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THE SUPPLY OF GENDER STEREOTYPES AND DISCRIMINATORY BELIEFS

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ABSTRACT

What determines beliefs about the ability and appropriate role of women? An overwhelming majority of men and women born early in the 20th century thought women should not work; a majority now believes that work is appropriate for both genders. Betty Friedan (1963) postulated that beliefs about gender were formed by consumer good producers, but a simple model suggests that such firms would only have the incentive to supply error, when mass persuasion is cheap, when their products complement women's time in the household, and when individual producers have significant market power. Such conditions seem unlikely to be universal, or even common, but gender stereotypes have a long history. To explain that history, we turn to a second model where parents perpetuate beliefs out of a desire to encourage the production of grandchildren. Undersupply of female education will encourage daughters' fertility, directly by reducing the opportunity cost of their time and indirectly by leading daughters to believe that they are less capable. Children will be particularly susceptible to persuasion if they overestimate their parents' altruism toward themselves. The supply of persuasion will diminish if women work before childbearing, which may explain why gender-related beliefs changed radically among generations born in the 1940s.

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I. Introduction

Why do gender-related beliefs emerge and shift over time? According to the General Social Survey waves of 2003, 2004 and 2007, 47 percent of women born before 1946 (and 59 percent of men) agree or strongly agree with the statement "It is much better for everyone involved if the man is the achiever outside the home and the woman takes care of the home and family." Only 29 percent of women born after 1945 share that view. A full 50 percent of female respondents (from all cohorts) agreed with that statement in America's West South Central Region, while only 26 percent of New Englanders shared the view.

We have less survey evidence on discriminatory beliefs about women's ability in the workforce than we do about women's "proper" role in the home, perhaps because surveyors may not have trusted respondents to answer truthfully. Nonetheless, the evidence that does exist also suggests dramatic transformations about beliefs about women's capacity during the late 20th century. In 1953, Gallup asked "If you were taking a new job and had your choice of a boss, would you prefer to work for a man or for a woman?" In the 1953, 57 percent of women and 79 percent of men expressed a preference for a male boss, as opposed to only 8 percent of women and 2 percent of men who expressed a preference for a female boss. By 1987, the share of female and male respondents expressing a preference for a male boss had dropped to 37 and 29 percent respectively, with men now preferring a female boss (Simon and Landis 1989).

Moreover, an abundance of personal histories, ethnographic work and field-specific statistical research suggests that men, and often women as well, have often believed that women are less capable in many-workplace relevant tasks (e.g. Lerner 1987). The literature on women and perceived math ability is voluminous, and suggests that men and women often both believe that women are less able in mathematics (see Gunderson et al. 2012). The women who pioneered their way up corporate ladders have often described a common male presumption that their talents were limited. Literature is replete with stories (such as Ibsen's "A Dollhouse") of women who experienced low expectations in their own homes, from both spouses and parents. Major thinkers from Aristotle to Freud have often depicted women as severely lacking in vital decision-making areas.

This paper does not attempt to add new measurement of discriminatory beliefs, nor does it attempt to quantify the effect that such beliefs may have had on women's labor force successes or family outcomes. In Section II, we discuss some of the survey, ethnographic and literary sources that persuade us to accept that patriarchal, discriminatory beliefs have existed and that they are important enough to investigate further.

Section II also discusses why we also assume that these gender-related stereotypes cannot be understood as a purely Bayesian response to commonly available facts, but that they should instead be understood as the product of persuasion. The surveys discussed above are taken in the same year, by respondents who observe the same labor markets, and yet respondents born before and after 1945 have markedly different opinions about the working women, suggesting that an impact of upbringing on beliefs is far stronger that it should be in a perfectly rational world. There are abundant reasons to believe that male attitudes toward the innate abilities of females have shifted dramatically since the Victorian past, and it is hard to see how women's "innate" characteristics could have experienced any such dramatic shift.

As in Glaeser (2005), we assume that beliefs reflect persuasion rather than reality, and we focus on the supply of persuasion. Gender-related stereotypes and ethnic hatred differ in major ways. Hostile racial, ethnic, and religious beliefs emphasize the threat created by the bad motives of an out-group (which sociologists often term "untrustworthiness"), which then generates an incentive to harm or avoid that group. Gender-related stereotypes rarely imply that women are innately evil, which would be a hard sell given the common experience of maternal altruism, or that women should be universally avoided, which would run against common male preferences. While discriminatory gender-related beliefs may certainly harm women, the holders of these beliefs often believe that women are morally superior, not evil, and rarely intend any intentional harm to women as a group.

To understand the supply of erroneous beliefs, we must understand who has the incentives to spread falsehood. In Section III of this paper, we discuss potential sources of error: politicians, companies, co-workers, spouses and parents. This discussion motivates our decision to focus on parents and market entrepreneurs. While many politicians, such as Theodore Bilbo and Adolph Hitler, devoted a significant amount of their public proselytizing to spreading ethnic hatred, we know of none who built a public career on spreading misperceptions about female competence.[2] Indeed, it is easy to understand why politicians have so rarely had incentives to spread gender-based stereotypes.

Friedan (1963) suggested that patriarchic beliefs were spread by magazine publishers and companies selling products for the home that complemented women's time in the household. Our model of market-supplied beliefs, discussed in Section IV suggests that the conditions needed for this to explain patriarchy are unlikely to hold, but the story seems common and plausible enough to formally model. Co-workers do have an incentive to persuade hiring authorities that women are less able in the workforce, but the power of this force should be muted, since employers have both the incentives and should have the resources to see through their own employees' talk.[3]

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[2] The efforts of Phyllis Schlaffly and others against the Equal Rights Amendment certainly defended traditional family arrangements, but rarely suggested any lack of female competence.

[3] It is, of course, possible that employers have "preferences" against hiring women, as in Becker (1957), but the spirit of this paper is to focus on the causes of such "preferences" or beliefs. We find it more plausible to believe that employers beliefs were themselves shaped by pre-adult influences than by their workers' persuasion.

Parents with a strong preference for own grandchildren may also have an interest in persuading daughters that the formal labor force is not for them, and other sources have suggested that parents or teachers have helped perpetuate gender stereotypes (Gunderson et al. 2012). Moreover, parents have far greater resources available with which to influence beliefs than co-workers or even spouses. Parents have some control over children's time and experiences for many years, during periods where children are less likely to have strong alternative sources of information. This combination of incentives and power leads us to believe that parents are a primary source of gender stereotypes and we model that process in Section IV.

Section IV, and V, discuss the two models that follow from this discussion. Our first model follows Friedan's (1963) emphasis on the generation of beliefs by sellers of household goods. As in Glaeser and Ujhelyi (2010), we assume that companies have access to a technology that broadcast stories which are taken as evidence about the relative returns of working in or out of the household. We aim to capture Friedan's description of magazine articles, generated by male publishers perhaps eager to please their advertisers, which depict happy housewives and miserable career women. We assume that women discount many of these stories, but as long as the marginal story is given some credence, then it will have some effect on the amount of time women spend at home or work.

The model suggests that gender stereotypes will be strongly dependent upon the market structure in consumer goods production, the effectiveness of communication technology and the naiveté of women about third party stories. In order for Friedan's model to work, the costs of persuasion must be low, the household goods market must be extremely oligopolistic, and household products must complement, not substitute, for women's time in the home. Since manufacturing lifestyle norms is an industry-wide public good, a highly competitive industry will not engage in much of that sort of persuasion.

Arguably, the late 1950s was an era that combined large market power held by a few large home product producers, more effective means of communication and limited skepticism about that communication, yet those conditions are unlikely to have held throughout the long history of patriarchal beliefs. Moreover, if this force was so strong, there should have been more counter-persuasion during that period by producers of technologies, like the washing machine, the dishwasher, the microwave and the vacuum cleaner, that substitute for women's time in the home.

Moreover, it is hard to see why fomenting gender related stereotypes is the cheapest means of persuading married women to buy new cookbooks or cooking appliances. It would seem more likely that the providers of home-related products, whether complements or substitutes for women's time, will broadcast a simpler message: a nice home is nice. Indeed, such banal messages often seem like the theme of home product advertisements, which really imply little about whether women are capable of finding fulfillment in the workforce. The model leaves us wondering whether corporate interests had either the incentives or the power to shape widespread beliefs in the way that Friedan (1963) suggests.

Section V starts with an alternative model on the parental formation of beliefs for female children. Parents typically have far more ability to force stories with morals on their children, and perhaps parental persuasion has been similar to the corporate persuasion suggested by Friedan (1963). Our model in Section V follows more standard economic assumptions and links the persuasion process to a Bayesian signaling model. We do not assume that parents have access to a persuasion technology, but rather that they can send costly signals, including altering the education of their children or their own workplace behavior, which may shape children's beliefs, either about their own ability or about the ability of women as a whole.

We focus on differential education choices by gender. If young women believe that parents have access to private information about their own ability, then education choices will be seen as a signal about their own individual ability. If young women and men believe that parents lack such private information, but base their decisions on their assessment of female ability more generally, the education given to daughters has the ability to generate beliefs about an entire gender. We focus on the first assumption in the first part of Section V and the second assumption in the second part of Section V.

In the model, parents are altruistic toward their children but they have an independent desire to have more grandchildren. This desire creates an incentive for them to try to generate beliefs that lead to more childbearing. If education increases the returns from working in the labor force relative to childbearing, this will generate lower levels of women's education, even if women know their ability levels with certainty.

The under-provision of education effect gets more pronounced if parents, but not their daughters, have private information about the ability of their own daughters or of women generally. Parents of skilled daughters may have an incentive to try to imitate parents of less able children by giving them less education, which may persuade daughters that their own time is best spent in childbearing. If daughters have rational beliefs, this will cause more able women to think that they are merely average, but will not lead to any aggregate misperception about women's ability.

In the second part of Section V, we turn to persuasion of both sons and daughters. If parents are making decisions based primarily on the ability distribution of women as a whole, then their investment decisions may persuade both sons and daughters that women are generally less capable in the workforce. This will lead to an added benefit of underinvestment for parents hoping to encourage more fertility in both sons and daughters.

This effect will be particularly strong if children are credulous Bayesians (Glaeser and Sunstein 2009) who make the understandable error of overestimating their parents' altruism toward themselves then the situation can become more extreme. Trusting their parents too much leads daughters to underestimate their parents' incentive to act strategically. This tendency will heighten the parents' incentive to behave in a strategic manner, by under-investing in education. Daughters may end up believing that they are in a separating equilibrium, when only the parents of the less able provide little schooling, while they are actually in a pooling equilibrium, where all parents provide little education to their daughters.

At the end of Section V, we discuss the timing of work and childrearing. In this model, women who have been educated or not, have the choice of when to schedule a continuous term of home production for producing children. One disadvantage of postponing childbearing is that it leads to a shorter time of continuous work, which limits human capital accumulation. A second disadvantage reflects potential health risks from delaying childbearing. In the model, the major advantage of postponing is that women can learn their ability levels if they work during an earlier period, which enables them to make better decisions about the tradeoff between parenting and work.

As long as the desire to eliminate breaks in work history is not too strong, then women have children immediately or wait depending on the state of medical technology, as discussed by Goldin (2006). In the model, reduced risks from late childbearing will delay childbearing and lead to more information at that decision-making stage. The critical implication is that parental investment in misinformation makes sense when women have kids early but not late. This fact implies that the shifts in the timing of women's childbearing should have had a major effect on the supply of gender stereotypes. As such, over a longer time period, technologies such as the pill (Goldin and Katz 2000) may have reduced the incentive to persuade daughters that their time is better spent bearing children.

Section VI concludes and discusses the interplay between sources of incorrect information and real world experience. Working before childbearing means that there is enough information to counteract persuasion. In a similar fashion, gender-related quotas that limit the number of women on the job seem unlikely to persist in the same way as glass-ceilings that prevent women from rising above a certain level. Gender related quotas should be unstable, if they are sustained with incorrect beliefs, because the few women hired for the job end up providing information that counteracts false beliefs. Glass ceilings, by contrast, provide no such evidence, which allow false beliefs to persist and maintain the incentives to perpetuate such beliefs.

II. Discrimination and the Social Formation of Beliefs

We have a great deal of information about women in the workforce, including the relative productivity of men and women in the household, the availability of market-provided household services, and perceived workplace discrimination against women (e.g. Goldin 1990; Blau et al. forthcoming). We have less evidence on beliefs about female competence. Perhaps this dearth of information is understandable. In late 20th or early 21st century America, we would hardly expect many respondents to honestly admit to thinking that women are less capable. Nonetheless, the relative absence of polling data about female competence makes it difficult to fully document shifts in beliefs about women and their capacities.

There is however a great deal of more "anecdotal" evidence suggesting that women have often faced strong belief-related barriers to employment. Men have often held strong opinions that women were just not up to certain jobs. Often, these beliefs have crumbled in the face of reality, but certainly some of these beliefs persist.

Attitudes toward Women and Work

In this subsection, we briefly review the polling data that is available about gender stereotypes from the General Social Survey (GSS) and other sources. The General Social Survey, and other surveyors, has been asking questions about traditional gender roles since the early 1970s. Unfortunately, these gender-role related questions do not map clearly into any particular taste or belief. A patriarchal viewpoint can reflect a higher opinion of female productivity in the household sector, or a belief that employers discriminate unfairly against women.

Figure 1, for example, shows the average responses to the question "It is much better for everyone involved if the man is the achiever outside the home and the woman takes care of the home and family" by birth year for men and women separately. The graph shows a strong downward pattern for both men and women. For cohorts born at the start of the 20th century, almost all men and women thought that traditional gender roles were best. The share of respondents sharing that view declines to about 30 percent by 1950 and then levels off. There are some odd positive upticks in the responses to the question in the most recent cohorts, but this may reflect measurement error. Certainly, the basic pattern documents a profound change across cohorts born in the first half of the last century, and this pattern presents itself during every year in which the survey question was asked.

The second figure shows a similar response to the GSS question asking whether mothers working outside the home are harmful or harmless for young children. Again, cohorts born at the start of the 20th century almost uniformly believed that children were hurt by women working outside the home. By 1960, almost half of respondents did not state this belief. Even though an overwhelming majority of respondents say that women working are just fine overall, a modest majority still say that working while children are young harms children.

There are far fewer questions that seem to directly capture assessments of female competence, and most that are relevant concern very particular tasks or occupations. The General Social Survey asks some highly specialized questions, in individual years only, that would seem to relate to female competence: the first (asked in 1974 and 1982) asked if men make better political leaders.

The cohort pattern, shown in Figure 3, is clear. About 40 percent of people born earlier in the 20th century think that men make better political leaders. By the latter decades of the century, this belief is down to 20 percent. We cannot generalize from political competence to competence in the workplace, but the effects are still quite striking.

Another question that is potentially related to ability was asked in 1996. Men and women were both asked if women earn less than men because they work less hard. This question about female work effort shows a striking non-linearity (shown in Figure 4), where beliefs about greater male effort decline with year of birth during the first half of the twentieth century and then a rise after that date. We have no real explanation for this pattern, but it does suggest that cohort does have an impact on these beliefs.

The Social Formation of Beliefs

Why do discriminatory beliefs differ radically over groups and across time? The economics of discrimination began when Gary Becker (1957) presented a model of discrimination based on the preferences of employers, customers, and fellow workers. Becker's approach posits that some members of one group dislike working with or buying from members of another group. The Becker model describes the reality of the mid-1950s, and provides many keen insights, like the negative impact on profits generated by an employer's discriminatory tastes.[4]

Even if whites had no innate dislike of blacks and men were willing to work with women, members of one group might still benefit if they were able to coordinate to expropriate the rights of another group (Krueger 1963; Thurow 1969), or if there was a society-wide equilibrium that restricts the choices of a disadvantaged group (Akerlof 1976).[5] The South's Jim Crow system was not merely the decentralized preferences or beliefs of ordinary people. It was socially and legally organized, and seems in many contexts to have generated transfers from blacks to whites. Those transfers were perhaps most obvious in the case of segregated schools, which allowed tax dollars to be spent far more heavily on white, rather than black children, especially when blacks were particularly immobile (Margo 1991).

These models certainly fit many aspects of the Jim Crow south, and they may also reflect some forms of gender-based discrimination as well. As Myrdal (1944) discussed in his classic study of American segregation, integration-oriented whites were no more allowed to travel in black railcars than blacks were allowed to travel white cars. Firms proudly trumpeted their whites only policies, and the system only changed with massive legal intervention from the federal government, which can be seen as breaking the old equilibrium with outside force. Margo

[4] Lazear's (1999) model of culture and language provides a complementary communication-based explanation for some forms of discrimination in the labor market. Difference cultures, or ways of speaking, can make coordination difficult and lead to lower productivity.

[5] Akterlof (1976) presents a model where a caste system, such as the Jim Crow South, was an unfortunate but stable equilibrium that reflected a society-wide rule where members of one clique are punished for interacting with members of a second clique.

(1991) predicts that centralized discriminatory behavior would change as blacks could move north, and indeed that seems to have happened.

It is less clear that there was an organized conspiracy against women in the mid-20th century, that was similar to the Jim Crow system in the south, or that the legal pressure exerted by the Equal Pay Act of 1963 or the Civil Rights Act of 1964 had the same cathartic impact for women that it did for African-Americans. Moreover, neither centralized discrimination models nor the Beckerian taste-based discrimination model can explain the changing nature of views toward African-Americans and women, because they were not intended to endogenize beliefs or preferences.[6] In centralized discrimination models, members of the ruling clique rationally respond to incentives, and have neither negative opinions nor disproportionate ill will toward either women or minorities.

Arrow's statistical discrimination model (1973) provides an alternative model that can explain discriminatory hiring practices and beliefs. The model suggests that employers and ordinary people have a low opinion of certain groups and these low opinions lead to discriminatory behavior. Certainly, it appears to be the case that at various times employers have held a low opinion of the competence of both blacks and women.

However, the great challenge of statistical discrimination models is that they typically also assume that people are fairly rational in their belief formation. This implies that attitudes need to be tethered to reality. Yet it is difficult to accept that there was much evidence to suggest that either women or blacks were as inept as many mid-century employers appear to have thought. Previous work (Glaeser 2005) focusing on beliefs about malevolence (rather than competence) emphasized that while Southern voters a century ago seem to have been convinced that African-Americans were a great threat to their safety, it was whites, not blacks, who had systematically enslaved, brutalized, sexually assaulted and even killed members of the other group. It is harder to document the error in beliefs about competence, but it seems quite likely that many people had beliefs about women and minorities that were not based on any real evidence and that bore little resemblance to the truth.

If it is true that beliefs about blacks and women systematically differed from reality, it becomes necessary to focus on theories that can generate widespread divergence between the truth and beliefs. There are at least two well-known systematic biases that can potentially generate such beliefs internally, without any external persuasion: the fundamental attribution error and self-serving biases. If the fundamental attribution error leads observers to associate the negative outcomes of others with intrinsic personal characteristics, rather than external constraints, that individuals could readily believe that poor labor market outcomes for either blacks or women

[6] Subsequent work by Becker and Murphy (2000) endogenizes preferences, and this paper is strongly indebted to their work. Our decision to focus on belief rather than preference formation reflects our own preference for the greater discipline created by belief-formation models, as in Section V, that require at least some Bayesianism. In the case of the model in part two of Section V, results would be identical if we allowed preference formation.

represent low levels of innate ability rather than discrimination. Self-serving biases, which lead people to prefer views that make them see themselves in a positive light, could also lead white men to have negative views of blacks and women, because such views prop up white self-esteem.

While these behavioral quirks may have contributed to negative assessment of blacks and women, there are limits to the power of these theories. For example, females' own belief in gender stereotypes, discussed in the previous subsection, cannot be the result of self-serving biases, since the beliefs do not seem to be self-serving. Moreover, the fundamental attribution error suggests that adverse outcomes for others are attributed to intrinsic factors, but that personal disappointments are blamed on external constraint. Yet women themselves seem to share patriarchal beliefs.

Here, we focus on the social formation of error, and our critical assumption is that human beings are sensitive to social persuasion. In the discussion and two models that follow, individuals will be reasonably rational, but they will not totally discount falsely generated signals about the characteristics of out-groups.

On one level, the social formation of error runs against a long-standing tendency of economists to assume a high level of rationality and even accuracy in beliefs. Yet if we accept that mid-20th century white males had erroneous opinions of the ability levels of blacks and women, we must consider at least the possibility that some beliefs have little basis in reality. While our approach runs against the economist's predilection for hyper-rationality, it fully embraces the role that incentives can play in the generation of all sorts of outcomes, including incorrect beliefs.

Naturally, those incentives must battle against the incentives of listeners to learn the truth. In the political context, those incentives may be quite weak. After all, no individual voter has a strong incentive to ascertain the truth about any particular story, if the truth will only serve to make his or her vote a bit wiser. In the labor force context, those incentives may be quite stronger.

Moreover, we will assume that widely spread falsehoods will not persist if there is obvious evidence to the contrary. In any sensible learning model, this fact will suggest that racial or sex-based quotas are not typically stable, while glass ceilings may be. The existence of a glass ceiling toward women (or perhaps a low dark roof for blacks in the Jim Crow South) ensures that there is no hard evidence on how women or blacks can perform in higher positions. The absence of information allows incorrect beliefs to persist.

Discrimination vs. Hatred

These models also help us to distinguish discrimination from hatred. Hatred is modeled as a belief that an out-group is malevolent, and prone to engage in harmful behavior if they are empowered. Discrimination is a belief that an out-group is different and perhaps less capable, but not necessarily harmful or malign. Hatred leads to policies such as segregation and genocide, as in-groups attempt to shield themselves from the perceived threat. Discrimination will lead to different hiring practices and perhaps even exclusion from political decision-making. Yet policies based on beliefs about lesser ability levels will not attempt to explicitly harm the out-group, because the out-group is not perceived as dangerous. While we might try to harm people who are perceive to be malevolent, before they harm us, we have little incentive to attack people who are merely somewhat dim.

Historically, African-Americans have suffered from both discrimination and hatred. They have been perceived as being less competent, and they have also been perceived as being a threat. These beliefs were able to persist, arguably, because blacks were excluded from positions where they might do harm and kept out of jobs where they could have demonstrated ability.

Women have suffered from discrimination but not typically from hatred. The primary experience of extraordinary altruism in the lives of most men is the self-sacrificing behavior of their own mothers, which would make it hard to accept that women are somehow naturally malevolent. Indeed, many of the most profound opponents of women in the workplace or in politics, who certainly subscribe and even promulgate views about female competence, have also held up women as the fairer sex that is more generous and good-hearted than men. When Senator Vest of Missouri opposed women's suffrage in 1887, he said "I believe that [women] are better than men, but I do not believe they are adapted to the political work of this world."

It is historically rare for out-groups to be simultaneously depicted as malign and incompetent. Indeed, such views would be counter-productive if a hate-producer is looking to generate support for policies that are harmful to the out-group. If a group is incompetent, then it is less threatening and that would mean less need to engage in defensive mechanisms. Jews, for example, have historically been depicted as both malign and powerful, which together justified the use of extreme anti-Semitic policies. The Soviet Union was depicted as an Evil Empire, which called for massive US military spending. If the Soviet Union was merely an evil bumbling bureaucracy (arguably a more accurate description during the Reagan era) then there would have been far less need for military spending.

In the case of patriarchic beliefs, it is possible to conceptually distinguish beliefs about ability and societal norms. A woman, for example, might stay in the home because she believes that her workplace productivity is relatively low in comparison to her productivity in the household. Alternatively, she may believe that staying home is just the "right" thing to do. But while these two notions may differ in some deep sense, they will be typically practically indistinguishable, and even conceptually the distinction is murky. In a sense, believing that remaining in the household sector is "right" is not all that different from believing that productivity in that sector is higher than in the workplace. There is a conceptual distinction between believing that women are less able in the workplace or more able in the household sector, and surely both beliefs have existed, but when it comes to time allocation decisions the beliefs are interchangeable.

III. The Entrepreneurs of Error

If common beliefs are socially formed, then they are unlikely to be produced by accidents. Instead, interested individuals must have incentives to spread falsehood. In this section, we discuss four potential sources of misinformation about female ability levels, and explain our decision to model only two sources of falsehood. One primary distinction is whether incentives exist to minimize the workplace talents only of a particular woman or to minimize the ability level of all women.

We focus on cases where spreading misinformation is intentional and instrumental. There have certainly been countless instances where politicians, for example, have uttered gender stereotypes, but most of the time, this seems more likely to reflect a pre-existing norm, rather than any conscious political strategy. It is of course possible that those politicians are part of a social echo chamber which amplifies existing opinions, but we are interested in where the opinions get started. We therefore look for a setting where someone with the power to persuade also has the motive to depict women as either less capable or more suited for work outside the labor market.

Political Entrepreneurs

In Glaeser (2005), political entrepreneurs spread hatred against an out-group because hatred complemented the policies proposed by those politicians. Southern conservative politicians in the 1890s had an incentive to spread anti-black hatred because their proposed policies would be harmful to disproportionately poor African-Americans relative to the policies proposed by their populist opponents (Woodward 1955). The model suggests that there were several factors needed for a steady supply of erroneous beliefs, including low costs of supply, persistent policy differences between parties that disproportionately impacted an out-group, political weakness of the out-group, and the relative segregation of that out-group to reduce alternative sources of information.

None of these conditions holds for gender-related policies throughout most history prior to 1950. The politician had to have access to the pulpit, cheap political persuasion, which, outside of cities, required both the printing press and voter literacy. For this reason, politically-induced hatred of groups, as opposed to religiously induced hatred, appears to have been a largely 19th century innovation. There just was not enough actively popular proselytizing politicians prior to 1750 to be responsible for patriarchal beliefs prior to that period.

Two prominent gender-related issues emerged in US politics during the 19th century: women's suffrage and temperance (eventually prohibition). Prominent leaders in women's suffrage, like Elizabeth Cady Stanton, also led temperance organizations and prominent temperance leaders, like Frances Willard, were also suffragists. Prohibition was partially justified as a policy that would protect wives and children from abusive, drunken husbands, and suffrage was justified as the means of passing prohibition. Both issues culminated in constitutional amendments ratified immediately after World War I.

The early connection between these issues and abolitionism (Fogel 2000) may have made them a more natural fit for the GOP than with the Democratic Party, and Republicans were stronger supporters of the bills that eventually led to the Nineteenth Amendment, but neither issue became a major party plank until 1916, when both parties' platforms supported extending voting rights to women.[7] Neither party's platform endorsed prohibition before the passage of the Eighteenth Amendment.

Beliefs about female competence would be far more relevant to the issue of female suffrage, and arguments about female incapacity were routinely made by the opponents of suffrage. By 1916, a large number of states already allowed women's suffrage, especially in presidential elections, making it politically unwise to insult a large voting bloc.

Yet even in that case, the political language was limited perhaps because the parties never split decisively on suffrage, and politicians had far less chance of changing male beliefs about women, than they did of conjuring the fear of a race riot. While many rural Germans in 1925 had little experience of Jews, allowing Nazi propaganda a clean field to shape anti-Semitic beliefs, most men have known plenty of women, thereby limiting the ability of any political voice to shape beliefs.

Moreover, since the Nineteenth Amendment passed, women have gone from being politically absent to the second largest and now the largest voting bloc. Telling a majority of voters that they are stupid (or evil) would seem to be immense electoral folly, which perhaps explains why the politicians have rarely led the charge promoting gender stereotypes. Since 1950, primary gender-related political issues have included legislation banning gender discrimination, including the Equal Rights Amendment (ERA), and abortion-related policies. While there has been plenty of vilification on both sides of the abortion, the suggestion that abortion limitations are justified by broad limits on female decision-making ability has been fairly rare (suggestions that teenage girls are incapable of making wise decisions are more common), presumably because there are so many female voters.[8]

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[7] In 1916, the Republican platform "favors the extension of the suffrage to women, but recognizes the right of each state to settle this question for itself," while the Democrats "recommend the extension of the franchise to the women of the country by the States upon the same terms as to men." The Republicans are endorsing suffrage, but not an amendment to force it on unwilling states, while it is unclear if the Democrats are supporting such an amendment or not. In 1872, the Republican platform provided the amorphous words "The Republican party is mindful of its obligations to the loyal women of America for their noble devotion to the cause of freedom " and "the honest demand of any class of citizens for additional rights should be treated with respectful consideration."

[8] Democrats do, of course, assert that Republicans are waging a "war on women," a charge that Republicans hotly deny.

Presumably for the same reason, ERA opponents were more likely to oppose allegedly unnecessary federal regulation, rather than to say that discrimination was broadly justified on ability related grounds. The Republican platform of 1980 affirmed "our Party's historic commitment to equal rights and equality for women," and supported "equal opportunities for women," but also claimed that "states have a constitutional right to accept or reject a constitutional amendment without federal interference or pressure," and that federal "pressure against states which refused to ratify ERA" must cease. Phyllis Schlafly was the most prominent political entrepreneur opposed to the amendment, and she based her opposition both on a defense of traditional family structure and by claiming that the amendment would strip women of traditional privileges, such as avoiding the draft. While there have been instances where politicians do seem to have actively promoted gender stereotypes, particularly around the issue of female suffrage, this seems to have been a relatively minor phenomenon, at least relative to the spread of stereotypes by other actors.

Market Entrepreneurs

A belief that women are less capable in the market place has one obvious beneficiary: competing male co-workers. This would suggest that men should have the incentive to spread the idea that women are less competent. Within a corporate hierarchy, presumably the sensible strategy would be to emphasize the limits of a particular woman. In other settings, where no single female competitor exists, then it may make more sense to disparage women more broadly.

Spreading false beliefs will be more common when women really are a potential threat, and this means that we can make sense of the rise and female of discrimination in certain jobs that is discussed by Goldin (2000). During the early 20th century, the threat of a female competitor was small and this meant that men spent little effort on persuading prospective bosses not to hire women. During the middle years of the 20th century, the threat became more obvious and men began to persuade more assiduously. At the end of the 20th century, there were enough examples of real women working that misinformation had much less effect.

Several factors would be necessary for this persuasion to represent a dominant force. First, people making hiring decisions would need to be susceptible to persuasion from the subordinates who will compete with the new hiree. This is not inconceivable—deans, for example, are quite reliant on faculty members when hiring—and junior faculty members are often allowed to weigh in on junior faculty hires. This process does suggest that persuasion would be occupation specific. It may be possible to persuade a superior that one's particular task (mathematics, construction work) requires male attributes, but it is unlikely to be as easy or as sensible to try to persuade the superior that women are less capable at all workplace tasks. However, if women are accepted as being less able in enough occupations, presumably the natural inference is that there is something more general going on.

Second, the persuaders would need to solve the free rider problem. No one worker has much of an incentive to persuade. The propagation of these beliefs would be therefore be more likely in small firm settings, or in cases where other organizations exist to collectively represent the interests of male workers. For example, in 1941, the United Auto Workers (UAW) filed a strike against the Kelsey-Hayes wheel plant, demanding "the removal of girl employees from machine work," (Milkman 1982). But while the UAW might demand segregation-by-job in particular plants, and would regularly fight for equal pay provisions that reduced the possibility of men losing jobs to lower cost female employees, the union was far more interested in representing female employees than disparaging them.

Third, if beliefs have some connection to evidence and Bayesian reasoning, then discriminatory beliefs in the workforce can only persist when there is no evidence to the contrary, which is true even if beliefs come from other sources. So hard discriminatory barriers, justified by these beliefs may be able to persist, while quotas, based on incorrect beliefs, seem unlikely to be stable. Many have argued that women working at typically male jobs during World War II helped dispel the idea that they were incapable of doing these typically male activities. The relative durability of glass ceilings may be connected with the formation of beliefs, because they ensure that there is no direct evidence on upper level administrators in one particular company, and advocates of discrimination can more plausibly argue that upper level jobs are more heterogeneous across firms than lower level jobs. That heterogeneity makes it easier to deny the relevance of female achievements in other firms.

Individual workers might disparage women, and unions might occasionally strike against female employment, but overall co-worker-spread discriminatory beliefs does not appear to have been a major force, presumably because of the relatively weak incentives and limited ability for workers to spread discriminatory beliefs to employers. Industrialists have every incentive to see through male claims about female incompetence and look for low cost labor, as Lowell did when starting his textile mills almost two centuries ago. While co-workers may have served as an occasional source of discriminatory beliefs, they are unlikely to be that significant, especially in more traditional societies.

The alternative, but far less natural, market entrepreneur who has an incentive to promulgate gender stereotypes is the consumer goods company. Friedan (1963) is the primary proponent of this view, and given her significance in this literature, we will explicitly model this process. Yet, as we will show, typical companies usually have only weak incentives to invest in such major life decisions as choosing to work vs. home-making. An industry must be oligopolistic, consumer goods must strongly complement not substitute for women's time at home, and the costs of persuasion must be low.

It is possible that these conditions existed with Friedan (1963), although they seem unlikely to hold today. Many important home products—the dishwasher, pre-made meals—substitute rather than complement time spent in the home, suggesting that their sellers should have been advocates of women working, not the opposite.

There is little doubt that magazines and advertisements provided many examples of the joys of home-making, but the instrumental aim of those examples seems far more likely to generate positive associations for using a particular product. Even a washing machine company has the incentive to show a happy woman at home with her washing machine, not because the company wants to her to stay at home, but rather because it wants her to think about how wonderful having a washing machine can be.

Family Entrepreneurs

The long history of patriarchal attitudes, before mass media, before widespread democracy, before even the possibility of significant female integration into the workforce, suggests that these attitudes ultimately have a deeper source. Perhaps, the deepest source of all is the family or clan itself, and ancient institutions, such as the church, that are often allied with adults in the family. If patriarchic views are common, if not ubiquitous, then it seems reasonable to believe that they are delivered for deep reasons and there is no deeper motivation than the perpetuation of the gene pool.

A particularly natural reason for supplying patriarchic beliefs is that these beliefs increase childbearing. Fertility is typically seen as a complement toward being in the home and substitute with being away from home. Children typically need childcare and that is typically most cheaply provided at home. Multiple pregnancies are often more difficult for working mothers to fit into their schedules. Given that fathers always bear far less of the costs of pregnancies and often bear far less of the cost of child-rearing, empowering men within the household may also lead to higher levels of fertility, especially in cultures that lack cheap forms of birth control.

For basic biological reasons, grandparents will often want more children than their own children will independently desire. For example, assume a Beckerian dynastic utility function for grandparents that equals U(Own Consumption) + V(Children's Utility, Number of Children) + W (Grandchildren's Utility, Number of Grandchildren). In this case, the grandparents will want more grandchildren than their children would independently produce, because the grandparents receive a direct benefit from grandchildren, over and above the indirect impact that grandchildren have through their children's own welfare. The envelope theorem implies that if children have maximized their own welfare with respect to their own progeny, then grandparents will always want more. There are multiple means of prodding children to be fertile, including bribes and verbal haranguing, but investing in beliefs may be a reasonable tool.

Parents have both a strong motive and abundant means of influencing children's beliefs, such as exposing children to gender stereotypes in childhood literature. Weitzman et al. (1972) examines children's storybooks in the US, and finds pervasive differences in the ways that genders are depicted, with boys being adventurous and girls being pretty and passive. Bereaud

(1975) examines French children's books and similarly finds that they portray girls as "timid, passive and dependent" and women "in the traditional housewife role or in low-paid, unskilled occupations." Children's books are bought by parents, so it is reasonable to believe that parents want such images broadcast to their own children.

In the pluralistic US today, parents can also choose other influences, such as religion. If parents want to encourage childbearing, then they can take their children to religious institutions that encourage childbearing. Some of the most extreme examples of pro-natalist religious entities are the Mormon Church and various ultra-orthodox groups. These institutions and the traditional Catholic Church also encouraged large families and traditional female lifestyles. Religious support for childbearing may reflect both a desire to cater to parents who want grandchildren, but also a desire to fill the pews in decades to come. Religious groups that did not support childbearing, such as the Shakers, tend to disappear over time.

As we will model, parents can also engage in more costly signals to children about their abilities. A mother may herself adopt a traditional lifestyle to convince her daughters to do the same and her sons to marry someone who acts similarly. Providing little education for daughters is another means of suggesting that her possibilities in the workplaces are limited, and that she should focus more on producing grandchildren. We will formally model under-education of women.

We will focus on the signaling choices of individuals, which will inevitably lead to some heterogeneity in the population. That heterogeneity may be smoothed out by institutions, such as churches, which will lead to a more ubiquitous set of attitudes. A state may also embrace traditional lifestyle choices for pro-natalist reasons, which may in turn be motivated by the desire for a large army. Hitler's Germany for example, pushed a strong ideology of motherhood and traditional female roles (Rossy 2011).

Empirically, demographers have documented that parental preferences do affect children's preferences and decisions on marriage and childbearing. Axinn, Clarkberg and Thornton (1994) show that mothers' preference for the size of their children's family is significantly positively correlated with the children's family size preferences when the children are young adults. Barber (2000) shows that both sons and daughters whose mothers prefer early marriage, large families, and low minimum education for their children end up entering parenthood earlier. This effect is significant controlling for family income, parental education, the mother's work choice and other family background variables. Such evidence corroborates our idea that parental influence is possibly quite powerful.

An Aside on Homophobia

The core assumption of our model is that parents want more grandchildren than their sons and daughters naturally will give to them. The same parental preferences should also generate incentives to engage in other forms of belief investment, most notably inculcating opposition to homosexual lifestyles. If homosexuality leads to less own grandchildren, then parents who value own grandchildren will invest in their children's beliefs to that end. They will attempt to convince them that homosexuality will lead to unhappiness and perhaps worse.

In this setting as well, religious organization may offer parents a means of perpetuating beliefs that serve their biological interests. If the church supports traditional lifestyles and opposes homosexuality, then parents may have an incentive to take their children to church despite their own private religious beliefs.

IV. Corporate Investment in Gender Stereotypes

Betty Friedan's *The Feminine Mystique* depicts advertisers and magazine editors as colluding to persuade women that they will be happier in the home than in the workplace. The essential assumption for this idea to make economic sense is that spending on consumer goods is a complement to time spent in the household. If producers want to sell more consumer goods, they also have an incentive to spend to persuade women that they will be unproductive in the workplace and happy in the home.

Throughout this paper, we will focus on beliefs about women's productivity in the workplace, and we will generally treat this productivity as purely pecuniary. This is at best a simplification, and at worst wrong. Friedan is at least as focused as beliefs about the non-pecuniary nature of work in the home and job. Such beliefs are perfectly analogous to the beliefs about female ability in the workforce that we focus on here.

In this model, we assume that women choose work hours to maximize a household utility function equal to $C_{NH} + \alpha V(T_H, C_H)$, where C_{NH} refers to non-household consumption which has a price of one, T_H refers to time spent working in the household, and C_H refers to household consumption, which is purchased at an endogenously determined price P_H. Time allocations and earnings occur before consumption decisions, but the quasi-linear structure crucially implies that realized productivity does not impact the optimal level of household consumption. Both husband and wife have a time budget of one. The husband's earnings equal $W(H_M)A_M(1 - T_{HM})$, where H_M refers to human capital level and A_M refers to ability level and T_{HM}, refers to the amount of time that the husband spends working in the household. The function V(.,.) is assumed to be concave.

The wife's earnings will equal $\delta W(H_F)A_F(1 - T_{HF})$. The term δ is meant to capture any potential discrimination in the labor market, H_F refers to her human capital level and A_F refers to ability level and T_{HF}, refers to the amount of time that the wife spends working in the household. As long as $W(H_M)A_M > \delta W(H_F)A_F$, and that it is not optimal to have more than one person's entire life spent in the household, then the husband will specialize entire in the formal work and the wife will specialize in the household sector, but will also spend some time working.

We think of this static model as an approximation to a lifetime time allocation problem. The assumption that less than a full life is spent working in the household means the wife will spend some part of her lifetime in the labor market, perhaps before or after peak child-rearing periods. This assumption is broadly compatible with the fact that even during the 1950s, large numbers of women worked before marriage (Goldin 1990). The assumption that $W(H_M)A_M > \delta W(H_F)A_F$ is broadly compatible with the fact that husbands are far more likely to be in the labor force than wives, especially during the Friedan period. In principle, that specialization could instead reflect greater female productivity in the household sector, and equivalent productivity in the labor force, which would have little impact on the model.

If the wife's expected workplace-related ability is denoted $\hat{A}_F$, then she will set $\delta W(H_F)\hat{A}_F = \alpha V_T(T_H, C_H)$. The condition $P_H = \alpha V_C(T_H, C_H)$ will also hold. This produces our first result:

Proposition 1: Holding prices constant, time spent in the household declines with δ, H_F, and $\hat{A}_F$ and decrease with P_H if and only if $V_{CT} > 0$. Spending on household consumption will always decline with P_H and decrease with δ, H_F, and $\hat{A}_F$ and P_H if and only if $V_{CT} > 0$.

This proposition provides the core logic behinds Friedan's logic that companies selling consumer goods might benefit if women spent more time on household production, and could even potentially gain from discrimination against women in the labor market. If consumer goods and time in the household are complements, this will mean that total spending on consumer goods increases if women face stronger discrimination in the labor market, or have lower levels of human capital or higher levels of ability.

This proposition implies that a key implication of Friedan's model is that pro-traditional household messages will only be broadcast by companies that produce goods that complements women's time, yet Fox (1990) documents that mechanical appliance advertisements "involved promotion of an ideology about housework that reinforced women's dedication to it."

Since such dedication runs somewhat counter to the incentives of many mechanical appliance producers, it seems more likely that these advertisements were intended simply to portray housework and housework appliance in an appealing light, rather than to promote home-making as a lifestyle.

The implications of Proposition 1 are not completely general. The quasi-linear form eliminates the role that income effects can play in spending on consumer goods, and means that lower levels of household income, from women not working, does not yield lower levels of spending. More significantly, many household consumption items, like dishwashers, are a substitute rather than a complement with women's time in the workplace. The purveyors of such goods will have no incentive to persuade women to stay home, but rather to emphasize the returns to working hard.

We will now assume a strong form of complementarity: the function V(.,.) has the form $T_H^{\varphi\sigma}\ C_H^{\varphi(1-\sigma)}$, where $0<\varphi<1$ and $0<\sigma<1$. With this assumption it follows that $T_H = \left(\alpha\varphi\sigma^{1-(1-\sigma)\varphi}(1-\sigma)^{(1-\sigma)\varphi}\right)^{\frac{1}{1-\varphi}} P_H^{\frac{-\varphi(1-\sigma)}{1-\varphi}} \left(\delta W(H_F)\hat{A}_F\right)^{\frac{\varphi(1-\sigma)-1}{1-\varphi}}$ and $C_H = (\alpha\varphi\sigma^{\sigma\varphi}(1-\sigma)^{1-\sigma\varphi})^{\frac{1}{1-\varphi}} P_H^{\frac{\varphi\sigma-1}{1-\varphi}} \left(\delta W(H_F)\hat{A}_F\right)^{\frac{-\varphi\sigma}{1-\varphi}}$. We assume this Cobb-Douglas functional form, not because we believe it to be generally true, but rather because it simplifies the subsequent algebra, and because Proposition 1 already highlights the important role that complementarity plays in the incentive to persuade.

To capture the role of persuasion, we assume that there is uncertainty about A_F, but about no other parameter. This assumption would be most reasonable if women had experienced home production, but not life in the formal labor force. Learning about labor force returns seems somewhat more reasonable to us than learning about productivity in the home sector, since presumably the magazine readers of the period experienced life in the home sector on a daily basis. It is perhaps more reasonable to believe that there was uncertainty about the impact of work in the home on longer term outcomes, such as children's success or marital survival. Incorporating learning about such long-run outcomes, or about the home sector more generally, would do little to change the intuition of the model but would add considerable complexity.

If the uncertainty is not resolved before consumption and work decision are made, then in the relevant first order condition we must use not the actual value of A_F but its expected value. The quasi-linear utility function implies risk neutrality, and that all of the uncertainty gets taken on in the form of consumption of the numéraire good. The decision-maker sets the expected value of time in the workforce equal to the marginal utility of household time spent.

We assume that women must choose their hours of work before observing their ability level and that women are able with probability "1-p" and in this case $A_F = 1$. They are less able with probability "p" and in this case their workforce productivity equals 1-a. While women actually care about their own productivity level, they must infer this probability level based on their beliefs about the general competence of women. While this assumption may seem strange, it is vital for the Friedan hypothesis to be correct, for the magazine writers and advertisers that she discusses do not have the ability to provide person-specific information to each reader, but only to convey general information about the "usual" outcomes of women in the workforce or at home.

Specifically, we assume that women are born believing that with probability one-half women are able in the workforce with probability $p_0-\Delta$ and with probability one-half they have ability equal to $p_0 + \Delta$. If they had no further information they would deduce that their probability of being less able equals p_0.

In equilibrium, we assume that women observe N_G examples of women being successful in work and N_B examples of less successful labor force outcomes. We also assume that women are sufficiently savvy to recognize the possibility that people are trying to persuade them, so that they believe that only some of the bad stories are true, while others may be discarded as manufactured. We let N_T denote the total number of believed stories. Bayes' rule implies that women believe the probability that women are generally less able with probability $p_0 + \Delta$ equals $\frac{(p_0+\Delta)^{N_B}(1-p_0-\Delta)^{N_G}}{(p_0+\Delta)^{N_B}(1-p_0-\Delta)^{N_G}+(p_0-\Delta)^{N_B}(1-p_0+\Delta)^{N_G}}$, which we denote $p(N_B)$.[9]

Stories can be both true and manufactured. We assume that the decision-maker is aware of $N_{G,0}$ examples of true positive labor market outcomes for women, and that women also understand that none of these stories are manufactured.

There are $N_{B,0}$ bad stories that are real and S_T total manufactured bad stories. We adopt a flexible belief model that assumes that women believe that $N_{B,1} + \theta(N_{B,0} + S_T)$ of the bad stories are true. This assumption nests a number of possibilities about the credulity of the listeners. If $N_{B,1} = N_{B,0}$ and $\theta = 0$, then women know exactly the number of stories that are true and added stories will have no effect on their beliefs. If $N_{B,1} = 0$ and $\theta = 1$, then listeners are credulous Bayesians, as in Glaeser and Sunstein (2009), believing everything is true. For intermediate values, women put some, but not full, weight on the manufactured stories. This leads to Lemma 1 (proofs to all lemmas and propositions are in the appendix).

Lemma 1: As long as $\theta > 0$ then the posterior probability that women are typically low ability is increasing and concave with respect to S_T.

There are Q total suppliers of the household product in the market, who first manufacture false examples of female failure in the labor force at a cost of k. After promulgating these stories, they sell household products, and engage in Cournot competition with the other firms. All firms have a unit cost of one. If the number of women equals M, then there optimal firm behavior means that post-advertising profits will equal: $\omega M Q^{\frac{\varphi\sigma-1}{1-\varphi}}(\delta W(H_F)A_F)^{\frac{-\varphi\sigma}{1-\varphi}}$, where ω is a constant.[10]

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[9] The probability that women are generally less able with probability $p_0 - \Delta$ equals $\frac{(p_0-\Delta)^{N_B}(1-p_0+\Delta)^{N_G}}{(p_0+\Delta)^{N_B}(1-p_0-\Delta)^{N_G}+(p_0-\Delta)^{N_B}(1-p_0+\Delta)^{N_G}}$.

[10] The value of ω equals $\left(1-\varphi(1-\sigma)(1-\varphi\sigma)\right)(\alpha\varphi^{2-\varphi\sigma}\sigma^{\sigma\varphi}(1-\varphi\sigma)^{1-\varphi\sigma}(1-\sigma)^{2-2\sigma\varphi})^{\frac{1}{1-\varphi}}$ which is $\left(1-\right.$ $\varphi(1-\sigma)1-\varphi\sigma$ times each firm's output.

Each firm has the opportunity to invest in stories, at a cost k, per story documenting some instance where a woman has entered the workforce and been unsuccessful. In practice, this may take the form of stories illustrating the bliss of staying at home. This assumption is far simpler than the relatively complicated worldview suggested by Friedan, in which magazine editors are part of a general conspiracy to promote non-working women, but it may be less accurate. The total number of stories S_T sums the investment of each individual firm.

Thus, each firm j chooses S_j to maximize $\omega M Q^{\frac{\varphi\sigma-1}{1-\varphi}}\left(\delta W(H_F)\left(1-p\left(N_{B,1}+\theta N_{B,0}+\theta\sum_{i=1}^{Q}S_i\right)a\right)\right)^{\frac{-\varphi\sigma}{1-\varphi}}-kS_j$. The function $(1-pa)^{\frac{-\varphi\sigma}{1-\varphi}}$ is increasing, but convex in p, implying that spending on household goods displays increasing returns with respect to doubts about female labor force competence. We assume that the second derivative of this with respect to S_i is negative around the first order condition, so that this first order conditions characterize a maximum, which implies that

$$(1)\qquad \left|\frac{p"\left(N_{B,1}+\theta N_{B,0}+\theta\sum_{i=1}^{Q}S_i\right)}{p\prime\left(N_{B,1}+\theta N_{B,0}+\theta\sum_{i=1}^{Q}S_i\right)}\right| > \left(\frac{1-\varphi+\varphi\sigma}{1-\varphi}\right)\frac{ap\prime\left(N_{B,1}+\theta N_{B,0}+\theta\sum_{i=1}^{Q}S_i\right)}{1-p\left(N_{B,1}+\theta N_{B,0}+\theta\sum_{i=1}^{Q}S_i\right)a}$$

This requirement is essentially that the diminishing returns of persuasion on beliefs is stronger than the increasing returns to doubts about competence. With this assumption, Proposition 2 follows:

Proposition 2: Total investment per firm is rising with M and declining with k, Q, δ and $W(H_F)$, and total market investment is falling with Q, An increase in θ causes the total number of negative stories that are believed to rise. As such, women's assessment of their ability in the workforce is rising with k, Q, δ, $W(H_F)$, and θ and rising with M.

Proposition 2 presents the comparative statics related to the belief formation by producers eager to sell consumer goods. Perhaps most notably, investment by firms is a function of market structure. Firms in highly competitive industries will not have the incentive to invest in industry-level public goods, like gender-specific norms. As such, if Friedan's model is right, then the model suggests that the market for consumer goods during her day must have been almost monopolistic in nature.

It is of course possible that some external agents, such as the magazine editors that Friedan discusses, managed to coordinate across disparate firms. However, they would need some mechanism for solving the free rider problem, and it hard to see how they had that much clout.

The proposition also delivers other comparative statics that are less surprising. Higher values of k, the cost of transmission, will reduce the spread of misinformation. That effect may explain the rise of misinformation in Friedan's era when magazines had become more common and more effective. The rise of television could also have played a role in reducing the costs of persuasion.

An alternative explanation is that M increased because of national markets for goods (and media). The nationalization of the market makes the returns to persuasion higher and increases the returns from persuasion.

Friedan may have been right about her era, but if she was right, then she lived in a highly unusual period that combined a number of requirements for corporate supply of patriarchal beliefs. First, consumer goods need to complement rather than supplement women's time. Second, the consumer goods industry had to be highly oligopolistic. Third, the ratio k/M needed to be low, which means that the cost of transmitting ideas was low relative to the size of the market.

In earlier time periods, k/M is likely to have been much higher, because of more fragmented regional markets and greater costs of transmitting ideas. In later periods, industrial organization surely became more competitive and goods seem more likely to be substitutes for women's time. This model provides conditions under which Friedan's hypothesis is true, and they might have held in America in 1960. Still, they are unlikely to be common enough to explain a phenomenon which is common across many societies in many different levels of development. As such, we now turn to the development of patriarchy within the family, which seems far more likely to explain an extremely widespread social phenomenon.

V. Teaching Gender Stereotypes to Sons and Daughters

We now turn to our primary model that examines persuasion by parents of daughters and sons. The critical assumption is that the parents care both about the welfare of their children and directly about their grandchildren. Parents have many tools for influencing beliefs about female competence in the workforce, including telling stories, attending religious services, maternal behavior and so forth. But we will focus on the provision of education for daughters. Female education is a particularly important signal that parents can send to daughters about their productivity outside the home, and it is relatively measurable.

Our model concerns three generations, called grandparents, parents, and children. The children make no choices in the model and are assumed to be homogeneous. The parents' generation decides only on the number of children, and their children's human capital, but we will not be focusing on their human capital level investment. The grandparents' generation selects the investment in human capital for a specific child in the second generation. We assume that we are looking at the decision of grandparents after their fertility decision has been made.

When they are adults, daughters in the second generation choose fertility levels, N, human capital levels for their boy children (H_{CM}) and human capital levels for their girl children (H_{CF})

to maximize: Consumption + $\alpha\big(V(N) + .5Ng_M(H_{CM}) + .5Ng_F(H_{CF})\big)$, where V(.),$g_M(.)$ and $g_F(.)$ are all increasing, concave functions. We are assuming that ½ of all children are male and that the benefits of skill may be different between boys and girls, perhaps because of the signaling issues that we will turn to when we consider the grandparents generation.

We focus on the case where all daughters will marry homogeneous husbands, and that consumption is $\delta W(H_F)A_F(1 - T_{HF}) + Y_0 - .5NH_{CM} - .5NH_{CF}$, where Y_0, reflects any other income including husbands' earnings, which we assume is independent of the women's education, N reflects the number of children and H_C reflects the human capital per child. Household time is proportional to the number of children, so $T_{HF} = N\underline{t}_C$. We assume that women make fertility decisions before observing their workplace productivity, although we relax this assumption in the last part of Section V. As before, they base their decisions on their estimate of workplace ability $\hat{A}_F$.

The first order conditions that determine human capital level investments are $g_M{}'(H_{CM}) = g_F{}'(H_{CF}) = 1$. We let G_T denote $.5\big(g_M(H_{CM}) + g_F(H_{CF})\big)$, and H_T denote $.5H_{CM} + .5H_{CF}$ evaluated at the welfare maximizing levels of human capital investment. We assume that $G_T \geq H_T$. We have made three assumptions that together ensure that the investment in children's human capital is independent of the number of children: quasi-linear preferences, the benefits from investing in children scales up linearly with the number of children and the costs of human capital investment similarly scale up linearly with the number of children. Quality and quantity of children are not completely independent, however, as the net benefit from investing in quality will impact the incentive to have more children.

The first order condition that determines fertility is $\delta W(H_F)\hat{A}_F\underline{t}_C = \alpha(V'(N) + G_T) - H_T$, which implies that the number of children is increasing with α and decreasing with δ, H_F, $\underline{t}_C$ and $\hat{A}_F$. Every one of these last parameters increases the opportunity cost of having more children. We use this equation to implicitly define a function $N(W(H_F)\hat{A}_F)$, which represents the number of children that a women will have depending on her level of human capital and beliefs about her workplace ability. The other elements that determine utility have been suppressed because they are fixed. We further assume that W(.) is increasing, concave and that $lim_{x\to 0}W(x) = \infty$.

Holding $\hat{A}_F$ and other parameters constant, the derivative of N with respect to H_F is $\frac{\delta W\prime(H_F)\hat{A}_F\underline{t}_C}{\alpha V''(N)} < 0$. The second derivative of N with respect to H_F is $\frac{\delta W\prime(H_F)\hat{A}_F\underline{t}_C}{\alpha V''(N)}\Big(\frac{W''(H_F)}{W\prime(H_F)} - \frac{V\prime\prime\prime(N)}{V\prime\prime(N)}\frac{\delta W\prime(H_F)\hat{A}_F\underline{t}_C}{\alpha V''(N)}\Big)$, which is negative as long as $V'''(N)$ is not too negative, as it will not be if V(.) has a standard form such as υN^{σ}, with $\sigma < 1$. We now turn to the grandparents' generation, and focus on their choice of investment in human capital for a single, female child in the second generation.

Assuming that parents accurately assess the daughter's ability level A_F, the welfare of parents (that is related to a specific child) equals:

(3) $$\alpha_1\big(Y_0 + \delta W(H_F)A_F\big(1 - N\underline{t}_C\big) + \alpha(V(N) + NG_T) - NH_T\big) + \alpha_2(V(N) + \mathrm{NG_T}) - H_F$$

where H_F refers to the investment of human capital in the parent. The parameter α_1 reflects the direct impact of the second generation's welfare on the welfare of the first generation. The parameter α_2 reflects the impact of the third generation's welfare on the welfare of the first generation. This independent impact of grandchildren on grandparents' utility is the critical assumption of the model. Because of this link that leaps a generation, grandparents want to raise children who will produce more grandchildren and invest more in those grandchildren.

These preferences are grounded in both evolutionary biology and everyday experience. Standard evolutionary preferences suggest that animals act as if they care about reproducing their gene pool, not just for a single generation but for generations to come. The grandparents who prod children to wed and have children of their own have become a cliché. Many grandparents also explicitly subsidize grandchildren, if they have the resources, by providing funds for education or even buying a house in a neighborhood with a good school district.

Given these preferences, grandparents will always want their children to have more progeny than they will naturally choose on their own. Grandparents will internalize the benefits that parents themselves get from childbearing and then will still want at least a little bit more. They will also want parents to spend a bit more investing in children's human capital.

We have chosen a stark and simple case to highlight how a desire for own grandchildren may lead to lower human capital investment in girls, and the generation of beliefs about female inability in the workplace, but we are well aware that reasonable perturbations of the model could generate alternative predictions. We have structured these preferences and production functions so that there is no tradeoff between quantity and quality. If one existed, then grandparents could conceivably care so much about grandchildren's quality that they might actually not want higher fertility levels. We have also assumed that maternal human capital only impacts childbearing by increasing opportunity costs. If maternal skills help generate human capital in the next generation, then this would create more of a grandchild-related incentive for investment in daughters.

We first focus on investments in a daughter's human capital, assuming that A_F is known at every point. We then turn to the possible scenario in which the parent, but not the daughter has received a private signal about the daughter's ability, in which case investing in education can serve as a costly signal to the daughter of her skills. Finally, we address sexist indoctrination of sons.

When the future mother's ability level is known to all, then the first order condition for the grandparent is:

(2) $\alpha_1 \delta W'(H_F) A_F \left(1 - N\underline{t}_C\right) + \alpha_2 (V'(N) + \mathrm{G_T}) \frac{\delta W\prime(H_F) A_F \underline{t}_C}{\alpha V"(N)} = 1$

We assume that second order conditions, described in the Appendix, hold for this to be a maximum.

Given our assumption on the second order condition, Proposition 3 follows:

Proposition 3: Parents will invest a positive amount in daughters' education if $-\frac{N_0 V"(N_0)}{V\prime(N_0)+\mathrm{G_T}} > \frac{\alpha_2}{\alpha\alpha_1} \frac{N_0 \underline{t}_C}{1-N_0 \underline{t}_C}$ where $\mathrm{N_0}$ represents the number of children chosen by a daughter with no education. If this condition holds, and parents do invest in a positive amount of education, then the level of education is declining with α_2 and increasing with α_1. The level of education will increase with δ and A_F if and only $\frac{(-\alpha V"(N))}{(\delta A_F \underline{t}_C)^2 W\prime(H_F) W(H_F)} > \left(\frac{\alpha_2}{\alpha} - \alpha_1 - \frac{\alpha_2 V'''(N)(V\prime(N)+\mathrm{G_T})}{\alpha (V"(N))^2}\right)$.

Proposition 3 implies that parents will always invest a positive amount in their daughter's education if α_2 is sufficiently small, and that the amount of education that their daughter receives is decreasing as α_2 rises. The incentive to under-invest in daughters is directly a function of the altruism toward grandchildren, but of course, this would diminish if daughters' human capital were an input into the human capital of the next generation. By contrast, as the grandparents care more about their daughters relative to their grandchildren, investment in the daughters' education will rise.

The parameters δ and A_F are complements to daughters' education, and they will typically cause the investment in the daughters' education to rise, as long as α_2 is relatively small, so the dominant effect of these parameters is to increase the payoff to daughters' education. A somewhat less intuitive possibility is that if α_1 is sufficiently low, higher values of δ and A_F which increase the returns to work may actually reduce the tendency to invest in daughters' education. If α_1 is low enough, then the grandparent only cares about investing in human capital because it impacts the supply of eventual grandchildren. As higher values of δ and A_F reduces the number of grandchildren directly, this may sufficiently increase the grandparents' demand for more grandchildren that they may offset these higher labor market returns with less investment in human capital.

Belief Formation

We now turn to the core of the model: the formation of daughters' beliefs. We assume that the first generation parents know their daughters' ability, but that daughters themselves only infer their talents from their parental investment in their human capital. As such, a daughter whose parents invest heavily, both personally and through external investments, will typically infer that she has raw skill, since skill is a complement with investment in the model. If parents ignore a daughter's education, then she will naturally infer that she has little innate talent. At this point, we focus on the formation of beliefs by a single individual, but in the next section, we discuss the implication of this for beliefs by sons and by society as a whole.

As before we assume that there are two possibilities for A_F: 1 and 1-a. It is useful for us to define the investment level chosen under perfect information as H_F^{Skill} for skilled daughters and $H_F^{Unskill}$ for unskilled daughters. These produce fertility levels N_{Skill} and $N_{Unskill}$ respectively, which generate parental welfare levels U_{Skill}^{Skill} and $U_{Unskill}^{Unskill}$. The values of H_F^{Skill} and N_{Skill} satisfy $\delta W\left(H_F^{Skill}\right)\underline{t}_C + H_T = \alpha(V'(N_{Skill}) + G_T)$, and $1 = W'\left(H_F^{Skill}\right)\delta\left(\alpha_1\left(1 - N_{Skill}\underline{t}_C\right) + \frac{\alpha_2(V'(N_{Skill})+G_T)t_C}{\alpha V''(N_{Skill})}\right)$, and other values are defined similarly. We also define a value $H_{Skill}^{Unskill}$ as the level of human capital that would be chosen by parents of unskilled daughters if their daughters believe erroneously that they are skilled. We also assume a minimum level of investment that parents are legally required to make denoted $\underline{H}_F$, which is less than $H_F^{Unskill}$.

We define $N_S(H)$ by $\delta W(H)\underline{t}_C + H_T = \alpha\left(V'\left(N_S(H)\right) + G_T\right)$, and $N_U(H)$ by $(1 - a)\delta W(H)\underline{t}_C + H_T = \alpha\left(V'\left(N_U(H)\right) + G_T\right)$. These are the fertility levels implied by human capital level H and the belief of the daughter that she is either skilled or unskilled respectively. Differentiation shows that $N_S(H) < N_U(H)$. As before, parents always prefer more children in equilibrium because the children who make the decision do not internalize that impact that their fertility has on parents' welfare through α_2. We go forward, and assume that for any given H_F:

$$(3)\ \left(1 + \frac{\alpha_2}{\alpha\alpha_1}\right)\left(\frac{V(N_U)-V(N_S)}{N_U-N_S} + G_T\right) > \delta W(H_F) + H_T$$

This assumption guarantees that parents of both skilled and unskilled children would prefer the fertility levels that result if the daughter believes she is unskilled to the fertility levels that result if the daughter believes that she is skilled. This will lead the parents of skilled daughters to want to imitate the parents of less skilled daughters, and it creates a signaling game involving parental investment in human capital.

We will assume a Bayesian equilibrium, where daughters' beliefs are such that if only parents of skilled daughters or unskilled daughters choose an investment level in equilibrium, the daughters who receive that level of investment will infer that they are either skilled or unskilled respectively. If both types of parents choose a level of investment in equilibrium, then daughters will believe that they are skilled with some probability weakly between zero and one. We will refine equilibrium beliefs as we proceed.

If parents of skilled daughters choose an investment level that is not chosen by parents of unskilled daughters, then they will only choose H_F^{Skill}. Any other investment level, chosen independently, will not change daughters' self-assessments, and will only reduce parental welfare. Parents of both types may both choose the same investment level, but they cannot both choose two or more investment levels. Generically, the beliefs that make one type of parent indifferent between two levels of investment will not make the other type of parent indifferent between two types of investment. We assume that the parents of skilled daughters strictly prefer investing H_F^{Skill}, and the daughter knowing that she is skilled, to investing nothing and convincing the daughter that she is unskilled. From this is follows that:

Lemma 2: There exists one value of H, denoted $\widehat{H}$, at which parents of skilled daughters are indifferent between choosing $\widehat{H}$ and appearing to be the parents of unskilled daughters and choosing H_F^{Skill}. There is no value of H greater than $\widehat{H}$ at which parents of unskilled daughters are indifferent between choosing H and appearing to be the parents of skilled daughters and choosing $\widehat{H}$ and being thought to be the parents of unskilled daughters. At $H = \widehat{H}$, holding beliefs constant, the welfare of parents of skilled daughters is strictly increasing in H, and this value of H is rising with δ and α_1 and falling with α_2.

If we place no further restrictions on off-the-equilibrium-path beliefs, then multiple equilibria are possible. For example, it is possible for there to be a continuum of pure separating equilibria, where parents of skilled daughters choose H_F^{Skill} and parents of unskilled daughters choose any value of H below $\widehat{H}$, as long as the parents of unskilled daughters prefer that value of H and being known to have an unskilled daughter to choosing $H_{Skill}^{Unskill}$, which is the best that they can do if their daughters believe that they are skilled. It is also possible for the unskilled parents to choose two levels of H that yield equal utility levels, as long as one is above $H_F^{Unskill}$ and one is below H_F^{Skill} and both yield equal utility. It is also possible for there to be a pooling equilibrium, where both parents of skilled and unskilled daughters choose a common level of H, as long as the payoff for the parents of skilled daughters is better off than if they choose H_F^{Skill} and the parents of unskilled daughters are better off than if they chose $H_{Skill.}^{Unskill}$. Semi-pooling equilibria are also possible, where some fraction of both groups mix and choose a common equilibrium, as well also choosing some separate investment level.

To generate a unique equilibrium when $\widehat{H} > H_F^{Unskill}$, it is sufficient to assume a Perfect Bayesian Equilibrium where daughters believe that if one type of parents would never deviate to a human capital level H, for any rational fertility response, then the deviation came from the other group. This assumption leads to proposition 3a:

Proposition 3a: If $\widehat{H} > H_F^{Unskill}$, then there is no pooling equilibrium, skilled parents choose to invest H_F^{Skill} and unskilled parents choose $H_F^{Unskill}$.

To generate a single equilibrium for a wider range of parameter values, we now assume a variant of the D1 refinement (Cho and Kreps 1987) which requires that if an off-the-equilibrium path investment level is more attractive for one type of parent, given any set of beliefs by children, then children assume that this type of parent has generated this deviation with probability one. In this model, the children's response to the parents' human capital investment is their fertility level. If $N_S^*(H)$ and $N_U^*(H)$ makes the parents of skilled and unskilled children respectively indifferent between their equilibrium payoff and any deviation H, then if $N_S^*(H) > N_U^*(H)$ the deviation seems more likely to have come from a parent of a skilled child. If $N_S^*(H) < N_U^*(H)$ then the deviation seems more likely to have come from a parent of an unskilled child. The D1 refinement requires to think that the deviation comes, with probability one, from the parent of an unskilled child if and only if $N_S^*(H) < N_U^*(H)$. This assumption produces:

Proposition 3b: If $H_F^{Unskill} > \widehat{H} > \underline{H_F}$, then skilled parents choose H_F^{Skill} and unskilled parents choose $\widehat{H}$. If $\widehat{H} < \underline{H_F}$, then all unskilled and some skilled parents choose $\underline{H_F}$. The number of parents of skilled daughters choosing $\underline{H_F}$ will decrease with α_1 and increase with α_2.

Following Lemma 2, the value of $\widehat{H}$ can be conceived as a representation of the level of labor market discrimination or δ. When δ is high, and there is little labor market discrimination, then $\widehat{H}$ will also be high and a separating equilibrium will exist with no distortion of parental incentives. The parents of the less skilled educate less—the parents of the more skilled educate more. The desire to persuade plays little role in the education of young women.

For lower levels of δ, where labor market discrimination has gotten more severe, the desire to distort views influences the education choices of the parents of the less skilled, but not the parents of the more skilled. The parents of the less skilled provide less and less education for their daughters in order to distinguish themselves from the parents of the more skilled. Their daughters end up having more children both because their opportunity cost of time is less and also because they know themselves are less able in the workforce.

For even lower levels of δ, some parents of the skilled begin imitating the parents of the unskilled. Ultimately, there can be a complete pooling equilibrium where all parents end up providing girls the legal minimum of education. Any deviation upwards will be seen as an indication that the girl is skilled, and will generate lower fertility levels. This force essentially traps society in a world where women are less educated and unable to distinguish among the more or less skilled.

Parental altruism works throughout this model. As parents care about their daughters more, relative to their grandchildren, pooling is less likely and skilled daughters receive more education. But if parents are particularly focused on their long-run genetic legacy, then daughters pay the cost in lower educational outcomes.

The model has several implications. When labor market discrimination is strong, then parents of skilled and unskilled daughters alike choose to provide them with minimal education. The skilled daughters may particularly suffer, because their parents are trying to ensure that they do not realize their skills.

As women are less discriminated against, this leads to more investment in the skilled daughters, and there can be a discrete jump in educational investment for this group. Previously, some members of the skilled group will be treated like less skilled children, and a lucky few will receive more schooling. Afterwards, all members of the skilled group will get more schooling. We think of this as capturing the gradual rise in women's college education in the US during the early 20th century.

Eventually, signaling concerns lose power in a pure separating equilibrium, and the skills essentially serve to maximize welfare as described in Proposition 3. Of course, all daughters will still be undereducated, because parents are trying to engender more fertility, but they will at least become informed about their talents. The underinvestment in female education may vanish altogether, if parents lose control over educational investment, or female education does little to reduce fertility, or daughter's education leads to more investment in grandchild quality, which her parents value.

This model suggested that the population would have only two levels of education for women, but that would not be the case if there were visible differences in parameters across the population. In that case, different parameters will lead to different equilibria, although for any given set of observable parameters, parents will still use education to influence their daughters' beliefs.

If taken literally, then in the parameter space when pooling occurs, skilled daughters do not know that they are skilled, but at least they, and everyone else, correctly infers the share of women in the population who are skilled. Yet children may be unable to actually know the true share of skilled daughters, since they do not observe any daughters being well educated. Since there is little hard evidence on skills, parents may be able to persuade sons and daughters alike that skilled daughters are rare even if they are common. Such stories would not be falsified by anything in the children's experience.

Implications for Sons and Credulous Bayesianism

The previous discussion focused on the formation of beliefs of daughters, but given our assumptions about preferences, parents will also have an incentive to induce sons to favor traditional family structures. Moreover, the opinions that men hold about women's ability in the workforce are important outcomes that deserve their own explanation.

Parents presumably have many tools to influence sons' beliefs, including literature, fertility choices and maternal behavior. We will continue to focus on the education of daughters. However, in the previous subsection, we assumed that parents were perfectly informed about the ability levels of daughters, but that daughters themselves only learned their productivity levels after they made their own fertility choices. We now assume that neither parents nor daughters know the daughters' productivity levels exactly. Instead, we assume that parents, but not children, are perfectly informed about the share of women in the population who are more able.

This knowledge is presumably provided by a lifetime's worth of experience. In this case, the parental investment decision is not just a signal of one daughter's ability, but a signal of the overall competence of womankind.

To simplify matters, we assume that daughters are homogeneous—either all high ability or all low ability—and children assume that parents know the truth. Ex ante, children believe that women are less able with probability "p." We assume that in the family—and across society—sons and daughters observe the investment decisions of parents in both sexes. As before, the fertility of the next children will equal $\delta W(H_F)\hat{A}_F \underline{t}_C = \alpha(V'(N) + G_T) - H_T$, and this will be true for both sons and daughters. But since each child will have a spouse with their own views, we assume that in a family decision making unit, the collective belief about $\hat{A}_F$ equals $\gamma\hat{A}_{F,Wife} + (1-\gamma)\hat{A}_{F,Husband}$ so that the parental generation takes the spouses beliefs as fixed, so belief formation's power is somewhat muted, but spread over more children. If the parents have exactly equal number of sons and daughters and $\gamma = .5$, then the impact of investment in daughters' human capital on total fertility of grandchildren will be the same as before. We will simplify the discussion by considering a family with exactly one son and one daughter.

If children correctly understand their parents' preferences, then we are in a situation quite similar to the one discussed above. Just as before, we will define $H_F^{Unskill}$ as the human capital investment of parents in daughters if those parents are known to know that all daughters have ability level 1-a and H_F^{Skill} as the human capital investment of parents in daughters if those parents are known to know that all daughters have ability level 1. We also define $\widehat{H}$ as the level of human capital investment in daughters that makes the parents who know that daughters are skilled indifferent between investing H_F^{Skill} and having children who believe women are more capable and investing $\widehat{H}$ and having children who believe that women are less capable.

Children believe that they are in a signaling game, and assuming our version of D1, Proposition 4 follows:

Proposition 4: If $\widehat{H} > \underline{H}_F$, then parents who know that girls are equally capable as boys will invest in H_F^{Skill}, but if $\widehat{H} < \underline{H}_F$, then some or all parents who know that daughters are equally capable as boys will invest $\underline{H}_F$, and children of both genders will believe that there is some chance that women are less capable then men even when this is not the case.

This proposition, which is essentially identical to 3b, emphasizes that when the uncertainty is about the overall nature of womankind, rather than a particular daughter, parents may perpetuate stereotypes by under-educating their female children. If $\widehat{H} < \underline{H}_F$, then it is possible for parents who know perfectly well that women are as capable as men, will make education decisions as if their daughters were less capable, in order to persuade both sons and daughters that women are less effective in the workforce. So even if the reality is that women are equally capable, children will think that there is some probability of a real difference in abilities.

There are reasons, however, that we might doubt this version of the world. This suggests a very stark difference between female and male education. It suggests that as discrimination decreases, there should be sharp societal jump from under-educating women.

An alternative approach, which is somewhat less attractive given strict assumptions about hyper-rationality but which perhaps lies closer to the truth, is that children misperceive parental preferences, and believe that $a_2 = 0$. As such, they believe that parents make decisions only to better children's welfare, and not to manipulate their children's beliefs. This a version of Glaeser and Sunstein's (2009) "Credulous Bayesianism" where agents use Bayes' rule to infer but they underestimate the incentives of people around them to persuade. Given the pervasive altruism that exists in parent-child relationships it would be particularly natural for children to think that parents are particularly benign.

We might also assume that children merely underestimate a_2, rather than thinking that it is equal to zero, but that would leave us with the complications of a signaling model. If a_2 is thought to be zero exactly, then children will look at parents' investment in daughters and believe that these investments maximize: $\alpha_1\left(Y_0 + \delta W(H_F)\hat{A}_F\left(1 - N\underline{t}_C\right) + \alpha(V(N) + NG_T) - NH_T\right) - H_F$, and hence $\hat{A}_F(H_F) = \frac{1}{\alpha_1 \delta W'(H_F)\left(1-N\left(W(H_F)\hat{A}_F(H_F)\right)\underline{t}_C\right)}$ and $\frac{\hat{A}_F{\prime}(H_F)}{\hat{A}_F(H_F)}$ equals $\frac{-\frac{W''(H_F)}{W'(H_F)}\left(1-N(W(H_F)\hat{A}_F)\underline{t}_C\right)+W'(H_F)\hat{A}_F N{\prime}(W(H_F)\hat{A}_F)\underline{t}_C}{1-N(W(H_F)\hat{A}_F)\underline{t}_C - W(H_F)\hat{A}_F N{\prime}(W(H_F)\hat{A}_F)\underline{t}_C}$, which is positive as long as children believe that their parents' maximization problem is concave.

Children may recognize that they will form their ultimate beliefs based on their prospective spouses' knowledge, but we appeal to the law of iterated expectations and assume that children think that their parents' beliefs (being correct) will also equal their spouses' beliefs.

We assume that parents know that $\hat{A}_F = 1$, but that they are trying to influence the beliefs of both their sons and daughters in their educational investment decisions. Parents also do not believe that their investments will influence the beliefs of their children's prospective spouses so they expect their daughter's fertility to be $N\left(W(H_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\right)$ and their son's fertility to be $N\left(W(\widetilde{H}_F)(\gamma\tilde{A}_{F,} + (1-\gamma)\hat{A}_F)\right)$, where $\widetilde{H}_F$ and $\tilde{A}_F$ are the prevalent levels of investment in daughters and beliefs about women's ability level in the wider population.

If they have Q_F daughters and Q_M sons, then the following proposition follows:

Proposition 5: If children believe that $\alpha_2 = 0$, then parents will choose to invest in daughters' education so that both sons and daughters believe that women are less talented than men. Investment in daughters will decline with α_2 and Q_M and so will beliefs about daughters' ability levels, while investment will rise with Q_F. If $\hat{A}_F = \tilde{A}_F$and $\widehat{H}_F = \widetilde{H}_F$ and hence $N_F = N_M$, then investment in daughters is rising with γ if and only if $Q_M > Q_F$.

Since children believe that parents lack ulterior motives, underinvestment in daughters is interpreted as meaning that daughters are expected to be less able. This belief will occur in both daughters and sons, and it will become more extreme if parents have stronger preferences toward grandchildren.

The underinvestment in daughters becomes more extreme when there are more boys in the household and less extreme when there are more girls. The logic of this effect is not that boys take up girls' resources (there are no income effects in this model), but rather that boys present an added target for indoctrination. When it comes to daughters, underinvestment carries a cost (too little human capital) and comes with a benefit (daughters choose to have more children). When it comes to sons, this underinvestment only creates a benefit (sons choose to have more children).

While the logic of this argument appears clear, it runs counter to the finding of Butcher and Case (1994) that women with only female siblings receive less, not more, education. Of course, these findings could reflect forces outside of the model, such as spillovers from boys to girls during the middle 20th century. Within the context of this model, two things would need to change to rationalize the Butcher and Case finding. First, indoctrinating boys through investment in women would need to be unimportant, perhaps because there are other ways to indoctrinate or perhaps because boys have little control in ultimate fertility decisions. Second, there would need to be some spillover in beliefs across women, so that the payoff of indoctrination was higher when there are more girls.

The final comparative static concerns "γ," which reflects the weight put on mothers' beliefs in the ultimate fertility decision. When a family has more boys, then higher values of "γ" will tend to increase investment in girls, and when a family has more girls then higher values of "γ" will decrease investment in girls.

Timing of Work and Persuasion

The models of persuasion that we have discussed ultimately assume that women are choosing their fertility levels with little direct knowledge of their workplace ability, but that would seem to depend on the timing of work and childbearing. If the mother works initially, she will surely have a better assessment of her talents from that direct source than from anything she may have inferred from either refrigerator advertisements or even her parents' investment in her human capital. That knowledge will then essentially eliminate the incentive to persuade initially.

For simplicity, we continue to assume that each child requires a fixed time investment of $\underline{t}_c$, although we ignore investment in children's human capital. We assume that mothers maximize the expected value of $y_0 + \int_{t=0}^{1} y(t)dt + \alpha V(N)$, where y(t) is the earnings at each t, so there are no discounting issues. Women end up being paid their expected or realized productivity level times $\delta W(H_F)$.

We only consider two options in childbearing. First, the mother has children immediately, basing her fertility decision on her expected workplace earnings. Second, the mother delays childbearing to the point where she has learned her actually productivity in the workplace and then decides on fertility knowing her actual ability level. We ignore more complicated strategies, and assume that the primary costs of delay are health or time related, so that the expected time cost of N children, when childbearing begins at time t_0, will be $g(t_0)N\underline{t}_C$ where $g(0) = 1$ and $g'(t_0) > 0$. These assumptions capture both the added difficulty of having children when older and that the ability to produce own children has historically been impossible for women at some age. We ignore other benefits of later childbearing (more experience in life) and other costs (more human capital may depreciate during the childbearing period).

Moreover we continue to assume A_F equals either 1 or 1-a (hence the probability that ability equals 1 equals $1 - \frac{1-\hat{A}_F}{a}$, where $\hat{A}_F$ is the women's expected ability level. The expected payoff from having children immediately will be $y_0 + \delta W(H_F)\hat{A}_F(1 - N\underline{t}_C) + \alpha V(N)$, meaning that as before N satisfies $\delta W(H_F)\hat{A}_F\underline{t}_C = \alpha V'(N)$. With delay, the woman learns her true ability level and eventually chooses fertility to maximize $g_\psi\, \delta W(H_F)A_F\underline{t}_C = \alpha V'(N)$, where we let g_ψ denote this health cost of delaying fertility until the point of knowledge. This leads to Proposition 6.

Proposition 6: There exists a value of g_ψ at which women are indifferent between postponing work or postponing childbearing, and for higher values of g_ψ women will postpone childbearing and for lower values of g_ψ, women will postpone work. If women postpone childbearing, then changes in the initial beliefs about workplace will have no impact on their fertility decision.

Proposition 6 suggests that medical changes which delay childbearing may have far-reaching impact on social beliefs. If women are making fertility decisions early in life, then those decisions will be based not on their actual workplace productivity, but rather on the information that they have gleaned about the relative pleasure of working and childbearing. That position of ignorance creates a possible role for persuasion for grandparents interested in encouraging fertility, or anyone else interested in persuading men and women.

But if women experience work first, then the impact of any such persuasion is highly muted. The knowledge gained in the labor force will surely swamp the knowledge inferred from parental education decisions or the persuasion of consumer goods companies. As such, the delay in childbearing can powerfully change the incentives to persuade. This effect links the time series of women working with the time series of opinions about female competence at work. As Goldin (2006) describes, women were initially prone to work after marriage and then the pattern switched and more women worked earlier. That switch should, if the model's assumptions are correct, act to reduce the incentive to invest in gender-related beliefs and stereotypes. If women are waiting to learn their type before having children, then they are likely to be less responsive to parental misinformation about their ability level or likelihood of enjoying work.

VI. Conclusion

This paper has produced a series of models which attempt to capture the different possible sources of gender stereotypes. We have discussed belief formation potentially influenced by politicians and co-workers, and modeled in detail the possible influence on gender specific beliefs by consumer good producers and by parents. We argued that Friedan's model of belief formation by consumer goods producers required assumptions that seemed unlikely to hold during most time periods. Parental persuasion seems like a likely factor driving the perpetuation of gender-related beliefs.

We recognize that this model will run counter to the experience of many daughters, who experienced parents who pushed them to succeed, and gave them nothing but positive affirmation of their own talents. While such occurrences do run counter to the literal structure of the model, we do not believe that they are incompatible with a somewhat richer view of the world.

In many cases, parents may have been more interested in grandchildren quality than in grandchildren quantity. If daughters' human capital, and even workplace success, ended up leading toward more investment in grandchildren, then grandparents would indeed have an incentive to push their daughters toward education and success in the workforce.

Moreover, we have treated parents as the only source of information available to children. Consider a world in which there are a variety of social institutions which broadcast messages about women's ability in work. We may even assume that these institutions exist to cater to parents who want allies in prodding children toward childbearing. Churches, for example, often seem to have served that role.

If parents believe that their daughters or sons are already exposed to information depicting women as less competent, and they also believe that their daughters will invest too little in themselves if they adopt those social beliefs (even given the parents' pro-grandchildren preferences), then the parents may work against those social beliefs. For example, assume that the prevalent social belief is that women have ability level $\underline{A}$ and that parents know that their own daughter has ability level $\overline{A}$. Those parents may not want the daughter to behave as if she knew her full ability level, but they may still want to think that she as an ability level higher than $\underline{A}$. They will then tell their daughter to disregard the negative stereotype, even if they would prefer it if she thought her own ability was slightly less than $\overline{A}$.

Gender-related beliefs do seem to have had an impact on labor markets and family choices. Those beliefs do not seem to have always been based on reality. We have adopted an economic approach to error that emphasizes the incentives to mislead. We hope that further work develops further models along this line, and does more to subject our and similar models to serious empirical tests.

References

Akerlof, George. 1976. "The Economics of Caste and of the Rat Race and Other Woeful Tales." *The Quarterly Journal of Economics* 90(4): 599-617.

Arrow, Kenneth J. 1973. "The Theory of Discrimination." In *Discrimination in Labor Markets*, edited by Ashenfelter Orley and Albert Rees, 3-33. Princeton, N.J.: Princeton University Press.

Axinn, William G., Marin Clarkberg, and Arland Thornton. 1994. "Family Influences on Family Size Preferences." *Demography* 31(1): 65-79.

Barber, Nigel. 2000. *Why parents matter: parental investment and child outcomes*. Westport, CT: Bergin & Garvey.

Becker, Gary. 1957. *The Economics of Discrimination.* Chicago: University of Chicago Press.

Becker, Gary S. and Kevin M. Murphy. 2000. *Social Economics: Market Behavior in a Social Environment*. Cambridge, MA: Harvard University Press.

Bereaud, Susan R. 1975. "Sex role images in French children's books." *Journal of Marriage and the Family* 37: 194-207.

Blau, Francine, Peter Brummund, and Albert Yung-Hsu Liu. "Trends in Occupational Segregation by Gender 1970-2009: Adjusting for the Impact of Changes in the Occupational Coding System." *Demography* (forthcoming).

Butcher, Kristin F. and Anne Case. 1994. "The Effect of Sibling Sex Composition on Women's Education and Earnings." *Quarterly Journal of Economics* 109(3): 531-563.

Cho, In-Koo and David M. Kreps. 1987. "Signaling Games and Stable Equilibria." *Quarterly Journal of Economics* 102(2): 179-221.

Fogel, Robert William. 2000. *The Fourth Great Awakening and the Future of Egalitarianism.* Chicago: University of Chicago Press.

Fox, Bonnie J. 1990. "Selling the Mechanized Household: 70 Years of Ads in Ladies Home Journal" *Gender & Society* 4(1): 25-40.

Friedan, Betty. 1963. *The Feminine Mystique.* New York: W.W. Norton.

Glaeser , Edward L. 2005. "The Political Economy of Hatred." *Quarterly Journal of Economics* 120(1): 45-86.

Glaeser, Edward L. and Cass Sunstein. 2009. "Extremism and Social Learning." *Journal of Legal Analysis* 1(1): 263-324.

Glaeser, Edward L. and Gergely Ujhelyi. 2010. "Regulating Misinformation." *Journal of Public Economics* 94(3-4): 247-257.

Goldin, Claudia. 2000. “Labor Markets in the Twentieth Century.” In *Cambridge Economic History of the United States, Vol. III, The Twentieth Century*, edited by Stanley J. Engerman and Robert E. Gallman, 549-625. New York: Cambridge University Press.

———. 2006. “The quiet revolution that transformed women’s employment, education, and family.” *American Economic Review* 96: 1–21.

———. 1990. *Understanding the gender gap: An economic history of American women.* Oxford: Oxford University Press.

Goldin, Claudia and Lawrence F. Katz. 2000. “Career and Marriage in the Age of the Pill.” *American Economic Review* 90(2): 461-465.

Gunderson, Elizabeth, Gerardo Ramirez, Susan Levine, and Sian Beilock. 2012. "The Role of Parents and Teachers in the Development of Gender-Related Math Attitudes." *Sex Roles* 66(3/4): 153.

Krueger, Anne O. 1963. “The Economics of Discrimination.” *Journal of Political Economy* 71(5): 481-486.

Lazear, Edward P. 1999. “Culture and Language.” *Journal of Political Economy* 107(6): 95-126.

Lerner, Gerda. 1987. *The Creation of Patriarchy*. Oxford, UK: Oxford University Press.

Margo, Robert A. 1991. "Segregated Schools and the Mobility Hypothesis: A Model of Local Government Discrimination." *The Quarterly Journal of Economics* 106(1): 61-73.

Milkman, Ruth. 1982. “Redefining ‘Women's Work’: The Sexual Division of Labor in the Auto Industry during World War II.” *Feminist Studies* 8(2): 336-372.

Myrdal, Gunnar. 1944. *An American Dilemma*: *The Negro Problem and Modern Democracy.* New York: Harper & Brothers.

Rossy, Katherine M. 2011. "Politicizing Pronatalism: Exploring the Nazi Ideology of Women through the Lens of Visual Propaganda, 1933-1939." *The Graduate History Review* 3(1): 49-77.

Simon, Rita J. and Jean M. Landis. 1989. "A Report: Women's and Men's Attitudes about a Woman's Place and Role." *The Public Opinion Quarterly* 53(2): 265-276.

Thurow, Lester C. 1969. *Poverty and Discrimination*. Washington D.C.: Brookings Institution.

Weitzman, Lenore J., Deborah Eifler, Elizabeth Hokada, and Catherine Ross. 1972. “Sex-role socialization in picture books for preschool children.” *American Journal of Sociology* 77(6): 1125-1150.

Woodward, C. Vann. 1955. *The Strange Career of Jim Crow*, Oxford: Oxford University Press.

Proofs of Propositions:

Proof of Proposition 1: Consider the two relevant first order conditions $\delta W(H_F)\hat{A}_F = \alpha V_T(T_H, C_H)$, and $P_H = \alpha V_C(T_H, C_H)$. Let Z denote $\delta W(H_F)A_F$, and then differentiate both conditions totally with respect to Z. This produces the derivative $\frac{\partial T_H}{\partial Z} = \frac{V_{CC}}{\alpha(V_{CC}V_{TT} - V_{CT}^2)}$ and $\frac{\partial C_H}{\partial Z} = \frac{-V_{CT}}{\alpha(V_{CC}V_{TT} - V_{CT}^2)}$. The first term is negative, proving the results for δ, H_F, and A_F which increase Z and have no other impact on the conditions. The second term is negative if and only if $V_{CT} > 0$. Differentiating with respect to P_H yields: $\frac{\partial T_H}{\partial P_H} = \frac{-V_{CT}}{\alpha(V_{CC}V_{TT} - V_{CT}^2)}$ and $\frac{\partial C_H}{\partial P_H} = \frac{V_{YT}}{\alpha^2(V_{CC}V_{TT} - V_{CT}^2)}$. The second term is unambiguously negative and the first term is negative if and only if $V_{CT} > 0$.

Proof of Lemma 1: The posterior probability that the women is low ability equals $\frac{(p_0+\Delta)^{N_{B,1}+\theta(N_{B,0}+S_T)+1}(1-p_0-\Delta)^{N_G}+(p_0-\Delta)^{N_{B,1}+\theta(N_{B,0}+S_T)+1}(1-p_0+\Delta)^{N_G}}{(p_0+\Delta)^{N_{B,1}+\theta(N_{B,0}+S_T)}(1-p_0-\Delta)^{N_G}+(p_0-\Delta)^{N_{B,1}+\theta(N_{B,0}+S_T)}(1-p_0+\Delta)^{N_G}}$ and the derivative of this with respect to S_T equals:

$$\frac{2\theta\Delta Log\left(\frac{p_0+\Delta}{p_0-\Delta}\right)(p_0+\Delta)^{N_{B,1}+\theta(N_{B,0}+S_T)}(1-p_0-\Delta)^{N_G}(p_0-\Delta)^{N_{B,1}+\theta(N_{B,0}+S_T)}(1-p_0+\Delta)^{N_G}}{\left((p_0+\Delta)^{N_{B,1}+\theta(N_{B,0}+S_T)}(1-p_0-\Delta)^{N_G}+(p_0-\Delta)^{N_{B,1}+\theta(N_{B,0}+S_T)}(1-p_0+\Delta)^{N_G}\right)^2} > 0.$$

The second derivative is negative since $\left(\frac{p_0+\Delta}{p_0-\Delta}\right)^{N_{B,1}+\theta(N_{B,0}+S_T)} > \left(\frac{1-p_0-\Delta}{1-p_0+\Delta}\right)^{N_G}$.

Proof of Proposition 2: The firm's first order condition for investing in misleading stories about women in the workforce is:

$$\omega M Q^{\frac{\varphi\sigma-1}{1-\varphi}} \frac{\varphi\sigma}{1-\varphi}\left(\delta W(H_F)\right)^{\frac{-\varphi\sigma}{1-\varphi}}\left(1 - p\left(N_{B,1} + \theta N_{B,0} + \theta\sum_{i=1}^{Q} S_i\right)a\right)^{\frac{\varphi(1-\sigma)-1}{1-\varphi}} a\theta p'\left(N_{B,1} + \theta N_{B,0} + \theta\sum_{i=1}^{Q} S_i\right) = k$$

The second derivative is
$\omega M Q^{\frac{\varphi-1}{1-\varphi}} \frac{\varphi\sigma}{1-\varphi}\left(\delta W(H_F)\right)^{\frac{-\varphi\sigma}{1-\varphi}} a\left(1 - p\left(N_{B,1} + \theta N_{B,0} + \theta\sum_{i=1}^{Q} S_i\right)a\right)^{\frac{\varphi(2-\sigma)-2}{1-\varphi}}$ times

$\frac{1-\varphi(1-\sigma)}{1-\varphi}\left(\theta p'\left(N_{B,1} + \theta N_{B,0} + \theta\sum_{i=1}^{Q} S_i\right)\right)^2 a +$
$\left(1 - p\left(N_{B,1} + \theta N_{B,0} + \theta\sum_{i=1}^{Q} S_i\right)a\right)\theta^2 p''\left(N_{B,1} + \theta N_{B,0} + \theta\sum_{i=1}^{Q} S_i\right)$ and we have assumed that this is negative, which requires that the second term is larger in absolute value than the first term.

This expression can be rewritten as: $h_S(\theta QS_i, Z) = k$ where Z represents a vector of exogenous characteristics. If $h_{SS}(\theta QS_i, Z) < 0$, which we assume, then S_i is declining with Z if $h_{SZ}(\theta QS_i, Z) < 0$. For any other exogenous characteristic, the derivative of S_i with respect to Z is positive if and only if $h_{SZ}(\theta QS_i, Z) > 0$. This means that stories are declining with Q, δ and $W(H_F)$ and rising with M. The total number of stories QS_i must also be declining with Q, because $\theta h_{SS}(\theta QS_i, Z)(Q\frac{\partial S_i}{\partial Q} + S_i) + h_{SQ}(\theta QS_i, Z) = 0$, or $\frac{\partial S_i}{\partial Q}Q + S_i = -\frac{h_{SQ}(\theta QS_i, Z)}{\theta h_{SS}(\theta QS_i, Z)} < 0$.

The derivative with respect to θ satisfies $Qh_{SS}(\theta QS_i, Z)(\theta\frac{\partial S_i}{\partial \theta} + S_i) + h_{S\theta}(\theta QS_i, Z) = 0$, where $h_{S\theta} > 0$. As such $\theta\frac{\partial S_i}{\partial \theta} + S_i > 0$, or the total number of messages heard must be rising with θ.

The second order condition needed for this to be a maximum is that $W"(H_F)\left(\alpha_1\alpha V"(N)(1 - N\underline{t}_C) + \alpha_2\underline{t}_C(V'(N) + \mathrm{G_T})\right) + \left(\frac{\alpha_2}{\alpha} - \alpha_1 - \alpha_2\frac{(V'(N)+\mathrm{G_T})V'''(N)}{\alpha(V"(N))^2}\right)\delta A_F\left(\underline{t}_C W'(H_F)\right)^2 > 0$, which we assume. Sufficient conditions for this to hold are that $\frac{W"(H_F)}{W'(H_F)}\frac{(1-N\underline{t}_C)}{N\underline{t}_C}\frac{NV"(N)}{V'(N)}$ is greater than $\frac{\alpha(V'(N)+G_T)-H_T}{\alpha V'(N)}$, which ensures concavity even when α_2 is near zero, and $(V"(N))^2 > (V'(N) + \mathrm{G_T})V'''(N)$, which holds as long as $V'''(N)$ is small or negative.

Proof of Proposition 3: The first order condition for parental investment in a daughter's education is that

$$1 = \alpha_1\left(\delta W'(H_F)A_F(1 - N\underline{t}_C)\right) + \left((\alpha_1\alpha + \alpha_2)(V'(N) + \mathrm{G_T}) - \alpha_1\left(\delta W(H_F)A_F\underline{t}_C + \mathrm{H_T}\right)\right)\frac{\partial \mathrm{N}}{\partial H_F}$$

The childbearing choices during the next generation set $\delta W(H_F)A_F\underline{t}_C + \mathrm{H_T} = \alpha(V'(N) + \mathrm{G_T})$, so $\frac{\partial \mathrm{N}}{\partial H_F}$ equals $\frac{\delta W'(H_F)A_F\underline{t}_C}{\alpha V"(N)}$ and the first order condition for parental investment can be written as

$1 = W'(H_F)\delta A_F\left(\alpha_1(1 - N\underline{t}_C) + \frac{\alpha_2(V'(N)+\mathrm{G_T})\underline{t}_C}{\alpha V"(N)}\right)$ which can be written as
$1 = W'(H_F(x))g(N(H_F(x), x), x)$, where x represents any of the other exogenous variables, and g(.,.) represents $\delta A_F\left(\alpha_1(1 - N\underline{t}_C) + \frac{\alpha_2(V'(N)+\mathrm{G_T})\underline{t}_C}{\alpha V"(N)}\right)$.

As $lim_{x\to 0}W'(x) = \infty$, then the derivative of parental well-being with respect to H_F when that variable equals zero, will be positive as long as $\alpha_1(1 - N\underline{t}_C) + \frac{\alpha_2(V'(N)+\mathrm{G_T})\underline{t}_C}{\alpha V"(N)}$ is positive at that point, which requires $-\frac{N_0V"(N_0)}{V'(N_0)+\mathrm{G_T}} > \frac{\alpha_2}{\alpha\alpha_1}\frac{N_0\underline{t}_C}{1-N_0\underline{t}_C}$

Differentiating $W'(H_F(x))g(N(H_F(x),x),x) = 1$ totally with respect to any "x" produces $-\left(W"(H_F(x))g(N,x) + W'(H_F(x))\frac{\partial g}{\partial N}\frac{\partial N}{\partial H_F}\right)\frac{\partial H_F}{\partial x} = W'(H_F(x))\left(\frac{\partial g}{\partial N}\frac{\partial N}{\partial x} + \frac{\partial g}{\partial x}\right)$. For the second order conditions to hold, we need $W"(H_F)g + W'(H_F)\frac{\partial g}{\partial N}\frac{\partial N}{\partial H_F} < 0$ or

$$W"(H_F)\left(\alpha_1(1 - N\underline{t}_C) + \frac{\alpha_2(V\prime(N)+G_T)\underline{t}_C}{\alpha V"(N)}\right) + \left(\frac{\alpha_2}{\alpha} - \alpha_1 - \frac{\alpha_2 V'''(N)(V\prime(N)+G_T)}{\alpha(V"(N)^2}\right)\frac{\delta A_F\left(W\prime(H_F)\right)^2\underline{t}_C^2}{\alpha V"(N)} < 0,$$

which we assume. Sufficient conditions for this include that $\frac{W"(H_F)}{W\prime(H_F)}\frac{(1-N\underline{t}_C)}{N\underline{t}_C}\frac{NV"(N)}{V\prime(N)}$ is greater than $\frac{\alpha(V\prime(N)+G_T)-H_T}{\alpha V\prime(N)}$, which ensures concavity even when α_2 is near zero, and $(V"(N))^2 > (V'(N) + G_T)V'''(N)$, which holds as long as $V'''(N)$ is small or negative. If α_2 and V'''(N) both equal zero then the condition is that $1 > -\frac{W"(H_F)}{W\prime(H_F)}$.

With that assumption, the sign of $\frac{\partial H_F}{\partial x}$ depends on the sign of $\frac{\partial g}{\partial N}\frac{\partial N}{\partial x} + \frac{\partial g}{\partial x}$. The terms α_1 and α_2 do not directly impact N, and since $\frac{\partial g}{\partial \alpha_1} = \delta A_F(1 - N\underline{t}_C) > 0$ and $\frac{\partial g}{\partial \alpha_2} = \frac{V'(N)\delta A_F \underline{t}_C}{\alpha V"(N)} < 0$, we know that investment in the daughter's human capital is rising in direct altruism and declining with altruism toward the number of grandchildren.

The derivative $\frac{\partial g}{\partial N}$ equals $\delta A_F\left(-\alpha_1\underline{t}_C + \frac{\alpha_2\underline{t}_C}{\alpha} - \frac{V'''(N)(V\prime(N)+G_T)}{(V"(N))^2}\frac{\alpha_2\underline{t}_C}{\alpha}\right)$. The term δA_F can be labeled y, since these two terms always enter together. In that case, $\frac{\partial g}{\partial y}$ equals $\left(\alpha_1(1 - N\underline{t}_C) + \frac{\alpha_2(V\prime(N)+G_T)\underline{t}_C}{\alpha V"(N)}\right)$ and $\frac{\partial N}{\partial y}$ equals $\frac{\underline{t}_C W(H_F)}{\alpha V"(N)}$. Plugging these values in implies that $\frac{\partial g}{\partial N}\frac{\partial N}{\partial y} + \frac{\partial g}{\partial y}$ equals:

$$\alpha_1(1 - N\underline{t}_C) + \frac{\alpha_2(V'(N) + G_T)\underline{t}_C}{\alpha V"(N)} - \left(\alpha_1 - \frac{\alpha_2}{\alpha} + \frac{\alpha_2 V'''(N)(V'(N) + G_T)}{\alpha(V"(N)^2}\right)\frac{\delta A_F W(H_F)\underline{t}_C^2}{\alpha V"(N)}$$

which is positive if and only if

$$\frac{(-\alpha V"(N))}{\left(\delta A_F \underline{t}_C\right)^2 W'(H_F)W(H_F)} > \left(\frac{\alpha_2}{\alpha} - \alpha_1 - \frac{\alpha_2 V'''(N)(V'(N) + G_T)}{\alpha(V"(N)^2}\right)$$

or

$$\alpha_1\alpha\left(1 - N\underline{t}_C - \frac{\delta A_F W(H_F)\underline{t}_C^2}{\alpha V"(N)}\right) > \left(2\delta A_F W(H_F)\underline{t}_C + H_T - \frac{V'''(N)\delta A_F W(H_F)\underline{t}_C(V\prime(N)+G_T)}{(V"(N)^2}\right)\frac{\alpha_2\underline{t}_C}{(-\alpha V"(N))}.$$

If V'''(.) is small, then this condition always holds if $\frac{\alpha_2}{\alpha} < \alpha_1$, but doesn't hold if α_1 is sufficiently close to zero.

Proof of Lemma 2: Define skilled parental welfare at H given a deviation that is perceived as having come from an unskilled child's parent as $G(H, N_U(H), x)$. This function is continuous in H, and by continuity, for H sufficiently close to H_F^{Skill}, $G(H, N_U(H), x)$ is greater than the payoff to choosing H_F^{Skill} and for H sufficiently close to 0, the parents of skilled children strictly prefer choosing H_F^{Skill}. By continuity, there must exist at least one value of H so such that $G(H, N_U(H), x)$ equals $G\left(H_F^{Skill}, N_S\left(H_F^{Skill}\right), x\right)$, the welfare of parents of skilled children when they choose H_F^{Skill}.

The derivative of G with respect to H is $\frac{\partial G}{\partial H} + \frac{\partial G}{\partial N_U}\frac{\partial N_U}{\partial H}$ which equals $\alpha_1\big(\delta W'(H)(1 - N_U(H)\underline{t}_C\ )) + \alpha_2(V'(N_U) + G_T)\frac{\delta W'(H)(1-a)\underline{t}_C}{\alpha V"(N_U)} - 1$.

The derivative of this expression with respect to H is $\delta(1-a)$ times

$W"(H)\left(\frac{\alpha_1}{1-a}\left((1 - N_U\underline{t}_C\ )\right) + \frac{\alpha_2(V'(N_U)+G_T)\underline{t}_C}{\alpha V"(N_U)}\right) + \left(\frac{\alpha_2}{\alpha} - \frac{\alpha_1}{1-a} - \frac{\alpha_2(V'(N_U)+G_T)V'''(H)}{\alpha(V"(N_U))^2}\right)\frac{\delta\left(\underline{t}_C W'(H)\right)^2(1-a)}{\alpha V"(N_U))}$ which is equal to $W"(H)\left(\alpha_1\left((1 - N_U\underline{t}_C\ )\right) + \frac{\alpha_2(V'(N_U)+G_T)\underline{t}_C}{\alpha V"(N_U)}\right) + \left(\frac{\alpha_2}{\alpha} - \alpha_1 - \frac{\alpha_2(V'(N)+G_T)V'''(H)}{\alpha(V"(N_U))^2}\right)\frac{\delta\left(\underline{t}_C W'(H)\right)^2(1-a)}{\alpha V"(N_U))}$ plus $W"(H)\frac{\alpha_1 a}{1-a}\left((1 - N_U\underline{t}_C\ )\right) - \frac{a\alpha_1}{1-a}\frac{\delta\left(\underline{t}_C W'(H)\right)^2(1-a)}{\alpha V"(N_U))}$. The first expression is negative by assumption (we assumed the maximization problem was everywhere concave for skilled and unskilled parents). The second expression is negative as long as $W"(H)\left((1 - N_U\underline{t}_C\ )\right) < \frac{\delta\left(\underline{t}_C W'(H)\right)^2(1-a)}{\alpha V"(N_U))}$, which also must be true since $W"(H)\left(\alpha_1\left((1 - N_U\underline{t}_C\ )\right) + \frac{\alpha_2(V'(N_U)+G_T)\underline{t}_C}{\alpha V"(N_U)}\right) + \left(\frac{\alpha_2}{\alpha} - \alpha_1 - \frac{\alpha_2(V'(N)+G_T)V'''(H)}{\alpha(V"(N_U))^2}\right)\frac{\delta\left(\underline{t}_C W'H\right)^2(1-a)}{\alpha V"(N_U))}$ must be negative at $\alpha_2 = 0$.

Since the function $G(H, N_U(H), x)$ is concave and begins below $G\left(H_F^{Skill}, N_S\left(H_F^{Skill}\right), x\right)$ and ends above $G\left(H_F^{Skill}, N_S\left(H_F^{Skill}\right), x\right)$ for H close to H_F^{Skill}, the line can only cross once. The first time $G(H, N_U(H), x)$ crosses $G\left(H_F^{Skill}, N_S\left(H_F^{Skill}\right), x\right)$ it must be upward sloping. If $G(H, N_U(H), x)$ crossed a second time, it would have a negative slope at that point, and by concavity, must lie below $G\left(H_F^{Skill}, N_S\left(H_F^{Skill}\right), x\right)$ for all higher levels of H, and this is false. As such there exists exactly one crossing point, which defines $\widehat{H}$, and $G(H, N_U(H), x)$, the welfare of skilled parents at the point whose daughters believe themselves to be unskilled, is strictly increasing with H at that point.

The term $\widehat{H}$ satisfies $\alpha_1\left(\delta W\left(H_F^{Skill}\right)\left(1 - N_S\left(H_F^{Skill}\right)\underline{t}_C\ \right) - N_S\left(H_F^{Skill}\right)H_T\right) + (\alpha_1\alpha + \alpha_2)\left(V\left(N_S\left(H_F^{Skill}\right)\right) + N_S\left(H_F^{Skill}\right)G_T\right) - H_F^{Skill} = \alpha_1\left(\delta W\left(\widehat{H}\right)\left(1 - N_U\left(\widehat{H}\right)\underline{t}_C\ \right) - N_U\left(\widehat{H}\right)H_T\right) +$

$(\alpha_1\alpha + \alpha_2)\left(V\left(N_U(\widehat{H})\right) + N_U(\widehat{H})G_T\right) - \widehat{H}$. If G(H,N) represents the welfare of unskilled parents at human capital level H and fertility level N, then the welfare of skilled parents at the same point would equal $G(H,N) + a\delta W(H)(1 - N\underline{t}_C)$. The value of H_F^{Skill} maximizes $G(H, N_S(H)) + a\delta W(H)(1 - N_S(H)\underline{t}_C)$, and that this maximizes level the welfare of skilled parents equals $G(\widehat{H}, \widehat{N}) + a\delta W(\widehat{H})(1 - \widehat{N}\underline{t}_C)$. Imagine that the there was a value of H'>$\widehat{H}$ at which $G(H', N_S(H')) > G(\widehat{H}, \widehat{N})$. For this to be the case, it would need to be true that $a\delta W(\widehat{H})(1 - \widehat{N}\underline{t}_C) > a\delta W(H')(1 - N_S(H')\underline{t}_C)$. But differentiation gives us that the derivative of $a\delta W(H)(1 - N_S(H)\underline{t}_C)$ is strictly positive, so this is impossible. Hence, if skilled parents are indifferent between $\widehat{H}$ and being thought to have unskilled children or H_F^{Skill} and being thought to have skilled children, then the parents of unskilled children will never prefer an H above $\widehat{H}$ and being thought to have skilled children.

The expression that defines $\widehat{H}$ can be written as $G\left(H_F^{Skill}, N_S(H_F^{Skill}), x\right) = G\left(\widehat{H}, N_U(\widehat{H}), x\right)$, and the derivative of this with respect to any parameter x satisfies: $\left(\frac{\partial G}{\partial H_F^{Skill}} + \frac{\partial G}{\partial N_S}\frac{\partial N_S}{\partial H_F^{Skill}}\right)\frac{\partial H_F^{Skill}}{\partial x} + \frac{\partial G(H_F^{Skill}, N_S(H_F^{Skill}), x)}{\partial x} = \left(\frac{\partial G}{\partial \widehat{H}} + \frac{\partial G}{\partial \widehat{H}}\frac{\partial N_U}{\partial H_F^{Skill}}\right)\frac{\partial \widehat{H}}{\partial x} + \frac{\partial G(\widehat{H}, N_U(\widehat{H}), x)}{\partial x}$. Since $G\left(H_F^{Skill}, N_S(H_F^{Skill}), x\right)$ is maximized over H_F^{Skill}, then the envelope theorem applies, and $\frac{\partial G}{\partial H_F^{Skill}} + \frac{\partial G}{\partial N_S}\frac{\partial N_S}{\partial H_F^{Skill}} = 0$. Moreover, the derivative $\frac{\partial G}{\partial \widehat{H}} + \frac{\partial G}{\partial N_U}\frac{\partial N_U}{\partial \widehat{H}}$ is positive since the slope of $G\left(\widehat{H}, N_U(\widehat{H}), x\right)$ is positive at the crossing point. That means that the sign of $\frac{\partial \widehat{H}}{\partial x}$ equals the sign of $\frac{\partial G(H_F^{Skill}, N_S(H_F^{Skill}), x)}{\partial x} - \frac{\partial G(\widehat{H}, N_U(\widehat{H}), x)}{\partial x}$.

For x=δ, the value of $\frac{\partial G(H_F^{Skill}, N_S(H_F^{Skill}), x)}{\partial x} - \frac{\partial G(\widehat{H}, N_U(\widehat{H}), x)}{\partial x}$ equals α_1 times $W(H_F^{Skill})(1 - N_S(H_F^{Skill})\underline{t}_C) - W(\widehat{H})(1 - N_U(\widehat{H})\underline{t}_C)$, which is strictly positive since $W(H_F^{Skill}) > W(\widehat{H})$ and $N_S(H_F^{Skill}) < N_U(\widehat{H})$.

For x=α_2, the value of $\frac{\partial G(H_F^{Skill}, N_S(H_F^{Skill}), x)}{\partial x} - \frac{\partial G(\widehat{H}, N_U(\widehat{H}), x)}{\partial x}$ equals $V\left(N_S(H_F^{Skill})\right) + N_S(H_F^{Skill})G_T - V\left(N_U(\widehat{H})\right) - N_U(\widehat{H})G_T$ and is strictly negative.

For x=α_1, the value of $\frac{\partial G(H_F^{Skill}, N_S(H_F^{Skill}), x)}{\partial x} - \frac{\partial G(\widehat{H}, N_U(\widehat{H}), x)}{\partial x}$ equals $\delta W(H_F^{Skill})(1 - N_S(H_F^{Skill})\underline{t}_C) + \alpha V\left(N_S(H_F^{Skill})\right) + N_S(H_F^{Skill})(\alpha G_T - H_T) - \delta W(\widehat{H})(1 - N_U(\widehat{H})\underline{t}_C) - \alpha V(N_U(\widehat{H})$- $N_U(\widehat{H})(\alpha G_T - H_T)$. This is positive since this is the difference in the daughters' welfare level and she benefits both from having more human capital and from knowing her ability level correctly.

Proof of Proposition 3a:

If $H_F^{Unskill} < \widehat{H}$, then there exists a separating equilibrium entails the unskilled choosing $H_F^{Unskill}$ and the skilled choosing H_F^{Skill}. These are equilibria, and they are stable, because neither group wants to imitate each other. The parents of less skilled daughters have no incentive to deviate since they are already receiving their best possible outcome. As such, any deviation can only come from the skilled, and if the skilled deviate, as long as they are believed to be skilled they will receive a lower payoff.

Are there other separating equilibria when $H_F^{Unskill} < \widehat{H}$? The parents of the skilled are always better off at H_F^{Skill}, than at any other separating human capital level, so there can be no equilibrium where the parents of the skilled choose a different level of H. That fact is true even without the equilibrium refinement.

Consider any other choice $H_F^{'}$ by the parents of the unskilled and consider a deviation to $H_F^{Unskill}$. There is no set of beliefs by children, or fertility response, that could induce the parent of the skilled to make that deviation, and so only the unskilled will make it and this means that the only reasonable belief by children is that they are unskilled given a choice of $H_F^{Unskill}$. That response means that the unskilled will deviate to $H_F^{Unskill}$. As such, the unskilled can only choose $H_F^{Unskill}$. The parents of the skilled will not choose this investment level, even if they are thought to be parents of the unskilled, and as the unskilled all choose this level, the skilled will go ahead and choose H_F^{Skill}.

It is similarly impossible to generate a pooling or semi-pooling equilibrium in this range of parameter values since the unskilled can always deviate to $H_F^{Unskill}$. In this pooling equilibrium, the parents of the unskilled are always doing worse than in the separating equilibrium (since it is their best outcome) and the parents of the skilled are always doing better (since they always have the option to separate). The parents of the skilled will therefore happily deviate if the fertility response is $N_U\left(H_F^{Unskill}\right)$ while the parents of the unskilled will require a higher fertility level to deviate. This means that children will infer that such a deviation can only come from the less skilled, and this inference will cause to want to deviate.

Proof of Proposition 3b:

If $H_F^{Unskill} > \widehat{H} > \underline{H}_F$ then the only separating equilibrium will again require that the parents of the skilled invest H_F^{Skill} as discussed above. A separating equilibrium does exist where the parents of the unskilled choose $\widehat{H}$ and the parents of the skilled choose H_F^{Skill}. The parents of the skilled will not choose to deviate to $\widehat{H}$, since they are getting equal welfare by choosing H_F^{Skill}. The parents of the unskilled will not choose to deviate to any lower level of H, because their welfare is locally increasing in H, assuming that children think that they are unskilled.

Consider the deviation to any level of H between $\widehat{H}$ and H_F^{Skill}. Let $G(\widehat{H},\widehat{N})$ define the welfare of unskilled parents choosing this level of human capital investment. The welfare of parents of the skilled choosing that level of human capital investment would equal $G(\widehat{H},\widehat{N}) + a\delta W(\widehat{H})(1-\widehat{N}\underline{t}_C\ )$, which also equals their welfare when they invest H_F^{Skill}, by construction. Consider any deviation to H'>$\widehat{H}$ and let $N_{Unskill}^{'}$ denote the associated fertility level which makes the unskilled indifferent between that level of human capital investment and investing $\widehat{H}$. Since the welfare of the skilled is always increasing in fertility, they will require a strictly lower level of fertility to deviate if and only if $W(\text{H'})(1-N_{Unskill}^{'}\underline{t}_C\ ) > W(\widehat{H})(1-\widehat{N}\underline{t}_C\ )$.

If $N_{Unskill}^{'} < \widehat{N}$, then it follows that the skilled will deviate for a lower fertility response, and hence any deviation must come from the skilled. Since there is no H that will induce a parent of an unskilled daughter to deviate from $\widehat{H}$ if they are thought to be parents of a skilled daughter, they will not deviate.

There does not exist a separating equilibrium where the parents of the less skilled choose on their own a value of H greater than $\widehat{H}$ because the parents of the more skilled would choose to imitate them and that does not require our refinement. There does not exist a separating equilibrium where the parents of the less skilled choose a lower level of H, because if they deviate to $\widehat{H}-\varepsilon$, then children must realize that if they choose $N_U(\widehat{H}-\varepsilon\ )$, the parents of the unskilled would strictly prefer this outcome. Moreover, if they choose $N_U(\widehat{H}-\varepsilon\ )$, the parents of the skilled would strictly prefer to remain investing $N_U(H_F^{Unskill})$. As a result, children must infer that the deviation came from the parents of the less skilled and that will induce the parents of the less skilled to deviate.

As such, there is a unique separating equilibrium, where parents of the skilled invest H_F^{Skill} and parents of the unskilled invest $\widehat{H}$. Without the D1 refinement a semi-pooling equilibrium would also be possible. It is impossible for the skilled independently to choose an investment level below H_F^{Skill} (because the skilled could always do better choose H_F^{Skill}) or for the parents of the unskilled to choose an investment level below $\widehat{H}$ (because they could always raise their investment level to that amount and do better). If the skilled and unskilled both choose some level H below H_F^{Skill} and above $\widehat{H}$, then this could in principle be an equilibrium. This pooling point would have to have the exact share of skilled and unskilled so that either the skilled parents are indifferent between choosing this level and choosing H_F^{Skill} or the unskilled would be indifferent between choosing this investment level and choosing $\widehat{H}$ or possibly both.

However, such a pooling or semi-pooling equilibrium would fail the D1 refinement. Consider a candidate pooling point H*>$\widehat{H}$. At the pooling point, fertility is denoted N*. We have assumed that both welfares are increasing with N, and note immediately that such a point must deliver weakly higher welfare to the parents of the unskilled than they receive by choosing $\widehat{H}$ and weakly higher to the parents of the skilled than they receive by choosing H_F^{Skill}, since both groups can deviate to those levels and reveal their types. The welfare level must also be lower than the unskilled parents receive at $H_F^{Unskill}$ if their children recognize themselves as being unskilled, since that is the best outcome possible for the parents of the unskilled.

Given any human capital investment H and associated fertility level N, the welfare of the parents of the unskilled is denoted $\alpha_1 \delta W(H_F)(1-a)(1-N\underline{t}_C) - H_F + \mathrm{Q(N)}$, where Q(N) equals $\alpha_1(Y_0 + \alpha(V(N) + NG_T) - NH_T) + \alpha_2(V(N) + \mathrm{NG_T})$.

At the same levels the welfare of the parents of the skilled equals $\alpha_1 \delta W(H_F)(1-N\underline{t}_C) - H_F + \mathrm{Q(N)}$, as such the derivative of the welfare of the skilled parents with respect to H is always greater than the derivative of the unskilled parents with respect to H, and the derivative with respect to N is always lower. Define $N_S(H^{**})$ as the fertility level that makes more skilled parents indifferent between pooling at H* and deviating and $N_U(H^{**})$ as the fertility level that makes less skilled parents indifferent between pooling at H* and deviating. Note that these alternative fertility levels are defined as the levels that would make parents indifferent, not the fertility levels that children would themselves choose. The derivative at (H*, N*) of $N_U(H^{**})$ with respect to H** satisfies: $N_U'(H^*) = -\frac{G_H(H*,N*)}{\mathrm{Q(N)}} == -\frac{\alpha_1 \delta(1-a)W'(H^*)(1-N^*\underline{t}_C)-1}{\mathrm{Q}'(N^*)-\alpha_1 \delta W(H^*)(1-a)\underline{t}_C}$. The derivative at (H*, N*) of $N_S(H^{**})$ with respect to H** satisfies:

$N_S'(H^*) = -\frac{\alpha_1 \delta W'(H^*)(1-N^*\underline{t}_C)-1}{\mathrm{Q}'(N^*)-\alpha_1 \delta W(H^*)\underline{t}_C}$.

If $\alpha_1 \delta(1-a)W'(H^*)(1-N^*\underline{t}_C) \geq 1$, as it will be for any H less than or equal to $H_F^{Unskill}$ (since that point is defined as maximizing welfare with respect to H for less skilled, and at the pooling equilibrium, the returns to H will be higher for the less skilled both because fertility is lower and because $G_H(H*, N*)$ does not include the negative impact that H has on fertility), then $N_S'(H^{**}) < N_U'(H^{**}) \leq 0$, because the denominator of $N_S'(H^{**})$ is smaller and the numerator is farther from zero (or negative). In that case, a marginal downward deviation will be attractive for a wider range of fertility levels for the less skilled, and D1 implies that a slight downward deviation will be thought to come from the less skilled, and hence the less skilled will deviate.

If $\alpha_1 \delta(1-a)W'(H^*)(1-N^*\underline{t}_C) < 1$, then locally, the welfare of the unskilled parents is decreasing in H. There must, however, exist a value of H^{**} less than H* and greater than $\widehat{H}$ at which the parents of the less skilled are indifferent between (H^*, N^*) and (H^{**}, N^*). Locally, we know that locally any deviations to (H^{**}, N^*) will yield higher welfare than the equilibrium level, since welfare is locally falling in H. But we also know that deviating to $(\widehat{H}, N^*)$ yields lower utility than the equilibrium level, since the equilibrium must be at least as good as $(\widehat{H}, \widehat{N})$

and $N^* < \widehat{N}$, both because human capital is higher and because the children do not believe that they are unskilled at H^*, and welfare is decreasing in fertility. Hence be continuity, there exists some value of (H^{**}, N^*) that makes the less skilled indifferent, but at that point $\alpha_1 \delta W(H^{**})(1 - a)(1 - N^* \underline{t}_C) - H^{**} + \mathrm{Q}(N^*) = \alpha_1 \delta W(H^*)(1 - a)(1 - N^* \underline{t}_C) - H^* + \mathrm{Q}(N^*)$, then it follows that $\alpha_1 \delta W(H^{**})(1 - N^* \underline{t}_C) - H^{**} + \mathrm{Q}(N^*) < \alpha_1 \delta W(H^*)(1 - N^* \underline{t}_C) - H^* + \mathrm{Q}(N^*)$, and the more skilled would not deviate downwards at fertility level N*, and hence the deviation is thought to come from parents of the less skilled, and hence those parents will deviate.

When $\widehat{H} < \underline{H}_F$, then the equilibrium involves pooling at this point, and some parents of the skilled may remain at H_F^{Skill}. We can rule out pure separating, since the parents of the more skilled will always choose H_F^{Skill} and will strictly prefer imitating the less skilled at any feasible level of H. We can rule out any pooling at a higher level of H, by the argument made immediately above, since the parents of the less skilled will always choose to deviate downwards. That leaves us with pure pooling, or semi-pooling. Pure pooling can only exist if the welfare of the parents of the skilled at the pooling point $\underline{H}_F$ is greater than the welfare they would have by choosing H_F^{Skill} and being known to be more skilled.

At the pooling point, any deviation upwards will be thought to come from the more skilled, by a variant of the argument made above, and again, following roughly the algebra above, the less skilled will be less likely to deviate, and hence any deviation is thought to come from the parents of the more skilled daughters, and hence no deviation will occur.

The semi-pooling equilibrium will ensure that the parents of the skilled are indifferent between choosing H_F^{Skill} and choosing $\underline{H}_F$. Let $N_{Mix}(\underline{H}_F)$ denote the fertility level at that point, which must satisfy the first order condition $\delta W(\underline{H}_F)(1 - (1 - p_{Mix})a)\underline{t}_C + H_T = \alpha \left(V'\left(N_{Mix}(\underline{H}_F) \right) + G_T \right)$, where p_{Mix} indicates the proportion of skilled parents is the mix of parents who are providing $\underline{H}_F$. In this case, $\frac{\partial N_{Mix}}{\partial p_{Mix}} = \frac{\delta W(\underline{H}_F) a \underline{t}_C}{\alpha V''(N_{Mix})} < 0$. The value of $N_{Mix}(\underline{H}_F)$ must also satisfy

$$\alpha_1\left(\delta W\left(H_F^{Skill}\right)\left(1 - N_S\left(H_F^{Skill}\right)\underline{t}_C\right) - N_S\left(H_F^{Skill}\right)H_T\right) + (\alpha_1 \alpha + \alpha_2)\left(V\left(N_S\left(H_F^{Skill}\right)\right) + N_S\left(H_F^{Skill}\right)G_T\right) - H_F^{Skill} = \alpha_1\left(\delta W\left(\underline{H}_F\right)\left(1 - N_{Mix}\left(\underline{H}_F\right)\underline{t}_C\right) - N_{Mix}\left(\underline{H}_F\right)H_T\right) + (\alpha_1 \alpha + \alpha_2)\left(V\left(N_{Mix}\left(\underline{H}_F\right)\right) + N_{Mix}\left(\underline{H}_F\right)G_T\right) - \underline{H}_F.$$

For any parameter that does not impact the fertility choice by daughters, the equality can be written as $G\left(H_F^{Skill}(x), N_S\left(H_F^{Skill}(x)\right), x\right) = G\left(\underline{H}_F, N_{Mix}\left(\underline{H}_F, p_{Mix}(x)\right), x\right)$. Using $\frac{\partial G}{\partial H_F^{Skill}} \frac{\partial H_F^{Skill}}{\partial x} + \frac{\partial G}{\partial N_S}\left(\frac{\partial N_S}{\partial H_F^{Skill}} \frac{\partial H_F^{Skill}}{\partial x}\right) = 0$, differentiating this expression totally with respect to x yields:

$$\frac{dG\left(H_F^{Skill}(x),N_S\left(H_F^{Skill}(x)\right),x\right)}{dx} - \frac{dG\left(\underline{H_F},N_{Mix}\left(\underline{H_F},p_{Mix}(x)\right)\right),x)}{dx} =$$
$$\frac{\partial G\left(H_F^{Skill}(x),N_S\left(H_F^{Skill}(x)\right),x\right)}{\partial x} - \frac{\partial G\left(\underline{H_F},N_{Mix}\left(\underline{H_F},p_{Mix}(x)\right)\right),x)}{\partial x} - \frac{\partial G\left(\underline{H_F},N_{Mix}\left(\underline{H_F},p_{Mix}(x)\right)\right),x)}{\partial N_{Mix}}\frac{\partial N_{Mix}}{\partial p_{Mix}}\frac{\partial p_{Mix}}{\partial x} = 0$$

We know that $\frac{\partial N_{Mix}}{\partial p_{Mix}} < 0$ and $\frac{\partial G\left(\underline{H_F},N_{Mix}\left(\underline{H_F},p_{Mix}(x)\right)\right),x)}{\partial N_{Mix}} > 0$, so the sign of $\frac{\partial p_{Mix}}{\partial x}$ must be the opposite of the sign of $\frac{\partial G\left(H_F^{Skill}(x),N_S\left(H_F^{Skill}(x)\right),x\right)}{\partial x} - \frac{\partial G\left(\underline{H_F},N_{Mix}\left(\underline{H_F},p_{Mix}(x)\right)\right),x)}{\partial x}$. When a parameter makes the high skill option relatively more attractive, the fertility given low skill must rise, which means that the share of skilled parents investing in lower skills must fall.

When x=α_1 then $\frac{\partial G\left(H_F^{Skill}(x),N_S\left(H_F^{Skill}(x)\right),x\right)}{\partial x} - \frac{\partial G\left(\underline{H_F},N_{Mix}\left(\underline{H_F},p_{Mix}(x)\right)\right),x)}{\partial x}$ equals $\left(\delta W\left(H_F^{Skill}\right)\left(1 - N_S\left(H_F^{Skill}\right)\underline{t}_C\right) - N_S\left(H_F^{Skill}\right)H_T + \alpha\left(V\left(N_S\left(H_F^{Skill}\right)\right) + N_S\left(H_F^{Skill}\right)G_T\right)\right) - \left(\delta W\left(\underline{H_F}\right)\left(1 - N_{Mix}\underline{t}_C\right) - N_{Mix}H_T + \alpha\left(V(N_{Mix}) + N_{Mix}G_T\right)\right)$ which must be positive since $\left(\delta W\left(H_F^{Skill}\right)\left(1 - N_S\left(H_F^{Skill}\right)\underline{t}_C\right) - N_S\left(H_F^{Skill}\right)H_T + \alpha\left(V\left(N_S\left(H_F^{Skill}\right)\right) + N_S\left(H_F^{Skill}\right)G_T\right)\right)$ is strictly increasing with H and $\delta W\left(\underline{H_F}\right)\left(1 - N_S\left(\underline{H_F}\right)\underline{t}_C\right) - N_S\left(\underline{H_F}\right)H_T + \alpha\left(V\left(N_S\left(\underline{H_F}\right)\right) + N_S\left(\underline{H_F}\right)G_T\right) > \delta W\left(\underline{H_F}\right)\left(1 - N_{Mix}\underline{t}_C\right) - N_{Mix}H_T + \alpha\left(V(N_{Mix}) + N_{Mix}G_T\right)$. Hence the number of parents of more skilled children choosing $\underline{H_F}$ must decline with α_1.

When x=α_2 then $\frac{\partial G\left(H_F^{Skill}(x),N_S\left(H_F^{Skill}(x)\right),x\right)}{\partial x} - \frac{\partial G\left(\underline{H_F},N_{Mix}\left(\underline{H_F},p_{Mix}(x)\right)\right),x)}{\partial x}$ equals $V\left(N_S\left(H_F^{Skill}\right)\right) + N_S\left(H_F^{Skill}\right)G_T - V(N_{Mix}) - N_{Mix}G_T$ which must be negative. Hence the number of parents of more skilled children choosing $\underline{H_F}$ must increase with α_2.

Proof of Proposition 4: The proof of this proposition follows exactly the logic of Proposition 3a and 3b. If $\widehat{H} > \underline{H_F}$, then it is impossible to have a pooling equilibrium at any value of H less than or equal to $\widehat{H}$, because the parents who know that women are skilled will choose H_F^{Skill}. It is impossible to have a pooling equilibrium at any value of H greater than $\widehat{H}$ because the parents who thought women were less skilled will deviate downwards to a point where they will benefit given a lower fertility rate than the fertility needed to generate deviation for parents who know that women are more skilled. As such only separating equilibria are possible, and the parents who know that daughters are more capable will always choose H_F^{Skill} in a separating equilibrium. If $\widehat{H} < \underline{H_F}$, then it is impossible to have a pure separating equilibrium, because at any value of $H \geq \underline{H_F}$, the skilled would prefer imitating the less skilled to choosing H_F^{Skill}. It is however, possible to have a semi-pooling or pure pooling equilibrium, where enough of the skilled choose $\underline{H_F}$ to make them indifferent (or strictly prefer) choosing that quantity to choosing H_F^{Skill} or any other level of H. Any upwards deviation in H will benefit the parents who know that women are skilled more than the parents who think that women are less capable, given the same fertility response, as such upward deviations are thought to come from the skilled, and this ensure that both types of agents are optimally choosing $\underline{H}_F$.

Proof of Proposition 5: For notational convenience we let B(N) denote $\alpha_1(\alpha(V(N) + NG_T) - NH_T) + \alpha_2(V(N) + \mathrm{NG_T})$

If they have Q_F daughters and Q_M sons, then parents will be choosing H_F to maximize.

$$\alpha_1\left(\delta W(H_F)\big(1 - N\big(W(H_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big)\underline{t}_C\big) + \frac{Q_M}{Q_F}\delta W(\widetilde{H}_F)\big(1 - N\big(W(\widetilde{H}_F)(\gamma\tilde{A}_F + (1-\gamma)\hat{A}_F)\big)\underline{t}_C\big)\right) + B(N\big(W(H_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big)) + \frac{Q_M}{Q_F}B(N\big(W(\widetilde{H}_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big)) - H_F$$

This produces the first order condition

$$\alpha_1\delta W'(H_F)\big(1 - N_F\underline{t}_C\big) - 1 + \hat{A}_F{}'(H_F)\left(\alpha_2(V'(N_F) + \mathrm{G_T})W(H_F)\gamma N'\big(W(H_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big) + \frac{Q_M}{Q_F}\alpha_2(V'(N_M) + \mathrm{G_T})W(\widetilde{H}_F)(1-\gamma)N'\big(W(\widetilde{H}_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big)\right) = 0\ .$$

The problem as a whole can be written as $U(H_F, Z)$ where $U_{H_F}(H_F, Z) = 0$, at a maximum. Assuming that second order conditions hold, the impact of any parameter Z on H_F depends on $U_{H_F Z}(H_F, Z)$. The function $\hat{A}_F(H_F)$ depends on α_1, δ and $\underline{t}_C$, but not on , α_2, γ or $\frac{Q_M}{Q_F}$ and hence we will concentrate on these latter parameters, especially since changes in these parameters will have no direct impact on the beliefs of the children.

The children's assessment of their ability level satisfies $\hat{A}_F(H_F) = \frac{1}{\alpha_1\delta W'(H_F)\left(1 - N\left(W(H_F)\hat{A}_F(H_F)\right)\underline{t}_C\right)}$, which will be the inverse of $1 - \hat{A}_F{}'(H_F)\left(\alpha_2(V'(N_F) + \mathrm{G_T})W(H_F)\gamma N'\big(W(H_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big) + \frac{Q_M}{Q_F}\alpha_2(V'(N_M) + \mathrm{G_T})W(\widetilde{H}_F)(1-\gamma)N'\big(W(\widetilde{H}_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big)\right)$ which is greater than one. Hence, daughters and sons alike will underestimate the ability of women. For parameters that do not directly impact $\hat{A}_F(H_F)$, changes in beliefs reflect only changes in H_F, with reductions in H_F causing reductions in beliefs of women's competence.

The cross-partial $U_{H_F\alpha_2}$ equals

$$\hat{A}_F{}'(H_F)\left((V'(N_F) + \mathrm{G_T})W(H_F)\gamma N'\big(W(H_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big) + \frac{Q_M}{Q_F}(V'(N_M) + \mathrm{G_T})W(\widetilde{H}_F)(1-\gamma)N'\big(W(\widetilde{H}_F)(\gamma\hat{A}_F + (1-\gamma)\tilde{A}_F)\big)\right) < 0\ .$$

The cross-partial $U_{H_F Q_M}$ equals

$\frac{\hat{A}_F{}'(H_F)}{Q_F}\alpha_2(V'(N_M)+\mathrm{G_T})W(\widetilde{H}_F)(1-\gamma)N'\big(W(\widetilde{H}_F)(\gamma\hat{A}_F+(1-\gamma)\tilde{A}_F)\big)<0$, and the cross-partial $U_{H_FQ_F}$ equals $-\frac{\hat{A}_F{}'(H_F)Q_M}{Q_F^2}\alpha_2(V'(N_M)+\mathrm{G_T})W(\widetilde{H}_F)(1-\gamma)N'\big(W(\widetilde{H}_F)(\gamma\hat{A}_F+(1-\gamma)\tilde{A}_F)\big)>0$.

If $\hat{A}_F=\tilde{A}_F$ and $\widehat{H}_F=\widetilde{H}_F$ and hence $N_F=N_M$ then the derivative with respect to γ is $\hat{A}_F{}'(H_F)W(\widetilde{H}_F)N'\big(W(\widetilde{H}_F)\hat{A}_F\big)\alpha_2(V'(N_F)+\mathrm{G_T})\left(1-\frac{Q_M}{Q_F}\right)$, which is positive if and only if $Q_M>Q_F$.

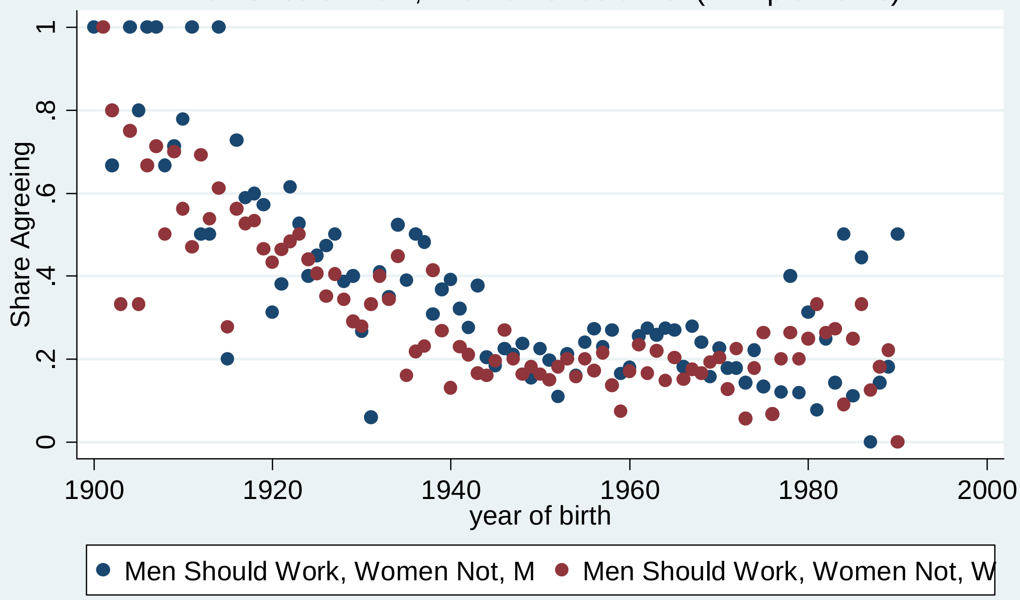
Proof of Proposition 6: The welfare from having children immediately equals, $y_0+\delta W(H_F)\hat{A}_F(1-N^0\underline{t}_C)+\alpha V(N^0)$, where N^0 satisfies $\delta W(H_F)\hat{A}_F\underline{t}_C=\alpha V'(N^0)$.

The expected welfare from waiting to have children is $y_0+\left(1-\frac{1-\hat{A}_F}{a}\right)\left(\delta W(H_F)(1-g_\psi N^+\underline{t}_C)+\alpha V(N^+)\right)+\left(\frac{1-\hat{A}_F}{a}\right)\left(\delta W(H_F)(1-a)(1-g_\psi N^-\underline{t}_C)+\alpha V(N^-)\right)$, where $g_\psi\delta W(H_F)\underline{t}_C=\alpha V'(N^+)$, and $g_\psi\delta(1-a)W(H_F)\underline{t}_C=\alpha V'(N^-)$.

When $g_\psi=1$, then waiting to have children is clearly preferable, as $\left(1-\frac{1-\hat{A}_F}{a}\right)\left(\delta W(H_F)(1-N^+\underline{t}_C)+\alpha V(N^+)\right)+\left(\frac{1-\hat{A}_F}{a}\right)\left(\delta W(H_F)(1-a)(1-N^-\underline{t}_C)+\alpha V(N^-)\right)>\delta W(H_F)\hat{A}_F(1-N^0\underline{t}_C)+\alpha V(N^0)$. When $g_\psi=\infty$, then welfare is $\hat{A}_F(\delta W(H_F)+\alpha V(0))$ which is less than $\delta W(H_F)\hat{A}_F(1-N^0\underline{t}_C)+\alpha V(N^0)$. Moreover $y_0+\left(1-\frac{1-\hat{A}_F}{a}\right)\left(\delta W(H_F)(1-g_\psi^*N^+\underline{t}_C)+\alpha V(N^+)\right)+\left(\frac{1-\hat{A}_F}{a}\right)\left(\delta W(H_F)(1-a)(1-g_\psi^*N^-\underline{t}_C)+\alpha V(N^-)\right)$, is monotonically and continuously decreasing with ψ, and as a result, there must exist a crossing point denoted g_ψ^* between zero and k at which mothers are indifferent between waiting or not. For all lower levels of g_ψ^*, mothers prefer waiting and for all higher levels of g_ψ^*, mothers prefer early fertility.

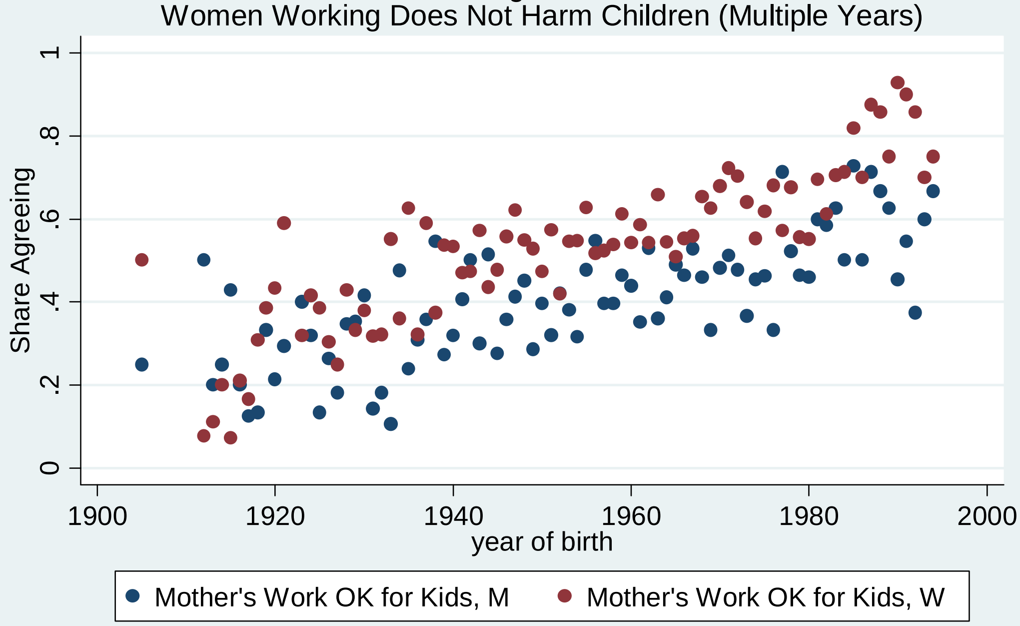
If $V(N)=vN^\mu$, then $N^+=\left(\frac{\alpha v}{g_\psi^*\underline{t}_C\delta W(H_F)}\right)^{\frac{1}{1-\mu}}$, $N^-=\left(\frac{\alpha v}{g_\psi^*\underline{t}_C(1-a)\delta W(H_F)}\right)^{\frac{1}{1-\mu}}$ and $N^0=\left(\frac{\alpha v}{\underline{t}_C\hat{A}_F\delta W(H_F)}\right)^{\frac{1}{1-\mu}}$, so with a bit of algebra, the crossing point is defined by $\left(1+\left(\frac{1-\hat{A}_F}{a}\right)\left((1-a)^{-\frac{\mu}{1-\mu}}-1\right)\right)^{\frac{1-\mu}{\mu}}\hat{A}_F=g_\psi^*$.

Figure 1:
Men Should Work, Women Should Not (Multiple Years)



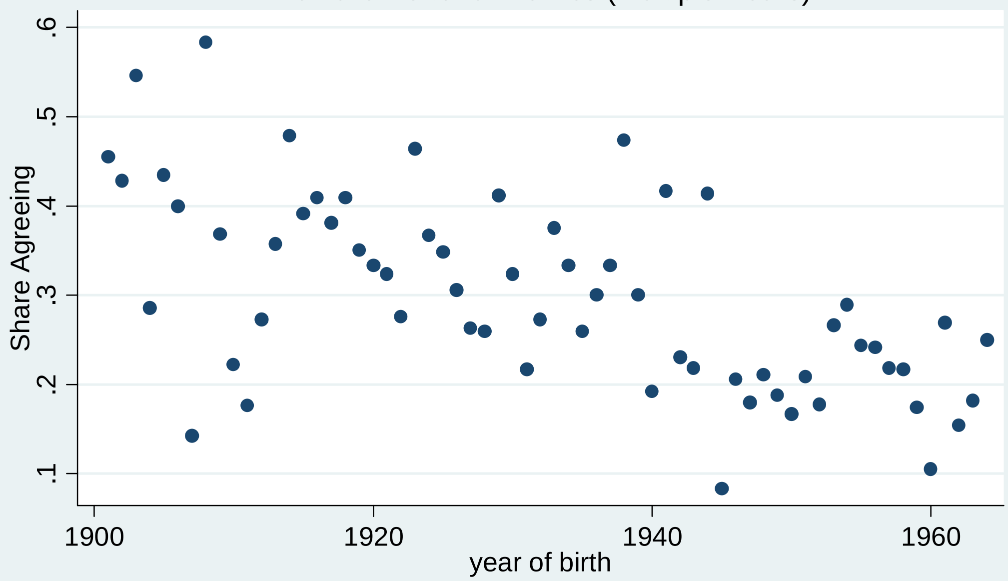
Source: General Social Survey
Text: A husband's job is to earn money; a wife's job is to look after the home and family.

Figure 2:
Women Working Does Not Harm Children (Multiple Years)



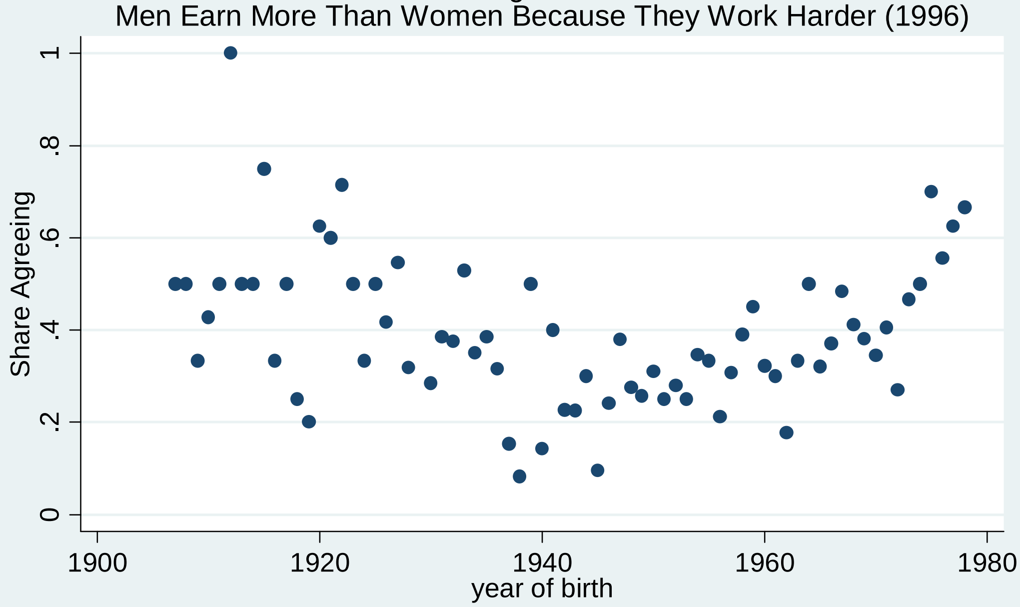
Source: General Social Survey
Text: A pre-school child is likely to suffer if his or her mother works.

Figure 3:
Men are Better at Politics (Multiple Years)



Source: General Social Survey
Text: Would you say that most men are better suited emotionally for politics than are most women, that men and women are equally suited, or that women are better suited than men in this area?

Figure 4:
Men Earn More Than Women Because They Work Harder (1996)



Source: General Social Survey
Text: Men work harder on the job than women do.
How important do you think this reason is for explaining why women earn less?