

# Price Dynamics for Durable Goods

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Advances in Macroeconomics  
May 2011

## Motivation

- Durables play a crucial role in business cycle fluctuations
  - $\sim 60\%$  of non-service consumption, all of investment
  - most volatile component of GDP
- Standard macro models assume marginal cost or constant markup pricing for durables
  - DSGE models with durables
  - Barsky, House and Kimball (2007)
- Endogenous price dynamics can affect the cyclical properties of durables
- Pass-through and markup dynamics with durable good pricing
- (Interesting time inconsistency problem)

# Motivation

Gopinath, Itskhoki and Neiman (2011)

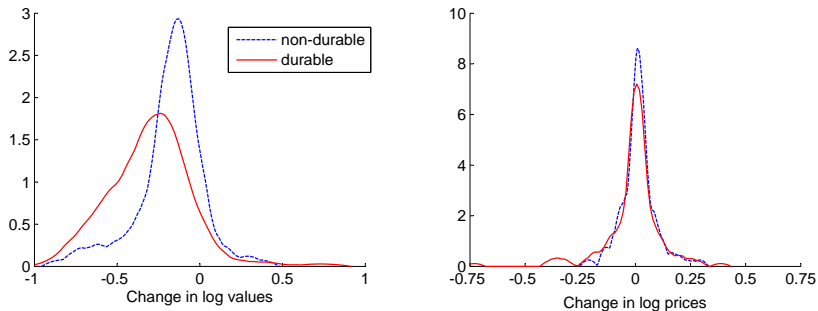


Figure: Change in US Import Values and Prices, 2008:07–2009:06

# Main Findings

- Assumptions
  - Some degree of monopolistic power
  - Lack of commitment by firms
  - Discrete time periods between price setting
- Results
  - ① Endogenous markup dynamics
    - markups decrease with the stock of durables
  - ② 'Countercyclical' markups in response to cost shocks
    - incomplete pass-through
  - ③ 'Procyclical' markups in response to demand shocks
  - ④ Adjustment-cost-like effect on quantities

# Literature

## Durable Monopoly Pricing

- Coase conjecture
  - Coase (1972), Stokey (1981), Bulow (1982), Gul et al. (1986), Bond and Samuelson (1984)
  - We focus on:  $\Delta t \gg 0$ ,  $\delta > 0$ , dynamics
- Durable-good oligopoly pricing
  - Gul (1987), Esteban (2003), Esteban and Shum (2007)
  - We focus on: dynamics of markups, GE
- Macro models
  - Caplin and Leahy (2006), Parker (2001)
  - We focus on: general demand and market structures, GE

## Demand

- Representative agent solves:

$$\max_{\{C_t, D_t, X_t, \dots\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_t, D_t) \quad \text{s.t.} \quad \begin{aligned} P_{Ct} C_t + P_t X_t &\leq E_t \\ D_t &= (1 - \delta) D_{t-1} + X_t \end{aligned}$$

Denote  $\Lambda_t$  the LM on expenditure constraint

- Partial durability,  $\delta \in (0, 1)$
  - Discrete time,  $\beta < 1$
- Optimal choice of  $D_t$  satisfies:

$$u'(D_t; \xi_t) = P_t - \beta(1 - \delta) \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1} \right\},$$

where  $u'(D_t, \xi_t) = U_D(C_t, D_t)/\Lambda_t$  and  $\xi_t$  is a stand-in for an arbitrary demand shock

- Approximation:  $\Lambda_t \approx \text{const}$  (implies constant interest rate)

# Demand

## Two special cases

- Constant-elasticity demand:

$$u'(D, \xi) = \xi \cdot D^{-1/\sigma}$$

— in the limit  $\delta \rightarrow 1$  results in constant markup pricing

- Linear demand:

$$u'(D, \xi) = a + \xi - bD$$

— yields simple closed-form solutions

# Market Structure

- Market structure:
  - Monopoly
  - Monopolistic competition
  - Homogenous-good Oligopoly
  - Next time: differentiated-good oligopoly
- Equilibrium concept:
  - Commitment (benchmark)
  - Discretion (Markov Perfect Equilibrium)
  - Not for now: reputational equilibria under oligopoly



# Durable Good Monopoly Commitment

- Optimal pricing with commitment

$$V^C(D_{-1}) = \max_{\{P_t, X_t, D_t\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (P_t - W_t) X_t$$

subject to durable stock dynamics

$$D_t = X_t + (1 - \delta)D_{t-1}$$

and durable-good demand

$$u'(D_t, \xi_t) = P_t - \beta(1 - \delta)\mathbb{E}_t P_{t+1}$$

and initial condition  $D_{-1} = 0$

## Commitment

(continued)

- First-order optimality:

$$\begin{aligned}P_0 : \quad D_0 - (1 - \delta)D_{-1} &= \lambda_0, \\P_t, t \geq 1 : \quad D_t - (1 - \delta)D_{t-1} &= \lambda_t - (1 - \delta)\lambda_{t-1}, \\D_t, t \geq 0 : \quad (P_t - W_t) - \beta(1 - \delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} \\ &= -\lambda_t u''(D_t, \xi_t),\end{aligned}$$

where  $\lambda_t$  is LM on demand constraint

- Given initial condition ( $D_{-1} = 0$ ), we have  $\lambda_t \equiv D_t$   
(commitment  $\sim$  leasing)
- Optimality condition:

$$\begin{aligned}(P_t - W_t) - \beta(1 - \delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} &= \underbrace{-D_t u''(D_t, \xi_t)}_{\equiv \frac{1}{\sigma_t} u'(D_t, \xi_t)}\end{aligned}$$

## Commitment

(continued)

- Combining optimality condition with demand:

$$\begin{aligned}u'(D_t, \xi_t) + D_t u''(D_t, \xi_t) &= W_t - \beta(1 - \delta) \mathbb{E}_t W_{t+1} \\ P_t - \beta(1 - \delta) \mathbb{E}_t P_{t+1} &= u'(D_t, \xi_t)\end{aligned}$$

- Contrast with marginal cost pricing:

$$u'(D_t, \xi_t) = W_t - \beta(1 - \delta) \mathbb{E}_t W_{t+1}$$

### Proposition

*Durable pricing with commitment features*

*no endogenous dynamics:*

- $P_t \equiv \bar{P}$  when there are no shocks ( $W_t$  and  $\xi_t$  constant)
- $D_{t-1}$  does not affect  $P_t$ , controlling for  $W_t$  and  $\xi_t$
- $P_t$  inherits the exogenous persistence of  $W_t$  and  $\xi_t$

▶ details

# Commitment

Two special cases

- Constant-elasticity demand  
→ constant markup pricing

$$P_t = \frac{\sigma}{\sigma - 1} W_t$$

- Linear demand

$$P_t = \frac{1}{2} \left[ \frac{a}{1 - \beta(1 - \delta)} + \frac{\xi_t}{1 - \rho_\xi \beta(1 - \delta)} + W_t \right]$$

- response to cost shocks does not depend on  $\delta$
- level of markup increases with durability

# Durable Good Monopoly

## Discretion

- Time inconsistency problem:
  - demand depends on expected price tomorrow
  - firm wants to promise high price tomorrow
  - but tomorrow it fails to internalize the effect of price on previous-period demand
  - firm competes with itself across time and in the limit of continuous time firm loses all monopoly power (Coase)
- Solution concept:
  - consumers are infinitesimal, form rational expectations about future prices and purchase durables according to demand
  - the firm set today's price to maximize value anticipating its inability to commit to future prices
  - accumulated stock of durables is the state variable
  - Markov Perfect Equilibrium
- Optimal price duration? Commitment versus flexibility

## Discretion

(continued)

- Formally, the problem of the firm:

$$V(D_{-1}, W, \xi) = \max_{(P, X, D)} \left\{ (P - W)X + \beta \mathbb{E} V(D, W', \xi') \right\}$$

$$\text{s.t.} \quad D = X + (1 - \delta)D_{-1},$$

$$u'(D, \xi) = P - \beta(1 - \delta)\mathbb{E}_t p(D, W', \xi')$$

- Equilibrium requirement:

$$p(D_{-1}, W, \xi) = \arg \max_{(P, X, D)} \left\{ (P - W)X + \beta \mathbb{E} V(D, W', \xi') \right\}$$

is the equilibrium strategy of the firm given state variable

## Discretion

(continued)

- Optimality condition:

$$\begin{aligned} & (P_t - W_t)^{-\beta} (1 - \delta) \mathbb{E}_t \{ P_{t+1} - W_{t+1} \} \\ &= (D_t - (1 - \delta) D_{t-1}) \frac{1}{-\varphi'(P_t, W_t, \xi_t)}, \end{aligned}$$

where demand slope is

$$\varphi'(P_t, W_t, \xi_t) = \frac{1}{u''(D_t, \xi_t) + \beta(1 - \delta) \mathbb{E}_t p'(D_t, W_{t+1}, \xi_{t+1})}$$

- Perturbation argument
- Lack of commitment (contrast with leasing)
- State variable dynamics:

$$D_t = \varphi(p(D_{t-1}, W_t, \xi_t), W_t, \xi_t) = f(D_{t-1}, W_t, \xi_t)$$

## Proposition

(a) *Steady state:*

$$\bar{P} = \frac{\bar{\sigma}}{\bar{\sigma} - \delta \bar{\kappa}} W,$$

where  $\bar{\sigma} \equiv \frac{-u'(\bar{D})}{\bar{D}u''(\bar{D})}$ ,  $\bar{\kappa} \equiv 1 + \frac{\beta(1-\delta)p'(\bar{D})}{u''(\bar{D})} > 1$ ,  $u'(\bar{D}) = [1 - \beta(1 - \delta)]\bar{P}$ .

(b) *Endogenous dynamics:*

$D_{t-1}$  is state variable for pricing at  $t$  and  $p'(\cdot, W, \xi) < 0$ .



### Proposition

*With linear demand and AR(1) demand and cost shocks, there exists a linear equilibrium:*

$$\begin{aligned}P_t &= \bar{P} - \alpha(D_{t-1} - \bar{D}) + \gamma(W_t - \bar{W}) + \omega\xi_t, \\D_t &= \bar{D} + \phi(D_{t-1} - \bar{D}) - \psi(W_t - \bar{W}) + \chi\xi_t,\end{aligned}$$

*with  $\alpha > 0$ ,  $\phi \in (0, 1 - \delta)$ ,  $\gamma \in (0.5, 1)$ ,  $\omega, \psi, \chi > 0$ .* [▶ details](#)

### Corollary

- (i)  $D_t$  increases over time, as prices and markups fall.
- (ii) markups increase (*procyclical*) with demand shocks and decrease (*countercyclical*) with cost shocks.

## Monopolistic competition

- $D$ -good is a CES aggregator of varieties:

$$D_t = \left( \int_0^1 D_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

- Two alternative assumptions:

(i) Durable aggregator:  $D_t = X_t + (1 - \delta)D_{t-1}$ .

Constant markup pricing (Barsky et al., 2007)

(ii) Durable varieties:  $D_{it} = X_{it} + (1 - \delta)D_{i,t-1}$ .

Problem isomorphic to that of a monopolist with  $\xi_t$  related to the equilibrium dynamics of  $D_t$

# Durable Good Oligopoly

## Commitment (Cournot-Nash)

- Consider  $N$  symmetric firms producing a homogenous durable good with constant marginal cost and no shocks
- Durable good dynamics

$$D_t = (1 - \delta)D_{t-1} + \sum_{i=1}^N x_{it}$$

- A given firm commits to a sequence  $\{\tilde{x}_{it}\}$  given the symmetric strategy of the other  $N - 1$  firms  $\{x_t\}$ . In equilibrium,  $\tilde{x}_t = x_t$
- In equilibrium,  $x_t = \frac{1}{N}(D_t - (1 - \delta)D_{t-1})$  and  $\lambda_t = D_t/N \Rightarrow$

$$(P_t - W_t) - \beta(1 - \delta)\mathbb{E}_t\{P_{t+1} - W_{t+1}\} = -\frac{D_t}{N}u''(D_t, \xi_t)$$

# Durable Good Oligopoly

Discretion (Cournot-MPE)

- Under discretion, both competition within firm over time and between firms at a given  $t$  reduces markups
- A firm chooses  $\tilde{x}(D_-)$  given the symmetric strategy  $x(D_-)$  of the other  $N - 1$  firms and equilibrium price next period  $p(D)$ :

$$v(D_-) = \max_{\tilde{x}, D, P} \{(P - W)\tilde{x} + \beta v(D)\}$$

$$\text{s.t.} \quad D = (1 - \delta)D_- + (N - 1)x(D_-) + \tilde{x}$$
$$P = u'(D) + \beta(1 - \delta)\mathbb{E}p(D)$$

- The solution to this problem in equilibrium yields:

$$\tilde{x}(D_-) = x(D_-), \quad P = p(D_-),$$

$$D = f(D_-) = (1 - \delta)D_- + Nx(D_-)$$

# Durable Good Oligopoly

Discretion (Cournot-MPE)

- Optimality condition for a firm:

$$\begin{aligned}(P - W) - \beta[(1 - \delta) + (N - 1)x'(D)](P' - W') \\ = \tilde{x}(D_-)(-u''(D) - \beta(1 - \delta)p'(D))\end{aligned}$$

- Impose equilibrium:

$$\tilde{x}(D_-) = x(D_-) = \frac{1}{N}(f(D_-) - (1 - \delta)D)$$

- Then equilibrium dynamics is characterized by

$$\begin{aligned}u'(D_t) &= P_t - \beta(1 - \delta)P_{t+1}, \\ (P_t - W) - \beta(P_{t+1} - W) &\left( \frac{1 - \delta}{N} + \frac{N - 1}{N}f'(D_t) \right) \\ &= \frac{D_t - (1 - \delta)D_{t-1}}{N} \frac{1}{-\varphi'(P_t)},\end{aligned}$$

where  $D_t = f(D_{t-1})$ ,  $P_t = p(D_{t-1})$  and  $\varphi(\cdot) = f(p^{-1}(\cdot))$ .

# Durable Good Oligopoly

## Linear Demand

### Proposition

*With linear demand, there exists a linear oligopoly equilibrium:*

$$D_t = \bar{D} + \phi^{(N)}(D_{t-1} - \bar{D}) \quad \text{and} \quad P_t = \bar{P} - \alpha^{(N)}(D_{t-1} - \bar{D}).$$

$\phi^{(N)}$  and  $\alpha^{(N)}$  decrease in  $N$ . [▶ details](#)

- As number of firms increases, prices are closer to marginal cost and there is less endogenous dynamics

# General Utility Functions

## Approximation

- Steady state markup cannot be solved for without  $p'(\bar{D})$ .
- To compute the steady state markup exactly, we need to know all derivatives of the policy function  $p(D)$  at  $\bar{D}$ .
- Similar problem arises in hyperbolic discounting
  - Krusell, Kuruscu, and Smith (2002)
  - Judd (2004)
  - Polynomial approximations
- In the case of durables, polynomial approximations should work perfectly.
- Each additional higher order term is suppressed by  $\phi^n$ .

# General Utility Functions

## Approximation

- In the case of monopoly, the transition function  $f(D)$  satisfies

$$\begin{aligned} & \frac{(1 - \beta(1 - \delta)) W - u'(f(D))}{f(D) - (1 - \delta) D} - u''(f(D)) \\ &= \beta(1 - \delta) \frac{(1 - \beta(1 - \delta)) W - u'(f(f(D)))}{f(f(D)) - (1 - \delta) f(D)} f'(f(D)) \end{aligned}$$



# General Utility Functions

## Approximation

- Express  $f(D)$  as a power series.
- When Taylor expanded, the functional equation gives an infinite number of conditions for the derivatives of  $f(D)$  at  $\bar{D}$
- The first one links  $\bar{D}$  and  $f'(\bar{D})$ .
- The second one links  $\bar{D}$ ,  $f'(\bar{D})$ , and  $f''(\bar{D})$ .
- The third one links  $\bar{D}$  and the first three derivatives.
- Etc.

# General Utility Functions

## Approximation

- If we set  $f^{(n)}(\bar{D})$  to zero and solve the system, we make only a small mistake proportional to  $\phi^n$ , where  $\phi \equiv f'(\bar{D})$
- In practice, only a couple of terms will be needed.
- When translated to the GE context, this means that it is possible to solve GE models with durables and discretion, for arbitrary utility functions.

## Numerical Example

- $\beta = 0.9$
- $\delta = 0.2$
- Constant elasticity  $\sigma = 2$
- Value function iteration on a grid, polynomial smoothing:

$$V(D_-) = \max_D \left\{ (u'(D) + \beta(1 - \delta)p(D) - W)(D - (1 - \delta)D_-) + \beta V(D) \right\}$$

Update  $\tilde{V}(D_-)$  and  $D = \tilde{f}(D_-)$ , and calculate

$$\tilde{p}(D_-) = u'(f(D_-)) + \beta(1 - \delta)p(f(D_-))$$

Polynomially smooth  $f(\cdot)$  and  $p(\cdot)$

# Numerical Example

Dynamics with no shocks

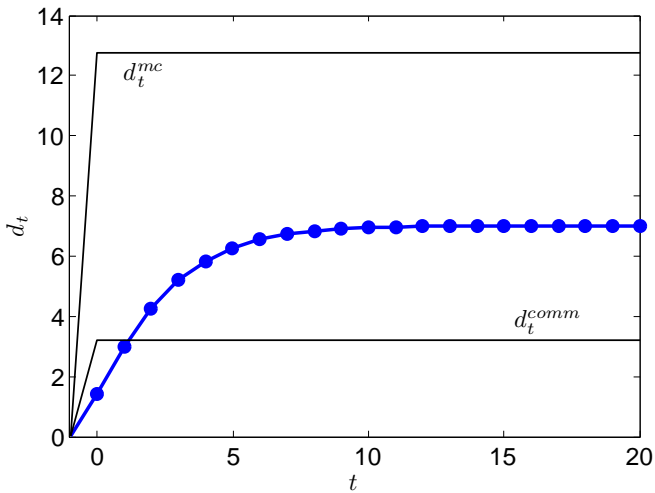


Figure: Dynamic path of  $D_t$

# Numerical Example

Dynamics with no shocks

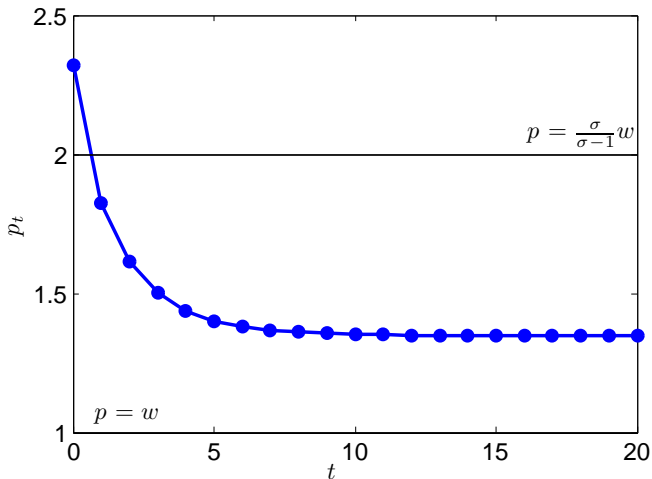


Figure: Dynamic path of  $P_t$

# Numerical Example

Unexpected permanent cost increase

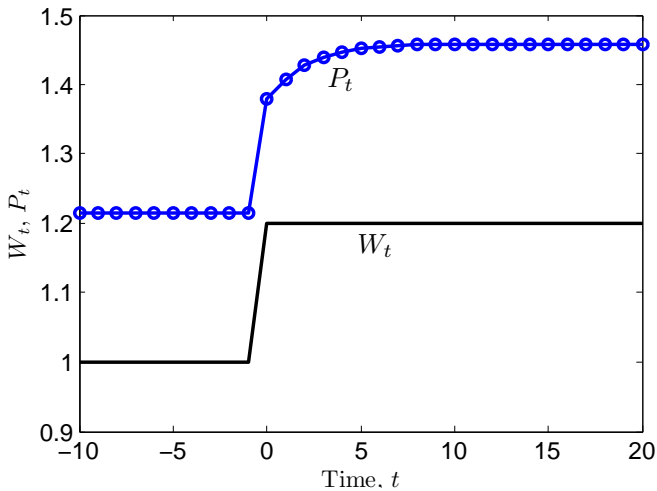


Figure: Response of  $P_t$

# Numerical Example

Unexpected permanent cost increase

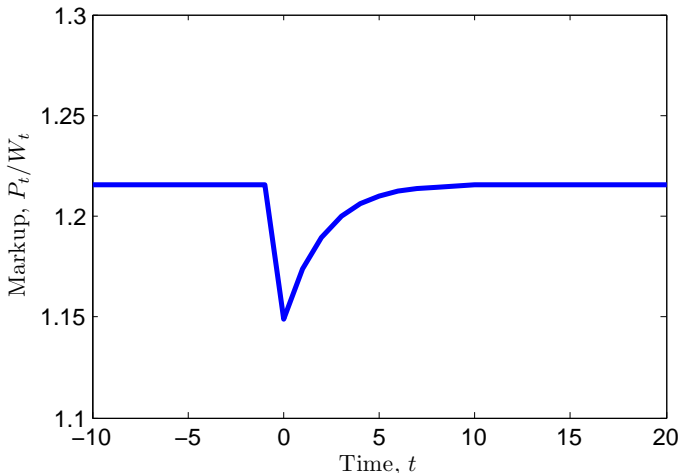


Figure: Response of markup,  $P_t/W_t$

# Numerical Example

Unexpected permanent cost increase

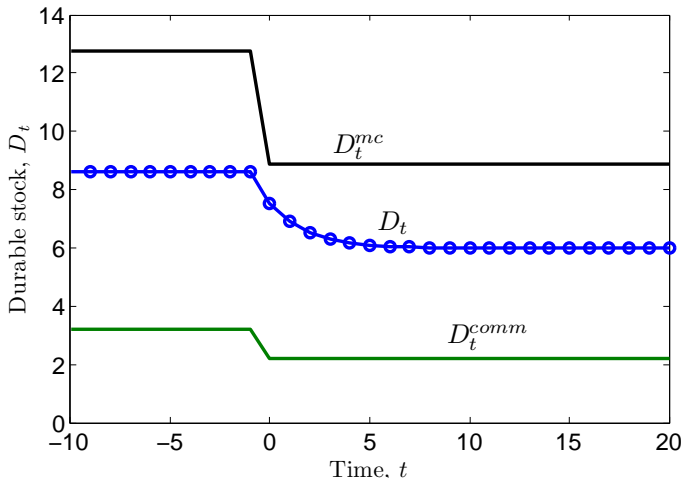


Figure: Response of  $D_t$



# Numerical Example

Unexpected permanent demand increase

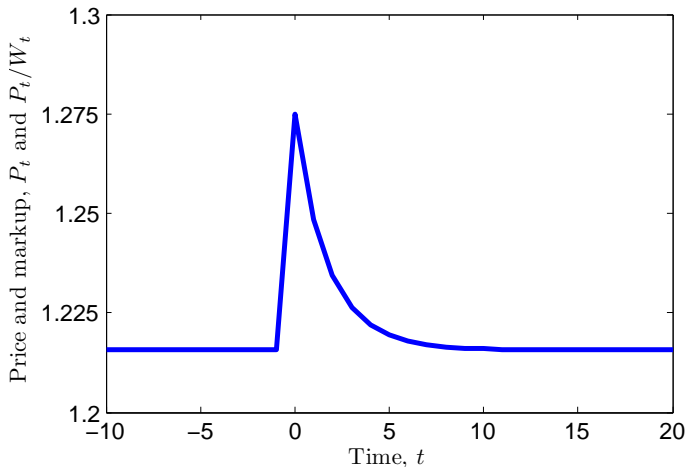


Figure: Response of  $P_t$  and markup  $P_t/W_t$

# Numerical Example

Unexpected permanent demand increase

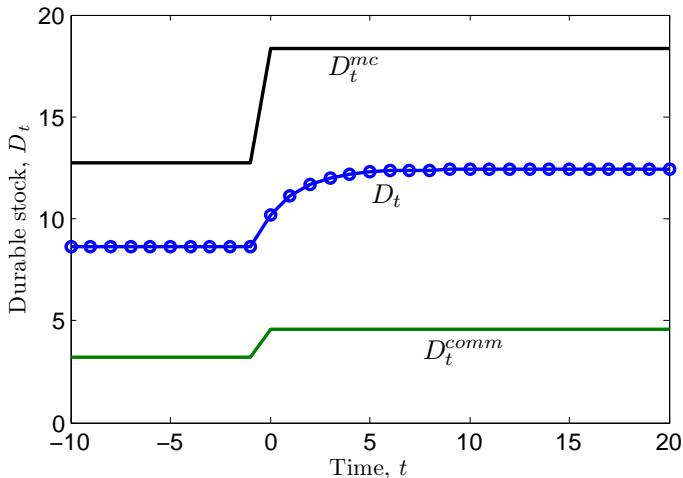


Figure: Response of  $D_t$

# Numerical Example

Stochastic cost shocks

Table: Statistical properties

$\log(\cdot)$	$\sigma$ (%)	$\rho$	$\text{corr}(\cdot, \log W_t)$
Wage, $W_t$	4.9	0.80	1.00
Price, $P_t$	5.1	0.90	0.88
Markup, $P_t/W_t$	2.2	0.69	-0.19
Durable stock, $D_t$			
— constant markup	15.5	0.79	-0.99
— discretion	12.2	0.95	-0.75
— ratio (disc/comm)			0.29
Durable purchases, $X_t$			
— constant markup	70.7	-0.08	-0.31
— discretion	21.4	0.57	-0.91
— ratio (disc/comm)			0.16

# Numerical Example

Stochastic cost shocks

Table: Pass-through

	$\log W_t$	$\log W_{t-1}$
$\log P_t$	0.91	
$\log P_t$	0.65	0.34
	$\Delta \log W_t$	$\Delta \log W_{t-1}$
$\Delta \log P_t$	0.61	
$\Delta \log P_t$	0.63	0.15

# Numerical Example

## Stochastic demand shocks

Table: Statistical properties

$\log(\cdot)$	$\sigma$ (%)	$\rho$	$\text{corr}(\cdot, \log \xi_t)$
Demand, $\xi_t$	4.8	0.77	1.00
Price and markup, $P_t/W$	1.9	0.79	-0.18
Durable stock, $D_t$			
— constant markup	9.7	0.77	1.00
— discretion	7.2	0.94	0.66
— ratio (disc/comm)			-0.22
Durable purchases, $X_t$			
— constant markup	36.1	-0.03	0.91
— discretion	13.6	0.56	0.55

## Conclusion

- Durable monopoly pricing results in **endogenous dynamics**
- Procyclical markups in response to **demand shocks**
- Countercyclical markups in response to cost shocks (**incomplete pass-through**)
- Oligopoly: endogenous dynamics dies out with  $N$
- Next steps: general equilibrium, quantitative evaluation