Math 191 Spring 2016

Short & Final Projects

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1. Logistics

The point of these projects is to get your hands dirty (and your feet wet) with mathematical self-study. It provides a gateway into mathematical research, a gentle introduction to the pursuit of finding and extending the boundaries of mathematical knowledge. As mathematics is a subset of "the art of arguing and convincing," there will be a collaborative aspect to these projects, as well as presentations.

Once a project topic has been decided:

1) Learn all you can about it – Delve into multiple sources, ask fellow mathematicians, check the proofs carefully and do exercises. This will certainly involve sidetracking to understand background material that you may not have learned yet or forget the details of.

2) Question everything you learn – Why do I (or why does someone) care about this theorem? Does this theorem need this particular assumption in its statement? Where does this proof hinge on a given hypothesis? Can this result be generalized? Is this theorem more easily proved if I restrict my attention to a particular class of objects?

3) Come up with some (potentially open) problems – Think of your own and attack them, no matter how brief and/or unsuccessful. Or find an unsolved question, track its historical progress, and see why there is not a complete answer yet.

Work in either *pairs* or *groups of three*.
Formally write up your projects in LaTeX.
Give an in-class presentation on your studies/results.
Submit at least one "informal progress report," interpreting that phrase in any way.
Do not hesitate to contact me when you are stuck, but allow days of being stuck.

Roughly speaking, the differences between the **Short Project** and **Final Project** are:

1) *Length* – Although arbitrarily set and it is not strict, the writeup for the short project should be at least 7 pages long and the writeup for the final project should be at least 14 pages long. The title page and bibliography do not contribute towards the page count.

2) *Thoroughness* – Neither project should simply skim a topic. The final project should consist of more references, more information, and more complete proofs of statements than the short project.

3) *Originality* – Both projects will have an expository aspect. While there will be some regurgitation of what you have learned, you should internalize the material and come up with your own viewpoint on it (via presenting analogies to various mathematical constructions, expressing your thought on the importance or usefulness or lack thereof of various definitions/theorems, and providing computations that may not be written down in the current literature). The short project may be more expository than the final project.

2. Some Suggestions

2.1. **Complexity and "Hard Unknots".** What makes them hard in a computing sense? Can you untangle your favorite one?

2.2. **(Amphi)Chiral Knots.** What is the probability that a knot out of a given collection of knots is chiral? What are (and are not) some invariants of chiral knots? What do various polynomials say about chiral knots?

2.3. **Wild Knots.** What is wild about them? What theorems concerning tame knots don't work for wild ones, and why? What can you learn from them, or what can you characterize about them? What are they related to?

2.4. **Higher-Dimensional Knotting and Various Ambient Spaces.** What can be said about $S^n \hookrightarrow \mathbb{R}^{n+2}$? About genus Riemann surfaces in $\mathbb{R}^3$? About (1-dimensional) knots in a given manifold, such as $\mathbb{R}^4$ or S^3 or $\mathbb{R}P^3$ or $S^1 \times S^2$ or $S^1 \times \mathbb{D}^2$? Do you gain/lose information by working in $\mathbb{R}^3$ versus $S^3 = \mathbb{R}^3 \cup \{\infty\}$?

2.5. **Knots in Physics.** Why do they arise? How can they be useful, or disastrous?

2.6. **Invariants and Numbers.** Be it the genus, crossing number, bridge number, unknotting number, fundamental group, (favorite) polynomial, or your own new one. Pick one or a few, and find/analyze relations between them. For example, it is not currently known (I think!) if the minimal crossing number of a knot is additive with respect to the connect-sum operation.

2.7. **Proofs of Various Theorems.** Pick a theorem, or a sequence of related ones, and analyze the proof in amazing detail. Try to generalize the theorem, or find a different proof that has not been found in the literature. Squeeze as much as you can out of the theorem, in terms of corollaries and (potentially unsolved) exercises.

2.8. **Braids versus Knots versus Links.** Differences and similarities? Particular theorems which work for some but not others, or have natural analogs?

2.9. **Analyzing Seifert matrices.** Perform linear algebra on Seifert matrices. Compute signatures of knots. Relate to polynomials.