Harvard University Extension School
Computer Science E-121

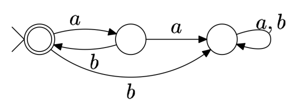
Problem Set 1

Due Friday, September 20 at 11:59 PM Eastern Time.
Submit your solutions in a single PDF called lastname+ps1.pdf emailed to cscie121@seas.harvard.edu.
LATE PROBLEM SETS WILL NOT BE ACCEPTED.
See syllabus for collaboration policy.

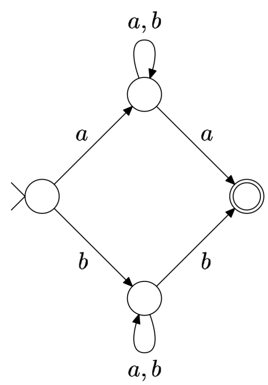
PROBLEM 1 (5+5 points)

Informally describe the languages recognized by the following finite automata:

(A) This DFA:



(B) This NFA:



PROBLEM 2 (5 points)

Write the formal description $(Q, \Sigma, \delta, q_0, F)$ for the DFA in Problem 1(A).

PROBLEM 3 (5 points)

Use the subset construction to make a DFA for the NFA in Problem 1(B). Show your work.

PROBLEM 4 (5 + 5 + 5 points)

Are the following statements true or false? Justify your answers with a proof or counterexample.

(A) For any languages L_1 and L_2, $(L_1 \cap L_2)^* = L_1^* \cap L_2^*$.

(B) For any languages L_1 and L_2, $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$.

(C) Complementing all states in a DFA M (making the final states non-final and vice-versa) will result in a new DFA M' such that $L(M') = \Sigma^* - L(M)$.

PROBLEM 5 (5+5 points)

An NFA M contains a *cycle* if there is a state q and a string x such that if M is in state q and reads string x, M can return to state q. Prove or disprove the following statements:

(A) If M recognizes an infinite language, then M has a cycle.

(B) If M has a cycle, then M recognizes an infinite language.

PROBLEM 6 (5+15 points)

For any language L, let $ExtraB(L) = \{sbt : s, t \in \{a, b\}^* \text{ and } st \in L\}$.

(A) What is $ExtraB(\{aba\}^*)$?

(B) Show that if L is regular, then so is $ExtraB(L)$. (Hint: Show how, given a DFA for L, you can construct an NFA for $ExtraB(L)$).

PROBLEM 7 (3 bonus points)

CHALLENGE: For any string s and language L, let $P_s(L) = \{t : st \in L\}$. Show that L is regular if and only if there are finitely many unique sets $P_s(L)$.