Defaultable Debt, Interest Rates and the Current Account*

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Abstract

World capital markets have experienced large scale sovereign defaults on a number of occasions. In this paper we develop a quantitative model of debt and default in a small open economy. We use this model to match four empirical regularities regarding emerging markets: defaults occur in equilibrium, interest rates are countercyclical, net exports are countercyclical, and interest rates and the current account are positively correlated. We highlight the role of the stochastic trend in emerging markets, in an otherwise standard model with endogenous default, to match these facts.

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1 Introduction

World capital markets have experienced large scale sovereign defaults on a number of occasions, the most recent being Argentina's default in 2002. This latest crisis is the fifth Argentine default or restructuring episode in the last 180 years.[1] While Argentina may be an extreme case, sovereign defaults occur with some frequency in emerging markets. A second set of facts about emerging markets relates to the behavior of the interest rates at which these economies borrow from the rest of the world and their current accounts. Interest rates and the current account are strongly countercyclical and positively correlated to each other. That is, emerging markets tend to borrow more in good times and at lower interest rates as compared to slumps. These features contrast with those observed in developed small open economies.

In this paper we develop a quantitative model of debt and default in a small open economy, which we use to match the above facts. Our approach follows the classic framework of Eaton and Gersovitz (1981) in which risk sharing is limited to one period bonds and repayment is enforced by the threat of financial autarky. In all other respects the model is a standard small open economy model where the only source of shocks are endowment shocks. In this framework, we show that the model's ability to match certain features in the data improve substantially when the productivity process is characterized by a volatile stochastic trend as opposed to transitory fluctuations around a stable trend. In a previous paper (Aguiar and Gopinath (2004)), we document empirically that emerging markets are indeed more appropriately characterized as having a volatile trend. The fraction of variance at business cycle frequencies explained by permanent shocks is shown to be around 50% in a small developed economy (Canada) and more than 80% in an emerging market (Mexico).

To isolate the importance of trend volatility in explaining default, we first consider a standard business cycle model in which shocks represent transitory deviations around a stable trend. We find that default is extremely rare, occurring roughly twice every 2,500 years. The weakness of the

[1] See Reinhart, Rogoff and Savastana (2003).

standard model begins with the fact that autarky is not a severe punishment, even adjusting for the relatively large income volatility observed in emerging markets. The welfare gain of smoothing transitory shocks to consumption around a stable trend is small. This in turn prevents lenders from extending debt, which we demonstrate through a simple calculation a la Lucas (1987). We can support a higher level of debt in equilibrium by assuming an additional loss of output in autarky. However, in a model of purely transitory shocks, this does not lead to default at a rate that resembles those observed in many economies.

The intuition behind why default occurs so rarely in a model with transitory shocks and a stable trend is described in Section 3. The decision to default rests on the difference between the present value of utility (value function) in autarky versus that of financial integration. Quantitatively, the level of default that arises in equilibrium depends on the *relative* sensitivity of the two value functions to endowment shocks. When the endowment process is close to a random walk there is limited need to save out of additional endowment, leaving little difference between financial autarky and a good credit history, regardless of the realization of income. At the other extreme, if the transitory shock is *iid* over time, then there is an incentive to borrow and lend, making integration much more valuable than autarky. However, an *iid* shock has limited impact on the entire present discounted value of utility, and so the difference between integration and autarky is not sensitive to the particular realization of the *iid* shock. At either extreme, therefore, the decision to default is not sensitive to the realization of the shock. Consequently, when shocks are transitory, the level of outstanding debt – and not the realization of the stochastic shock –is the primary determinant of default. This is reflected in financial markets by an interest rate schedule that is extremely sensitive to quantity borrowed. Borrowers internalize the steepness of the "loan supply curve" and recognize that an additional unit of debt at the margin will have a large effect on the cost of debt. Agents therefore typically do not borrow to the point where default is probable.

On the other hand, a shock to trend growth has a large impact on the two value functions (because of the shock's persistence) and on the *difference* between the two value functions. The latter effect arises because a positive shock to trend implies that income is higher today, but even higher tomorrow, placing a premium on the ability to access capital markets to bring forward anticipated income. In this context, the decision to default is relatively more sensitive to the particular realization of the shock and less sensitive to the amount of debt. Correspondingly, the interest rate is less sensitive to the amount of debt held. Agents are consequently willing to borrow to the point that default is relatively likely. This theme is developed in Section 4.

The next set of facts concerns the phenomenon of countercyclical current accounts and interest rates. In the current framework where all interest rate movements are driven by changes in the default rate, the steepness of the interest rate schedule makes it challenging to even qualitatively match the positive correlation between interest rates and the current account. This is because, on the one hand, an increase in borrowing in good states (countercyclical current account) will, all else equal, imply a movement along the heuristic "loan supply curve" and a sharp rise in the interest rate. On the other hand, if the good state is expected to persist, this lowers the expected probability of default and is associated with a favorable shift in the interest rate schedule. To generate a positive correlation between the current account and interest rates we need the effect of the shift of the curve to dominate the movement along the curve. A stochastic trend is again useful in matching this fact since the interest rate function tends to be less steeply sloped and trend shocks have a significant effect on the probability of default. Accordingly, in our benchmark simulations, a model with trend shocks matches the empirical feature of a positive correlation between the interest rate and the current account. The model with transitory shocks however fails to match this fact. The prediction for which both models perform poorly is in matching the volatility of the interest rate process.[2]

The model with shocks to trend generates default roughly once every 125 years, which is a ten-fold

[2] The additional observed volatility may be due to a volatile risk premium, as suggested by Broner, Lorenzoni, and Schmukler (2004).

improvement over the standard model but still shy of the observed pattern for chronic defaulters. We bring the default rate closer to that observed empirically for Latin America by introducing third-party bail-outs. Realistic bailouts raise the rate of default dramatically – bailouts up to 18% of GDP lead to defaults once every 27 years. However, the subsidy implied by bailouts breaks the tight linkage between default probability and the interest rate. Interest rate volatility is therefore an order of magnitude below that observed empirically.

The business cycle behavior of markets in which agents can choose to default has received increasing attention in the literature. The approach we adopt here is a dynamic stochastic general equilibrium version of Eaton and Gersovitz (1981) and is similar to the formulations in Chatterjee et al (2002) on household default and Arellano (2003) on emerging market default. Our point of distinction from the previous literature is the emphasis we place on the role of the stochastic trend in driving the income process in emerging markets. We find that the presence of trend shocks substantially improves the ability of the model to generate empirically relevant levels of default. Moreover, we obtain the coincidence of countercyclical net exports, countercyclical interest rates and the positive correlation between interest rates and current account observed in the data. This is distinct from what is obtained in Arellano (2003) and Kehoe and Perri (2002).

In the next section we describe empirical facts for Argentina. Section 2 describes the model environment, parameterization and solution method. Section 3 describes the model with a stable trend and its predictions. Section 4 describes the model with a stochastic trend and performs sensitivity analysis. Section 5 examines the effect of third party bailouts on the default rate and Section 6 concludes.

1.1 Empirical Facts

Reinhart et al (2003) document that among emerging markets with at least one default or restructuring episode between 1824 and 1999, the average country experienced roughly 3 crises every 100 years. The same study documents that the external debt to GDP ratio at the time of default or restructuring averaged 71%. A goal of any quantitative model of emerging market default is to generate a fairly high frequency of default coinciding with an equilibrium that sustains a large debt to GDP ratio.

Table 1 documents business cycle features for Argentina over the period 1983.1 to 2000.2 using an HP filter with a smoothing parameter of 1600 for quarterly frequencies. A striking feature of the business cycle is the strong countercyclicality of net exports (-0.89) and interest rates spreads (-0.59). Interest rates and the current account are also strongly positively correlated (0.68). These features regarding interest rates, in addition to the high level of volatility of these rates, have been documented by Neumeyer and Perri (2004) to be true for several other emerging market economies and to contrast with the business cycle features of Canada, a developed small open economy. Aguiar and Gopinath (2004b) document evidence of the stronger countercyclicality of the current account for emerging markets relative to developed small open economies. In the model we emphasize the distinction between shocks to the stochastic trend and transitory shocks. This is motivated by previous research (Aguiar and Gopinath (2004b)) that documents that emerging markets are subject to more volatile shifts in stochastic trend as compared to a developed small open economy.

2 Model Environment

To model default we adopt the framework of Eaton and Gersovitz (1981). Specifically, we assume that international assets are limited to one period bonds. If the economy refuses to pay any part of the debt that comes due, we say the economy is in default. Once in default, the economy is forced into financial autarky for a period of time as punishment.

We begin our analysis with a standard model of a small open economy that receives a stochastic endowment stream, y_t. (We discuss a production economy in Section 4.1.). The economy trades a single good and single asset, a one period bond, with the rest of the world. The representative agent has CRRA preferences over consumption of the good:

$$u = \frac{c^{1-\gamma}}{1-\gamma}. \tag{1}$$

The endowment y_t is composed of a transitory component z_t and a trend Γ_t:

$$y_t = e^{z_t}\Gamma_t. \tag{2}$$

The transitory shock, z_t, follows an $AR(1)$ around a long run mean μ_z

$$z_t = \mu_z(1-\rho_z) + \rho_z z_{t-1} + \varepsilon_t^z \tag{3}$$

$|\rho_z| < 1$, $\varepsilon_t^z \sim N(0, \sigma_z^2)$, and the trend follows

$$\begin{aligned} \Gamma_t &= g_t\Gamma_{t-1} && (4) \\ \ln(g_t) &= (1-\rho_g)(\ln(\mu_g) - c) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g && (5) \end{aligned}$$

$\left|\rho_g\right| < 1$, $\varepsilon_t^g \sim N(0, \sigma_g^2)$, and $c = \frac{1}{2}\frac{\sigma_g^2}{1-\rho_g^2}$.

We denote the *growth rate* of trend income as g_t, which has a long run mean μ_g. The log growth rate follows an $AR(1)$ process with AR coefficient $\left|\rho_g\right| < 1$. Note that a positive shock ε^g implies a permanently higher level of output, and to the extent that $\rho_g > 0$, a positive shock today implies that the growth of output will continue to be higher beyond the current period. We assume that $E\left\{\lim_{t\to\infty} \beta^t(\Gamma_t)^{1-\gamma}\right\} = 0$ to ensure a well defined problem, where $0 < \beta < 1$ denotes the agent's discount rate.

Let a_t denote the net foreign assets of the agent at time t. Each bond delivers one unit of the good next period for a price of q this period. We will see below that in equilibrium q depends on a_t and the state of the economy. We denote the value function of an economy with assets a_t and access to international credit as $V(a_t, z_t, g_t)$ (see the Appendix for a formal derivation with more complete notation). At the start of the period, the agent decides whether to default or not. Let V^B

denote the value function of the agent once it defaults. The superscript B refers to the fact that the economy has a bad credit history and therefore cannot transact with international capital markets (i.e. reverts to financial autarky). Let V^G denote the value function given that the agent decides to maintain a good credit history this period. The value function of being in good credit standing at the start of period t with net assets a_t can then be defined as $V(a_t, z_t, g_t) = \max \left\langle V_t^G, V_t^B \right\rangle$. At the start of period t, an economy in good credit standing and net assets a_t will default only if $V^B(z_t, g_t) > V^G(a_t, z_t, g_t)$.

An economy with a bad credit rating must consume its endowment. However, with probability λ it will be "redeemed" and start the next period with a good credit rating and renewed access to capital markets. If redeemed, all past debt is forgiven and the economy starts off with zero net assets. We also add a parameter δ that governs the additional loss of output in autarky[3].

In recursive form, we therefore have:

$$V^B(z_t, g_t) = u((1-\delta)y_t) + \lambda\beta E_t V(0, z_{t+1}, g_{t+1}) + (1-\lambda)\beta E_t V^B(z_{t+1}, g_{t+1}) \tag{6}$$

where E_t is expectation over next period's endowment and we have used the fact that λ is independent of realizations of y. If the economy does not default, we have:

$$\begin{aligned} V^G(a_t, z_t, g_t) &= \max_{c_t} \{u(c_t) + \beta E_t V(a_{t+1}, z_{t+1}, g_{t+1})\} \\ s.t. c_t &= y_t + a_t - q_t a_{t+1} \end{aligned} \tag{7}$$

The Appendix derives some key properties of the value functions.

The international capital market consists of risk neutral investors that are willing to borrow or lend at an expected return of r^*, the prevailing world risk free rate. The default function $D(a_t, z_t, g_t) = 1$ if $V^B(z_t, g_t) > V^G(a_t, z_t, g_t)$ and zero otherwise. Then equilibrium in the capi-

[3]Rose (2002) finds evidence of a significant and sizeable (8% a year) decline in bilateral trade flows following the initiation of debt renegotiation by a country.

tal market implies

$$q(a_{t+1}, z_t, g_t) = \frac{E_t\{(1 - D_{t+1})\}}{1 + r^*}. \tag{8}$$

The higher the expected probability of default the lower the price of the bond.

To emphasize the distinction between the role of transitory and permanent shocks we present two extreme cases of the model described above. Model I will correspond to the case when the only shock is the transitory shock z_t and Model II to the case when the only shock has permanent effects, g_t.[4] Since few results can be analytically derived we discuss at the outset the calibration and solution method employed.

2.1 Calibration and Model Solution

Benchmark parameters that are common to all models are reported in Table 2A. Each period refers to a quarter with a quarterly risk free interest rate of 1%. The coefficient of relative risk aversion of 2 is standard. The probability of redemption $\lambda = 0.1$ implies an average stay in autarky of 2.5 years, similar to the estimate by Gelos *et al* (2003). The additional loss of output in autarky is set at 2%. We will see in our sensitivity analysis (Section 4.1) that high impatience is necessary for generating reasonable default in equilibrium. Correspondingly, our benchmark calibration sets $\beta = 0.8$. Authors such as Arellano (2003) and Chatterjee et. al. also employ similarly low values of β to generate default. The mean quarterly growth rate is calibrated to 0.6% to match the number for Argentina.

The remaining parameters characterize the underlying income process and therefore vary across models (Table 2B). To focus on the nature of the shocks, we ensure that the HP filtered income volatility derived in simulations of both models approximately match the same observed volatility in the data. In Model I, output follows an $AR(1)$ process with stable trend and an autocorrelation

[4] One of the reasons we consider the two extremes is to minimize the dimensionality of the problem, which we solve employing discrete state space methods. Using insufficient grids of the state space can generate extremely unreliable results in this set up.

coefficient of $\rho_z = 0.9$, which is similar to the values used in many business cycle models and $\sigma_z = 3.4\%$. We set the mean of log output equal to $-1/2\sigma_z^2$ so that average detrended output in levels is standardized to one. In Model II, $\sigma_z = 0$, $\sigma_g = 3\%$ and $\rho_g = 0.17$.

To solve the model numerically we use the discrete state-space method. We first recast the Bellman equations in detrended form and then discretize the state space. See the Appendix for proofs regarding the equivalence of the detrended and original problem. We approximate the continuous $AR(1)$ process for income with a discrete Markov chain using 25 equally spaced grids[5] of the original processes steady state distribution. We then integrate the underlying normal density over each interval to compute the values of the Markov transition matrix.

The asset space is discretized into 400 possible values. We ensured that the limits of our asset space never bind along the simulated equilibrium paths. The solution algorithm involves the following (see the Appendix for proofs regarding convergence of the algorithm):

(i) Assume an initial price function $q^0(a, z, g)$. Our initial guess is the risk free rate at each point in the state space.

(ii) Use this q^0 and an initial guess for $V^{B,0}$ and $V^{G,0}$ to iterate on the Bellman equations (6) and (7) to solve for the optimal value functions V^B, V^G, $V = \max\left\langle V^G, V^B \right\rangle$ and the optimal policy functions.

(iii) For the initial guess q^0, we now have an estimate of the default function $D^0(a, z, g)$. Next, we update the price function as $q^1 = \frac{E_t\{(1-D_{t+1})\}}{1+r^*}$ and using this q^1 repeat steps (ii) and (iii) until $|q^{i+1} - q^i| < \varepsilon$, where i represents the number of the iteration and ε is a very small number.

[5] It is important to span the stationary distribution sufficiently so as to include large negative deviations from the average even if these are extremely rare events because default is more likely to occur in these states.

3 Model I: Stable Trend

Model I assumes a deterministic trend ($\Gamma_t = (\mu_g)^t$) and the process for z_t is given by (3). In Figure 1A, we plot the difference between the value function with a good credit rating (V^G) and that of autarky (V^A) as a function of z for our calibration. The agent defaults when output is relatively low. The top panel of Figure 2A plots the region of default in (z, a) space. The line that separates the darkly shaded from the lightly shaded region represents combinations of z and a along which the agent is indifferent between defaulting and not defaulting. The darkly shaded region represents combinations of low productivity and negative foreign assets for which it is optimal to default.

For a given realization of z, clearly the agent is more likely to default at lower values of a. Since V^B refers to financial autarky, its value is invariant to a. Conversely, V^G is strictly increasing in assets. This follows straightforwardly from the budget constraint and strict monotonicity of utility. For each z, there is a unique point of intersection, say $\overline{a}(z)$, and the agent will default if foreign assets lie below $\overline{a}$.

The default decision as a function of z is less clear-cut. In the case when $\lambda = 0$, it must be the case that V^G should be at least as steep as V^B at the indifference point, for a given a. To see this, consider the value of an additional unit of endowment at the indifference point. The agent in autarky must consume this additional income. The agent with a good credit standing can consume it or save it. The larger opportunity set implies that the value of the additional endowment is at least as great for the agent in good credit standing. Hence, the slope of V^G relative to z must be (weakly) greater than the slope of V^B relative to z, at the indifference point. However, if $\lambda > 0$, then the agent in autarky has an outcome not available to the one in good credit standing, i.e. redemption with debt forgiveness. In this case, the previous argument's premise does not hold.

The fact that financial autarky is relatively attractive in bad states of the world is not a feature shared by alternative models based on Kehoe and Levine (1993). In that model, optimal "debt" contracts are structured so that the agent never chooses autarky. However, the participation constraint binds strongest in good states of nature, i.e. the states in which the optimal contract calls for payments by the agent. In the bond-only framework, contracted payments are not state contingent and therefore the burden of repayment is greatest in low endowment states.

3.1 Debt and Default Implications in Model I

A simple calculation *a la* Lucas (1987) quickly reveals that it is difficult to sustain a quantitatively realistic level of debt in a standard framework without recourse to additional punishment.[6] Consider our endowment economy in which the standard deviation of shocks to detrended output are roughly 4%. For this calculation, we stack the deck against autarky by assuming no domestic savings (capital or storage technology), that shocks are *iid*, and that autarky lasts forever. We stack the deck in favor of financial integration by supposing that integration implies a constant consumption stream (perfect insurance). In order to maintain perfect consumption insurance, we suppose that the agent must make interest payments of rB each period. We now solve for how large rB can be before the agent prefers autarky. We then interpret B as the amount of sustainable debt when interest payments are equal to r.

Specifically, let $Y_t = \overline{Y} e^{z_t} e^{-(\frac{1}{2})\sigma_z^2}$ where $z \sim N(0, \sigma_z^2)$ and *iid* over time. We ensure that $EY_t = \overline{Y}$ regardless of the volatility of the shocks. Then,

$$V^B = E \sum_t \beta^t \frac{Y_t^{1-\gamma}}{1-\gamma} = \frac{(\overline{Y} e^{-(\frac{1}{2})\gamma\sigma_z^2})^{1-\gamma}}{(1-\gamma)(1-\beta)}. \tag{9}$$

Assuming that financial integration results in perfect consumption insurance,

$$V^G = E \sum_t \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} = \frac{(\overline{Y} - rB)^{1-\gamma}}{(1-\gamma)(1-\beta)}. \tag{10}$$

The economy will not default as long as $V^G \geq V^B$, or $\frac{rB}{\overline{Y}} \leq 1 - \exp(-(\frac{1}{2})\gamma\sigma_z^2)$. The volatility of detrended output for Argentina is 4.08% (i.e. $\sigma_z^2 = 0.0408^2 = 0.0017$). For a coefficient of relative

[6] See Obstfeld and Rogoff (1996) and Mendoza (1992) for alternative calculations of the welfare gains from financial integration.

risk aversion of 2, this implies the maximum debt payments as a percentage of GDP is 0.17%. Or, at a quarterly interest rate of 2%, debt cannot exceed 8.32% of output.

Our simulated model will be shown to support higher debt levels because we impose an additional loss of δ percent of output during autarky. Introducing such a loss into the above calculation implies a debt cutoff of $\frac{rB}{\overline{Y}} \leq 1 - (1-\delta)\exp(-(\frac{1}{2})\gamma\sigma_z^2)$. If $\delta = 0.02$, we can support debt *payments* of 20% of GDP, which implies a potentially large debt to GDP ratio. It is clear that to sustain any reasonable amount of debt in equilibrium in a standard model, we need to incorporate punishments beyond the inability to self-insure, particularly since in reality financial integration does not involve full insurance and autarky does not imply complete exclusion from markets.

A second implication of the model is that default rarely occurs in equilibrium. This arises because of the steepness of the interest rate schedule and the fact that the agent internalizes the effect of his borrowing on the interest rate he must pay. Figure 3A plots the q schedule as a function of assets for the highest and lowest realizations of z. Over asset regions for which agents never default, the implied interest rate is the risk free rate ($q = \frac{1}{1+r^*}$). However, the schedule is extremely steep over the range of assets for which default is possible. Consequently, even when the current endowment shock is below average the agent does not borrow much. At the margin, the borrower recognizes that an additional unit of debt raises the average cost of debt by the slope of the interest rate schedule.

The steepness of the interest rate schedule is ultimately tied to the persistence of the endowment process. Let $\overline{z}(a)$ denote the threshold endowment below which the agent defaults for the given asset level. That is, $\overline{z}$ is the line separating the shaded region from the unshaded region in Figure 2. For a given a_{t+1}, we can then express the probability of default at time $t+1$ as $\Pr(z_{t+1} < \overline{z}(a_{t+1})|z_t)$, and correspondingly $q_t(a_{t+1}, z_t) = \frac{(1-\Pr(z_{t+1}<\overline{z}(a_{t+1})|z_t))}{(1+r^*)}$. Suppose the investor is considering saving an additional Δa of assets. The change in the interest rate is given by

$$q(a_{t+1}+\Delta a, z_t) - q(a_{t+1}, z_t) = \frac{-1}{1+r^*} \sum_{\overline{z}(a) \le z_{t+1} \le \overline{z}(a+\Delta a)} \pi(z_{t+1}|z_t). \tag{11}$$

where $\pi(z_{t+1}|z_t)$ represents the probability that z_{t+1} will happen conditional on z_t. From Figure 2, we see that quantitatively, the distance between $\overline{z}(a)$ and $\overline{z}(a+\Delta a)$ for small Δa is extremely large. That is, the steepness of the $\overline{z}(a)$ translates into the steepness of the q schedule.

We now link the shape of $\overline{z}(a)$ to the nature of the shock process. Recall that $\overline{z}(a)$ represents combinations of a and z for which the agent is indifferent to default, i.e., $V^G = V^B$. An increase in a raises only V^G. Therefore, to maintain equality, we need to lower $\bar{z}$ (recall that V^G is more sensitive to z at the indifference point). How much $\bar{z}$ needs to fall depends on the difference in sensitivity: $\frac{\Delta V^G}{\Delta z} - \frac{\Delta V^B}{\Delta z}$, where $\Delta V^j, j = G, B$, is the change in V^j induced by the change in z. In the case of Model I, this difference tends to be a very small number. This can be seen from Figure 1A that plots the difference between V^G and V^B across z (for a given a). This implies that the slopes of V^G and V^B with respect to z are not that different.

The similarity in slopes results from the underlying process for z. Suppose that z is a random walk. In this case, a shock to z today is expected to persist indefinitely and will have a large impact on expected lifetime utility. However, with a random walk income process there is limited need to save out of additional endowment (the only reason would be precautionary savings and the insurance provided by the option to default). This implies an additional unit of endowment will be consumed, leaving little difference between financial autarky and a good credit history. This issue arises for strictly transitory shock processes as well. Consider the other extreme and suppose that z is *iid* over time. Then there is a stronger incentive to borrow and lend. However, the lack of persistence implies the impact of an additional unit of endowment today is limited to its effect on current endowment, resulting in a limited impact on the entire present discounted value of utility. That is, both ΔV^G and ΔV^B are relatively small and therefore so is the difference. Therefore, whether a shock follows a random walk or is *iid*, a given shock realization has little impact on the difference between V^G

and V^B. Consequently, a large movement in $\overline{z}$ is necessary to maintain equality between V^G and V^B for a given change in assets.

The key point is that in our benchmark Model I, the q schedule is extremely steep over the relevant range of borrowing. Moreover, the slope increases dramatically as we move into the range of potential default. Therefore, a large implicit demand for borrowing does not result in additional borrowing and a high probability of default. Instead, it generates minimal borrowing and a large movement in the slope of the interest rate function. The flip side of this is that net exports are extremely stable.

3.2 Business Cycles Implications in Model I

Table 3, column 3A, reports key business cycle moments from Model I.[7] Default is a rare event as it occurs on average only two times in 10,000 periods (i.e., once every 2,500 years). Net export and interest volatility is much lower than in the data. The model supports a maximum debt to GDP ratio of 26%.

A typical feature of these models is that the current account and the interest rate tend to be negatively correlated, a counterfactual implication. This follows from the steepness of the interest rate function. That is, if the agent borrows more in good states of the world (countercyclical current account) then one effect is for the interest rate to increase as the agent moves up the "loan supply curve". The countering effect is that a persistent good state can imply a lower probability of default and therefore a shift down in the q schedule. To generate the empirical fact that countries borrow more in good times at lower interest rates we need the second effect to dominate the first. However, the steepness of the q schedule in Model I makes this a less likely outcome. Consequently, in

[7] To analyze the model economy's stationary distribution we simulate the model for 10,000 periods and extract the last 500 observations to rule out any effect of initial conditions. We then log and HP filter the simulated series in the same way as the empirical data, using a HP filter smoothing parameter of 1600. The reported numbers are averages over 500 such simulations.

our parameterization of Model I, we obtain a countercyclical current account as the data suggests, however the interest rate process is now procyclical.

We have adopted a relatively persistent process for income that generates a countercyclical current account, but at the cost of procyclical interest rates. That the current account can be countercyclical even in an endowment economy with purely transitory shocks highlights the endogenous response of interest rates. When the income shock is temporarily high, there is the typical incentive to save to smooth consumption. In addition there is the interest rate effect that works through shifts in the interest rate function. That is, all else equal, the expected probability of default is lower when the current income state is high and expected to persist. This was seen in Figure 3A where the price function shifts in for high z. What is less visible, but is also the case, is that the slope of the interest rate function also is reduced. While q is countercyclical in equilibrium, we can calculate the *total* marginal cost of borrowing an additional unit along the equilibrium path and this is found to be countercyclical. This is then consistent with households wanting to borrow more when income is temporarily high. There are alternative parameterizations of Model I that produces a countercyclical interest rate process, as called for by the data. However, this occurs only when the current account is procyclical.[8]

4 Model II: Stochastic Trend

In Model II: $y_t = \Gamma_t$ where we let Γ_t vary stochastically as in (4) and (5). The behavior of this model is captured by the simulation results reported in Table 3, column 3B. One important distinction between the model with stochastic trend and Model I is the rate of default in equilibrium. Specifically, the rate of default increases by a factor of ten. The reason for this can be seen by

[8] For example, the countercyclical interest rate found in Arellano (2004) comes with a pro-cyclical current account. While introducing investment may generate a countercyclical current account, it is not clear that the countercyclical interest rate will be invariant to this extension.

contrasting the behavior of the difference between V^B and V^G between Figure 1A (Model I) and Figure 1B (Model II). High and low states of the growth shock will have substantially different effects on life time utility. Moreover, with persistent shocks, the value of financial integration will be high. Consequently, (V^G-V^B) has a greater slope with respect to the g shocks (Figure 1B) as compared to z shocks (Figure 1A). By the logic introduced in the context of Model I, this in turn suggests $\overline{z}(a)$ has a smaller slope. This is seen in Figure 2. Compared to the figure in the top panel, the region of default is larger and the slope of the line of indifference is smaller in the case of growth shocks. This then translates into a less steep q function as seen in Figure 3B. Moreover, it also suggests that default will occur in equilibrium with more frequency. That is, an implicit increase in the demand for borrowing is not completely offset by a change in slope of the interest rate function, but rather translates into additional borrowing and a higher rate of default.[9]

The results of the simulation of the model are reported in Table 3. Some improvements over Model I are immediately apparent. Both the current account and interest rates are countercyclical and positively correlated (though the magnitudes are lower than in the data). A positive shock increases output today, but increases output tomorrow even more (due to the persistence of the growth rate). This induces agents to borrow in good times. In Figure 3, we plot the q schedule as a function of (detrended) a for a low and high value of g. We see that a positive shock lowers the interest rate (raises q) for all levels of debt. In the reported parameterization, the shift in the q schedule dominates the movement along the schedule induced by additional borrowing. Correspondingly, the volatility of interest rates increases by a factor of 2.5. Net export volatility rises by a factor of 5 to match more closely the volatility in the data. However, interest rate volatility still remains well below that observed empirically for Latin America.

[9] It is also the case that a volatile trend generates volatility in consumption and an increased demand for the insurance a defaultable bond provides, implying higher default rates as well. However, quantitatively, this effect is small relative to that induced by the change in the shape of the interest rate function. See Aguiar and Gopinath (2004a) for a full discussion.

4.1 Sensitivity Analysis

In this section we perform sensitivity tests within the framework of Model II. First, we consider a case with endogenous labor supply. The production function now takes the form $y_t = e^{z_t}\Gamma_t L_t^{\alpha}$. We set $\alpha = 0.68$. The utility function takes the GHH form (from Greenwood et al 1988)

$$u_t = \frac{\left(c_t - \frac{1}{\omega}\mu_g \Gamma_{t-1} L_t^{\omega}\right)^{1-\gamma}}{1-\gamma}, \tag{12}$$

The ω parameter is calibrated to 1.455 implying an elasticity of labor supply of 2.2. This is the value employed in previous studies (Mendoza(1991), Neumeyer and Perri (2004)). The results for the simulation are reported in Table 4. The number of defaults are lower compared to the endowment model, however the business cycle moments line up as in the data.

The importance of a high level of impatience in generating reasonable levels of default is also seen in Table 4 where we report the results for higher levels of β. As β increases the number of defaults drop from 23 to 11 per 2500 years.

Default has an insurance (i.e. state contingent) component absent from a risk-free bond. The agent only repays in the good (nondefault) states of nature. While the agent cannot explicitly move resources across states of nature in the next period, she can move resources from good states next period forward to today, leaving resources available in bad/default states next period unaffected. Given that default provides insurance, one would expect that the value of this insurance would increase with uncertainty. Table 5 explores this conjecture. For each model, we doubled the innovation variance relative to the case reported in Table 3. We see that the rate of default roughly doubles relative to the benchmark case for both models. It is interesting to note that despite the increased rate of default, the higher volatility economies do not hold more debt in equilibrium, reflecting that these economies face a different interest rate function.

Finally, we explore the sensitivity of our results to the cost of autarky parameter δ. The last two columns of Table 5 report simulations of the two model economies where we have set $\delta = 0.005$

rather than 0.02. As expected, the lower cost of autarky results in very low debt levels observed in equilibrium, where instead of roughly 25% debt to income ratios, we now have ratios closer to 5%. This lack of borrowing is also reflected in the lower volatility of net exports and interest rates.

5 Third Party Bailouts

Incorporating shocks to trend has improved the model's performance on a number of dimensions. However, the default rate of once every 125 years remains low, at least compared to the track record of many Latin American countries. In this section, we try to improve on Model II by augmenting the model with a phenomenon observed in many default episodes – bailouts. For example, Argentina received a $40 billion bailout in 2001 from the IMF, an amount nearly 15% of Argentine GDP in 2001. Such massive bailouts must influence the equilibrium debt market studied in the previous two sections.

We model bailouts as a transfer from an (unmodelled) third party to creditors in the case of default. While in practice bailouts often tend to nominally take the form of loans, we assume that bailouts are grants. To the extent that such loans in practice are extended at below market interest rates, they incorporate a transfer to the defaulting country. We also assume that creditors view bailouts as pure transfers. Again, it may be the case in practice that creditors ultimately underwrite the bailouts through tax payments. A reasonable assumption is that creditors do not internalize this aspect of bailouts.

Specifically, bailouts take the following form. Creditors are reimbursed the amount of outstanding debt up to some limit a^*. Any unpaid debt beyond a^* is a loss to the creditor. From the creditors viewpoint, therefore, every dollar lent up to a^* is guaranteed. Any lending beyond that has an expected return determined by the probability of default. Specifically, the break even price of debt solves

$$q_t = \frac{1}{1+r^*}\left\{\min\left\langle 1, \frac{a^*}{a}\right\rangle + E_t\{1-D_{t+1}\}\max\left\langle 1-\frac{a^*}{a}, 0\right\rangle\right\}.$$

The presence of bailouts obviously implies that debt up to a^* carries a risk free interest rate. Moreover, the probability of default is used to discount only that fraction of debt that exceeds the limit. The net result is to shift up and flatten the q schedule. To consider why the q schedule is flatter, consider the case without bailouts, i.e. $a^* = 0$. Each additional dollar of debt raises the probability of default. As default implies zero repayment, this lowers the return on all debt. However, with bailouts, an increase in the probability of default affects only the return on debt beyond a^*. While this may have a large impact on the return of the marginal dollar, the sensitivity of the average return is mitigated by the fact that part of the debt is guaranteed. From the agent's perspective, bailouts subsidize default. The increase in the rate of default is not surprising given that bailouts are a pure transfer from a third party.

This intuition is confirmed in our simulation results reported in Column 3C of Table 3. We calibrate a^* so that the maximum bailout is 18% of (mean detrended) output and we now set the time preference rate at 0.95 so that impatience rates are not as high as in the previous benchmark simulations. We see that the agent now defaults roughly once every 27 years, similar to the Argentine default rate. The increased rate of default however does not raise interest rate volatility. This reflects the fact that bailouts insulate interest rates from changes in the probability of default. In this sense the model with bailouts still leaves unexplained the high interest rate volatility that emerging markets face.

6 Conclusion

We present a model of endogenous default that emphasizes the role of switches in growth regimes in matching important business cycle features of Emerging Markets and in generating default levels that are closer to the frequency observed in the data. The reason why a model with growth shocks performs better is that in such an environment a given probability of default is associated with a smaller borrowing cost at the margin. This in turn rests on the fact that trend shocks have a greater impact on the propensity to default than do standard transitory shocks, making interest rates relatively less sensitive to the amount borrowed and relatively more sensitive to the realization of the shock.

Further, to match the business cycle features of the interest rate and current account, the model should predict that agents borrow more at lower interest rates during booms and the reverse during slumps. Since the interest rate schedule tends to be very steep in these models, the typical prediction is for the interest rate and current account to be negatively correlated. In the model with growth shocks, however, we find that for plausible parameterizations the shift of the interest rate schedule in good states, in anticipation of lower default probabilities, dominates the increase in interest rates that arise because of additional borrowing. While the predictions still remain short of matching quantitatively the magnitudes obtained in the data, the predicted sign of the correlations of income, net exports, and the interest rate are in line with empirical facts.

We have left unexplained the question of what precisely are the default penalties economies face, what underlies the stochastic trend and what are realistic impatience rates to be used to understand sovereign default. This we leave as questions for future research.

Appendix

This appendix characterizes and derives properties of the representative agent's problem. The primary technical issues that separate our problem from standard optimizations concern the option to default and the presence of trend growth. At any point in time, the state of the economy is given by assets a_t, a credit standing $h_t \in \{B, G\} \equiv H$, and an income process governed by $y_t = e^{z_t}\Gamma_t$ where $\Gamma_t = g_0 * g_1 * ... * g_t$. Let θ denote the shocks $\{z, g\}$, which are Markov processes drawn from

$\overline{z} = [z_1, ..., z_{N_z}]$ and $\overline{g} = [g_1, ..., g_{N_g}]$, respectively. We assume that $\overline{z}$ and $\overline{g}$ are strictly positive, so there is never a state in which the endowment is zero. These transition probability functions for z and g satisfy monotonicity (i.e., if f is nondecreasing in (z, g), then so is $E(f(z', g')|z, g))$. The state of the income process is then characterized by (Γ_{t-1}, θ_t). When omitting time subscript, this will be written as $(\Gamma_{(-1)}, \theta)$ to remind the reader that we have separated the pre-existing trend component Γ_{t-1} and its current innovation g_t.

There is one more random variable in the system – the redemption of a bad credit rating. We therefore split the credit standing variable h into its random and nonrandom components. Let $\tilde{h}_t$ denote the credit rating as of the end of $t-1$ which is determined by the default decisions in $t-1$. Then $h_t = G$ with probability one if $\tilde{h}_t = G$ and with probability λ if $\tilde{h}_t = B$. Let $\rho \in 0, 1$ denote the random variable with realization $\rho = 1$ denoting redemption. In sum, the state variable $s_t = (a_t, \tilde{h}_t, \rho_t, \Gamma_{t-1}, \theta_t)$.

Define $\Phi(s_t)$ as the budget correspondence mapping the state s into the range of possible consumption, assets and credit standings $\tilde{h}_{t+1}$. If $\tilde{h}_t = B, \rho = 0$, then $\Phi(s) = (c_t \leq e^{z_t}\Gamma_t,\ \ \tilde{h}_{t+1} = B,\ \ a_{t+1} = 0)$. If $\tilde{h}_t = G$ or $\rho = 1$, the agent has the choice of defaulting ($\tilde{h}_{t+1} = B$) and facing the budget set of the previous sentence, or choosing not to default and selecting from $\Phi(s) = (c_t \leq e^{z_t}\Gamma_t + a_t - q(a_{t+1}, \Gamma_{t-1}, \theta_t)a_{t+1}, \tilde{h}_{t+1} = G,\ a_{t+1} \in [\underline{a}, \bar{a}]\mu_g\Gamma_t)$.

We restrict next period's assets normalized by $\mu_g\Gamma_t$ to lie is some finite interval $[\underline{a}, \bar{a}]$. That is, debt and asset growth are bounded by trend growth. This rules out Ponzi schemes that entail debt growing faster than the economy. It also rules out the possibility that agent's accumulate a large stock of assets relative to income.[10] In our numerical simulations, we ensure that the asset limits never bind in equilibrium. It will be convenient later to define the choice set for assets as $g_t^{-1}a_{t+1} \in [\underline{a}, \bar{a}]\mu_g\Gamma_{t-1}$, which is the same as above divided through by g_t.

The price of a bond, $q : (a_{t+1}, \Gamma_{t-1}, \theta_t) \rightarrow [0, 1/(1 + r^*]$, depends on the amount borrowed and

[10] The technical assumption needed to ensure that consumption normalized by income is stationary in our context is $E\left\{\beta^t (1 + r^*)^t (\Gamma_t)^{-\gamma}\right\} \rightarrow 0$.

the state of the income process. This function will be determined in equilibrium, but is taken as given by the agent. For now, we take the function as homogenous of degree zero in a_{t+1} and Γ_{t-1}. We will show below that such an interest rate schedule exists in equilibrium.

The fact that q is homogenous of degree zero plus how we define the range of possible assets and the nature of the endowment process ensures the budget set for assets is homogenous of degree one in a_t and Γ_{t-1}. That is, if $(c_t, g_t^{-1} a_{t+1})$ is feasible given $s_t = (a_t, \tilde{h}_t, \rho_t, \Gamma_{t-1}, \theta_t)$, then $(\alpha c_t, \alpha g_t^{-1} a_{t+1})$ is feasible given $(\alpha a_t, \tilde{h}_t, \rho_t, \alpha\Gamma_{t-1}, \theta_t)$ for any $\alpha \geq 0$.

Preferences are given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma > 1$. The consumer's problem for a given initial state s_0 is then

$$\begin{aligned} v^*(s_0) \quad &= \quad sup_{\{c_t, a_{t+1}, \tilde{h}_{t+1}\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \qquad (13) \\ & \qquad s.t. \;\; (c_t, g_t^{-1} a_{t+1}, \tilde{h}_{t+1}) \in \Phi(s_t) \quad t = 0, 1, ..., \end{aligned}$$

where E_0 is expectation conditional on s_0. As $u < 0$, the infinite sum is bounded above by zero. To show it is bounded below, it suffices to show that there is a feasible path that yields finite expected utility. The fact that the agent can always choose to default and enter autarky implies one such feasible path conditional on a growth restriction. Specifically, if $c_t = e^{z_t}\Gamma_t$, every period, we need that $E_0 \sum_t \beta^t \left(e^{z_t}\Gamma_t\right)^{1-\gamma}$ is well defined, which requires $\lim_{t\to\infty} E_0 \beta^t \left(\Gamma_t\right)^{1-\gamma} = 0$ for all starting s_0.

Homogeneity of v^* in (a, Γ): Let $\mathbf{s}(s_0)$ be the optimal plan across all possible paths of s_t starting at s_0, with a corresponding consumption plan $\mathbf{c}(s_0)$ and utility $v^*(s_0)$. Now consider a starting state s_0' which is the same as s_0 in $(\tilde{h}, \rho, \theta)$, but the (a_0, Γ_{-1}) associated with s_0 have been replaced by $(\alpha a_0, \alpha\Gamma_{-1})$. The plan $\mathbf{c}(s_0') = \alpha\mathbf{c}(s_0)$ is feasible given the homogeneity of the budget set. Moreover, it is optimal given the homogeneity of the utility function.[11] The homogeneity of u implies that this consumption plan results in a utility level of $\alpha^{1-\gamma} v^*$, establishing that v^* is homogenous of degree $1 - \gamma$ in (a_t, Γ_{t-1}).

[11] If not, then the alternative, preferred plan scaled by $1/\alpha$ would have been preferable at s_0.

Continuity of v^* in assets: The continuity of v^* in assets follows from the continuity of utility and the budget constraints. In particular, consider an initial state s_0 associated with an asset position a_0. Let $\mathbf{s}(s_0)$ be the optimal plan starting from s_0. We can rewrite the utility from this program as $v^* = u(c_0^*) + \beta E_0 U(\mathbf{s}(s_1))$, where c_0^* is the optimal first period consumption and $\beta E_0 U(\mathbf{s}(s_1)$ represents the remaining terms in the summation of (13).

Let a_0^k converge to a_0, and denote the associated sequence of states s_0^k (where all other elements are the same as s_0). Suppose that a_1 is the amount of assets carried over to period 1 under $\mathbf{s}(s_0)$. Now consider a plan starting from s_0^k that follows $\mathbf{s}(s_0)$ at every point after the initial period. This requires next period assets be a_1. The budget constraint requires an initial consumption of $c_0^k = e^{z_0}\Gamma_0 + a_0^k - qa_1$. As period zero consumption under $\mathbf{s}(s_0)$, $c_0^* = e^{z_0}\Gamma_0 + a_0 - qa_1$, we have $c_0^k = c_0^* + a_0^k - a_0$. This is feasible as a_1 is in the choice set under both s_0 as well as s_0^k. Optimality implies that $v^*(s_0^k) \geq u(c_0^k(s_0)) + \beta E_0 U(\mathbf{s}(s_1))$. This in turn implies: $\lim_{k\to\infty}(v^*(s_0) - v^*(s_0^k)) \geq \lim_{k\to\infty}(u(c_0^*) - u(c_0^k)) = 0$, where the last equality comes from the continuity of the period utility function and $a_0^k \to a_0$. A similar line of reasoning that involves following the optimal plan starting from s_0^k after the initial period generates the reverse inequality.

Recursive Formulation: We now discuss the recursive formulation of the agent's problem (13). Recall that at time t, the state of our economy is $s_t = (a_t, \tilde{h}_t, \rho_t, \Gamma_{t-1}, \theta_t)$ which resides in the space S. We also include the interest rate schedule q in the value function to make explicit that the agent's actions and choices are predicated on this function.

$$V(s_t; q) = \tag{14}$$

if $h_t = B$ ($\tilde{h}_t = B$ and $\rho_t = 0$): $u((1-\delta)y_t) + \beta E_t V(0, \tilde{h}_{t+1} = B, \rho_{t+1}, \Gamma_t, \theta_t; q)$.

if $h_t = G$ ($\tilde{h}_t = G$ or $\rho_t = 1$):

$$\max \left\langle \begin{array}{c} u((1-\delta)y_t) + \beta E_t V(0, \tilde{h}_{t+1} = B, \rho_{t+1}, \Gamma_t, \theta_t; q), \\ \max_{c_t, a_{t+1}} \left\{ u(c_t) + \beta E_t V(a_{t+1}, \tilde{h}_{t+1} = G, \rho_{t+1}, \Gamma_t, \theta_t; q) \right\} \\ s.t. \;\; c_t \leq y_t + a_t - q_t a_{t+1} \\ a_{t+1}/(\mu_g \Gamma_t) \in [\underline{a}, \overline{a}] \end{array} \right\rangle$$

Let T denote the operator defined by (14). Let $H(S, \gamma)$ denote the space of functions on S that are homogenous of degree $1 - \gamma$ in $x \equiv (a, \Gamma_{-1})$ for each $\tilde{h}$, ρ, and θ, and bounded and continuous in the norm $\|f\| = \sup_{\{\tilde{h}, \rho, \theta\}} \sup_{\|x\|_2 = 1} |f|$.

Given that the optimum is finite, any solution to the Bellman equation that satisfies the transversality condition will equal v^*. A few characteristics of T should be noted. First, the lower bound on output and the option to default implies that the agent is never forced into a state with zero consumption. This implies u is always finite and our operator maps bounded functions into bounded functions. Moreover, the homogeneity of u and the budget set ensures that Tv is homogenous if $v \in H(S, \gamma)$. The Theorem of the Maximum implies that T preservers continuity.[12] That is, T maps $H(S, \gamma)$ into itself.

Finally, iteration of the Bellman equation yields the fixed point. To see this, note that T is monotone. That is, if $v' \geq v$, then $Tv' \geq Tv$. Moreover, as $u \leq 0$, $Tf \leq f$, where f is the zero function ($f = 0$). Thus by induction, $v^* \leq Tv^* \leq T^n f \leq Tf$. The sequence $T^n f$ is monotone and bounded below, so the limit exists and satisfies the Bellman equation.

Detrended Form: The fact that T maps $H(S, \gamma)$ into itself implies that we can solve the Bellman equation in detrended form. That is, suppose $V \in H(S, \gamma)$ solves (14), that is $V(s_t) = \max_{(c_t, g_t a_{t+1}, \tilde{h}_{t+1}) \in \Phi(s_t)} \{u(c) + \beta E_t V(s_{t+1})\}$, where we have suppressed the dependence on q. Now consider a scale factor $\alpha_t > 0$:

[12] There is one technical issue with the application of the Theorem of the Maximum. The usual assumption is that the objective function is continuous over the state space. However, the utility function is not defined at zero. Nevertheless, while zero consumption is in the feasible set, it is never forced on the consumer and will never be chosen by the consumer. Therefore, the usual proof that the maximum is continuous goes through.

$$\alpha_t^{1-\gamma} V(s_t) = \max_{(c_t, g_t a_{t+1}, \tilde{h}_{t+1}) \in \Phi(s_t)} \left\{ \alpha_t^{1-\gamma} u(c_t) + \beta E_t \alpha_t^{1-\gamma} V(s') \right\}. \tag{15}$$

Let $\hat{c}_t = \alpha_t c_t$ and $\hat{a}_t = \alpha_t a_t$. Similarly, let $\hat{s}_t = (\hat{a}_t, \tilde{h}_t, \rho_t, \alpha\Gamma_{t-1}, \theta_t)$ if $s_t = (a_t, \tilde{h}_t, \rho_t, \Gamma_{t-1}, \theta_t)$. Note that $\alpha_t a_{t+1} = \frac{\alpha_t}{\alpha_{t+1}} \hat{a}_{t+1}$.

Linear homogeneity of the budget constraint ensures that $(c_t, g_t^{-1} a_{t+1}, \tilde{h}_{t+1}) \in \Phi(s_t)$ if and only if $(\hat{c}_t, \frac{\alpha_t}{\alpha_{t+1}} g_t^{-1} \hat{a}_{t+1}, \tilde{h}_{t+1}) \in \Phi(\hat{s}_t)$ for any $\alpha_t \geq 0$. Homogeneity of degree $1-\gamma$ of $V \in H(S, \gamma)$ implies that $\alpha_t^{1-\gamma} V(s_t) = V(\hat{s}_t)$ and $\alpha_t V(s_{t+1}) = \frac{\alpha_t}{\alpha_{t+1}} V(\hat{s}_{t+1})$.

We can then write

$$V(\hat{s}) = \max_{(\hat{c}, \frac{\alpha_t}{\alpha_{t+1}} g_t^{-1} a_{t+1}, \tilde{h}_{t+1}) \in \Phi(\hat{s})} \left\{ u(\hat{c}) + \beta E_t \left(\frac{\alpha_t}{\alpha_{t+1}} \right)^{1-\gamma} V(\hat{s}_{t+1}) \right\}. \tag{16}$$

Letting $\alpha_t = (\mu_g \Gamma_{t-1})^{-1}$, we have $\frac{\alpha_t}{\alpha_{t+1}} = g_t$. Thus an equivalent formulation of (14) is:

$$V(\hat{s}_t) = \max_{(\hat{c}_t, \hat{a}_{t+1}, \tilde{h}_{t+1}) \in \Phi(s_t)} \left\{ u(\hat{c}_t) + \beta E_t g_t^{1-\gamma} V(\hat{s}_{t+1}) \right\}. \tag{17}$$

Note that $\hat{s}$ no longer depends explicitly on Γ_{t-1} and we can solve the Bellman equation without keeping track of this variable. However, to compute the level of utility, we need to rescale: $V(s_t) = (\mu_g \Gamma_{t-1})^{1-\gamma} V(\hat{s}_t)$.

Equilibrium: To close the model, we define equilibrium as an interest rate schedule that provides creditors with an expected return of r^* conditional on the actions of the representative agent consistent with q. Specifically, the equilibrium q satisfies

$$q = \frac{1}{1+r^*} \Pr\{V(a_{t+1}, \tilde{h}_{t+1} = G, \Gamma_t, \theta_{t+1}); q) > V(a_{t+1}, \tilde{h}_{t+1} = B, \Gamma_t, \theta_{t+1}), ; q)\} \tag{18}$$

Let T^q be the operator that maps Q into Q, where $T^q q$ equals the right hand side of (18) and Q is the space of candidate interest rate functions. Homogeneity of the value function implies that the inequality holds if and only if the same inequality holds for the detrended value function. This implies that the operator maps elements of Q that are homogenous of degree zero in next period's assets and $\Gamma_{(-1)}$ into homogenous functions. Moreover, the operator T^q is monotone. A lower interest rate schedule at every point implies a more attractive asset market for the agent. This in turn increases the value of having a good credit rating (it also increase the value of a bad credit rating, but only through the chance of redemption). This in turn implies a lower probability of default at each point in the state space. Thus, if $q' \geq q$, then $T^q q' \geq T^q q$. Moreover, if q^* is simply the inverse of the risk free rate at all points, $q^* = \frac{1}{1+r^*}$, then $T^q q^* \leq q^*$. By induction, we can iterate down from q^*. As q is bounded below by the zero function, monotonicity implies the sequence will converge to a fixed point.

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Table 1: Argentina Business Cycle Statistics (1983.1-2000.2)

Data	HP	SE
$\sigma(Y)$	4.08	(0.52)
$\sigma(R_s)$	3.17	(0.54)
$\sigma(TB/Y)$	1.36	(0.24)
$\sigma(C)/\sigma(Y)$	1.19	(0.04)
$\rho(Y)$	0.85	(0.08)
$\rho(R_s, Y)$	-0.59	(0.11)
$\rho(TB/Y, Y)$	-0.89	(0.10)
$\rho(R_s, TB/Y)$	0.68	(0.13)
$\rho(C, Y)$	0.96	(0.01)

The series were deseasonalized if a significant seasonal component was identified. We log the income, consumption and investment series and compute the ratio of the trade balance (TB) to GDP (Y) and the interest rate spread (R_s). R_s refers to the difference between Argentina dollar interest rates and US 3 month treasury bond rate (annualized numbers). All series were then HP filtered with a smoothing parameter of 1600. GMM estimated standard errors are reported in parenthesis under column SE. The standard deviations (Y, R_s , TB/Y) are reported in percentage terms.

Table 2A: Common Benchmark Parameter Values

Risk Aversion	γ	2
World Interest Rate	r^*	1%
Loss of Output in Autarky	δ	2%
Probability of Redemption	λ	10%
Mean (Log) Transitory Productivity	μ_z	$-\frac{1}{2}\sigma_z^2$
Mean Growth Rate	μ_g	1.006

Table 2B: Model Specific Benchmark Parameter Values

	Model I: Transitory Shocks	*Model II: Growth Shocks*	*Model II with Bail Outs*
σ_z	3.4%	0	0
ρ_z	0.90	NA	NA
σ_g	0	3%	3%
ρ_g	NA	0.17	0.17
β	0.8	0.8	0.95
Bail Out Limit	NA	NA	18%

Table 3: Benchmark Simulation Results

	Data	**Model I (3A)**	**Model II (3B)**	**Model II with Bail Outs (3C)**
$\sigma(y)$	4.08	4.32	4.45	4.43
$\sigma(c)$	4.85	4.37	4.71	4.68
$\sigma(TB/Y)$	1.36	0.17	0.95	1.10
$\sigma(R_s)$	3.17	0.04	0.32	0.12
$\rho(C,Y)$	0.96	0.99	0.98	0.97
$\rho(TB/Y,Y)$	-0.89	-0.33	-0.19	-0.12
$\rho(R_s,Y)$	-0.59	0.51	-0.03	-0.02
$\rho(R_s,TB/Y)$	0.68	-0.21	0.11	0.38
Rate of Default (per 10,000 quarters)	75	2	23	92
Mean Debt Output Ratio (%)		27	19	18
Maximum R_s *(basis points)*		23	151	57

Note: Simulation results reported are averages over 500 simulations each of length 500 (drawn from a stationary distribution). The simulated data is treated in an identical manner to the empirical data. Standard deviations are reported in percentages.

Table 4: Sensitivity Analysis

	Innovation Variance		Cost of Autarky		Time Preference		Endogenous Labor Supply
	Model I (σ_z=6.4%)	Model II (σ_g=6%)	Model I (δ=0.5%)	Model II (δ=0.5%)	Model II $\beta = 0.85$	Model II $\beta = 0.9$	Model II
$\sigma(y)$	8.60	8.25	4.32	4.42	4.43	4.43	4.44
$\sigma(c)$	8.68	8.56	4.33	4.46	4.67	4.66	4.85
$\sigma(TB/Y)$	0.36	1.28	0.07	0.25	0.85	0.76	0.79
$\sigma(R_s)$	0.16	0.64	0.08	0.32	0.24	0.16	0.24
$\rho(C,Y)$	0.99	0.99	0.99	0.99	0.98	0.99	0.99
$\rho(TB/Y,Y)$	-0.29	-0.18	-0.31	-0.17	-0.20	-0.23	-0.49
$\rho(R_s,Y)$	0.30	0.07	0.33	-0.03	-0.002	-0.05	-0.23
$\rho(R_s,TB/Y)$	-0.17	-0.05	-0.15	0.05	0.01	0.11	0.26
Rate of Default (per 10,000 quarters)	4	47	3	23	16	11	16
Mean Debt Output Ratio (%)	25	15	6	5	19	19	15
Maximum R_s *(basis points)*	52	301	40	185	120	80	191
$\sigma(g)$	0	3.0	0	3.0	3.0	3.0	2.3

Note: Simulation results reported are averages over 500 simulations each of length 500 (drawn from a stationary distribution). Standard deviations are reported in percentages. In column 7 we consider the case of endogenous labor supply where the elasticity of labor supply is taken to be 2.2 implying an ω of 1.455

Figure 1A: Model I

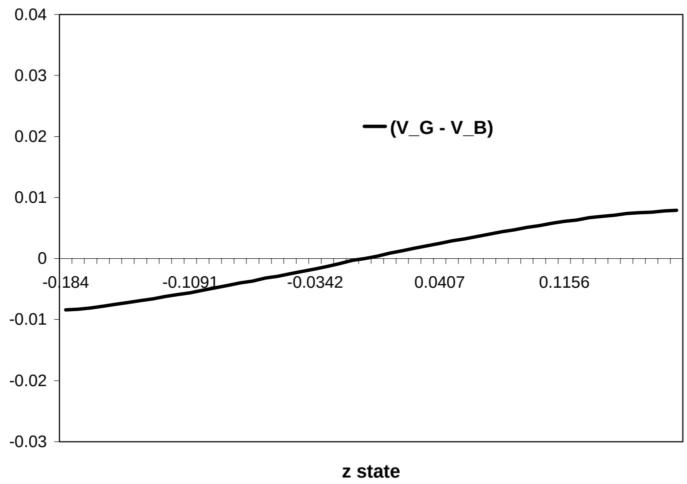
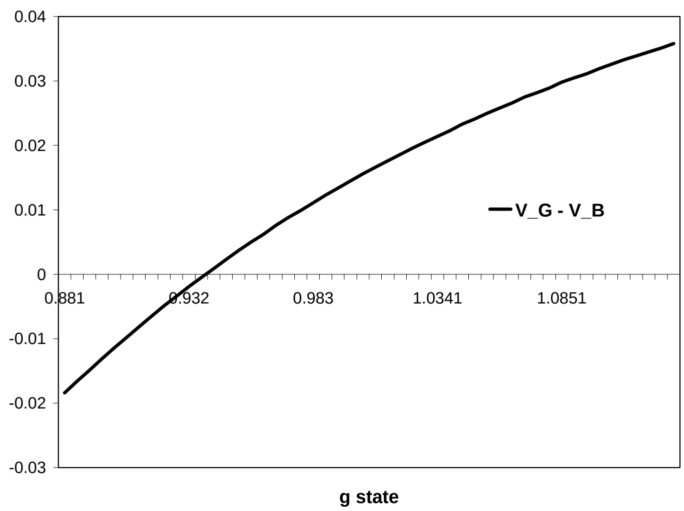
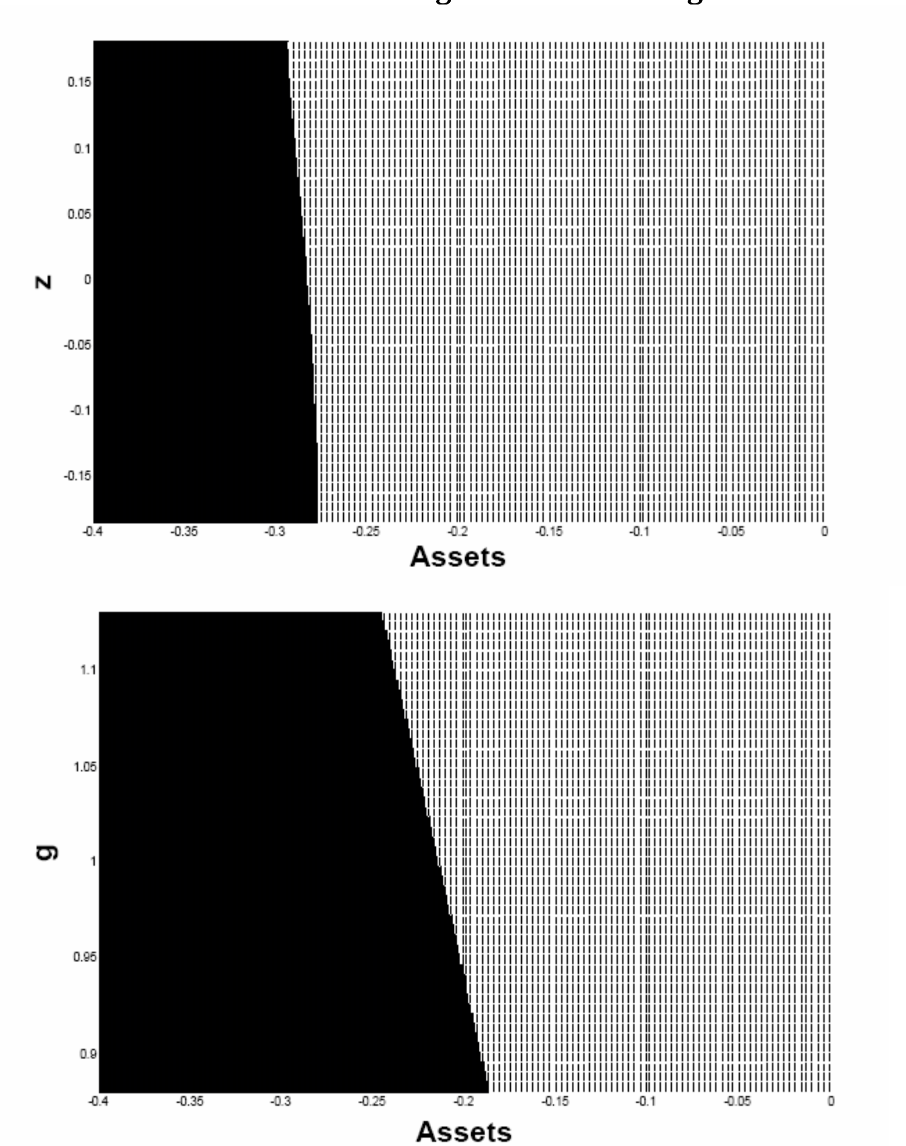


Figure 1B: Model II



Note: V_G represents the value function when the agent chooses to repay and is in good credit standing and V_B is the value function when the agent chooses to default. We have plotted here the difference between the two value functions for a given level of assets, across different productivity states. Figure 2A corresponds to the case when z varies and Figure 2B to the case when g varies. The (V_G-V_B) line is more steeply sloped in the case of g shocks.

Figure 2: Default Region



Note: The darkly shaded region represents combinations of the productivity state and assets for which the economy will prefer default. The lightly shaded region accordingly is the nondefault region. The vertical axis represents the realization of the productivity shock. The horizontal axis represents assets normalized by (mean) trend income. In both pictures, the agent is more likely to default when holding larger amounts of debt (negative assets) and when in worse productivity states. The line of indifference is less steeply sloped in the case of g shocks.

Figure 3A: Model I

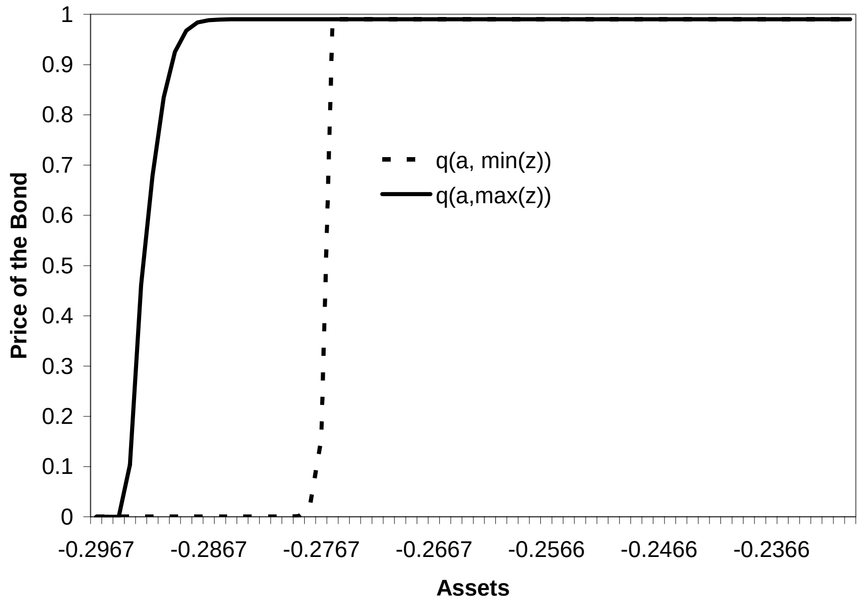
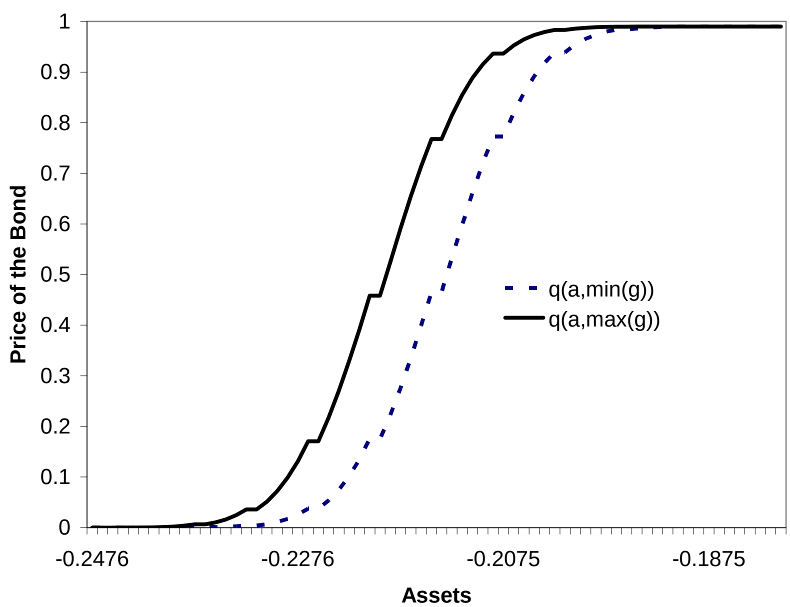


Figure 3B: Model II



Note: Figures 4A and 4B plot the Price of the Bond (inverse of one plus the interest rate) as a function of assets for the highest and lowest values of z in the case of Fig. 4A and the same for growth shocks in Fig. 4B. The price function is less sensitive to changes in borrowing in the case of g shocks (Fig. 4B).