The Role of Interest Rates and Productivity Shocks in Emerging Market Fluctuations

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Abstract

In this paper we use a quantitative model to explore the potential frictions that distinguish emerging market business cycles from developed small open economies. Following Aguiar and Gopinath (2007) we allow total factor productivity (TFP) to have a stationary and an integrated component. We also allow for shocks to the consumption and investment Euler Equations that operate through the interest rate. These “wedges” represent changes in the intertemporal marginal rate of transformation, which may be due to changes in observed interest rates, unobserved borrowing constraints, or other financial frictions. We estimate the model using data from Mexico and Canada. We show that interest rate shocks orthogonal to domestic TFP fail to explain the behavior of emerging markets. We then allow for interest rates to respond to/co-vary with productivity shocks. We find that emerging market business cycles appear to be driven by large shocks to trend income combined with relatively small transitory shocks that co-vary with the interest rate.
1 Introduction

Business cycles in Emerging Markets are characterized by high levels of volatility in income, investment and net exports. Consumption is more volatile than income and net exports are highly counter-cyclical. These facts are summarized in Aguiar and Gopinath (2007, henceforth AG (2007)). Further, the interest rates faced by these economies are highly volatile and negatively correlated with income. These features of the interest rate process are summarized in Neumeyer and Perri (2005). In this paper we adopt a standard stochastic small open economy business cycle model and allow the economy to be driven by productivity shocks that have permanent and transitory components as well as by shocks to the interest rate process. We then estimate the role of the different processes in explaining the business cycle behavior of emerging markets.

In AG (2007), we examined an economy driven exclusively by shocks to productivity. Productivity shocks in this context may be viewed as manifestations of deeper frictions in the economy such as changes in monetary, fiscal and trade policies. For instance, Restuccia and Schmitz (2004) provide evidence of a 50% drop in productivity within five years in the petroleum industry in Venezuela following its nationalization in 1975. Similarly, Schmitz and Teixeira (2004) document almost a doubling of productivity in the Brazilian Iron-Ore Industry following its privatization in 1991. It is such dramatic changes in productivity following reforms and undoing of reforms that we view as characterizing emerging markets. In addition, several emerging markets experience terms of trade shocks that display a high degree of persistence. In this set-up we provided a methodology for identifying the role of transitory versus trend shocks in explaining business cycles. The procedure relied on using the intuition behind the permanent income hypothesis.

In AG (2007), we adopted the standard small open economy assumption and modelled the interest rate as an exogenous international risk-free rate, which we held constant. In this environment the economy always repays its debt and there is never any default. In Aguiar and Gopinath (2006) we allow explicitly for default in an Eaton and Gersovitz set-up. We
specified an endowment economy driven by trend and stationary shocks. We show that incorporating trend shocks is important in generating empirically plausible rates of default as well as simultaneously matching key correlations between the interest rate, output, and the current account.

In this paper, we extend AG (2007) to allow for a stochastic interest rate process. We will consider three specifications. The first is the case of exogenous interest rate shocks that are independent of the productivity shocks. The second is the case where in addition to independent shocks the interest rate responds to transitory productivity shocks. The third case is where the interest rate also responds to trend productivity shocks. We will assume a reduced form specification for all these processes and provide intuition for the nature of the process.

It is important to note that we estimate the interest rate process from the Euler Equations and do not use observed interest rates. This mirrors our treatment of productivity shocks, where we do not use the Solow residual series to directly identify the underlying productivity process. We do this for two reasons. Firstly, the observed rates are not risk free rates given the probability of default. The promised rate observed in the data may therefore not be the relevant real rate governing behavior.\(^1\) Secondly, agents may be constrained in their access to financial markets. In that case, there is an implicit Lagrange multiplier that governs the consumption/investment decision rather than the observed market rate. Our estimation will pick up fluctuations in this multiplier. This approach is different from the work of Neumeyer and Perri (2005) in that NP take the observed interest rate process and feed this into the economy. This assumes that the euler equation with repayment is always satisfied at the observed interest rates.

We show that the model with interest rate shocks that are orthogonal to productivity shocks does poorly in matching the features of the data for emerging market countries. Movements in the interest rate affect consumption and investment by setting the price for

intertemporal substitution. An increase in the interest rate reduces consumption relative to the future as it increases the incentive to save. It also reduces investment in physical capital as the return from the bond is higher. Since in this exercise interest rate shocks are orthogonal to productivity shocks, the induced correlation between consumption and income and investment and income is low, contrary to the data. The response of output, on impact, to a rise in the interest rate will be small as productivity has not changed and capital takes time to adjust. Moreover, when consumption and leisure are non-separable, labor supply rises in response to a drop in consumption which generates an increase in output, which is counterfactual given that high interest rate periods have been associated with large declines in output. It is clearly the case that interest rate shocks that are not associated with movements in productivity will perform poorly in matching the facts for emerging markets. This point is similar in spirit to the work of Neumeyer and Perri (2005) and Chari and Kehoe (2006).

We next allow the interest rate to respond to productivity shocks—both transitory and trend shocks. The data suggest that a high level of productivity should be associated with a lower interest rate. A positive shock to productivity raises consumption and the increase is amplified by the contemporaneous decline in interest rates. This increases the relative volatility of consumption for a given income process. Also, investment increases following the rise in productivity and decline in interest rates. This implies that net-exports decrease, inducing a negative correlation between net exports and income. The precise moments of the stationary distribution will depend on the persistence in the income and interest rate process. For reasons explained below, the model performs better when the interest rate primarily responds to the transitory income shock.

Lastly, we use GMM and data from Mexico to estimate the parameters of a model that allows for both exogenous interest rate shocks and productivity shocks and for the interest rate shock to respond to the transitory income shock. In the benchmark case, where the model allows only for productivity shocks, the random walk component of the Solow residual is estimated to be 1.02. In AG (2007) we estimate that the random walk component for
Canada was far lower at 0.5. When we allow for the richer specification with interest rate shocks, we estimate the random walk component to be essentially the same at 1.01. This supports the conclusions in AG (2007) that emerging markets are subject to more volatile trend shocks as compared to developed markets. We also find evidence of a small negative covariance between productivity shocks and the implied interest rate.

It is important to emphasize that the differences in the Solow residual processes between developed and emerging markets may well be a manifestation of deeper frictions in the economy. Chari, Kehoe, and McGrattan (forthcoming), for instance, show that many frictions, including financial frictions, can be represented in reduced form as Solow residuals. From the perspective of private agents in our economy, these shocks appear as exogenous shifts in productivity. Our analysis provides support for models with frictions that are reflected in the persistence of Solow residuals, rather than frictions that distort the response of investment and consumption to underlying productivity. For instance, Guajardo (2007), describes a model with financial frictions and finds that it fits the data best when there are exogenous labor financing wedges which affect hiring decisions and are procyclical. That is, financing working capital requirements is easier in booms as compared to recessions. These financing wedges behave like productivity shocks. Our analysis shows that interest rate shocks that only affect the Euler Equation add little to matching the facts in the data for emerging markets. Clearly, one could argue that there are interest rate movements that interact with underlying financial frictions to generate shocks that look like productivity shocks. Our analysis is completely consistent with such a model.

We also present here evidence that Chile has features similar to other emerging markets documented in AG (2007). The correlation between HP-filtered net exports as a ratio of GDP and HP-filtered log of GDP is -0.82 for Chile. There does not exist a quarterly series on private consumption before 1996. For the 10 years from 1996-2006 the volatility of HP-filtered log GDP is 1.63 compared to a volatility of 1.89 for HP-filtered log of private consumption. This is similar to other emerging markets, which on average experience

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2 We thank David Rappoport for providing us with this data
consumption volatility that exceeds the volatility of income and net exports that are highly counter-cyclical. In Section 2 we describe the stochastic growth model. In Section 3 we describe the identification strategy and provide intuition by presenting impulse responses to various shocks. Lastly, in Section 4 we describe the results from a GMM estimation of the model.

2 Stochastic Growth Model

The model here is reproduced from AG (2007) and augmented to include a stochastic interest rate process. Technology is characterized by a Cobb-Douglas production function that uses capital, $K_t$, and labor, $L_t$, as inputs

$$Y_t = e^{zt}K_t^{1-\alpha}(\Gamma_tL_t)^\alpha,$$

where $\alpha \in (0, 1)$ represents labor’s share of output. The parameters $z_t$ and $\Gamma_t$ represent productivity processes. The two productivity processes are characterized by different stochastic properties. Specifically, $z_t$ follows an AR(1) process

$$z_t = \rho_z z_{t-1} + \varepsilon^z_t$$

with $|\rho_z| < 1$, and $\varepsilon^z_t$ represents iid draws from a normal distribution with zero mean and standard deviation $\sigma_z$.

The parameter $\Gamma_t$ represents the cumulative product of “growth” shocks. In particular,

$$\Gamma_t = e^{g_t} \Gamma_{t-1} = \prod_{s=0}^{t} e^{g_s}$$

$$g_t = (1 - \rho_g) \mu_g + \rho_g g_{t-1} + \varepsilon^g_t,$$

where $|\rho_g| < 1$ and $\varepsilon^g_t$ represents iid draws from a normal distribution with zero mean and standard deviation $\sigma_g$. The term $\mu_g$ represents productivity’s long-run mean growth rate. We loosely refer to the realizations of $g$ as “growth shocks” as they constitute the stochastic
trend of productivity. We use separate notation for shocks to the level of productivity \( (z_t) \) and the growth of productivity \( (g_t) \) to simplify exposition and calibration.

Given that a realization of \( g \) permanently influences \( \Gamma \), output is nonstationary with a stochastic trend. For any variable \( x \), we introduce a hat to denote its detrended counterpart:

\[
\hat{x}_t \equiv \frac{x_t}{\Gamma_{t-1}}.
\]

Note that we normalize by trend productivity through period \( t-1 \). This insures that if \( x_t \) is in the agent’s information set as of time \( t-1 \), so is \( \hat{x}_t \). The solution to the model is invariant to the choice of normalization.

Period utility is Cobb-Douglas,

\[
u_t = \left( \frac{C_t (1 - L_t)^{1-\gamma}}{1-\sigma} \right)^{1-\sigma}
\]

where \( 0 < \gamma < 1 \). A well-behaved steady state of the deterministic linearized model requires \( \beta(1 + r^*) = \mu g^{1-\gamma(1-\sigma)} \).

The equilibrium is characterized by maximizing the present discounted value of utility subject to the production function (1) and the per-period resource constraint:

\[
C_t + K_{t+1} = Y_t + (1 - \delta)K_t - \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - e^{\mu g} \right)^2 K_t - B_t + q_t B_{t+1}.
\]

Capital depreciates at the rate \( \delta \) and changes to the capital stock entail a quadratic adjustment cost \( \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - e^{\mu g} \right)^2 K_t \). We assume international financial transactions are restricted to one-period, risk-free bonds. The level of debt due in period \( t \) is denoted \( B_t \) and \( q_t \) is the time \( t \) price of debt due in period \( t + 1 \).

Fluctuations in the price of debt, \( q_t \), will be our focus. We assume that the interest rate is potentially driven by an exogenous process \( r_t \) as well as the domestic TFP shocks. Specifically, the price of debt \( q \) is given by the expression below.
\[
\frac{1}{q_t} = 1 + r^* + e^{\{r_t + a_z z_t + a_g (g_t - \mu_g)\}} + \psi \left[ e^{\frac{b_{t+1}}{r_t} - b} - 1 \right],
\]
where
\[
r_t = \rho r_{t-1} + \epsilon_t^r.
\]
The world interest rate is held constant at \( r^* \). The country-specific shock to the interest rate is given by \( \epsilon_t^r \), which is orthogonal to \( z \) and \( g \). The induced process \( r_t \) has an autocorrelation coefficient \( \rho_r \) and a long run mean of zero. The parameters \( a_z \) and \( a_g \) capture the sensitivity of the interest rate to the the transitory productivity shock and the trend productivity shock, respectively. We should note that the correlation of the interest rate and productivity does not imply a direction of causation between the two. See Aguiar and Gopinath (2006) for a model in which exogenous domestic productivity shocks drive an endogenous interest rate, while Neumeyer and Perri (2005) present a model in which exogenous (foreign) interest rate shocks drive domestic TFP. \( b \) represents the steady-state level of debt, and \( \psi > 0 \) governs the elasticity of the interest rate to changes in indebtedness. This sensitivity to the level of outstanding debt, takes the form used in Schmitt-Grohe and Uribe (2003).\(^3\) In choosing the optimal amount of debt, the representative agent does not internalize the fact that she faces an upward-sloping supply of loans.

In normalized form, the representative agent’s problem can be stated recursively:

\[
V(\hat{K}, \hat{B}, z, g, r) = \max_{(\hat{C}, L, \hat{K}', \hat{B}')} \left\{ \frac{(\hat{C}^\gamma (1 - L)^{1-\gamma})^{1-\sigma}}{1 - \sigma} + \beta e^{\sigma (1-\sigma)} EV(\hat{K}', \hat{B}', z', g', r') \right\}
\]

\(^3\)This adjustment is typically motivated by the need to make assets in the linearized model stationary. An alternative is to recognize that we are linearly approximating a non-linear economy for which a stationary distribution exists (for example, due to borrowing constraints and a world equilibrium interest that is lower than the discount rate, as in Aiyagari 1994). Quantitatively, since the elasticity of interest rate to changes in indebtedness is set close to 0 (0.001 to be exact), there is a negligible difference between the two approaches in terms of the HP-filtered or first-differenced moments of the model.
\[
\text{s.t. } \dot{C} + e^g \dot{K} = \dot{Y} + (1 - \delta) \dot{K} - \frac{\phi}{2} \left( e^g \frac{\dot{K}}{K} - e^{\mu_g} \right)^2 \dot{K} - \dot{B} + e^g q \dot{B}'.
\]

The evolution of the capital stock is given by,

\[
e^g \dot{K} = (1 - \delta) \dot{K} + \dot{X} - \frac{\phi}{2} \left( \frac{\dot{K}}{K} e^g - e^{\mu_g} \right)^2 \dot{K}.
\]

Given an initial capital stock, \(\dot{K}_0\), and debt level, \(\dot{B}_0\), the behavior of the economy is characterized by the first-order conditions of the problem (7), the technology (1) and budget (8) constraints, and the transversality conditions.

We solve the normalized model numerically by log-linearizing the first-order conditions and resource constraints around the deterministic steady state. Given a solution to the normalized equations, we can recover the path of the non-normalized equilibrium by multiplying through by \(\Gamma_{t-1}\). We also compute the theoretical moments of the model from the coefficients of the linearized solution.

3 Identification

The primary goal of this paper is to assess the relative importance of interest rate shocks, transitory productivity shocks and permanent shocks to productivity in explaining the behavior of emerging markets. In AG (2007) we described the methodology of exploiting decisions by informed, optimizing agents to identify the underlying shock process. In this paper, we extend that methodology to accommodate a richer process for the interest rate.

The methodology we employ selects parameters of the model to match key moments of the data. Below, we discuss which moments are particularly useful in identifying the parameters of interest. However, we note from the start that we do not use market interest rates on sovereign debt. The reason is that those interest rates represent the price of a defaultable bond. This is a different asset than that modelled above. To see this, consider
the Euler Equation for bonds from the above model:

\[
\frac{\beta}{q} E u'_{u_c} = 1. \tag{10}
\]

Note that while consumption is stochastic, the interest rate paid (conditional on information at the time of borrowing) is deterministic. In a model with defualtable debt, the consumer pays the interest rate \textit{conditional} on no-default, and pays zero (or some fraction) if default occurs. Therefore, the observed market interest rate cannot be used directly in a simple Euler Equation, but must be combined with a full specification of in which states default occurs and what payments are made conditional on default.

Our interest rate process \( q \) can be viewed as a wedge in the Euler Equations for consumption and investment. Our estimation will then back out the parameters governing the stochastic process of this wedge. In this sense, it is similar to the exercise of Chari, Kehoe, and McGrattan (forthcoming). It also captures unobserved frictions (to a linear approximation) such as additional borrowing costs or constraints beyond the market interest rate.

\section*{3.1 Interest Rates Shocks Orthogonal to Productivity Shocks}

We begin with an exploration of uncorrelated interest rate shocks. That is, shocks to the interest rate that are orthogonal to total factor productivity. Changes to the interest rate induce changes to consumption and investment for a given path of income due to inter-temporal substitution. This will raise the relative volatility of consumption and investment. Therefore, such shocks have the potential to explain the relatively high volatility of consumption in emerging markets.

However, by introducing shocks that move consumption and investment independently of income reduces the covariance of consumption and investment with income. This generates counterfactual implications for the cyclicality of net exports.

Figure 1 plots the impulse response of consumption, investment, net exports, and income
to a one percent shock to $\epsilon^r$. We set $\rho_r = 0.9$. As expected, an increase in the interest rate leads to a drop in consumption, with an initial decline of roughly 3 percent. Investment declines in an even more dramatic fashion. Output remains steady, declining slightly over the path due to the lagged declines in investment. This leads to a jump in net exports.

To see how orthogonal interest rate shocks affect key moments of the simulated model, consider a model in which we set $a_z = a_g = 0$, but set $\sigma_r \equiv \text{stdev}(\epsilon^r) > 0$. To be precise, we consider models with various $\sigma_r$ ranging from zero to one percent. For each environment, we compute key moments of the simulated economy and plot them in Figure 2. We fix all other parameters. We also set $\gamma = 1$ so that labor supply is fixed. All moments refer to HP-filtered variables. In Panel A of Figure 2 we see how the relative variance of consumption, investment, and net exports increases as we increase $\sigma_r$. This corresponds to the above intuition. In Panel B, we see that net exports become more pro-cyclical as we increase $\sigma_r$. This takes us further from the data. Correspondingly, consumption, investment become less correlated with income. This is because a positive interest rate shock lowers consumption and investment. As TFP has not changed, this reduces the correlation with income. In fact, when consumption and leisure are nonseparable, the decreased consumption is associated with higher labor and therefore higher income, inducing a negative correlation between consumption and income. In this set-up, a crisis which is associated with a large increase in interest rates, will reduce consumption but raise output, which is completely counterfactual.

It is clearly the case that exogenous interest rate shocks will do poorly in explaining the behavior of emerging markets. It will be hard to generate the large counter-cyclicality in the current accounts and the much larger responsiveness of consumption relative to income. This argument is in line with the results in Neumeyer and Perri (2005) and Chari and Kehoe (2006). A model where the interest rate process does not show up as affecting productivity will have little hope of matching moments of the business cycle.
3.2 Interest Rate that Co-vary with Productivity Shocks

From the previous section it is clear that we need to interact the interest rate shock with the productivity shock. As we have two productivity processes, there are two dimensions along which we can link the interest rate and productivity. We begin by setting $a_g = 0$ and considering the link between transitory productivity shocks and the interest rate. We then set $a_z$ equal to 0 and assume the interest rate responds only to the permanent shock $g$.

To gain some intuition, in Figure 3 we plot the impulse response of consumption and income to a shock to $\epsilon^z$ when $a_z = 0$ and when $a_z = -0.1$. This latter case generates a fall in the interest rate when productivity increases. This can be an implication of the Eaton Gersovitz style models of default in which default occurs during low income realizations (see Aguiar and Gopinath (2006) and Arellano (2004)). With persistent shocks, a high shock today implies on average high shocks tomorrow and a correspondingly low probability of default. This generates a negative relationship between productivity and the interest rate.

For the benchmark case of $a_z = 0$, we see the standard consumption smoothing result — consumption increases, but income increases much more. The case of $a_z < 0$ combines the income response with a substitution response that favors initial consumption. This generates a larger initial jump in consumption and a subsequent declining profile of consumption. Given the transitory nature of the shock, the net effect is that consumption tracks the shape of the income impulse response. Not depicted is the response of investment, which has a similar intuition as consumption.

The impulse responses indicate that allowing the interest rate and productivity to co-move overcome some of the limitations of transitory productivity. Namely, consumption and investment respond more to income and in a way that makes net exports negatively associated with income. To see how this extension affects business cycle moments, we plot the key moments as a function of $a_z$ in Figure 4. As $a_z$ becomes increasingly negative, this raises the volatility of consumption relative to income. A positive productivity shock lowers interest rates, generating an increase in consumption above and beyond the income
effect. Unlike the orthogonal interest rate process of Figure 2, the additional consumption volatility increases the correlation of consumption and income. This effect is driven by the fact that the interest rate moves one-for-one with productivity. A similar story holds for investment. These effects make net export countercyclical, a key feature of the data for emerging markets.

As noted above, an alternative approach is to allow the interest rate to respond to permanent productivity shocks, i.e. $a_g < 0$. In Figure 5 we plot the impulse response to a shock to $\epsilon^g$ in the benchmark case and in the case when $a_g = -1$. Given that $g$ has a permanent effect on income, we see that consumption responds strongly to the initial shock in the benchmark case, exceeding the initial response of income. Allowing the interest rate to respond as well heightens the initial response of consumption. However, subsequently, the interest falls back quickly to its initial level as $g$ is nearly iid. This generates a sharp fall in consumption and then a levelling out. However, income jumps and then continues to rise in response to a growth shock. Therefore, allowing $a_g < 0$ lowers the correlation of consumption with income, taking us further from the data.

This effect can be seen clearly in Figure 6. As we increase $a_g$ (in absolute value), while the variance of consumption and investment increase, the correlations with income at business cycle frequencies fall. This reduces the cyclicality of net exports, drawing us further from the data.

The poor performance of the model with $a_g < 0$ is due to the fact that growth rates have little persistence, generating interest rates with little persistence. An alternative would be that interest rates are a function of the level of the stochastic trend $\Gamma$. However, this would imply a nonstationary interest rate.
3.3 Productivity Shocks Alone

AG (2007) considered a model in which \( a_g = a_z = 0 \). We briefly summarize the intuition behind the identification of the relative variance \( \sigma_g / \sigma_z \). In response to a transitory shock to productivity, agents increase consumption, but by less than the increase in income since they expect income to be lower in the future and by saving they can smooth consumption. On the other hand, if the economy is hit by a growth shock which implies permanently higher income and depending on the persistence of the growth shock an upward sloping profile of income the agents will increase consumption by at least as much as the increase in income. Therefore consumption is more volatile relative to income in the world with permanent shocks relative to transitory shocks. This difference in the response of \( \sigma_c \) is observed in figure 7.

Therefore, by observing the behavior of consumption we can infer the relative importance of trend versus transitory shocks. Similarly, it follows that given the response of consumption and we should expect net exports to be far more countercyclical for the economy with trend shocks and the moment on net exports can be used to identify the underlying productivity shock.

3.4 Identification Strategy

Given the above results, we restrict \( \sigma_r = a_g = 0 \). That is, we consider a model in which the interest rate co-varies with transitory productivity shocks and allow for both transitory and trend shocks to productivity. The patterns depicted in figures 4 and 7 indicate how we can identify the key parameters. Increases in the magnitude of \( a_z \) and \( \sigma_g / \sigma_z \) have a similar impact on the cyclicality of the current account. However, while both raise the relative volatility of consumption, net exports, and investment, the relationships differ. Figure 4 indicate that \( a_z \) has an almost linear effect on the relative variances, while figure 7 indicates that the impact of \( \sigma_g / \sigma_z \) eventually dies out. In particular, for large enough \( a_z \), the relative
volatility of net exports exceeds that of consumption. This reflects the differential sensitivity of investment and consumption to interest rate shocks. Therefore, the empirical moments of \( \sigma(c) \) and \( \sigma(nx) \) combined with the empirical covariance of net exports with output pin down the relative magnitudes of \( a_z \) and \( \sigma_g/\sigma_z \). Given the relative variance of trend and transitory shocks, the level of income volatility then identifies the level of \( \sigma_z \) and \( \sigma_g \).

## 4 Estimates

Following the above identification strategy, in this section we estimate \( \sigma_g \), \( \sigma_z \), and \( a_z \) by matching the following (HP-filtered) moments of the data: the standard deviations of income, consumption, and net exports, as well as the covariance of net exports with income. We use data from Mexico as a representative emerging market and Canada as a representative developed open economy. We fix other parameters at the values listed in Table 5.

For each set of estimates, we report the relative importance of the random walk component of productivity. Beveridge-Nelson (1981) showed that any I(1) series can be decomposed into a random walk component (denoted \( \tau \)) and a stationary component. A natural measure of the importance of the random walk component is the ratio of the variance of the growth rate of the trend component to the growth rate of total TFP.

\[
\frac{\sigma^2_{\Delta \tau}}{\sigma^2_{\Delta TFP}} = \frac{\alpha^2 \sigma^2_g}{(1 - \rho_g)^2 \sigma^2_{\Delta TFP}} = \frac{\alpha^2 \sigma^2_g}{(1 - \rho_g)^2} \left[ \frac{2 \sigma^2 z}{1 + \rho_z} + \frac{\alpha^2 \sigma^2}{1 - \rho_g^2} \right]
\]

(11)

We report the estimates for \( \sigma_g \), \( \sigma_z \) and \( a_z \) in Table 5. In the columns denoted “benchmark” we restrict \( a_z = 0 \). This corresponds to the benchmark model of AG (2007). The other columns estimate \( a_z \). The first two columns consider a model in which labor is supplied exogenously. This corresponds to setting the Cobb-Douglas preference parameter on consumption (\( \gamma \)) to one, so that leisure does not enter utility. The next two columns allow labor supply to vary endogenously, setting \( \gamma = 0.36 \). The final two columns estimate the
model using Canadian data.

For the benchmark model using Mexican data (column 1), we see that $\sigma_g$ is larger than $\sigma_z$ and that the relative contribution of the random walk component to TFP is 1.02. This is similar to the results reported in AG (2007). In the second column of Table 2, we estimate $a_z$ along with $\sigma_z$ and $\sigma_g$. We find that $a_z < 0$, although we cannot reject $a_z = 0$ at standard significance levels. Even allowing for interest rate shocks, we estimate a relatively large $\sigma_g$, with the contribution of the random walk component estimated to be 1.01.

Allowing labor supply to vary endogenously does not overturn this pattern. In both specifications, the random walk component of productivity is estimated to be roughly 1.0. The coefficient $a_z$ is estimated to be small.

The case of Canada indicates a relatively small random walk component. In both specifications, we estimate the relative random walk component to be 0.4. The coefficient $a_z$ is also estimated to be small and not significantly different from zero.

In Table 5 we report the implied business cycle moments from the estimated models along with the corresponding empirical moments from Mexico and Canada. The implied moments for Mexico correspond to the first two columns of Table 5. In respect to Mexico, we see that both models perform well in matching key features of the data. The empirical relative volatility of consumption is 1.3, while the models with and without interest rate shocks both generate relative variances of 1.1. The cyclicity of net exports is -0.8 in the data and is -0.7 and -0.6 in the models without and with interest rate shocks, respectively. In general, allowing for interest rate shocks does not markedly improve the fit of the model. A similar story holds for Canada, as seen in the final columns of Table 5.

In the specification with interest rate shocks, we see that interest rates are countercyclical in Mexico and procyclical in Canada. However, the variance of the implied interest rates is negligible. This reflects that while consumption is volatile in emerging markets, it is not driven by inter-temporal substitution, but rather by income shocks.
5 Discussion and Conclusion

Emerging Markets are characterized by large volatility in their income and consumption and large countercyclicality in net exports as compared to developed small open economies. They also face a volatile interest rate process that is negatively correlated with the level of GDP in these economies. There is a large amount of literature that attempts to explain these features of the data and infer the importance of productivity and interest rate shocks in explaining the patterns observed in the data. In this paper we perform a similar exercise by extending the framework in AG(2007) which allowed only for productivity shocks to one that allows for a richer specification of interest rate shocks and for the interaction between productivity and interest rate shocks.

One finding, which supports other evidence in the literature, is that interest rate shocks that do not affect productivity cannot be the main explanation for the business cycles of emerging markets. These markets are characterized by large movements in output at business cycle frequencies that are associated with large movements in the Solow residual. Accordingly, interest rate shocks alone will do little to explain these large movements in output. It is important to uncover channels through which interest rate shocks affect productivity.

If the interest rate is negatively correlated with the productivity shock, then interest rates can indeed play an important role. It can explain, at least qualitatively, a consumption process that is more volatile than income and counter-cyclical net exports. When we estimate the model to allow for the interaction between interest rates and productivity we find a small negative correlation between productivity and interest rates. We also find that, even in this framework, we obtain a large role for trend shocks which supports the main result in AG (2007), namely, that an important characteristic of emerging markets is that shocks to trend productivity play a predominant role in explaining movements at business cycle frequencies, unlike developed markets.
In this paper we have taken a reduced form approach to modelling both the interest rate process and productivity shocks. Future work should examine the structural features of emerging markets that give rise to the particular form of these processes. In AG (2006) we explore a model with Eaton-Gersovitz style endogenous default. While this approach does generate default in equilibrium and can generate a countercyclical interest rate process, it fails to generate sufficient volatility in the market interest rate process. Further research is required to understand the source of the volatility in the interest rate process.
References


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<th>Parameter</th>
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<td>Cobb-Douglas Utility Parameter</td>
<td>$\gamma$</td>
<td>1, 0.36</td>
</tr>
<tr>
<td>Steady State Debt to GDP</td>
<td>$b$</td>
<td>10%</td>
</tr>
<tr>
<td>Coeff. on Interest Rate Premium</td>
<td>$\Psi$</td>
<td>0.001</td>
</tr>
<tr>
<td>Labor Exponent (Production)</td>
<td>$\alpha$</td>
<td>0.68</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>$\delta$</td>
<td>0.05</td>
</tr>
<tr>
<td>Capital Adjustment Cost</td>
<td>$\phi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Persistence in $z$ process</td>
<td>$\rho_z$</td>
<td>0.95</td>
</tr>
<tr>
<td>Persistence in $g$ process</td>
<td>$\rho_g$</td>
<td>0.01</td>
</tr>
</tbody>
</table>
## Table 2: Estimates

<table>
<thead>
<tr>
<th></th>
<th>Mexico</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exogenous Labor</td>
<td>Endogenous Labor</td>
</tr>
<tr>
<td></td>
<td>Benchmark</td>
<td>With $a_z$</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(2.42)</td>
<td>(0.79)</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>2.78</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>$a_z$</td>
<td>-0.40</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(1.85)</td>
<td>(0.55)</td>
</tr>
<tr>
<td>Random Walk</td>
<td>1.02</td>
<td>1.01</td>
</tr>
<tr>
<td>Component</td>
<td>(0.18)</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

Notes: Estimates obtained from matching empirical moments of Mexico and Canada for respective columns. Moments used were standard deviation of HP-filtered log income, log consumption, and net exports/GDP as well as the covariance of HP-filtered net exports/GDP and log income. Exogenous Labor model sets $\gamma = 1$. Endogenous Labor model sets $\gamma = 0.36$. 
Table 3: Implied Moments

<table>
<thead>
<tr>
<th></th>
<th>Mexico</th>
<th></th>
<th>Canada</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model I</td>
<td>Model II</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>2.40</td>
<td>2.69</td>
<td>2.63</td>
<td>1.55</td>
</tr>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>1.26</td>
<td>1.09</td>
<td>1.10</td>
<td>0.74</td>
</tr>
<tr>
<td>$\sigma(nx)/\sigma(y)$</td>
<td>0.90</td>
<td>0.74</td>
<td>0.84</td>
<td>0.57</td>
</tr>
<tr>
<td>$\sigma(i)/\sigma(y)$</td>
<td>4.15</td>
<td>3.52</td>
<td>3.81</td>
<td>2.67</td>
</tr>
<tr>
<td>$\sigma(r)$</td>
<td>NA</td>
<td>0.08</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$\rho(y_t, y_{t-1})$</td>
<td>0.82</td>
<td>0.78</td>
<td>0.78</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho(c, y)$</td>
<td>0.92</td>
<td>0.98</td>
<td>0.98</td>
<td>0.87</td>
</tr>
<tr>
<td>$\rho(nx, y)$</td>
<td>-0.75</td>
<td>-0.68</td>
<td>-0.61</td>
<td>-0.12</td>
</tr>
<tr>
<td>$\rho(i, y)$</td>
<td>0.91</td>
<td>0.86</td>
<td>0.82</td>
<td>0.74</td>
</tr>
<tr>
<td>$\rho(r, y)$</td>
<td>NA</td>
<td>-0.01</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Notes: Empirical moments and implied moments from alternative models. Model I and Model II for Mexico correspond respectively to the first two columns of estimates (exogenous labor supply model) of Table 2. Model I and Model II of Canada correspond to the respective columns of estimates for Canada from Table 2.
Figure 1: Impulse Response to Interest Rate Shock

Note: Impulse response to a 1 percent shock to $\varepsilon^r$

Figure 2: Business Cycle Moments and $\sigma_r$
Panel A: Standard Deviation of Investment, Consumption, and Net Exports

Note: Standard Deviation of (HP-filtered, log) consumption, investment, and net exports relative to income as a function of $\sigma_r$. 

Panel B: Cyclicality of Investment, Consumption, and Net Exports

![Graph showing the cyclicality of investment, consumption, and net exports.](image)

Note: Correlation of (HP-filtered, log) consumption, investment, and net exports with income as a function of $\sigma_r$.

Figure 3: Impulse Response to $z$ Shock

![Graph showing the impulse response to a 1 percent shock to $z$.](image)

Note: Impulse response to a 1 percent shock to $z$. Benchmark model sets $a_z=0$. “$az$” model sets $a_z=-0.1.$
Figure 4: Business Cycle Moments and $a_z$

Panel A: Standard Deviation of Investment, Consumption, and Net Exports

Note: Standard Deviation of (HP-filtered, log) consumption, investment, and net exports relative to income as a function of $a_z$.

Panel B: Cyclicality of Investment, Consumption, and Net Exports

Note: Correlation of (HP-filtered, log) consumption, investment, and net exports with income as a function of $a_z$. 
Figure 5: Impulse Response to g Shock

Note: Impulse response to a 1 percent shock to $\varepsilon^g$. Benchmark model sets $a_g = 0$. “ag” model sets $a_g = -1$.

Figure 6: Business Cycle Moments and $a_g$
Panel A: Standard Deviation of Investment, Consumption, and Net Exports

Note: Standard Deviation of (HP-filtered, log) consumption, investment, and net exports relative to income as a function of $a_g$. 
Panel B: Cyclicality of Investment, Consumption, and Net Exports

Figure 7: Business Cycle Moments and $\sigma_g/\sigma_z$

Panel A: Standard Deviation of Investment, Consumption, and Net Exports

Note: Correlation of (HP-filtered, log) consumption, investment, and net exports with income as a function of $a_g$.

Note: Standard Deviation of (HP-filtered, log) consumption, investment, and net exports relative to income as a function of $\sigma_g/\sigma_z$. 
Panel B: Cyclicality of Investment, Consumption, and Net Exports

Note: Correlation of (HP-filtered, log) consumption, investment, and net exports with income as a function of $\sigma_g/\sigma_z$. 