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PART VII

Supplements for Selected Chapters

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SUPPLEMENT TO CHAPTER 2: The Equations of Exchange Equilibrium

This supplement introduces the notation and structure of the formal models that we develop in subsequent supplements.

For notation, D refers to demands and x to production. Thus D_F signifies the home country's demand for food, and x_C^* the foreign country's production of clothing. The asterisk symbolizes foreign variables, as in the text. The price of commodity j is denoted by p_j if a monetary unit of account is used for the home country or p_j^* if the foreign country uses a different unit of account or if the foreign price differs. In the two-commodity food and clothing example, the home country's prices are p_F and p_C. The relative price of food is p_F/p_C. Because the phrase "terms of trade" is so prominent in the real models of trade, the simple p (in the home country) and p^* (in the foreign country, if prices are different) denote the terms of trade, the relative price of food.

The use of equations is not completely forsaken in the text. For this reason a different numbering scheme is required for the supplements. Thus Equation 2.S.4 refers to the fourth equation in the supplement to Chapter 2.

This account of the basic model begins by stating prices in monetary units. The budget constraint for this model posits that for each country the value of aggregate demand must be restricted to, and equal to, the value of production. Thus:

$$p_C D_C + p_F D_F = p_C x_C + p_F x_F \tag{2.S.1}$$

$$p_C^* D_C^* + p_F^* D_F^* = p_C^* x_C^* + p_F^* x_F^* \tag{2.S.2}$$

Assume that in a trading context the home country will import food. Then rewrite these two equations to highlight, on the left side, the country's demand for imports and, on the right side, the corresponding supply of exports.

$$p_F(D_F - x_F) = p_C(x_C - D_C) \tag{2.S.3}$$

$$p_C^*(D_C^* - x_C^*) = p_F^*(x_F^* - D_F^*) \tag{2.S.4}$$

The importance of *relative* prices is brought out by dividing Equation 2.S.3 by p_C and Equation 2.S.4 by p_C^*. Furthermore, in a free-trade equilibrium with no barriers to costless movement of commodities between countries, relative prices in the two countries are brought into line so that

$$p(D_F - x_F) = (x_C - D_C) \tag{2.S.5}$$

$$(D_C^* - x_C^*) = p(x_F^* - D_F^*) \tag{2.S.6}$$

The symbol p represents the relative price of food.

Suppose the terms of trade, p, clear the world market for food. That is, the home country's excess demand, $(D_F - x_F)$, equals the foreign country's excess supply, $(x_F^* - D_F^*)$. In such a case it is obvious from Equations 2.S.5 and 2.S.6 that the world's clothing market must be cleared as well: $(D_C^* - x_C^*)$ will equal $(x_C - D_C)$.

One consequence of this phenomenon is that free-trade market equilibrium can be expressed by the statement that *either* world demand and supply are equal for food (as in Equation 2.S.7) *or* they are equal for clothing (as in Equation 2.S.8):

$$D_F + D_F^* = x_F + x_F^* \tag{2.S.7}$$

$$D_C + D_C^* = x_C + x_C^* \tag{2.S.8}$$

If the budget constraints in Equations 2.S.5 and 2.S.6 are always satisfied, Equation 2.S.7 implies Equation 2.S.8, or vice versa. Oddly enough, neither market-clearing equation is typically used in the literature of the pure theory of trade. Rather, they are replaced by the equivalent statement that in free-trade equilibrium, the value of the home country's imports equals the value of the foreign country's imports. This balance of payments equilibrium condition, in Equation 2.S.9, follows from the two budget constraints, Equations 2.S.5 and 2.S.6, and either Equation 2.S.7 or 2.S.8.

$$p(D_F - x_F) = (D_C^* - x_C^*) \tag{2.S.9}$$

This redundancy in stating equilibrium conditions is two-sided. On the one hand, it reveals that the model is more simple than a mere scanning of equations might reveal: There is only one market, and if world demand for clothing balances world production at specified terms of trade, then the food market must be cleared as well. Furthermore, the value of each country's demand for imports would, at those market-clearing terms of trade, equal the other country's demand for imports. On the other hand, it implies there are several ways to describe the same equilibrium: The food market is cleared, the clothing market is cleared, or the home country's demand for imports equals, in value, the foreign country's demand for imports. Saying the same thing in three different ways can be confusing.

Changes in Real Incomes

Throughout, assume that a community's level of satisfaction or real income depends only on the bundle of commodities it consumes. For the two-commodity example this can be stated formally as

$$u = u(D_C, D_F)$$

The symbol u represents some arbitrary index used to measure utility or the level of welfare. Differentiate this expression to obtain

$$du = \frac{\partial u}{\partial D_C}\, dD_C + \frac{\partial u}{\partial D_F}\, dD_F$$

which states that when the amounts consumed are altered, utility changes by an amount that depends on the marginal utility of a commodity (e.g., $\partial u/\partial D_F$ for food) multiplied by the change in the quantity of it consumed. The arbitrariness of the utility index can be removed by dividing both sides of this equation by the marginal utility of clothing.

$$\frac{du}{\partial u/\partial D_C} = dD_C + \frac{\partial u/\partial D_F}{\partial u/\partial D_C}\, dD_F$$

The left-hand term is positive only if utility has increased. Furthermore, it is a measure of the change in utility expressed in units of clothing (the *utils* cancel out). Call this change in real income in clothing units dy. The right-hand side can be simplified by noticing that the coefficient of dD_F is the *marginal rate of substitution,* the amount of clothing that must be added to compensate for a loss of one unit of food along an indifference curve. In a market equilibrium, however, this amount corresponds to the relative price of food, p. Thus, Equation 2.S.10 can be derived as the basic expression for a change of real income.

$$dy = dD_C + pdD_F \tag{2.S.10}$$

It could almost be taken as a *definition* of real income changes—the sum of consumption changes with each such change weighted by the relative price of that commodity.

The budget constraint,

$$D_C + pD_F = x_C + px_F \tag{2.S.11}$$

reveals that the *source* of any change in real income must reside in either a change in the endowment bundle or a change in the terms of trade. To see this, differentiate to obtain

$$dD_C + pdD_F + D_F dp = dx_C + pdx_F + x_F dp$$

Subtract $D_F dp$ from both sides, and use Equation 2.S.10 for dy to obtain

$$dy = -(D_F - x_F)dp + (dx_C + pdx_F) \tag{2.S.12}$$

This basic expression for the change of real income in the home country provides the following breakdown.

1. The term $-(D_F - x_F)dp$ is the *terms-of-trade effect*. Assume the home country is a net importer of food, and let M denote $(D_F - x_F)$. If the terms of trade deteriorate for the home country, dp is positive and real income at home falls by Mdp, an amount proportional to the volume of imports.

2. The term $dx_C + pdx_F$, the price-weighted sum of any change in the home country's production bundle, enters directly into the measure of a change in real income.

A Basic Production Relationship

If production possibilities are shown by a bowed-out transformation schedule (as in Figure 2.3), a rise in food's relative price, p, would encourage food production and discourage clothing output. Nonetheless, for output movements along the transformation schedule,

$$dx_C + pdx_F = 0 \tag{2.S.13}$$

The reason is simple: At a competitive equilibrium (e.g., point B in Figure 2.3) the absolute value of the slope of the transformation schedule, $-(dx_F/dx_C)$, must equal clothing's relative price, $1/p$. Of course in the special case of rigid production [Figure 2.4(a)], both dx_C and dx_F are separately zero.

Substitution and Income Effects

Appendix B to Chapter 2 suggested that any change in price has both a substitution and an income effect on quantity demanded. The decomposition into these two effects can be expressed algebraically for small price changes, making use of Equation 2.S.12's expression for the change in real income, which is simplified by Equation 2.S.13's relationship among outputs.

The demand for any commodity depends on all prices and income. Alternatively, in a two-commodity model it depends on relative price, p, and real income, y.[1] For example, consider the home country's demand for food, written as in Equation 2.S.14.

$$D_F = D_F(p, y) \tag{2.S.14}$$

Differentiate this with respect to food's relative price, p, to obtain

$$\frac{dD_F}{dp} = \frac{\partial D_F}{\partial p} + \frac{\partial D_F}{\partial y} \cdot \frac{dy}{dp}$$

The first term is the substitution effect of a price rise—as p rises, food demand falls along an indifference curve. The second composite term shows the two aspects of the income effect described in the text. The term dy/dp shows how real income at home has been affected by the rise in food's relative price. Equation 2.S.12 reveals that dy/dp is just $-(D_F - x_F)$ because any output response along the transformation curve has negligible impact on real incomes (by Equation 2.S.13). If food is imported, dy/dp is negative. The other term, $\partial D_F/\partial y$, expresses the change in demand for food as a consequence of a unit rise in incomes with prices constant. This is not a pure number because D_F is measured in food units and y in clothing units. Therefore define α_F as p times $(\partial D_F/\partial y)$, so that α_F is the home country's marginal propensity to consume food. This is a pure number, between 0 and 1 if neither commodity is "inferior." Therefore Equation 2.S.15 depicts the breakdown of dD_F/dp into substitution and income effects.

$$\frac{dD_F}{dp} = \frac{\partial D_F}{\partial p} - \frac{(D_F - x_F)}{p} \cdot \alpha_F \tag{2.S.15}$$

This breakdown of demand shows the importance of the direction of trade. If food is imported at home, both income and substitution terms combine to reduce food demand as the relative price of food rises. However, if food were exported, the income

[1] The change in real income, dy, has been defined by Equation 2.S.10. Mathematical liberties are taken here in using the symbol y for real income itself. However, this supplement requires only the expression for dy because it considers only "small" changes in prices and demands.

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effect of a rise in food's price would be positive, running counter to the substitution effect and, in some cases, resulting in more food being demanded locally.

The Hat Notation

It will often prove convenient to express the change in a variable, dx, as a fraction of the original value of that variable, x. A hat symbol, ^, denotes this relative change. Thus, for any variable, x,

$$\hat{x} \equiv \frac{dx}{x}$$

The Elasticity of Demand for Imports

This discussion of the components of demand behavior can be added to a consideration of production changes to investigate the *elasticity of demand for imports,* ε, defined as

$$\varepsilon \equiv -\frac{\hat{M}}{\hat{p}} \tag{2.S.16}$$

where the minus sign is used to make ε a positive number. M, of course, refers to home imports of food,

$$M = D_F - x_F$$

We argued in the text that three ingredients are involved in the expression for ε, the elasticity of demand for imports. As we now show, ε can be expressed as the simple sum of (1) $\overline{\eta}$, the pure substitution elasticity of demand, (2) m, the marginal propensity to import, and (3) e, the elasticity of supply for import-competing production:

$$\varepsilon = \overline{\eta} + m + e \tag{2.S.17}$$

To see this, differentiate the expression for M and use hat notation:

$$-\frac{\hat{M}}{\hat{p}} = -\frac{D_F}{M} \cdot \frac{\hat{D}_F}{\hat{p}} + \frac{x_F}{M} \cdot \frac{\hat{x}_F}{\hat{p}}$$

The expression for

$$-\frac{D_F \hat{D}_F}{M \hat{p}}$$

follows readily from Equation 2.S.15. Let $\overline{\eta}$ represent the (negative of the) pure substitution term in demand,

$$\overline{\eta} \equiv \frac{p}{-M} \cdot \frac{\partial D_F}{\partial p}$$

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and m the marginal propensity to import, which is the marginal propensity to consume the imported good (food) at home, α_F. Finally, define the elasticity of import-competing production, e, as[2]

$$e \equiv -\frac{p}{M} \cdot \frac{dx_F}{dp}$$

Combining yields the final breakdown for the elasticity of demand for imports, ε.

Comparative Advantage and the Gains from Trade

A basic line of argument reveals how competitive behavior leads to gains from international trade when countries take advantage of world markets to import commodities that are relatively inexpensive compared to autarky. In striving for generality, this discussion removes the two-commodity (food and clothing) limitation and considers a country originally consuming and producing many commodities before international trade. Let autarky market-clearing prices and quantities be indicated by the ^{0} superscript, so that before trade, item by item

$$D_i^0 = x_i^0 \tag{2.S.18}$$

International trade frees a country from the necessity of providing all its own requirements; imposed instead is a balance-of-payments constraint that the overall value of consumption match that of national production. Letting the superscript ^{1} denote free-trade variables,

$$\Sigma p_i^1 D_i^1 = \Sigma p_i^1 x_i^1 \tag{2.S.19}$$

A country is considered to gain from international trade if, in a trade equilibrium, it chooses a consumption bundle, D^1, that (at free-trade prices, p^1) costs at least as much to purchase as does the autarky bundle, D^0. Such a choice is taken to *reveal* a preference for the consumption choice available with trade because it is selected either (1) despite the higher price tag, or (2) if the price tag is the same but the bundle chosen, D^1, is different from D^0.[3]

$$\text{Gains if} \quad \Sigma p_i^1 D_i^1 \geqq \Sigma p_i^1 D_i^0 \tag{2.S.20}$$

This criterion provides one of the two fundamental building blocks for the general argument. The other compares the aggregate value of production at a given set of prices with any alternative production pattern along a given production-possibilities

[2]An equivalent expression for e is

$$\frac{\hat{X}}{(\widehat{1/p})}$$

with demands constant, which could be termed the elasticity of export supply. (X represents $x_C - D_C$ for the home country.)

[3]In Figure 2.6 consumption point F is preferred to E even though they cost the same. We are assuming strictly bowed-in indifference curves.

schedule. The basic production relationship states that a price line is tangent to the transformation curve at the point chosen. The bowed-out shape of the transformation curve implies that should any other production combination have been chosen at the same prices, it would have a lower aggregate value. This statement holds for any number of commodities and any set of prices. In particular, at free-trade prices, p^1, the value of production bundle x^1 is greater than that of autarky bundle x^0 at those same prices. That is,

$$\Sigma p_i^1 x_i^1 > \Sigma p_i^1 x_i^0 \tag{2.S.21}$$

if the transformation schedule is smoothly bowed out.

These results provide the basis for two propositions. First, the production relationship shown in Inequality 2.S.21 is used to prove that Inequality 2.S.20 is indeed satisfied. Adding up the value (at free-trade prices) of autarky consumption and production from Equation 2.S.18,

$$\Sigma p_i^1 D_i^0 = \Sigma p_i^1 x_i^0$$

Now substitute this and Equation 2.S.19 into Inequality 2.S.21 to establish Inequality 2.S.20. Free trade leads to gains.

The second proposition concerns the *pattern of trade* according to comparative advantage that leads to these gains from trade. It generalizes the notion that to obtain gains when trading, a country should export commodities produced relatively cheaply at home and import commodities that are relatively inexpensive on world markets. Because it is established that free trade leads to gains, at autarky prices the consumption bundle purchased with free trade must have been out of consumers' reach. They could not afford to purchase the superior bundle, D^1, or they would have done so. This implies that

$$\Sigma p_i^0 D_i^1 > \Sigma p_i^0 D_i^0 \tag{2.S.22}$$

As for production comparisons at autarky prices, the notion that at *any* given prices production responds to maximize the aggregate value of produced income leads to the following:

$$\Sigma p_i^0 x_i^1 < \Sigma p_i^0 x_i^0 \tag{2.S.23}$$

The logic is the same as that leading to Inequality 2.S.21, except that at autarky prices, p^0, the production bundle x^0 has greater value than x^1. Let E_i^1 be defined as imports of commodity i in the trade situation, $D_i^1 - x_i^1$. Because the right sides of Inequalities 2.S.22 and 2.S.23 are the same, subtraction reveals that

$$\Sigma p_i^0 E_i^1 > 0 \tag{2.S.24}$$

That is, if evaluated at autarky prices, imports in the aggregate exceed exports. At free-trade prices, of course, they must have the same value if trade is balanced.

$$\Sigma p_i^1 E_i^1 = 0 \tag{2.S.25}$$

(This restates Equation 2.S.19.)

The final step involves subtracting Inequality 2.S.24 from Equation 2.S.25 to obtain

$$\Sigma(p_i^1 - p_i^0)E_i^1 < 0 \tag{2.S.26}$$

This states that *on average* any commodity, i, imported with free trade has an autarky price, p_i^0, higher than its trade price, p_i^1. It is not possible to establish such a relationship item by item, but Inequality 2.S.26 shows that in the aggregate with trade, a country imports goods that are relatively cheaper and exports goods that are relatively expensive.[4]

The line of argument developed here is pursued in the supplement to Chapter 12, where we consider situations in which tariffs or export taxes distort home prices from world prices.

SUPPLEMENT TO CHAPTER 3: Stability and Comparative Statics in the Basic Trade Model

Stability in the two-commodity world trade model requires that an increase in the relative price of food reduces world excess demand for food. Conditions sufficient to guarantee stability can be derived and presented in two alternative, but equivalent, ways.

The Marshall-Lerner Stability Condition

This form of the condition concentrates on the elasticity of each country's demand for imports. World excess demand for food is the difference between the home country's excess demand, M, and the foreign country's intended exports of food. Assuming a budget balance, these intended food exports have a value equivalent to foreign import demand (for clothing). This value is M^*/p. (The division by p is to change from clothing units to food units.) Therefore, stability requires an increase in p to lower $(M - M^*/p)$. That is, the condition for stability is

$$\frac{dM}{dp} < \frac{d(M^*/p)}{dp}$$

This inequality can be slightly modified (1) by dividing the denominators of both sides by p to highlight the *relative* price change, dp/p. (A circumflex—hat—denotes relative changes: dp/p is written as $\hat{p}$). Then, (2) divide the numerator on the left side by M and the numerator on the right side by M^*/p (which equals M at the initial equilibrium). Making use of the hat notation for relative changes, the inequality becomes

$$\frac{\hat{M}}{\hat{p}} < \frac{\widehat{(M^*/p)}}{\hat{p}} \tag{3.S.1}$$

[4] To see why you should not expect an item-by-item correspondence, suppose that commodity *17* is slightly more expensive with trade than it is at home in autarky. Some major items of consumption that are good substitutes for commodity *17* might become even more expensive with trade, thus deflecting demand onto commodity *17*. Additionally, or alternatively, resources could be drained away from commodity *17* toward other commodities that have risen in price with trade. The net result? Commodity *17* might end up as an import instead of an export.

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By definition, the elasticity of home demand for imports along the offer curve is $\varepsilon \equiv -\hat{M}/\hat{p}$; foreign ε^* is $-\hat{M}^*/(\widehat{1/p})$, which is equivalent to $\hat{M}^*/\hat{p}$.[1] Because Inequality 3.S.1 can be written as

$$\frac{\hat{M}}{\hat{p}} < \frac{\hat{M}^* - \hat{p}}{\hat{p}}$$

substituting for ε and ε^* yields

$$\varepsilon + \varepsilon^* > 1 \tag{3.S.2}$$

This is known as the *Marshall-Lerner condition for stability.* It suggests that for the market to be stable, offer curves cannot be too inelastic. The offer curves in Figure 2.A.3 intersect at stable equilibrium point Q. Note that at that point ε is less than 1 but ε^* exceeds unity, so the Marshall-Lerner condition is obviously satisfied. To illustrate an unstable equilibrium, both offer curves must be inelastic. Instability requires the offer curves to cut each other in the direction opposite to that shown in Figure 2.A.3, as at point Q in Figure 3.B.1(b).

An Alternative Form for the Stability Condition

Concentrate on the excess world demand curve for food, but generalize by assuming many countries in the trading world. Some will be food importers, others exporters. The condition for market stability is that the slope of the excess world demand curve for food be negative, or

$$\Sigma \frac{dD_F^i}{dp} - \Sigma \frac{dx_F^i}{dp} < 0$$

Multiply each term by $-p$, which also changes the direction of the inequality sign. Next, divide and multiply each term in the first sum by D_F^i, country i's demand for food, and each term in the second sum by x_F^i. Finally, divide all terms by total world demand, ΣD_F^i, or by the equivalent (in the neighborhood of equilibrium) total world supply, Σx_F^i. At this stage the condition for stability is

$$\Sigma \lambda_F^i \left\{ -\frac{p}{D_F^i} \cdot \frac{dD_F^i}{dp} \right\} + \Sigma \rho_F^i \left\{ \frac{p}{x_F^i} \cdot \frac{dx_F^i}{dp} \right\} > 0 \tag{3.S.3}$$

In Inequality 3.S.3 two sets of weights appear in the summations. λ_F^i is the fraction of total world food consumption represented by country i's demand, $D_F^i/\Sigma D_F^i$. Similarly, the ρ_F^i are production weights; ρ_F^i equals $x_F^i/\Sigma x_F^i$. The λ and the ρ sums each add to unity.

The final step involves breaking down the demand elasticities into income and substitution terms and defining the appropriate supply elasticities. The breakdown of

[1] The relative change in a ratio, such as $(\widehat{x/y})$, is the difference between the relative change in the numerator and denominator: $\hat{x} - \hat{y}$. Because 1 is a constant, $(\widehat{1/p})$ equals $-\hat{p}$.

home food demand in response to price was shown in Equation 2.S.15, and is repeated here for country i as Equation 3.S.4.

$$\frac{dD_F^i}{dp} = \frac{\partial D_F^i}{\partial p} - \frac{(D_F^i - x_F^i)}{p}\alpha_F^i \tag{3.S.4}$$

Multiply Equation 3.S.4 by $-p/D_F^i$ and define the pure substitution term, $-\frac{p}{D_F^i} \cdot \frac{\partial D_F^i}{\partial p}$, as $\overline{\omega}_F^i$, which must be positive.[2] This yields

$$-\frac{p}{D_F^i} \cdot \frac{dD_F^i}{dp} = \overline{\omega}_F^i + \frac{(D_F^i - x_F^i)}{D_F^i}\alpha_F^i \tag{3.S.5}$$

Similarly, define

$$\frac{p}{x_F^i} \cdot \frac{dx_F^i}{dp}$$

as e_F^i. This own-supply response to price must be positive.

Sweeping countries together, let S be defined as

$$S \equiv \Sigma\lambda_F^i\overline{\omega}_F^i + \Sigma\rho_F^i e_F^i$$

That is, S is the sum of two terms: The first is the positive-weighted average of each nation's substitution elasticity of demand, and the second is the weighted average of own-production elasticities. In similar fashion for income effects, let γ be defined as

$$\gamma \equiv \Sigma(\lambda_F^i - \rho_F^i)\alpha_F^i$$

Each country's marginal propensity to consume food, α_F^i, has as a weight in γ the fraction of total world food production represented by that country's net *imports* of food. If country i exports food, $\lambda_F^i - \rho_F^i$ would be a negative fraction. Substituting these terms into Inequality 3.S.3 yields Inequality 3.S.6 as an alternative basic stability condition.

$$S + \gamma > 0 \tag{3.S.6}$$

This form of the stability condition is in some ways more revealing than the equivalent Marshall-Lerner expression, Inequality 3.S.2. Substitution effects both in consumption and production are contained in the term S and must be positive. Thus high values help ensure stability. As the price of food rises, in every country consumers substitute away from demanding food and resources are attracted to food production. γ captures the effect of a rise in food's price in redistributing real incomes toward countries exporting food and away from food importers. If all countries share identical marginal propensities to consume food, α_F^i, γ must vanish and the market will be

[2]Note that for the home country importing F, $\overline{\omega}_F$ is smaller than the trade substitution elasticity, $\overline{\eta}$, defined in the supplement to Chapter 2. Indeed, $\overline{\omega}_F$ is (M/D_F) times $\overline{\eta}$. They would be equal only if no food were produced at home.

stable. If, on average, food importers have a higher marginal propensity to consume food, γ would be positive and market stability would be guaranteed. Returning to the two-country case in which the home country imports food (denoted by M), we have

$$\gamma = \frac{M}{D_F + D_F^*}(\alpha_F - \alpha_F^*)$$

Thus stability would be endangered if foreign food exporters had a higher α_F^* than home food importers. Note, however, that γ's absolute size tends to be small if the volume of trade is small relative to total world consumption. In such a case the market is apt to be stable regardless of taste differences.

Comparative Statics

This chapter discussed several comparative statics exercises involving changes in tastes, the composition of outputs, growth, and international transfers. The basic equilibrium relationship for all these exercises (except transfers) is the balance-of-payments condition (see also Equation 2.S.9)

$$pM = M^* \tag{3.S.7}$$

The method of comparative statics involves seeing how a disturbance to the market causes prices to change so as to restore the equilibrium relationship shown by Equation 3.S.7. That is, anything that causes imports in either country to change must bring about an equilibrating price response.

Proceed formally by differentiating Equation 3.S.7, making use of the hat notation for relative changes.

$$\hat{p} + \hat{M} = \hat{M}^* \tag{3.S.8}$$

Imports in either country respond to a change in the terms of trade—this is what the offer curves describe. In addition, a disturbance may *shift* one or more offer curves. Let the relative change in imports at home that would take place at *constant terms of trade* be denoted by $\hat{M}|_{\bar{p}}$. This is the shift in the home offer curve. Similarly, $\hat{M}^*|_{\bar{p}}$ denotes the relative shift in the foreign offer curve. Putting these two sources of import change together,

$$\hat{M} = -\varepsilon\hat{p} + \hat{M}|_{\bar{p}} \tag{3.S.9}$$

$$\hat{M}^* = \varepsilon^*\hat{p} + \hat{M}^*|_{\bar{p}}$$

Substitute these into Equation 3.S.8 and solve for the relative change in the terms of trade that serves to clear markets to get

$$\hat{p} = \frac{(\hat{M}|_{\bar{p}} - \hat{M}^*|_{\bar{p}})}{\Delta} \quad \text{where } \Delta \equiv \varepsilon + \varepsilon^* - 1 \tag{3.S.10}$$

This is a basic, and readily understandable, result. From the Marshall-Lerner stability expression, Inequality 3.S.2, the denominator, Δ, must be positive. This shows that the less sensitive imports are to price changes (small Δ), the more the relative price must adjust to clear markets. Furthermore, the numerator of Equation 3.S.10 has a ready interpretation. It shows the relative increase in world excess demand for the home country's import commodity (food) at the initial prices. In other words, Equation 3.S.10 shows that the equilibrium relative price of food rises if the excess world demand curve for food shifts to the right and the market is stable.

In many applications of the basic trade model, the aim is to analyze how real incomes at home and abroad are affected. The expression for real income changes at home was developed in the supplement to Chapter 2. A slight rewriting of Equation 2.S.12 yields

$$dy = -pM \cdot \hat{p} + (dx_c + pdx_F) \tag{3.S.11}$$

There is a terms-of-trade effect and a direct effect from production changes. Recall that for movements along the transformation curve, $dx_c + pdx_F$ equals zero. Therefore, the second part of dy in Equation 3.S.11 picks up the value of *shifts* in the transformation curve.

Now consider the following scenarios, in each of which there is a shock or disturbance to a preexisting world trade equilibrium balancing home and foreign import demands. The first involves only a change in the *composition* of outputs at home, the next two applications involve *growth,* and the final scenario deals with the *transfer* problem. In each case focus on the change in the terms of trade and on the consequent effects on real incomes:

1. *A change in the composition of home outputs.* In this case assume that at constant prices food output increases ($dx_F > 0$), and clothing output falls ($dx_C < 0$), but at initial prices there is no change in the value of aggregate production ($dx_C + pdx_F = 0$). This means that at the initial price there is no alteration in home *demand* for food importables (both price and income are constant at the initial price). Yet *production* rises, and this causes demand for imports to fall ($dM = -dx_F$). Abroad no changes take place at the initial prices. Substitution into Equation 3.S.10 reveals that

$$\hat{p} = -\frac{1}{M \cdot \Delta} dx_F \tag{3.S.12}$$

With the terms of trade improving, so must real income at home. Equation 3.S.11 thus gives

$$dy = \frac{pdx_F}{\Delta} \tag{3.S.13}$$

The more inelastic are world demand and supply, the more successful would be a policy of substituting import-competing production, x_F, for exportables, x_C. This is a theme picked up by the tariff literature.

2. *Export-led growth.* Suppose growth is biased, so that at initial prices only the output of exportables at home expands ($dx_C > 0$), but at constant prices $dx_F = 0$. No *shifts* in demand or supply take place abroad. At initial prices there is no change in production of food (importables), but because incomes expand at *initial* prices, so does demand. That is, $dM = dD_F$, and $dD_F = (m/p)(dx_C)$. Demand for food rises by an amount determined by the marginal propensity to import food, m, and the increase in initial incomes in food units, dx_C/p. Substituting into Equation 3.S.10 yields

$$\hat{p} = \frac{m}{pM \cdot \Delta} dx_C \tag{3.S.14}$$

The terms of trade have deteriorated and, by Equation 3.S.11, this deterioration offsets at least a part of the initial growth effect on real incomes.

$$dy = \left(\frac{\Delta - m}{\Delta}\right) dx_C \tag{3.S.15}$$

The expression in parentheses provides the condition for immiserizing growth. Stability ensures that Δ is positive, but if elasticities are nonetheless low, Δ may not exceed the home marginal propensity to import. In such a case, real incomes at home would fall despite output growth.

3. *Balanced growth.* The kind of growth just discussed was quite biased—at initial prices only the home country's export good expanded, which ensures a deterioration in its terms of trade. Yet what about balanced growth? Suppose the home country's transformation schedule shifts outward uniformly at rate μ—both dx_C/x_C and dx_F/x_F equal μ at initial prices. Assume also that demand for both goods expands in a balanced fashion at initial prices. Then imports (at initial prices) must also expand at rate μ. Substitute into Equation 3.S.10 to show that neutral growth must cause a deterioration in the terms of trade (assuming no growth abroad).

$$\hat{p} = \frac{\mu}{\Delta} \tag{3.S.16}$$

It proves convenient to express the change in real income (given in Equation 3.S.11) in relative terms. $\hat{y}$ is dy divided by initial income, $(x_C + px_F)$. That is,

$$\hat{y} = -\theta_M \hat{p} + \mu$$

where θ_M represents the share of imports in the national income and μ, of course, is the growth rate at initial prices. This expression is perfectly general. Substituting the terms-of-trade change shown by Equation 3.S.16 for the case of balanced growth yields

$$\hat{y} = \left(\frac{\Delta - \theta_M}{\Delta}\right)\mu \tag{3.S.17}$$

This result shows that even balanced growth can be immiserizing, for it does worsen the terms of trade. If elasticities are sufficiently low, their sum may not exceed unity by more than the share of imports in the national income. Equation 3.S.17 should be compared with Equation 3.S.15. Retaining the assumption that at constant prices growth in demand is proportional, the marginal propensity to import, m, is the same as the fraction of total income spent on importables (including domestic production as well as imports). Unless production of importables is nonexistent, this must exceed the share of income represented by total imports, θ_M. Export-led growth is more apt to worsen real incomes than is balanced growth.

4. *The transfer problem.* Discussion of the transfer problem requires a bit more preparation. The basic equilibrium relationship set out in Equation 3.S.7 rests on the classical form of the budget constraint: In each country all earned income is spent. The transfer process has the home country spending less than its produced income by the amount of transfer (call it T in units of clothing), matched by an equal amount of excess spending (over earned income) abroad. This implies that the value of spending on imports at home must also be cut below the value of foreign imports by the amount of the transfer.[3] This is the following basic relationship.

$$pM = M^* - T \tag{3.S.18}$$

Assume that initially there is no transfer ($T = 0$). Differentiation of Equation 3.S.18 yields

$$\hat{p} + \hat{M} = \hat{M}^* - \frac{dT}{pM} \tag{3.S.19}$$

Proceeding as before (in the development of Equation 3.S.10), the result is

$$\hat{p} = \left(\hat{M}|_{\bar{p}} - \hat{M}^*|_{\bar{p}} + \frac{dT}{pM} \right) / \Delta \tag{3.S.20}$$

With a transfer of purchasing power there are no production changes at the initial terms of trade.[4] Demand for imports falls at home and rises abroad, however. That is, $\hat{M}|_{\bar{p}} = -m dT/pM$, and $\hat{M}^*|_{\bar{p}} = m^* dT/pM$. In other words, the impact of the direct redistribution of income on the terms of trade is shown by

$$\hat{p} = \frac{-(m + m^* - 1)}{\Delta} \cdot \frac{dT}{pM} \tag{3.S.21}$$

This expression confirms Chapter 3's statement that with transfer the terms of trade might go in either direction. Note that the numerator can also be written as $[(1 - m^*) - m]$ or, to use the earlier terminology, as $\alpha_F^* - \alpha_F$. Whether the real income transfer is a consequence of a change in the terms of trade (as in the stability expression, Inequality 3.S.6) or of a direct transfer of purchasing power (as in Equation 3.S.21), the same comparison between foreign α_F^* and home α_F, the marginal propensities to consume a particular commodity in the two countries, is required.

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[3]The home budget constraint becomes $D_C + pD_F = x_C + px_F - T$. Rewriting, $p(D_F - x_F) = (x_C - D_C) - T$. When markets clear, home-intended exports equal foreign imports, M^*.

[4]Ignored here is the chapter's discussion of a possible transfer of real resources. A general treatment of the transfer problem, which includes possible supply reactions, is R. W. Jones, "Presumption and the Transfer Problem," *Journal of International Economics* (August 1975): 263–274, reprinted in his *International Trade: Essays in Theory* (Amsterdam: North Holland, 1979), Chapter 10.

This supplement concludes by confirming Chapter 3's argument that even if the terms of trade move in favor of the transferor, real income for the transferor cannot improve. The equivalent of Equation 3.S.11 for the transfer problem is[5]

$$dy = -pM\hat{p} - dT$$

Direct substitution of $\hat{p}$ into this expression yields

$$dy = -\left\{\frac{\varepsilon + \varepsilon^* - (m + m^*)}{\Delta}\right\} dT$$

However, the supplement to Chapter 2 decomposed the elasticity of import demand (ε, and, by analogy, ε^*) into a substitution term in consumption ($\bar{\eta}$ and $\bar{\eta}^*$), a positive elasticity in production (e and e^*), and the import propensity (m and m^*). Therefore, with transfer, the expression for dy can finally be given as follows:

$$dy = -\frac{\{\bar{\eta} + \bar{\eta}^* + e + e^*\}}{\Delta} dT \qquad (3.S.22)$$

Real income for the transferor must decline, as is demonstrated in Figure 3.4.

SUPPLEMENT TO CHAPTER 5: The Specific-Factors Model of Production

This supplement provides a formal analytic treatment of the model of production described in Chapter 5. The community produces two commodities, clothing and food. Labor (L) and capital (K) are combined to produce clothing. The input requirements *per unit* output of clothing are denoted by a_{LC} and a_{KC}. Labor is also used to produce food, in cooperation with land (T). Thus, the per-unit output requirements in the food sector are a_{LF} and a_{TF}. Capital and land are each used specifically only in one sector, whereas labor is mobile between sectors.

The Distribution of Income

Pure competition is assumed to prevail, assuring that commodity prices (p_C and p_F) reflect units costs of production. These costs, in turn, depend in part on the input mix used in production (the a_{ij}'s) and in part on factor prices. The wage rate is denoted by w, and the amount that must be paid per unit rental on capital is given by r_K and the rental on land by r_T. The competitive profit conditions are thus summarized as follows.

$$a_{LC}w + a_{KC}r_K = p_C \qquad (5.S.1)$$

$$a_{LF}w + a_{TF}r_T = p_F \qquad (5.S.2)$$

[5]Here dy is interpreted as the change in current real consumption. Left out of this account is the possibility that the transfer represents a loan, which will be repaid in the future. Presumably this does not by itself lower real income for the transferor. Also left out of this account in the expression that follows is the possibility that trade involves other countries in addition to the transferor and transferee. In such a case a transfer welfare paradox is possible, wherein real income may improve for the transferor. For a general discussion of this issue, with references to the literature, see R. W. Jones, "Income Effects and Paradoxes in the Theory of International Trade," *Economic Journal* (June 1985): 330–334.

Techniques of production are chosen so as to minimize the costs of producing a unit of output in the face of prevailing factor prices. To see what this entails, consider the clothing sector. The assumption of constant returns to scale implies that the *unit isoquant* captures all there is to know about techniques of production. At the point of cost minimization, the isocost line—with slope given by (minus) the ratio of factor prices, $-(w/r_K)$—is tangent to the unit isoquant, with slope da_{KC}/da_{LC}. That is, cost minimization entails that

$$wda_{LC} + r_K da_{KC} = 0$$

Once again it proves convenient to write these changes in *relative* terms (denoted by ^). Thus $\hat{a}_{LC}$ is da_{LC}/a_{LC}. Also, write the factor *distributive shares* as θ_{LC} and $\theta_{KC,}$ respectively, where, for example, θ_{LC} is wa_{LC}/p_C. Therefore, in the clothing sector, cost minimization entails

$$\theta_{LC}\hat{a}_{LC} + \theta_{KC}\hat{a}_{KC} = 0 \qquad (5.S.3)$$

Similarly, in the food sector,

$$\theta_{LF}\hat{a}_{LF} + \theta_{TF}\hat{a}_{TF} = 0 \qquad (5.S.4)$$

Each of these expressions states that if labor is used more intensively, less of the specific factor need be used along the unit isoquant. The left side in Equations 5.S.3 and 5.S.4 shows, for each industry, the relative change in unit costs involved in substituting one input for another. At a point of cost minimization this change must be zero: All cost reductions have already been taken at the minimum cost point.

It is now possible to confirm Chapter 5's argument that each commodity price change is flanked by the changes in the returns to productive factors used in that industry. Differentiate Equations 5.S.1 and 5.S.2, put into relative terms, and simplify by using Equations 5.S.3 and 5.S.4 to obtain

$$\theta_{LC}\hat{w} + \theta_{KC}\hat{r}_K = \hat{p}_C \qquad (5.S.5)$$

$$\theta_{LF}\hat{w} + \theta_{TF}\hat{r}_T = \hat{p}_F \qquad (5.S.6)$$

Thus each commodity price change must be a weighted average of factor price changes, with the weights given by distributive shares—reflections of the importance of each factor in unit costs. Suppose now that commodity prices are disturbed—that clothing's price rises while the price of food remains unchanged. Equations 5.S.5 and 5.S.6 then suggest that some factor's return will rise relatively by more than p_C has while some other factor's return will fall absolutely (because $\hat{p}_F = 0$). As is easily shown, capitalists are the clear gainers and landlords the losers. This is established by first showing that the wage rate must rise, but not as much, relatively, as the price of clothing.

The wage rate is determined by the condition that the labor force be fully employed. The clothing sector's demand for labor is written as $a_{LC}x_C$, where x_C shows the scale of output. Output is restricted by the availability of capital, however. If a_{KC} denotes the quantity of capital used per unit and if K units of capital are all the economy possesses, clothing output must be given by

$$x_C = \frac{K}{a_{KC}}$$

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Therefore, the clothing sector's labor demand can be written as $(a_{LC}/a_{KC}) \cdot K$. In similar fashion the food sector's demand for labor must be $(a_{LF}/a_{TF}) \cdot T$. Thus the following is the statement that all the economy's labor force is fully employed:

$$\frac{a_{LC}}{a_{KC}} \cdot K + \frac{a_{LF}}{a_{TF}} \cdot T = L \tag{5.S.7}$$

Differentiate this, assuming now that K and T remain constant but L may change, to obtain

$$\lambda_{LC}(\hat{a}_{LC} - \hat{a}_{KC}) + \lambda_{LF}(\hat{a}_{LF} - \hat{a}_{TF}) = \hat{L} \tag{5.S.8}$$

where the λ's correspond to the fraction of the economy's labor force used in each sector.

To proceed, reconsider the relationship between the wage rate and the value of labor's marginal product in each sector. These must be equal. Figure 5.S.1 illustrates how the quantity of labor used per unit of capital (a_{LC}/a_{KC}) depends inversely on the real wage in the clothing sector (w/p_C). (For a given clothing price this curve is the same as that drawn in Figure 5.3, reading from right to left.) The curve shows the marginal physical product of labor in clothing. Define the *elasticity* of labor's marginal product curve, γ_{LC}, as

$$\gamma_{LC} \equiv \frac{-(\hat{a}_{LC} - \hat{a}_{KC})}{(\hat{w} - \hat{p}_C)} \tag{5.S.9}$$

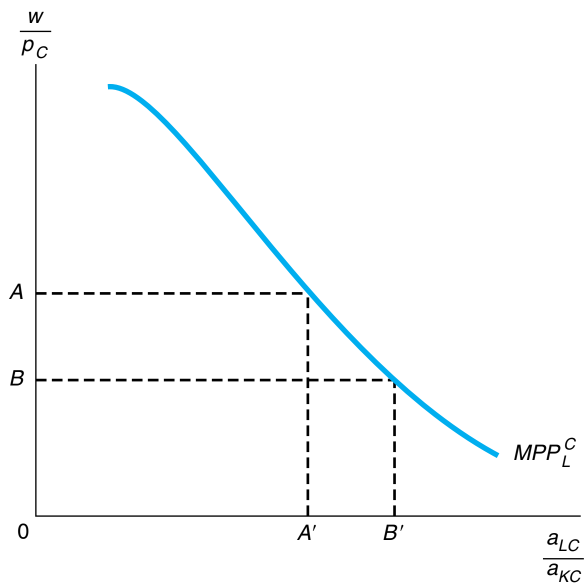
Similarly, in the food industry,

$$\gamma_{LF} \equiv \frac{-(\hat{a}_{LF} - \hat{a}_{TF})}{(\hat{w} - \hat{p}_F)} \tag{5.S.10}$$

FIGURE 5.S.1

The Elasticity of Labor's Marginal Product

A drop in the real wage from 0A to 0B would encourage labor to be used more intensively—an increase in the labor/capital ratio from 0A' to 0B'. The elasticity of labor's marginal product in clothing, γ_{LC}, is defined as $-(\hat{a}_{LC} - \hat{a}_{KC})$ divided by $(\hat{w} - \hat{p}_C)$.



These concepts are crucial. Substitute the expressions for the elasticities γ_{LC} and γ_{LF} into Equation 5.S.8 to obtain

$$\lambda_{LC}\gamma_{LC}(\hat{w} - \hat{p}_C) + \lambda_{LF}\gamma_{LF}(\hat{w} - \hat{p}_F) = -\hat{L} \tag{5.S.11}$$

Solving explicitly for the change in the wage rate in terms of the commodity price changes and any change in the labor force,

$$\hat{w} = \beta_C\hat{p}_C + \beta_F\hat{p}_F - \frac{1}{\gamma}\hat{L} \tag{5.S.12}$$

where

$$\beta_C \equiv \lambda_{LC}\frac{\gamma_{LC}}{\gamma}$$

$$\beta_F \equiv \lambda_{LF}\frac{\gamma_{LF}}{\gamma}$$

and

$$\gamma \equiv \lambda_{LC}\gamma_{LC} + \lambda_{LF}\gamma_{LF}$$

γ_{LC} and γ_{LF} are the elasticities of labor's marginal product curve in each sector, and γ is the economywide weighted average of these two elasticities. γ directly provides the answer to the following question: If commodity prices are constant and the wage rate rises by 1 percent, by what percentage will the entire economy's demand for labor fall? If γ is large, the answer is that the economy's demand for labor would be reduced by a relatively large amount. Conversely, Equation 5.S.12 shows that a given increase in the labor force would, at constant commodity prices, reduce the wage rate, but not by very much if γ is large. The β coefficients, which add to unity, reveal the power of each separate commodity price to influence the wage rate. At constant overall factor endowments, the wage rate change is trapped between (i.e., is a positive weighted average of) the commodity price changes. Therefore, if clothing's relative price increases ($\hat{p}_C > \hat{p}_F$), this relationship, coupled with Equations 5.S.5 and 5.S.6, establishes that

$$\hat{r}_K > \hat{p}_C > \hat{w} > \hat{p}_F > \hat{r}_T$$

The specific factors are most radically affected by price changes. The mobile factor (labor) finds its return rising in terms of one sector and falling in terms of the other. The algebraic demonstration supplements the diagrammatic illustration of a price change in Figure 5.2.

The expression for each β coefficient in Equation 5.S.12 allows a further refinement. Consider only β_C, the relative effect of an increase in clothing's price on the wage rate. This coefficient was explicitly defined in Equation 5.S.12, but rewrite it as

$$\beta_C = \theta_C \cdot \frac{\lambda_{LC}}{\theta_C} \cdot \frac{\gamma_{LC}}{\gamma}$$

Reading from right to left, the term γ_{LC}/γ can be considered the *relative* degree of substitutability of the demand for labor in the clothing sector—a comparison of γ_{LC} with the economywide average, γ. Call this term s_C. Next is the expression λ_{LC}/θ_C, where θ_C denotes the share of clothing production in the national income, $p_Cx_C/(p_Cx_C + p_Fx_F)$.

This expression also reflects a "relative" for the clothing industry; it is a measure of *relative labor intensity* for clothing. The concept of relative factor intensity in a two-factor setting comes into its own in Chapter 6 and the supplement to Chapter 6. Here it is used to compare λ_{LC}, the fraction of the labor force used in clothing, with θ_C, the fraction of the economy's entire input base used in clothing. (Thus, if λ_{LC}/θ_C were unity, clothing would be neither labor intensive nor labor unintensive.) Call this term i_C. Then β_C is the product of three terms:

$$\beta_C = \theta_C \cdot i_C \cdot s_C$$

That is, a price rise in clothing has a more severe impact on the wage rate (1) the more elastic is the demand for labor in clothing compared with the economywide average (i.e., the higher is s_C), (2) the more labor intensive is the clothing sector (i.e., the higher is i_C), and (3) the more important is production of clothing as a fraction of national income produced (i.e., the higher is θ_C).[1]

Outputs, Prices, and Factor Endowments

Outputs respond to changes in relative prices along the transformation schedule. Outputs also respond to changes in factor endowments (at constant commodity prices). Chapter 5 showed how an ample supply of capital lends a presumption that relatively much clothing is produced. By contrast, plentiful land encourages food production. Now endowments of capital and land are kept constant, but the implication for outputs (and thus for positions of comparative advantage) of changes in labor abundance are explored.

If, as assumed, the total capital stock is kept fixed, clothing output can expand only by using capital less intensively. Similarly, because x_F equals T/a_{TF}, food output can change only if a_{TF} is altered, given that overall land is fixed in supply. Combining shows that

$$\hat{x}_C - \hat{x}_F = \hat{a}_{TF} - \hat{a}_{KC} \qquad (5.S.13)$$

The ingredients are at hand to solve separately for $\hat{a}_{TF}$ and $\hat{a}_{KC}$. From Equations 5.S.4 and 5.S.10,

$$\hat{a}_{TF} = \theta_{LF}\gamma_{LF}(\hat{w} - \hat{p}_F)$$

Similarly, Equations 5.S.3 and 5.S.9 can be solved for $\hat{a}_{KC}$:

$$\hat{a}_{KC} = \theta_{LC}\gamma_{LC}(\hat{w} - \hat{p}_C)$$

The change in the wage rate is provided by Equation 5.S.12, so that Equation 5.S.13 can be written as

$$\hat{x}_C - \hat{x}_F = \sigma_s(\hat{p}_C - \hat{p}_F) + \frac{1}{\gamma}(\theta_{LC}\gamma_{LC} - \theta_{LF}\gamma_{LF})\hat{L} \qquad (5.S.14)$$

[1]This decomposition is discussed and applied in R. W. Jones, *Globalization and the Theory of Input Trade* (Cambridge, MA: MIT Press, 2000). An application to the question of the effect of tariffs on real wages in the specific-factors model is found in R. Ruffin and R. Jones, "Protection and Real Wages: The Neoclassical Ambiguity," *Journal of Economic Theory* (April 1977): 337–348.

where

$$\sigma_s \equiv \beta_C[\theta_{LF}\gamma_{LF}] + \beta_F[\theta_{LC}\gamma_{LC}] > 0$$

The effect of a change in relative commodity prices on relative outputs along the transformation schedule (i.e., for given factor endowments) is captured by the positive term σ_s, the elasticity of supply of relative outputs. Because the β's add to unity this is generally larger the greater are the elasticities of labor's marginal product curves in the two sectors.[2] The coefficient of $\hat{L}$ reveals that two distinct features of the technology determine the composition of output. As the labor supply expands (at given terms of trade), clothing output will tend to expand more than does the food sector if the elasticity of labor's marginal product is higher in clothing (i.e., if $\gamma_{LC} > \gamma > \gamma_{LF}$). This is one feature. However, the comparison of labor's distributive shares, θ_{LC} and θ_{LF}, is also important. The clothing sector tends to expand relative to food if θ_{LC} exceeds θ_{LF}. As the supplement to Chapter 6 reveals, this comparison of distributive shares is a comparison of *relative labor intensity* in the two sectors.[3] In the Heckscher-Ohlin model in Chapter 6, these factor intensity comparisons assume critical importance.

SUPPLEMENT TO CHAPTER 6: The Two-Sector Heckscher-Ohlin Model

The two-sector Heckscher-Ohlin model of production assumes each of two outputs (clothing, food) is produced in a constant returns to scale competitive setting with the use of two primary inputs (labor, capital). The productive factors are each homogeneous and mobile between sectors. Prices are flexible and both inputs are fully employed.

$$a_{LC}x_C + a_{LF}x_F = L \tag{6.S.1}$$

$$a_{KC}x_C + a_{KF}x_F = K \tag{6.S.2}$$

Furthermore, unit costs in each sector are equated to the prevailing commodity price (if output is positive):

$$a_{LC}w + a_{KC}r = p_C \tag{6.S.3}$$

$$a_{LF}w + a_{KF}r = p_F \tag{6.S.4}$$

Again, w refers to the wage rate, and now the common return to capital in the economy is denoted by r.

[2]The supplement to Chapter 6 compares this expression for σ_s with the comparable elasticity in the Heckscher-Ohlin model by making further simplifying assumptions.

[3]With reference to the definition of the relative degree of substitutability, s_c (and s_F for the food sector), on the one hand, and i_C (and i_F) for relative labor intensities, on the other hand, the coefficient of $\hat{L}$ in Equation 5.S.14 can also be written as $\theta_L\ [i_C\ s_C - i_F\ s_F]$, where θ_L is labor's distributive share in the national income. Thus, as the labor force expands at constant commodity prices, clothing output is apt to rise relatively more than food output to the extent that clothing is relatively labor intensive and has a relatively high elasticity of demand for labor.

Equations of Change: Prices

As in the specific-factors model of Chapter 5, techniques of production are chosen so as to minimize unit costs. This condition implies Equations 6.S.5 and 6.S.6: The distributive-share weighted average of changes in input-output coefficients along the unit isoquant in each industry must vanish near the cost-minimization point.[1]

$$\theta_{LC}\hat{a}_{LC} + \theta_{KC}\hat{a}_{KC} = 0 \tag{6.S.5}$$

$$\theta_{LF}\hat{a}_{LF} + \theta_{KF}\hat{a}_{KF} = 0 \tag{6.S.6}$$

These relationships are crucial, for they suggest that differentiating Equations 6.S.3 and 6.S.4 totally yields

$$\theta_{LC}\hat{w} + \theta_{KC}\hat{r} = \hat{p}_C \tag{6.S.7}$$

$$\theta_{LF}\hat{w} + \theta_{KF}\hat{r} = \hat{p}_F \tag{6.S.8}$$

These conditions state that in each industry the distributive-share weighted average of factor-price changes equals the relative commodity-price change. They correspond to Equations 5.S.5 and 5.S.6 for the specific-factor models. Yet now more can be said: This pair of equations links the commodity-price changes $(\hat{p}_C, \hat{p}_F)$ to the pair of factor-price changes $(\hat{w}, \hat{r})$. Factor prices are determined *uniquely* by commodity prices, as long as both commodities are produced, and assuming the techniques used in clothing and food differ.

This qualification about techniques refers to the capital/labor ratio employed in the two sectors. As in the text, assume that food always is produced with a higher capital/labor ratio than clothing. This comparison must then be revealed in a ranking of distributive shares. Specifically, labor's distributive share in labor-intensive clothing, θ_{LC}, must exceed that in capital-intensive food, θ_{LF}. To see this, compute the determinant of coefficients in Equations 6.S.7 and 6.S.8. Call this determinant $|\theta|$. By definition,

$$|\theta| \equiv \theta_{LC}\theta_{KF} - \theta_{LF}\theta_{KC}$$

Substitute the formal definition of each distributive share (e.g., θ_{LC} is wa_{LC}/p_C) to obtain

$$|\theta| = \frac{wr}{p_C p_F}(a_{LC}a_{KF} - a_{LF}a_{KC})$$

Therefore, $|\theta|$ is positive if clothing is labor intensive. However, because distributive shares in any industry add to unity, θ_{KF} is $1 - \theta_{LF}$, and θ_{KC} is $1 - \theta_{LC}$. Therefore, $|\theta|$ can be written as

$$|\theta| = \theta_{LC} - \theta_{LF}$$

[1]This states that an isocost line is tangent to the unit isoquant. Details are provided in the supplement to Chapter 5.

The relationships shown by Equations 6.S.7 and 6.S.8 underlie the shape of the curve in Figure 6.4. Subtract Equation 6.S.8 from Equation 6.S.7 to obtain

$$|\theta| \cdot (\hat{w} - \hat{r}) = (\hat{p}_C - \hat{p}_F) \tag{6.S.9}$$

Thus an increase in labor-intensive clothing's relative price must raise the wage/rent ratio by a magnified amount. Even more can be said: If $\hat{p}_C$ is greater than $\hat{p}_F$ and clothing is labor intensive,

$$\hat{w} > \hat{p}_C > \hat{p}_F > \hat{r}$$

The factor-price changes are magnified reflections of the commodity-price changes. The *Stolper-Samuelson theorem* asserts that an increase in labor-intensive clothing's price (with food prices constant) must unambiguously raise the *real wage.* This follows directly from this chain of inequalities.

If two countries share the same technology and produce both goods in common, free trade in commodities will not only equate commodity prices, it will also result in *factor-price equalization.* Simply treat the variables in Equations 6.S.7 and 6.S.8 as relative differences between countries. Thus, if $\hat{p}_C = \hat{p}_F = 0$ with free trade, then $\hat{w}$ and $\hat{r}$ must be zero.

Equations of Change: Outputs

The pair of full-employment equations suggests that outputs respond both to factor endowment changes and to changes in intensity of techniques. Differentiate Equations 6.S.1 and 6.S.2 totally, and let λ_{ij} refer to the fraction of the total supply of factor *i* that is employed in commodity *j*.

$$\lambda_{LC}\hat{x}_C + \lambda_{LF}\hat{x}_F = \hat{L} - (\lambda_{LC}\hat{a}_{LC} + \lambda_{LF}\hat{a}_{LF}) \tag{6.S.10}$$

$$\lambda_{KC}\hat{x}_C + \lambda_{KF}\hat{x}_F = \hat{K} - (\lambda_{KC}\hat{a}_{KC} + \lambda_{KF}\hat{a}_{KF}) \tag{6.S.11}$$

Each equation points out the limitation on outputs provided by the overall endowment of the factor, as well as the intensity with which that factor is used. Consider the changed techniques in clothing: $\hat{a}_{LC}$ and $\hat{a}_{KC}$. Equation 6.S.5 provided one relationship between these two changes. Another follows from the definition of the *elasticity of substitution* between labor and capital in the clothing sector.[2]

$$\sigma_C \equiv \frac{\hat{a}_{KC} - \hat{a}_{LC}}{\hat{w} - \hat{r}} \tag{6.S.12}$$

Solve Equations 6.S.5 and 6.S.12 to obtain

$$\hat{a}_{LC} = -\theta_{KC}\sigma_C(\hat{w} - \hat{r})$$

$$\hat{a}_{KC} = \theta_{LC}\sigma_C(\hat{w} - \hat{r}) \tag{6.S.13}$$

[2]You may wonder how the elasticity of substitution, σ_C, is related to the elasticity of labor's marginal product in clothing, γ_{LC}, defined in Equation 5.S.9. Because $\hat{w} - \hat{p}_C$ is equal to $\theta_{KC}(\hat{w} - \hat{r})$, from Equation 6.S.7 or 5.S.5, γ_{LC} equals σ_C divided by θ_{KC}.

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Comparable solutions are obtained for changes in the labor and capital coefficients in the food sector—merely replace C with F in the subscripts of Equation 6.S.13.

With these solutions now in hand, reconsider expressions such as $\lambda_{LC}\hat{a}_{LC} + \lambda_{LF}\hat{a}_{LF}$, which shows for the economy as a whole how much of an increase or reduction in labor is required at unchanged outputs. Suppose the wage/rent ratio rises. Both industries will economize on labor. Thus Equations 6.S.10 and 6.S.11 can be rewritten as

$$\lambda_{LC}\hat{x}_C + \lambda_{LF}\hat{x}_F = \hat{L} + \delta_L(\hat{w} - \hat{r}) \tag{6.S.14}$$

$$\lambda_{KC}\hat{x}_C + \lambda_{KF}\hat{x}_F = \hat{K} - \delta_K(\hat{w} - \hat{r}) \tag{6.S.15}$$

where

$$\delta_L \equiv \lambda_{LC}\theta_{KC}\sigma_C + \lambda_{LF}\theta_{KF}\sigma_F$$

$$\delta_K \equiv \lambda_{KC}\theta_{LC}\sigma_C + \lambda_{KF}\theta_{LF}\sigma_F$$

Subtract Equation 6.S.15 from Equation 6.S.14 and let

$$|\lambda| \equiv \lambda_{LC} - \lambda_{KC}$$

Then

$$(\hat{x}_C - \hat{x}_F) = \frac{1}{|\lambda|}(\hat{L} - \hat{K}) + \frac{(\delta_L + \delta_K)}{|\lambda|}(\hat{w} - \hat{r}) \tag{6.S.16}$$

If clothing is labor intensive, $|\lambda|$ is a positive fraction.[3] Finally, substitute the link between factor and commodity prices provided by Equation 6.S.9 to obtain

$$(\hat{x}_C - \hat{x}_F) = \frac{1}{|\lambda|}(\hat{L} - \hat{K}) + \sigma_S(\hat{p}_C - \hat{p}_F) \tag{6.S.17}$$

where

$$\sigma_S \equiv \frac{\delta_L + \delta_K}{|\lambda||\theta|} > 0$$

Several features of the two-sector production model are revealed by Equation 6.S.17. First, note that σ_S must be positive because δ_L and δ_K are each positive and $|\lambda|$ and $|\theta|$ must have the same sign. If, as is assumed, clothing is labor intensive, both $|\lambda|$ and $|\theta|$ are positive. If clothing were capital intensive, each would be negative, making the product $|\lambda|\ |\theta|$ positive once again. σ_S denotes the elasticity of supply along the bowed-out transformation curve. Second, note that at constant prices the coefficient of $\hat{L} - \hat{K}$ in Equation 6.S.17 reveals how the transformation schedule shifts as factor

[3] $|\lambda|$ is clearly the determinant of coefficients in Equations 6.S.14 and 6.S.15. The argument is similar to the one used in discussing $|\theta|$.

endowments change. It confirms the magnification effect of uneven growth of factor endowments on outputs if the terms of trade are constant. If $\hat{L}$ exceeds $\hat{K}$,

$$\hat{x}_C > \hat{L} > \hat{K} > \hat{x}_F$$

If only labor expands, one output must actually fall—the Rybczynski result.[4]

Output Responses to Price Changes: Sector-Specific and Heckscher-Ohlin Models

Outputs are more responsive to price signals in the Heckscher-Ohlin model than in the specific-factor model because all factors are mobile between sectors. The following discussion probes more deeply into each model's expression for the elasticity of supply along the transformation curve, σ_S, to point out the basic similarity and the basic difference between models.[5]

In the Heckscher-Ohlin model, the elasticity of supply with respect to prices was shown by σ_S in Equation 6.S.17. δ_L and δ_K each contain a blend of information on the degree of factor substitutability in the two sectors, σ_C and σ_F. Thus σ_S can be rewritten as

$$\sigma_S = \frac{Q_C\sigma_C + Q_F\sigma_F}{|\lambda||\theta|} \tag{6.S.18}$$

where

$$Q_C \equiv \theta_{LC}\lambda_{KC} + \theta_{KC}\lambda_{LC}$$

$$Q_F \equiv \theta_{LF}\lambda_{KF} + \theta_{KF}\lambda_{LF}$$

Clearly, σ_S is larger the greater is the elasticity of factor substitution for either sector. To simplify, suppose $\sigma_C = \sigma_F = \sigma$. Furthermore, note that

$$Q_C + Q_F + |\lambda||\theta| = 1$$

Therefore in the Heckscher-Ohlin model the assumption of a common degree of factor substitutability in each sector leads to the following as the expression for σ_S:

$$\sigma_S = \frac{1 - |\lambda||\theta|}{|\lambda||\theta|}\sigma \tag{6.S.19}$$

Two features of the model lead to elastic responses of outputs along the transformation schedule: first, a high degree of factor substitutability in each sector (σ), and second, fairly similar factor proportions, as shown by low values for $|\lambda||\theta|$. If factor

[4] See the reference in footnote 2 of Chapter 6. This supplement is based on R. W. Jones, "The Structure of Simple General Equilibrium Models," *Journal of Political Economy,* 73 (December 1965): 557–572, reprinted in his *International Trade: Essays in Theory* (Amsterdam: North-Holland, 1979).

[5] More details of this comparison are provided in R. W. Jones, *International Trade: Essays in Theory* (Amsterdam: North-Holland, 1979), Chapter 7.

proportions were identical, $|\lambda||\theta|$ would equal zero. By contrast, if labor were used only in one sector and capital in the other, $|\lambda||\theta|$ would equal 1 and σ_S would be zero.

In the sector-specific model, the expression for σ_S was given in Equation 5.S.14. The elasticities of labor's marginal product curves, γ_{Lj}, are related to the elasticity of factor substitution.[6] Thus γ_{LC} equals σ_C/θ_{KC}, and γ_{LF} equals σ_F/θ_{TF}. As in the Heckscher-Ohlin case, simplify by assuming a common value for $\sigma = \sigma_C = \sigma_F$ because intersectoral differences between σ_C and σ_F do little to change the value of σ_S (in either model). Furthermore, simplify by equating labor shares between sectors. The rationale here is that the Heckscher-Ohlin model focuses on the difference between factor intensities in the two sectors and assumes the *same* degree of factor mobility between sectors. (It assumes that labor and capital are each perfectly mobile between sectors.) By contrast, the sector-specific model focuses on the different degree of factor mobility between factors (labor perfectly mobile, capital—or land—completely immobile). It seems fair, therefore, to allow the same degree of labor intensity between the two sectors as captured by θ_{LC} and θ_{LF}. Thus the share of the specific factor in each industry is the same. Let θ_S denote the common value of θ_{KC} and θ_{TF}.

These simplifications allow σ_S for the sector-specific model in Equation 5.S.14 to be rewritten as

$$\sigma_S = \frac{1 - \theta_S}{\theta_S}\sigma \qquad (6.S.20)$$

A comparison with Equation 6.S.19 for the Heckscher-Ohlin model reveals (1) the common role in the two models played by the elasticity of factor substitution, σ, and (2) the focus in the sector-specific model on the importance of sector specificity as captured by θ_S, the share in the national income of specific factors. A greater degree of factor specificity implies a lower value for σ_S, precisely as (in the Heckscher-Ohlin model) a greater disparity in factor proportions implies a low σ_S. Each model is designed to focus on a different feature of the technology, with somewhat analogous results in terms of the response of outputs to prices.

SUPPLEMENT TO CHAPTER 10: Real Incomes, Prices, and the Tariff

Real Incomes and the Optimum Tariff

Recall from the supplement to Chapter 2 the basic expression for the change in the home country's level of real income, dy, in terms of the domestic price-weighted sum of consumption changes. This was Equation 2.S.10, reproduced here.

$$dy = dD_C + pdD_F \qquad (10.S.1)$$

This expression needs no modification in the case of tariffs, for it rests on the simple notion that real income depends only on the quantities of each commodity consumed

[6]See footnote 2.

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and the relative valuation at the margin of one commodity in terms of another, as reflected in the *domestic* relative price of food, p.

The home country's budget constraint indicates the source of a change in real incomes. With a tariff, however, the budget constraint can be written either in terms of domestic or world prices. It is instructive to look at each in turn.

In terms of domestic prices, aggregate spending at home, $D_C + pD_F$, is limited to the value of income, which is derived both from income earned in producing commodities, $x_C + px_F$, and from the proceeds of the tariff revenue. In the case of ad valorem tariffs, revenue depends on the home country's quantity of food imports, M, the foreign relative price of imports, p^*, and the tariff rate, t, and is the product of these three terms:

$$D_C + pD_F = x_C + px_F + tp^*M \tag{10.S.2}$$

Figure 10.4 illustrates this form of the budget constraint with all items measured in food units instead of clothing units. With post-tariff consumption at J, the aggregate value of incomes at domestic prices is $0E$, the value of incomes earned in production is shown by $0C$, and CE is the tariff revenue.

Consider, now, a small change in the tariff rate. This change leads to changes in prices, the consumption bundle, and production so that

$$dD_C + pdD_F + D_F dp = dx_C + pdx_F + x_F dp + d(tp^*M)$$

Shift $D_F dp$ to the right-hand side to obtain

$$dD_C + pdD_F = -Mdp + (dx_C + pdx_F) + d(tp^*M) \tag{10.S.3}$$

Note that the left-hand side is, by the definition given in Equation 10.S.1, the increase in the home country's real income, dy. Furthermore, the expression $dx_C + pdx_F$ on the right-hand side must vanish, because the slope of the transformation schedule, dx_F/dx_C, must equal the negative of clothing's relative *domestic* price, $1/p$.[1] Thus Equation 10.S.3 can be simplified as

$$dy = -Mdp + d(tp^*M) \tag{10.S.4}$$

That is, the sources of any real income gain to the home country are to be found in (1) a change in the domestic relative price of imports, dp, where any decrease in this price will raise real incomes at home by a factor given by the volume of imports, M; and (2) any increase in the tariff revenue, $d(tp^*M)$.

This provides one decomposition of real income changes, highlighting *domestic* prices and tariff revenue. An alternative, but equivalent expression, one emphasizing *world* prices (the terms of trade), is more frequently used in the literature. Expenditure and income are related by world prices. The domestic relative price of food, p, is given by $(1 + t)p^*$; substituting this quantity into Equation 10.S.2, and noticing that M is given by excess food demand, $D_F - x_F$, results in

$$D_C + p^*D_F = x_C + p^*x_F \tag{10.S.5}$$

[1]See the supplement to Chapter 2 for a more complete account.

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This equation states that at *world* prices the value of the home country's consumption bundle exactly equals the value of its production bundle. This equality is illustrated in Figure 10.4 by the fact that the post-tariff consumption bundle, J, and production bundle, B, both lie on line 4, whose slope, $-(1/p^*)$, indicates the world terms of trade. Differentiate 10.S.5 to obtain

$$dD_C + p^*dD_F = -Mdp^* + (dx_C + p^*dx_F)$$

Add and subtract pdD_F on the left-hand side and pdx_F on the right-hand side. This yields

$$(dD_C + pdD_F) + (p^* - p)dD_F = -Mdp^* + (dx_C + pdx_F) + (p^* - p)dx_F$$

As already explained, $dD_C + pdD_F$ is the definition of the increase in real income at home, and $dx_C + pdx_F$ vanishes if resources are allocated at the optimal point along the transformation schedule. Because the change in imports, dM, is equal to $dD_F - dx_F$, the entire expression reduces to

$$dy = -Mdp^* + (p - p^*)dM \qquad (10.S.6)$$

It is difficult to overestimate the importance of the breakdown represented by Equation 10.S.6 in understanding the welfare significance of tariffs. The first term, $-Mdp^*$, is the terms-of-trade effect, now stated in terms of world prices. Any policy that depresses the relative price at which the home country can purchase its imports in the world market will favorably affect welfare at home by an amount proportional to the volume of imports. If trade is impeded, however, as it will be if a tariff exists, the second term, $(p - p^*)dM$, must also be taken into account. $(p - p^*)$ is the tariff wedge—it is the discrepancy (tp^*) between the relative domestic price of imports and the world price of imports. This second term indicates that any increase in the home country's level of imports must increase real income if the cost of obtaining imports in the world market (as shown by p^*) falls short of the relative value of imports in the local market (as shown by p). Any policy pursued by the home country that restricts imports entails a welfare loss if a tariff wedge has raised the domestic (relative) price of imports over the world level. This loss is directly proportional to the extent of the tariff rate.

We are now in a position to develop a formula for the *optimum tariff rate.* In Equation 10.S.6 the expression for dy can be set equal to zero if we are considering small variations in the tariff rate around the optimal rate that maximizes real income. (In Figure 10.6, $dy = 0$ at the optimal tariff rate t_0.) Replace $p - p^*$ by the equivalent expression, tp^*:

$$Mdp^* = tp^*dM$$

Dividing both sides by p^*M, and recalling the use of the hat notation to express relative changes (e.g., $\hat{M}$ is defined as dM/M), the optimal tariff can be expressed as

$$t = \frac{1}{\hat{M}/\hat{p}^*} \qquad (10.S.7)$$

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The foreign offer curve remains stationary, but the home offer curve does not. Therefore, if $\hat{M}$, the relative change in the home country's import demand, could be linked to $\hat{M}^*$, the relative change in foreign import demand, the expression for the optimal tariff given by Equation 10.S.7 could be translated into an expression involving ε^*, the elasticity of import demand along the foreign offer curve.

The relationship between M and M^* is simple—it is given by the equilibrium condition of Equation 10.S.8, which states that at world prices the value of the home country's imports is equated to the value of foreign imports (or home country exports).

$$p^*M = M^* \tag{10.S.8}$$

Taking relative changes in Equation 10.S.8 yields

$$\hat{p}^* + \hat{M} = \hat{M}^* \tag{10.S.9}$$

Therefore $\hat{M}/\hat{p}^*$ equals $(\hat{M}^*/\hat{p}^*) - 1$. But $\hat{M}^*/\hat{p}^*$ is merely the definition of ε^*, the elasticity of the foreign country's demand for imports along its offer curve.[2] This shows that the formula for the optimum tariff given in Equation 10.S.7 can be rewritten as

$$t = \frac{1}{\varepsilon^* - 1} \tag{10.S.10}$$

This formula needs to be interpreted carefully. It seems to state that if the foreign offer curve is inelastic ($\varepsilon^* < 1$) the tariff should be negative. This interpretation of the relationships underlying the formula would be incorrect. Reconsider Equation 10.S.6. If the foreign offer curve is inelastic, an increase in the tariff would cause home imports to increase. The terms of trade improve for the home country, and with ε^* less than 1, foreigners offer more food for export. (See the discussion in the appendix to Chapter 10.) On both counts dy in Equation 10.S.6 must be positive. The home country should raise its tariff until it reaches the elastic stretch of the foreign offer curve. Only then will a favorable movement in the terms of trade be countered by an unfavorable cutback in the volume of imports.

The Impact of Tariffs on World and Domestic Prices

Tariffs create wedges between domestic import prices and world prices. A natural presumption is that the imposition of a tariff drives up the price of imports at home relative to other goods while it depresses the world price. As we shall see, this may not always follow. What is required is an explicit solution for each of these price changes, and a sharp distinction must be drawn between shifts of demand curves and movements along demand curves.

[2]This elasticity formulation was introduced in Chapter 2. Because $1/p^*$ is the relative price of the foreign country's import (clothing), ε^* is defined as *minus* $\hat{M}^*$ divided by $(\widehat{1/p^*})$, which is equivalent to *plus* $\hat{M}^*/\hat{p}^*$.

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Equation 10.S.9 revealed the equations of change that can be used to solve for the change in world prices, $\hat{p}^*$. The change in foreign imports, $\hat{M}^*$, is captured by movements along the foreign offer curve because our tariff does not cause their demand curve to shift. Thus

$$\hat{M}^* = \varepsilon^* \hat{p}^* \qquad (10.S.11)$$

The expression for $\hat{M}$ is more complicated. A change in the tariff rate shifts the home country's offer curve. Therefore $\hat{M}$ will exhibit a mixture of such a shift and a move along the home country's offer curve. Specifically, the home offer curve is shown as $M = M(p^*, t)$ and the rate of change can be decomposed as follows:

$$\hat{M} = -\varepsilon\, \hat{p}^* + \beta dt \qquad (10.S.12)$$

where β, defined literally as $(1/M)(\partial M/\partial t)$, is the shift in the home country's offer curve at given world terms of trade. One of the primary objectives is to develop an explicit expression for β to guarantee that it is negative. Figure 10.5 showed that an increase in t would reduce imports at given world terms of trade.

Substituting Equation 10.S.11 for $\hat{M}^*$ and Equation 10.S.12 for $\hat{M}$ into Equation 10.S.9 yields the following solution for the effect of a tariff on world terms of trade.

$$\hat{p}^* = \frac{1}{\Delta} \beta dt \qquad (10.S.13)$$

where

$$\Delta = \varepsilon + \varepsilon^* - 1$$

The expression Δ captures the Marshall-Lerner condition for market stability discussed in the supplement to Chapter 3. Assuming the market to be stable, the sum of import-demand elasticities must exceed unity, and Δ must therefore be positive. Thus, if the home country's offer curve shifts inward (β will be shown to be negative), the world relative price of our import falls.

Home prices are linked to foreign prices by the tariff rate: $p = (1 + t)p^*$. Assuming that trade is initially free, taking relative changes in these terms yields

$$\hat{p} = \hat{p}^* + dt \qquad (10.S.14)$$

With the solution for the terms-of-trade change, $\hat{p}^*$, given by Equation 10.S.13, the next step is to substitute to obtain the solution for the change in the relative domestic price of imports, $\hat{p}$:

$$\hat{p} = \frac{1}{\Delta}(\Delta + \beta)dt \qquad (10.S.15)$$

Although Δ is positive, this discussion has maintained (and will subsequently prove) that β is negative. This argument underscores the doubts expressed in the text concerning whether an increase in t must protect the import-competing industry.

Elasticity and Shift of the Home Offer Curve

To simplify matters at this stage, continue to assume that initially there is free trade so the initial value of t is zero.[3] The forces at work along the home country's offer curve were displayed in Equation 2.S.17 for the home elasticity of import demand:

$$\varepsilon = \overline{\eta} + e + m$$

An improvement in the terms of trade encourages imports by (1) causing consumers to *substitute* toward the now-cheaper imports ($\overline{\eta}$); (2) causing resources to be allocated away from now-cheaper import-competing goods toward exports (e); and (3) raising real incomes, with part of the gain spilling over to importables (m).

The *shift* in the home offer curve reveals the forces encouraging a reduced volume of imports *at the initial terms of trade* as the tariff is raised. The hike in t at initial p^* raises domestic p and thus reduces imports via a substitution effect in consumption, $\overline{\eta}$, and a substitution effect in production, e. However, because the terms of trade are unchanged, so is real income; thus the (m) term in ε is missing from the shift. The reason: Because trade is initially free ($p^* = p$ initially), the expression for real income changes reduces to the terms-of-trade effect,

$$dy = -Mdp^*$$

which is zero if the terms of trade are held constant. That is, the *shift* in the offer curve is shown by

$$\beta = -(\overline{\eta} + e) \qquad (10.S.16)$$

The Metzler Tariff Paradox

It is now possible to develop an explicit criterion for the paradoxical case in which a tariff so depresses the terms of trade that the relative domestic price of imports falls as well. Substitute the expression for β in Equation 10.S.16 into the expression for $\hat{p}$ in Equation 10.S.15 to obtain

$$\hat{p} = \frac{1}{\Delta}(\varepsilon + \varepsilon^* - 1 - \overline{\eta} - e)dt$$

Given the breakdown of home ε, the solution for $\hat{p}$ is

$$\hat{p} = \frac{1}{\Delta}(\varepsilon^* + m - 1)dt \qquad (10.S.17)$$

The argument in Chapter 10 suggested that a tariff could fail to protect if the foreign import demand elasticity, ε^*, were sufficiently small. Equation 10.S.17 reveals that the critical value for this elasticity is $(1 - m)$ or, more simply, the country's propensity to consume its export commodity.

[3]A more general treatment is provided in R. W. Jones, "Tariffs and Trade in General Equilibrium: Comment," *American Economic Review*, 59 (June 1969): 418–424.

The appendix to Chapter 10 shows, in Figure 10.A.2, an offer curve diagram in which the Metzler tariff paradox may hold. The razor's-edge case in which the income-consumption curve is tangent at Q to the foreign offer curve, $0_T R^*$, corresponds to ε^* being equal to $1 - m$ in Equation 10.S.17.

SUPPLEMENT TO CHAPTER 11: Tariffs, Growth, and Welfare

This supplement continues the algebraic analysis of tariffs initiated in the supplement to Chapter 10. It provides a formal proof of the fact that the maximum-revenue tariff rate exceeds the optimal rate. For a given degree of protection, a criterion is developed relating growth to welfare changes. Finally, a broader analysis of the tariff, making use of matrix algebra, allows an easy overview of the question of gains from trade and commercial policy.

The Maximum-Revenue Tariff

The supplement to Chapter 10 expressed the home country's budget constraint in terms of domestic prices (see Equation 10.S.2). When differentiated, this expression led to an expression for the change in real income in terms of the change in the domestic price ratio and the tariff revenue. This was Equation 10.S.4, reproduced here.

$$dy = -Mdp + d(tp^*M) \tag{11.S.1}$$

Consider this expression in conjunction with Figure 11.1. The optimal tariff rate is t_0, and the optimal tariff formula (Equation 10.S.10) showed that near t_0, the foreign offer curve must be elastic. This means that the tariff must be protective in the sense of raising p with a small further increase in t. Thus the $-Mdp$ term in Equation 11.S.1 is negative in the neighborhood of the optimum tariff, where dy equals zero. As a consequence, $d(tp^*M)$ must be positive. That is, at rate t_0 in Figure 11.1, the curve plotting the tariff revenue against the tariff rate must be positively sloped. Tariff revenue reaches a maximum at the higher rate, t_2.

Growth with Protection

The supplement to Chapter 3 analyzed the possibility of *immiserizing growth*—a situation in which expansion of a country's production of exportables during the growth process causes such a deterioration in the terms of trade that the community's welfare actually falls.

Examine here the case of a country with fixed tariff rates and *given world prices*. For some reason (growth of resources, improvement in technology) the country's transformation schedule shifts outward, so that at the fixed domestic prices (given world prices adjusted for fixed tariff rates) aggregate output expands. The budget constraint is shown by

$$D_C + p^*D_F = x_C + p^*x_F \tag{11.S.2}$$

(This repeats Equation 10.S.5.) Differentiation leads to

$$dD_C + p^*dD_F = dx_C + p^*dx_F \tag{11.S.3}$$

Note there is no terms-of-trade effect because p^* is assumed constant. Add and subtract pdD_F on the left-hand side.

$$(dD_C + pdD_F) + (p^* - p)dD_F = dx_C + p^*dx_F$$

The first expression in parentheses is, of course, the change in home real income, dy. The change in home consumption of food, dD_F, can only be explained by income effects because domestic prices are constant. That is, with the home country's marginal propensity to import food denoted by m,

$$dD_F = \frac{m}{p}dy$$

The fraction $(p^* - p)/p$ is minus $t/(1 + t)$ so that

$$\left(1 - m\frac{t}{1 + t}\right)dy = dx_C + p^*dx_F \tag{11.S.4}$$

This expression provides the criterion with which to judge growth in a protected economy. Real income gains are registered only if growth results in a greater aggregate production *evaluated at world prices.* This may seem paradoxical. The criterion for judging an increase in welfare is to measure consumption changes at *domestic prices,* yet production changes should be evaluated at world prices because world prices measure the trade-off between production and consumption (see Equation 11.S.2). Figure 11.4 illustrated how various possibilities of output expansion from point A—points D, B, C, or E, all showing a 25 percent gain in output at domestic prices—resulted in different real income gains. For point E the value of output actually fell at world prices.

Tariffs, Gains from Trade, and Welfare: A General Analysis

Turn, now, to a different question: How can welfare or real income of an economy be compared in two situations in which prices, quantities traded, and trade restrictions may differ by more than a small amount? There is no restriction on the number of commodities produced or consumed at home or abroad. For notation, x is the vector of quantities produced at home, D is the vector of quantities demanded or consumed, p is the vector of prices ruling in the home country, and p^* is the vector of prices ruling abroad.[1] Not all commodities need be produced at home, so that in the vector $x = (x_1, x_2, \ldots, x_n)$ some entries may be zero. Similarly, not all commodities produced need be demanded locally, so that in the vector $D = (D_1, D_2, \ldots, D_n)$ some entries may also be zero. The two situations to be compared are denoted by a single prime and a double prime. Thus, in the initial situation, home prices are given by the vector $p' = (p'_1, p'_2, \ldots, p'_n)$. This vector may or may not represent a situation in which some international trade takes

[1]The analysis in this section rests heavily on Michihiro Ohyama, "Trade and Welfare in General Equilibrium," *Keio Economic Studies,* 9 (1972): 37–73.

place. In the second situation prices have altered at home to $p'' = (p_1'' \ p_2'', \ldots, p_n'')$. Let the vector E represent the home country's set of *excess demands:*

$$E \equiv D - x$$

An element E_i in the vector E is positive if commodity i is imported at home, negative if i is exported, and zero if high transport costs or tariffs result in no international exchange of the ith commodity.

The basic criterion by which welfare in the double-prime situation is contrasted to welfare in the single-prime situation involves a comparison of the value of aggregate demand in each, when the prices used for the evaluation are in both instances those of the double-prime situation. Thus welfare is deemed to have risen if

$$p''D'' - p''D' > 0 \tag{11.S.5}$$

This inequality states that if the initial bundle of goods consumed, D', could have been purchased in the double-prime situation, the community is assumed to have increased its real income.

This assumption is illustrated for the two-commodity case in Figure 11.S.1. The fact that the consumption bundle in the single-prime situation, D', lies below the line showing prices in the double-prime situation (and supporting demand, D'') is taken as a sufficient criterion for establishing that point D'' represents a higher level of welfare. Clearly, if indifference curves do not intersect, point D' must lie on a lower indifference curve than point D''.

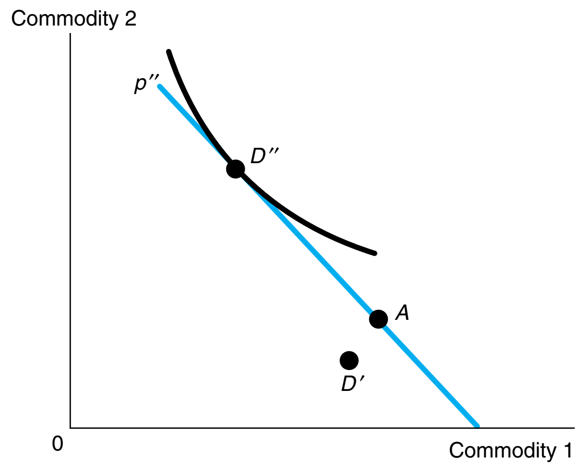
The vector of excess demands equals the vector of total demands minus the vector of production. Turn this equation around to state that demand equals excess demand *plus* production. Making this substitution for both the single-prime and the double-prime situations in the improvement in welfare criterion, Inequality 11.S.5, yields the following inequality as an equivalent expression.

$$p''(E'' - E') + p''(x'' - x') > 0 \tag{11.S.6}$$

FIGURE 11.S.1

The Welfare Criterion for Two Commodities

Two alternative consumption bundles are illustrated: D' and D''. The prices ruling when D'' is consumed are shown by line p''. The welfare criterion whereby situation double-prime is superior to situation single-prime is shown by the fact that D' lies below line p'', which means $p''D'' - p''D' > 0$.



Prices at home in the double-prime situation will differ from prices abroad for any traded commodity that is subject to a tariff, an export tax, or a subsidy in the home country. Let the matrix T'' represent these taxes and/or trade subsidies. T'' is a *diagonal matrix,* all of whose elements are zero except the diagonal terms. What does the entry t_i'' represent? This depends on whether commodity i is imported (E_i positive), in which case a positive t_i'' represents a tariff and a negative t_i'' an import subsidy, or exported (E_i negative), in which case an export tax is a negative t_i'' and an export subsidy a positive t_i''. In short, $t_i'' p_i^{*''} E_i''$ is positive if the government collects tax revenue and negative if the government is subsidizing a trade flow. For any commodity i

$$p_i'' = (1 + t_i'')p_i^{*''}$$

where $p_i^{*''}$ is the world price of commodity i. This can be summarized in matrix notation by making use of the identity matrix, I, whose off-diagonal elements are all zero, and with 1's all along the diagonal.

$$p'' = (I + T'')p^{*''} \tag{11.S.7}$$

The home country's budget constraint states that the value at world prices of aggregate excess demand is zero, both for the double-prime and single-prime situations.

$$p^{*''}E'' = 0 \tag{11.S.8}$$

$$p^{*'}E' = 0 \tag{11.S.9}$$

Furthermore, if the single-prime situation refers to the pretrade situation at home, each element of the vector E' would have to equal zero because in equilibrium, local demand would have to be balanced by local sources of supply.

All the ingredients are now at hand to transform the welfare criterion, Inequality 11.S.6, into an explicit listing of the sources of an improvement in real incomes. To proceed, merely substitute the relationship shown in Equation 11.S.7 between domestic and world prices into the first term in Inequality 11.S.6.

$$p''(E'' - E') = (I + T'')p^{*''}(E'' - E')$$

This expression, in turn, equals

$$p^{*''}E'' - p^{*''}E' + T''p^{*''}(E'' - E')$$

Notice, however, that by Equation 11.S.8, $p^{*''}E''$ vanishes. This statement of the budget constraint at world prices applies as well to the single-prime situation (shown by Equation 11.S.9), and thus allows $p^{*'}E'$ (equal to zero) to be added to the expression. Thus rewritten, the expression becomes

$$-(p^{*''} - p^{*'})E' + T''p^{*''}(E'' - E')$$

Substitute this expression for $p''(E'' - E')$ back into Inequality 11.S.6 to obtain the basic welfare criterion.

$$-(p^{*''} - p^{*'})E' + T''p^{*''}(E'' - E') + p''(x'' - x') > 0 \tag{11.S.10}$$

Each of the three terms in this inequality should be familiar from the preceding discussion.

1. The term $-(p^{*\prime\prime} - p^{*\prime})E'$ is the terms-of-trade effect. If the two primed situations represent different trading equilibria that are very close to each other, and if only one relative price (because only two commodities) exists, it is shown by the $-Mdp^*$ term in Equation 10.S.6. The general expression states that the community's welfare improves to the extent that the world price falls for any commodity imported ($E_i' > 0$), or rises for any commodity exported ($E_i' < 0$).

2. The term $T''p^{*\prime\prime}(E'' - E')$ measures the change in the volume of trade for all commodities for which domestic prices, p'', differ from world prices, $p^{*\prime\prime}$. The term $T''p^{*\prime\prime}$ is the tariff wedge. Returning again to the case in which only two commodities are traded (and the two situations are very close to each other), we see that this term reduces to the $(p - p^*)dM$ term in Equation 10.S.6. It states in general that real income is improved if the level of imports increases for any commodity worth more at home (as indicated by p'') than it costs to obtain in world markets (as indicated by $p^{*\prime\prime}$).

3. The term $p''(x'' - x')$ must in any case be greater than or equal to zero. It shows the change in real income attributable to the change in production. In the absence of distortions, x'' is the point on the transformation schedule that maximizes the value of output at domestic prices when these are given by p''. Therefore, the value of any other production possibility, say x', at these prices (p''), must be less. If the single-prime and double-prime situations are very close together in the two-commodity model, this term reduces to $dx_C + pdx_F$. As was argued in Chapter 2 and subsequently, this reduction approaches zero as an expression of the equality between the domestic price ratio and the slope of the transformation schedule.

This line of reasoning has been useful in comparing two states of trade, differing from each other in prices—perhaps as a result of changes in tariffs. It is also useful in comparing a state of trade (in the double-prime situation) with the pretrade situation. In such a case each element in the vector E' goes to zero. The welfare criterion, Inequality 11.S.10, then assumes the special form

$$T''p^{*\prime\prime}E'' + p''(x'' - x') > 0 \tag{11.S.11}$$

Because the production term, $p''(x'' - x')$, must be nonnegative, as was just argued, this criterion yields a powerful result. Suppose that a complex mixture of tariffs and trade subsidies exists. Is the community better off than with no trade? The question needs to be raised because an export subsidy by itself can reduce welfare at home—this is akin to giving something away. The term $T''p^{*\prime\prime}E''$ represents the *net* tariff and subsidy revenue to the home government. The criterion reveals that regardless of the pattern of subsidies, if this net revenue is positive, trade must be superior to no trade.[2] Note that it

[2]This result is derived in M. Ohyama, "Trade and Welfare in General Equilibrium."

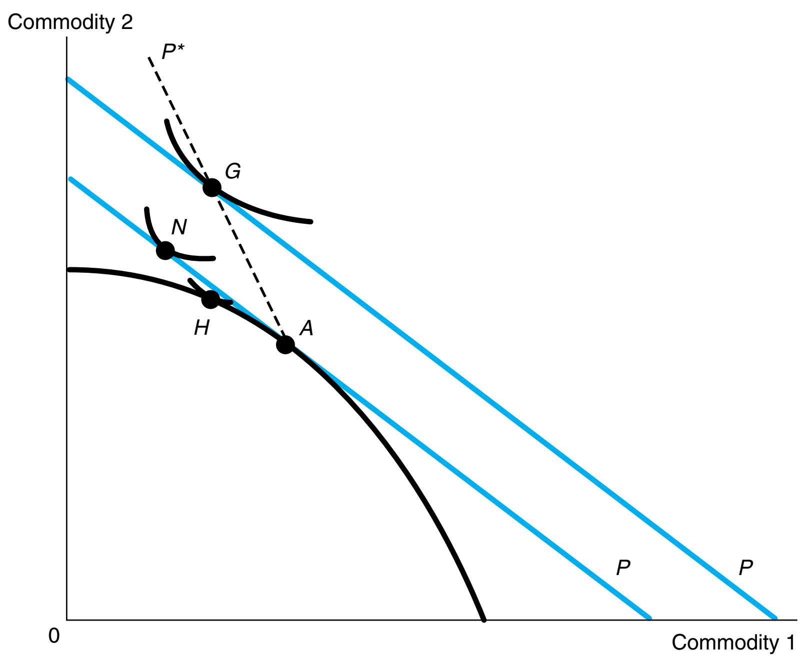
is *sufficient* that the net revenue be positive for the double-prime situation to represent an improvement. However, even if net revenue is negative, it is possible for D'' to be preferred to D'.

This result, that trade distorted by the presence of trade taxes and subsidies is nonetheless superior to autarky as long as the net tariff revenue is positive, is illustrated in Figure 11.S.2. The tax-distorted consumption equilibrium at point *G* is similar to that illustrated in Figure 10.4's standard depiction of the effect of a tariff on real incomes. Price lines labeled *P* show a higher relative domestic price for importables than does price line *P**, which reflects world prices. Behind the tax barriers, producers select point *A* and consumers choose *G*; these points have equal value at world prices, but the value of the consumption bundle at domestic prices exceeds the value of production by the amount of the net tariff revenue. Key to the argument that distorted trade with positive net tariff revenue is superior to autarky is the comparison between consumption point *N*, which would be selected if domestic price line *P* indicated world prices (i.e., tariff revenues were zero), and autarky bundle *H*. If the country were offered terms-of-trade *P*, differing from autarky prices (the slope at *H*), the country would gain. If, in addition, consumers were provided a boost to their disposable incomes in the form of a positive net tariff revenue, real incomes (at *G*) would rise even further. Indeed, even if, on net, the budget line were reduced slightly below line *NA*, reflecting a small negative tax revenue (subsidies exceeding taxes), trade might be preferable to autarky, but a sufficient condition for distorted trade to lead to gains over autarky is a positive value for net tariff revenue.

FIGURE 11.S.2

Positive Tax Revenue Leads to Gains

Domestic prices, represented by the *P* lines, are distorted from world prices, shown by the *P** line. Production is at *A*, consumption at *G*. *N* is superior to autarky bundle *H*, as is *G*, as long as net tariff revenue is positive.



SUPPLEMENT TO CHAPTER 12: Imperfect Competition, Trade Restrictions, and Welfare

The supplement to Chapter 10 developed an expression for the way in which a country's aggregate real income is affected by changes in levels of protection when markets are perfectly competitive. The basic statement was contained in Equation 10.S.6, which showed how an increase in the rate of protection might aid by improving a country's *terms of trade* (lowering p^*, the relative world price of imports), but most likely at the expense of lowering the *volume of trade* (a negative dM), in a situation in which protection has raised the domestic price of imports, p, above the world price, p^*. The domestic price reflects the value to the home country of obtaining another unit of imports, whereas the foreign price indicates the real cost of obtaining another unit of imports. When elements of imperfect competition characterize home markets for importables or exportables, the breakdown of real income changes for competitive markets shown in Equation 10.S.6 needs to be supplemented to take into account the fact that any change in the economy's composition of outputs also affects aggregate welfare.

A useful starting point is the statement that, when evaluated at world prices, the economy's aggregate consumption bundle must match the value of aggregate production. (The rationale: At world prices the value of exports equals the value of imports under the assumption that trade is balanced.) Returning to the standard two-commodity model, this is the relationship shown in Equation 10.S.5, reproduced here:

$$D_C + p^*D_F = x_C + p^*x_F \tag{12.S.1}$$

Proceeding as in the supplement to Chapter 10, total differentiation of both sides and a subsequent addition and subtraction of pdD_F on the left-hand side and pdx_F on the right-hand side yields

$$(dD_C + pdD_F) + (p^* - p)dD_F = -Mdp^* + (dx_C + pdx_F) + (p^* - p)dx_F$$

As in previous discussions, the country is assumed to import food (M). The expression can be simplified, as the change in the economy's level of real income, dy, is the first expression, $(dD_C + pdD_F)$, and the wedge separating foreign and home food prices, $(p^* - p)$, multiplied by the changes in consumption, dD_F, and local production, dx_F, can be combined to yield

$$dy = -Mdp^* + (p - p^*)dM + (dx_C + pdx_F) \tag{12.S.2}$$

The first two terms of Equation 12.S.2 are familiar from the tariff analysis in the supplement to Chapter 10, corresponding to the terms-of-trade effect and the volume-of-trade effect, respectively. Of course, a tariff that improves the terms of trade usually does so at the expense of a cutback in imports. The optimal rate of tariff in the absence of monopoly pricing involves a trade-off between the terms-of-trade effect and the volume-of-trade effect. In a competitive market setting, the final term, $(dx_C + pdx_F)$, vanishes. However, if competition locally is less than perfect, the relative price does not

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reflect the ratio of marginal costs. Let c denote the ratio of the marginal cost of producing food to the marginal cost of producing clothing, that is, the marginal opportunity cost of producing food (much as p denotes the relative domestic price of food). The term $(dx_C + cdx_F)$ equals zero because the slope of the transformation schedule indicates, in general, marginal opportunity costs. Therefore, Equation 12.S.2 can be written as

$$dy = -Mdp^* + (p - p^*)dM + (p - c)dx_F \qquad (12.S.3)$$

The last term in this equation reveals that if markets are imperfectly competitive at home, even small changes in the composition of output have welfare consequences. This exposes the basis for *industrial policy* in managing a nation's commercial policy instruments. Suppose, for example, that a local monopoly in producing food has caused its price to exceed marginal costs. Consider a tariff initially set at a rate that would be optimal for a *competitive* economy—that is, a rate for which any further increase in the tariff would cause a volume-of-trade loss just balancing the terms-of-trade gain. Equation 12.S.3 suggests that if competition is less than perfect in the import-competing food sector, a further increase in the tariff rate would still contribute positively to national welfare because it would encourage a reallocation of resources toward producing more of the importable (food)—the value of food locally, p, exceeds its marginal cost of production, c. Tariff policy thus may have a further dimension—it provides a second-best means of encouraging output in a sector in which the existence of monopoly power has curtailed output below the competitively optimal level.

Figure 12.1 showed an initial equilibrium at point B or B' on the transformation curve when food production is characterized by local monopolistic behavior. (The clothing sector is competitive, with price equal to marginal cost.) With the domestic price of food exceeding marginal cost, the budget line showing domestic prices at B or B' is flatter than the transformation schedule, so any policy encouraging a reallocation of resources in favor of food production tends to raise national income.

Suppose, instead, that some element of local monopoly control exists in the export sector (clothing), so that at an initial free-trade equilibrium the relative domestic price of food falls short of its relative marginal cost. If the country can improve its terms of trade with a tariff, Equation 12.S.3 reveals that the temptation to pursue a protectionist policy is limited both by the volume-of-trade effect once the tariff is sufficiently high and by the deleterious effect on welfare of an expansion in the competitive import-competing sector (food). If the volume of trade is somewhat limited (small value of M), and if the discrepancy between relative food price, p, and cost, c, is relatively large in absolute value (p lies below c if clothing is the monopolistic sector), the country may find free trade a better policy than any tariff level despite the forgone terms-of-trade improvement.

Expressions such as Equation 12.S.3 are useful in appraising the welfare consequences of the use of various instruments of commercial policy. In some cases, simplifications of the expression are allowed or modifications required. For example, suppose a small country has no influence on its terms of trade—then the first term in Equation 12.S.3 vanishes. If an import quota has been imposed and is binding, a loosening up of quota restrictions directly raises welfare as the volume of allowed imports increases. In such a setting the domestic price of food will fall as foreign sources supply a larger share of the home market, and this encourages local demand (a welfare gain). Because the change in imports equals the change in demand less the change in local supply, the last two terms in Equation 12.S.3 can be rewritten as

$$(p - p^*)dD_F + (p^* - c)dx_F$$

Thus a loosening of quota restrictions encourages demand, which raises welfare but cuts back on local production. Of crucial relevance in appraising the consequences of such a cut is the relationship between the *world* price of food and local marginal costs. Even if domestic price exceeds marginal cost (with a local imperfectly competitive food industry), if world price is lower than marginal cost, the cutback in food production further improves welfare.

Finally, note the modifications required if the country has restricted imports with VERs (voluntary export restrictions urged on foreign suppliers) instead of quotas. In such a case the home country receives none of the revenue represented by the gap between home and foreign prices. This implies that the first term in Equation 12.S.3 should be replaced by $-Mdp$ because p represents the price the country must pay to foreigners when VERs are imposed, and the second term, $(p - p^*)dM$, is deleted because the spread between home and foreign prices no longer accrues to the home government. One immediate consequence for a small open economy in a competitive setting (so that p^* remains constant and the last term in Equation 12.S.3 can be ignored): Reductions in levels of protection must raise real incomes whether imports have been restricted by quotas or VERs. Reductions in import quotas lead to positive values for $(p - p^*)dM$; reductions in VERs lower the domestic price of importables and thus lead to positive values of $-Mdp$.

Subsidies with International Duopoly

The appendix to Chapter 12 described the potential for a strategic use of a subsidy on exports in a global market characterized by duopoly.[1] Here we provide some of the algebraic underpinning for the result that an export subsidy may be used to raise the real income in the home country.

The setting: Home and foreign country each have a single firm producing a commodity not consumed in either country. In each case the commodity is exported to a third market in which there are no competing producers. Each firm is assumed to have constant marginal costs—c at home and c^* in the foreign country. The home country's government, however, supports its home firm by granting an export subsidy of amount s

[1]The pioneering work in this area is by James Brander and Barbara Spencer. In particular see their "International R&D Rivalry and Industrial Strategy," *Review of Economic Studies,* 50 (October, 1983): 707–722, and "Export Subsidies and International Market Share Rivalry," *Journal of International Economics,* 18 (1985): 83–100.

per unit of output, so that *net* marginal costs at home are $c - s$. Demand for the commodity in the third market is given by

$$p = p(q + q^*)$$

That is, price depends on total home and foreign output. Each firm is attempting to maximize its profits by picking the appropriate output level, assuming its rival's output remains unchanged (this is the so-called Cournot assumption). Thus marginal revenue for the home firm is $(p + qp')$ and for the foreign firm is $(p + q^*p')$, where p' denotes the slope of the market demand curve and is negative. Profit maximization for each firm under the Cournot assumption involves setting marginal revenue equal to marginal cost. The reaction function shown for the home firm in Figure 12.A.1 illustrates how home output rises (for a given subsidy rate) for each unit fall in q^*. In the case of linear demand (p' is a constant), each unit fall in q^* leads to a one-half unit rise in home output, q. The Cournot market equilibrium position is at the point of intersection of these two reaction functions.

An increase in the home government's rate of subsidy, s, to the home firm serves to shift outward the home reaction function in Figure 12.A.1. For a small change in s, ds, this process can be depicted algebraically by differentiating each firm's first-order condition for profit maximization (i.e., an equation matching marginal revenue with (net) marginal cost). Assuming linear demand (i.e., $p'' = 0$), this leads to

$$2p'dq + p'dq^* = -ds$$

$$p'dq + 2p'dq^* = 0$$

Solving for the change in each firm's output shows dq equal to $\left\{\frac{2}{3p'}\right\}ds$ and dq^* equal to $\left\{\frac{1}{3p'}\right\}ds$. Thus an increase in the subsidy rate raises home output by twice as much as it lowers the foreign firm's output. The subsidy clearly is of benefit to consumers in the third market.

Subsidizing the home firm obviously benefits the home firm as well. But how about real incomes at home, where the increase in the home firm's profits must be offset by the increased export subsidy? The profits of the home firm are given by $[p - (c - s)]q$, and the change in these profits as a consequence of the raised subsidy rate is shown by

$$[p + qp' - (c - s)]dq + qp'dq^* + qds$$

The coefficient of dq vanishes by the first-order condition for profit maximization. Because both p' and dq^* are negative, the next term, sometimes called the *strategic effect,* must be positive, whereas the last term is clearly positive because it shows the raised subsidy rate on the firm's initial output. Against this expression must be set $(qds + sdq)$, the increase in the government's outlay on export subsidies. The first term, qds, just cancels the last term in the expansion in firm profits. If the subsidy rate were initially zero, the net welfare change for the home country would be just the positive term, $qp'dq^*$, the strategic effect. Thus some subsidy is beneficial to the country as a whole. But all good things come in moderation, and too high a rate of subsidy would

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not be optimal. In Figure 12.A.1 the optimal point for the home country is where one of its isoprofit curves is tangent to the foreign reaction function. The appendix discusses possible counterarguments to the logic that a country can advance its cause by subsidizing the export activities of one of its firms; certainly one such warning would be the possibility that the foreign government retaliates.

SUPPLEMENT TO CHAPTER 16: Import and Export Elasticities

Under what conditions does a devaluation improve the trade balance? The answer when producers are assumed to supply exports and imports with infinite elasticity is the Marshall-Lerner condition. First we prove this. Then we relax the assumption of infinite elasticities.

Proof of the Marshall-Lerner Condition

For notational simplicity, we adopt the normalization $\overline{P} = 1$ and $\overline{P}^* = 1$ in the proof of this proposition. Then the trade balance expressed in foreign currency is

$$TB^* = (1/E)X_D(E) - M_D(E)$$

Differentiate with respect to E.

$$dTB^*/dE = -(1/E^2)X + (1/E)(dX_D/dE) - dM_D/dE$$

Multiply by E^2/X. The derivative is positive if

$$-1 + (E/X)(dX_D/dE) - (E^2/X)(dM_D/dE) > 0$$

Using the definitions of the elasticities,

$$\varepsilon_X \equiv (dX_D/dE)E/X \qquad \varepsilon_M = -(dM_D/dE)E/M,$$

the condition becomes

$$-1 + \varepsilon_X + (EM/X)\varepsilon_M > 0$$

Starting from a position of balanced trade, $EM = X$, the equation reduces to the Marshall-Lerner condition asserted in Chapter 16.

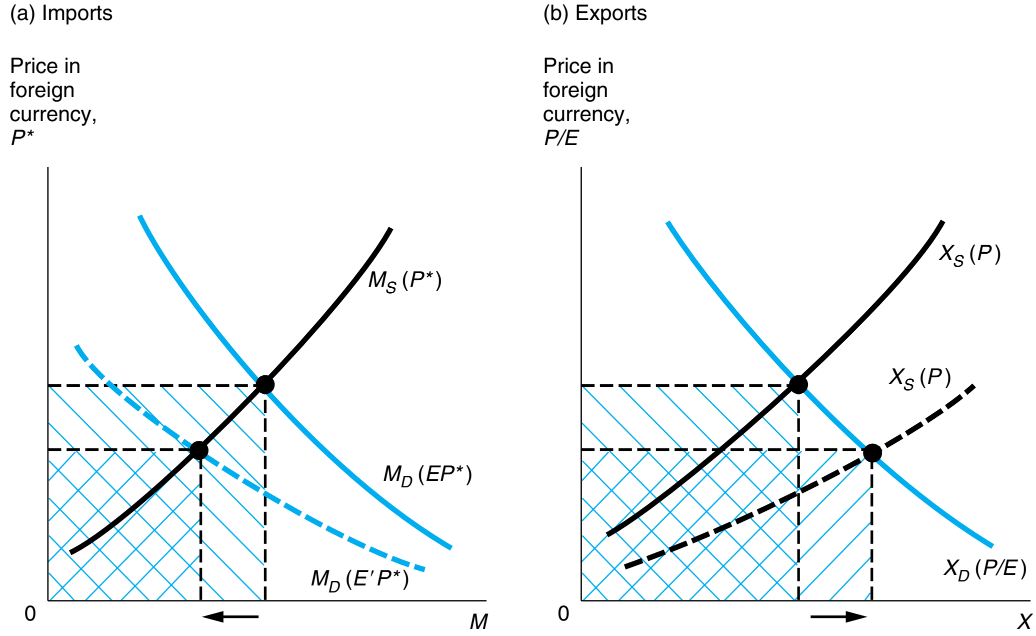
When Supply Elasticities Are Finite

Consider the case where supply of X and M is not infinitely elastic. Figure 16.S.1 illustrates this general case. True, a devaluation still shifts the import demand curve and the export supply curve (which was a horizontal line in Figure 16.2) down. In addition, it remains true that import spending falls and that the effect on export revenue is ambiguous because any given quantity of exports translates into a smaller value when expressed in foreign currency. Thus the basic conclusions are similar, but the relevant condition necessary for the trade balance to improve includes supply as well as demand elasticities. (See Problem 6 at the end of Chapter 16.)

FIGURE 16.S.1

Effect of a Devaluation with Less Than Infinitely Elastic Supply

The devaluation can lower prices when expressed in foreign currency. Panel (a) shows the effect on imports, and (b) shows the effects on exports.



SUPPLEMENT TO CHAPTER 19: The Monetarist Two-Country Model of the Balance of Payments

Chapter 19 assumed that the home country's money supply is too small to affect substantially the world money supply or world price level. To be sure, when international reserves are flowing out through a balance-of-payments deficit, the rest of the world is running a balance-of-payments surplus. However, it was assumed that the reserve flow is just a drop in the ocean so far as the rest of the world is concerned. This supplement relaxes the small-country assumption and moves to a two-country world. A domestic monetary expansion will succeed in raising the price level in the world to the extent that it raises the world money supply. As Appendix B mentioned, the two-country model is useful for understanding the gold standard, as well as for understanding the Bretton Woods system, with the United States in the 1960s increasingly playing the role of the country with a balance-of-payments deficit, and Europe the role of the surplus country.[1]

[1]The model in this supplement is drawn from the first half of Rudiger Dornbusch, "Devaluation, Money, and Nontraded Goods," *The American Economic Review,* 65, no. 5 (1973): 871–880.

Determination of the Balance of Payments in the Two Countries

We model the foreign country just as we modeled the domestic country in the chapter. The rate of increase of the foreign money supply, H^*, is related to the foreign excess demand for money. The foreign excess demand for money is, in turn, an increasing function of foreign nominal income, or of the foreign price level, with foreign real income determined at $\overline{Y}^*$ by exogenous supply factors, and a decreasing function of the foreign money supply, M^*.

$$H^* \equiv \dot{M}^* = \delta K\overline{Y}^*P^* - \delta M^* \qquad (19.S.1)$$

We multiply through by the exchange rate to work in terms of domestic currency.

$$EH^* = \delta K\overline{Y}^*P^* - \delta EM^* \qquad (19.S.2)$$

Here we have applied PPP ($P = EP^*$). Equation 19.S.2 represents the foreign payments surplus measured in domestic currency. Its negative is the domestic payments surplus measured in domestic currency:

$$BP = -EH^* = -\delta K\overline{Y}^*P + \delta EM^* \qquad (19.S.3)$$

This is a second equation describing the balance of payments, in addition to Equation 19.5. It represents, for a given foreign money supply, M^*, a negative dependence of the domestic balance of payments on the price level. An increase in the domestic price level under fixed exchange rates is an increase in the world price level. As far as the foreign country is concerned, it raises the foreign demand for money and leads to a foreign payments surplus, which is a domestic payments deficit.

The downward-sloping $BP = -EH^*$ schedule, Equation 19.S.3, is shown in Figure 19.S.1 on the same axes as the upward-sloping $BP = H$ schedule, Equation 19.5. Because both equations must hold, short-run equilibrium is given by the intersection of the two schedules, at point A initially.

Determination of the World Price Level

It is possible to solve the two simultaneous equations for the world price level expressed in domestic currency.

$$\delta K\overline{Y}P - \delta M = \delta K\overline{Y}^*P + \delta EM^*$$

$$P = \frac{M + EM^*}{K(\overline{Y} + \overline{Y}^*)} \qquad (19.S.4)$$

The numerator is the total world money supply measured in domestic currency. Considered in the aggregate, Planet Earth is, after all, a closed economy, so it makes sense that the world price level should be proportionate to the world money supply. Equation 19.S.4 is shown in Figure 19.S.1 as a vertical line at the price level P.

The Effect of an Increase in One Country's Money Supply

An increase in the domestic money supply shifts the country's H schedule down by $\delta\Delta M$, precisely the same as in the small-country model of Figure 19.16: An excess supply of money leads to "dishoarding." The H schedule also can be viewed as shifting

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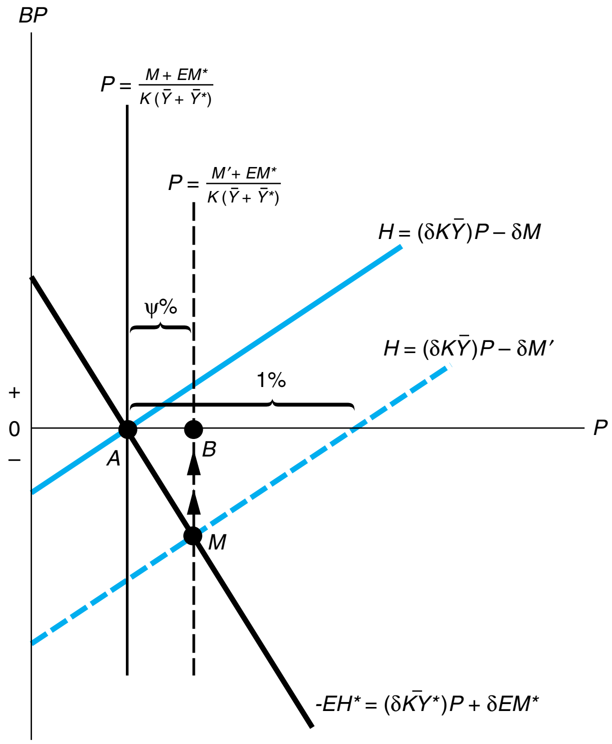
FIGURE 19.S.1

Monetary Expansion in the Monetarist, Two-Country Model

A monetary expansion of 1 percent will raise the world price level by ψ percent, where ψ is the domestic country's fraction of the world's money supply. As with a small country (where $\psi = 0$), the expansion shifts the H schedule downward leading to a temporary excess supply of money and balance-of-payments deficit.



horizontally to the right by the same proportion as the increase in the money supply: If the price level were for some reason to increase by the same proportion as the money supply, the excess supply of money would remain at zero.

When the country was small, the world price level was unchanged; but now it is recognized that the monetary expansion will raise the world price level to the extent that the domestic country is large. Define the domestic country's share in the world money supply.

$$\psi \equiv M/(M + EM^*)$$

A 1 percent increase in the domestic money supply is a ψ percent increase in the world money supply. As shown in Equation 19.S.4, it raises the world price level by ψ percent, whether that is measured in terms of domestic currency, P, or foreign, P^*. In Figure 19.S.1, the monetary expansion shifts to the right not only the domestic H line but the price level line as well. This means that the money demand function must be evaluated at a higher price level, at point M. Under the previous small-country case, ψ was negligible, and so the price level rose negligibly. There was an increase in the money supply of, say, 1 percent, with no increase in money demand. Now there is a 1 percent increase in the money supply with a ψ percent increase in money demand. There is still an excess supply of money (equal to $1 - \psi$ percent of the original money supply), and therefore a balance-of-payments deficit, but they are not as large as in the small-country case.

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What is happening in the foreign country? Its money supply has not changed, but it is faced with a ψ percent increase in the price level. Therefore, its demand for money goes up by ψ percent. It has an excess demand for money (equal to ψ percent of its money supply) that is the counterpart of the domestic country's excess supply of money, causing a foreign balance-of-payments surplus that is the counterpart of the domestic country's balance-of-payments deficit.

Over time, the domestic country loses gold to the foreign country. Under the nonsterilization assumption, the domestic money supply falls and the foreign money supply rises. The domestic H schedule shifts upward and the *negative* foreign schedule, $-EH^*$, shifts upward as well. The economy follows a sequence of intersections, moving upward from M along the new price level line. The transfer of money from the home country to the foreign country gradually alleviates the excess demand in the foreign country. Long-run equilibrium is reached when both countries return to a zero balance of payments, at point B. There the supply of money equals the demand for money in both countries. Because the price level has risen by ψ percent in both countries and the demand for money is proportional to the price level, this can only mean that the supply of money has increased by ψ percent in both countries. The world money supply has increased by ψ percent. In the short run the expansion took place entirely in the domestic country, but in the long run it is distributed equiproportionately across both countries.

SUPPLEMENT TO CHAPTER 24: Debt Dynamics

An explosive or unsustainable path for debt is defined as one where the expected ratio of debt to GDP rises without limit. The general framework is known as debt dynamics. We will apply it here to domestic government debt, but it could as easily be applied to foreign debt. For simplicity, assume no inflation.

Define $b \equiv \frac{Debt}{Y}$. We are concerned with the rate of change of this ratio:

$$\begin{aligned} \frac{db}{dt} &= \frac{dDebt/dt}{Y} - \frac{Debt}{Y^2}\, dY/dt \\ &= \frac{TotalFiscalDeficit}{Y} - \frac{Debt}{Y}\frac{dY/dt}{Y} \\ &= \frac{primarydeficit + rDebt}{Y} - bn \end{aligned}$$

where $n \equiv$ *growth rate*, and the primary deficit is the name for the budget deficit exclusive of interest payments. Therefore,

$$\begin{aligned} \frac{db}{dt} &= d + rb - bn \\ &= d + (r - n)\, b, \end{aligned}$$

where $d \equiv primary\ deficit/Y$. The equation says that to keep the debt ratio from exploding (to avoid $\frac{db}{dt} > 0$), we need either $d < 0$ *or* $r < n$.

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Consider the implications. A country may have a sustainable budget balance one day, and yet if it is hit by a sudden rise in interest rates or fall in the growth rate, through no fault of its own, it can find itself on an unsustainable path. To get back on a sustainable path, it will have to respond by raising the primary budget balance. This means adopting a contractionary fiscal policy at precisely the time when the economy has already slowed down anyway. But the government may have no choice because of the exigencies of international capital markets. This may help explain why so many developing countries exhibit procyclical fiscal policies, rather than the countercyclical policies that would be desired to stabilize the growth rate. The ultimate conclusion is that countries are better off if they do not make themselves vulnerable by high debt ratios in the first place.

SUPPLEMENT TO CHAPTER 25: The Locomotive Theory

This supplement explores the theory of international macroeconomic policy coordination. Table 25.3 illustrated the game of exporting unemployment for the simple case where each country faces a binary choice of expand or contract. Here we present the complete analysis with a continuous range of policy options.

Assume that the United States and Europe seek to attain two objectives, internal balance, $Y = \overline{Y}$, and external balance, $TB = 0$, and that each has only one policy instrument, the money supply, M.[1] Figure 25.S.1 shows how the two countries set their monetary policies, with Europe's money supply, M_E, on the horizontal axis and America's, M_A, on the vertical axis.

First consider the problem from the U.S. point of view. There is some combination of the two money supplies that is optimal from the American viewpoint, represented by point A. A is in the lower right area, indicating that the United States would prefer that the other country do the expanding, enabling the United States to run a trade surplus while maintaining high output.[2] Of course, the other country will not generally set its money supply at the level desired. How should the United States set M_A, if it has to take M_E as given? Radiating out from A are a series of concentric indifference curves representing successive levels of American economic welfare further and further from the optimum. For any given level of M_E, the United States should choose the level of

[1]We do the theory with two targets and one instrument, to keep it simple. We could introduce additional policy targets for each country, such as the exchange rate or the CPI. We could also introduce additional policy instruments for each country, such as fiscal policy. But one point to keep in mind is that if each country has as many independent policy instruments as policy targets, then it can obtain its optimum regardless of what the other country does. (Think back to our analysis of internal and external balance in section 22.6.) In this case, a change in American policy has an effect in Europe if European policy makers do not change their policy settings, but it is an effect that they can fully offset if they choose, without cooperation from the United States. Issues of conflict and coordination arise if each country has more targets than it has independent instruments, the usual case.

[2]We will assume for purposes of discussing Figure 25.S.1 that a monetary expansion in one country has a positive effect on the other country's trade balance and income, even though this is true only in some of the models shown in Table 25.2. Otherwise, the curves might look different.

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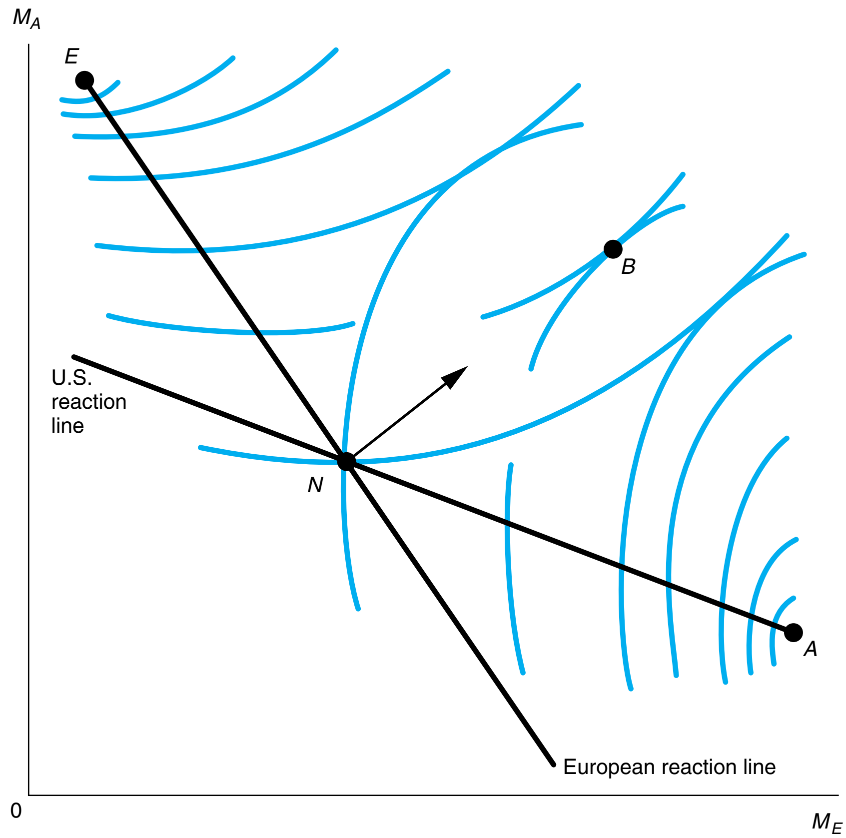
FIGURE 25.S.1

The Gains from International Monetary Coordination

Each country's reaction function indicates how it would set its money supply if it took the other's as given. The Nash noncooperative equilibrium occurs at *N*. In this case, cooperation would take the form of both countries agreeing to joint monetary expansion. Higher welfare is attained at a cooperative point such as *B*.



M_A that brings it to the highest indifference curve possible. This will be the point where the vertical line corresponding to M_E is tangent to an indifference curve. Thus, tracing out the set of points where the U.S. indifference curves run vertically will trace out its *reaction line,* which shows how it will set its money supply as a function of Europe's. Notice that, in this diagram, the reaction line is downward sloping: the more Europe withholds monetary expansion, the more America expands to compensate. This is half the story.

Now consider the problem from Europe's viewpoint. Europe's optimum is the point *E*, located in the upper left area, indicating that Europe, too, would prefer that its trading partner be the one to expand. Successive indifference curves radiate out from

E. How should Europe set M_E, if it takes M_A as given? To get as close to the optimum as possible, it should choose the point where the horizontal line corresponding to M_A is tangent to one of its indifference curves. Thus tracing out the set of points where Europe's indifference curves run horizontally will trace out Europe's reaction line, which shows how it will set its money supply as a function of that of the United States.

The situation without coordination is the Nash noncooperative equilibrium, defined as the point at which each country is setting its money supply at the optimal level, given what the other country is doing. This is represented by point N in Figure 25.S.1, where the two reaction lines intersect. It is now clear why the noncooperative point is suboptimal. There is a package of policy changes that will leave both countries better off. As the diagram is drawn, the Pareto-superior package consists of joint expansion by the two countries, moving in the northeastward direction. This illustrates the locomotive theory—that is, each country is afraid to expand on its own for fear of adverse trade balance consequences. (The figure could also have been drawn so that coordination dictated some other combination of policy changes, for example, cooperative discipline in the competitive depreciation game.) This package raises welfare in both countries because it moves both to higher indifference curves. Ideally they will agree to a bargain that is Pareto-optimal—such as point B where the indifference curves are tangent—that is, a bargain that maximizes some weighted sum of the two countries' welfares as an omniscient world social planner would do. Any point in the lens-shaped area (the area bounded by the two indifference curves that run through point N) will entail gains from cooperation for both countries.

SUPPLEMENT TO CHAPTER 26: Real Wage Indexation

This supplement considers what happens when wages are indexed (either partially or completely) to the CPI:

$$W = \overline{w}\text{CPI}^{\delta} \tag{26.S.1}$$

where δ is the degree of indexation. If $\delta = 1$, then indexation is complete and the real wage—expressed in terms of the CPI—is fixed at the target level, $W/\text{CPI} = \overline{w}$.

From Equation 26.1, the supply relationship is now

$$(Y/\overline{Y}) = (wP/\overline{w}\text{CPI}^{\delta})^{\sigma} \tag{26.S.2}$$

Assume that the target real wage, $\overline{w}$, is appropriately set to the warranted real wage, w, the one consistent with full employment. Also assume an open economy in which imports have a weight of α in the CPI.

$$\text{CPI} = (SP^*)^{\alpha}P^{1-\alpha} \tag{26.S.3}$$

where the price of imports is given by the exchange rate, S, times the foreign price level, P^*. Substituting Equation 26.S.3 in Equation 26.S.2, the general supply relationship is

$$(Y/\overline{Y}) = [P/(SP^*)^{\alpha\delta}P^{(1-\alpha)\delta}]^{\sigma} \tag{26.S.4}$$

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For simplicity, consider the case where indexation is complete: $\delta = 1$. Then

$$(Y/\overline{Y}) = (P/SP^*)^{\alpha\sigma} \tag{26.S.5}$$

We readily see that real depreciation is contractionary, not expansionary as in a nonindexed economy. A 1 percent decrease in P/SP^* reduces output by $\alpha\sigma$ percent. The reason, as explained in the text, is that a real depreciation that leaves W/CPI unchanged necessarily raises W/P when it raises W. The result is that changes in fiscal policy have real effects. A domestic fiscal expansion that causes a real appreciation because of high capital mobility raises domestic output, Y. Notice that if imports are not important ($\alpha = 0$), there is little effect on Y.

Now consider international transmission in a two-country model. We model the foreign country just like the domestic country.

$$(Y^*/\overline{Y}^*) = (SP^*/P)^{\alpha^*\sigma^*} \tag{26.S.6}$$

Looking at Equations 26.S.5 and 26.S.6 together reveals a remarkable property. The only circumstance that allows an increase in domestic output—a decrease in the real exchange rate SP^*/P—is also the only circumstance that allows a decrease in foreign output. Y and Y^* can vary from their potential output levels, but to the extent that output goes up in one country, it must go down in the other! A fiscal expansion in the foreign country, which raises foreign output to the extent it raises SP^*/P, reduces domestic output to the same extent. The only scope for variation in the real wage comes from real variation in the real exchange rate. This is why what goes up in one country, goes down in the other. This is an extreme case of inverse transmission of policy.

What about monetary policy? Although it remains true that any policy that changes the real exchange rate changes output, a monetary expansion in a completely indexed economy does not succeed in changing the real exchange rate. Rather, a 10 percent increase in the money supply raises S and P proportionately, with no real effects in either country, assuming indexation is complete ($\delta = 1$).[†]

SUPPLEMENT TO CHAPTER 27: The Monetary Model of the Exchange Rate

Flexible-Price Version

This first part of the supplement presents formally the complete model described in Section 27.2: the monetary approach to exchange rate determination with perfectly flexible goods prices.[1] Logarithms are used so that equations that would otherwise be multiplicative come out linear (additive). The PPP equation (Equation 27.3) thus becomes

$$s_t = p_t - p_t^* \tag{27.S.1}$$

[†]This point is explored in Problem 3 in the chapter problems.

[1]See Michael Mussa, "The Exchange Rate, the Balance of Payments, and Monetary and Fiscal Policy Under a Regime of Controlled Floating," and other papers, in Jacob Frenkel and Harry Johnson, eds., *The Economics of Exchange Rates* (Reading, MA: Addison-Wesley, 1978).

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where s_t is the log of the exchange rate, p_t is the log of the domestic price level, and p_t^* is the log of the foreign price level. (The equation implies that the percentage change of the exchange rate is equal to the percentage change of the domestic price level minus the percentage change of the foreign price level.) The money demand equations (Equations 27.4 and 27.6) become

$$m_t - p_t = y_t - \mu i_t \tag{27.S.2}$$

$$m_t^* - p_t^* = y_t^* - \mu_t^* \tag{27.S.3}$$

where m_t and m_t^* are the logs of the countries' money supplies, y_t and y_t^* are the logs of their income levels, and μ is the semielasticity of money demand with respect to the interest rate. (For simplicity, this parameter is assumed to be the same in both countries. Also, the elasticity of money demand with respect to income has been assumed equal to 1, as in the text.) Combining the three equations gives the equation of exchange rate determination:

$$s_t = (m_t - m_t^*) - (y_t - y_t^*) + \mu(i_t - i_t^*) \tag{27.S.4}$$

This is just the logarithmic version of Equation 27.8; indeed, it furnishes the justification for entering the interest rates in difference form in the text. It is easy to see how a 1 percent increase in the domestic money supply causes a 1 percent depreciation of the domestic currency and how anything that causes an increase in the demand for domestic money (a rise in income or fall in the interest rate) has the opposite effect.

The uncovered interest parity condition is

$$i_t - i_t^* = \Delta s_t^e$$

Substitute into Equation 27.S.4 to get the logarithmic version of Equation 27.9.

$$s_t = (m_t - m_t^*) - (y_t - y_t^*) + \mu(\Delta s_t^e) \tag{27.S.5}$$

If the currency is expected to depreciate over the coming period ($\Delta s_t^e > 0$), the result is a low demand for the currency today and a high exchange rate. Under rational expectations it is possible to substitute the rationally expected future exchange rate, $E_t s_{t+1}$ (conditional on information available at time t), in place of the investors' expected rate, s_t^e.

$$s_t = \tilde{m}_t + \mu(E_t s_{t+1} - s_t) \tag{27.S.6}$$

where for ease of notation $\tilde{m} \equiv (m_t - m_t^*) - (y_t - y_t^*)$. The equation can be solved for the current exchange rate, which in Equation 27.S.6 appears on both sides.

$$s_t = \frac{1}{1 + \mu}\tilde{m}_t + \frac{\mu}{1 + \mu} E_t s_{t+1} \tag{27.S.7}$$

This equation shows clearly how a change in expectations can cause today's exchange rate to change, even in the absence of any change in today's macroeconomic fundamentals.

What determines the expected value of next period's exchange rate? Equation 27.S.7 itself. Move it one period into the future and then take the expectation

$$s_{t+1} = \frac{1}{1+\mu}\tilde{m}_{t+1} + \frac{\mu}{1+\mu}E_{t+1}s_{t+2}$$

$$E_t s_{t+1} = \frac{1}{1+\mu}E_t\tilde{m}_{t+1} + \frac{\mu}{1+\mu}E_t s_{t+2} \tag{27.S.8}$$

Equation 27.S.8 can be substituted into Equation 27.S.7 to get today's exchange rate as a function of two-period-ahead expectations.

$$s_t = \frac{1}{1+\mu}\tilde{m}_t + \frac{\mu}{1+\mu}\frac{1}{1+\mu}E_t\tilde{m}_{t+1} + \left(\frac{\mu}{1+\mu}\right)^2 E_t s_{t+2} \tag{27.S.9}$$

Pushing the expectation one step further into the future may not seem very helpful, except that the process can be repeated.

$$E_t s_{t+2} = \frac{1}{1+\mu}E_t\tilde{m}_{t+2} + \frac{\mu}{1+\mu}E_t s_{t+3} \tag{27.S.10}$$

Substitute Equation 27.S.10 into Equation 27.S.9 and continue iteratively to get the following infinite series:

$$s_t = \frac{1}{1+\mu}\left[\tilde{m}_t + \frac{\mu}{1+\mu}E_t\tilde{m}_{t+1} + \left(\frac{\mu}{1+\mu}\right)^2 E_t\tilde{m}_{t+2} + \left(\frac{\mu}{1+\mu}\right)^3 E_t\tilde{m}_{t+3} + \cdots\right] \tag{27.S.11}$$

It is now clear that the entire expected future path of the relative money supply matters for determining today's exchange rate. The sum of the series is not infinite (assuming the money process itself is not explosive) because each stage multiplies by a factor $\mu/(1+\mu)$, which is less than 1. Today's exchange rate can be considered as a present discounted sum of future money supplies.

We will use Equation 27.S.11 for three experiments that were also considered in the text. First, what happens if people suddenly decide today that the money supply will be increased by 1 percent at some point T periods into the future? It is immediately clear from Equation 27.S.11 that today's exchange rate will increase by $[1/(1+\mu)][\mu/(1+\mu]^T$ percent. The chapter explained why: Forward-looking investors realize that the currency will lose value in the future, so they seek to shift out of it today. This is the case illustrated in Figure 27.1(c).

Second, what happens if the current money supply goes up by 1 percent? It depends how the expectations of future money supplies are affected. If the change in the current money supply is purely transitory (i.e., if the level is expected to go back down next period), then the current exchange rate goes up by $1/(1+\mu)$ percent. The current depreciation is less than proportionate because speculators increase their

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demand for the currency in the knowledge that it will be gaining value over the *coming* period, thus partly offsetting the effect of the increase in supply.

What if all the future money supplies are expected to be higher by the same 1 percent as the current money supply? This will be the case if the money supply is thought to follow a random walk. (The increase in the level of the expected money supply is permanent, but the increase in the growth rate is transitory.) When $\Delta\tilde{m}_t = \Delta E_t\tilde{m}_{t+1} = \Delta E_t\tilde{m}_{t+2} = \cdots$, then Equation 27.S.11 becomes the sum of a geometric series.[2]

$$\Delta s_t = \frac{1}{1+\mu}\left[1 + \frac{\mu}{1+\mu} + \left(\frac{\mu}{1+\mu}\right)^2 + \cdots\right]\Delta\tilde{m}_t$$

$$= \frac{1}{1+\mu}\left[\frac{1}{1-(\mu/1+\mu)}\right]\Delta\tilde{m}_t = \Delta\tilde{m}_t \qquad (27.S.12)$$

In other words, the exchange rate goes up by the same 1 percent as the relative money supply. This is the case illustrated in Figure 27.1(a). When the money supplies follow random walks, the exchange rate moves in proportionate lockstep.

Finally, consider the case where the money supply is expected to rise at a new steady-state growth rate of 1 percent per annum (relative to the foreign money supply and to the countries' real incomes). Then for any period T years in the future, the money supply is expected to be T percent higher. The answer, although we omit its derivation from Equation 27.S.11, is that the effect on today's exchange rate is a depreciation of μ percent. The depreciation occurs at the moment that investors revise their expectation of the money growth rate, as illustrated in Figure 27.1(b). Subsequently, if the money supply does indeed grow at a 1 percent faster rate, as expected, then the exchange rate increases at a 1 percent faster rate from then on. Equation 27.S.5 shows that a 1 percent increase in the rate of expected depreciation causes a μ percent depreciation today.

Recall that μ is the semielasticity of money demand with respect to the rate of return on alternative assets. John Bilson, for example, obtained an estimate of 2.3, for the long-run semielasticity of the mark/pound exchange rate.[3] This estimate implies that when news about faster money growth raises the expected inflation rate by 1.0 percent per annum, the immediate impact on the equilibrium exchange rate is a depreciation of 2.3 percent (even before taking into account any overshooting, of the type discussed in Section 27.4 and the chapter appendix).

The Overshooting Model of the Exchange Rate

We can continue to use logs to represent the monetary model when goods prices are sticky. The assumption that expected real depreciation is formed regressively is written as,

[2]Recall that the sum of a geometric series is 1 over the quantity 1 minus the factor that multiplies each term to get the next.

[3]John Bilson, "Rational Expectations and the Exchange Rate," in Jacob Frenkel and Harry Johnson, eds., *The Economics of Exchange Rates*, p. 92.

$$\Delta s^e_{real} = -\theta(s - \bar{s}) \tag{27.S.13}$$

Expected real depreciation is set equal to the real interest differential by international equalization of expected rates of return. Then, solving for the exchange rate shows how the percentage "undervaluation" is related to the real interest differential:

$$s - \bar{s} = -(1/\theta)(r - r^*) \tag{27.S.14}$$

Equation 27.S.14 describes the magnitude of overshooting relative to long-run equilibrium. An increase in the real interest rate makes domestic assets more attractive and causes the currency to appreciate.

An increase in the level of the money supply causes a proportionate increase in the long-run equilibrium exchange rate, as we know from the earlier flexible-price model, and in addition causes the exchange rate to overshoot. The magnitude of the overshooting is $1/\theta$ times the decrease in the interest rate (by Equation 27.S.14), which in turn is $1/\mu$ times the percentage increase in the money supply (by Equation 27.S.2), assuming that y_t as well as p_t is slow to respond. Thus a 1 percent increase in the money supply has a total initial impact on the exchange rate of $[1 + (1/\mu\theta)]$ percent.

SUPPLEMENT TO CHAPTER 28: The Optimally Diversified Portfolio

This supplement develops the theory of optimal portfolio diversification described in Chapter 28.[1] To simplify, assume there are only two assets, euros and dollars. The problem is how investors should allocate their portfolios between these two assets.

Use x to denote the share of the portfolio that investors decide to allocate to euros and $(1 - x)$ to dollars. The ex post real rate of return on the investors' total portfolio, r, is given by

$$r = xr^{€} + (1 - x)r^{\$} \tag{28.S.1}$$

where $r^{€}$ is the ex post real return on euros and $r^{\$}$ is the ex post real return on dollars. The investors care about two things: the mean or average return on their overall portfolio (they want it to be high) and the risk or uncertainty in their overall portfolio (they want it to be low). The average return is measured by the statistical concept of the *expected value,* represented by E; the expected return on the portfolio is given by

$$E(r^{€}) = xE(r^{€}) + (1 - x)E(r^{\$}) \tag{28.S.2}$$

(The E passes right through the x and $1 - x$: The expected value of half of the Dow Jones index is equal to half the expected value of the Dow Jones index.) The risk is measured by the statistical concept of the *variance,* represented by V. Basic properties

[1]Two of the papers that spell out optimal diversification of the international portfolio in more detail are Michael Adler and Bernard Dumas, "International Portfolio Choice and Corporation Finance: A Survey," *Journal of Finance,* 38 (1983): 925–984; and Jeffrey Frankel, "In Search of the Exchange Risk Premium: A Six-Currency Test Assuming Mean-Variance Optimization," *Journal of International Money and Finance,* 1 (December 1982): 255–274.

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of the variance can be used to show how the variance of the overall portfolio depends on the allocation share, x, and on the individual variances.[2]

$$V(r) = x^2V(r^{€}) + (1 - x)^2V(r^{\$}) + x(1 - x)2\text{Cov}(r^{€}, r^{\$}) \quad (28.S.3)$$

The last term represents the covariance, which reflects the correlation between the return on euros and the return on dollars. One lesson to be drawn from Equation 28.S.3 is that overall risk, $V(r)$, will be greater if the two returns are highly correlated. Chapter 28 mentioned that investors should be happy if they can hold a pair of assets that have a low correlation.

Consider first the case where the two currencies happen to have the same variances: $V(r^{€}) = V(r^{\$})$, which will be represented by $\overline{V}$. Is the overall risk in the portfolio the same regardless of the allocation x because each asset has the same variance? The answer is no. Diversification among assets allows investors to reduce their risk.[3] If $x = 1$ (the portfolio is allocated entirely to euros), then $V(r) = V(r^{€}) = \overline{V}$; and if $x = 0$ (the portfolio is allocated entirely to dollars), then $V(r) = V(r^{\$}) = \overline{V}$; but if x is anything in between, $V(r)$ is lower than $\overline{V}$. This is an example of the gains from diversification.[4] Risk-averse investors will not put all their portfolio into euros, even if the expected return on euros is greater than the expected return on dollars.

Now take the case where the dollar is considered completely safe. This will be the case if the investors are American residents seeking to minimize the risk of their position expressed in terms of dollars. (Perhaps they consume only U.S. goods with prices predetermined in dollars, or perhaps they represent a corporation seeking to minimize variability in terms of dollars for accounting reasons.) The returns expressed in dollars are given by $r^{\$} = i^{\$}$ (the U.S. interest rate) and $r^{€} = i^{€} + \Delta s$ (the European nominal interest rate plus the rate of appreciation of the euro against the dollar), respectively. The interest rates are already determined at the time the investors make their decision; this means that their variances are zero. Only the change in the spot exchange rate is uncertain. Equation 28.S.2 for the mean becomes

$$E(r) = x(i^{€} + E\Delta s) + (1 - x)(i^{\$}) \quad (28.S.4)$$

Equation 28.S.3 for the variance reduces to

$$V(r) = x^2V(\Delta s) \quad (28.S.5)$$

The expressions for the mean and variance can be used to see how investors will choose x. If they are extremely risk averse, caring little for expected returns and seeking only to minimize variance, then they will choose $x = 0$ because that way they can

[2]The variance of r is defined as $E(r - Er)^2$. If this concept is unfamiliar, notice that it indicates how far away (by the square of the distance) r is from Er, on average. Two properties are needed to derive Equation 28.S.3: The variance of x times a random variable is equal to x^2 times the variance of the variable; and the variance of the sum of two variables is equal to the sum of the variances of the two variables, plus 2 times the covariance.

[3]The one exception arises where the returns on the two securities are perfectly correlated. In that case, it is not possible to reduce risk at all by diversification because holding one is just like holding the other. [Exercise: Find $V(r)$ in Equation 28.S.3 when $\text{Cov}(r^{€}, r^{\$}) = V(r^{€}) = V(r^{\$})$. Does it depend on x?]

[4]Assume for simplicity that the covariance is zero. Then $V(r) = x^2\overline{V} + (1 - x)^2\overline{V}$. The variance of the overall portfolio is minimized by setting $x = \frac{1}{2}$. You are asked to show this in Problem 3 at the end of Chapter 28.

attain $V(r) = 0$. In other words, they will hold no euros at all, only dollars. This makes sense because of the assumption that they view the dollar as entirely safe.

In general, however, investors care about both the mean and variance. Assume that they seek to maximize a function, Φ, of the mean and variance $\Phi[E(r), V(r)]$. To choose the value of x that maximizes welfare, differentiate Φ with respect to x,

$$d\Phi/dx = [d\Phi/dE(r)][dE(r)/dx] + [d\Phi/dV(r)][dV(r)/dx] \tag{28.S.6}$$

and substitute derivatives of the mean from Equation 28.S.4 and the variance from Equation 28.S.5.

$$d\Phi/dx = [d\Phi/dE(r)][i^{€} + E\Delta s - i^{\$}] + [d\Phi/dV(r)][2xV(\Delta s)]$$

Finally, set the derivative equal to zero, and solve for x to find the investor's optimal portfolio allocation.

$$x = \frac{[i^{€} + E\Delta s - i^{\$}]}{\{[-d\Phi/dV(r)]/[2d\Phi/dE(r)]\}V(\Delta s)} \tag{28.S.7}$$

The expression inside the curly brackets measures how much the investors dislike risk relative to how much they like expected gains. It is often known as the coefficient of relative risk aversion, and so is denoted here by $\text{RRA} \equiv \{[-d\Phi/dV(r)]/[2d\Phi/dE(r)]\}$. Recall also the definition of the risk premium on euros.

$$rp \equiv [i^{€} + E\Delta s - i^{\$}] \tag{28.S.8}$$

Thus the expression for the optimal portfolio can be written more compactly as:

$$x = \frac{rp}{\text{RRA}\ V(\Delta s)} \tag{28.S.9}$$

This equation states that the share of the portfolio allocated to euros (x) depends (1) positively on the expected rate of return relative to dollars (rp); (2) inversely on the coefficient of relative risk aversion (RRA); and (3) inversely on the variance of the change in the exchange rate. Notice again that if the investors are highly risk averse (RRA is large), then they will hold few euros. What happens if the investors do not mind risk at all? They are said to be risk neutral. When RRA = 0, the denominator is zero. Of course, x cannot be infinite, but the investors are infinitely responsive to expected rates of return. This is the case when euros and dollars are perfect substitutes.

The consequences are seen more clearly by inverting Equation 28.S.8.

$$rp = [\text{RRA}\ V(\Delta s)]x \tag{28.S.10}$$

Now it is clear that if investors have zero risk aversion, then their infinite sensitivity to expected returns ensures that the risk premium is zero. This is the case of uncovered interest parity. The same holds if there is no uncertainty regarding the future exchange rate, $V(\Delta s) = 0$. In general, however, with nonzero uncertainty and nonzero risk aversion, the risk premium should also be nonzero. Consider, finally, what happens if x, the share of the portfolio consisting of euros, increases (e.g., because European governments issue more bonds, which someone in the market must hold). Then rp increases: Euros have to pay a higher expected return to induce investors to hold them.

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