Harvard University Extension School
Computer Science E-121

Problem Set 3

Due Friday, October 4, 2013 at 11:59 PM Eastern Time.
Submit your solutions in a single PDF called lastname+ps3.pdf emailed to cscie121@seas.harvard.edu.
LATE PROBLEM SETS WILL NOT BE ACCEPTED.

Problem set by ** ENTER YOUR NAME HERE **

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

PROBLEM 1 (3+3+3 points)

Prove the following statements, or provide a counterexample.
(A) If $L \subseteq L'$ and L' is regular, then L is regular.
(B) If $L \setminus L'$ is cofinite, then L' is regular.
(C) If $L \cap L'$ is regular and infinite, then either L or L' is regular.

PROBLEM 2 (4+4+2 points)

Consider the language $L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

(A) Prove that L satisfies the conditions of the pumping lemma. That is, show that there is a number p where, if s is any string in L of length at least p, then s may be written as $s = xyz$ such that:

1. for each $i \geq 0$, $xy^i z \in L$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

(B) Prove that L is non-regular.

(C) Explain why this fact does not contradict the pumping lemma.

PROBLEM 3 (3+3+3+3+3 points)

For each of the following languages, determine whether the language is regular or non-regular, and prove your answer.

(A) $\{xyx^R : x, y \in \Sigma^*\}$.

(B) $\{a^i b^j a^j b^i \colon\ i, j \geq 0\}$.

(C) $\{w \ : \ w$ is, for some $n \geq 1$, the decimal notation for $10^n\}$.

(D) $\{w \ : \ w$ is, for some $n \geq 1$, the unary notation for $10^n\}$.

(E) $\{R \ : \ R$ is a regular expression for a language over $\Sigma\}$ for some alphabet Σ.

PROBLEM 4 (3+2 points)

Let

$$L_1 = \{a^n b^m a^m b^n : m, n \geq 0\}$$
$$L_2 = \{a^n b^m : m, n \geq 0, m \neq n\}$$

(A) Construct a context-free grammar G_1 for L_1 and a context-free grammar G_2 for L_2.
(B) Draw a parse tree for the string *aababb* for the CFG G_1.

PROBLEM 5 (3+3+3 points)

Show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not regular:

(A) $L = \{w \in \Sigma^* : w$ contains twice as many as as bs$\}$

(B) $L = \{ww^R : w \in \Sigma^*\}$

(C) $L = \{w \in \Sigma^* : w = w^R\}$

PROBLEM 6 (10 points)

Show that the language $L = \{a^p : p$ is a prime number$\}$ is not regular.

PROBLEM 7 (2 bonus points)

CHALLENGE: Show that the language $L = \{a^{p+q} : p$ and q are prime numbers$\}$ is not regular.