption, which is impossible to assess. A key research question is therefore how to assess validity of the principal stratification modeling assumptions in different practical settings. The nonparametric identification results and link between the principal stratification and finite mixtures provide some promising avenues for future work.

Finally, our results support the argument that further efforts to expand publicly-funded preschool programs should be targeted toward those who are currently not enrolling their children in center-based programs (for discussion, see Ludwig and Phillips, 2010; Bassok et al., 2013; Cascio and Schanzenbach, 2013). Nationwide, over 40 percent of eligible children are served by Head Start programs (Schmit et al., 2013). Although the availability of state and local prekindergarten has grown in recent years, many low-income children still spend their preschool years in home-based settings. In 2011, approximately 42 percent of three- and four-year-old children from low-income families enrolled in center-based prekindergarten compared to 59 percent of their non-low income peers (Burgess et al., 2014). Based on our results, shifting children from home-based care into formal care will likely lead to much larger effects than shifting children between preschool programs.

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Table 1: Covariate Balance at Baseline

	Control Mean	T-C Diff.	Norm. Diff.
Child Characteristics			
PPVT pre-test (std.)	0.03	-0.05	-0.04
Bottom third by pre-test	0.32	0.02	0.03
Three-year old	0.55	—	0.01
Male	0.51	—	-0.01
Black	0.30	0.01	0.02
Hispanic	0.37	0.01	0.01
Dual-Language Learner	0.29	0.01	0.03
Special needs	0.11	0.03	0.08
Caregiver and Family Characteristics			
Caregiver age: <25	0.32	-0.02	-0.05
Caregiver age: 25-29	0.31	—	—
Caregiver age: 30-39	0.29	0.01	0.02
Caregiver age: 40+	0.07	0.02	0.06
Teen mother	0.19	-0.03	-0.07
High school dropout	0.39	-0.02	-0.04
Only high school diploma/GED	0.33	0.01	0.02
Married	0.45	-0.01	-0.01
Previously married	0.16	—	—
Urban	0.84	—	—
Family risk: medium/high	0.22	0.03	0.06
Lives with both biological parents	0.49	—	—
Recent immigrant	0.19	—	0.01
Any older sibling attended Head Start	0.37	0.04	0.09
Oldest child	0.45	-0.03	-0.06
Head Start Center of Random Assignment Characteristics			
Provides transportation	0.63	—	—
At least four home visits per year	0.21	—	-0.01
Full day child care	0.64	—	0.01
Student-teacher ratio	6.75	-0.02	-0.01
All teachers certified in early childhood	0.41	—	—
All teachers have mentors	0.46	—	—
Center is always filled	0.48	—	—
Number of children randomized	17	—	—
Neighborhood and State Characteristics			
Percent in poverty	0.25	—	—
Percent minority	0.44	—	—
Percent unemployed	0.11	—	—
Percent commute by car	0.82	—	—
Number of crimes per 1000 people	44	0.1	0.01
State has DOE Pre-K	0.64	—	0.01
State per-child spending ($'000)	3.9	—	0.01
State Head Start teacher salary ($'000)	21.8	—	0.01

Notes: Section 3.3 discusses the Normalized Difference. For clarity, 0.00 is denoted by '—'.

Table 2: Child care setting by treatment group

	Treatment	Control	Difference
Head Start	0.77	0.11	0.66
Other center-based care	0.08	0.26	-0.18
Home-based care	0.09	0.47	-0.38
Missing	0.06	0.16	-0.10
Head Start (admin.)	0.81	0.12	0.69

Notes: Child care setting is based on responses from the Spring 2003 parent reports. "Head Start (admin.)" refers to the administrative records collected as part of HSIS and is the compliance rate used in Puma et al. (2010a).

Table 3: Possible principal strata in the Head Start Impact Study

		$Z = 0$: **Head Start**	$Z = 0$: **Not Head Start**
$Z = 1$	**Head Start**	Always Head Start	Complier
$Z = 1$	**Not Head Start**	*(Defier)*	Never Head Start

(a) **Binary D^*: Head Start vs. No Head Start.**

		$Z = 0$: **Head Start**	$Z = 0$: **Center Care**	$Z = 0$: **Home Care**
$Z = 1$	**Head Start**	Always Head Start	Center Complier	Home Complier
$Z = 1$	**Center Care**	*(A)*	Always Center Care	*(B)*
$Z = 1$	**Home Care**	*(C)*	*(D)*	Always Home Care

(b) **Multi-valued D: Head Start, Other Center-based care, Home-based care.**

Table 4: Distribution of Principal Strata

Noncompliers			Compliers	
Always HS	Always Center	Always Home	Center Complier	Home Complier
0.11	0.11	0.12	0.20	0.45

Notes: Posterior medians, with missing care type imputed. See Appendix C.2.

Table 5: Relationship between Observed Care Type and Principal Strata

Z	D^*	Possible Principal Strata
1	HS	Always Head Start, Complier (treat)
1	Not HS	Never Head Start
0	HS	Always Head Start
0	Not HS	Never Head Start, Complier (control)

(a) Binary D^*: Head Start vs. No Head Start.

Z	D	Possible Principal Strata
1	HS	Always Head Start, Center Complier (treat), Home Complier (treat)
1	Center	Always Center
1	Home	Always Home
0	HS	Always Head Start
0	Center	Always Center, Center Complier (control)
0	Home	Always Home, Home Complier (control)

(b) Multi-valued D: Head Start, Other Center-based care, Home-based care.

Table 6: Covariate Means by Principal Stratum

	Always Head Start	Always Center	Always Home	Center Complier	Home Complier
Child Characteristics					
PPVT pre-test (std.)	-0.20	0.24	-0.03	0.12	-0.05
Bottom third by pre-test	0.36	0.28	0.35	0.30	0.35
Three-year old	0.63	0.43	0.53	0.47	0.59
Male	0.54	0.53	0.57	0.48	0.48
Black	0.35	0.40	0.23	0.31	0.30
Hispanic	0.42	0.33	0.39	0.35	0.37
Dual-Language Learner	0.37	0.28	0.27	0.33	0.29
Special needs	0.15	0.17	0.12	0.13	0.11
Caregiver and Family Characteristics					
Caregiver age: <25	0.29	0.31	0.38	0.24	0.32
Caregiver age: 25-29	0.29	0.31	0.30	0.35	0.31
Caregiver age: 30-39	0.34	0.28	0.26	0.30	0.29
Caregiver age: 40+	0.08	0.09	0.07	0.11	0.08
Teen mother	0.16	0.19	0.20	0.15	0.17
High school dropout	0.44	0.27	0.47	0.35	0.38
Only high school diploma/GED	0.28	0.35	0.31	0.30	0.36
Married	0.46	0.42	0.47	0.45	0.44
Previously married	0.14	0.18	0.16	0.17	0.16
Urban	0.90	0.87	0.86	0.86	0.81
Family risk: medium/high	0.27	0.16	0.22	0.24	0.25
Lives with both biological parents	0.51	0.47	0.48	0.48	0.51
Recent immigrant	0.23	0.20	0.18	0.22	0.17
Any older sibling attended Head Start	0.40	0.34	0.38	0.34	0.43
Oldest child	0.43	0.47	0.45	0.50	0.39
Head Start Center of Random Assignment Characteristics					
Provides transportation	0.44	0.61	0.60	0.62	0.68
At least four home visits per year	0.15	0.17	0.21	0.18	0.25
Full day child care	0.69	0.75	0.59	0.67	0.61
Student-teacher ratio	6.66	6.89	6.58	7.07	6.64
All teachers certified in early childhood	0.50	0.43	0.41	0.44	0.38
All teachers have mentors	0.38	0.49	0.43	0.46	0.48
Center is always filled	0.50	0.43	0.44	0.48	0.49
Number of children randomized	14	18	15	16	18
Neighborhood and State Characteristics					
Percent in poverty	0.27	0.25	0.23	0.27	0.24
Percent minority	0.55	0.49	0.40	0.45	0.40
Percent unemployed	0.12	0.11	0.10	0.11	0.10
Percent commute by car	0.72	0.77	0.82	0.81	0.85
Number of crimes per 1000 people	49	45	42	47	43
State has DOE Pre-K	0.72	0.69	0.59	0.68	0.62
State per-child spending ($'000)	3.4	3.8	3.4	4.1	4.2
State Head Start teacher salary ($'000)	21.1	21.7	21.3	21.9	22.1

Notes: Covariate means based on multiply imputed stratum membership.

Table 7: Impacts in the Head Start Year

ITT	**0.14** **(0.11, 0.16)**
Overall $LATE$	**0.18** **(0.14, 0.23)**
$LATE$ for Center Compliers	0.00 (-0.13, 0.14)
$LATE$ for Home Compliers	**0.23** **(0.15, 0.30)**
$\mathbb{P}\{LATE_{\text{hc}} > LATE_{\text{cc}}\}$	0.99

Notes: Point estimates are posterior medians, with 2.5 and 97.5 quantiles of posterior distribution in parentheses. 95% posterior intervals that exclude zero are printed in bold. See Appendix B for estimation details. All treatment effects, including $LATE_{\text{cc}}$ and $LATE_{\text{hc}}$, are allowed to vary by Head Start center of random assignment. Note that the IV and Principal Stratification models are estimated separately.

Table 8: Impacts in the Head Start Year for Select Subgroups

	Center Compliers	Home Compliers
Panel A. Bottom Third on Pre-Test		
Bottom Third	0.19	**0.30**
	(-0.09, 0.47)	**(0.16, 0.45)**
Not Bottom Third	-0.06	**0.21**
	(-0.24, 0.16)	**(0.08, 0.31)**
Panel B. DLL Status		
DLL Students	0.06	**0.36**
	(-0.33, 0.42)	**(0.23, 0.49)**
Non-DLL Students	-0.04	**0.15**
	(-0.20, 0.12)	**(0.08, 0.23)**

Notes: Point estimates are posterior medians, with 2.5 and 97.5 quantiles of posterior distribution in parentheses. 95% posterior intervals that exclude zero are printed in bold. Estimates are shown in effect size units, so point estimates might not average to the pooled estimate due to different outcome standard deviations. See Appendix B for estimation details. All treatment effects are allowed to vary by Head Start center of random assignment.

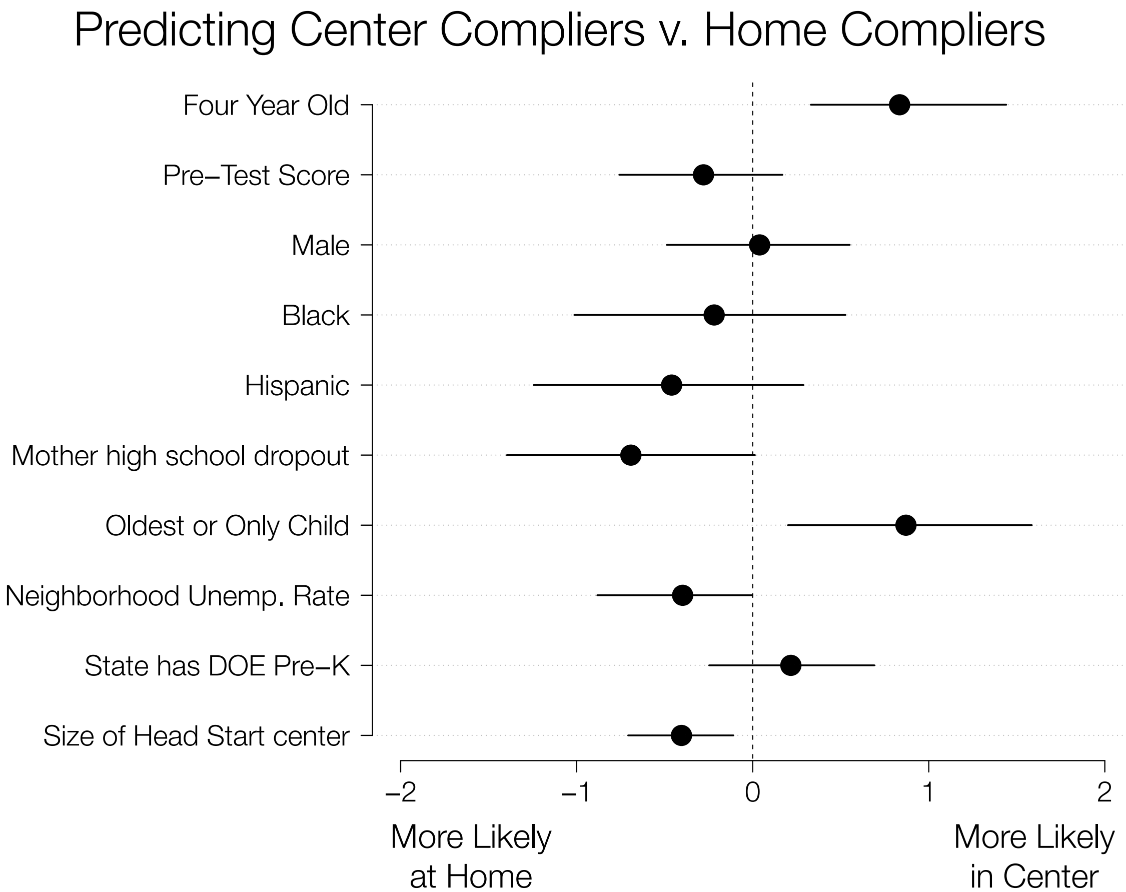


Figure 1: Logistic regression coefficients predicting Center vs. Home Compliers, generated from a multinomial logistic regression predicting all types. All continuous covariates are standardized. Point estimates and error bars show posterior medians and 95% credible intervals.

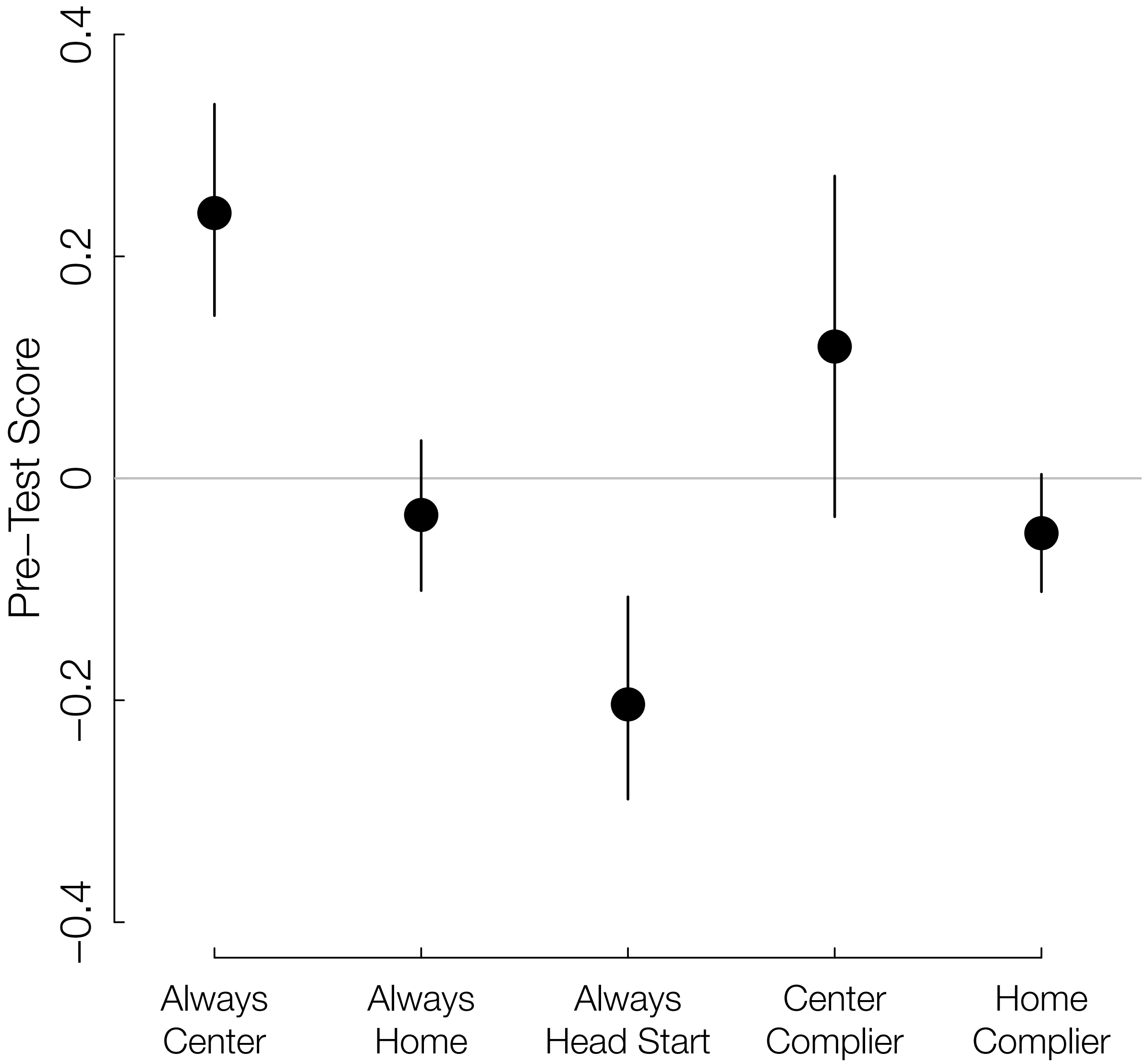
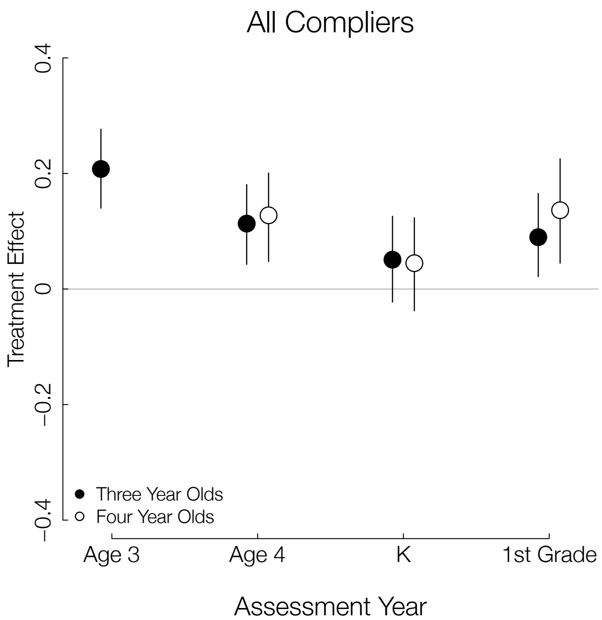
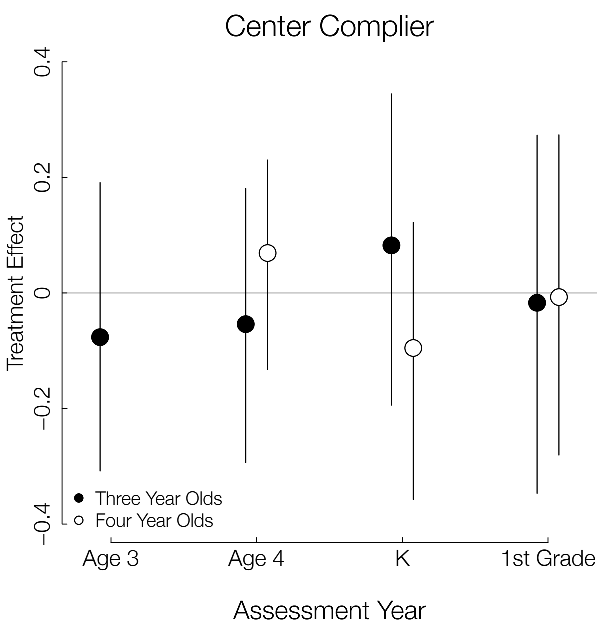


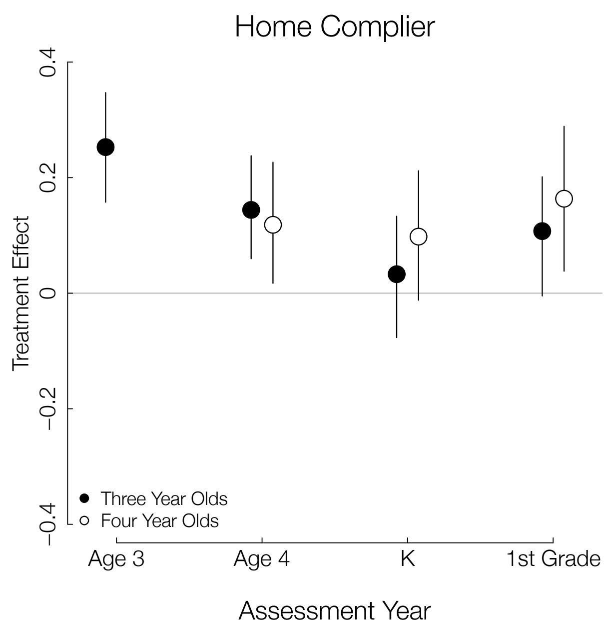
Figure 2: PPVT pre-test score by principal stratum. Point estimates and error bars show posterior medians and 95% credible intervals.



(a) All Compliers.

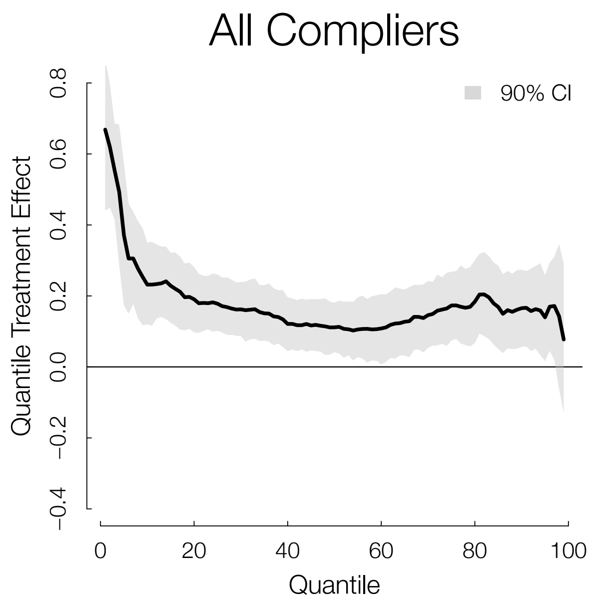


(b) Center Compliers.

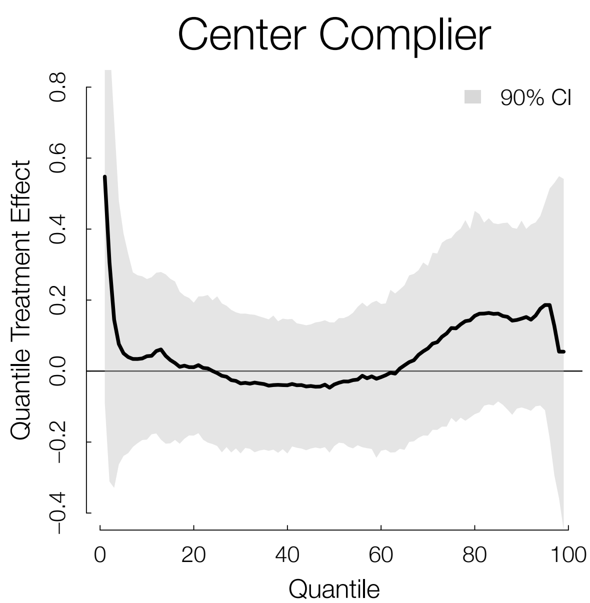


(c) Home Compliers.

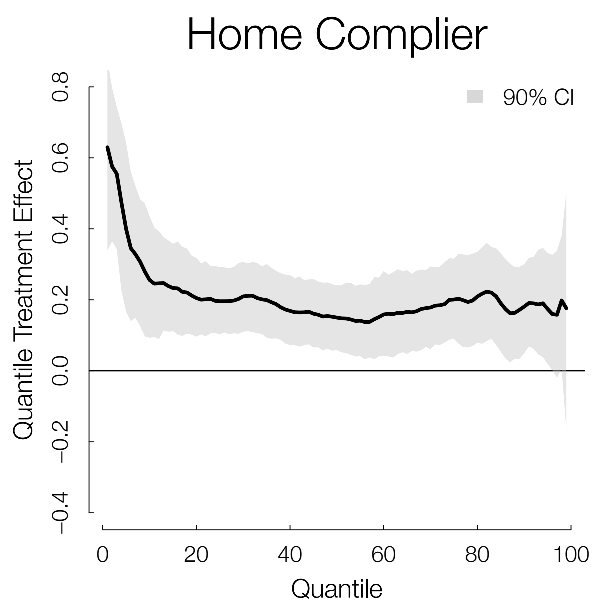
Figure 3: Impact estimates on PPVT by principal stratum and by three- and four-year-old cohort for each assessment year. Point estimates and error bars show posterior medians and 95% credible intervals. Effect sizes are calculated separately for each cohort in each assessment year.



(a) All Compliers.



(b) Center Compliers.



(c) Home Compliers.

Figure 4: Quantile treatment estimates on PPVT by principal stratum for the Head Start year, with approximate 90 percent credible intervals.

Online Appendix

A Identification for Outcome Distributions

A.1 Instrumental Variables

We begin with the IV case, following Imbens and Rubin (1997b) and Abadie (2003). Formally, let $g_{sz}(y)$ be the distribution of $Y_i(z)$ for principal stratum s. Table 3a shows the possible strata for binary D^* under the monotonicity assumption (Assumption IV-1): Always Head Start, Never Head Start, and Compliers. Under the exclusion restrictions (Assumption IV-2), the following outcome distributions are equal: $g_{\text{ahs}\,0}(y) = g_{\text{ahs}\,1}(y)$ and $g_{\text{nhs}\,0}(y) = g_{\text{nhs}\,1}(y)$. This yields four possible stratum outcome distributions: $g_{\text{ahs}}(y)$, $g_{\text{nhs}}(y)$, $g_{\text{co}\,0}(y)$, and $g_{\text{co}\,1}(y)$. Finally, let $f_{zd}(y)$ be the distribution of Y_i^{obs} in the subsample defined by $Z_i = z$ and $D_i^{*,obs} = d$. Table 5a shows the relationship between the observed and latent distributions. We can write these relationships mathematically:

$$\begin{aligned}
f_{11}(y) &= \frac{\pi_{\text{co}}}{\pi_{\text{co}} + \pi_{\text{ahs}}} g_{\text{co}\,1}(y) + \frac{\pi_{\text{ahs}}}{\pi_{\text{co}} + \pi_{\text{ahs}}} g_{\text{ahs}}(y) \\
f_{10}(y) &= g_{\text{nhs}}(y) \\
f_{01}(y) &= g_{\text{ahs}}(y) \\
f_{00}(y) &= \frac{\pi_{\text{co}}}{\pi_{\text{co}} + \pi_{\text{nhs}}} g_{\text{co}\,0}(y) + \frac{\pi_{\text{nhs}}}{\pi_{\text{co}} + \pi_{\text{nhs}}} g_{\text{nhs}}(y)
\end{aligned}$$

Simply re-arranging terms yields:

$$\begin{aligned}
g_{\text{ahs}}(y) &= f_{01}(y) \\
g_{\text{nhs}}(y) &= f_{10}(y) \\
g_{\text{co}\,0}(y) &= \frac{\pi_{\text{co}} + \pi_{\text{nhs}}}{\pi_{\text{co}}} f_{00}(y) - \frac{\pi_{\text{nhs}}}{\pi_{\text{co}}} g_{\text{nhs}}(y) \\
g_{\text{co}\,1}(y) &= \frac{\pi_{\text{co}} + \pi_{\text{ahs}}}{\pi_{\text{co}}} f_{11}(y) - \frac{\pi_{\text{ahs}}}{\pi_{\text{co}}} g_{\text{ahs}}(y)
\end{aligned}$$

Since we can non-parametrically identify π_s for all s in $\{\text{ahs}, \text{nhs}, \text{co}\}$ and $f_{zd}(y)$ for $Z_i \in \{0, 1\}$ and $D^*i \in \{0, 1\}$, we can therefore non-parametrically identify the relevant $g_{sz}(y)$.

A.2 Principal Strata

We now extend this logic to the full principal stratification problem. Table 3b shows the five possible principal strata under Assumptions PS-1a and PS-1b. Under the exclusion restrictions (Assumption PS-2), we have the following equalities: $g_{\text{ahs}\,0}(y) = g_{\text{ahs}\,1}(y)$, $g_{\text{ac}\,0}(y) = g_{\text{ac}\,1}(y)$, and $g_{\text{ah}\,0}(y) = g_{\text{ah}\,1}(y)$. Table 5b shows the relationship between the observed and latent distributions with multi-valued D. We again describe these mathematically:

$$\begin{aligned}
f_{1\,\mathrm{HS}}(y) &= \frac{\pi_{\mathrm{ahs}}}{\pi_{\mathrm{ahs}}+\pi_{\mathrm{cc}}+\pi_{\mathrm{hc}}}\ g_{\mathrm{ahs}}(y) + \frac{\pi_{\mathrm{cc}}}{\pi_{\mathrm{ahs}}+\pi_{\mathrm{cc}}+\pi_{\mathrm{hc}}}\ g_{\mathrm{cc}\,1}(y) + \frac{\pi_{\mathrm{hc}}}{\pi_{\mathrm{ahs}}+\pi_{\mathrm{cc}}+\pi_{\mathrm{hc}}}\ g_{\mathrm{hc}\,1}(y) \\
f_{1\,\mathrm{Center}}(y) &= g_{\mathrm{ac}}(y) \\
f_{1\,\mathrm{Home}}(y) &= g_{\mathrm{ah}}(y) \\
f_{0\,\mathrm{HS}}(y) &= g_{\mathrm{ahs}}(y) \\
f_{0\,\mathrm{Center}}(y) &= \frac{\pi_{\mathrm{ac}}}{\pi_{\mathrm{ac}}+\pi_{\mathrm{cc}}}\ g_{\mathrm{ac}}(y) + \frac{\pi_{\mathrm{cc}}}{\pi_{\mathrm{ac}}+\pi_{\mathrm{cc}}}\ g_{\mathrm{cc}\,0}(y) \\
f_{0\,\mathrm{Home}}(y) &= \frac{\pi_{\mathrm{ah}}}{\pi_{\mathrm{ah}}+\pi_{\mathrm{hc}}}\ g_{\mathrm{ah}}(y) + \frac{\pi_{\mathrm{hc}}}{\pi_{\mathrm{ah}}+\pi_{\mathrm{hc}}}\ g_{\mathrm{hc}\,0}(y)
\end{aligned}$$

Re-arranging terms yields five of the seven needed distributions:

$$\begin{aligned}
g_{\mathrm{ac}}(y) &= f_{1\,\mathrm{Center}}(y) \\
g_{\mathrm{ah}}(y) &= f_{1\,\mathrm{Home}}(y) \\
g_{\mathrm{ahs}}(y) &= f_{0\,\mathrm{HS}}(y) \\
g_{\mathrm{cc}\,0}(y) &= \frac{\pi_{\mathrm{ac}}+\pi_{\mathrm{cc}}}{\pi_{\mathrm{cc}}} f_{0\,\mathrm{Center}}(y) - \frac{\pi_{\mathrm{ac}}}{\pi_{\mathrm{cc}}} g_{\mathrm{ac}}(y) \\
g_{\mathrm{hc}\,0}(y) &= \frac{\pi_{\mathrm{ah}}+\pi_{\mathrm{hc}}}{\pi_{\mathrm{hc}}} f_{0\,\mathrm{Home}}(y) - \frac{\pi_{\mathrm{ah}}}{\pi_{\mathrm{hc}}} g_{\mathrm{ah}}(y)
\end{aligned}$$

Following the same logic as in the IV case, these outcome distributions are non-parametrically identified. However, we still need the outcome distributions for Compliers under treatment, $g_{\mathrm{cc}\,1}(y)$ and $g_{\mathrm{hc}\,1}(y)$. Since we observe $g_{\mathrm{ahs}}(y)$ in the control group, we can isolate these outcome distributions by further "backing out" $g_{\mathrm{ahs}}(y)$ from the three-component mixture of $f_{1\,\mathrm{HS}}(y)$:

$$f^*_{1\,\mathrm{HS}}(y) = \frac{\pi_{\mathrm{ahs}}+\pi_{\mathrm{cc}}+\pi_{\mathrm{hc}}}{\pi_{\mathrm{ahs}}} f_{1\,\mathrm{HS}}(y) - \frac{\pi_{\mathrm{cc}}+\pi_{\mathrm{hc}}}{\pi_{\mathrm{ahs}}} g_{\mathrm{ahs}}(y).$$

We are then left with a classic two-component finite mixture model for $f^*_{1\,\mathrm{HS}}(y)$:

$$f^*_{1\,\mathrm{HS}}(y) = \phi\ g_{\mathrm{hc}\,1}(y) + (1-\phi)\ g_{\mathrm{cc}\,1}(y),$$

with known mixing proportion $\phi = \frac{\pi_{\mathrm{hc}}}{\pi_{\mathrm{cc}}+\pi_{\mathrm{hc}}}$. In general, the component densities in a two-component finite mixture model are not identifiable without additional restrictions (see, e.g., Hall and Zhou, 2003). The standard econometric solution to this problem is to find a new source of "exogenous" variation and impose an additional conditional independence assumption (e.g., Henry et al., 2014). As we discuss in Appendix A.4, however, there are no clear candidate covariates for such exogenous variation in the Head Start setting. We therefore turn to finite mixture modeling to obtain identification.

A.3 Finite Mixture Models

There is a long literature on identification in finite mixture modeling, which differs in important but subtle ways from identification in more standard settings. For reviews, see Titterington et al. (1985), Lindsay (1995), McLachlan and Peel (2004), and Frühwirth-Schnatter (2006). Rather than address these additional complications here, we note that many of the associated pathologies do not apply in our case, since the mixing proportion, ϕ, is directly estimable and differs from $\frac{1}{2}$.

As we discuss in Appendix B, our estimation approach assumes that the outcome distribution is Normal. Therefore, a sufficient condition for identification is simply that a two-component Gaussian mixture model is identified. This is a classic result due to Pearson (1894). Titterington et al. (1985) give a more modern treatment of this result, including a discussion of the weak technical restrictions necessary for identification (see also, Frumento et al., 2012).

While this result is formally enough for our purposes, it is useful to know how sensitive our identification results are to the assumption of Normality. First, we can obtain identification under a broad range of parametric models. Lindsay (1995) proves identification for a surprisingly large class of single-parameter component densities; distributions in the one-parameter natural exponential family (NEF-QVF) are special cases. Frühwirth-Schnatter (2006) discusses extensions to multi-parameter parametric families. With these results, we also have point identification for the principal stratification model in Appendix B.6 that uses Student's t distribution rather than Normality.

Second, we can depart from parameteric assumptions altogether to obtain semi-parametric identification, and, in particular, can use the assumption of symmetry on the outcome distribution. In the general case with unknown mixing proportion, Bordes et al. (2006) and Hunter et al. (2007) proved semi-parametric identifiability for the following mixture of location-shifted distributions,

$$f^*_{1\,\mathrm{HS}}(y) = \phi\ g^*(y - \mu_{\mathrm{hc}\,1}) + (1-\phi)\ g^*(y - \mu_{\mathrm{cc}\,1}),$$

where $g^*(\cdot)$ is a completely unspecified distribution function, except for the assumption that it is symmetric about 0.

In our setting, we also know the mixing proportion, ϕ, and are focused on the component means, $\mu_{\mathrm{cc}\,1}$ and $\mu_{\mathrm{hc}\,1}$, rather than the entire distribution. We therefore provide a simple variation on the above result. Let m_k be the kth central moment of the identified marginal distribution, $f^*_{1\,\mathrm{HS}}(y)$, with the mean denoted as m_1. Additionally, define $\sigma^2_{\mathrm{cc}\,1}$ and $\sigma^2_{\mathrm{hc}\,1}$ as the variances of the component densities, and $\gamma^2_{\mathrm{cc}\,1}$ and $\gamma^2_{\mathrm{hc}\,1}$ as the skewness of the component densities. Finally, define $\Delta = \mu_{\mathrm{hc}\,1} - \mu_{\mathrm{cc}\,1}$. By standard results for mixture moments:

$$\begin{aligned}
m_1 &= \mu_{\mathrm{cc}\,1} + \phi\Delta \\
m_2 &= \phi\ \sigma^2_{\mathrm{hc}\,1} + (1-\phi)\ \sigma^2_{\mathrm{cc}\,1} + \phi(1-\phi)\Delta^2 \\
m_3 &= \phi(1-\phi)(1-2\phi)\Delta^3 + 3\phi(1-\phi)\left(\sigma^2_{\mathrm{hc}\,1} - \sigma^2_{\mathrm{cc}\,1}\right)\Delta + \phi\gamma_{\mathrm{hc}\,1} + (1-\phi)\gamma_{\mathrm{cc}\,1},
\end{aligned}$$

where m_k and ϕ are known. For point identification, we make two assumptions. First, we assume that the component densities are symmetric: $\gamma_{\text{hc}\,1} = \gamma_{\text{cc}\,1} = 0$. Second, we assume that the densities have equal variance, $\sigma_*^2 = \sigma_{\text{hc}\,1}^2 = \sigma_{\text{cc}\,1}^2$, which is strictly weaker than the assumption of equal distributions. We then have the following moment equations:

$$
\begin{aligned}
m_1 &= \mu_{\text{cc}\,1} + \phi\Delta \\
m_2 &= \sigma_*^2 + \phi(1-\phi)\Delta^2 \\
m_3 &= \phi(1-\phi)(1-2\phi)\Delta^3
\end{aligned}
$$

Re-arranging terms, we obtain these simple identification results:

$$
\begin{aligned}
\Delta &= \sqrt[3]{\frac{m_3}{\phi(1-\phi)(1-2\phi)}} \\
\mu_{\text{cc}\,1} &= m_1 - \phi\Delta \\
\sigma_*^2 &= m_2 - \phi(1-\phi)\Delta^2
\end{aligned}
$$

Therefore, $\mu_{\text{cc}\,1}$ and $\mu_{\text{hc}\,1}$ are semi-parametrically identified under symmetry and equal variance.

An additional option is to incorporate information on the variances from elsewhere in the data. For example, we could make a homoskedasticity assumption for all principal strata outcome distributions, strengthening the identification of the common variance term (Gallop et al., 2009). Alternatively, Imbens and Rubin (1997a) assume the variance depends on treatment received, which would lead to $\sigma_{\text{cc}\,1}^2 = \sigma_{\text{hc}\,1}^2 = \sigma_{\text{ahs}}^2$ in this context. While this is a stronger assumption than the equal variance case above, invoking this assumption reduces the number of unknown parameters in the mixture model by one. Finally, we can take the more general view that the variances are unequal and unknown, which requires an additional assumption about the kurtosis of the component distribution.

In the end, there are a range of possibilities for combining identification results from mixture modeling with principal stratification models. In particular, we can strengthen estimation and identification by making a variety of assumptions about connections between different aspects of the observed and latent distributions. This is also true for recent developments in Bayesian non-parametric modeling. For example, Rodríguez and Walker (2014) show promising computational results—though no formal identification proofs—under a broad class of uni-modal densities.

A.4 Alternative Identifying Assumptions

As we mention above, the classic econometric approach utilizes additional conditional independence assumptions to obtain point identification. We briefly survey some alternatives here, with a focus on connections between causal inference and mixture modeling approaches. For further discussion, see Feller and Miratrix (2014).

Selection on Observables. The key assumption in this approach is that, given an observed vector of covariates, $\mathbf{X}$, the outcome and principal stratum memberhsip are conditionally independent. In this setting, Jo and Stuart (2009) refer to this assumption as *Principal Ignorability*. Mathematically:

$$(Y_i(0), Y_i(1)) \perp\!\!\!\perp S_i \mid \mathbf{X}_i. \qquad \text{(Principal Ignorability)}$$

In other words, the covariates are "good enough" at predicting stratum membership, such that any unexplained variation is unrelated to the outcome. This can be a sensible assumption in some settings (e.g., Hill et al., 2002; Schochet and Burghardt, 2007; Scott-Clayton and Minaya, 2014), but seems somewhat implausible here, as we do not observe critical variables like parental preference for care type prior to randomization. See Jo and Stuart (2009) for further discussion of inference in this setting.

Multiple, Independent Outcomes. A growing literature in mixture modeling relies on the use of repeated measurements to nonparametrically identify the component densities. In a seminal paper, Hall and Zhou (2003) show that, in general, a two-component finite mixture model is non-parametrically identified if each unit has at least three observations. Importantly, this and other repeated measures approaches assume that the observations are independent, which is plausible in a repeated panel under stationarity, but is not plausible in the Head Start example, where we are especially concerned about fadeout. Note that this is closely related to the literature on the use of secondary outcomes for identification in principal stratification models (Mealli and Pacini, 2013).

Conditional Independence for a Given Covariate. Finally, a range of approaches explore the identifying power of a covariate that is predictive of stratum membership but is otherwise independent of the outcome. Stated this way, the assumption clearly requires an additional instrument, though this is not generally explicit. This idea has also emerged in the mixture modeling literature. For example, Henry et al. (2014) show that such a covariate, combined with weak restrictions on the tail behavior of the densities, yields non-parametric identification for the component densities. Finally, Feller and Miratrix (2014) explore connections between these conditional independence assumptions and a method known as Multi-Site, Multi-Mediator IV (Raudenbush et al., 2012), which could prove promising in the Head Start setting.

A.5 Identification and Bayesian Estimation

There is an extensive debate about whether formal identification results have a role to play in Bayesian inference. In general, we view identification as a useful tool for understanding what "drives" the model and, in particular, those aspects of the model might be especially sensitive to a given specification. For example, consider identification for the average outcome for Home-based Compliers under treatment and control. As we note above, identification for the control mean relies on strictly weaker assumptions than identification for the treatment mean, suggesting that it is especially important to assess sensitivity of the treatment mean to different specifications. See Stan Development Team (2014) for a thoughtful discussion of identification in a Bayesian setting.

B Estimation

B.1 Bayesian Computation and Estimation

We estimate parameters using the Bayesian programming language, Stan (Stan Development Team, 2014), which is a fast implementation of a Hamiltonian Monte Carlo (HMC) sampler. Unlike, say, a classic Gibbs sampler, HMC-based samplers explore the space of the (log) posterior far more efficiently than more standard Markov chain Monte Carlo approaches, dramatically increasing the effective sample size of the same number of draws (Hoffman and Gelman, 2014). One drawback of the HMC approach is that the log-posterior must have globally smooth gradients. As a result, Stan/HMC cannot incorporate discrete latent parameters, such as indicators for principal stratum membership that would be standard in a data augmentation scheme. We sidestep this issue by maximizing the observed data log-posterior rather than the complete data log-posterior; in other words, we directly estimate $\pi_{s|\mathbf{x}}$ for each unit rather than first impute each child's stratum membership. While it is possible to couple a data augmentation Gibbs step with a bespoke HMC sampler, doing so would lose many of Stan's key advantages, including optimized C++ code and a powerful, flexible programming language. Moreover, both approaches remain significantly faster than standard Gibbs or Metropolis-Hastings MCMC samplers. In the end, it is unlikely that this project would have been feasible without the development of Stan.

Each model was run with five separate chains with 500 "warm up" draws and 500 posterior draws. We assess model convergence in the usual way via traceplots, via Gelman-Rubin $\widehat{R}$ statistics at or near 1, and via measures of the effective sample size from each chain. All models reported here showed excellent convergence for parameters of interest. As with all hierarchical models, some hyperparameters were poorly estimated. We do not report those here.

Before computation, the outcome was centered by the global mean and re-scaled by the standard deviation of the observed outcomes for the control group, so that all units were in terms of "effect size." Throughout we use default, weakly informative priors on all model parameters (Gelman et al., 2008). For regression coefficients, all main coefficients have independent $N(0, 1.5^2)$ priors; stratum-by-covariate interaction terms have a tighter $N(0, 0.25^2)$ prior. For multinomial logistic regressions, all coefficients have Cauchy(0, 2.5) priors (Gelman et al., 2008). All standard deviations have half-Cauchy priors, with the scale set to 1, unless otherwise noted (Gelman, 2006). This effectively places a flat prior over the space of 0 to 2, while allowing for much larger standard deviations if there is a strong signal in the data. As the outcome has already been standardized to have a global standard deviation of 1 (for the control group), an observed model standard deviation greater than 2 would suggest an especially bad model fit. The prior for the site-level standard deviation is also given a half-Cauchy(0,1) distribution. Following the recommendation among Stan developers, all random effect correlation matrices are given weak LKJ priors (Lewandowski et al., 2009), which have more attractive properties than the more standard inverse-Wishart distribution and which slightly favor an identity correlation matrix (Stan Development Team, 2014). Finally, missing pre-tests and outcomes have a $N(0, 1)$ prior, on the same order as the standardized outcome distribution.

For the ITT model, we use a standard varying intercept/varying slope model (i.e., a "random effects" model), which accounts for center-level variation via multilevel modeling. See, for example, Gelman and Hill (2006).[33] Following Bloom et al. (2014), we also estimate separate residual variances for the treatment and control groups. Both the intercept, α_j, and the treatment effect, τ_j, vary by site. Finally, we include the proportion assigned to treatment, $\bar{z}_j$, as a site-level predictor in order to account for differing proportions randomized to treatment by site (Bafumi and Gelman, 2006; Raudenbush and Bryk, 2002).

B.2 Estimating LATE

Imbens and Rubin (1997a) proposed a model-based estimation strategy for instrumental variables models as an alternative to the standard Wald/Two-Stage Least Squares estimator. See Imbens and Rubin (2014) for a textbook discussion of this approach. The key idea is that the usual ratio estimators ignore individual-level information about compliance status, since they are based solely on sample averages.

Incorporating this information requires the use of a full likelihood:

$$\begin{aligned}
\mathcal{L}_{obs}(\theta \mid \mathbf{y}, \mathbf{x}, \mathbf{d}, \mathbf{z}) \quad = \quad & \prod_{i:D_i=0,Z_i=1} \pi_{i:\text{nt}}\ g_{\text{nt}}(y_i \mid \theta, \mathbf{x}_i) \times \\
& \prod_{i:D_i=1,Z_i=0} \pi_{i:\text{at}}\ g_{\text{at}}(y_i \mid \theta, \mathbf{x}_i) \times \\
& \prod_{i:D_i=0,Z_i=0} \Big\{ \pi_{i:\text{nt}}\ g_{\text{nt}}(y_i \mid \theta, \mathbf{x}_i) + \\
& \qquad \pi_{i:\text{co}}\ g_{\text{co0}}(y_i \mid \theta, \mathbf{x}_i) \Big\} \times \\
& \prod_{i:D_i=1,Z_i=1} \Big\{ \pi_{i:\text{at}}\ g_{\text{at}}(y_i \mid \theta, \mathbf{x}_i) + \\
& \qquad \pi_{i:\text{co}}\ g_{\text{co1}}(y_i \mid \theta, \mathbf{x}_i) \Big\}
\end{aligned}$$

In our setting, we incorporate weak prior information and estimate these parameters in a Bayesian framework. Computationally, we jointly estimate two sub-models. The first is a multinomial logistic regression predicting principal stratum membership as a function of covariates:

$$\pi_{s|\mathbf{x}} \equiv \mathbb{P}(S_i = s \mid \theta, \mathbf{x}_i) = \frac{\exp(\gamma_s + \delta_s' \mathbf{x}_i)}{\sum_{s=1}^{K} \exp(\gamma_s + \delta_s' \mathbf{x}_i)}.$$

While we do not explore additional models here, we note that we could replace the multinomial logit model with other discrete choice models, such as a multinomial probit model.

[33] Bloom et al. (2014) propose a hybrid approach, with fixed effects for the site-specific intercepts and random effects for the site-specific treatment effects.

The second is an outcome model, effectively a separate regression for each compliance type:

$$
\begin{aligned}
y_i \mid (S_i^* = \text{nt}, \theta, \mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{nt}} + \beta_{\text{nt}}'\mathbf{x}_i, \sigma_{\text{nt}}^2\right) \\
y_i \mid (S_i^* = \text{at}, \theta, \mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{at}} + \beta_{\text{at}}'\mathbf{x}_i, \sigma_{\text{at}}^2\right) \\
y_i \mid (S_i^* = \text{co}, \theta, \mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{co}} + \beta_{\text{co}}'\mathbf{x}_i + \tau z_i, \sigma_{\text{co},z}^2\right)
\end{aligned}
$$

where we partially pool the coefficients, $\beta_{s,k} \sim N\left(\mu_{\beta,k}, \eta_k^2\right)$, for $k = 1, \ldots, K$. The variance term differs by principal stratum and, among Compliers, by treatment assignment (Bloom et al., 2014).

We then make two modifications to extend model-based IV to a multi-level setting. First, we estimate a varying-intercept/varying-slope model separately for each principal stratum, where $j[i]$ indicates the site j corresponding to child i:

$$
\begin{aligned}
y_i \mid (S_i^* = \text{nt}, \theta, \mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{nt}} + \beta_{\text{nt}}\mathbf{x}_i + \psi_{j[i]}, \sigma_{\text{nt}}^2\right) \\
y_i \mid (S_i^* = \text{at}, \theta, \mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{at}} + \beta_{\text{at}}\mathbf{x}_i + \psi_{j[i]}, \sigma_{\text{at}}^2\right) \\
y_i \mid (S_i^* = \text{co}, \theta, \mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{co}} + \beta_{\text{co}}\mathbf{x}_i + \psi_{j[i]} + \tau z_i + \omega_{j[i]} z_i, \sigma_{\text{co},z}^2\right)
\end{aligned}
$$

$$
\begin{pmatrix} \gamma_j \\ \delta_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \beta^{ctr}\mathbf{w}_j \\ 0 \end{pmatrix}, \begin{pmatrix} \eta_\psi^2 & \rho\eta_\psi\eta_\omega \\ \rho\eta_\psi\eta_\omega & \eta_\omega^2 \end{pmatrix}\right)
$$

Second, we adjust the multinomial logistic regression to include a site-specific intercept. This simple varying-intercept model is repeated across all three principal strata:

$$
\mathbb{P}(S_i = s \mid \theta, \mathbf{x}_i) = \frac{\exp(\gamma_{s,j[i]} + \delta_s'\mathbf{x}_i)}{\sum_{s=1}^K \exp(\gamma_{s,j[i]} + \delta_s'\mathbf{x}_i)}
$$

$$
\begin{pmatrix} \gamma_{n,j} \\ \gamma_{a,j} \\ \gamma_{c,j} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_{\gamma,n} + \delta_n^{ctr}\mathbf{w}_j \\ \mu_{\gamma,a} + \delta_a^{ctr}\mathbf{w}_j \\ \mu_{\gamma,c} + \delta_c^{ctr}\mathbf{w}_j \end{pmatrix}, \begin{pmatrix} \eta_{\gamma,n}^2 & 0 & 0 \\ 0 & \eta_{\gamma,a}^2 & 0 \\ 0 & 0 & \eta_{\gamma,c}^2 \end{pmatrix}\right)
$$

where the site-level coefficients, δ_s^{ctr}, vary across strata, as in the non-hierarchical model.

B.3 Estimating the Principal Stratification Model

To estimate the full principal stratification model, we simply expand the likelihood from the instrumental variable model to account for the additional latent groups:

$$
\begin{aligned}
\mathcal{L}_{obs}(\theta \mid \mathbf{y},\mathbf{x},\mathbf{d},\mathbf{z}) \quad = \quad & \prod_{i:D_i=HS,Z_i=0} \Big\{\pi_{i:\text{ahs}}\ g_{\text{ahs}}(y_i \mid \theta,\mathbf{x}_i)\Big\} \times \\
& \prod_{i:D_i=Center,Z_i=0} \Big\{\pi_{i:\text{cc}}\ g_{\text{cc}\,0}(y_i \mid \theta,\mathbf{x}_i) + \pi_{i:\text{ac}}\ g_{\text{ac}}(y_i \mid \theta,\mathbf{x}_i)\Big\} \times \\
& \prod_{i:D_i=Home,Z_i=0} \Big\{\pi_{i:\text{hc}}\ g_{\text{hc}\,0}(y_i \mid \theta,\mathbf{x}_i) + \pi_{i:\text{ah}}\ g_{\text{ah}}(y_i \mid \theta,\mathbf{x}_i)\Big\} \times \\
& \prod_{i:D_i=?,Z_i=0} \Big\{\pi_{i:\text{cc}}\ g_{\text{cc}\,0}(y_i \mid \theta,\mathbf{x}_i) + \pi_{i:\text{hc}}\ g_{\text{hc}\,0}(y_i \mid \theta,\mathbf{x}_i) + \pi_{i:\text{ah}}\ g_{\text{ah}}(y_i \mid \theta,\mathbf{x}_i)\Big\} \times \\
& \prod_{i:D_i=HS,Z_i=1} \Big\{\pi_{i:\text{ahs}}\ g_{\text{ahs}}(y_i \mid \theta,\mathbf{x}_i) + \pi_{i:\text{cc}}\ g_{\text{cc}\,1}(y_i \mid \theta,\mathbf{x}_i) + \pi_{i:\text{hc}}\ g_{\text{hc}\,1}(y_i \mid \theta,\mathbf{x}_i)\Big\} \times \\
& \prod_{i:D_i=Center,Z_i=1} \Big\{\pi_{i:\text{ac}}\ g_{\text{ac}}(y_i \mid \theta,\mathbf{x}_i)\Big\} \times \\
& \prod_{i:D_i=Home,Z_i=1} \Big\{\pi_{i:\text{ah}}\ g_{\text{ah}}(y_i \mid \theta,\mathbf{x}_i)\Big\} \times \\
& \prod_{i:D_i=?,Z_i=1} \Big\{\pi_{i:\text{ac}}\ g_{\text{ac}}(y_i \mid \theta,\mathbf{x}_i) + \pi_{i:\text{ah}}\ g_{\text{ah}}(y_i \mid \theta,\mathbf{x}_i)\Big\}
\end{aligned}
$$

Table A5 gives this mapping in words. Section C describes the assumptions for an ignorable missingness mechanism. The corresponding outcome models are:

$$
\begin{aligned}
y_i \mid (S_i^* = \text{ahs},\theta,\mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{ahs}} + \beta_{\text{ahs}}\mathbf{x}_i, \sigma^2_{\text{ahs}}\right) \\
y_i \mid (S_i^* = \text{ac},\theta,\mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{ac}} + \beta_{\text{ac}}\mathbf{x}_i, \sigma^2_{\text{ac}}\right) \\
y_i \mid (S_i^* = \text{ah},\theta,\mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{ah}} + \beta_{\text{ah}}\mathbf{x}_i, \sigma^2_{\text{ah}}\right) \\
y_i \mid (S_i^* = \text{cc},\theta,\mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{cc}} + \beta_{\text{cc}}\mathbf{x}_i + \tau_{\text{cc}} z_i, \sigma^2_{\text{cc},z}\right) \\
y_i \mid (S_i^* = \text{hc},\theta,\mathbf{x}_i) &\sim \mathcal{N}\left(\alpha_{\text{hc}} + \beta_{\text{hc}}\mathbf{x}_i + \tau_{\text{hc}} z_i, \sigma^2_{\text{hc},z}\right)
\end{aligned}
$$

where the variance terms for the two complier groups under treatment are assumed to be equal, $\sigma^2_{\text{cc}\,1} = \sigma^2_{\text{hc}\,1}$. Relaxing this assumption gives comparable results but leads to poor model fit, since identification for these variance terms is rather weak. See also Griffin et al. (2008). As in the IV case, we then add in the same multilevel structure, with two treatment effect random slopes rather than just one.

B.4 Estimating the Principal Score Model

To illustrate principal score estimation, it is useful to start with the simpler instrumental variables case—that is, for binary D^*.

First, under one-sided noncompliance (i.e., only Compliers and Never Takers), principal score estimation proceeds almost identically to propensity score estimation in the usual observational setting. In particular, due to randomization, the principal score in the treatment group is the same as the overall principal score: $\mathbb{P}[S_i = s \mid Z_i = 1, \mathbf{X}_i = \mathbf{x}] = \mathbb{P}[S_i = s \mid \mathbf{X}_i = \mathbf{x}]$. Since we can directly observe compliance type in the treatment group, we simply estimate a model using covariates to predict compliance type among treated units. See Hill et al. (2002) and Jo and Stuart (2009).

Second, under two-sided noncompliance (i.e., Compliers, Always Takers, and Never Takers), we observe mixtures of compliance types under both treatment and control, which complicates estimation. One approach is to proceed non-parametrically, using kernel density estimation to estimate $(\pi_{a|\mathbf{x}}, \pi_{c|\mathbf{x}}, \pi_{n|\mathbf{x}})$. In high dimensions, however, this is impractical. Instead, we recommend a simple data augmentation strategy, due to Ibrahim (1990) and applied to causal inference by, among others, Aronow and Carnegie (2013) and Hsu and Small (2014). The essential idea is to use the same model as in Section B.2, except without any outcomes:

- **Estimate the principal score.** Given the vector of compliance types, estimate the principal score via multinomial logistic regression, ignoring treatment assignment.

- **Impute compliance type.** Given the principal score model, impute compliance types for all individuals with unknown type.

The principal score approach for the full model (i.e., multi-valued D) proceeds exactly as under the two-sided noncompliance setting. In addition, we extend the multinomial logistic regression to account for center-level variation, as in Appendix B.2.

B.5 Estimating Quantile Treatment Effects

We provide a brief overview of our approach for estimating the quantile treatment effects (QTE) by principal stratum.

We begin with the simpler setting without covariates. In this case, the overall QTE is simply the differences in observed quantiles under treatment and control. To estimate the QTE by compliance type, we use results from Imbens and Rubin (1997b) and Section 6, non-parametrically estimating the qth quantile, $G_{sz}^{-1}(q)$, for a given stratum outcome distribution, $g_{sz}(y)$ (see also Abadie et al., 2002). The QTE for Compliers, also known as IV-QTE, is therefore

$$\tau_{\text{co}}(q) = G_{\text{co}\,1}^{-1}(q) - G_{\text{co}\,0}^{-1}(q)$$

for quantile q. In practice, we instead might want to estimate the first two moments of the outcome distributions, μ_{sz} and σ_{sz}^2, and then plug these into the Normal quantile function, $\Phi^{-1}(q)$, to obtain the QTE.

There are two main options for estimating the QTE for the principal strata, PS-QTE, which is no longer possible non-parametrically. First, we can use the approach in Section 6 to obtain the mean and variance for the principal stratum outcome distributions under treatment and control, and then plug these estimates directly into the Normal quantile function. Alternatively, we can use this same model to impute stratum membership for each MCMC iteration. We then directly estimate the PS-QTE for all Center Compliers and Home Compliers for a given MCMC iteration, and combine the resulting estimates across iterations. While the second approach does not depend as directly on the assumption of Normality, the estimates are still sensitive to the parametric model, especially in the tails.

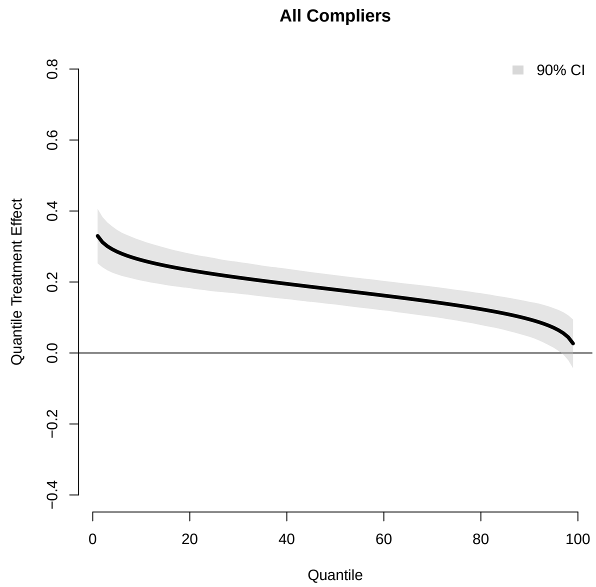
Estimation is more complicated with covariates. In particular, there are two possible objects of interest: conditional and unconditional QTE. For conditional QTE, the impacts are on the outcome distributions conditional on covariates. Estimation of the conditional QTE is straightforward via quantile regression (e.g., Angrist and Pischke, 2008). Abadie et al. (2002) proposed a weighted quantile regression estimator of the conditional IV-QTE. For unconditional QTE, the impacts are on the marginal outcome distributions. Firpo (2007) proposes to estimate these effects via the difference between weighted treatment and control quantiles, with inverse propensity score weights. Frölich and Melly (2013) proposes a slightly different set of weights to estimate the unconditional IV-QTE. Following Bitler et al. (2014), we focus on the unconditional QTE here.

As in the no covariates case, there are two main approaches. First, we can use the full principal model to estimate the marginal μ_{sz} and σ^2_{sz} for each principal stratum, using covariates to improve the precision of these estimates. The unconditional IV-QTE and PS-QTE are then differences between Normal quantiles. Second, we can use the model to impute stratum membership for each MCMC iteration. We can then use the inverse propensity score weighting approach of Firpo (2007) to estimate the covariate-adjusted unconditional IV-QTE and PS-QTE for each MCMC draw, combining estimates across iterations for final inference. As in the no covariate case, this approachrelies less on the Normality assumption, but is still likely to be sensitive to the particular model choice.

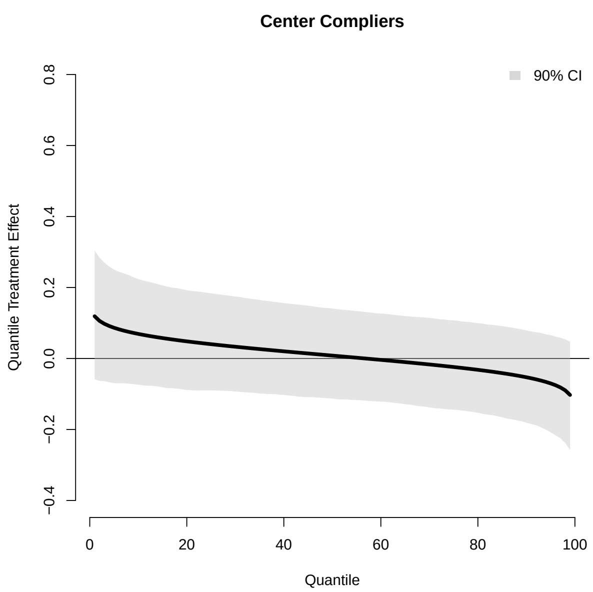
Figure A1 shows the results of the Normal approximation approach for the Head Start year for all Compliers, Center Compliers, and Home Compliers. Figure 4 shows comparable estimates using the inverse propensity score weighting approach. As expected, the results are very similar between the two approaches, though the impacts at the bottom of the distribution are larger without relying directly on the Normal quantiles.

B.6 Sensitivity Checks

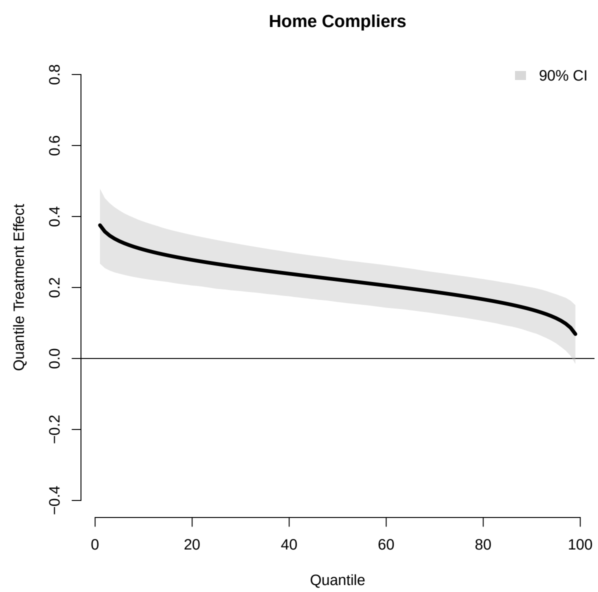
We briefly discuss robustness checks for our main model. First, we assess sensitivity to our handling of missing data and re-fit the principal stratification model using only observed outcomes, approximately 80 percent of the overall sample. Table A1 shows the resulting complete case estimates, which are essentially unchanged from the full version. Second, as we discuss in Section 6, the Normality assumption plays a critical role in both identification and estimation. Table A1 shows



(a) All Compliers.



(b) Center Compliers.



(c) Home Compliers.

Figure A1: Quantile treatment estimates by compliance type for the Head Start year using the Normal approximation, with approximate 90 percent credible intervals.

Table A1: Sensitivity Analysis; First Year

	Normal; Complete case	Student t_7; All observations
$LATE$ for Center Compliers	0.03 (-0.07, 0.15)	0.04 (0.08)
$LATE$ for Home Compliers	**0.21** **(0.15, 0.27)**	**0.21** **(0.15, 0.26)**
$\mathbb{P}\{LATE_{\text{hc}} > LATE_{\text{cc}}\}$	0.98	0.96

Notes: Point estimates are posterior medians, with 2.5 and 97.5 quantiles of posterior distribution in parentheses. 95% posterior intervals that exclude zero are printed in bold. See Section B for estimation details, and Section **??** for a list of all individual- and center-level covariates. All treatment effects, including $LATE_{\text{cc}}$ and $LATE_{\text{hc}}$, vary by Head Start center of random assignment and by cohort.

the same model using a heavy-tailed Student t_7 distribution rather than a Gaussian. Again, the results are consistent under both models.

B.7 Assessing the Normality Assumption

A key assumption of the model is that the component densities follow a Normal distribution. While we cannot directly assess Normality for all seven outcome distributions, in Appendix A, we demonstrate non-parametric identification for the outcome distributions of the three "always" strata and the two Complier strata under control. We can therefore estimate these distributions via kernel density estimation, as discussed in Imbens and Rubin (1997b). We can then use the moment equations from Appendix A to estimate the mean and variance for these five distributions. To illustrate this, we focus on the mixture of Home-based Compliers under control and Always Home children:

$$m_1 = \phi\mu_{\text{hc}\,0} + (1-\phi)\mu_{\text{ah}}$$
$$m_2 = \phi\;\sigma^2_{\text{hc}\,0} + (1-\phi)\;\sigma^2_{\text{ah}} + \phi(1-\phi)(\mu_{\text{hc}\,0} - \mu_{\text{ah}})^2$$

where $\phi = \frac{\pi_{\text{hc}}}{\pi_{\text{ah}}+\pi_{\text{hc}}}$. We directly observe the sample moments for the Always Home strata. We can then obtain the corresponding sample estimates for the Home-based Compliers via:

$$\widehat{\mu}_{\text{hc}\,0} = \frac{1}{\widehat{\phi}}\widehat{m}_1 - \frac{1-\widehat{\phi}}{\widehat{\phi}}\widehat{\mu}_{\text{ah}}$$
$$\widehat{\sigma}^2_{\text{hc}\,0} = \frac{1}{\widehat{\phi}}\widehat{m}_2 - \frac{1-\widehat{\phi}}{\widehat{\phi}}\widehat{\sigma}^2_{\text{ah}} - (1-\widehat{\phi})(\widehat{\mu}_{\text{hc}\,0} - \widehat{\mu}_{\text{ah}})^2.$$

Figure A2 shows the corresponding kernel density estimates and Normal approximations for the PPVT. Overall, the nonparametric estimates and Normal approximations show excellent agreement, though there is some moderate skewness in the Always Head Start outcome distribution. Finally, for the Always Head Start, Always Center-based, and Always Home-based subgroups, we can also check Normal quantile-quantile plots, shown in Figure A3. These also show very good fit overall, though the small sample sizes make this difficult to assess.

B.8 Posterior Predictive Checks

Following Rubin et al. (1984) and Gelman et al. (2013), we use posterior predictive checks to assess the fit of our full model to the observed data. Formally, let y be the observed data and θ be the parameter vector. Define y^{rep} as the replicated data that could have been observed if the study were replicated with the same model and the same value of θ that produced y. We can estimate the distribution of y^{rep} via the posterior predictive distribution,

$$p(y^{\text{rep}} \mid y) = \int p(y^{\text{rep}} \mid \theta) p(\theta \mid y) d\theta.$$

The intuition is to assess whether the replicated data produced from the model are similar to the observed data.

First, we can visually compare the overall distributions of y and y^{rep}. Figure A4 shows a histogram of the observed data, y, and five replicated data sets, y^{rep}, for PPVT score. The histograms are indistinguishable from each other, suggesting that the model correctly captures the main features of the outcome distribution.

Second, we compare specific features of the distributions for y and y^{rep}. Define $T(y)$ as a test statistic that only depends on the data. We then determine whether the observed value of the test statistic, $T(y)$, is similar to the test statistics for the replicated data, $T(y^{\text{rep}})$. We can assess this numerically via a posterior predictive p-value:

$$p_{\text{pp}} = \mathbb{P}\{T(y^{\text{rep}}) \geq T(y) \mid \text{data}\}.$$

Intuitively, this is the proportion of replicated test statistics that are more extreme than the observed test statistic. We can also assess this discrepancy using visual summaries.

A key issue is choosing an appropriate test statistic. Following Barnard et al. (2003) and Mattei et al. (2013), we use the following three test statistics, which aim to assess whether the model captures broad features of the signal and noise available in the data:

- $T_{\text{signal}}(y) = |\overline{Y}_{s1} - \overline{Y}_{s0}|$, where $\overline{Y}_{sz}$ is the outcome mean for all children in stratum s and condition z
- $T_{\text{noise}}(y) = \sqrt{\frac{s^2_{s1}}{N_{s1}} + \frac{s^2_{s0}}{N_{s0}}}$, where s^2_{sz} is the outcome variance for all children in stratum s and condition z
- $T_{\text{ratio}}(y) = \frac{T_{\text{signal}}(y)}{T_{\text{noise}}(y)}$

We then compare $T(y^{\text{rep}})$ to $T(y)$ for each test statistic and each stratum to compute a posterior predictive p-value, where values close to 0 or 1 indicate poor model fit. As shown in Table A2, all

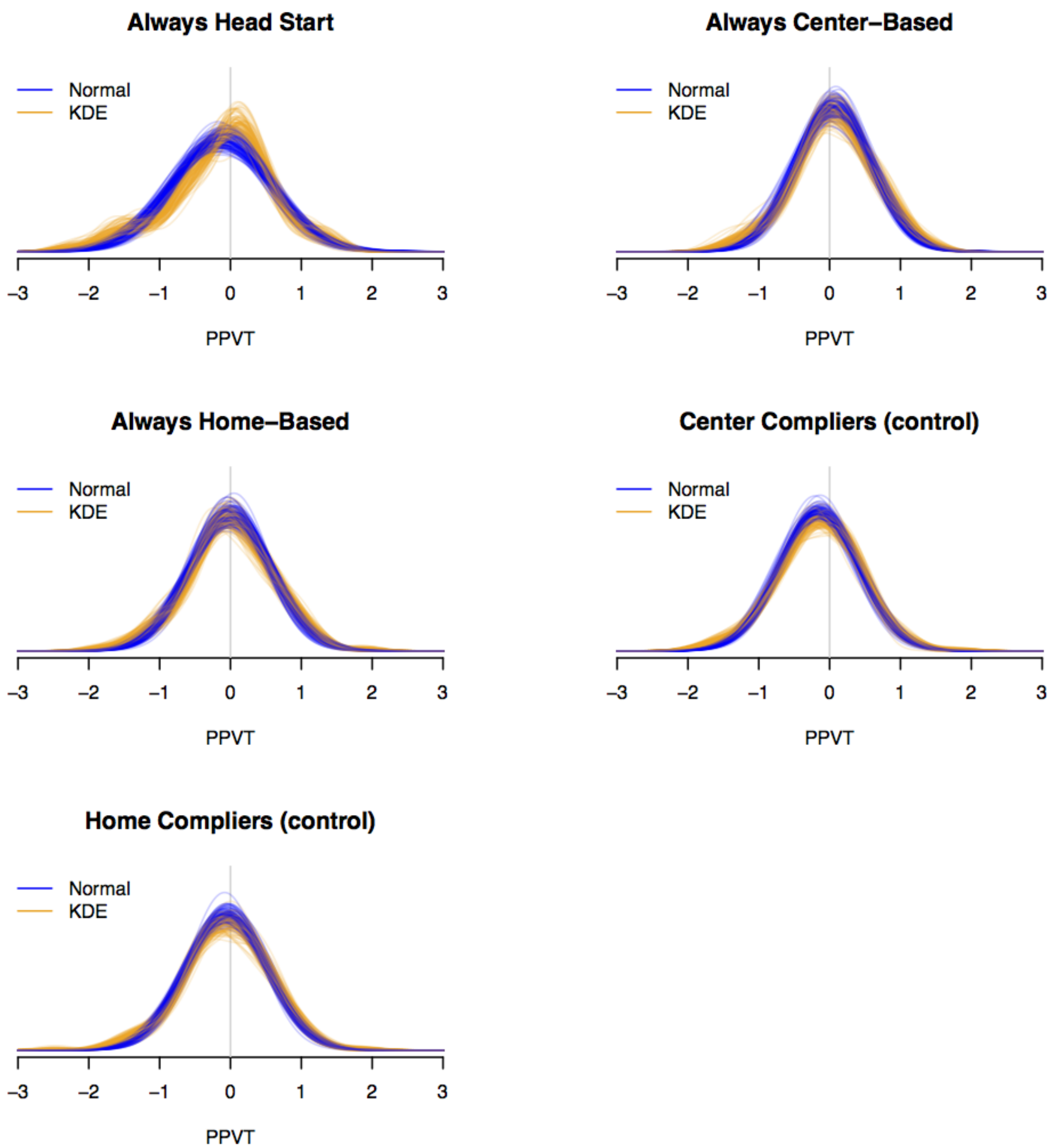
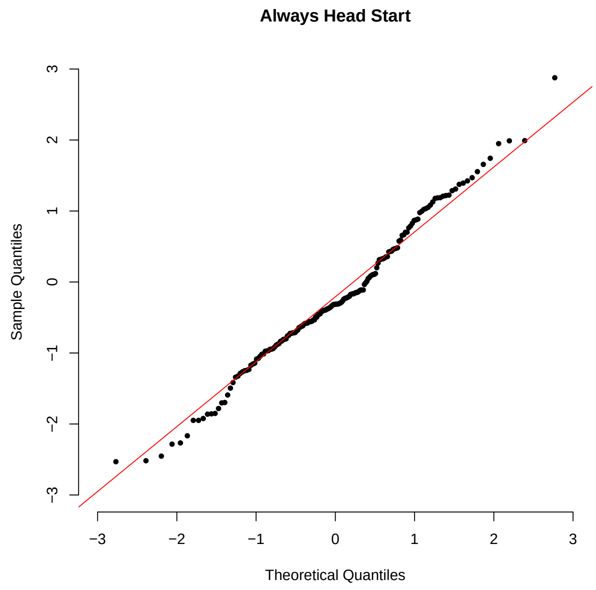
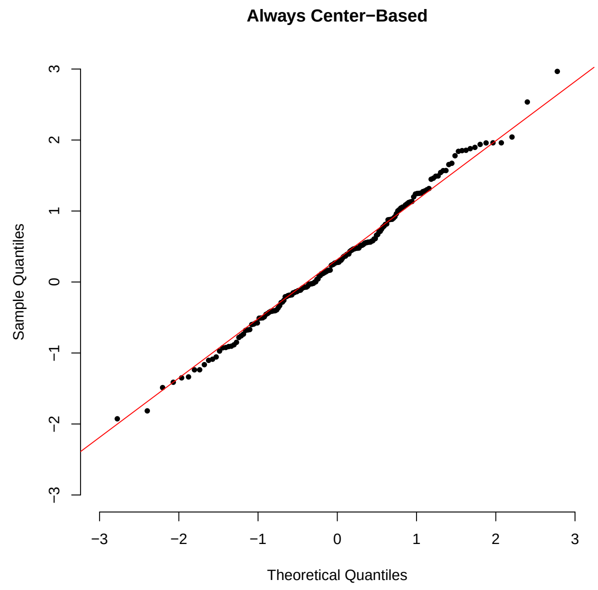


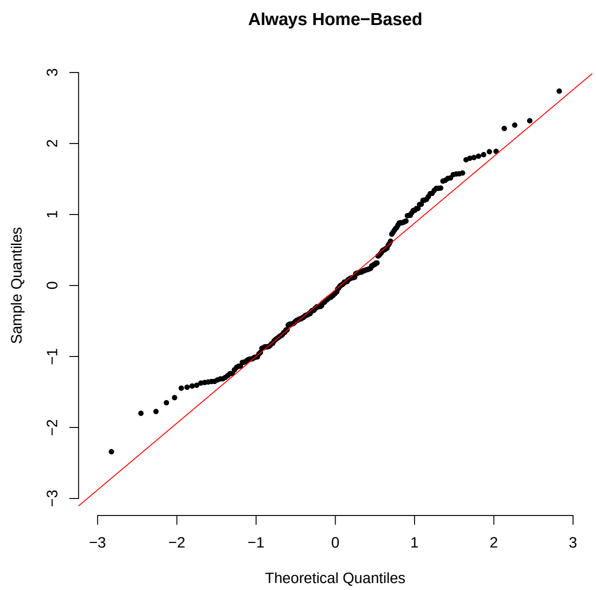
Figure A2: Comparison of kernel density estimates and Normal approximations for the five non-parametrically identified outcome distributions for the PPVT. Each plot shows 100 bootstrapped estimates.



(a) Always Head Start.



(b) Always Center-Based.



(c) Always Home-Based.

Figure A3: Normal quantile-quantile plots for the outcome distributions of three principal strata for the PPVT score.

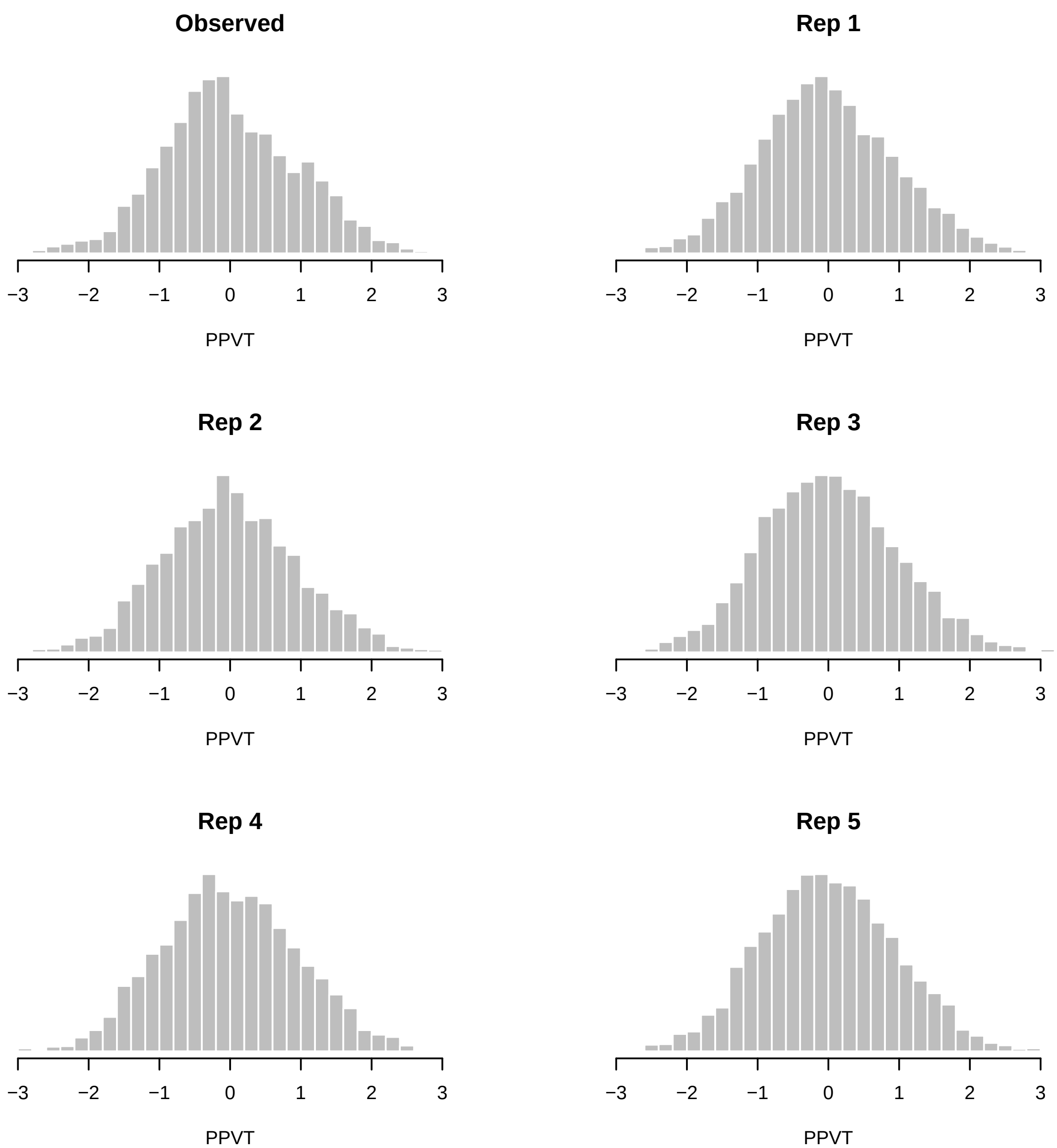


Figure A4: Overall outcome distributions for the observed data, y, and five replicated data sets, y^{rep}, using PPVT score.

posterior predictive p-values are away from the extremes, showing excellent model fit overall.

Table A2: Posterior Predictive p-values

Stratum	$T_{\text{signal}}(y)$	$T_{\text{noise}}(y)$	$T_{\text{ratio}}(y)$
Always Head Start	0.77	0.41	0.77
Always Center-based	0.21	0.38	0.22
Always Home-based	0.40	0.20	0.41
Center-based Compliers	0.59	0.48	0.60
Home-based Compliers	0.40	0.09	0.46

C Missing Data

Survey nonresponse and general data missingness present major hurdles in the analysis of the Head Start Impact Study. In general, we account for three types of missing data in our analysis: missing outcomes, missing care type, and missing covariates. We focus in particular on missing pre-test scores, since this variable presents a unique set of challenges.

C.1 Missing Outcomes

As shown in Table A3, a substantial proportion of test scores scores are missing, with differential missingness by treatment status. Moreover, this missingness pattern is non-monotone; for example, some children have missing outcomes in the second year of observation but observed outcomes in the third year.

Table A3: Percent Missing PPVT Score.

	Control	Treatment	Difference
Pre-Test	0.33	0.19	-0.13
HS Year	0.24	0.13	-0.11
Pre-K/K	0.24	0.16	-0.08
K/1st	0.27	0.19	-0.08

Failing to account for such missingness, especially the differential missingness across experimental conditions, can lead to biased estimates (Frangakis and Rubin, 1999). A common approach to address this issue is to assume that outcomes are Missing at Random (MAR) (Rubin, 1976):

$$M_i \perp\!\!\!\perp Y_i \mid \mathbf{X}_i, Z_i, D_i^{obs} \qquad \text{(Missing at Random)}$$

where M_i is an indicator for missing outcome. We can re-write this as: $\mathbb{P}\{M_i \mid Y_i, \mathbf{X}_i, Z_i, D_i^{obs}\} = \mathbb{P}\{M_i \mid \mathbf{X}_i, Z_i, D_i^{obs}\}$. In other words, given covariates, treatment assignment, and observed child care setting, missing outcomes are just as likely to be low test scores as high test scores. This is a very sensible assumption in the Head Start Impact Study, as the data collection procedure depended heavily on the child's care setting.

Although implicit, this is the assumption behind the nonresponse adjustment in the official HSIS report. Specifically, Puma et al. (2010a) estimate a model predicting item nonresponse as a function of child and family covariates, geography, Head Start offer, whether the child enrolled in Head Start, and other missingness indicators (e.g., missing pre-test score). The authors then weight units by the inverse of the predicted probability of nonresponse. We make the same key assumption but implement this via the likelihood rather than via nonresponse weights (Mealli et al., 2004; Frumento et al., 2012). In that case, the missingness mechanism is *ignorable* (Rubin, 1976) since the likelihood factors such that the distribution of missingness indicators can be ignored in subsequent estimation. For computational reasons, we explicitly impute the missing outcomes rather than simply drop these terms from the likelihood, though the parameter estimation is the same.

Finally, we note that other missingness mechanisms are possible here. One promising but more technical assumption is *Latent Ignorability* (Frangakis and Rubin, 1999):

$$M_i \perp\!\!\!\perp Y_i \mid \mathbf{X}_i, Z_i, S_i. \qquad \text{(Latent Ignorability)}$$

Here, the missingness mechanism depends not only on observed care type, but also on the child's principal stratum. In other words, for control group children in center care, the probability of missingness could differ for Center Compliers and Always Center children. Nonetheless, there is no reason to believe that this relaxation is necessary here.

C.2 Missing Care Type

Based on survey responses alone, approximately 15 percent of children are missing information on their focal care setting. Using information elsewhere in the data, we can re-classify around one-third of these children. First, we utilize administrative data for "no shows" and "crossovers" collected for purposes of determining the IV estimate, which re-classifies 118 children. Second, we utilize Fall 2002 survey responses for parents who did not respond to the Spring 2003 survey, which re-classifies an additional 99 children. Table A4 shows detailed information for this procedure.

Table A4: Missing Care Type

	Raw Survey	+ Admin. Data	+ Fall 2002 Survey
Head Start	2,083	2,201	2,202
Other Center-Based Care	634	634	654
Other Home-Based Care	1,014	1,014	1,092
Unknown	654	536	437

Even after this effort, there is still substantial substantial missingness: with 16 percent of control group children and 6 percent of treatment group are missing care type, as shown in Table 2.

Following Frumento et al. (2012), we assume that children with missing care type would belong to one of the otherwise existing principal strata, which means that children with missing care type are simply added to the likelihood components for these other strata. These relationships are shown in Table A5. Importantly, we assume that children with missing care type could be in center care or home care, but not in Head Start; this is sensible since HSIS staff kept exhaustive records of Head Start attendance. Therefore, treatment group children with missing care type could be in either the Always Center or Always Home strata, but could not be Compliers, with the probability of being in the Always Center vs. Always Home stratum dependent on covariates and the outcome.

Table A5: Expanded version of Table 5b: Relationship between care type and principal stratum

Z_i	D_i^{obs}	Possible Principal Strata
1	HS	Always Head Start, Center Complier, Home Complier
1	Center	Always Center
1	Home	Always Home
1	?	Always Center, Always Home
0	HS	Always Head Start
0	Center	Always Center, Center Complier
0	Home	Always Home, Home Complier
0	?	Always Home, Home Complier, Always Center, Center Complier

C.3 Missing Covariates

As with outcomes and care setting, many children are missing a number of key covariates. The public use file for HSIS imputes these missing covariates, primarily via hot deck imputation. Importantly, the observed "donor cases" chosen for the hot deck were selected not only on the basis of covariates available in the HSIS file, but also on geographic and other programmatic variables not available to the public.

Ideally, we would multiply impute missing covariates alongside missing outcomes and missing care type, such as in Frumento et al. (2012). However, given the computational demands of multiple imputation and the non-public factors utilized for hot deck imputation, we use the imputed variables in the public use files. Note that this ignores the uncertainty associated with this imputation, though this uncertainty is likely quite small.

Finally, two of the 340 Head Start centers were missing all geocoded data. Note that while these were coded in the data file as centers, these are actually two of the grantees for which randomization was performed at the grantee level, rather than at the center/center group level. Given the small proportion of missingness here, we use simple mean value imputation to create a complete center-level data file.

C.4 Missing Pre-Test

The pre-test introduces several complications into the HSIS analysis. First, the pre-test is not, in fact, a true pre-test, since the test was conducted up to halfway through the Head Start year. However, we can reasonably assume the tests were administered early enough in the year that there was no meaningful treatment effect. The observed pre-tests are consistent this assumption.

Second, missingness is substantially higher in the control group than in the treatment group. As we do with missing outcomes, we multiply impute missing pre-test scores under the assumption of Missingness at Random:

$$M_i \perp\!\!\!\perp Pre_i \mid \mathbf{X}_i, Z_i, D_i^{obs}.$$

While this is a reasonable assumption for the missingness mechanism, the resulting imputations are conditional on D^{obs}. As a result, we cannot simply include these imputed scores in a later outcome regression, since the dependence on a post-treatment outcome would induce bias. At the same time, both M and Pre show large differences by observed care type, which suggests that dropping the dependence on D^{obs} altogether is not appropriate. Fortunately, we can sidestep this issue by conditioning on principal stratum membership, $S_i \equiv (D_i(0), D_i(1))$, as well as $\mathbf{X}$. Formally, we can factor the joint distribution of missingness and pre-test as follows under the MAR assumption:

$$\begin{aligned}
\mathbb{P}\{M_i, Pre_i \mid \mathbf{X}_i, Z_i, D_i^{obs}\} &= \mathbb{P}\{M_i \mid \mathbf{X}_i, Z_i, D_i^{obs}\} \cdot \mathbb{P}\{Pre_i \mid \mathbf{X}_i, Z_i, D_i^{obs}\} \\
&= \mathbb{P}\{M_i \mid \mathbf{X}_i, Z_i, D_i^{obs}\} \cdot \mathbb{P}\{Pre_i \mid \mathbf{X}_i, S_i\}
\end{aligned}$$

Ideally, we would incorporate the uncertainty in the imputation by drawing a new value of Pre_i for every MCMC iteration, using an imputation model that does not condition on either Z or Y. This is difficult in practice, however, especially with Stan. As a result, we adopt a hybrid approach similar to Frumento et al. (2012). First, we generate five separate data sets, which are identical except for different draws of the imputed pre-test score. Second, we run separate MCMC chains on each data set and combine these for final inference. Overall, we find very good convergence with this approach.

D Proofs

Proof of Lemma 1 (Non-Parametric Identification of the Distribution of Principal Strata).

The proof below is a simple extension of Lemma 3.1 in Abadie (2003). Under monotonicity and valid randomization, we have the following series of equalities, relating oberved population proportions to principal strata proportions. We repeatedly condition on X to emphasize the role of covariates, though for complete randomization, this equality holds unconditionally.

$$\begin{aligned}
\mathbb{P}\{D_1 = \text{HS} \mid Z = 1, X\} &= \mathbb{P}\{S = \text{Always Head Start} \mid X\} \\
&\quad + \mathbb{P}\{S = \text{Center Complier} \mid X\} \\
&\quad + \mathbb{P}\{S = \text{Home Complier} \mid X\} \\
\mathbb{P}\{D_1 = \text{Center} \mid Z = 1, X\} &= \mathbb{P}\{S = \text{Always Center} \mid X\} \\
\mathbb{P}\{D_1 = \text{Home} \mid Z = 1, X\} &= \mathbb{P}\{S = \text{Always Home} \mid X\}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}\{D_0 = \text{HS} \mid Z = 0, X\} &= \mathbb{P}\{S = \text{Always HS} \mid X\} \\
\mathbb{P}\{D_0 = \text{Center} \mid Z = 0, X\} &= \mathbb{P}\{S = \text{Always Center} \mid X\} \\
&\quad + \mathbb{P}\{S = \text{Center Complier} \mid X\} \\
\mathbb{P}\{D_0 = \text{Home} \mid Z = 0, X\} &= \mathbb{P}\{S = \text{Always Home} \mid X\} \\
&\quad + \mathbb{P}\{S = \text{Home Complier} \mid X\}
\end{aligned}$$

We can then immediately identify the following:

$$\begin{aligned}
\mathbb{P}\{S = \text{Always HS} \mid X\} &= \mathbb{P}\{D^{obs} = \text{HS} \mid Z = 0, X\} \\
\mathbb{P}\{S = \text{Always Center} \mid X\} &= \mathbb{P}\{D^{obs} = \text{Center} \mid Z = 1, X\} \\
\mathbb{P}\{S = \text{Always Home} \mid X\} &= \mathbb{P}\{D^{obs} = \text{Home} \mid Z = 1, X\}
\end{aligned}$$

Substituting these into the first set of equations:

$$\begin{aligned}
\mathbb{P}\{S = \text{Center Complier} \mid X\} &= \mathbb{P}\{D^{obs} = \text{Center} \mid Z = 0, X\} - \mathbb{P}\{S = \text{Always Center} \mid X\} \\
&= \mathbb{P}\{D^{obs} = \text{Center} \mid Z = 0, X\} - \mathbb{P}\{D^{obs} = \text{Center} \mid Z = 1, X\} \\
\mathbb{P}\{S = \text{Home Complier} \mid X\} &= \mathbb{P}\{D^{obs} = \text{Home} \mid Z = 0, X\} - \mathbb{P}\{S = \text{Always Home} \mid X\} \\
&= \mathbb{P}\{D^{obs} = \text{Home} \mid Z = 0, X\} - \mathbb{P}\{D^{obs} = \text{Home} \mid Z = 1, X\}
\end{aligned}$$

Therefore, the principal score, $\mathbb{P}\{S = s \mid X\}$, is non-parametrically identified for all principal strata, s.

Proof of Lemma 2 (Distribution of Covariates by Principal Stratum). The proof is immediate. We want to identify the quantity $\mathbb{P}\{\mathbb{X}_i = \mathbf{x} \mid S_i = s\}$. From Bayes' Rule:

$$\mathbb{P}\{\mathbb{X}_i = \mathbf{x} \mid S_i = s\} = \frac{\mathbb{P}\{S_i = s \mid \mathbb{X}_i = \mathbf{x}\} \cdot \mathbb{P}\{\mathbb{X}_i = \mathbf{x}\}}{\mathbb{P}\{S_i = s\}}$$

From Lemma 1, we can non-parametrically identify $\mathbb{P}\{S_i = s\}$. Due to randomization, we can also identify $\mathbb{P}\{S_i = s \mid \mathbb{X}_i = \mathbf{x}\}$. Finally, we can identify the overall distribution of covariates in the sample, $\mathbb{P}\{\mathbb{X}_i = \mathbf{x}\}$. Therefore, $\mathbb{P}\{\mathbb{X}_i = \mathbf{x} \mid S_i = s\}$ is non-parametrically identified.

See Abadie (2003) for a similar argument.