

**Harvard University
Computer Science 121**

Problem Set 5

Due Tuesday, October 22, 2013 at 1:20 PM.

Submit your solutions electronically to `cs121+ps5@seas.harvard.edu` with "ps5 submission" in the subject line. The solutions to Parts A and B should be attached as separate PDF files, called `lastname+ps5a.pdf` and `lastname+ps5b.pdf`.

Late problem sets may be turned in until Friday, October 25, 2013 at 1:20 PM with a 20% penalty.

Problem set by ****ENTER YOUR NAME HERE****

Collaboration Statement: ****FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)****

See syllabus for collaboration policy.

PART A (Graded by Gabe)

PROBLEM 1 (5+7 points)

Recall that given a function $f : \Sigma \rightarrow \Delta^*$, the function $h : \Sigma^* \rightarrow \Delta^*$ defined recursively by $h(\epsilon) = \epsilon$ and $h(w\sigma) = h(w)f(\sigma)$ is called a homomorphism.

(A) Show that if L is a context-free language over the alphabet Σ and $h : \Sigma^* \rightarrow \Delta^*$ is a homomorphism, then $h(L) = \{h(w) : w \in L\}$, is a context-free language over the alphabet Δ .

(B) Show that if L is a context-free language over the alphabet Δ and $h : \Sigma^* \rightarrow \Delta^*$ is a homomorphism, then $h^{-1}(L) = \{w : h(w) \in L\}$ is a context-free language over the alphabet Σ .

PROBLEM 2 (10 points)

Given a PDA M with the symbol $\$$ in its stack alphabet, let $\mathcal{N}(M)$ be the set of strings w such that there is a path through M on w starting at the start state of M with the symbol $\$$ on the stack and ending with an empty stack. Show that a language L is context free if and only if there is a PDA such that $\mathcal{N}(M) = L$.

PROBLEM 3 (10 points)

Show that if A is context free and B is regular then $A/B = \{w : wx \in A \text{ for some } x \in B\}$ is context free.