

# **Banks Are Where The Liquidity Is**

**Oliver Hart**

Harvard University & NBER

and

**Luigi Zingales\***

University of Chicago, NBER & CEPR

February 2014  
(revised, June 2015)

## *Abstract*

What is so special about banks that their demise often triggers government intervention? In this paper we show that, even ignoring interconnectedness issues, the failure of a bank causes a larger welfare loss than the failure of other institutions. The reason is that agents in need of liquidity tend to concentrate their holdings in banks. Thus, a shock to banks disproportionately affects the agents who need liquidity the most, reducing aggregate demand and the level of economic activity. The optimal fiscal response to such a shock is to help people, not banks, and the size of this response should be larger if a bank, rather than a similarly-sized nonfinancial firm, fails.

Key Words: liquidity, bailout, banking.  
JEL Codes: E41 G21, E51.

\* We would like to thank Robert Barro, Arnoud Boot, Christian Leuz, Alp Simsek, Vania Stavrakeva, Rene Stulz, Robert Vishny, and participants in seminars at Harvard- MIT, the University of Chicago, Cass Business School, LSE (finance), Boston College, the University of Amsterdam, European Summer Symposium in Economic Theory, 2014 (Gerzensee), Yonsei University, and Sungkyunkwan University for useful comments, and Kirill Borusyak for research assistance. Oliver Hart gratefully acknowledges financial support from the U.S. National Science Foundation through the National Bureau of Economic Research. Luigi Zingales gratefully acknowledges financial support from the Center for Research in Security Prices (CRSP) and the Initiative on Global Markets at the University of Chicago.

During the 2007-2008 financial crisis industrial firms, including major ones like General Motors, were allowed to go bankrupt. By contrast, financial firms, with the notable exception of Lehman, were bailed out. One possible reason for this differential treatment can be found in the political clout of these two industries. Financial firms were and are major donors of recent administrations. Many of the recent Treasury Secretaries and White House Chiefs of Staff had close ties to the financial industry. The greater attention shown by the government toward the financial industry, thus, might be purely a matter of politics. While not denying this possibility, in this paper we explore an alternative interpretation: that government intervention is justified by an intrinsic difference in the welfare consequences when a bank, rather than an equally-sized industrial firm, fails.

An often-mentioned rationale for this difference is the degree of interconnectedness of financial institutions. But while there is no doubt that large financial institutions tend to be highly interconnected, large industrial firms like General Motors and Ford are very interconnected too. To quote Ford's CEO Alan Mulally: "The domestic auto industry is highly interdependent. A collapse of one of our competitors would not only affect Ford and our transformation plan, but would have a devastating ripple effect across the economy."<sup>1</sup>

Another popular interpretation among financial economists for the difference between large manufacturing firms and banks focuses on the ability of depositors to run (e.g., Diamond and Dybvig, 1983), raising the possibility of inefficient liquidation. Yet, suppliers and customers of GM can run too, a concern that led the U.S. Government to intervene in 2008 to guarantee GM warranties. So what makes banks different?

---

<sup>1</sup> Alan Mulally's testimony to the United States Senate Committee on Banking, Housing and Urban Affairs, November 18, 2008.

In this paper, we focus on a different dimension: banks are special because they are where people in need of liquidity keep their wealth.<sup>2</sup> In a world where future wealth is not fully pledgeable, there is a natural demand for safe (or relatively safe) assets to support trading. Banks arise to satisfy this need. As a result, we will show that agents who need liquidity for transaction purposes will hold their wealth disproportionately in the form of bank deposits (or similar financial securities—see below).

If future wealth is not fully pledgeable, the failure of any firm implies two losses: a direct loss of wealth, and an indirect loss in liquidity resulting from the reduction in pledgeable assets. This dual effect is present both for a bank and for an industrial firm. Yet, the impact of a loss of pledgeable assets is different depending on the liquidity needs of the holders of those assets. A bank failure disproportionately hits agents who are liquidity constrained (more so than if an industrial firm was to fail), causing a larger drop in the demand for labor services that was supported by that liquidity, and a larger fall in GDP. (Similarly, a loss borne by debt-holders affects GDP disproportionately more than an equally-sized loss borne by equity investors.)

Unlike the classic literature (e. g., Baumol (1952) and Tobin (1956)) that focuses on the transactional demand for *money*, our model derives a transactional demand for *safe assets*. It is a way to formalize the source of the shortage of safe assets stressed by Caballero (2006). The fundamental friction is the same as that noted by Caballero et al. (2008): as a result of limited pledgeability, the world’s ability to generate wealth has outpaced its ability to generate safe stores of value and to credibly transfer that wealth. As we show in Section 5 this “shortage”

---

<sup>2</sup> A focus on consumers seems particularly germane given the growing evidence (Mian and Sufi (2014)) that during the Great Recession firms had plenty of liquidity, while consumers were severely liquidity constrained. Indeed, the 2004 Survey of Consumer Finances finds that 37% of families are financially constrained, where constrained is defined as a family that applied for credit and has been rejected or has been discouraged from applying by the fear of being rejected. By contrast, the 2003 Survey of Small Business Finances finds that only 15% of small firms were constrained, using the same definition. Since small firms are more likely to be constrained than big firms, this evidence seems to suggest that financial constraints are more likely to be a problem for consumers than for firms.

persists even in the presence of government debt. Our approach is also in the Clower (1967) tradition of cash-in-advance models, but unlike this tradition we focus not just on cash holdings, but on all pledgeable wealth.

Building on Hart and Zingales (forthcoming), we consider a simple general equilibrium economy where security markets are complete, but consumers cannot pledge future income or wealth. There are two groups of agents and the lack of a simultaneous double coincidence of wants between the two groups generates the need for a relatively safe asset for transaction purposes. In this context we show that agents with liquidity needs (in our model, those who have a purchase motive) will choose to hold a disproportionate amount of (Arrow) securities that pay off in low states of the world. We also show that a one dollar loss occurring in bad states of the world or incurred by buyers has a welfare cost bigger than one.

In this simple framework the demand for liquidity can be satisfied by senior claims in individual projects. When we add to this simple framework idiosyncratic shocks and asymmetry of information, however, we can prove that buyers' liquidity needs are best satisfied by a debt claim on a diversified pool of projects, while the sellers' needs by an equity claim on the same set of projects. We can interpret the institution holding the diversified pool of projects as a bank.

A bank is a way the claim desired by buyers can be manufactured (not necessarily the only way). If this way is used, we can compare a bank failure with a failure of an industrial firm of similar size. Given that a bank will fail in bad states of the world (where welfare losses are bigger), a bank failure will naturally have worse economic consequences. To compare banks' and firms' failure for a given state of the world, we consider an exogenous idiosyncratic shock, something like a major fraud a la Madoff. In this context we can prove that the same loss hits liquidity-constrained agents more (and other agents less) if it is suffered by a bank than if it is

suffered by a firm and as a result produces worse welfare effects. These results can explain why the bursting of the internet bubble had relatively mild macroeconomic effects, while the (milder) loss in subprime mortgages had a devastating impact: the former shock was concentrated on equity instruments held by people with no compelling liquidity needs, while the latter shock impacted relatively senior claims held by people with compelling liquidity needs (see also Mian and Sufi, 2014).

A bank deposit is a way in which the desired claim can be manufactured. Others are possible. For example, instead of a senior claim on a portfolio, one can build a portfolio of senior claims (something that resembles a money market fund). We show that unless we allow for state contingent debt, a money market fund (i.e., a portfolio of senior claims) is an inferior instrument to satisfy liquidity needs vis-à-vis a bank deposit.

According to this view, banks are nothing but a cost-effective way to manufacture safe assets needed for transaction purposes. This simple theory of banking is able to explain why banks need to have deposits that do not fluctuate in value. Depositors are the agents with the highest need for liquidity and thus they demand insurance against possible falls in the value of their investments, even if they are risk neutral. In our model this insurance is provided by the agents less in need of liquidity.

This very simple theory of banking is able to explain not only why the default of a bank is worse than the default of a similarly-sized industrial company, but also why these effects are not unique to banks: they are common to all the most senior securities. This might also explain why governments are so reluctant to impose losses on bonds, especially secured bonds. The reason is the same: they are held disproportionately by people in need of liquidity. Finally, the theory can explain why the size of the fiscal response to a shock is larger if a bank, rather than a

similarly-sized nonfinancial firm, fails. Yet, our model would suggest that the government intervention should be targeted to help people with liquidity needs, not banks per se.

Note that we would not have obtained the same results had we assumed that the demand for safe assets came from agents' risk aversion. In a complete markets model, agents with different risk aversion will trade so as to equalize their marginal rates of substitution. Thus, on the margin the welfare loss is the same regardless of which group of agents is hit by the shock. By contrast, in our model even a marginal loss has welfare effects that depend upon who is hit.<sup>3</sup>

A risk-aversion based model would also be unable to explain why riskless debt has a yield lower than that predicted by a risk-return tradeoff (Krishnamurthy and Vissing-Jorgensen (2012)). To explain riskless debt's lower yield, several papers have postulated a special demand for riskless securities (e.g., DeAngelo and Stulz (2015), Hanson et al. (2014), Krishnamurthy and Vissing-Jorgensen (2012), and Stein (2012)). In this paper we derive this demand from first principles, in so doing providing foundations for the monetary policy results obtained by Stein (2012), the banking capital requirement implications derived by DeAngelo and Stulz (2015), and the differences between traditional and shadow banks emphasized by Hanson et al. (2014).

Our model analyzes one source of demand for safe assets, others exist. For example, the explosion in the use of derivative contracts has increased the demand for safe assets for collateral purposes (e.g., Gorton, 2010). Therefore, we do not claim that our model by itself explains the ratio of 3.3 of safe assets to GDP in 2010 found by Gorton, Lewellen, and Metrick (2012).

The rest of the paper proceeds as follows. Section 2 presents the framework. Section 3 characterizes the non-pledgeable equilibrium. Section 4 introduces idiosyncratic shocks and asymmetric information. Section 5 considers the role of fiscal policy. Section 6 concludes.

---

<sup>3</sup> If, in addition to heterogeneity in risk aversion, one adds also catastrophic events (as Barro and Mollerus, 2014), then one can obtain similar welfare implications.

## 2. The Framework

There are many agents; ex ante they are identical. At the beginning of period 1 each agent learns his type. Each agent is equally likely to be a type that buys (b) or a type that sells (s). Both buyers and sellers consume wheat in period 3 and there is no discounting. Each agent has an endowment of wheat in period 1 equal to  $e$ , which can be invested in projects that yield wheat in period 3. We will assume that  $e > 0$ . In addition the buyers will receive a non-pledgeable large endowment of wheat in period 3. This extra endowment can be seen as a reduced form version of the non-pledgeable labor income generated by the buyers when they turn into sellers of their services, as in Hart and Zingales (forthcoming).

The time-line is as in Figure 1.

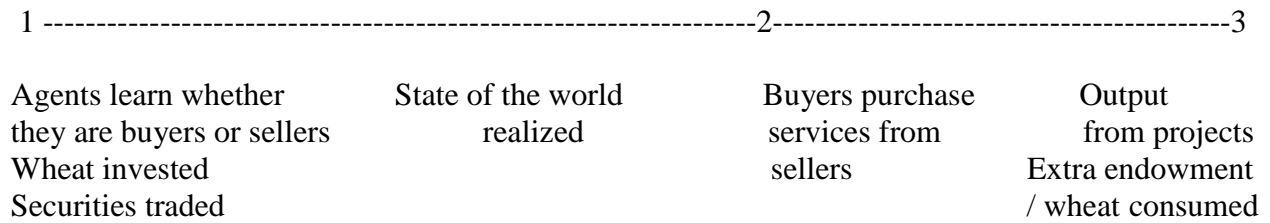


Figure 1

We write agents' utilities as:

$$\text{Buyers: } U^b = w^b + s^b$$

$$\text{Sellers: } U^s = w^s - \frac{1}{2}(l^s)^2$$

where  $s^b$  is the quantity of a seller's services consumed by a buyer;  $l^s$  is the labor supplied by a seller; and  $w^i$  is the quantity of wheat consumed by individual  $i = b, s$  in period 3. We assume constant returns to scale: one unit of labor yields one unit of services. In other words buyers are

indifferent between wheat and sellers' services and sellers have a quadratic disutility of labor. Agents are risk neutral. The market for sellers' services is perfectly competitive.

There is a risky constant returns to scale technology that transfers endowment between periods 1 and 3. There are  $n$  (aggregate) states of the world, which are verifiable. With probability  $\pi_i > 0$ , one unit of period 1 wheat is transformed into  $R_i$  units of period 3 wheat ( $i=1, \dots, n$ ). Without loss of generality we label the states so that  $0 < R_1 < R_2 < \dots < R_n$ . Agents learn about the state of the world between periods 1 and 2. We also assume that there is free entry of firms possessing the technology described and that the firms' technologies are perfectly correlated.

This framework is similar to Hart and Zingales (forthcoming), except in four respects. One is that there are  $n$  states instead of just 2. The second is that the model is not fully symmetric. The buyers do not sell services and vice versa. The third is that we do not assume that an agent can insure against becoming a buyer rather than a seller before period 1; a justification is that period 1 endowment cannot be pledged in advance (ex post an agent can hide it). Fourth, in this model there is no riskless storage. As we show in Hart and Zingales (forthcoming), the presence of multiple investment choices creates a distortion between private and social incentives. Since this distortion has already been analyzed in our other paper, we want to eliminate it here.

The timing is as follows. Agents learn their type at the beginning of period 1. Markets for Arrow securities then open, and firms invest (recall that the aggregate state of the world is verifiable). The state of the world is learned at the end of period 1. In period 2 buyers buy sellers' services. In period 3 investments pay off and wheat is consumed.



As in Hart and Zingales(forthcoming), if buyers *can* pledge their period 3 wealth, this economy has a unique (rational expectations) equilibrium. In this equilibrium, there is a separation between consumption and production. The price of sellers' services and the wage rate of sellers in period 2, the price of Arrow security  $i$  in period 2 in state  $i$ , and the price of wheat in periods 1 and 3, can all normalized to be 1. At these prices each buyer purchase one unit of services and each seller incurs a labor cost of  $1/2$ , and receives a consumer surplus of  $1/2$ . Agents also receive expected surplus  $e \sum_i^n \pi_i R_i$  from investing their endowment, and so the utility of sellers is  $e \sum_i^n \pi_i R_i + \frac{1}{2}$  and the utility of buyers is  $e \sum_i^n \pi_i R_i$ . Finally, since the rate of return in the economy is  $\bar{R} = \sum_i^n \pi_i R_i$ , the period 1 price of Arrow security  $i$  is  $\pi_i / \bar{R}$ .

### 3. Non-pledgeable Equilibrium with only Aggregate Uncertainty

Consider now the case where buyers cannot pledge their period 3 endowment of wheat ( they can hide it). We continue to assume that project returns *can* be pledged (the firms carrying out the projects might be public companies, say, and managers cannot steal project returns). That is, firms can invest period 1 wheat in the risky project and issue securities collateralized by the project returns. These securities will be purchased in period 1 by buyers and used as a means of payment for services in period 2. We assume that firms issue a full set of Arrow securities backed by their projects, where security  $i$ ,  $i=1, \dots, n$ , pays off a unit of wheat in period 3 if and only if state  $i$  occurs.

We suppose that  $2eR_n < 1$ , i.e., there is not enough pledgeable wealth even in the best state of the world. To compute the non-pledgeable equilibrium, normalize so that the prices of

wheat in period 1, wheat in period 3, Arrow security  $i$  in period 2 (if state  $i$  occurs), are all one. Consider a buyer's utility maximization problem. In equilibrium the price of services in period 2 cannot exceed 1 since otherwise buyers would strictly prefer to use their securities to purchase period 3 wheat rather than sellers' services, and so the services market would not clear. Thus, we can assume for the purpose of calculating utility that buyers use all their Arrow securities to buy services.

It follows that in period 1 a buyer chooses his holdings of Arrow securities  $(x_i^b, i=1, \dots, n)$  to solve:

$$(*) \quad \text{Max} \sum_i \pi_i \left[ \frac{x_i^b}{p_i^s} \right]$$

$$\text{s.t.} \sum_i^n q_i x_i^b \leq e,$$

where  $p_i^s$  is the price of sellers' services in state  $i$  and  $q_i$  is the period 1 price of the  $i$ th Arrow security. Note that firm profits are zero in equilibrium given constant returns to scale, and so we do not need to keep track of any dividends received by consumers. Note also that  $x_i^b \geq 0$ , since a short seller cannot credibly commit to pay ex post: he can always hide his endowment of wheat in period 3.

In period 2 a seller chooses her labor supply  $l^s$  to maximize  $p^s l^s - \frac{1}{2}(l^s)^2$ , where  $p^s$  is the price of sellers' services, yielding  $l^s = p^s$ . A seller's net utility from work is  $\frac{1}{2}(p^s)^2$ . Hence in period 1 a seller chooses her holdings of Arrow securities  $(x_i^s, j=1, \dots, n)$  to solve:

$$(**) \quad \text{Max} \sum_i \pi_i \left[ x_i^s + \frac{1}{2}(p_i^s)^2 \right]$$

subject to

$$\sum_i^n q_i x_i^s \leq e,$$

where  $x_i^s \geq 0$  for the same reason as above.

As noted, profit maximization and constant return to scale imply zero profit:

$$(3.1) \quad \sum_i^n q_i R_i = 1 .$$

Since all the wheat is invested, the supply of the  $i$ th Arrow security is  $2eR_i$ . Hence, the securities market clearing conditions are

$$(3.2) \quad x_i^b + x_i^s = 2eR_i, \text{ for } i=1, \dots, n.$$

It is easy to see that in equilibrium  $x_i^b > 0$  for all  $i$ : if  $x_i^b = 0$ , the price of services in state  $i$  would be zero and the return to a buyer of purchasing an Arrow security in that state would be infinite. Also, since we have assumed that  $2eR_n < 1$ , even in the best state the supply of liquidity is not enough to support the Arrow-Debreu level of trade and so the price of services will be strictly below 1. It follows that buyers will spend all their available wealth on services in period 2. Given the supply function for services obtained earlier, we can write the market clearing conditions for services in state  $i$  as

$$(3.3) \quad \frac{x_i^b}{p_i^s} = p_i^s \text{ for } i=1, \dots, n.$$

or,  $p_i^s = (x_i^b)^{\frac{1}{2}}$ .

(\*), (\*\*), (3.1) - (3.3) characterize a non-pledgeable equilibrium.

*Proposition 1:* If  $R_1 \geq \frac{\bar{R}}{2}$ , then there is a unique non-pledgeable equilibrium given by, for

all  $i$ ,

$$x_i^b = e\bar{R}, x_i^s = 0,$$

$$p_i^s = (e\bar{R})^{\frac{1}{2}},$$

$$q_i = \frac{\pi_i}{\bar{R}}.$$

*Proof:* The first order conditions for (\*) and (\*\*) when the seller is at an interior solution are:

$$(3.4) \quad \frac{\pi_i}{p_i^s} = \lambda q_i,$$

$$(3.5) \quad \pi_i = \mu q_i,$$

for some  $\lambda, \mu > 0$ .

From (3.1), it follows that

$$(3.6) \quad p_i^s = \frac{\pi_i}{\lambda q_i} = \frac{\bar{R}}{\lambda},$$

$$(3.7) \quad q_i = \pi_i / \bar{R}.$$

Hence by (3.3)  $x_i^b$  is constant. Given the buyer's budget constraint in (\*), this implies

$$(3.8) \quad x_i^b = e\bar{R}.$$

Given  $R_1 \geq \frac{\bar{R}}{2}$ ,  $2eR_1 \geq e\bar{R}$ , which implies  $x_i^s \geq 0$  for all  $i$ . Hence this is a non-pledgeable

equilibrium. The argument in the appendix establishes uniqueness.

*Q.E.D.*

Note that the buyer holds a fixed claim in every state of the world, while the seller gets the residual. Thus, it is as if the buyer invests in a debt claim of each project and the seller in an equity claim.

Let us now look at the case where the seller is not at an interior solution for every  $i$  (i.e.,  $2eR_i < e\bar{R}$ ). Proposition 2 characterizes the equilibrium in this case (it also applies to the interior case of Proposition 1).

*Proposition 2:* There is a unique non-pledgeable equilibrium, which can be characterized as follows: there exists  $\lambda > 0$ ,  $\mu > 0$ ,  $\hat{i} \in \{1, \dots, n-1\}$ , such that for  $i < \hat{i}$

$$x_i^b = 2eR_i, x_i^s = 0,$$

$$p_i^s = (2eR_i)^{\frac{1}{2}},$$

$$q_i = \frac{\pi_i}{\lambda(2eR_i)^{\frac{1}{2}}},$$

for  $i \geq \hat{i}$

$$x_i^b = \left(\frac{\mu}{\lambda}\right)^2, \quad x_i^s = 2eR_i - \left(\frac{\mu}{\lambda}\right)^2,$$

$$p_i^s = \frac{\mu}{\lambda},$$

$$q_i = \pi_i / \mu,$$

where

$$2eR_{i-1} < \left(\frac{\mu}{\lambda}\right)^2 \leq 2eR_i,$$

and we set  $R_0 = 0$  if  $\hat{i} = 1$ .

*Proof:* See Appendix.

Proposition 1 is a special case of Proposition 2 with  $\hat{i}=1$ . The key result -- that the portfolio of Arrow securities held by the buyers is equivalent to a fixed senior claim on project returns and the one held by the sellers to a junior claim -- holds even when the solution is not always in the interior. The only difference is that now the fixed claim does not always pay a fixed amount, but is equivalent to risky debt that pays the minimum of the face value of debt and the value of the firm. Similarly, the “equity tranche” held by sellers sometimes is out of the money, i.e. does not pay anything.

Thus, we have established our first result:

*Result 1:* Buyers will hold senior securities, while sellers junior ones.

So far nothing in our model prevents senior securities held by buyers from being senior debt in individual firms. Since the only source of risk is an aggregate shock and all projects are perfectly correlated, there is no difference between holding a senior tranche in an individual project and holding a senior tranche in a pool of projects. We will discuss the difference in the next section.

Before doing so, we want to analyze how welfare is affected by the realization of returns in a particular state. The welfare function in each state  $i$  is given by

$$(3.9) \quad W_i = \left[ \frac{x_i^b}{p_i^s} + x_i^s + \frac{1}{2}(p_i^s)^2 \right] = (x_i^b)^{\frac{1}{2}} + x_i^s + \frac{1}{2}(x_i^b).$$

Here  $W_i$  is the sum of buyer and seller utilities, a reasonable measure of welfare since each agent is risk neutral and equally likely to be a buyer or seller.

For  $i < \hat{i}$  we can write the welfare function as

$$(3.10) \quad W_i = (2eR_i)^{\frac{1}{2}} + 0 + (eR_i).$$

Differentiating (3.10) w.r.t.  $R_i$  we have:

$$\frac{dW_i}{dR_i} = e(2R_i)^{-\frac{1}{2}} + e > 2e.$$

Therefore, we have

*Corollary 1:* In a bad state of the world ( $i < \hat{i}$ ) a fall in return of one dollar causes a welfare loss bigger than \$1.

Corollary 1 captures the idea that a loss of wealth has a disproportionate impact when agents are liquidity constrained. Note in contrast that in a good state of the world  $i > \hat{i}$   $W_i$  is given by

$$(3.11) \quad W_i = \frac{\mu}{\lambda} + 2eR_i - \frac{1}{2} \frac{\mu^2}{\lambda^2},$$

which yields  $\frac{dW_i}{dR_i} = 2e$ , i.e., a fall in return of one dollar generates a welfare loss of one dollar.

Let us now consider the welfare effects of an *unexpected* wealth loss. Suppose that the economy arrives in period 2 in state  $i$  and agents learn that there has been an (unanticipated) wealth loss, stemming from a loss in investment returns, of  $\nu$ . Assume that a fraction  $\alpha$  of this loss is borne by the buyers and  $1 - \alpha$  by the sellers. How does the overall welfare loss depend on  $\alpha$  and  $\nu$ ?

Consider first  $i \geq \hat{i}$ . The welfare function is given by

$$(3.12) \quad W_i = (x_i^b - \alpha v)^{\frac{1}{2}} + x_i^s - (1 - \alpha)v + \frac{1}{2}(x_i^b - \alpha v).$$

Differentiating (3.12) w.r.t.  $v$  yields:

$$(3.13) \quad \frac{\partial W_i}{\partial v} = -\frac{\alpha}{2}(x_i^b - \alpha v)^{-\frac{1}{2}} - (1 - \alpha) - \frac{1}{2} < -1 \text{ if } \alpha > 0.$$

(3.13) implies that a one dollar loss has a welfare cost bigger (in absolute terms) than one dollar. We can also see that the size of this loss (in absolute terms) increases with the share of the loss borne by the buyers. To establish this, differentiate (3.13) w.r.t.  $\alpha$  :

$$\frac{\partial^2 W_i}{\partial v \partial \alpha} \Big|_{v=0} = -\frac{1}{2}(x_i^b)^{-\frac{1}{2}} + \frac{1}{2}\alpha < 0.$$

We obtain the same result for  $i < \hat{i}$ . In this case the welfare function is given by

$$(3.14) \quad W_i = (2eR_i - v)^{\frac{1}{2}} + \frac{1}{2}(2eR_i - v)$$

since the buyers have all the wealth and so experience 100% of the wealth loss. Differentiating w.r.t.  $v$  we obtain:

$$(3.15) \quad \frac{\partial W_i}{\partial v} = -\frac{1}{2}(2eR_i - v)^{-\frac{1}{2}} - \frac{1}{2} < -1,$$

as before.

Finally, if we differentiate (3.15) with respect to  $R_i$ , we obtain

$$\frac{\partial^2 W_i}{\partial v \partial R_i} = (e/2)(2eR_i - v)^{-3/2} > 0.$$

Thus, the effect of an unexpected wealth loss on welfare is worse in the states where  $R_i$  is low.

*Corollary 2:* While an unexpected \$1 loss suffered by sellers has a welfare cost of one dollar, a \$1 loss suffered by buyers has a welfare cost bigger than one dollar. Also the effect of an unexpected wealth loss on welfare is greater in bad states of the world.



#### 4. Non-pledgeable Equilibrium with Idiosyncratic Uncertainty

So far we have considered only aggregate risk. Now we introduce idiosyncratic risk as well. With risk averse agents idiosyncratic shocks would be sufficient to explain why buyers prefer a senior tranche in a pool of projects rather than a single project. Our agents, however, are risk neutral. Hence, to explain why they prefer a pool of projects we rely on an adverse selection argument in the spirit of Gorton and Pennacchi (1990) and Dang et al. (2012).

##### 4.1 Symmetric information

We introduce a shock idiosyncratic to each investment project. With probability  $\Pi$  each project delivers  $\frac{R_i(1-\varepsilon)}{\Pi}$  and with probability  $1-\Pi$ , it delivers  $\frac{\varepsilon R_i}{1-\Pi}$ , with  $\varepsilon$ ,  $\Pi$  being small. We suppose that the number of projects is large enough so that the law of large numbers applies and we can ignore uncertainty within each aggregate state.

For simplicity we assume that even the idiosyncratic shock is verifiable and whereas before there was an Arrow security paying \$1 in state  $i$ , now each project (firm) issues a security paying  $\frac{1-\varepsilon}{\Pi}$  if state  $i$  occurs and the project succeeds (which happens with probability  $\Pi$ ) and  $\frac{\varepsilon}{1-\Pi}$  when state  $i$  occurs and the project does not succeed (which happens with probability  $1-\Pi$ ). Thus, the expected payoff of the security in state  $i$  is the same as before and equal to 1.

With this small modification, the results of the previous section still hold as long as there is symmetry of information about the realization of the idiosyncratic shock. To establish this result it is enough to note that, if the demands for Arrow securities  $x_i^b, x_i^s$  do not change, the

aggregate equilibrium does not change and vice versa that if the aggregate equilibrium does not change, the demand for Arrow securities  $x_i^b, x_i^s$  will not change. The important point is that the aggregate liquidity in each state remains the same.

#### 4.2 Asymmetric information

Now let us assume –in the spirit of Gorton and Pennacchi (1990) -- that there is asymmetry of information about the realization of the idiosyncratic shock: before period 2 the buyers find out which projects have paid off well and which ones have not, while the sellers do not know this.

In this context, when a buyer is offering a security to purchase the seller's services, the seller will have to compute the expected payoff of that security conditional on the buyer offering the security. The buyer will offer a security only if the value of the services received in exchange exceeds the payoff of that security in terms of future wheat. We have

*Proposition 3:* For  $\varepsilon, \Pi$  sufficiently small, in every state  $i$  there exists a separating equilibrium given by  $p_i^s = (2\varepsilon R_i)^{\frac{1}{2}}$  for all  $i$ , with the buyer offering only the low-value securities. Also there is no pooling equilibrium.

*Proof:* Let us first prove that there is no pooling equilibrium where the buyers use all their securities to purchase the services and the sellers value these securities at their expected value. If this is the case, the price of services in state  $i$  will be given by  $p_i^s = (2\varepsilon R_i)^{\frac{1}{2}}$ . Then, the buyer has the choice between keeping the high-value security paying  $\frac{1-\varepsilon}{\Pi}$  in terms of period 3 wheat or offering it as a payment to the seller, who will value it at its expected value, i.e. 1, to purchase

services at a price  $p_i^s = (2eR_i)^{\frac{1}{2}}$ . Hence, pooling is not an equilibrium if  $\frac{1-\varepsilon}{\Pi} > \frac{1}{p_i^s}$ , or

$\Pi < (2eR_i)^{\frac{1}{2}}(1-\varepsilon)$ , which is true for  $\varepsilon, \Pi$  small.

In a separating equilibrium only the low-paying securities are offered as a payment. To check whether this is indeed an equilibrium we need to show that a buyer is better off retaining the high-value securities rather than using them to buy services. Given a price for service  $p_i^s$ , the

utility derived from using securities to buy services is  $\frac{\varepsilon}{p_i^s}$ , while the benefit (in terms of

period 3 wheat) of keeping them is  $\frac{1-\varepsilon}{\Pi}$ , thus the condition is  $\frac{\varepsilon}{p_i^s} \leq \frac{1-\varepsilon}{\Pi}$ . Given that the total

amount of liquidity if only the low-paying securities are used to buy services will be  $\varepsilon$  and the

price will be  $p_i^s = (2e\varepsilon R_i)^{\frac{1}{2}}$ , this condition corresponds to  $\frac{\varepsilon^{\frac{1}{2}}}{1-\Pi} \leq \frac{1-\varepsilon}{\Pi} (2eR_i)^{1/2}$ , which is

satisfied for  $\varepsilon, \Pi$  small.

QED

Since the welfare obtained in the separating equilibrium is inferior to the welfare obtained in the equilibrium in Section 3, it is natural to ask whether there is a way to overcome the adverse selection problem caused by the asymmetry of information regarding the idiosyncratic shocks. The answer is that this can be achieved by bundling a large number of projects, so that the law of large numbers will apply. A natural way to do this is through a third party, which will ensure that the bundle is properly diversified.

Suppose then that the buyers' liquidity needs are backed by a fixed claim (equal to  $(\mu / \lambda)^2$ , as in Proposition 2) against a diversified pool of all the projects. For  $i < \hat{i}$ , these securities will absorb all the proceeds. For  $i \geq \hat{i}$  the proceeds of the pool of projects will exceed the buyers' claims. The leftover will be absorbed by the securities held by sellers. Thus, the portfolio of securities held by buyers is equivalent to a senior claim equal to  $(\mu / \lambda)^2$  on a diversified portfolio of projects and the portfolio of securities held by sellers is equivalent to a junior claim on the same diversified portfolio of projects.

Thus, we have

*Result 2:* The buyers' liquidity needs are best satisfied by a debt claim on a diversified pool of projects, while the sellers' needs by an equity claim on the same set of projects.

#### *4.3 Banks' failure vs. failure of industrial firms*

In 2008 GM and Chrysler survived bankruptcy, but their creditors suffered heavy losses. By contrast, Citigroup did not go through bankruptcy and its creditors did not suffer any loss. Thus, the key difference in the policy response during the 2008 financial crisis was not that banks were saved and industrial firms were not (all of the major firms survived), but that banks' creditors did not suffer any loss (except for Lehman's), while GM's and Chrysler's did. To study whether there is any rationale in this differential treatment we explore whether there is any welfare difference between a loss borne by a bank's creditors and a loss borne by the creditors of an industrial firm.

There is a trivial sense in which it is true that the welfare consequences of a loss borne by a bank's creditors are larger than the welfare consequences of an identical loss suffered by the creditors of an industrial firm. Given the way deposit claims are manufactured, a bank tends to

fail in bad states of the world, where welfare losses are bigger simply because there is less endowment. By contrast, an industrial firm can fail for idiosyncratic reasons, regardless of the aggregate state. Hence, on average bank failures have worse welfare consequences than failures of industrial firms.

Our main goal, though, is to try to understand whether there is a difference between the failure of an industrial firm like GM and of a bank like Citigroup in a given state of the world, such as the 2008 financial crisis. To carry out this comparison, we need to assume an exogenous idiosyncratic shock that can push a large firm or a large bank into default. For this reason, we consider fraud, a sort of a Madoff shock, which makes a significant fraction of the assets disappear.

Consider a very simple case with 2 firms and 2 banks and suppose that each bank has a debt equal to  $d$ . Each bank holds 50% of the equity of each firm. The buyers hold the bank debt. The value of bank 1's and bank's 2 debt claim is given by

$$\min \left\{ \frac{1}{2} X_1 + \frac{1}{2} X_2, d \right\}.$$

Suppose firm 1 has an unexpected "Madoff" shock of size  $\nu$ : the amount  $\nu$  disappears. Then the total value of fixed claims held by buyers is

$$2 \min \left\{ \frac{1}{2} X_1 + \frac{1}{2} X_2 - \frac{1}{2} \nu, d \right\}.$$

By contrast, suppose that bank 1 has a similar "Madoff" shock  $\nu$ . Then the total value of fixed claims held by buyers is

$$\min \left\{ \frac{1}{2} X_1 + \frac{1}{2} X_2 - \nu, d \right\} + \min \left\{ \frac{1}{2} X_1 + \frac{1}{2} X_2, d \right\}.$$

To compare the two, let  $Y = \frac{1}{2}X_1 + \frac{1}{2}X_2$ . Then, the total value of fixed claims held by buyers will be affected less by a shock in an industrial firm if

$$2 \min \left\{ Y - \frac{1}{2}v, d \right\} > \min \{ Y - v, d \} + \min \{ Y, d \}$$

or

$$\min \left\{ Y - \frac{1}{2}v, d \right\} > \frac{1}{2} \min \{ Y - v, d \} + \frac{1}{2} \min \{ Y, d \}$$

which is true since the function  $\min\{\cdot\}$  is concave.

*Result 3:* An unexpected loss caused by fraud in a bank has worse welfare consequences than the same loss caused by fraud in an industrial firm.

The intuition for this result is very simple. A loss suffered by a firm affects the total value of senior claims outstanding less, because it affects only one senior claim out of many, thus its impact is averaged out. By contrast, a loss suffered by a bank has a direct impact on the value of all senior claims. Since the senior claims are held by the buyers who are liquidity constrained, the losses that affect the buyers have larger welfare consequences. Hence, the welfare consequences of a loss suffered by a bank will be larger than those of a loss suffered by an industrial firm.

#### *4.4 Banks vs. money market funds*

A bank deposit is just a way in which the claim desired by buyers can be manufactured. Others are possible. Shares in money market funds are often thought as a substitute. A share in a money market fund is a portfolio of senior tranches of  $n$  projects, while a deposit is a senior tranche in a portfolio of  $n$  projects. In practice this difference seems to be mitigated by the fact

that banks' assets are mostly in the form of senior tranches (debt). Yet, they are not exclusively in the form of senior tranches, since universal banks also own equity. Even when banks hold only debt, they tend to be engaged in long term relationships with their clients, so that they capture some of their upside (e.g., Rajan, 1992). Given this, is a portfolio of senior tranches of  $n$  projects different from a senior tranche in a portfolio of  $n$  projects?

When individual projects pay off well, the senior claim will be paid in full. When they do not, the senior claimholders become residual claimants and receive  $\frac{\varepsilon R_i}{1 - \Pi}$ , i.e. an amount contingent on the state  $i$ . Thus, unlike a senior claim on a portfolio, a portfolio of senior claims will have a payoff that is state contingent in most states of the world. But this is not what buyers want. To satisfy the buyers' demand, thus, it is not sufficient to have a portfolio of senior claims, we need to have a portfolio of state-contingent claims.

Note that money markets funds are portfolios of senior claims, not portfolios of state-contingent claims. Thus, in our model money markets funds are an inferior instrument to satisfy buyers' liquidity needs vis-à-vis bank deposits.

## 5. Fiscal Policy

So far we have not considered how the government might respond to the liquidity problems that we have highlighted. In this section we analyze fiscal policy along the lines of Hart and Zingales (forthcoming). Specifically, we assume that there is a milling technology in period 3 that allows buyers and sellers to convert wheat into flour, and that they enjoy consuming flour as well as wheat. The government can impose a per unit sales tax on flour – something that the private sector cannot do – and can issue bonds in period 2, after the state of the world is realized, backed by this sales tax. As in Hart and Zingales (forthcoming), we suppose that the government bonds

( $b$  units of them) are handed directly to *buyers* (nothing in our analysis relies on the idea that the identity of an agent is nonverifiable).

The details of the milling technology and preferences for flour versus wheat can be found in Hart and Zingales (forthcoming). For our purposes it is enough to rely on the following result from that paper: the government can increase the liquidity of a buyer in period 2 in state  $i$  from  $x_i^b$  to  $x_i^b + b$ , but this imposes a loss on the economy in period 3 of  $b + \Delta(b)$ , where the first term reflects the bond repayment and  $\Delta(b)$  is the deadweight loss of the sales tax required to raise  $b$ .

Here  $\Delta(b)$  satisfies

$$(5.1) \quad \Delta(0) = 0, \quad \Delta'(0) = 0, \quad \Delta''(b) > 0 \text{ for all } b \geq 0.$$

In other words, the marginal deadweight loss is zero when the tax rate is zero but is strictly positive and increasing when the tax rate is positive.

We will be particularly interested in how the government should respond to an unexpected shock. However, given that there is a shortage of liquidity in the economy, the government will want to respond even in the absence of such a shock. Thus, we will analyze the optimal fiscal policy with and without a shock.

We will suppose that the government chooses fiscal policy in state  $i$  in period 2 to maximize the sum of buyers' and sellers' utilities. We will also assume that the government *cannot commit* to its fiscal policy in advance: the government will choose  $b_i$  in state  $i$  to maximize the sum of buyer and seller utilities after  $x_i^b, x_i^s$  are determined.

### 5.1 Equilibrium in the Absence of a Shock

Let us consider the case where there is no shock and focus on the non-intervention interior equilibrium in Proposition 1. If the government issues  $b_i$  units of bonds to a buyer (each



bond pays one unit of wheat in period 3 in state  $i$ ), then since  $x_i^b = e\bar{R}$ ,  $x_i^s = 2eR_i - e\bar{R}$ , the new equilibrium in the goods market is given by

$$(5.2) \quad \frac{e\bar{R} + b_i}{p_i^s} = p_i^s,$$

that is,

$$(5.3) \quad p_i^s = (e\bar{R} + b_i)^{\frac{1}{2}}.$$

From (\*) and (\*\*), the sum of buyers' and sellers' utilities in state  $i$  is

$$(5.4) \quad W_i = \left[ \frac{x_i^b + b_i}{p_i^s} + x_i^s + \frac{1}{2}(p_i^s)^2 - b_i - \Delta(b_i) \right]$$

$$= \left[ (e\bar{R} + b_i)^{\frac{1}{2}} + (2eR_i - e\bar{R}) + \frac{1}{2}(e\bar{R} + b_i) - b_i - \Delta(b_i) \right].$$

The government will choose  $b_i$  to maximize  $W_i$ . Since  $W_i$  is strictly concave in  $b_i$ , the following first order condition is necessary and sufficient:

$$(5.5) \quad \frac{1}{2}(e\bar{R} + b_i)^{-\frac{1}{2}} - \frac{1}{2} = \Delta'(b_i).$$

Note that the left-hand side of (5.5) is strictly positive when  $b_i = 0$  and zero when  $e\bar{R} + b_i = 1$ .

Hence the solution to (5.5) is unique and will satisfy  $0 < b_i < 1 - e\bar{R}$ .

It is interesting to note that (5.5) is independent of the state  $i$ . Thus, the fiscal response will be the same across all states.

Now suppose that agents anticipate that the government will choose  $b_i$  to satisfy (5.5).

Will they change their ex ante behavior? We argue that they will not: in the rational expectations equilibrium  $x_i^b = (e\bar{R})$ ,  $x_i^s = 2eR_i - (e\bar{R}_i)$ . To see this set the new price in state  $i$  to be as in (5.3).

Also let

$$(5.6) \quad q_i = \frac{\pi_i}{\bar{R}}, \quad .$$

Then, it is easy to see that (3.4) and (3.5) are satisfied and (3.2) is satisfied as well.

Thus, we have constructed a new equilibrium with government intervention where the demand for Arrow securities is as in Proposition 1 and the government optimizes accordingly.

## 5.2 Fiscal Response to an Unexpected Shock

We can now analyze how the government will respond in case of an unexpected shock. Let  $\alpha$  be the fraction of a shock  $v$  borne by the buyers. Then, we want to see how the fiscal response will change as a function of  $\alpha$ .

To this purpose, we can rewrite (5.4) as

$$W_i = \left[ (e\bar{R} + b_i - \alpha v)^{\frac{1}{2}} + 2eR_i - e\bar{R} - (1 - \alpha)v + \frac{1}{2}(e\bar{R} + b_i - \alpha v) - b_i - \Delta(b_i) \right].$$

The first order condition is given by:

$$\frac{1}{2}(e\bar{R} + b_i - \alpha v)^{\frac{1}{2}} = \frac{1}{2} + \Delta'(b_i).$$

By using the implicit function theorem we can obtain

$$\frac{db_i}{dv} \Big|_{v=0} = \frac{\frac{1}{4}\alpha(e\bar{R} + b_i)^{-\frac{1}{2}}}{\frac{1}{4}(e\bar{R} + b_i)^{-\frac{1}{2}} + \Delta''(b_i)} < \alpha,$$

because  $\Delta''(b_i) > 0$ .

Thus we have:

*Proposition 4:* In response to an unexpected shock, the government will find it optimal to respond with a larger fiscal intervention, the higher is the fraction of the shock born by the buyers.

In other words, in our model the government should always intervene because there is always a shortage of liquidity, which causes a reduction in aggregate demand. However, this liquidity shortage is particularly severe after an unexpected negative shock. Hence, the fiscal response of the government should be greater than average after an unexpected negative shock

## **6. Conclusions**

This paper explains why – in a complete markets framework with pledgeability constraints -- there is a demand for relatively safe assets for transaction purposes. It also explains why this demand can be satisfied by a senior claim on a portfolio of projects , i.e. by a bank deposit. An equity claim on a portfolio of senior claims (i.e. a money market fund) does not achieve the same goal.

We also show that a negative unexpected loss suffered by a bank generates greater welfare losses than an equal size loss suffered by an industrial firm. The reason is that a loss in an industrial firm has a smaller impact on the total value of senior claims than a similar loss in a bank. Since senior claims (deposits) are held by people in need of liquidity, the welfare consequences of an identical dollar loss in a bank will be greater. In other words, banks are special because banks are where the liquidity is.

This welfare effect provides a justification for greater fiscal intervention when there is an unexpected wealth shock. Yet, it does not necessarily justify banks' bailouts. If – as in our model—the government can target liquidity-constrained individuals, it should aim at transferring resources to them. In practice, it is not always obvious how to do this. During the 2008 financial crisis a natural way to do it would have been to forgive underwater mortgages (Posner and Zingales, 2009). More generally, the optimal way to intervene will depend on the ease of

identifying the buyers in need of liquidity and the relative severity of the moral hazard engendered by bailing out individuals versus institutions.

## References

- Barro, Robert J. and Andrew Mollerus, 2014, "Safe Assets", Harvard University working paper
- Baumol, W, 1956, "The Transactions Demand for Cash: An Inventory Theoretic Approach", *Quarterly Journal of Economics*, November Vol. 66, p.545-556
- Caballero, Ricardo J., 2006, "On the Macroeconomics of Asset Shortages" in *The Role of Money: Money and Monetary Policy in the Twenty-First Century. The Fourth European Central Banking Conference 9-10 November 2006*, Andreas Beyer and Lucrezia Reichlin, editors. Pages 272-283.
- Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas. 2008. "An Equilibrium Model of "Global Imbalances" and Low Interest Rates." *American Economic Review*, 98(1): 358-93.
- Clower, R W, 1967, " A Reconsideration of the Microfoundations of Monetary Theory," *Western Economic Journal*, 6(1), pp. 1-8
- Dang, Tri Vi, Gary Gorton, and Bengt Holmström (2012), *Ignorance, Debt, and Financial Crises*, manuscript Yale University
- DeAngelo, Harry and Stulz, René M., 2015, "Liquid-Claim Production, Risk Management, and Bank Capital Structure: Why High Leverage is Optimal for Banks," *Journal of Financial Economics*, 2015, vol. 116, issue 2, pages 219-236
- Diamond, Douglas W. and Philip Dybvig, 1983, "Bank runs, deposit insurance and Liquidity", *Journal of Political Economy* 91, 401-419.
- Gorton, Gary, 2010, "Slapped by the Invisible Hand: The Panic of 2007," Oxford University Press.
- Gorton, Gary, Stefan Lewellen, and Andrew Metrick, 2012. "The Safe-Asset Share," *American Economic Review*, vol. 102(3), pages 101-06, May.
- Gorton Gary and Guillermo Ordonez, 2013, "The Supply and Demand for Safe Assets", NBER working paper.
- Gorton, Gary and George Pennacchi, 1990, "Financial intermediaries and liquidity creation," *Journal of Finance*, 45(1): 49–71.
- Hanson, Samuel G., Andrei Shleifer, Jeremy C. Stein, and Robert W. Vishny, 2014, "Banks as Patient Fixed Income Investors," Fed working paper.

Hart, O., and L. Zingales (forthcoming) “Liquidity and Inefficient Investment”, *Journal of the European Economic Association*.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, 2012, “The Aggregate Demand for Treasury Debt.” *Journal of Political Economy*, 120, 233-267.

Mian, Atif and Amir Sufi, 2014, *House of Debt*, University Of Chicago Press

Posner, Eric and L. Zingales, 2009, “The Housing Crisis and Bankruptcy Reform: The Prepackaged Chapter 13 Approach”. *American Law and Economic Association Review*.

Rajan, Raghuram, 1992, “Insiders and Outsiders: The Choice between Informed and Arm’s-Length Debt”, *Journal of Finance* 47, 1367-1400.

Stein, Jeremy. 2012, “Monetary Policy as Financial-Stability Regulation”, *Quarterly Journal of Economics* 127, no. 1: 57-95.

Tobin, James, 1956, “The Interest Elasticity of the Transactions Demand for Cash”, *Review of Economics and Statistics*, August vol 38, n.3, p 241-247.

## Appendix

A non-pledgeable equilibrium exists by standard fixed point arguments.

As noted in the text  $x_i^b > 0$  for all  $i$ . Hence the buyer's first order conditions are, for all  $i$ ,

$$(A1) \quad \frac{\pi_i}{p_i^s} = \lambda q_i$$

for some  $\lambda > 0$ . The seller's first order conditions are, for all  $i$ ,

$$(A2) \quad \begin{aligned} \pi_i &\leq \mu q_i \\ \pi_i < \mu q_i &\text{ implies } x_i^s = 0 \end{aligned}$$

for some  $\mu > 0$ .

Suppose that (A2) holds with equality for some  $i$ . Consider  $j > i$ . If  $x_j^s = 0$ , then

$$x_j^b = 2eR_j > 2eR_i \geq x_i^b. \text{ Hence by (3.3) } p_j^s > p_i^s, \text{ which implies, given (A1)–(A2), } \frac{\pi_j}{q_j} > \frac{\pi_i}{q_i} = \mu,$$

which is a contradiction. Hence,  $x_j^s > 0$  and  $\pi_j = \mu q_j$ . We may conclude that there is a cut-off: if (A2) holds with equality for some  $i$ , then it holds with equality for higher values of  $i$ .

Let  $\hat{i}$  be the smallest value of  $i$  such that  $\pi_i = \mu q_i$ . Then, for  $i < \hat{i}$ ,  $x_i^s = 0$  and so  $x_i^b = 2eR_i$ ,

$$p_i^s = (2eR_i)^{1/2}, \text{ as in Proposition 2. On the other hand, for } i \geq \hat{i}, \frac{\pi_i}{p_i^s} = \lambda q_i, \pi_i = \mu q_i \text{ and so, } p_i^s = \frac{\mu}{\lambda},$$

$$x_i^b = \left(\frac{\mu}{\lambda}\right)^2, \text{ again as in Proposition 2. Finally, for } i = \hat{i} - 1, \pi_i < \mu q_i, \pi_i = \lambda q_i p_i^s \text{ and so}$$

$$p_i^s = (2eR_{\hat{i}-1})^{1/2} < \frac{\mu}{\lambda}, \text{ while, for } i = \hat{i}, \frac{\mu}{\lambda} = x_i^b \leq 2eR_i, \text{ since } x_i^s \geq 0. \text{ Hence,}$$

$$(A3) \quad 2eR_{\hat{i}-1} < \left(\frac{\mu}{\lambda}\right)^2 \leq 2eR_{\hat{i}},$$

where we set  $R_0 = 0$  if  $\hat{i} = 1$ . Thus we have established the characterization of equilibrium in Proposition 2.

To prove uniqueness of equilibrium, set  $\frac{\mu}{\lambda} = h$  and write (3.1) and the buyer's budget constraint as

$$(A4) \quad \frac{1}{\lambda} \left[ \sum_{i < \hat{i}} \frac{\pi_i R_i}{(2eR_i)^{\frac{1}{2}}} + \sum_{i \geq \hat{i}} \frac{\pi_i R_i}{h} \right] = 1,$$

$$(A5) \quad \frac{1}{\lambda} \left[ \sum_{i < \hat{i}} \pi_i (2eR_i)^{\frac{1}{2}} + \sum_{i \geq \hat{i}} \pi_i h \right] = e.$$

We show that, if there is a solution to (A3)-(A5), it is unique. Fix  $\hat{i}$  and graph (A4) and (A5). It is easy to see that the locus of  $(\lambda, h)$  satisfying (A4) is downward-sloping and the locus of  $(\lambda, h)$  satisfying (A5) is upward-sloping. Moreover, the first curve asymptotes to the horizontal axis as  $\lambda \rightarrow \infty$  and to a vertical line at  $\lambda > 0$  as  $h \rightarrow \infty$ ; while the second curve satisfies  $\lambda \rightarrow \hat{\lambda} > 0$  as  $h \rightarrow 0$  and  $\lambda \rightarrow \infty$  as  $h \rightarrow \infty$ . It follows that the curves have a unique intersection.

Now increase  $\hat{i}$ . It is straightforward to show that the downward-sloping curve moves in, while the upward-sloping curve moves to the right. Hence,  $h$  falls. But this means that if (A3) is satisfied at the first level of  $\lambda$ , it cannot be satisfied at the second.

Hence the equilibrium is unique.

QED