

# CAPITAL ALLOCATION AND PRODUCTIVITY IN SOUTH EUROPE\*

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Starting in the early 1990s, countries in southern Europe experienced low productivity growth alongside declining real interest rates. We use data for manufacturing firms in Spain between 1999 and 2012 to document a significant increase in the dispersion of the return to capital across firms, a stable dispersion of the return to labor, and a significant increase in productivity losses from capital misallocation over time. We develop a model with size-dependent financial frictions that is consistent with important aspects of firms' behavior in production and balance sheet data. We illustrate how the decline in the real interest rate, often attributed to the euro convergence process, leads to a significant decline in sectoral total factor productivity as capital inflows are misallocated toward firms that have higher net worth but are not necessarily more productive. We show that similar trends in dispersion and productivity losses are observed in Italy and Portugal but not in Germany, France, and Norway. *JEL Codes:* D24, E22, F41, O16, O47.

## I. INTRODUCTION

Beginning in the 1990s, so-called imbalances emerged across countries in Europe. Countries in the South received large capital inflows. During this period productivity diverged, with countries in the South experiencing slower productivity growth than other European countries. Economists and policy makers often

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conjecture that low productivity growth resulted from a misallocation of resources across firms or sectors in the South.

This article has two goals. First, we bring empirical evidence to bear on the question of how the misallocation of resources across firms evolves over time. Between 1999 and 2012, we document a significant increase in the dispersion of the return to capital and a deterioration in the efficiency of resource allocation across Spanish manufacturing firms. Second, we develop a model with firm heterogeneity, financial frictions, and capital adjustment costs to shed light on these trends. We demonstrate how the decline in the real interest rate, often attributed to the euro convergence process, led to an increase in the dispersion of the return to capital and to lower total factor productivity (TFP) as capital inflows were directed to less productive firms operating within relatively underdeveloped financial markets.

Our article contributes to the literatures on misallocation and financial frictions. Pioneered by [Restuccia and Rogerson \(2008\)](#) and [Hsieh and Klenow \(2009\)](#), the misallocation literature documents large differences in the efficiency of factor allocation across countries and the potential for these differences to explain observed TFP differences. But so far there is little systematic evidence on the dynamics of misallocation within countries. Models with financial frictions, such as [Kiyotaki and Moore \(1997\)](#), have natural implications for the dynamics of capital misallocation at the micro level. Despite this, there exists no empirical work that attempts to relate the dynamics of capital misallocation at the micro level to firm-level financial decisions and to the aggregate implications of financial frictions. Our work aims to fill these gaps in the literature.

We use a firm-level data set from ORBIS-AMADEUS that covers manufacturing firms in Spain between 1999 and 2012. Our data cover roughly 75% of the manufacturing economic activity reported in Eurostat (which, in turn, uses census sources). Further, the share of economic activity accounted for by small and medium-sized firms in our data is representative of that in Eurostat. Unlike data sets from census sources, our data contain information on both production and balance sheet variables. This makes it possible to relate real economic outcomes to financial decisions at the firm level over time in a large and representative sample of firms.

We begin our analysis by documenting the evolution of misallocation measures within four-digit level manufacturing

industries. We examine trends in the dispersion of the return to capital, as measured by the log marginal revenue product of capital (MRPK), and the return to labor, as measured by the log marginal revenue product of labor (MRPL). As emphasized by [Hsieh and Klenow \(2009\)](#), an increase in the dispersion of a factor's return across firms could reflect increasing barriers to the efficient allocation of resources and be associated with a loss in TFP at the aggregate level. We document an increase in the dispersion of the MRPK in Spain in the precrisis period between 1999 and 2007 that further accelerated in the postcrisis period between 2008 and 2012. By contrast, the dispersion of the MRPL does not show a significant trend throughout this period. Importantly, we document that the increasing dispersion of the return to capital is accompanied by a significant decline in TFP relative to its efficient level.

To interpret these facts and evaluate quantitatively the role of capital misallocation for TFP in an environment with declining real interest rates, we develop a parsimonious small open economy model with heterogeneous firms, borrowing constraints, and capital adjustment costs. Firms compete in a monopolistically competitive environment and employ capital and labor to produce manufacturing varieties. They are heterogeneous in terms of their permanent productivity and also face transitory idiosyncratic productivity shocks. Firms save in a bond to smooth consumption over time and invest to accumulate physical capital.

The main novelty of our model is that financial frictions depend on firm size. We parameterize the borrowing constraint such that the model matches the positive relationship between firm leverage and size in the microdata. We compare the model to the data along various firm-level moments not targeted during the parameterization. We show that the model generates within-firm and cross-sectional patterns that match patterns of firm size, productivity, MRPK, capital, and net worth in the data. A size-dependent borrowing constraint is important for understanding firms' behavior. Nested models, such as when financial frictions do not depend on firm size or are absent, do worse than our model in terms of matching firm-level moments.

When subjected to the observed decline in the real interest rate that started in 1994, our model generates dynamics that resemble the trends in the manufacturing sector in Spain between 1999 and 2007 characterized by an inflow of capital, an increase in MRPK dispersion across firms, and a decline in sectoral TFP. In our model firms with higher net worth increase their capital

in response to the decline in the cost of capital. For these unconstrained firms, the real interest rate drop generates a decline in their MRPK. On the other hand, firms that happen to have lower net worth despite being potentially productive delay their adjustment until they can internally accumulate sufficient funds. These firms do not experience a commensurate decline in their MRPK. Therefore, the dispersion of the MRPK between financially unconstrained and constrained firms increases. Capital flows into the sector but not necessarily to the most productive firms, which generates a decline in sectoral TFP.

Quantitatively, our model generates large increases in firm capital, borrowing, and MRPK dispersion and a significant fraction of the observed decline in TFP relative to its efficient level between 1999 and 2007. We argue that a size-dependent borrowing constraint is crucial in generating these aggregate outcomes. We show that the model without a size-dependent borrowing constraint fails to generate significant changes in firm capital, borrowing, MRPK dispersion, and TFP in response to the same decline in the real interest rate.

To further corroborate the mechanism of our model, we present direct evidence that firms with higher initial net worth accumulated more capital during the precrisis period conditional on their initial productivity and capital. Our model generates an elasticity of capital accumulation with respect to initial net worth of similar magnitude to the elasticity estimated in the firm-level data. Informatively for our mechanism, we additionally document that MRPK dispersion in the data does not increase in the subsample of larger firms. Our model also implies that MRPK dispersion does not increase within larger firms because, with a size-dependent borrowing constraint, larger firms are more likely to overcome their borrowing constraint than smaller firms.

We illustrate that alternative narratives of the precrisis period, such as a relaxation of borrowing constraints or transitional dynamics that arise purely from capital adjustment costs, do not generate the trends observed in the aggregate data. In addition, we show that the increase in the dispersion of the MRPK in the precrisis period cannot be explained by changes in the stochastic process governing firm productivity. During this period, we actually find a decline in the dispersion of productivity shocks across firms. By contrast, changes in financial conditions and uncertainty shocks at the micro level may be important for the postcrisis dynamics characterized by reversals of capital flows, by even larger

increases in the dispersion of the MRPK, and by declines in TFP relative to its efficient level. Indeed, we find that idiosyncratic shocks became significantly more dispersed across firms during the postcrisis period.

We conclude by extending parts of our empirical analyses to Italy (1999–2012), Portugal (2006–2012), Germany (2006–2012), France (2000–2012), and Norway (2004–2012). We find interesting parallels between Spain, Italy, and Portugal. As in Spain, there is a trend increase in MRPK dispersion in Italy before the crisis and a significant acceleration of this trend in the postcrisis period. Portugal also experiences an increase in MRPK dispersion during its sample period that spans mainly the postcrisis years. By contrast, MRPK dispersion is relatively stable in Germany, France, and Norway throughout their samples. Finally, we find significant trends in the loss in TFP due to misallocation in some samples in Italy and Portugal, but do not find such trends in Germany, France, and Norway. We find these differences suggestive, given that firms in the South are likely to operate in less well-developed financial markets.

Our article contributes to a recent body of work that studies the dynamics of dispersion and misallocation. [Oberfield \(2013\)](#) and [Sandleris and Wright \(2014\)](#) document the evolution of misallocation during crises in Chile and Argentina, respectively. [Larrain and Stumpner \(2013\)](#) document changes in resource allocation in several Eastern European countries during financial market liberalization episodes. [Bartelsman, Haltiwanger, and Scarpetta \(2013\)](#) examine the cross-country and time-series variation of the covariance between labor productivity and size as a measure of resource allocation. [Kehrig \(2015\)](#) presents evidence for a countercyclical dispersion of (revenue) productivity in U.S. manufacturing.

[Asker, Collard-Wexler, and De Loecker \(2014\)](#) show how risk and adjustment costs in capital accumulation can rationalize dispersion of firm-level revenue productivity. Following their observation, our model allows for the possibility that an increase in the dispersion of firm-level outcomes are driven by changes in second moments of the stochastic process governing idiosyncratic productivity. [Bloom et al. \(2012\)](#) demonstrate that increases in the dispersion of plant-level productivity shocks is an important feature of recessions in the United States.

[Banerjee and Duflo \(2005\)](#) discuss how capital misallocation can arise from credit constraints. An earlier attempt to link

productivity and financial frictions to capital flows in an open economy is [Mendoza \(2010\)](#). Recently, several articles have endogenized TFP as a function of financial frictions in dynamic models ([Midrigan and Xu 2014](#); [Moll 2014](#); [Buera and Moll 2015](#)). A typical prediction of these models is that a financial liberalization episode is associated with capital inflows, a better allocation of resources across firms, and an increase in TFP growth (see, e.g., [Buera, Kaboski, and Shin 2011](#); [Midrigan and Xu 2014](#)). This shock, however, does not match the experience of countries in South Europe where TFP growth declined.

One important difference between our article and these articles is that we focus on transitional dynamics generated by a decline in the real interest rate. Contrary to a financial liberalization shock, the decline in the real interest rate is associated with an inflow of capital and a decline in TFP in the short run of our model.<sup>1</sup> Relative to the environment considered in these articles, our model produces larger TFP losses during the transitional dynamics because the borrowing constraint depends on firm size.

The problems associated with current account deficits and declining productivity growth in the euro area were flagged early on by [Blanchard \(2007\)](#) for the case of Portugal. [Reis \(2013\)](#) suggests that large capital inflows may have been misallocated to inefficient firms in Portugal in the 2000s. [Benigno and Fornaro \(2014\)](#) suggest that the decline in aggregate productivity growth resulted from a shift in resources from the traded sector, which is the source of endogenous productivity growth, to the nontraded sector following the consumption boom that accompanied the increase in capital inflows. In contemporaneous work, [Dias, Marques, and Richmond \(2014\)](#) and [Garcia-Santana et al. \(2016\)](#) present descriptive statistics on trends in resource allocation within sectors, including construction and services, for Portugal (1996–2011) and Spain (1995–2007), respectively.

1. Consistent with our narrative, [CETTE, Fernald, and Mojon \(2016\)](#) provide VAR and panel-data evidence in a sample of European countries and industries linking lower real interest rates to lower productivity in the prerecession period. [Fernandez-Villaverde, Garicano, and Santos \(2013\)](#) also note the decline in interest rates and the inflow of capital fostered by the adoption of the euro and discuss sluggish performance in peripheral countries in the context of abandoned structural reforms. [Buera and Shin \(2016\)](#) study countries undergoing sustained growth accelerations and attribute capital outflows from countries with higher TFP growth to economic reforms that remove idiosyncratic distortions.

## II. DESCRIPTION OF THE DATA

Our data come from the ORBIS database. The database is compiled by the Bureau van Dijk Electronic Publishing (BvD). ORBIS is an umbrella product that provides firm-level data for many countries worldwide. Administrative data at the firm level are initially collected by local Chambers of Commerce and, in turn, relayed to BvD through roughly 40 different information providers including business registers. Given our article's focus, we also use the AMADEUS data set, which is the European subset of ORBIS. One advantage of focusing on European countries is that company reporting is regulatory even for small private firms.

The data set has financial accounting information from detailed harmonized balance sheets, income statements, and profit and loss accounts of firms. Roughly 99% of companies in the data set are private. This crucially differentiates our data from other data sets commonly used in the finance literature, such as Compustat for the United States, Compustat Global, and Worldscope, that only contain information on large listed companies.

Our analysis focuses on the manufacturing sector for which challenges related to the estimation of the production function are less severe than in other sectors. In the countries that we examine, the manufacturing sector accounts for roughly 20% to 30% of aggregate employment and value added. The ORBIS database allows us to classify industries in the manufacturing sector according to their four-digit NACE Rev. 2 industry classification.

A well-known problem in ORBIS-AMADEUS is that, while the number of unique firm identifiers matches the number in official data sources, key variables, such as employment and materials, are missing once the data are downloaded. There are several reasons for this. Private firms are not required to report materials. Additionally, employment is not reported as a balance sheet item but in memo lines. Less often, there are other missing variables such as capital or assets. Variables are not always reported consistently throughout time in a particular disk or in a download, either from the BvD or the Wharton Research Data Services (WRDS) website. BvD has a policy by which firms that do not report during a certain period are automatically deleted from their later vintage products, creating an artificial survival bias in the sample. An additional issue that researchers face is that online downloads (BvD or WRDS) cap the amount of firms that can be downloaded in a given period of time. This cap translates into



missing observations in the actual download job instead of termination of the download job.

We follow a comprehensive data collection process to address these problems and maximize the coverage of firms and variables for our six countries over time.<sup>2</sup> Broadly, our strategy is to merge data available in historical disks instead of downloading historical data at once from the WRDS website. We rely on two BvD products, ORBIS and AMADEUS. These products have been developed independently and, therefore, they follow different rules regarding the companies and years that should be included. AMADEUS provides data for at most 10 recent years for the same company while ORBIS only reports data for up to 5 recent years. In addition, AMADEUS drops firms from the database if they did not report any information during the last five years while ORBIS keeps the information for these companies as long as they are active. We merge data across several vintages of these two products (ORBIS disk 2005, ORBIS disk 2009, ORBIS disk 2013, AMADEUS online 2010 from WRDS, and AMADEUS disk 2014).<sup>3</sup>

Finally, it is sometimes the case that information is updated over time, and the value of variables that was not available in early disks is made available in later vintages. In addition, because of reporting lags the coverage in the latest years of a certain disk can be poor. To maximize the number of firms in the sample and the coverage of variables we merge across all products using a unique firm identifier and we update information missing in early vintages by the value provided in later vintages. An issue when merging data across disks is that there can be changes in firm identifiers over time. We use a table of changes in official identifiers provided by BvD to address this issue.

**Table I** summarizes the coverage in our data for Spain between 1999 and 2012.<sup>4</sup> The columns in the table represent the

2. See also [Kalemli-Özcan et al. \(2015\)](#) for a description of how to use ORBIS to construct representative firm-level data sets for various countries.

3. For example, consider a company that files information with BvD for the last time in 2007. Suppose that BvD has information from the Business Registry that this company is still active. In AMADEUS disk 2014 this company will not be included in the database. However, information for the period 2002–2007 for this company will still be available when we combine ORBIS disks 2005 and 2009.

4. We begin our analysis in 1999 as the coverage in ORBIS-AMADEUS between 1995 and 1998 is, in most cases, extremely low. There is no representative data set with financial information going back to the beginning of the 1990s. The ESEE (Encuesta Sobre Estrategias Empresariales) data set for Spain has the



TABLE I  
 COVERAGE IN ORBIS-AMADEUS RELATIVE TO EUROSTAT (SBS):  
 SPAIN MANUFACTURING

	Employment	Wage bill	Gross output
1999	0.56	0.69	0.75
2000	0.58	0.71	0.76
2001	0.61	0.73	0.77
2002	0.65	0.75	0.79
2003	0.65	0.74	0.78
2004	0.66	0.75	0.78
2005	0.66	0.74	0.77
2006	0.67	0.74	0.77
2007	0.67	0.74	0.77
2008	0.65	0.72	0.72
2009	0.71	0.72	0.75
2010	0.68	0.73	0.74
2011	0.69	0.74	0.75
2012	0.65	0.71	0.72

ratio of aggregate employment, wage bill, and gross output recorded in our sample relative to the same object in Eurostat as reported by its Structural Business Statistics (SBS). The data in Eurostat are from census sources and represent the universe of firms. The coverage statistics we report are conservative because we drop observations with missing, zero, or negative values for gross output, the wage bill, capital stock, and materials, that is the variables necessary for computing productivity at the firm level.<sup>5</sup> As [Table I](#) shows, the coverage in our sample is consistently high and averages roughly 75% for the wage bill and gross output and typically more than 65% for employment.<sup>6</sup>

[Figure I](#) plots the aggregate real wage bill and the aggregate real gross output in our ORBIS-AMADEUS data set. It compares these aggregates to the same aggregates as recorded by Eurostat. Except for the wage bill in the first two years of the sample, these series track each other closely.

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required variables beginning in 1993 but surveys mostly large firms and therefore is not representative of the population of firms.

5. [Online Appendix A](#) provides a detailed description of the process we follow to clean the data and presents summary statistics of the main variables used in our analysis. It also presents coverage statistics for the other countries.

6. A difference between our sample and Eurostat is that we do not have data on the self-employed. While this has little impact on our coverage of the wage bill and gross output relative to Eurostat, it matters more for employment.

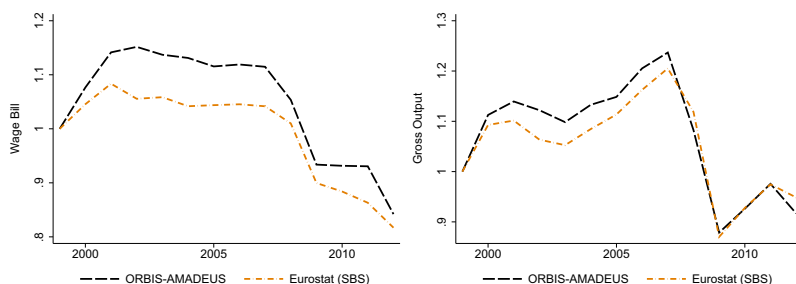


FIGURE I

Aggregates in ORBIS-AMADEUS and Eurostat (SBS)

TABLE II

SHARE OF TOTAL MANUFACTURING ECONOMIC ACTIVITY BY SIZE CLASS IN SPAIN (2006)

		Employment	Wage bill	Gross output
ORBIS-AMADEUS	1–19 employees	0.24	0.19	0.14
	20–249 employees	0.50	0.47	0.42
	250+ employees	0.26	0.34	0.45
Eurostat (SBS)	0–19 employees	0.31	0.20	0.14
	20–249 employees	0.43	0.43	0.38
	250+ employees	0.26	0.37	0.49

Table II presents the share of economic activity accounted for by firms belonging in three size categories in 2006.<sup>7</sup> Each column presents a different measure of economic activity, namely employment, wage bill, and gross output. The first three rows report statistics from ORBIS-AMADEUS and the next three from Eurostat. The entries in the table denote the fraction of total economic activity accounted for by firms belonging to each size class. For example, in our data from ORBIS-AMADEUS, firms with 1–19 employees account for 19% of the total wage bill, firms with 20–249 employees account for 47% of the total wage bill, and firms with 250 or more employees account for 34% of the total wage bill. The corresponding numbers provided by Eurostat's SBS are 20%, 43%, and 37%.

7. The share of economic activity by size category in our sample relative to Eurostat is relatively stable over time. We show year 2006 in Table II for comparability with our analyses of other countries below that also start in 2006. The sum of entries across rows within each panel and source may not add up to 1 because of rounding.

Our sample is mainly composed of small and medium-sized firms that account for a significant fraction of economic activity in Europe and the majority of economic activity in the South. [Table II](#) illustrates that our sample is broadly representative in terms of contributions of small and medium-sized firms to manufacturing employment, the wage bill, and gross output. This feature is an important difference of our article relative to the literature that works with both financial and real variables at the firm level. Most of this literature focuses on listed firms that account for less than 1% of the observations in our data.

### III. DISPERSION AND MISALLOCATION FACTS

In this section we document the evolution of measures of dispersion and misallocation for the manufacturing sector in Spain. We build our measurements on the framework developed by [Hsieh and Klenow \(2009\)](#). We consider an industry  $s$  at time  $t$  populated by a large number  $N_{st}$  of monopolistically competitive firms.<sup>8</sup> We define industries in the data by their four-digit industry classification.

Total industry output is given by a CES production function:

$$(1) \quad Y_{st} = \left[ \sum_{i=1}^{N_{st}} D_{ist} (y_{ist})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where  $y_{ist}$  denotes firm  $i$ 's real output,  $D_{ist}$  denotes a demand shifter for firm  $i$ 's variety, and  $\varepsilon$  denotes the elasticity of substitution between varieties. We denote by  $p_{ist}$  the price of variety  $i$  and by  $P_{st}$  the price of industry output  $Y_{st}$ . Firms face an isoelastic demand for their output given by  $y_{ist} = \left(\frac{p_{ist}}{P_{st}}\right)^{-\varepsilon} (D_{ist})^{\varepsilon} Y_{st}$ .

Firms' output is given by a Cobb-Douglas production function:

$$(2) \quad y_{ist} = A_{ist} k_{ist}^{\alpha} \ell_{ist}^{1-\alpha},$$

8. In our analysis we model entrepreneurs as single-plant firms. In the ESEE data set for Spain that generally covers only large firms, we find that firms with more than a single plant constitute roughly 15% of all firms in the data. Importantly, there is no time series variation in this share. Given that large firms tend to have more plants than small firms, we expect the share of multiplant firms to be even smaller in our data set.

where  $k_{ist}$  is capital,  $\ell_{ist}$  is labor,  $A_{ist}$  is physical productivity, and  $\alpha$  is the elasticity of output with respect to capital. As a baseline and for comparability with our dynamic model below that features a single sector we set  $\alpha = 0.35$  for all industries, corresponding to the average capital share in a relatively undistorted economy such as the United States. Our measures of dispersion of factor returns are not affected by the assumption that  $\alpha$  is homogeneous across industries because these measures use within-industry variation of firm outcomes. In [Online Appendix B](#) we show that our estimated trends in TFP losses do not change meaningfully when using either Spanish or U.S. factor shares to construct elasticities  $\alpha_{s,t}$  that vary by sector and time.

We measure firm nominal value added,  $p_{ist}y_{ist}$ , as the difference between gross output (operating revenue) and materials. We measure real output,  $y_{ist}$ , as nominal value added divided by an output price deflator. Given that we do not observe prices at the firm level, we use gross output price deflators from Eurostat at the two-digit industry level. We measure the labor input,  $\ell_{ist}$ , with the firm's wage bill deflated by the same industry price deflator. We use the wage bill instead of employment as our measure of  $\ell_{ist}$  to control for differences in the quality of the workforce across firms. We measure the capital stock,  $k_{ist}$ , with the book value of fixed assets and deflate this value with the price of investment goods.<sup>9</sup> In fixed assets we include both tangible and intangible fixed assets.<sup>10</sup>

9. Deflating fixed assets matters for our results only through our measures of capital and TFP at the aggregate level. We choose to deflate the book value of fixed assets because in this article we are interested in measuring changes (rather than levels) of capital and TFP. Changes in book values across two years reflect to a large extent purchases of investment goods valued at current prices. For plots that cover the whole sample period until 2012, we use country-specific prices of investment from the World Development Indicators to deflate the book value of fixed assets, as we do not have industry-specific prices of investment goods for the whole sample period. For our quantitative application to Spain between 1999 and 2007, we construct a manufacturing-specific investment deflator based on the prices of investment goods for eight types of assets provided from KLEMS.

10. Our results do not change in any meaningful way if we measure  $k_{ist}$  with the book value of tangible fixed assets, with one exception. In 2007 there was a change in the accounting system in Spain and leasing items that until 2007 had been part of intangible fixed assets were from 2008 included under tangible fixed assets. If we measure  $k_{ist}$  with tangible fixed assets, we observe an important discontinuity in some of our dispersion measures in Spain between 2007 and 2008 that is entirely driven by this accounting convention.

Denoting the inverse demand function by  $p(y_{ist})$ , firms choose their price, capital, and labor to maximize their profits:

$$(3) \quad \max_{p_{ist}, k_{ist}, \ell_{ist}} \Pi_{ist} = (1 - \tau_{ist}^y) p(y_{ist}) y_{ist} - (1 + \tau_{ist}^k) (r_t + \delta_{st}) k_{ist} - w_{st} \ell_{ist},$$

where  $w_{st}$  denotes the wage,  $r_t$  denotes the real interest rate,  $\delta_{st}$  denotes the depreciation rate,  $\tau_{ist}^y$  denotes a firm-specific wedge that distorts output, and  $\tau_{ist}^k$  denotes a firm-specific wedge that distorts capital relative to labor. For now we treat wedges as exogenous and endogenize them later in the model of Section IV.

The first-order conditions with respect to labor and capital are given by:

$$(4) \quad \text{MRPL}_{ist} := \left( \frac{1 - \alpha}{\mu} \right) \left( \frac{p_{ist} y_{ist}}{\ell_{ist}} \right) = \left( \frac{1}{1 - \tau_{ist}^y} \right) w_{st},$$

$$(5) \quad \text{MRPK}_{ist} := \left( \frac{\alpha}{\mu} \right) \left( \frac{p_{ist} y_{ist}}{k_{ist}} \right) = \left( \frac{1 + \tau_{ist}^k}{1 - \tau_{ist}^y} \right) (r_t + \delta_{st}),$$

where  $\mu = \frac{\varepsilon}{\varepsilon - 1}$  denotes the constant markup of price over marginal cost. Equation (4) states that firms set the marginal revenue product of labor (MRPL) equal to the wage times the wedge  $\frac{1}{1 - \tau_{ist}^y}$ . Similarly, in equation (5) firms equate the marginal revenue product of capital (MRPK) to the cost of capital times the wedge  $\frac{1 + \tau_{ist}^k}{1 - \tau_{ist}^y}$ . With the Cobb-Douglas production function, the marginal revenue product of each factor is proportional to the factor's revenue-based productivity.

Following the terminology used in Foster, Haltiwanger, and Syverson (2008) and Hsieh and Klenow (2009), we define the revenue-based total factor productivity (TFPR) at the firm level as the product of price  $p_{ist}$  times physical productivity  $A_{ist}$ :

$$(6) \quad \text{TFPR}_{ist} := p_{ist} A_{ist} = \frac{p_{ist} y_{ist}}{k_{ist}^\alpha \ell_{ist}^{1-\alpha}} = \mu \left( \frac{\text{MRPK}_{ist}}{\alpha} \right)^\alpha \left( \frac{\text{MRPL}_{ist}}{1 - \alpha} \right)^{1-\alpha}.$$

Firms with higher output distortions  $\tau_{ist}^y$  or higher capital relative to labor distortions  $\tau_{ist}^k$  have higher marginal revenue products and, as equation (6) shows, a higher  $\text{TFPR}_{ist}$ .

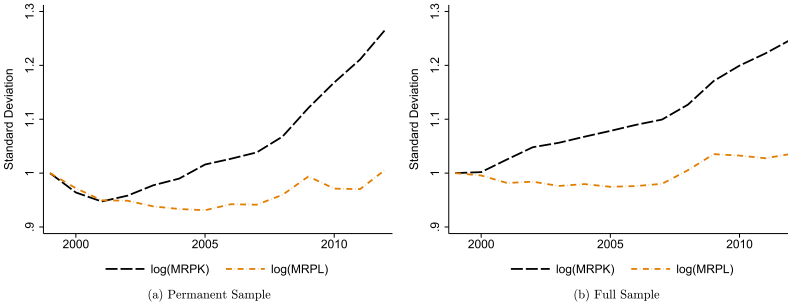


FIGURE II

Evolution of MRPK and MRPL Dispersion

In this economy, resources are allocated optimally when all firms face the same (or no) distortions in output ( $\tau_{ist}^y = \tau_{st}^y$ ) and capital relative to labor ( $\tau_{ist}^k = \tau_{st}^k$ ). In that case, more factors are allocated to firms with higher productivity  $A_{ist}$  or higher demand shifter  $D_{ist}$ , but there is no dispersion of the returns to factors, that is the MRPL and the MRPK are equalized across firms.<sup>11</sup> On the other hand, the existence of idiosyncratic distortions,  $\tau_{ist}^y$  and  $\tau_{ist}^k$ , leads to a dispersion of marginal revenue products and a lower sectoral TFP.

In Figure II we present the evolution of the dispersion of the  $\log(\text{MRPK})$  and  $\log(\text{MRPL})$  in Spain. To better visualize the relative changes over time, we normalize these measures to 1 in the first sample year. The left panel is based on the subset of firms that are continuously present in our data. We call this subset of firms the “permanent sample.” The right panel is based on the “full sample” of firms. The full sample includes firms that enter or exit from the sample in various years and, therefore, comes closer to matching the coverage of firms observed in Eurostat.<sup>12</sup>

11. Without idiosyncratic distortions,  $\text{TFPR}_{ist} = p_{ist}A_{ist}$  is equalized across firms since  $p_{ist}$  is inversely proportional to physical productivity  $A_{ist}$  and does not depend on the demand shifter  $D_{ist}$ . This also implies that capital-labor ratios are equalized across firms.

12. We calculate that in 2000 the entry rate among firms with at least one employee is 6.5%. The entry rate declines over time to 2% by the end of our sample. These numbers match closely the entry rates calculated from Eurostat. Our permanent sample of firms differs from the full sample both because of real entry and exit and because firms with missing reporting in at least one year are excluded from the permanent sample but are included in the full sample during

The time series of the dispersion measures are computed in two steps. First, we calculate a given dispersion measure across firms  $i$  in a given industry  $s$  and year  $t$ . Second, for each year we calculate dispersion for the manufacturing sector as the weighted average of dispersions across industries  $s$ . Each industry is given a time-invariant weight equal to its average share in manufacturing value added. We always use the same weights when aggregating across industries. Therefore, all of our estimates reflect purely variation within four-digit industries over time.

Figure II shows a large increase in the standard deviation of log(MRPK) over time. With the exception of the first two years in the permanent sample, we always observe increases in the dispersion of the log(MRPK). The increase in the dispersion of the log(MRPK) accelerates during the postcrisis period between 2008 and 2012. We emphasize that we do not observe similar trends in the standard deviation of log(MRPL).<sup>13</sup> The striking difference between the evolution of the two dispersion measures argues against the importance of changing distortions that affect both capital and labor at the same time. For example, this finding is not consistent with heterogeneity in price markups driving trends in dispersion because such an explanation would cause similar changes to the dispersion of both the log(MRPK) and the log(MRPL).<sup>14</sup> Finally, we note that while we use standard deviations of logs to represent dispersion, we obtain similar results when we measure dispersion with either the 90-10 or the 75-25 ratio.

The framework of Hsieh and Klenow (2009) that we adopt for measuring trends in the dispersion of returns to factors relies on the Cobb-Douglas production function. Under a Cobb-Douglas production function, an increasing dispersion of the log(MRPK) together with stable dispersion of the log(MRPL) implies that the covariance between  $\log(\text{TFPR})$  and  $\log(\frac{k}{l})$  across firms is

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years with nonmissing reporting. See [Online Appendix A](#) for more details on the construction of the two samples.

13. We obtain a similar result if we use employment instead of the wage bill to measure  $\ell_{ist}$ .

14. The relationship between markups and misallocation has been recently the focus of articles such as [Fernald and Neiman \(2011\)](#) and [Peters \(2013\)](#).



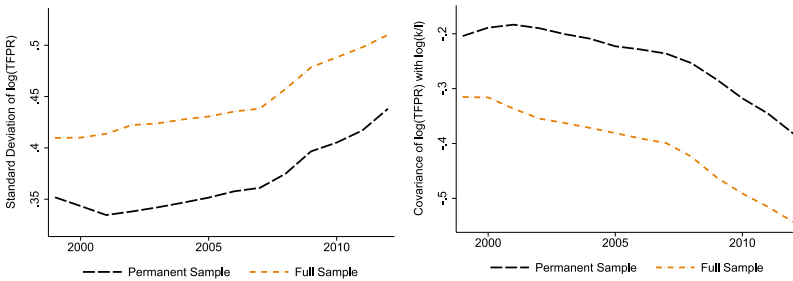


FIGURE III  
TFPR Moments

decreasing over time. To see this point, write:

$$(7) \quad \begin{aligned} \text{Var}(\text{mrpk}) &= \text{Var}(\text{tfpr}) + (1 - \alpha)^2 \text{Var}\left(\log\left(\frac{k}{\ell}\right)\right) \\ &\quad - 2(1 - \alpha) \text{Cov}\left(\text{tfpr}, \log\left(\frac{k}{\ell}\right)\right), \end{aligned}$$

$$(8) \quad \begin{aligned} \text{Var}(\text{mrpl}) &= \text{Var}(\text{tfpr}) + \alpha^2 \text{Var}\left(\log\left(\frac{k}{\ell}\right)\right) \\ &\quad + 2\alpha \text{Cov}\left(\text{tfpr}, \log\left(\frac{k}{\ell}\right)\right), \end{aligned}$$

where we define  $\text{mrpk} = \log(\text{MRPK})$ ,  $\text{mrpl} = \log(\text{MRPL})$ , and  $\text{tfpr} = \log(\text{TFPR})$ . Figure III confirms that the dispersion of  $\text{tfpr}$  is increasing over time and that the covariance between  $\text{tfpr}$  and  $\log\left(\frac{k}{\ell}\right)$  is decreasing over time. The variance of the log capital-labor ratio (the second term) is also increasing over time.

We now discuss measures of productivity and misallocation. Total factor productivity at the industry level is defined as the wedge between industry output and an aggregator of industry inputs,  $\text{TFP}_{st} := \frac{Y_{st}}{K_{st}^\alpha L_{st}^{1-\alpha}}$ , where  $K_{st} = \sum_i k_{ist}$  is industry capital and

$L_{st} = \sum_i \ell_{ist}$  is industry labor. We can write TFP as:<sup>15</sup>

$$\begin{aligned}
 \text{TFP}_{st} &= \frac{Y_{st}}{K_{st}^\alpha L_{st}^{1-\alpha}} = \frac{\overline{\text{TFPR}}_{st}}{P_{st}} \\
 (9) \quad &= \left[ \sum_i \left( \underbrace{(D_{ist})^{\frac{\varepsilon}{\varepsilon-1}} A_{ist}}_{Z_{ist}} \frac{\overline{\text{TFPR}}_{st}}{\text{TFPR}_{ist}} \right)^{\varepsilon-1} \right]^{\frac{1}{\varepsilon-1}}.
 \end{aligned}$$

We note that for our results it is appropriate to only track a combination of demand and productivity at the firm level. From now on we call “firm productivity,”  $Z_{ist} = (D_{ist})^{\frac{\varepsilon}{\varepsilon-1}} A_{ist}$ , a combination of firm productivity and the demand shifter.

To derive a measure that maps the allocation of resources to TFP performance, we follow [Hsieh and Klenow \(2009\)](#) and define the “efficient” level of TFP as the TFP level we would observe in the first-best allocation in which there is no dispersion of the MRPK, MRPL, and TFPR across firms. Plugging  $\text{TFPR}_{ist} = \overline{\text{TFPR}}_{st}$  into [equation \(9\)](#), we see that the efficient level of TFP is given by

$$\text{TFP}_{st}^e = \left[ \sum_i Z_{ist}^{\varepsilon-1} \right]^{\frac{1}{\varepsilon-1}}.$$

The difference in  $\log(\text{TFP})$  arising from misallocation,  $\Lambda_{st} = \log(\text{TFP}_{st}) - \log(\text{TFP}_{st}^e)$ , can be expressed as:

$$\begin{aligned}
 \Lambda_{st} &= \frac{1}{\varepsilon-1} \left[ \log \left( \mathbb{E}_i Z_{ist}^{\varepsilon-1} \mathbb{E}_i \left( \frac{\overline{\text{TFPR}}}{\text{TFPR}_{ist}} \right)^{\varepsilon-1} \right) \right. \\
 (10) \quad &+ \left. \text{Cov}_i \left( Z_{ist}^{\varepsilon-1}, \left( \frac{\overline{\text{TFPR}}}{\text{TFPR}_{ist}} \right)^{\varepsilon-1} \right) \right] - \frac{1}{\varepsilon-1} \log \left( \mathbb{E}_i Z_{ist}^{\varepsilon-1} \right).
 \end{aligned}$$

15. To derive [equation \(9\)](#), we substitute into the definition of TFP the industry price index  $P_{st} = (\sum_i (D_{ist})^\varepsilon (p_{ist})^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}$ , firms’ prices  $p_{ist} = \frac{\text{TFPR}_{ist}}{A_{ist}}$ , and an industry-level TFPR measure,  $\overline{\text{TFPR}}_{st} = \frac{P_{st} Y_{st}}{K_{st}^\alpha L_{st}^{1-\alpha}}$ . [Equation \(9\)](#) is similar to the one derived in [Hsieh and Klenow \(2009\)](#), except for the fact that we also allow for idiosyncratic demand shifters  $D_{ist}$ .

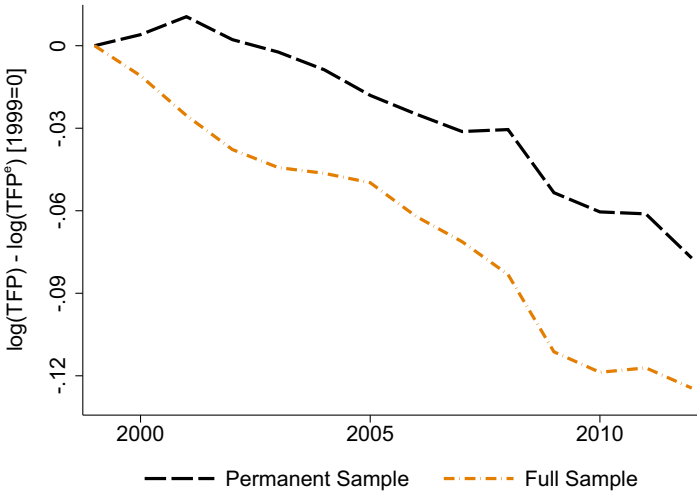


FIGURE IV  
Evolution of TFP Relative to Efficient Level

To construct this measure of misallocation, we need estimates of  $Z_{ist}$ . Employing the structural assumptions on demand and production used to arrive at equation (10), we estimate firm productivity as:<sup>16</sup>

$$(11) \quad \tilde{Z}_{ist} = \left( \frac{(P_{st} Y_{st})^{-\frac{1}{\varepsilon-1}}}{P_{st}} \right) \left( \frac{(p_{ist} y_{ist})^{\frac{\varepsilon}{\varepsilon-1}}}{k_{ist}^\alpha \ell_{ist}^{1-\alpha}} \right),$$

where  $p_{ist} y_{ist}$  denotes firm nominal value added and  $P_{st} Y_{st} = \sum_i p_{ist} y_{ist}$  denotes industry nominal value added.

Figure IV plots changes relative to 1999 in the difference in log(TFP) relative to its efficient level. For comparability with Hsieh and Klenow (2009), we use an elasticity of substitution between varieties equal to  $\varepsilon = 3$ . As with our measures of dispersion, we first estimate the difference  $\Lambda_{st}$  within every industry  $s$  and then use the same time-invariant weights to aggregate across industries. Between 1999 and 2007, we document declines in TFP

16. To derive equation (11), first use the production function to write  $\tilde{Z}_{ist} = A_{ist} D_{ist}^{\frac{\varepsilon}{\varepsilon-1}} = \frac{D_{ist}^{\frac{\varepsilon}{\varepsilon-1}} y_{ist}}{k_{ist}^\alpha \ell_{ist}^{1-\alpha}}$ . Then, from the demand function substitute in  $D_{ist}^{\frac{\varepsilon}{\varepsilon-1}} = \left( \frac{P_{ist}}{P_{st}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left( \frac{y_{ist}}{Y_{st}} \right)^{\frac{1}{\varepsilon-1}}$ .

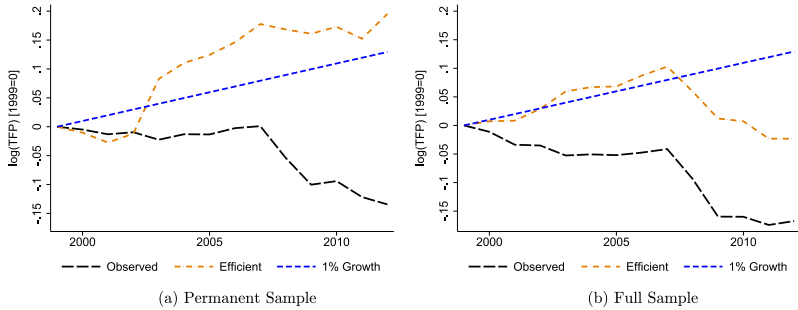


FIGURE V

Evolution of Observed TFP Relative to Benchmarks

relative to its efficient level of roughly 3 percentage points in the permanent sample and 7 percentage points in the full sample. By the end of the sample in 2012, we observe declines in TFP relative to its efficient level of roughly 7 percentage points in the permanent sample and 12 percentage points in the full sample.<sup>17</sup>

In Figure V we plot changes in manufacturing log(TFP) in the data. We measure  $\log(\text{TFP})$  for each industry as  $\log(\text{TFP}_{st}) = \log(\sum_i y_{ist}) - \alpha \log(K_{st}) - (1 - \alpha)\log(L_{st})$  and use the same time-invariant weights to aggregate across industries  $s$ . Manufacturing TFP could be changing over time for reasons other than changes in the allocation of resources (for example, labor hoarding, capital utilization, entry, and technological change). We therefore compare observed  $\log(\text{TFP})$  in the data to two baseline  $\log(\text{TFP})$  paths. The first path is the efficient path implied by the model,  $\log(\text{TFP}_{st}^e) = \left(\frac{1}{\varepsilon - 1}\right) \left(\log(N_{st}) + \log\left(\mathbb{E}_i \tilde{Z}_{ist}^{\varepsilon - 1}\right)\right)$ . The second path corresponds to a hypothetical scenario in which TFP grows at a constant rate of 1% per year. Figure V documents that observed  $\log(\text{TFP})$  lies below both baseline paths. Our loss measures in Figure IV suggest that an increase in the misallocation

17. The 1999 level of the difference  $\Lambda_{st}$  is roughly  $-0.21$  in the permanent sample and  $-0.28$  in the full sample. We also note that for an elasticity  $\varepsilon = 5$  we obtain declines of roughly 4 and 10 percentage points for the permanent and the full sample between 1999 and 2007 and declines of roughly 13 and 19 percentage points between 1999 and 2012. For an elasticity  $\varepsilon = 5$ , the 1999 level of  $\Lambda_{st}$  is roughly  $-0.36$  in the permanent sample and  $-0.46$  in the full sample.

of resources across firms is related to the observed lower productivity performance relative to these benchmarks.<sup>18</sup>

To explain the joint trends in MRPK dispersion and TFP relative to its efficient level, our model relates the decline in the real interest rate to inflows of capital that are directed to some less productive firms. We now present some first evidence supporting this narrative. It is useful to express the dispersion of the log (MRPK) in terms of dispersions in firm log productivity and log capital and the covariance between these two:

$$(12) \quad \begin{aligned} \text{Var}_i(\log \text{MRPK}_{ist}) &= \gamma_1 \text{Var}_i(\log Z_{ist}) + \gamma_2 \text{Var}_i(\log k_{ist}) \\ &\quad - \gamma_3 \text{Cov}_i(\log Z_{ist}, \log k_{ist}), \end{aligned}$$

for some positive coefficients  $\gamma$ 's.<sup>19</sup> Loosely, [equation \(12\)](#) says that the dispersion of the log (MRPK) increases if capital becomes more dispersed across firms for reasons unrelated to their underlying productivity. More formally, holding constant  $\text{Var}_i(\log Z_{ist})$ , an increase in  $\text{Var}_i(\log k_{ist})$  or a decrease in  $\text{Cov}_i(\log Z_{ist}, \log k_{ist})$  is associated with higher  $\text{Var}_i(\log \text{MRPK}_{ist})$ .

The left panel of [Figure VI](#) shows an increase in the dispersion of capital over time. The right panel shows the unconditional correlation between firm productivity (as estimated by  $\hat{Z}_{ist}$ ) and capital in the cross section of firms. In general, more productive firms invest more in capital. However, the correlation between productivity and capital declines significantly over time. This fact

18. The path of model-based TFP, as constructed in the last part of [equation \(9\)](#), does not in general coincide with the path of “Observed” TFP in [Figure V](#). We make use of the CES aggregator to move from the definition of TFP as a wedge between output and an aggregator of inputs to the last part of [equation \(9\)](#). The divergence between the two series is a measurement issue because “Observed” TFP does not use the CES aggregator or the price index. We use [Figure V](#) only to show that a measure of TFP in the data lies below some benchmarks and do not wish to make any quantitative statements about allocative efficiency based on this figure. Finally, we note that in [Figure V](#) the larger increase in  $\log(\text{TFP}_{st}^c)$  in the permanent sample relative to the full sample is explained by the fact that the latter includes new entrants that typically have lower productivity.

19. The coefficients are given by  $\gamma_1 = \left(\frac{\varepsilon-1}{1+\alpha(\varepsilon-1)}\right)^2$ ,  $\gamma_2 = \left(\frac{1}{1+\alpha(\varepsilon-1)}\right)^2$ , and  $\gamma_3 = \frac{2(\varepsilon-1)}{(1+\alpha(\varepsilon-1))^2}$ . [Equation \(12\)](#) is derived by substituting the solution for labor  $\ell_{ist}$  into the definition of MRPK and treating the choice of  $k_{ist}$  as given. In our model we justify treating  $k_{ist}$  as a predetermined variable with a standard technology that implies that investment becomes productive after one period.

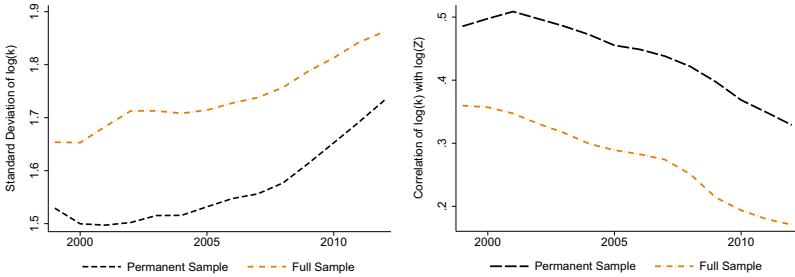


FIGURE VI  
Log Capital Moments

suggests that capital inflows may have been allocated inefficiently to less productive firms.<sup>20</sup>

#### IV. MODEL OF MRPK DISPERSION, TFP, AND CAPITAL FLOWS

To evaluate quantitatively the role of capital misallocation for TFP in an environment with declining real interest rates, we consider a small open economy populated by a large number of infinitely lived firms  $i = 1, \dots, N$  that produce differentiated varieties of manufacturing goods. The three elements of the model that generate dispersion of the MRPK across firms are a borrowing constraint that depends on firm size, risk in capital accumulation, and capital adjustment costs. Motivated by the fact that we did not find significant trends in the MRPL dispersion in the data, in our baseline model there is no MRPL dispersion across firms. We allow for MRPL dispersion in an extension of the baseline model.

##### IV.A. Firms' Problem

Firms produce output with a Cobb-Douglas production function  $y_{it} = Z_{it} k_{it}^\alpha \ell_{it}^{1-\alpha}$ , where  $Z_{it}$  is firm productivity,  $k_{it}$  is the capital stock, and  $\ell_{it}$  is labor. Labor is hired in a competitive market at an exogenous wage  $w_t$ . Varieties of manufacturing goods are supplied monopolistically to the global market. Each firm faces a downward

20. We present the correlation between log productivity and log capital to make the interpretation of the figure transparent. Both the covariance between log productivity and log capital and the elasticity of capital with respect to productivity are also generally decreasing. The  $\text{Var}_i(\log Z_{ist})$  is decreasing until 2007 and then it increases.

sloping demand function for its product,  $y_{it} = p_{it}^{-\varepsilon}$ , where  $p_{it}$  is the price of the differentiated product and  $\varepsilon$  is the absolute value of the elasticity of demand. We denote by  $\mu = \frac{\varepsilon}{\varepsilon-1}$  the markup of price over marginal cost.<sup>21</sup>

Firms can save and borrow in a bond traded in the international credit market at an exogenous real interest rate  $r_t$ . Denoting by  $\beta$  the discount factor, firms choose consumption of tradeables  $c_{it}$ , debt  $b_{it+1}$ , investment  $x_{it}$ , labor  $\ell_{it}$ , and the price  $p_{it}$  of their output to maximize the expected value of the sum of discounted utility flows:

$$(13) \quad \max_{\{c_{it}, b_{it+1}, x_{it}, \ell_{it}, p_{it}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{it}).$$

The utility function is given by  $U(c_{it}) = \frac{c_{it}^{1-\gamma} - 1}{1-\gamma}$ , where  $\gamma$  denotes the inverse of the elasticity of intertemporal substitution. This maximization problem is subject to the sequence of budget constraints:

$$(14) \quad c_{it} + x_{it} + (1 + r_t)b_{it} + \frac{\psi (k_{it+1} - k_{it})^2}{2k_{it}} = p_{it}y_{it} - w_t\ell_{it} + b_{it+1},$$

and the capital accumulation equation:

$$(15) \quad k_{it+1} = (1 - \delta)k_{it} + x_{it},$$

where  $\delta$  denotes the depreciation rate of capital. Firms face quadratic costs of adjusting their capital. The parameter  $\psi$  in the budget constraint controls for the magnitude of these costs.

Firms own the capital stock and augment it through investment. This setup differs from the setup in [Hsieh and Klenow \(2009\)](#) where firms rent capital in a static model. We do not adopt the convenient assumption in [Moll \(2014\)](#), [Midrigan and Xu \(2014\)](#), and [Buera and Moll \(2015\)](#) that exogenous shocks during period  $t + 1$  are known at the end of  $t$  before capital and borrowing

21. Since our model is partial equilibrium, we normalize both the demand shifter and the sectoral price index to 1 in the demand function  $y_{it} = p_{it}^{-\varepsilon}$ . It is appropriate to abstract from the determination of the sectoral price index because manufacturing in a small open economy accounts for a small fraction of global manufacturing production. Similarly to our analysis in [Section III](#), we call a combination of idiosyncratic productivity and demand “firm productivity” and denote it by  $Z_{it}$ .



decisions are made for  $t + 1$ . This timing assumption effectively renders the choice of capital static and generates an equivalence with the environment in [Hsieh and Klenow \(2009\)](#). Instead, in our model firms face idiosyncratic investment risk which makes capital and debt imperfect substitutes in firms' problem. Risk in capital accumulation is an additional force generating MRPK dispersion across firms in the model.

The main novelty of our model is to introduce a borrowing constraint that depends on firm size.<sup>22</sup> The amount of debt that firms can borrow is constrained by:

$$(16) \quad b_{it+1} \leq \theta_0 k_{it+1} + \theta_1 \Psi(k_{it+1}) = \underbrace{\left[ \theta_0 + \theta_1 \frac{\Psi(k_{it+1})}{k_{it+1}} \right]}_{\theta(k_{it+1})} k_{it+1},$$

where  $\Psi(k) = \exp(k) - 1$  is an increasing and convex function of capital and  $\theta_0$  and  $\theta_1$  are parameters characterizing the borrowing constraint. In [Online Appendix C](#) we write explicitly a model that yields the [constraint \(16\)](#) from the requirement that firms do not default in equilibrium. In this microfoundation, the  $\Psi(\cdot)$  function denotes an increasing and convex cost that firms incur from the disruption of their productive capacity if they decide to default.

The [constraint \(16\)](#) nests the standard model in the literature ([Midrigan and Xu 2014](#); [Moll 2014](#); [Buera and Moll 2015](#)) when  $\theta_1 = 0$ . In this case the maximum fraction of capital that can be borrowed,  $\theta(k_{it+1}) = \theta_0$ , is exogenous. Because  $\Psi(\cdot)$  is a convex function, a positive value for  $\theta_1$  implies that larger firms are more leveraged. We discipline the value of  $\theta_1$  from the positive cross-sectional relationship between leverage  $\frac{b_{it+1}}{k_{it+1}}$  and firm size that we find in our data. A key finding of our analysis is that a size-dependent borrowing constraint, with larger firms being more leveraged, is crucial for the ability of the model to account for the cross-sectional patterns of the return to capital in the data.<sup>23</sup>

22. [Guner, Ventura, and Xu \(2008\)](#) examine the effects of size-dependent input taxes on the size distribution of firms and argue that such taxes significantly reduce steady state capital accumulation and TFP.

23. [Arellano, Bai, and Zhang \(2012\)](#) also document a positive cross-sectional relationship between firm leverage and size for less financially developed European countries. In a sample of U.S. manufacturing firms with access to corporate bond markets, [Gilchrist, Sim, and Zakrajšek \(2013\)](#) document that larger firms face lower borrowing costs. In the European survey on the access to finance of enterprises ([European Central Bank 2013](#)), small and medium-sized firms were

We write firm productivity  $Z_{it} = Z_t^A z_i^P \exp(z_{it}^T)$  as the product of an aggregate effect  $Z_t^A$ , an idiosyncratic permanent effect  $z_i^P$ , and an idiosyncratic transitory effect  $z_{it}^T$ . Idiosyncratic transitory productivity follows an AR(1) process in logs:

$$(17) \quad z_{it}^T = -\frac{\sigma^2}{2(1 + \rho)} + \rho z_{it-1}^T + \sigma u_{it}^z, \quad \text{with } u_{it}^z \sim N(0, 1).$$

In [equation \(17\)](#),  $\rho$  parameterizes the persistence of the process and  $\sigma$  denotes the standard deviation of idiosyncratic productivity shocks  $u_{it}^z$ . The constant term in [equation \(17\)](#) normalizes the mean of transitory productivity to  $\mathbb{E} \exp(z_{it}^T) = 1$  for any choice of  $\rho$  and  $\sigma$ .

We define firm net worth in period  $t$  as  $a_{it} := k_{it} - b_{it} \geq 0$ . Using primes to denote next-period variables and denoting by  $\mathbf{X}$  the vector of exogenous aggregate shocks, we rewrite the firm's problem in recursive form as:

$$(18) \quad V(a, k, z^P, z^T, \mathbf{X}) = \max_{a', k', \ell, p} \{U(c) + \beta \mathbb{E}V(a', k', z^P, (z^T)', \mathbf{X}')\},$$

subject to the budget constraint:

$$(19) \quad c + a' + \frac{\psi (k' - k)^2}{2k} = p(y)y - w\ell - (r + \delta)k + (1 + r)a,$$

the production function  $y = Zk^\alpha \ell^{1-\alpha}$  and the demand function  $y = p^{-\varepsilon}$ . The borrowing constraint becomes:

$$(20) \quad k' \leq \lambda_0 a' + \lambda_1 \Psi(k') = \underbrace{\left[ \lambda_0 + \lambda_1 \frac{\Psi(k')}{a'} \right]}_{\lambda(k', a')} a',$$

where  $\lambda_0 = \frac{1}{1-\theta_0}$  and  $\lambda_1 = \frac{\theta_1}{1-\theta_0}$  with  $\lambda_0 + \lambda_1 \geq 1$ . The reformulation of the borrowing constraint in [equation \(20\)](#) shows that large firms are less constrained by their net worth in accumulating capital than small firms. The standard model is nested by [equation \(20\)](#) under  $\lambda_1 = 0$ , in which case  $\lambda(k', a') = \lambda_0$  becomes exogenous.

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more likely than larger firms to mention access to finance as one of their most pressing problems. In addition to disruptions in productive capacity that increase in size, there may be other reasons why larger firms have easier access to finance. [Khwaja and Mian \(2005\)](#) show that politically connected firms receive preferential treatment from government banks and [Johnson and Mitton \(2003\)](#) present evidence that ties market values of firms to political connections and favoritism.

IV.B. Dispersion in Factor Returns

We first solve for labor  $\ell$  for a given state vector  $(a, k, z^P, z^T, \mathbf{X})$ . Labor demand is given by:

$$(21) \quad \ell = Z^{\frac{\varepsilon-1}{1+\alpha(\varepsilon-1)}} \mu^{\frac{-\varepsilon}{1+\alpha(\varepsilon-1)}} \left( \frac{w}{1-\alpha} \right)^{\frac{-\varepsilon}{1+\alpha(\varepsilon-1)}} k^{\frac{\alpha(\varepsilon-1)}{1+\alpha(\varepsilon-1)}}.$$

Labor is increasing in capital  $k$  and productivity  $Z$  and decreasing in the wage  $w$  and the markup  $\mu$ .<sup>24</sup> The marginal revenue product of labor is equalized across firms, that is for any firm with a state vector  $(a, k, z^P, z^T, \mathbf{X})$  we obtain  $\text{MRPL} := \frac{(1-\alpha)p\gamma}{\mu\ell} = w$ . Therefore, the allocation of labor across firms is undistorted.

In general, the MRPK is not equalized across firms. We define  $\text{MRPK} := \frac{\alpha p\gamma}{\mu k} := (1 + \tau^k)(r + \delta)$ , where  $\tau^k$  denotes the percent deviation of MRPK from  $r + \delta$ . To illustrate the sources of MRPK dispersion, denote by  $\chi$  the multiplier on the borrowing constraint (20) and by  $\text{AC} = \frac{\psi(k'-k)^2}{2k}$  the adjustment cost technology and consider the first-order condition with respect to capital for a firm characterized by some state vector  $(a, k, z^P, z^T, \mathbf{X})$ :

$$(22) \quad \mathbb{E} \left[ \frac{\beta U'(c')}{U'(c)} \right] \left[ \text{MRPK}' - (r' + \delta) - \frac{\partial \text{AC}'}{\partial k'} \right] = \frac{\chi (1 - \lambda_1 \Psi'(k'))}{U'(c)} + \frac{\partial \text{AC}}{\partial k'}.$$

In the absence of borrowing constraints, risk in capital accumulation, and capital adjustment costs, there would be no dispersion of the MRPK across firms. More productive firms would choose higher capital stocks but would lower their price  $p$  one-to-one with their productivity  $Z$ , leading to an equalization of the MRPK across firms. Under these assumptions, equation (22) simplifies to  $\text{MRPK} = r + \delta$  for all firms  $(a, k, z^P, z^T, \mathbf{X})$ .

24. To obtain equation (21), we combine the first-order condition for labor ( $\frac{(1-\alpha)p\gamma}{\ell} = \mu w$ ) with the demand function, the production function, the expression for the marginal cost  $\text{MC} = \left( \frac{w}{Z(1-\alpha)} \right) \left( \frac{\ell}{k} \right)^\alpha$ , and the solution for prices  $p = \mu \text{MC}$ . Given that capital is predetermined at some level  $k$ , at the beginning of each period firms face decreasing returns to scale with respect to the variable input  $\ell$ . Therefore, the marginal cost MC is increasing in the scale of production. To solve for prices we substitute the solution for labor in equation (21) into  $p = \mu \text{MC}$ . Firms with high  $k$  and low  $Z$  have lower marginal cost MC and lower price  $p$ .

By contrast, binding borrowing constraints, risk in capital accumulation, or capital adjustment costs introduce dispersion of the MRPK across firms. Binding borrowing constraints are captured by a positive multiplier  $\chi$  in [equation \(22\)](#). Adjustment costs are captured by the derivatives of the adjustment cost function  $AC$  and  $AC'$ . Finally, similar to the analysis of [Asker, Collard-Wexler, and De Loecker \(2014\)](#), a capital stock determined in some previous period may not be optimal ex post, that is after productivity is realized. As a result, part of the dispersion of the MRPK across firms would also arise in an undistorted economy in which the capital stock is chosen under uncertainty and becomes productive in the next period.

#### *IV.C. The Real Interest Rate Decline*

We associate the secular decline in the real interest rate  $r_t$  to trends in MRPK dispersion, TFP losses, and capital flows in the precrisis period between 1999 and 2007. The decline in  $r_t$  started prior to 1999 and could reflect expectations of a reduction in exchange rate risk as countries converged to a common currency, declines in default risk from an implicit assumption that euro countries would be bailed out in a crisis, and the removal of barriers to capital mobility within the euro area. Following the small open economy literature, we model these forces as an exogenous decline in the real interest rate.

The solid line in [Figure VII](#) presents the evolution of the real interest rate  $r_t$  since the early 1990s in the data. We measure  $r_t$  as the difference between the nominal corporate lending rate to nonfinancial firms and next year's expected inflation. The lending rate comes from Eurostat and refers to loans with size less than one million euros that mature within one year. Expected inflation is given by the fitted values from an estimated AR(1) process for inflation.

The dashed line is the real interest rate that we feed as the driving process into the model. We start the economy in a stochastic steady state that corresponds to the period before 1994 with a constant  $r = 0.10$ . We define the stochastic steady state as an equilibrium of the model in which all aggregate shocks are constant, but firms are hit by idiosyncratic productivity shocks and change their production, savings, and investment decisions over time. Starting from the stochastic steady state, we then introduce the path of the real interest rate  $r_t$  depicted in [Figure VII](#). In the baseline results we assume that firms have perfect foresight and

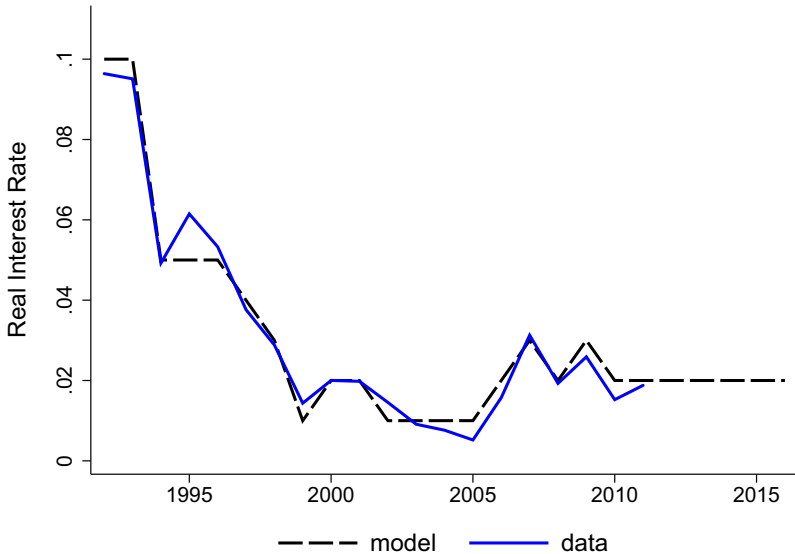


FIGURE VII  
Evolution of Real Interest Rate

are continuously surprised by shocks to  $r_t$ , that is any change in  $r_t$  is perceived by firms as an unexpected and permanent shock. In our robustness checks shown below, we obtain similar results when we assume that firms perceive that the real interest rate follows an AR(1) process that we estimate in the data after 1994.<sup>25</sup>

#### IV.D. Parameterization

Table III lists parameter values for the baseline version of our model.<sup>26</sup> There are three types of parameters. First, we choose

25. We solve the model with standard value function iteration methods. We discretize the space of net worth, capital, permanent productivity, transitory productivity, and the real interest rate into 120, 120, 2, 11, and 6 points, respectively. In Figure VII we do not match exactly the path of the real interest rate in the data because, to economize on computational speed for our many extensions and robustness checks, we define the grid for the real interest rate on 6 points. Our baseline results (labeled “HeF Model” below) remained unchanged in simulations in which we matched exactly the path of the real interest rate in the data by solving the model on a grid of 20 points for the real interest rate.

26. We have fixed the aggregate wage to  $w = 1$  and the aggregate component of firm productivity to  $Z^A = 1$ .

TABLE III  
BASELINE PARAMETERS

$\beta$	$\gamma$	$\delta$	$\varepsilon$	$\alpha$	$\rho$	$\sigma$	$\pi$	$z_L$	$z_H$	$\psi$	$\lambda_0$	$\lambda_1$
0.87	2.00	0.06	3.00	0.35	0.59	0.13	0.80	0.785	1.86	3.20	0.98	0.047

conventional values for five parameters ( $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\varepsilon$ , and  $\alpha$ ) before solving the model. Second, the productivity parameters ( $\rho$ ,  $\sigma$ ,  $\pi$ ,  $z_L$ , and  $z_H$ ) are estimated directly from the data without solving the model. Third, we calibrate the three remaining parameters ( $\psi$ ,  $\lambda_0$ , and  $\lambda_1$ ) by requiring that the model matches three moments in the data.

To estimate firm productivity, we use the [Wooldridge \(2009\)](#) extension of the [Levinsohn and Petrin \(2003\)](#) methodology and denote this estimate by  $\hat{Z}_{ist}$ .<sup>27</sup> In the estimation, we allow the elasticities of value added with respect to inputs to vary at the two-digit industry level. We discuss our estimates in more detail in [Online Appendix D](#). Here we note that we estimate reasonable elasticities, with their sum ranging from 0.75 to 0.91. Our estimate  $\hat{Z}_{ist}$  uncovers a combination of idiosyncratic productivity and demand as we do not separately observe firm prices.<sup>28</sup>

We estimate the productivity process at the firm level using the regression:

$$(23) \quad \log(\hat{Z}_{ist}) = d_i + d_{st} + \rho \log(\hat{Z}_{ist-1}) + u_{ist}^z,$$

where  $d_i$  denotes the firm permanent effect and  $d_{st}$  denotes a four-digit industry-year fixed effect. Using this regression, we find

27. [Olley and Pakes \(1996\)](#) and [Levinsohn and Petrin \(2003\)](#) use a two-step method to estimate production functions in which investment and intermediate inputs respectively proxy for unobserved productivity. [Akerberg, Caves, and Frazer \(2015\)](#) highlight that if a variable input (e.g., labor) is chosen as a function of unobserved productivity, then the coefficient on the variable input is not identified. [Wooldridge \(2009\)](#) suggests a generalized method of moments estimation to overcome some of the limitations of previous methods.

28. For this reason our elasticities are more appropriately defined as revenue elasticities. The correlation between  $\log(\hat{Z}_{ist})$  and  $\log(\hat{Z}_{ist-1})$ , which was defined in [equation \(11\)](#), in the cross section of firms ranges between 0.8 and 0.9 and is stable over time. Unless otherwise noted, we now use the  $\hat{Z}_{ist}$  to construct moments in the data.

that  $\rho = 0.59$ .<sup>29</sup> We use the cross-sectional standard deviation of residuals  $u_{ist}^z$  from regression (23) to choose a value for  $\sigma$ . The value of  $\sigma = 0.13$  corresponds to the average standard deviation over time.

The permanent component of productivity is drawn from the following distribution:

$$(24) \quad z_i^P = \begin{cases} z_L, & \text{with probability } \pi \\ z_H, & \text{with probability } 1 - \pi. \end{cases}$$

We normalize the average  $z_i^P$  to one and choose the values  $\pi = 0.80$ ,  $z_L = 0.785$ , and  $z_H = 1.86$  such that, together with our estimated  $\rho = 0.59$  and  $\sigma = 0.13$ , the standard deviation of  $\log(Z_{it})$  equals 0.38 as in the data.<sup>30</sup>

Finally, we calibrate the three remaining parameters,  $\psi$  in the adjustment cost technology and  $\lambda_0$  and  $\lambda_1$  in the borrowing constraint, to match three informative moments in the data between 1999 and 2007. We view the data as generated from transitional dynamics of our model following the decline in the real interest rate that started in 1994. We choose the three parameters such that our model generates:

- Moment 1. Within-firm regression coefficient of capital growth  $\frac{k'-k}{k}$  on productivity  $\log Z$  equal to 0.10.
- Moment 2. Fraction of firms that borrow equal to 0.90.
- Moment 3. Cross-sectional regression coefficient of firm leverage  $\frac{b}{k}$  on capital  $\log k$  equal to 0.15.

29. Including firm fixed effects in a regression with a lagged dependent variable and a short time series leads to a downward bias in the estimated persistence of a process. When we estimate the AR(1) process in equation (23) we obtain an estimated persistence parameter of 0.46. Therefore, we set  $\rho = 0.59$  such that, in model-generated data of 14 sample periods, the estimated persistence parameter equals 0.46. To maximize the length of the sample, our estimates of the productivity process are obtained from the permanent sample of firms between 1999 and 2012.

30. We have experimented with values of  $\pi$  ranging from 0.50 to 0.90 and found that, conditional on matching the same moments in the data, such alternative values for  $\pi$  do not alter significantly the dynamics of TFP following the decline in the real interest rate. Our choice of  $\pi = 0.80$  implies a natural comparison between the model and the data in terms of the share of aggregate variables accounted for by the top 20% of firms.



For our baseline model this procedure yields  $\psi = 3.20$ ,  $\lambda_0 = 0.98$ , and  $\lambda_1 = 0.047$ .<sup>31</sup>

#### IV.E. Model Evaluation

We evaluate the performance of our model by comparing it with two nested benchmark models. In our model with a size-dependent borrowing constraint, we choose three parameters,  $(\psi, \lambda_0, \lambda_1) = (3.2, 0.98, 0.047)$  to match the three moments discussed above. Because the borrowing constraint depends on firm size, financial frictions are heterogeneous across firms and in what follows we label the baseline model by “HeF.” The first comparison is with the standard model with  $\lambda_1 = 0$ . In this model, financial frictions are homogeneous across firms (so we call this model “HoF”) and we choose  $(\psi, \lambda_0, \lambda_1) = (3.2, 1.06, 0)$  to match the first two moments discussed above. Finally, we compare our model to a model without financial frictions (which we call “NoF”). We choose  $(\psi, \lambda_0, \lambda_1) = (3.5, \infty, \infty)$  to match only the first moment.

We evaluate the performance of our model by comparing it to these two nested models along moments that are not targeted during the parameterization of each model. Therefore, comparing HeF to HoF informs us by how much untargeted moments change when we ignore the cross-sectional relationship between leverage and size during the parameterization of the model. Comparing HoF to NoF informs us by how much untargeted moments change when we ignore the fact that financial frictions prevent some firms from borrowing.

### V. FIRM-LEVEL IMPLICATIONS OF THE MODEL

In this section we document the success of our model to generate firm-level outcomes that resemble those observed in the data. We summarize firm-level outcomes in the data in terms of statistics estimated during the pre-recession period of 1999 to 2007 for the permanent sample of firms. We construct these statistics similarly to the dispersion and misallocation measures presented in

31. During the transitional dynamics generated by our model between 1999 and 2007, the mean adjustment cost equals 2.8% of value added conditional on adjusting the capital stock and the mean frequency of adjustment is 20%. The value of 2.8% lies within the wide range of estimates that [Bachmann, Caballero, and Engel \(2013\)](#) report from the literature and is close to their preferred estimate of 3.6%.

TABLE IV  
FIRM-LEVEL MOMENTS: MODEL VS. DATA (1999–2007)

Moment	Data	HeF	HoF	NoF
Panel A: Distributional moments				
1. Std.dev.(log $Z$ )	0.37	0.38	0.38	0.38
2. Std.dev.(log $\ell$ )	1.13	0.80	0.74	0.71
3. Std.dev.(log $k$ )	1.52	0.91	0.75	0.66
4. Top 20% share of aggregate labor	0.69	0.63	0.59	0.58
5. Top 20% share of aggregate capital	0.79	0.71	0.61	0.56
Panel B: Within-firm moments				
6. Coefficient of $\frac{k'-k}{k}$ on log $Z$	0.10	0.10	0.10	0.10
7. Coefficient of $\frac{k'-k}{k}$ on log $a$	0.09	0.04	0.27	0.01
8. Coefficient of $\frac{k'-k}{k}$ on log $k$	-0.46	-0.07	-0.46	-0.19
9. Coefficient of $\frac{b'-b}{k}$ on log $Z$	-0.38	-0.04	-0.06	0.03
10. Coefficient of $\frac{b'-b}{k}$ on log $a$	0.15	0.09	0.72	0.02
11. Coefficient of $\frac{b'-b}{k}$ on log $k$	-0.34	0.07	-0.74	-0.22
Panel C: Cross-sectional moments				
12. Fraction borrowing	0.90	0.90	0.90	1.00
13. Coefficient of $\frac{b}{k}$ on log $k$	0.15	0.15	0.03	0.00
14. Corr (log $Z$ , log $k$ )	0.62	0.88	0.89	0.91
15. Corr (log $Z$ , log $a$ )	0.75	0.84	0.90	0.43
16. Corr (log MRPK, log $Z$ )	0.03	-0.08	0.28	0.51
17. Corr (log MRPK, log $k$ )	-0.62	-0.54	-0.18	0.10
18. Corr (log MRPK, log $a$ )	-0.14	-0.36	-0.15	0.20
Panel D: Model evaluation				
19. Root mean squared error		0.28	0.35	0.41
20. Mean absolute error		0.22	0.29	0.34

**Section III.** We compute the same statistics in the model during the transitional dynamics between 1999 and 2007 in response to the decline in the real interest rate shown in [Figure VII](#). Summary statistics both in the data and the model are averaged between 1999 and 2007. In [Online Appendix E.1](#) we show that our conclusions remain unchanged if we evaluate the model in terms of moments generated from the full sample instead of the permanent sample of firms.

#### V.A. Distributional Moments

Panel A of [Table IV](#) shows statistics related to the distribution of firm size in the data and in the various models. Row 1 shows

that all models feature the same standard deviation of  $\log Z$  by construction. Our model with a size-dependent borrowing constraint (HeF) generates roughly 70% of the standard deviation of  $\log l$  observed in the data (row 2) and roughly 60% of the standard deviation of  $\log k$  observed in the data (row 3). Rows 4 and 5 show that the model generates roughly 90% of the share of aggregate labor or capital accruing to the top 20% of firms.

The size-dependent borrowing constraint in model HeF implies that some firms find it profitable to accumulate a significant amount of capital in order to relax their borrowing limit in response to the decline in the real interest rate. Therefore, our model HeF generates more dispersion in labor and capital than the model with a homogeneous borrowing constraint (HoF) and the model without financial frictions (NoF).

### V.B. Within-Firm Moments

Panel B of [Table IV](#) focuses on the dynamics of capital, debt, and productivity at the firm level. We present results from two regressions that use within-firm variation over time. Rows 6 to 8 show estimated coefficients from a regression of capital growth on productivity, net worth, and capital:

$$(25) \quad \frac{k_{ist+1} - k_{ist}}{k_{ist}} = d_i + d_{st} + \beta_z \log(Z_{ist}) + \beta_a \log(a_{ist}) + \beta_k \log(k_{ist}) + u_{ist}^k,$$

where  $d_i$  denotes a firm fixed effect and  $d_{st}$  denotes a four-digit industry-year fixed effect. The second regression in rows 9 to 11 is a similar regression but uses the change in (net) debt to capital ratio  $\frac{b_{ist+1} - b_{ist}}{k_{ist}}$  as the left-hand side variable. The choice of regressors is motivated by our model in which productivity, net worth, and capital are the state variables summarizing firm capital and debt decisions. The first two regressors resemble sales and cash flow, commonly used by the finance literature in investment regressions. In [Online Appendix E.2](#) we report such regressions.<sup>32</sup>

32. We measure firm net worth  $a$  in the data as the difference between the book value of total assets and total liabilities and deflate this difference with the industry output price deflators previously described in [Section III](#). We measure (net) debt  $b$  with the book value of current liabilities minus cash holdings and also deflate this difference with the same price deflators. This short-term measure of

In the data column, we document that firm capital growth  $\frac{k'-k}{k}$  is positively related to productivity  $\log Z$  (row 6) and to net worth  $\log a$  (row 7). In row 8, the estimated coefficient of  $\frac{k'-k}{k}$  on  $\log k$  is negative and greater than minus one, which indicates that log capital is a persistent process. Rows 9 to 11 document that the change in firm debt to capital ratio,  $\frac{b'-b}{k}$ , is negatively related to  $\log Z$ , positively related to  $\log a$ , and negatively related to  $\log k$ . Given the large sample size with more than 100,000 observations, all estimated coefficients in rows 6 to 11 are statistically significant at levels below 1%. We present the standard errors of our estimates in [Online Appendix E.2](#).

Row 6 shows that all models match the responsiveness of  $\frac{k'-k}{k}$  to  $\log Z$  by construction. An increase in current productivity leads to an increase in next period's capital because productivity is a persistent process and firms expect the marginal product of capital to be higher in future periods. Capital adjustment costs ameliorate the responsiveness of capital growth and, thus, the choice of the adjustment cost parameter  $\psi$  in each model reflects the magnitude of adjustment costs that allow models to match the responsiveness of capital growth to productivity in the data. In row 9 we see that the change in debt is weakly negatively correlated with  $\log Z$  in models HoF and HeF and weakly positively correlated with  $\log Z$  in model NoF. The change in debt,  $b' - b$ , equals the difference between the flow of investment and saving. A persistent but mean-reverting increase in  $\log Z$  increases investment because firms expect a higher marginal product of capital in future periods and also saving because firms desire to smooth consumption. The negative coefficient in models HoF and HeF reflects the fact that, similarly to the data, firm saving responds more than investment to changes in productivity.

Contrary to the data, we find that net worth in model NoF is not a significant determinant of the within-firm variation in capital growth (row 7) and change in debt (row 10). Models with financial frictions will, in general, imply a more important role of net worth for capital and debt accumulation. Our model HeF comes close to matching quantitatively the role of net worth for capital growth and change in debt, while model HoF assigns a much stronger role to net worth than observed in the data. Net worth is a less important determinant of capital and debt decisions

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debt is our preferred one because our model abstracts from a maturity choice of debt and savings in long-term assets.

on average in model HeF than in model HoF because in model HeF some firms accumulate a significant amount of capital and become effectively unconstrained in response to the decline in the real interest rate.<sup>33</sup>

### *V.C. Cross-Sectional Moments*

Panel C of [Table IV](#) presents correlations between firm size, productivity, leverage, net worth, and MRPK in the cross section of firms. Row 12 shows the fraction of firms that are borrowing ( $b > 0$ ), a moment which both model HoF and model HeF match by construction. Row 13 shows the cross-sectional relationship between leverage  $\frac{b}{k}$  and  $\log k$ . In the data, we find that larger firms are more leveraged.<sup>34</sup> We calibrated the parameter  $\lambda_1$  in the borrowing constraint (20) such that model HeF generates the cross-sectional relationship between  $\frac{b}{k}$  and  $\log k$  observed in the data.

Rows 14 to 18 of Panel C show other (untargeted) cross-sectional moments. Rows 14 and 15 show that all models reproduce the positive relationship between firm productivity and capital in the data.<sup>35</sup> As shown in rows 16 to 18, the key difference between models is in their cross-sectional patterns of the return to capital. In the data,  $\log(\text{MRPK})$  is essentially uncorrelated with  $\log Z$  and negatively correlated with  $\log k$  and  $\log a$

33. Rows 8 and 11 show that model HeF fails to match the coefficient on  $\log k$  and performs worse than HoF and NoF in that dimension. Recalling that  $\frac{k'-k}{k} \approx \log(k') - \log(k)$ , the estimated coefficient in the HeF model implies that  $\log$  capital becomes a more persistent process when we allow for size-dependent borrowing constraints.

34. We obtain a similar result in the data when we use other measures of firm size such as the wage bill. Since our model does not consider the distinction between short-term and long-term liabilities or assets, the regressions in the data control for the difference between long-term liabilities and long-term assets (current assets minus cash). Additionally, cross-sectional regressions control for firm age. We define firm age in period  $t$  as  $t$  minus the date of incorporation plus 1. Firm age is a firm-specific linear time trend and, therefore, is absorbed by the firm fixed effect in regressions that use within-firm variation over time.

35. All models successfully replicate the positive and high correlation between  $\log$  productivity and firm size (measured either by  $\log$  labor or  $\log$  capital) and the positive and high correlation between firm  $\log$  productivity and share in sectoral economic activity (measured either by labor or capital). The correlations between  $\log$  productivity and firm share in economic activity resemble the measures of resource allocation emphasized by [Bartelsman, Haltiwanger, and Scarpetta \(2013\)](#).

in the cross section of firms.<sup>36</sup> However, model NoF implies that  $\log(\text{MRPK})$  is positively correlated with  $\log Z$ ,  $\log k$ , and  $\log a$ . In Model HoF, the distance between model-generated moments and the corresponding moments in the data is smaller. Model HeF comes closer than all models in matching the cross-sectional patterns of the return to capital, as it generates a weak negative correlation between  $\log(\text{MRPK})$  and  $\log Z$  and negative correlations of  $\log(\text{MRPK})$  with  $\log k$  and  $\log a$ .

To understand these differences, first we note that risk in capital accumulation tends to generate a positive correlation between  $\log(\text{MRPK})$  and either  $\log Z$  or measures of firm size. As an example, consider two firms that start with the same state vector  $(a, k, z^P, z^T, \mathbf{X})$  in some period and, therefore, choose the same capital for the next period  $k'$ . If in the next period one of these firms receives a higher productivity shock, then that firm will have higher revenues and MRPK ex post. This explains the high correlation between  $\log(\text{MRPK})$  and either  $\log Z$  or  $\log k$  in the NoF model. Model HeF differs crucially from the other models because, with a size-dependent borrowing constraint, some highly productive firms accumulate a significant amount of capital in order to relax their borrowing limit. These firms become effectively unconstrained and have a low  $\log(\text{MRPK})$ . The existence of large firms with low MRPK accounts for the negative correlation between  $\log(\text{MRPK})$  and either  $\log Z$  or  $\log k$  in the HeF model. The correlations in the HoF model are between those in the NoF model and those in the HeF model.

Finally, in the two models with financial frictions, HoF and HeF, higher net worth firms are less constrained and, therefore, tend to have a lower return to capital. The two models are successful in generating the negative correlation between  $\log(\text{MRPK})$  and  $\log a$  observed in the data. By contrast, model NoF generates a positive correlation.

#### *V.D. Model Evaluation*

Panel D of [Table IV](#) compares the models in terms of their distance from data moments not targeted during the parameterization of each model. These moments are in rows 2 to 5, 7 to 11, and 14 to 18. In rows 19 and 20 we present the root mean squared

36. [Hsieh and Olken \(2014\)](#) report that smaller firms in India, Indonesia, and Mexico have lower average product of capital than larger firms. We do not find this pattern in Spain.

TABLE V  
AGGREGATE RESPONSES: DATA VS. BASELINE HeF MODEL (1999–2007)

	Data	Model HeF
1. $\Delta \Lambda$	-0.031	-0.023
2. $\Delta$ (Dispersion)	0.034	0.074
3. $\Delta \log(\sum y)$	0.14	0.26
4. $\Delta \log K$	0.22	0.38
5. $\frac{\Delta B}{\Delta K}$	1.28	1.24

*Notes.* All changes are calculated between 1999 and 2007.  $\Delta \Lambda$  denotes the percentage point change in TFP relative to its efficient level, that is the difference between  $\log(\text{TFP}_{07}) - \log(\text{TFP}_{07}^e)$  and  $\log(\text{TFP}_{99}) - \log(\text{TFP}_{99}^e)$ .  $\Delta$  (Dispersion) is the  $\Delta(\text{std. dev.}(\log(\text{MRPK})))$ .  $\Delta \log(\sum y)$  is the change in the log of the sum of output and  $\Delta \log K$  is the change in the log of aggregate capital.  $\frac{\Delta B}{\Delta K}$  is the change in aggregate debt between 1999 and 2007 divided by the change in the aggregate capital between 1999 and 2007.

error and the mean absolute error of the three models. We find that the HeF model performs better than the HoF model and that the HoF model performs better than the NoF model.

## VI. MACROECONOMIC IMPLICATIONS OF THE MODEL

Having documented the success of the model to match aspects of firm-level behavior, we now present the evolution of aggregate outcomes in response to the decline in the real interest rate.

### VI.A. Aggregate Responses

Table V summarizes the aggregate responses. The first column shows changes in various statistics between 1999 and 2007 in the data for the permanent sample of firms.<sup>37</sup> In Section III we documented a decline in TFP relative to its efficient level and an increase in the dispersion of the log(MRPK) over time. We reproduce these changes in the first two rows of Table V that show a 3.1 percentage points decline in TFP relative to its efficient level and a 3.4 percentage points increase in the standard deviation of log(MRPK). The data column of the table also documents the evolution of total output and capital flows to the manufacturing

37. In a previous version of this article we considered an extension of the model with endogenous entry and exit. The model with endogenous entry and exit generates a decline in mean firm productivity over time when less productive firms choose to enter manufacturing in response to the decline in the real interest rate. See Lagos (2006) for a formalization of the idea that TFP depends on the productivity composition of firms that are active in equilibrium which, in turn, responds to labor market policy.

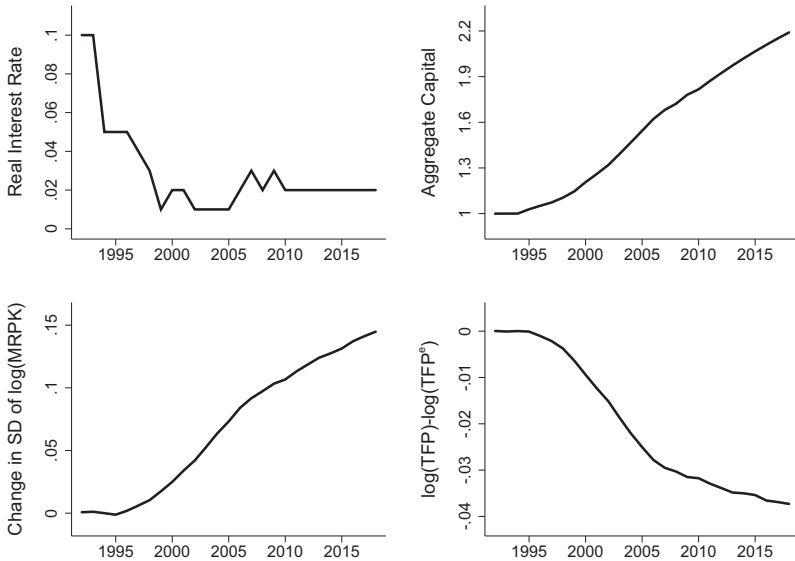


FIGURE VIII

Evolution of Aggregate Variables in Baseline HeF Model

sector from our data set. Between 1999 and 2007, we observe significant aggregate capital accumulation financed by an increase in short-term debt and a significant increase in aggregate output. Additionally, as the last row shows, debt increased more than one-to-one with capital over that time period (i.e.,  $\frac{\Delta B}{\Delta K} > 1$ ).

The second column of Table V shows the aggregate responses in the HeF model. Figure VIII plots some of the transitions to better visualize the dynamics generated by the model. Similarly to the data, our model driven by a decline in the real interest rate generates capital and debt accumulation, an increase in the dispersion of the log(MRPK), and a decline in TFP relative to its efficient level.<sup>38</sup> Quantitatively, the model generates roughly three-quarters of the observed decline in TFP relative to its efficient level. Consistently with the data, debt increases more than capital in the model following the decline in the real interest rate.

38. In Figure VI we documented that the increase in the dispersion of log capital was associated with a declining correlation between log capital and log productivity. We obtain a similar prediction in the HeF model. In the model, the standard deviation of  $\log k$  increases by roughly 20 percentage points and the correlation between  $\log k$  and  $\log Z$  declines by roughly 3 percentage points.



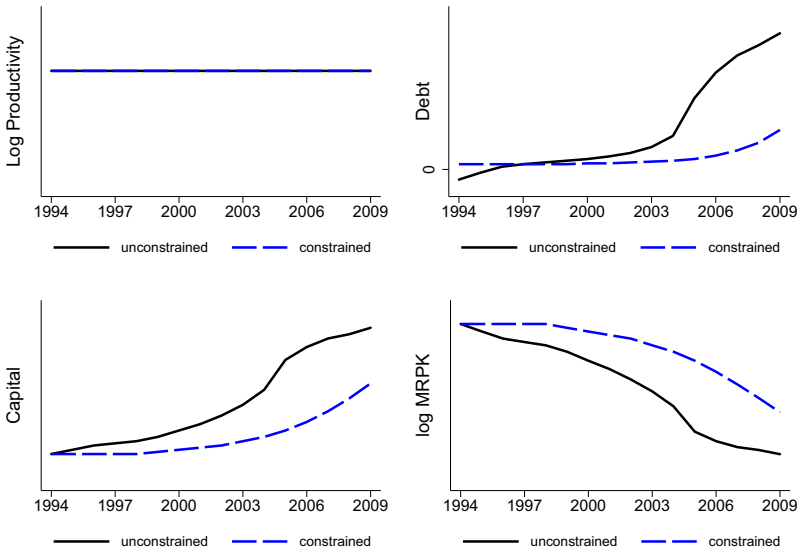


FIGURE IX

Real Interest Rates and Misallocation: An Illustrative Example (HeF Model)

The model generates a somewhat larger increase in the dispersion of  $\log(\text{MRPK})$ , in capital, and in output than observed in the data.<sup>39</sup> Below we show that increasing the magnitude of adjustment costs, so that the model reproduces the observed changes in capital and output, does not affect much the ability of the model to generate a significant decline in TFP.

### VI.B. Inspecting the Mechanism: Capital Allocation across Firms

To understand these aggregate responses, we begin with a simple example that illustrates the mechanism generating misallocation in our model in response to the decline in  $r_t$ . Figure IX depicts outcomes for two firms in model HeF. We label one firm as “unconstrained” and the other as “constrained.” The initial state

39. In the table and the figure we show the change in the level of the standard deviation of  $\log(\text{MRPK})$  rather than the percent change in this standard deviation. The reason is that the baseline model generates only roughly one third of the level of dispersion relative to the data, which makes percent comparisons uninformative. Below we show that adding measurement error in capital in order to match the level of MRPK dispersion does not change significantly the quantitative predictions of the model.

variables for the two firms in 1994 are drawn from the stationary distribution of the model. As the first panel of the figure shows, the two firms have the same constant productivity in all periods. The firms, however, differ in their initial net worth and debt.

The unconstrained firm is initially saving ( $b < 0$ ) and, therefore, its borrowing constraint is not binding when the  $r_t$  shock hits. The decline in  $r_t$  increases desired investment. In the first few periods after the shock, the firm uses internal savings to finance capital accumulation and adjustment costs. With a size-dependent borrowing limit, the borrowing constraint of this firm is relaxed as capital accumulates over time. As a result, the unconstrained firm borrows more over time and uses the inflow of debt to increase its capital even more and to finance its consumption. As the last panel of the figure shows, the decline in the real interest rate causes a decline in the MRPK of the unconstrained firm.

By contrast, the constrained firm has initially lower net worth and its borrowing constraint is binding when the  $r_t$  shock hits. This firm also desires to increase its capital. However, the lack of sufficient funds prevents the firm from doing so in the first few periods after the shock. Over time, the firm uses internal funds to increase its capital stock but, even 15 years after the initial decline in the real interest rate, the constrained firm's capital is significantly lower than the desired level. The financially constrained firm experiences a smaller decline in its MRPK; therefore, the dispersion of the MRPK between the two firms increases over time.

This example illustrates how a decline in the real interest rate in an environment with size-dependent borrowing constraints and adjustment costs causes capital inflows to be misallocated. Misallocation here means that capital is unequally allocated across firms despite both firms being equally productive. It is not crucial that both firms are equally productive. We would obtain the same result even if the unconstrained firm experienced a few negative productivity shocks along its transition.

This mechanism explains the results in [Table V](#). Firms with higher initial wealth are more able to finance capital accumulation. As these firms grow, they gradually overcome their borrowing constraints and accumulate a significant amount of debt. TFP declines because capital is not allocated to its most efficient use as some productive but financially constrained firms with low net worth are not able to grow in the short run.

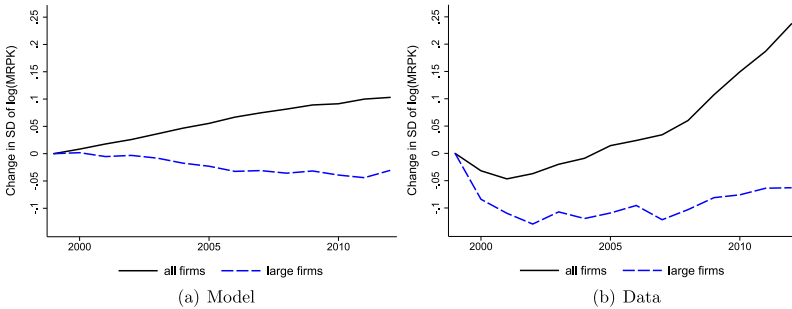


FIGURE X  
MRPK Dispersion and Large Firms

We now provide direct evidence that supports this mechanism. We run the cross-sectional regression in the data:

$$\begin{aligned}
 \frac{k_{i,s,07} - k_{i,s,99}}{k_{i,s,99}} = & \alpha_s + \beta_z \log(Z_{i,s,99}) + \beta_a \log(a_{i,s,99}) \\
 (26) \qquad \qquad \qquad & + \beta_k \log(k_{i,s,99}) + u_{i,s}^k,
 \end{aligned}$$

where the left-hand side denotes firm capital growth between 1999 and 2007. The result we want to emphasize is our estimate  $\beta_a = 0.17$  which implies that, conditional on initial productivity and capital, firms that in 1999 had a 1% higher net worth invested 0.17% more in capital by 2007. [Online Appendix E.2](#) shows that all estimated coefficients in regression (26) are statistically significant.

Model HeF comes close to matching the relationship between initial net worth and subsequent capital accumulation in the data. Running the same regression in model-generated data yields an estimated coefficient of  $\beta_a^{\text{HeF}} = 0.21$ . To set a benchmark for this estimate, we report that the corresponding coefficients in the two nested models are quite different as we obtain  $\beta_a^{\text{NoF}} = -0.03$  and  $\beta_a^{\text{HoF}} = 0.11$ .

At the core of the mechanism amplifying the misallocation of capital in our model is that larger firms are more likely than small firms to overcome their borrowing constraints. As a result, the increase in the dispersion of the MRPK should be found primarily between small and large firms and not within large firms. The left panel of [Figure X](#) shows that in response to the decline in the real interest rate, MRPK dispersion does not increase in our

model when we condition on the subsample of large firms. We define large firms as firms with capital in the top 5% of the capital distribution. The right panel confirms that in the data MRPK dispersion also does not increase in the subsample of large firms. This result highlights the importance of having smaller firms in the sample.<sup>40</sup>

### VI.C. *Impact of Misallocation on Aggregate Dynamics*

The inflow of capital in our model is associated with a deterioration in the allocation of resources across firms. We now ask what is the additional impact of this deterioration on aggregate dynamics following the decline in the real interest rate. To answer this question, we compare the transitional dynamics generated by the HeF model to the transitional dynamics generated by a model without financial frictions. This comparison allows us to isolate the effect of financial frictions on aggregate dynamics, holding constant the other two factors that generate MRPK dispersion (risk in capital accumulation and adjustment costs).

Figure XI shows that the transitional dynamics in the HeF model with parameters  $(\psi, \lambda_0, \lambda_1) = (3.2, 0.98, 0.047)$  differ significantly from the transitional dynamics generated by a model without financial frictions and parameters  $(\psi, \lambda_0, \lambda_1) = (3.2, \infty, \infty)$ . Output and capital grow by significantly less in the model with financial frictions. We also note an important difference in the initial growth of aggregate consumption. Consumption grows by substantially more in the model without financial frictions because permanent income grows by more in this model than in the HeF model. We conclude that in response to the decline in the real interest rate, there is an important quantitative effect of misallocation due to size-dependent financial frictions on the transitional dynamics of aggregate variables.<sup>41</sup>

40. We find a divergence in the change in MRPK dispersion between the whole sample and the subsample of large firms for various alternative definitions of large firms (firms in the top 1%, 3%, or 10% of the capital distribution or firms with more than 100 or 250 employees). We also note that we fail to detect significant increases in the MRPK dispersion in the ESEE data set that includes mostly large firms.

41. Consistently with the findings of Moll (2014) and Midrigan and Xu (2014), in our model the TFP loss due to financial frictions in the stochastic steady state is small because firm productivity is a persistent process. Our baseline experiment is a decline in the real interest rate in a model with financial frictions, rather than a change in financial frictions per se. The decline in the real interest rate generates a loss in TFP along the transitional dynamics that is substantially larger than the

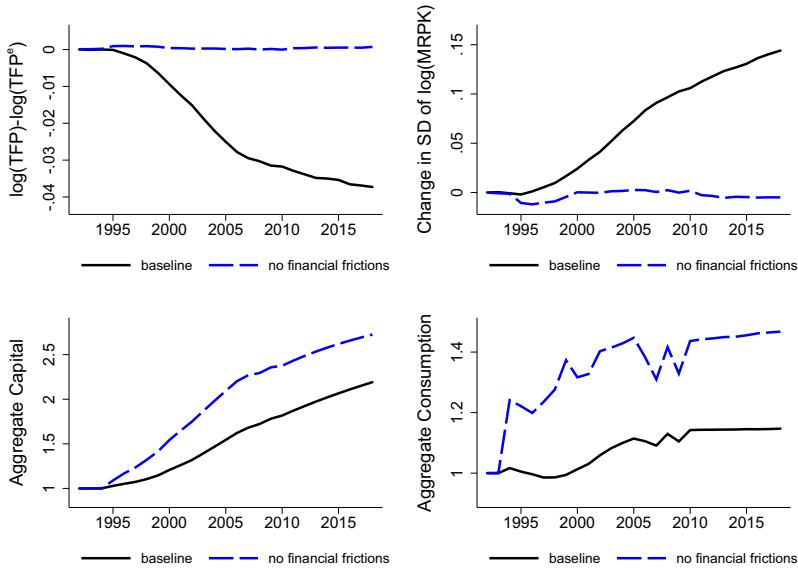


FIGURE XI

Impact of Misallocation on Evolution of Aggregate Variables

#### VI.D. Extensions

We next present aggregate responses to the real interest rate decline in the nested models HoF and NoF and in various extensions of the baseline HeF model. We summarize our results in [Table VI](#). The first two rows of [Table VI](#) repeat the aggregate responses in the data and the HeF model. For each extension, we present more details about the model environment, our choice of parameters, the firm-level moments, and metrics of model performance in [Online Appendix E.3](#).

1. *Models HoF and NoF.* Rows 3 and 4 of [Table VI](#) show the responses of aggregate variables in the model with homogeneous frictions and the model without frictions. The most important result is that both models fail to generate any significant change in TFP and MRPK dispersion over time. This result is striking considering that the two models generate very different output and capital dynamics in response to the decline in the real interest

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steady state TFP loss. Additionally, we show that there is a substantial impact of financial frictions on the transitional dynamics of aggregate variables in response to a decline in the real interest rate.

TABLE VI  
 AGGREGATE RESPONSES: DATA VS. VARIOUS MODELS (1999–2007)

	$\Delta\lambda$	$\Delta(\text{Dispersion})$	$\Delta\log(\sum y)$	$\Delta\log K$	$\frac{\Delta B}{\Delta K}$
1. Data	-0.031	0.034	0.14	0.22	1.28
2. Heterogeneous frictions	-0.023	0.074	0.26	0.38	1.24
3. Homogeneous frictions	0.000	0.003	0.01	0.01	0.86
4. No frictions	0.000	0.002	0.27	0.43	1.06
5. Higher adjustment costs	-0.019	0.050	0.13	0.22	0.88
6. Exogenous labor wedges	-0.026	0.057	0.27	0.39	1.17
7. Overhead labor	-0.023	0.075	0.26	0.38	1.24
8. Unmeasured capital	-0.027	0.048	0.31	0.47	1.29
9. Alternative $r$ process	-0.018	0.056	0.19	0.28	1.25

*Notes.* All changes are calculated between 1999 and 2007.  $\Delta\lambda$  denotes the percentage point change in TFP relative to its efficient level, that is the difference between  $\log(\text{TFP}_{07}) - \log(\text{TFP}_{99}^e)$  and  $\log(\text{TFP}_{99}) - \log(\text{TFP}_{99}^e)$ .  $\Delta(\text{Dispersion})$  is the  $\Delta[\text{std. dev.}(\log(\text{MRPK}))]$ .  $\Delta\log(\sum y)$  is the change in the log of the sum of output and  $\Delta\log K$  is the change in the log of aggregate capital.  $\frac{\Delta B}{\Delta K}$  is the change in aggregate debt between 1999 and 2007 divided by the change in the aggregate capital between 1999 and 2007.

rate. Output and capital barely change in model HoF whereas they increase significantly in model NoF.

To understand the difference between the HeF and the HoF model, Figure XII repeats in model HoF the example with the two firms first presented in Figure IX for model HeF. We keep the scale of the axes the same between the two figures to ease the comparisons. Figure XII shows muted differences in capital and debt accumulation between the two types of firms. Contrary to the HeF model, in the HoF model the initially unconstrained firm is not able to overcome its borrowing constraint over its transition. Therefore, capital and debt accumulation are relatively similar across firms. The small dispersion in firm outcomes explains why the HoF model fails to generate significant movements in TFP and MRPK dispersion following the decline in the real interest rate.

In response to the decline in the real interest rate, the NoF model generates significant capital and debt accumulation. In that respect, the NoF model is different from the HoF model. In the HoF model, most firms remain constrained and do not grow significantly. In the NoF model, all firms are unconstrained and grow significantly. Therefore, both models generate too little heterogeneity in firm capital and debt accumulation. This explains why both models fail to generate significant changes in TFP and MRPK dispersion over time.

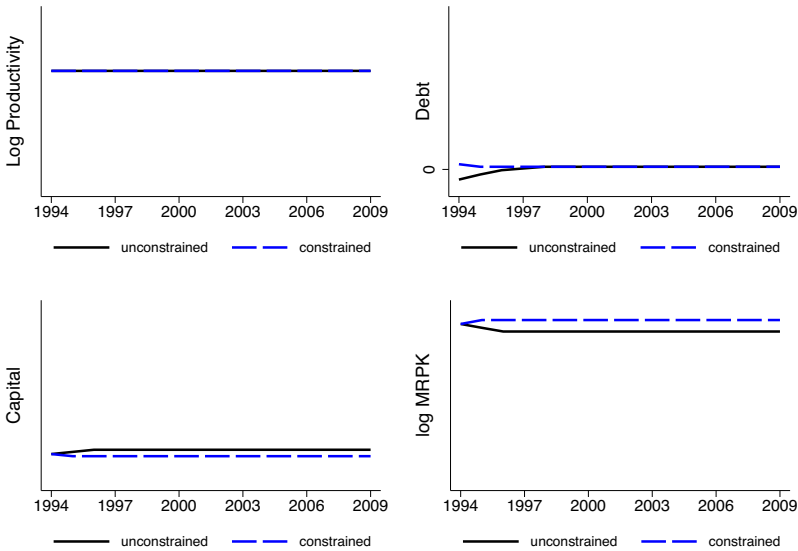


FIGURE XII

Real Interest Rates and Misallocation: An Illustrative Example (HoF Model)

*2. Higher Adjustment Costs.* The baseline HeF model generates larger increases in aggregate output, capital, and debt relative to the data. The magnitude of these increases is directly affected by the parameter  $\psi$  that controls for the magnitude of adjustment costs in capital accumulation. In the baseline HeF model we choose  $\psi$  such that the model replicates the within-firm responsiveness of capital growth to productivity observed in the microdata. To examine the robustness of our conclusions to alternative calibration strategies, we now choose  $\psi = 7.6$  such that the model reproduces the increase in aggregate capital observed in the data.<sup>42</sup>

Row 5 of Table VI shows that the model with higher adjustment costs generates a somewhat smaller decline in TFP relative to its efficient level than the baseline HeF model. The model with higher adjustment costs comes closer than the HeF model to matching the changes in MRPK dispersion, output, and debt

42. In the model with  $\psi = 7.6$ , the mean adjustment cost equals 1.8% of value added conditional on adjusting the capital stock, which is lower than the 2.8% we found with  $\psi = 3.2$ . This is because firms choose to adjust by less conditional on adjusting. With  $\psi = 7.6$ , the mean frequency of adjustment is 14%.

observed in the data. We note, however, that the model with higher adjustment costs performs worse than the baseline HeF model in terms of matching firm-level moments (see [Online Appendix E.3](#)).

3. *Exogenous Labor Wedges.* We now extend the baseline HeF model to allow for MRPL dispersion across firms. We begin our analysis with a model of MRPL dispersion arising from exogenous labor wedges. The labor wedge takes the form of a time-varying proportional tax that firms pay on their compensation to labor. The stochastic process for the labor wedge is independent of calendar time and, as a result, MRPL dispersion is constant in the model. We calibrate this process to match the standard deviation and the first-order autocorrelation of  $\log(\text{MRPL})$  in the firm-level data.

Row 6 of [Table VI](#) shows that the model with exogenous labor wedges performs better than the baseline HeF model in terms of generating a larger decline in TFP relative to its efficient level and a smaller increase in the dispersion of the  $\log(\text{MRPK})$ . The responses of aggregate output, capital, and debt are similar between the two models. We also note that the model with exogenous labor wedges performs roughly as well as the baseline model in terms of matching the micro moments in the data.

4. *Overhead Labor.* A model with overhead labor endogenously generates MRPL dispersion across firms. Such a model would, however, imply changes in MRPL dispersion over time in response to shocks. We, therefore, started with the simpler approach of specifying exogenous labor wedges at the firm level and assumed that the dispersion of these wedges is constant over time. To confirm the robustness of our results to the modeling choice that generates MRPL dispersion, we now adopt the production function in [Bartelsman, Haltiwanger, and Scarpetta \(2013\)](#) given by  $y_{it} = Z_{it}k_{it}^{\alpha}(\ell_{it} - \phi_{\ell})^{1-\alpha}$ , where  $\phi_{\ell}$  denotes overhead labor. With this production function, the measured marginal revenue product of labor varies across firms and over time.

Row 7 of [Table VI](#) shows that the model with overhead labor generates nearly identical aggregate responses as the baseline model. Overhead labor does not interact in a significant quantitative way with firm investment and debt decisions and, therefore, moments related to leverage, net worth, capital, and MRPK are similar between the model with overhead labor and the baseline model without MRPL dispersion.

5. *Unmeasured Capital.* The standard deviation of  $\log(\text{MRPK})$  in the baseline HeF model (0.26) is much lower than in the data (0.88). We incorporate measurement error in firms'



capital in order to generate the higher level of MRPK dispersion in the data. We consider the production function  $y_{it} = Z_{it} (k_{it} + q_{it})^\alpha \ell_{it}^{1-\alpha}$ , where  $q_{it}$  denotes measurement error in firms' capital reflecting, for example, unmeasured intangible assets or overhead capital in production.

Row 8 of [Table VI](#) shows aggregate responses in the model with measurement error in capital. The model matches closer the observed changes in TFP relative to its efficient level and in the dispersion of the log (MRPK) than the baseline model. However, the model also generates a stronger response in aggregate output, capital, and debt. In terms of matching untargeted firm-level moments, the model with unmeasured capital performs similarly to the baseline model.

6. *Alternative  $r$  Process.* In the baseline HeF model, all changes in the real interest rate  $r_t$  were unexpected to firms and perceived as permanent. We examine the robustness of our results to an alternative stochastic process for  $r_t$ . We assume that the initial decline in  $r_t$  in 1994 was unexpected but that from that date on firms expect  $r_t$  to evolve according to an AR(1) process,  $r_t = (1 - \rho^r)\bar{r} + \rho^r r_{t-1} + \sigma^r \epsilon_t^r$ , where  $\bar{r}$  denotes the mean real interest rate,  $\rho^r$  denotes the persistence of the process, and  $\sigma^r$  denotes the standard deviation of shocks to the real interest rate. Using data from our sample period, we estimate  $\bar{r} = 0.03$ ,  $\rho^r = 0.50$ , and  $\sigma^r = 0.009$ . We continue to feed into the model the path of  $r_t$  shown in [Figure VII](#).

Row 9 of [Table VI](#) shows that, relative to our baseline model, the model with the alternative process for  $r_t$  generates a somewhat smaller decline in TFP relative to its efficient level but comes closer to matching the change in MRPK dispersion observed in the data. The model also generates smaller responses of output, capital, and debt relative to the baseline model. Finally, firm-level moments generated by the model with the alternative  $r_t$  process are similar to those in the baseline model. Our conclusion is that changing the process for the real interest rate process does not affect significantly our results.

### *VI.E. Other Shocks and Postcrisis Dynamics*

We next compare the decline in the real interest rate to other shocks in terms of their ability to generate changes in MRPK dispersion, TFP, and capital flows that resemble the changes observed in the data. We do not assess quantitatively the

TABLE VII  
AGGREGATE RESPONSES AND VARIOUS SHOCKS

	$\frac{\text{TFP}}{\text{TFP}^e}$	Std. dev. (log MRPK)	$K$	$B - K$
1. Data (precrisis period)	↓	↑	↑	↑
2. Real interest rate decline ( $\downarrow r$ )	↓	↑	↑	↑
3. Financial liberalization ( $\uparrow \lambda_0$ or $\uparrow \lambda_1$ )	↑	↓	↑	↑
4. Uncertainty decline ( $\downarrow \sigma$ )	↑	↓	—	↑
5. Data (postcrisis period)	↓	↑	↓	↓

performance of the model under these other shocks because we simply wish to make the qualitative point that the decline in the real interest rate generates directional responses of dispersion, TFP, and capital flows similar to those observed in the data whereas these other shocks do not.

Table VII summarizes the results. In the precrisis period (row 1), we observe a decline in TFP relative to its efficient level, an increase in the dispersion of the return to capital, and capital inflows. As discussed before, our model with size-dependent financial frictions matches the dynamics of these variables over time (row 2).

It is often conjectured that countries in the South received large capital inflows following a financial liberalization associated with the euro convergence process. Row 3 of Table VII evaluates the implications of such a development through the lens of our baseline HeF model. A financial liberalization episode in our model, modeled as a relaxation of the borrowing constraint (an increase in either  $\lambda_0$  or  $\lambda_1$ ), is associated with an increase in borrowing that allows previously constrained firms to increase their capital accumulation. Therefore, this shock captures the common view that the adoption of the euro was associated with capital inflows to the South.

The relaxation of the borrowing constraint generates a more efficient allocation of resources and is associated with increases in TFP relative to its efficient level and a decrease in the dispersion of the return to capital. This contradicts the key fact that capital inflows in Spain were accompanied by a decline in TFP relative to its efficient level and an increase in MRPK dispersion. The prediction that financial liberalization episodes are associated with increasing productivity is common in models with financial frictions (see, e.g., Buera, Kaboski, and Shin 2011; Midrigan and Xu

2014).<sup>43</sup> While we do not deny that such a financial liberalization may have taken place, our results point out that the decline in the real interest rate is more important for understanding the evolution of productivity in Spain in the first few years after the adoption of the euro.<sup>44</sup>

Can changes in the process governing firm productivity explain Spain's precrisis experience? In a model with risky capital accumulation and adjustment costs, a higher dispersion of productivity across firms leads to higher MRPK dispersion (Asker, Collard-Wexler, and De Loecker 2014). However, we find that the standard deviation of productivity shocks across firms,  $\sigma$ , was generally decreasing between 1999 and 2007.<sup>45</sup> As row 4 of Table VII shows, a decrease in  $\sigma$  generates an increase in debt as the precautionary saving motive weakens but is associated with a lower dispersion of MRPK and a higher TFP relative to its efficient level.

To summarize, our model shows that the decline in the real interest rate generates dynamics in capital flows, dispersion, and TFP that resemble the dynamics observed in the data. Changes in financial frictions or in the productivity process are not able to match even qualitatively the trends observed in the data between 1999 and 2007.<sup>46</sup>

43. The directional response of the dispersion of the log (MRPK) to various shocks is a general feature of these models with financial frictions and not an artifact of specific features of our model. In Online Appendix F we examine a similar model to Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2015) without a size-dependent borrowing constraint, risky capital accumulation, and adjustment costs. Within this simpler environment, we derive closed-form solutions for the response of the dispersion of the log (MRPK) to various shocks and show that all responses have the same sign as the responses generated by our richer model.

44. Our results relate to Aoki, Benigno, and Kiyotaki (2010) who develop a model in which financial liberalization may lead to a decline in TFP. With an underdeveloped financial system, TFP in their model declines because capital inflows are intermediated by unproductive entrepreneurs who expand relative to productive entrepreneurs.

45. To obtain idiosyncratic productivity shocks, we follow Bloom et al. (2012) and estimate the firm-level AR(1) process shown in equation (23). We find that the dispersion of productivity shocks is decreasing between 1999 and 2007 using either the productivity measure  $\log \hat{Z}$  estimated with the Wooldridge (2009) extension of the Levinsohn and Petrin (2003) methodology or the productivity measure  $\log \tilde{Z}$  defined in equation (11). We find these patterns both in the permanent sample and the full sample of firms.

46. An increase in the mean level of firm productivity or demand  $Z^A$  generates an increase in capital accumulation, a higher dispersion of the MRPK, and lower TFP relative to its efficient level. However, the increase in  $Z^A$  would generate an

Uncertainty and deleveraging are likely to be important factors in the postcrisis period between 2008 and 2012. In [Section III](#) we documented an acceleration of the increase in the dispersion of the log(MRPK) in the postcrisis period and a continuation of the decline in TFP relative to its efficient level. Further, row 5 of [Table VII](#) shows a reversal of capital flows during the postcrisis period. While some changes in dispersion and TFP can plausibly reflect transitional dynamics from the decline in the real interest rate, the reversal of capital flows implies an important role for other shocks during the postcrisis period. Indeed, we find that the dispersion of productivity shocks  $\sigma$  increased sharply after 2007. Additionally, borrowing constraints were tightened during the crisis. As rows 3 and 4 of [Table VII](#) show, an increase in  $\sigma$  and a decline in  $\lambda_0$  or  $\lambda_1$  lead to capital outflows, higher MRPK dispersion, and lower TFP relative to its efficient level. We leave for future research an evaluation of the quantitative role of uncertainty and deleveraging shocks for the postcrisis dynamics.

## VII. EVIDENCE FROM OTHER EURO COUNTRIES

In this section we extend parts of our empirical analyses to Italy (1999–2012), Portugal (2006–2012), Germany (2006–2012), France (2000–2012), and Norway (2004–2012). In [Online Appendix A](#) we present coverage statistics and the size distribution of firms for all countries. The coverage is high and averages from roughly 60% to more than 90% of the coverage observed in Eurostat. The exception is Germany, for which we have roughly one-third of the wage bill starting in 2006. As with the case of Spain, our sample for other countries is also broadly representative in terms of contributions of small and medium-sized firms to economic activity.

We present the trends in dispersion of factor returns and TFP losses in [Online Appendix G](#). The conclusion emerging from this analysis is that countries in the South share some similar trends in the MRPK dispersion and the TFP loss due to misallocation. By contrast, these trends differ significantly in the North. Specifically, we find a significant increase in the standard deviation of log(MRPK) in Spain and Italy before the crisis. During the same period, France experienced a smaller increase. We

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increase in the level of TFP, which contradicts the fact that TFP in the data did not increase.

document significant increases in the dispersion of the log (MRPK) in all countries of the South during and after the crisis. By contrast, we do not observe such trends in the North. In addition, we do not see significant changes in the dispersion of the log (MRPL) in any country in our sample. This holds both during the pre-crisis period and during the postcrisis period. Finally, similarly to Spain, we observe significant declines in log (TFP) in Italy's full sample throughout the period, in Italy's permanent sample during the crisis, and in Portugal's permanent sample that mostly covers the crisis period. We do not observe trend declines in Germany, France, or Norway.

### VIII. CONCLUSIONS

The aim of this article is to shed light on the joint dynamics of capital flows, dispersion of factor returns, and productivity in South Europe following the adoption of the euro. The first contribution of our work is to bring empirical evidence on the dynamics of misallocation over time. Employing a large and representative sample of Spanish manufacturing firms, we document a significant increase in MRPK dispersion over time and a decline in TFP relative to its efficient level. Interestingly, we do not find an important role for a changing dispersion of the returns to labor.

Our second contribution is to link patterns of capital misallocation at the micro level to firm-level financial decisions and to the macroeconomic implications of financial frictions. We have developed a model with heterogeneous firms, financial frictions that depend on firm size, and capital adjustment costs that matches closely various moments estimated from production and balance sheet data. Using this calibrated model, we illustrate how the decline in the real interest rate that started in the early to mid-1990s generates transitional dynamics that are similar to the dynamics of MRPK dispersion, TFP relative to its efficient level, and capital flows during the precrisis period.

Finally, we have documented that trends in the dispersion of the return to capital and in productivity losses from misallocation differ significantly between countries in South Europe and countries in the North. We find these differences suggestive, given that firms in the South are likely to operate in less well-developed financial markets. A more complete analysis of the differences between the South and the North remains a promising avenue for future research.

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SUPPLEMENTARY MATERIAL

An [Online Appendix](#) for this article can be found at *The Quarterly Journal of Economics* online. Data and code replicating the tables and figures in this paper can be found in Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez (2017), in the Harvard Dataverse, [doi:10.7910/DVN/FGRWIG](https://doi.org/10.7910/DVN/FGRWIG).

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