LIQUIDITY AND INEFFICIENT INVESTMENT

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Abstract
We study consumer liquidity in a general equilibrium model where the friction is the nonpledgeability of future income. Liquidity helps to overcome the absence of a double coincidence of wants. Consumers over-hoard liquidity and the resulting competitive equilibrium is constrained inefficient. Fiscal policy following a large negative shock can increase ex-ante welfare. If the government cannot commit, the ex-post optimal fiscal policy will be too small from an ex-ante perspective. The model throws light on the holding of foreign reserves in international markets. (JEL: E41, G21, E51)

1. Introduction

The expansion of international capital flows and the impact on the United States of the large build-up of foreign reserves in Asia have spurred a renewed interest among economists in the role played by liquidity. Is there a fundamental inefficiency in the market provision of liquidity? If so, does the market provide too little liquidity or too much? Is government intervention justifiable only ex post, in the face of an unexpected shock, or are there some ex-ante benefits if it is anticipated? Will the government have an incentive to stick to the ex-ante optimal fiscal policy or is there a time inconsistency?

There is by now a very large literature on these questions. Most of this literature has focused on firms' liquidity needs in the face of unexpected funding shocks given that firms cannot pledge future returns; leading examples are Holmstrom and Tirole (1998,

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2011) and Lorenzoni (2008). In contrast, in this paper, we focus on consumers' need for a means of payment given an inability to pledge future labor income: specifically, the role of liquidity in overcoming a lack of a double coincidence of wants.

In a domestic setting, we can think about consumers who are unable to pledge a sufficient amount of future income to smooth their lifetime consumption optimally (see, e.g., Zeldes 1989).[1] In an international setting, we can think about countries (or their citizens) that are unable to pledge a sufficient amount of future income to invest optimally (see, e.g., Caballero et al. 2008).

To capture the essence of this general equilibrium problem, we build a very stylized finite-horizon model. It contains two groups of agents, whom we call financial advisors and secretaries. (Aide-memoire: **f**inancial advisors buy **f**irst and **s**ecretaries buy **s**econd.) Financial advisors buy secretarial services from secretaries and then secretaries buy financial advice from financial advisors. There are large numbers in each group and so markets are competitive. Each secretary requires a financial advisor at a different date and typically one with different skills from the financial advisor he is the secretary for, and vice versa for financial advisors. In other words, there is no simultaneous double coincidence of wants—see Jevons (1876) and, for a modern treatment, Kiyotaki and Wright (1989). We make the critical assumption that financial advisors cannot pledge their future labor income to pay for secretarial services. While extreme in a domestic setting, this assumption is meant to capture the fact that workers can move, shirk, and hide their future income, severely limiting what fraction of this future income can be seized. (In Section 4, we show that our analysis generalizes to the case where agents cannot be forced to work.) In an international setting, it captures the lack of jurisdiction to enforce contracts. This friction generates a need for a means of payment. There are no other imperfections.

Agents are endowed with a real commodity: wheat. Wheat can be invested by firms in a risky or a riskless project, where the risky project has a higher expected return. Agents are risk neutral. We assume that, in contrast to labor income, all project returns can be pledged—think of the firms as public companies. There are two states of the world, high and low, and risky returns are perfectly positively correlated—that is, there is an aggregate productivity shock. We suppose that uncertainty about returns is resolved before trading in secretarial and financial services takes place. In the first-best, where labor income can be pledged, there is no need for liquidity and so there is a complete separation between trade and investment: there is efficient trading of secretarial and financial services and all wheat is invested in the high-return risky project. In the second-best, liquidity is important: financial advisors will use claims on

1. A focus on consumers seems particularly germane given the growing evidence (Kahle and Stulz 2013; Mian and Sufi 2012) that during the Great Recession firms had plenty of liquidity, while consumers were severely liquidity constrained. Indeed, the 2004 Survey of Consumer Finances finds that 37% of families are financially constrained, where constrained is defined as a family that applied for credit and has been rejected or has been discouraged from applying by the fear of being rejected. By contrast, the 2003 Survey of Small Business Finances finds that only 15% of small firms were constrained, using the same definition. Since small firms are more likely to be constrained than big firms, this evidence seems to suggest that financial constraints are more likely to be a problem for consumers than for firms.

firms' returns to buy secretarial services given that they cannot pledge their future labor income. If all wheat is invested in the high-return project there will be little liquidity in the low state and so, to satisfy the demand for liquidity, for some parameter values the riskless project will also be employed in the second-best equilibrium. In a nutshell, the safe asset is used because it is good for liquidity.

We show that in the laissez-faire equilibrium there will still be too little liquidity in the low state and so trade between financial advisors and secretaries will be inefficiently low relative to the first-best. However, and somewhat counter-intuitively, from a second-best point of view, there will be *too much* liquidity: there will be excessive trade in the low state and the economy will *overinvest* in the riskless technology. The reason is that a financial advisor who invests in liquidity to buy secretarial services imposes a negative externality on other financial advisors since his actions increase the price of secretarial services. Because financial advisors are liquidity constrained, this pecuniary externality has welfare as well as distributional effects. As a result, too much wheat is invested in the safe asset to create liquidity instead of being invested in socially productive projects. This can explain why holdings of US Treasuries by Asian countries might be excessive and why this might reduce the overall return on investment.

In a domestic setting, the government can improve on the competitive equilibrium by restricting the amount of investment in the riskless asset.[2] Alternatively, the government can increase the efficiency of the economy by introducing its own riskless asset. (Both policies are difficult to enforce/coordinate in an international setting.) In our finite-horizon economy, fiat government money can exist only if a government can tax individuals. We assume that the government can impose sales taxes and that agents can pay these sales taxes with government notes. In our model government notes or government money are equivalent, because both of them must be backed by future taxes. Therefore, since the intervention we consider does not affect the aggregate wealth of consumers, but only the temporal distribution of this wealth, we label it *fiscal policy*.

We find that government fiscal policy in bad states can increase not only output more than one-to-one (fiscal multiplier), but also—absent any other use of the fiscal lever—ex-ante welfare. In fact, our model suggests that it is optimal to under-tax in normal times in order to retain the flexibility to use the fiscal lever for this countercyclical policy. If the government cannot commit to the optimal level of fiscal policy, it will do *too little* of it ex post—that is, the ex-post optimal fiscal policy is different from the ex-ante one. This is reminiscent of Kydland and Prescott's (1977) renowned result, with two differences: it applies to fiscal policy and, as in Debortoli and Nunes (2013)

2. In Hart and Zingales (2011), we showed that a similar inefficiency and role for government intervention arise in an economy with perfect certainty where there are two investments, one with a high return that is collateralizable and one with a low return that is not collateralizable. (As in the current paper future labor income cannot be pledged.) One weakness of that formulation is that some assets are *assumed* to be collateralizable and others not. In a way, the current paper can be seen as an attempt to endogenize collateralizability; an asset with a riskless return serves as better collateral/liquidity than an asset with an uncertain return.

and Song, Storesletten, and Zilibotti (2012), it goes in the opposite direction (too little rather than too much).

It should be emphasized that our inefficiency is driven by a single imperfection: the inability of consumers to pledge their future labor income. Markets are complete in other respects. Indeed our results continue to hold even if agents can insure against whether they buy or sell first. It turns out that an agent who buys first has a lower utility than an agent who sells first, but the same marginal utility of wealth. Thus there will be no demand for insurance.

There are parallels between our work and the literature on incomplete markets. In that literature, a competitive equilibrium is typically inefficient and a central planner operating under the same constraints as the market can do better (see, e.g., Hart 1975 and Geanakoplos and Polemarchakis 1986). One feature of this literature is that the market structure is taken as given. This raises the question: why can the private sector not create new securities to complete the market? Our work differs in that we endogenize the market structure: markets are complete with respect to verifiable events such as aggregate shocks, but the inability to borrow against human capital creates liquidity problems. Related to this, we focus on whether a market economy overinvests in safe assets, something that, as far as we know, the incomplete markets literature has not considered.[3]

Our work is closely related to Holmstrom and Tirole's recent (2011) book (HT) and Lorenzoni (2008). As noted, the main difference is that we focus on consumer liquidity rather than firm liquidity. Also, in contrast to HT and Lorenzoni (2008), our model requires one friction: the inability of consumers to pledge future income rather than two: the inability of firms to pledge future returns and the inability of consumers to pledge future endowments. Yet, several of the same forces operate in all these works. As in HT, the government's ability to tax gives it a role in improving matters by injecting liquidity—for example, by creating a riskless asset and taxing consumers later to finance repayments. As in Lorenzoni, the equilibrium without government money creation is second-best inefficient.

This difference in focus changes the sign of the pecuniary externality, leading to opposite results. In Lorenzoni, firms borrow too much, ignoring the fact that in the bad state of the world they have to sell capital to make debt payments, which drives down the price of capital and forces other agents to increase their capital sales (a "fire-sale" externality). In our paper, the externality goes the other way: agents acquire too much liquidity (which is like borrowing *too little*), ignoring the fact that their liquidity drives up the price of goods their fellow liquidity-constrained agents are trying to buy (an "inflation" externality). HT do consider an externality similar to ours in Chapter 7. They analyze a situation where ex post some distressed firms will be forced to liquidate and other healthy firms can purchase these firms at discounted prices. They show that, ex ante, firms will overinvest in safe securities

3. A recent paper by Dávila et al. (2012) analyzes the possibility of oversaving in an incomplete markets economy.

in order to take advantage of the opportunity to buy distressed firms' assets, thereby driving up the prices of these assets and imposing an externality on other buyers.

The different perspectives on the source of the externality lead to opposite policy implications: any ex-ante promise to push up prices during a crisis alleviates the inefficiency in the case of fire-sale externalities, while it worsens it in the case of our inflation externality. For example, in the context of the last recession, the fire-sale externality perspective would push toward a more aggressive monetary policy, while ours might suggest a renegotiation of underwater mortgages to help consumers who were liquidity constrained. The full difference in policy implications can be seen in Jeanne and Korinek (2013). They develop the ex-ante and ex-post policy implications of the fire-sale externalities' perspective. As in our case, they find a time inconsistency of fiscal policy. But in their case the fiscal authority would like to commit to intervene *less* than what is ex-post optimal to mitigate the overinvestment in *risky* assets, while in our case it would like to commit to intervene *more* than what is ex-post optimal to mitigate the overinvestment in *riskless* assets.

The role money plays in our model (i.e., to address the lack of a double coincidence of wants) is similar to that in Kiyotaki and Wright (1989). Their focus, however, is on what goods can become money and how. Our focus is to what extent private traders can provide the efficient quantity of medium of exchange.

Finally, our paper is linked to a vast and growing literature on the welfare effects of pecuniary externalities in the presence of financial constraints (see, e.g., Kehoe and Levine 1993; Gromb and Vayanos 2002; Allen and Gale 2004; Farhi, Golosov, and Tsyvinski 2009; Korinek 2012; He and Kondor 2012). With the exception of Gromb and Vayanos where the externality can go both ways, this literature typically finds that the competitive equilibrium delivers an inefficiently low investment in liquid/safe assets. We find the opposite: a competitive equilibrium delivers an inefficiently high level of liquidity and an excessively high level of investment in risky assets.

The paper is structured as follows. The model and applications are presented in Section 2. Section 3 analyzes fiscal policy. Section 4 considers an extension of the basic model. Conclusions follow in Section 5.

2. The Model

There are many agents. Ex ante each agent is equally likely to be a financial advisor or a secretary; agents learn their type at the beginning of period 1. **F**inancial advisors buy **f**irst (in period 2) and **s**ecretaries buy **s**econd (in period 3). Financial advisors and secretaries can also consume wheat in period 4 and there is no discounting. Each financial advisor and secretary has an endowment of wheat in period 1 equal to e. Wheat can be invested in projects; these projects yield wheat in period 4. We will assume that $e \geq 1$. The timeline is as in Figure 1.

1		2	3	4
Agents learn whether they are financial advisors or secretaries Wheat invested Securities traded	State of world realized	Trade of secretarial services	Trade of financial advising services	Output from wheat consumed

FIGURE 1. Timeline.

We write agents' utilities as

$$\text{Financial advisors:} \quad U_f = w_f + s_f - \frac{1}{2} l_f^2,$$
$$\text{Secretaries:} \quad U_s = w_s + f_s - \frac{1}{2} l_s^2,$$

where s_f is the quantity of secretarial services consumed by a financial advisor, l_f is the labor supplied by a financial advisor, f_s is the quantity of financial advice consumed by a secretary, l_s is the labor supplied by a secretary, and w_i is the quantity of wheat consumed by individual $i = f, s$ in period 4. We assume constant returns to scale: one unit of secretarial labor yields one unit of secretarial services and one unit of financial advisor's labor yields one unit of financial advice.

In other words, financial advisors (resp., secretaries) are indifferent between wheat and secretarial (resp., financial) services and have a quadratic disutility of labor.

Agents are risk neutral. The markets for secretarial and financial services are perfectly competitive. It is crucial for our analysis that there is *no simultaneous* double coincidence of wants: a secretary does not want to consume financial advice in period 2 from the financial advisor who is buying his secretarial services.

There are two (aggregate) states of the world, H (high) and L (low), which occur with probabilities π, $1 - \pi$, respectively, where $0 < \pi < 1$. There are two technologies. The riskless technology (storage) transforms one unit of wheat in period 1 into one unit of wheat in period 4; the risky technology transforms one unit of wheat in period 1 into $R^H > 1$ units of wheat in period 4 in state H and $R^L < 1$ units in state L, where $\bar{R} = \pi R^H + (1 - \pi) R^L > 1$. There is free entry of firms possessing the two technologies described previously and these firms face constant returns to scale. *The returns of the various risky projects are perfectly correlated.* Agents learn the state of the world between periods 1 and 2, after investment decisions have been made but before period 2 trading occurs.

Throughout the paper we assume that the states H and L are *verifiable.*

We have deliberately set up the model to be very symmetric; this helps with the welfare comparisons later.

2.1. A First Benchmark: the Walrasian Equilibrium

Ignore wheat for the moment (drop it from the utility functions) and investment, and consider the two-good economy in which financial advisors and secretaries trade secretarial and financial services with each other under the standard assumption that financial advisors *can* pledge their period 3 labor income to secretaries. Then it is easily seen that the economy has a unique Walrasian equilibrium. In this equilibrium, which is symmetric, the prices of financial and secretarial services and the wage rates of financial advisors and secretaries will be the same and we can normalize them to be 1; at these prices each financial advisor and secretary produces and consumes one unit of services and incurs a labor cost of 1/2. In this equilibrium, each agent receives a consumer surplus of 1/2.

2.2. A Second Benchmark: Storable Wheat as a Means of Payment

Now suppose that financial advisors *cannot* pledge their period 3 labor income. However, bring wheat back into the story and assume that the costless technology for storing wheat from periods 1 to 4 exists, but the risky technology does not. Then wheat becomes a medium of exchange: financial advisors can use their endowment of wheat to buy secretarial services and secretaries can store and use wheat to buy financial advice. Indeed, since agents are indifferent between wheat and services, if $e \geq 1$ (as we have assumed), there is enough liquidity in the system to support one unit of trade at price 1; while if $e < 1$ there is insufficient liquidity and trade will be inefficiently low. If $e = 0$, there will be no trade at all.

2.3. A Third Benchmark: the (Sequential) Arrow–Debreu Equilibrium

Now consider the full-blown model where both technologies are available. We will suppose that firms' returns can be pledged (asset returns cannot be stolen by the firms' managers) and hence firms can issue securities based on, and collateralized by, these returns. These securities (rather than wheat directly) can be used for liquidity purposes to buy secretarial and financial services. Since the state of the world is verifiable it is natural to assume that firms issue H and L Arrow securities, the first paying a unit of wheat in period 4 in state H and the second, a unit of wheat in period 4 in state L.

The economy proceeds as follows. In period 1, there is a market for wheat and for the two Arrow securities. Agents use their wheat endowment to buy Arrow securities *after* they learn whether they will be financial advisors or secretaries. Firms sell Arrow securities and use the proceeds to buy wheat as input for production. In period 2, after the state of the world is realized, markets open for secretarial services and in period 3 for financial advice. The Arrow securities can be used to buy these services. (Once period 2 arrives only one of these securities will have value—the H security in state H and the L security in state L.) The Arrow securities are redeemed and wheat is consumed in period 4. We normalize so that the price of wheat in periods 1 and 4 is 1.

We will be interested in the case where financial advisors *cannot* pledge their period 3 labor income. However, it is worth beginning with the case where they *can*. We then know that in both states the Walrasian equilibrium described in Section 2.1

will rule: financial advisors and secretaries will trade one unit of financial advice for one unit of secretarial services, and prices of both services will be 1. Given that agents do not require Arrow securities for liquidity purposes and that they are risk neutral it is easy to see that all wheat will be invested in the risky (high expected return) project. Thus the rate of return in the economy between periods 1 and 4 will be $\bar{R}$. Using the fact that each Arrow security price q^H, q^L will be proportional to the probability of the state of the world associated with it (again by risk neutrality), and that the risky technology breaks even, we must have

$$q^H = \pi/\bar{R},\ q^L = (1-\pi)/\bar{R}.$$

The utilities of the financial advisors and secretaries are $U_f = e\bar{R} + (1/2)$, $U_s = e\bar{R} + (1/2)$, respectively, where the first term represents investment return on their wheat endowment and the second term the consumer surplus from trading. This is the Arrow–Debreu equilibrium for the economy.

Note that since the utilities of a financial advisor and a secretary are the same there is no demand for insurance before an agent knows her type. Thus, not surprisingly, in light of the first welfare theorem, the Arrow–Debreu equilibrium achieves the first-best. In the first-best, the central planner maximizes the expected utility of an agent who does not know whether she will be a financial advisor or a secretary subject to the aggregate feasibility constraints. That is, the central planner solves

$$\begin{aligned}\max\ & \Big\{\pi\Big[w_f^H + s_f^H - \frac{1}{2}(l_f^H)^2 + w_s^H + f_s^H - \frac{1}{2}(l_s^H)^2\Big] \\ & + (1-\pi)\Big[w_f^L + s_f^L - \frac{1}{2}(l_f^L)^2 + w_s^L + f_s^L - \frac{1}{2}(l_s^L)^2\Big]\Big\}\end{aligned}$$

$$\begin{aligned}\text{subj. to:}\quad & s_f^H = l_s^H \\ & f_s^H = l_f^H \\ & s_f^L = l_s^L \\ & f_s^L = l_f^L \\ & w_f^H + w_s^H = y^s + y^r R^H \\ & w_f^L + w_s^L = y^s + y^r R^L \\ & y^s + y^r = 2e,\end{aligned}$$

where w_f^i, w_s^i stand for wheat consumption of financial advisors and secretaries in state $i = H, L$, the l_f^i, l_s^i, stand for labor services of financial advisors and secretaries in state i, and so forth, and y^s and y^r stand for the quantities of period 1 wheat invested respectively in the safe and risky technology. The solution is easily seen to be

$$s_f^H = l_s^H = f_s^H = l_f^H = s_f^L = l_s^L = f_s^L = l_f^L = 1,$$
$$y^s = 0 \text{ and } y^r = 2e,$$

as in the Arrow–Debreu equilibrium.

2.4. Nonpledgeable (NP) Equilibrium

Consider now the case where financial advisors *cannot* pledge their period 3 income.[4] Now Arrow securities will be used as a means of payment by financial advisors in periods 2 and by secretaries in period 3. Before analyzing the equilibrium, it is useful first to see whether the Arrow-Debreu equilibrium is still sustainable. Clearly each financial advisor must hold at least one unit of each Arrow security to purchase one unit of secretarial services at a price 1 in period 2 (secretaries can then use this Arrow security to purchase financial advice in period 3 at price 1). Given $q^H = \pi/\bar{R}$, $q^L = (1-\pi)/\bar{R}$ and $e \geq 1$ a financial advisor can afford this. But if $2eR^L < 1$, one unit of liquidity will not be available in the low state if all wheat is invested in the risky technology. Thus, as Proposition 1 will confirm, in this case the Arrow–Debreu equilibrium is not sustainable.

In order to solve for equilibrium in the general case when financial advisors cannot pledge their income—we call this the nonpledgeable Arrow–Debreu equilibrium or NP equilibrium for short—let x_f^H and x_f^L be the quantities of the two Arrow securities bought by financial advisors and x_s^H and x_s^L the quantities bought by secretaries. Since agents have no endowment of wheat in period 4 and there is no penalty for breach (such as jail), short sales are impossible: a short seller can always breach ex post with impunity. Hence $x_f^H, x_f^L, x_s^H, x_s^L \geq 0$.

As before, normalize so that the price of wheat in period 1, wheat in period 4, Arrow security i in periods 2 and 3 if state i occurs (i = H, L), are all one. Let p_s^H, p_s^L, p_f^H, and p_f^L be the prices of secretarial and financial services in the high and low state, respectively.

Consider a financial advisor's utility maximization problem. In equilibrium, the price of secretarial services in period 2 cannot exceed 1 since otherwise financial advisors would strictly prefer to use their securities to purchase period 4 wheat rather than secretarial services, and so the secretarial market would not clear. Thus, we can assume for the purpose of calculating utility that financial advisors use all their Arrow securities to buy secretarial services. (By a parallel argument the price of financial advice in period 3 cannot exceed 1 and so for purposes of calculating utility we can

4. Implicitly we are making the further assumption that the legal system does not make the following kind of contract feasible: a financial advisor promises to pay a secretary in period 3 and if he defaults firms who issued the Arrow securities that the financial advisor holds in period 4 must pay the proceeds to the defaulted-against secretary rather than to the financial advisor. The recording and administrative requirements of such a system would be considerable. One way to rule out such a contract in the present model is to assume that Arrow securities are in bearer form: anyone holding them can cash them in.

assume that secretaries spend all their Arrow securities on financial advice.) Next consider a financial advisor's labor supply decision in period 3. Suppose that we have arrived in one of the states, and ignore the superscript on the state. Then a financial advisor will choose his labor supply l_f to maximize $p_f l_f - (1/2)l_f^2$, that is, set $l_f = p_f$. Note that it is too late for the financial advisor to buy more secretarial services and so his marginal return from work is p_f (he will use the proceeds to buy wheat in period 4). A financial advisor's labor yields revenue p_f^2, which he redeems for wheat in period 4; in addition, he incurs an effort cost of $(1/2)p_f^2$, and so his net utility from work is $(1/2)p_f^2$.

It follows that in period 1 a financial advisor chooses x_f^H and x_f^L to solve

$$\max \quad \pi\left[\frac{x_f^H}{p_s^H} + \frac{1}{2}\left(p_f^H\right)^2\right] + (1-\pi)\left[\frac{x_f^L}{p_s^L} + \frac{1}{2}\left(p_f^L\right)^2\right], \tag{*}$$

$$\text{subj. to:} \quad q^H x_f^H + q^L x_f^L \le e,$$
$$x_f^{H,L} \ge 0.$$

Note that firm profits are zero in equilibrium given constant returns to scale, and so we do not need to keep track of any dividends received by consumers.

A similar calculation applies to secretaries. The difference is that a secretary in period 2 chooses his labor supply l_s to maximize $(p_s/p_f)l_s - (1/2)l_s^2$. The reason is that a secretary's marginal return from work is (p_s/p_f), since he will use his income to buy financial advice. Thus a secretary's net utility from work is $(1/2)(p_s/p_f)^2$. Hence in period 1 a secretary chooses x_s^H and x_s^L to solve

$$\max \quad \pi\left[\frac{x_s^H}{p_f^H} + \frac{1}{2}\left(\frac{p_s^H}{p_f^H}\right)^2\right] + (1-\pi)\left[\frac{x_s^L}{p_f^L} + \frac{1}{2}\left(\frac{p_s^L}{p_f^L}\right)^2\right], \tag{**}$$

$$\text{subj. to:} \quad q^H x_s^H + q^L x_s^L \le e,$$
$$x_s^{H,L} \ge 0.$$

Let y^s and y^r be the quantity of period 1 wheat invested respectively in the safe and risky technology. As noted, profit maximization and constant returns to scale imply zero profit: the value of the return stream of each technology cannot exceed the cost of investing in that technology (i.e., 1); and if the inequality is strict the technology will not be used. In other words,

$$q^H + q^L \le 1 \quad \text{where} \quad y^s = 0 \quad \text{if the inequality is strict;} \tag{1}$$

$$q^H R^H + q^L R^L \le 1 \quad \text{where} \quad y^r = 0 \quad \text{if the inequality is strict.} \tag{2}$$

The market clearing conditions in the securities and wheat markets in period 1 are given respectively by

$$x_f^H + x_s^H = y^s + y^r R^H, \tag{3}$$

$$x_f^L + x_s^L = y^s + y^r R^L, \tag{4}$$

$$y^s + y^r = 2e. \tag{5}$$

Finally, market clearing conditions for secretarial and financial services in periods 2 and 3 in each state are

$$p_s^H \leq 1\text{: if } p_s^H < 1, \text{ then } \frac{x_f^H}{p_s^H} = \frac{p_s^H}{p_f^H}\text{; if } p_s^H = 1, \text{ then } x_f^H \geq \frac{1}{p_f^H}; \tag{6}$$

$$p_s^L \leq 1\text{: if } p_s^L < 1, \text{ then } \frac{x_f^L}{p_s^L} = \frac{p_s^L}{p_f^L}\text{; if } p_s^L = 1, \text{ then } x_f^L \geq \frac{1}{p_f^L}; \tag{7}$$

$$\begin{gathered} p_f^H \leq 1\text{: if } p_f^H < 1, \text{ then } \frac{x_s^H + ((p_s^H)^2/p_f^H)}{p_f^H} = p_f^H; \\ \text{if } p_f^H = 1 \text{ then } x_s^H + (p_s^H)^2 \geq 1; \end{gathered} \tag{8}$$

$$\begin{gathered} p_f^L \leq 1\text{: if } p_f^L < 1, \text{ then } \frac{x_s^L + ((p_s^L)^2/p_f^L)}{p_f^L} = p_f^L; \\ \text{if } p_f^L = 1 \text{ then } x_s^L + (p_s^L)^2 \geq 1. \end{gathered} \tag{9}$$

Equations (6)–(9) reflect the fact that, if the price of secretarial (resp., financial) services is less than 1, financial advisors (resp., secretaries) want to spend all their income on secretarial (resp., financial) services. On the other hand, if the price of secretarial or financial services equals 1, consumers are indifferent between buying the service and wheat and so the market clears as long as liquidity is at least equal to supply.

In summary, this describes a standard Arrow–Debreu equilibrium with one wrinkle: consumers cannot pledge future income.

The following lemma and proposition characterize the NP equilibrium and provide a comparison to the first-best.

LEMMA 1. *In a NP equilibrium, the prices and trading levels of secretarial and financial services equal 1 in the high state* ($p_f^H = p_s^H = 1$, *and* $x_f^H \geq 1$).

Proof. See Appendix.

The (rough) intuition is that, given $R^H > 1$ and $e \geq 1$, then $2eR^H \geq 1$, and so there is enough liquidity in the high state (and a financial advisor can afford it) to support efficient trade at prices of 1.

PROPOSITION 1.

P1.1 *If* $2eR^L \geq 1$, *then a NP equilibrium delivers the first-best.*

P1.2 *If* $1 > 2eR^L \geq [((1-\pi)/\pi)((1-R^L)/(R^H-1))]^{4/3}$ *then a NP equilibrium is such that investment is first-best efficient (only the risky technology is used), but trading in secretarial and financial services is inefficiently low in the low state relative to the first-best.*

P1.3 *If* $2eR^L < [((1-\pi)/\pi)((1-R^L)/(R^H-1))]^{4/3}$ *then a NP equilibrium is such that investments and trading in labor services are both inefficient relative to the first-best: each technology is operated at a positive scale and trading in both services is inefficiently low in the low state.*

Proof. See Appendix.[5]

Proposition 1 characterizes three regions. In region 1, when $2eR^L$ exceeds 1, our earlier observation is confirmed: we achieve the first-best. To understand the equilibrium in the two other regions, when $2eR^L < 1$, it is useful to start by noticing that financial advisors will hold some liquidity in both states. If they did not, the price of secretarial services in that state would be zero and the financial advisors' return to holding an epsilon amount of liquidity in that state would be infinite.

Since financial advisors hold some liquidity in both states, their marginal return to investing in the two Arrow securities must be the same. The expected return for a financial advisor from investing one unit of wheat in a high Arrow security is π/q^H, where $1/q^H$ is the quantity purchased and π the probability the security will pay a unit of wheat. Similarly, the return from investing one unit of wheat in a low Arrow security is $(1-\pi)/p_s^L q^L$, where the only difference is that in the low state the one dollar claim is worth $(1/p_s^L)$ to the financial advisor, since the financial advisor can purchase secretarial services worth 1 to him at a price of $p_s^L < 1$. Hence, we have

$$\frac{\pi}{q^H} = \frac{1-\pi}{p_s^L q^L}. \tag{10}$$

In the Appendix, we show that (10) implies that secretaries do not want to hold the low Arrow security: $x_s^L = 0$.

Equation (10) explains why in region 3 both technologies will be used. Consider a situation where $2eR^L$ is very low, but all wheat is invested in the risky technology. Then, in the low state there will be very little liquidity and the price of secretarial services will be very low (financial advisors' demand for secretarial services is low

5. In each region the NP equilibrium is unique. See the Online Appendix for details.

since they are severely liquidity constrained and so prices have to be low to keep supply low). From (10), if p_s^L is very low, then q^L (the price of the low state Arrow security) is very high. But if q^L is large enough then it becomes profitable to use the riskless project.

To determine when this will occur, it is useful to notice that if both technologies are used, then (1) and (2) hold as equalities and $q^L = (R^H - 1)/(R^H - R^L)$ and $q^H = (1 - R^L)/(R^H - R^L)$.

Plugging these values into (10) we obtain

$$p_s^L = \frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1} < 1. \tag{11}$$

Using equations (7) and (9) and $x_s^L = 0$, we obtain $p_s^L = (x_f^L)^{3/4}$, or

$$x_f^L = \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{4/3}.$$

Therefore, when $2eR^L < 1$, the financial advisors' demand for liquidity in the low state can be satisfied without investing in the riskless technology only if $2eR^L \geq [((1-\pi)/\pi)((1-R^L)/(R^H-1))]^{4/3}$ (region 2). Otherwise, the riskless technology will be used and we will be in region 3.

Another way to understand this is to recognize that the nonpledgeability of future labor income creates a demand for relatively safe assets. Transactional needs generate a form of risk aversion even in risk-neutral people. When an agent has the opportunity/desire to buy, having a great deal of pledgeable wealth in some states does not compensate her for the risk of having very little pledgeable wealth in other states because there are diminishing returns to liquidity: in the former states the gains from trade have been exhausted and the marginal value of liquidity is zero, whereas in the latter states the agents are wealth constrained and the marginal value of liquidity is high. As a result, agents are willing to hold relatively safe assets even if they have a lower yield. The behavior induced by the nonpledgeability of future labor income, however, is different from that induced by simple risk aversion. Given the state of the world, each individual has a linear utility and no desire to insure himself. Yet, he is willing to pay a premium for a security that pays when the state of the world is low. This really is a liquidity premium.

Note that Proposition 1 tells us that a NP equilibrium never goes to the other extreme of having no production of the risky asset. To understand this, observe that if all resources are invested in the riskless technology there is enough liquidity to support one unit of trade in both states. But then $q^H = \pi$, $q^L = 1 - \pi$ are the break-even prices for the riskless technology and at these prices the risky technology becomes profitable.

How realistic is it that $2eR^L < 1$? If we assume that $e \geq 1$, we require that the gross returns fall by more than 50% in the low state relative to the high state. If we are talking of a generalized drop in the value of investment, this is a rather unlikely, but not unheard of, phenomenon. During the Great Depression, the Dow Jones fell 89% from 1929 to 1932. During the stock market crash of 1987, the Dow Jones dropped 22.6% in a single day. From 2000 to 2002, the Nasdaq Composite lost 78% of its value. Finally, during the 2008 financial crisis the Dow Jones index dropped 54% peak to trough.

Note also that this condition is so demanding because we assume that all agents have the same endowment. However, if financial advisors are not equally endowed, it is much more likely that some of them will be liquidity constrained even for small stock market declines. Of course, the aggregate impact of these constraints would be limited by the mass of people who find themselves in this situation. Thus, for given level of pledgeable wealth, wealth inequality will play a major role.

2.5. Insurance

Since individuals know their type before they trade they could in principle obtain insurance against their type. Insurance markets would open before period 1 and pay wheat in period 1 contingent on an agent's type, assumed verifiable. Financial advisors would receive wheat from insurance companies and secretaries would hand over wheat. (Of course, if period 1 wheat cannot be pledged in advance such an arrangement is infeasible.) We show in the Appendix that, while the utility of a financial advisor is below that of a secretary, their marginal utilities of wealth are the same. Thus there is no demand for insurance.

2.6. Second-Best Optimum: Overinvestment in Safe Assets

It is clear that there is overinvestment in safe assets relative to the first-best, where all endowment is invested in the risky technology. We now show that the NP equilibrium is constrained inefficient: there is also overinvestment in a second-best sense. Rather than characterize the second best, we simply show that a central planner can improve upon the competitive equilibrium outcome by restricting investment in the riskless technology.

We assume that the central planner can choose y^s (or equivalently y^r), but cannot interfere in markets in other ways. We will focus on the case $2eR^L < [((1-\pi)/\pi)((1-R^L)/(R^H-1))]^{4/3}$ (region 3 of Proposition 1). It is easy to see from the proof of Proposition 1 that in a neighborhood of the NP equilibrium both technologies are used and $x_f^L > 0, x_s^L = 0$. Therefore, from equations (4) and (5) there is a one-to-one relationship between y^s and x_f^L: a decrease in investment in the riskless technology corresponds to a decrease in the holding of low Arrow securities by financial advisors. In what follows, we therefore assume that the central planner picks $x_f^L = x^{CP}$ rather than y^s. We will show that the central planner can increase surplus by reducing x^{CP} below the NP equilibrium level.

Suppose that the central planner picks $x_f^L = x^{CP}$ in a neighborhood of the equilibrium. Then the market clearing conditions (7) and (9) yield

$$p_f^L = \left(x^{CP}\right)^{1/2},$$
$$p_s^L = \left(x^{CP}\right)^{3/4}.$$

Also q_H, q_L will satisfy equations (1) and (2) with equality:

$$q^L = (R^H - 1)/(R^H - R^L), q^H = (1 - R^L)/(R^H - R^L).$$

A financial advisor's utility, which is given by

$$\pi\left[x_f^H + (1/2)\right] + (1-\pi)\left[\left(x_f^L/p_s^L\right) + (1/2)\left(p_f^L\right)^2\right],$$

becomes

$$\pi\left[((e - q^L x^{CP})/q^H) + (1/2)\right] + (1-\pi)\left[\left(x^{CP}\right)^{1/4} + (1/2)x^{CP}\right].$$

Similarly, a secretary's utility, which is given by

$$\pi\left[x_s^H + (1/2)\right] + (1-\pi)\left[(1/2)\left(p_s^L/p_f^L\right)^2\right]$$

becomes

$$\pi\left[(e/q^H) + (1/2)\right] + (1-\pi)\left[(1/2)\left(x^{CP}\right)^{1/2}\right].$$

The central planner maximizes the expected utility of an agent who does not know whether he will be a financial advisor or a secretary, i.e., the central planner maximizes $U^f + U^s$. Differentiating the welfare function with respect to x^{CP} yields

$$-\pi\frac{q^L}{q^H} + (1-\pi)\left[\frac{1}{4}\left(x^{CP}\right)^{-(3/4)} + \frac{1}{2}\right] + (1-\pi)\frac{1}{4}\left(x^{CP}\right)^{-(1/2)}. \qquad (12)$$

We want to prove that (12) is negative when we evaluate it at the market equilibrium,

$$x^{CP} = \left[((1-\pi)/\pi)((1-R^L)/(R^H-1))\right]^{4/3}.$$

From equations (1) and (2) we know that

$$(1 - R^L)/(R^H - 1) = (q^H/q^L).$$

Hence, we can rewrite (12) calculated at

$$x^{CP} = [((1-\pi)/\pi)((1-R^L)/(R^H-1))]^{4/3}$$

as

$$\frac{\pi q^L}{q^H}\left[-1 + (x^{CP})^{3/4}((1/2) + (1/4)(x^{CP})^{-3/4} + (1/4)(x^{CP})^{-1/2})\right] < 0,$$

since $x^{CP} < 1$.

It follows that the central planner can increase surplus by reducing x^{CP} below the NP level or equivalently by reducing y^s.

PROPOSITION 2. *When* $2eR^L < [((1-\pi)/\pi)((1-R^L)/(R^H-1))]^{4/3}$*, starting at the NP equilibrium, a central planner can achieve an ex-ante Pareto improvement by restricting investment in safe assets.*

In other words, the NP equilibrium will be second-best inefficient as long as there is sufficiently high aggregate uncertainty before trading takes place:

$$2eR^L < \left[((1-\pi)/\pi)((1-R^L)/(R^H-1))\right]^{4/3}.$$

The intuition is that financial advisors who acquire liquidity impose a negative externality on other financial advisors. In deciding how to divide his wealth between low- and high-state Arrow securities a financial advisor ignores the fact that an increase in his holding of low-state Arrow securities raises the price of secretarial services in the low state. In contrast to a standard complete markets model, this pecuniary externality has a negative effect on other financial advisors, who are liquidity constrained, which is not purely a redistribution. In a standard complete markets model, a small increase in the price of secretarial services has only distributional effects: secretaries enjoy a gain from the increase, financial advisors enjoy a loss and these cancel out given that the marginal benefit of secretarial services equals the marginal cost of providing these services and both equal the price. In our model, since financial advisors are liquidity constrained in the low state the marginal value of secretarial services strictly exceeds the price and so the loss to financial advisors exceeds the gain to secretaries: an increase in the price of secretarial services creates a welfare loss.

In standard models of portfolio choice with labor income risk (Heaton and Lucas (1997) and Cocco et al. (2005)) but no pledgeability problems, agents invest more in risky assets because these provide insurance against labor income shocks. In our model, however, there is no labor income risk absent pledgeability problems. Furthermore, agents are not trying to insure against a fluctuation of their wealth since they are risk neutral. Hence, agents need to protect themselves only against shortages of liquidity, a need that leads them to overinvest in safe claims.

To summarize, compared to the first-best, the NP equilibrium has too much investment in the riskless technology and too little trade in the low state; while compared to the second-best the NP equilibrium has too much investment in the riskless technology and too much trade in the low state.

2.7. Applications

We have shown that too many resources are invested in manufacturing and holding relatively safe assets. In a domestic setting, a manifestation of this distortion is the amount of wealth invested in housing (which is perceived as relatively safe), especially by poorer consumers who are more likely to face financial constraints. An effect of this overinvestment is the high price (and lower return) of real estate. Another effect is the progressive decline in the average returns of investments.

Another example of the overinvestment in safe assets is given by the emerging economies' holdings of low-yielding US Treasury securities (see Caballero and Krishnamurthy 2009). The concern of these economies is precisely that they will not have sufficient collateralizeable wealth in a severe downturn. Hence, their desire to hold relatively safe securities—that is, securities that maintain their values in the most severe downturn. One effect produced by these massive holdings is a decline in the US Treasuries' yield and an increase in the equity premium.

To see that our model can explain the latter phenomenon, notice that in the case 3 of Proposition 1, when

$$2eR^L < \left[((1-\pi)/\pi)((1-R^L)/(R^H-1))\right]^{4/3},$$

then the high-state Arrow security will carry a discount and the low-state Arrow security will carry a premium, in spite of the linear utility of the agents. To see the discount it is sufficient to compare the price of the high-state Arrow security in case 3 ($q^H = (1-R^L)/(R^H-R^L)$), with its price in the efficient equilibrium in case 1 ($q^H = \pi/\bar{R}$). It is easy to see that $(1-R^L)/(R^H-R^L) < \pi/\bar{R}$. Correspondingly, the price of the low-state Arrow security in case 3 ($q^L = (R^H-1)/(R^H-R^L)$) exceeds the price in the efficient equilibrium in case 1 ($q^L = (1-\pi)/\bar{R}$), since $(R^H-1)/(R^H-R^L) > (1-\pi)/\bar{R}$.

This result is reminiscent of Barro (2006), who rationalizes the high equity premium and low risk-free rate on the basis of the historical occurrence of rare disasters. Our low state with

$$2eR^L < \left[((1-\pi)/\pi)((1-R^L)/(R^H-1))\right]^{4/3}$$

can be thought of as one of those disasters. The main difference is that Barro explains the equity premium and low risk-free rate in the context of a representative agent model with risk-averse investors, while we do so with heterogeneous agents who are risk neutral.

3. Fiscal Policy

So far we have ignored the role of the government in providing liquidity. We will now relax this assumption. Following Holmstrom and Tirole (1998, 2011) and Woodford (1990), we assume the government can exploit a power it has, which the private sector does not: the power to tax. In particular, the government can issue notes to consumers, and these notes will be valuable because they are backed by future tax receipts.

Holmstrom and Tirole (1998) justify the assumption that it is easier for the government to collect taxes than for creditors to collect debts from consumers on the grounds that the government can audit incomes or impose jail penalties. While realistic, this assumption can be criticized given that auditing and jail could be used as a penalty for the nonpayment of private debts as well—after all, debtors' prisons have existed in the past. We therefore adopt a different rationale. We suppose that the government can impose sales taxes on certain productive facilities that consumers use and which can be easily monitored. Private lenders cannot duplicate such an arrangement since they do not have the power to require (all) facilities to participate.

In our finite-horizon model notes or money are equivalent because both of them must be backed by future taxes. Therefore, since the intervention we consider does not affect the aggregate wealth of consumers, but only the temporal distribution of this wealth, we label it *fiscal policy*.

3.1. Taxing Technology

To allow for sales taxes, we assume that in period 4 our agents consume flour as well as wheat: one unit of flour yields one unit of utility. There is a milling technology for turning wheat into flour: each agent can obtain λ units of flour at the cost of $(1/2)c\lambda^2$ units of wheat, where $\lambda \geq 0$ is the agent's choice variable. This activity occurs at facilities (mills) that can easily be monitored by the government, and so the government can impose a per unit flour tax τ that cannot be avoided.

An agent's utility is now:

$$\begin{aligned} \text{Financial advisors:} \quad & U_f = w_f + s_f - \frac{1}{2}l_f^2 + (1-\tau)\lambda_f - \frac{1}{2}c\lambda_f^2, \\ \text{Secretaries:} \quad & U_s = w_s + f_s - \frac{1}{2}l_s^2 + (1-\tau)\lambda_s - \frac{1}{2}c\lambda_s^2, \end{aligned}$$

where τ is the tax rate on flour.

We now assume that in period 4 each agent has a large endowment of wheat (in addition to any labor income and dividends from investment). We also suppose that, like labor income, this wheat cannot be pledged in advance (or seized by the government). Given this large endowment no agent is at a corner solution, and hence λ_f, λ_s satisfy the first-order condition

$$\lambda_f = \lambda_s = \frac{1-\tau}{c}.$$

This yields

$$U_f = w_f + s_f - \frac{1}{2}l_f^2 + \frac{1}{2c}(1-\tau)^2,$$
$$U_s = w_s + f_s - \frac{1}{2}l_s^2 + \frac{1}{2c}(1-\tau)^2.$$

The total amount raised by taxes on financial advisors and secretaries is

$$T = \frac{2\tau(1-\tau)}{c}. \tag{13}$$

Since in the high state there is ample liquidity, it is natural to focus on the case where the government issues notes only in the low state. We will suppose that the government can target those who need the liquidity most: the financial advisors. (Our analysis so far is consistent with the assumption that it is verifiable in period 2 whether an agent has to buy or sell first.)[6] In summary, the government's fiscal policy consists of issuing m notes to each financial advisor in period 2 if and only if state L occurs. Each note promises one unit of wheat in period 4.

Given that government notes must be backed by taxes, we have

$$m = T = \frac{2\tau(1-\tau)}{c}. \tag{14}$$

Note that (14) implies that $T = 0$ when $\tau = 0$ and T reaches a maximum at $\tau = (1/2)$. Thus, it is never optimal to set $\tau > (1/2)$ since the deadweight loss increases in τ.

3.2. Ex-Post Intervention

Let us first consider the case of an unexpected ex-post intervention: when the state is low, the government intervenes with an (unexpected) hand-out m in period 2. This hand-out will have the effect of boosting the level of output by more than m. To see this, assume that x_f^L and x_s^L are fixed at their competitive equilibrium levels: $x_f^L < 1, x_s^L = 0$. Consider a small government hand-out of m to the financial advisors. Then, after the hand-out the new equilibrium becomes

$$\frac{x_f^L + m}{p_s^L} = \frac{p_s^L}{p_f^L},$$
$$\frac{x_f^L + m}{p_f^L} = p_f^L,$$

which implies $p_s^L = (x_f^L + m)^{3/4}$ and $p_f^L = (x_f^L + m)^{1/2}$. Since $l_f^L = p_f^L$ and $l_s^L = (p_s^L/p_f^L)$, the fiscal policy increases (the nominal value of) output (which we measure as $p_f^L l_f^L + p_s^L l_s^L = (p_f^L)^2 + ((p_s^L)^2/p_f^L))$ from $2x_f^L$ to $2(x_f^L + m)$.

6. The case where the government cannot target the constrained agents is not qualitatively different, but a bit more cumbersome.

In other words, in this model there is a fiscal multiplier equal to 2. The intuition behind the multiplier is very Keynesian: the fiscal stimulus enables the financial advisors to buy more secretarial services. This extra demand increases production by secretaries and therefore secretaries can afford to buy more from financial advisors, raising their production.

Not only does a fiscal policy following a big negative shock increase output more than one-to-one, but it also increases ex-ante welfare. To see this, it is sufficient to notice that ex-post welfare in state L is given by

$$W^L = \left[\frac{x_f^L + m}{p_s^L} + \frac{1}{2}\left(p_f^L\right)^2 + \frac{1}{2c}(1-\tau)^2 + \frac{x_s^L}{p_f^L} + \frac{1}{2}\left(\frac{p_s^L}{p_f^L}\right)^2 + \frac{1}{2c}(1-\tau)^2 \right], \tag{15}$$

and

$$\frac{dW^L}{dm} = \left[\frac{1}{4}(x_f^L + m)^{-(3/4)} + \frac{1}{2} + \frac{1}{4}(x_f^L + m)^{-(1/2)} - \frac{(1-\tau)}{(1-2\tau)} \right] > 0 \tag{16}$$

for m close to 0.

A more interesting question is what happens when the intervention is fully anticipated. We will consider two cases: one where the government can commit to m ex ante and the other where the government cannot commit. Also, in contrast to Section 2.6, we will suppose that the government cannot constrain the production decision of firms: its only policy tool is m.

3.3. *Anticipated Intervention: The Case of Commitment*

We will assume that agents have rational expectations about government actions. Suppose that agents anticipate that m will be injected in the low state. Then, the equilibrium of Section 2.4 changes as follows.

In period 1 a financial advisor chooses x_f^H and x_f^L to solve

$$\begin{aligned} \max \quad & \pi \left[\frac{x_f^H}{p_s^H} + \frac{1}{2}\left(p_f^H\right)^2 + \frac{1}{2c} \right] + (1-\pi) \left[\frac{x_f^L + m}{p_s^L} + \frac{1}{2}\left(p_f^L\right)^2 \right. \\ & \left. + \frac{1}{2c}(1-\tau)^2 \right], \qquad (*) \end{aligned}$$

$$\text{subj. to:} \quad q^H x_f^H + q^L x_f^L \le e,$$

where (*) reflects the fact that the flour tax is zero in state H and τ in state L and the financial advisors receive m in state L.

Similarly, a secretary chooses x_s^H and x_s^L to solve

$$\max \quad \pi\left[\frac{x_s^H}{p_f^H}+\frac{1}{2}\left(\frac{p_s^H}{p_f^H}\right)^2+\frac{1}{2c}\right]+(1-\pi)\left[\frac{x_s^L}{p_f^L}+\frac{1}{2}\left(\frac{p_s^L}{p_f^L}\right)^2+\frac{1}{2c}(1-\tau)^2\right], \tag{**}$$

subj. to:

$$q^H x_s^H + q^L x_s^L \le e.$$

The other equilibrium conditions (1)–(6) and (8)–(9) stay the same, while (7) becomes

$$p_s^L \le 1\text{: if } p_s^L < 1, \text{ then } \frac{x_f^L+m}{p_s^L} = \frac{p_s^L}{p_f^L}\text{; if } p_s^L = 1, \text{ then } x_f^L + m \ge \frac{1}{p_f^L}. \tag{17}$$

The government chooses in period 0 to maximize the expected utility of an agent who does not know whether he will buy or sell first (equivalently, whether he will be a financial advisor or a secretary). That is, the government chooses m to maximize the sum of financial advisor and secretary utilities:

$$\begin{aligned} W = \pi &\left[\frac{x_f^H}{p_s^H}+\frac{1}{2}\left(p_f^H\right)^2+\frac{1}{2c}+\frac{x_s^H}{p_f^H}+\frac{1}{2}\left(\frac{p_s^H}{p_f^H}\right)^2+\frac{1}{2c}\right]+(1-\pi) \\ &\times\left[\frac{x_f^L+m}{p_s^L}+\frac{1}{2}\left(p_f^L\right)^2+\frac{1}{2c}(1-\tau)^2+\frac{x_s^L}{p_f^L}+\frac{1}{2}\left(\frac{p_s^L}{p_f^L}\right)^2+\frac{1}{2c}(1-\tau)^2\right], \end{aligned} \tag{18}$$

where for each m the x and the p are given by the market equilibrium corresponding to that m.

The interesting case is when in the absence of fiscal policy investment and trading in labor services are both inefficient, namely

$$2eR^L < \left[((1-\pi)/\pi)((1-R^L)/(R^H-1))\right]^{4/3}$$

(see Case 3 of Proposition 1). In this case we have the following proposition.

PROPOSITION 3. *If* $2eR^L < [((1-\pi)/\pi)((1-R^L)/(R^H-1))]^{4/3}$, *a positive injection of notes* ($m > 0$) *in the low state is welfare improving.*

Proof. See Appendix.

Consider a small injection of m. From equation (11) we know that when both technologies are used, we have

$$p_s^L = \frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1} < 1.$$

Thus, a government injection of m does not affect p_s^L, which is determined only by technological parameters. Since the market clearing conditions require that $(p_s^L)^{4/3} = x_f^L + m$, this also implies that the sum $x_f^L + m$ stays the same after a government injection of liquidity. If the sum stays the same and m increases, it means that there is a 100% crowding out of x_f^L by m. Thus, when the government intervention is expected, the inefficient overinvestment in safe assets is reduced. Yet, the level of output in periods 2 and 3 is still inefficient. Since government liquidity completely crowds out private liquidity, the level of trade remains the same as in the original equilibrium without government intervention. Nevertheless, when the government does intervene in period 2, the multiplier is bigger than 1 as per the previous analysis.

What is the optimal level of m? Assume that c is small (the deadweight cost of taxation is large) and thus it is never desirable for the government to move the economy too far from the nonintervention equilibrium. We can then be confident that we will remain in a neighborhood of $m = 0$ and so the equilibrium conditions will continue to hold. As shown in the Appendix, the first-order condition for the optimal m then becomes

$$\frac{\pi}{(1-\pi)}\left[\frac{R^H - 1}{1 - R^L}\right] = \frac{(1-\tau)}{(1-2\tau)}. \tag{19}$$

Since the right-hand side is strictly increasing in τ and converges to ∞ as $\tau \to (1/2)$ from below, (19) has a unique solution and the optimal m can be deduced from (19).

Here we have ignored the possible existence of other government expenses (e.g., to finance valuable public goods). If they exist, then distortions of extra taxation are first order, not second order. In such a case it is not obvious that we want to intervene with a fiscal stimulus. In fact, if the government anticipates the need for such a stimulus in the future it might want to restrain the provision of public goods (and taxation) beyond what is optimal in isolation to retain the fiscal flexibility to intervene in case of a disaster.

3.4. Anticipated Intervention: The Case of Noncommitment

Suppose that m is characterized by (19) but now the government can change m ex post if state L occurs. Will it choose to do so? We assume that the government continues to be benevolent: it maximizes the sum of financial advisor and secretary utilities in the low state. The problem in terms of commitment is that x_f^L, x_s^L and production decisions are sunk.

Given that x_f^L and $x_s^L = 0$ are fixed, market prices for secretarial and financial services will be given by

$$\frac{x_f^L + m}{p_s^L} = \frac{p_s^L}{p_f^L}, \tag{20}$$

$$\frac{x_f^L + m}{p_f^L} = p_f^L, \tag{21}$$

where m now varies. Total welfare in the low state (see equation (15)) can be written as

$$W^L = \left[\left(x_f^L + m\right)^{1/4} + \frac{1}{2}\left(x_f^L + m\right) + \frac{1}{2}\left(x_f^L + m\right)^{1/2} + \frac{1}{c}(1-\tau)^2\right]. \quad (22)$$

Since x_f^L is fixed,

$$\frac{\partial W^L}{\partial m} = \left[\frac{1}{4}\left(x_f^L + m\right)^{-(3/4)} + \frac{1}{2} + \frac{1}{4}\left(x_f^L + m\right)^{-(1/2)} - \frac{(1-\tau)}{(1-2\tau)}\right]. \quad (23)$$

Now apply (11) and (19) and use (20)–(21) to write $x_f^L + m = (p_s^L)^{4/3}$. Then,

$$\frac{\partial W^L}{\partial m} = \left[\frac{1}{4}(p_s^L)^{-1} + \frac{1}{2} + \frac{1}{4}(p_s^L)^{-(2/3)} - (p_s^L)^{-1}\right] < 0, \quad (24)$$

since $p_s^L < 1$.

We see that, when the government considers its decision as of period 2, it has an incentive to renege on the previously announced level of fiscal intervention.[7] In other words, we have the following proposition.

PROPOSITION 4. *The government fiscal policy is time inconsistent.*

Ex post the government will want to give fewer hand-outs than it said it would. The reason is that the promise to give hand-outs in the low state helps address two problems: the inefficient investment in period 1 and the inefficiently low level of trade in periods 2 and 3. If the government can renege on its promise in period 2, however, it will find that at that time its actions affect only one inefficiency: the low level of trade in periods 2 and 3. Since the government finds it less beneficial to tax people to deal with one inefficiency rather than two, it will deviate in the direction of intervening less than promised.

Note that the optimal time-consistent fiscal policy is characterized by

$$\left[\frac{1}{4}(x_f^L + m)^{-(3/4)} + \frac{1}{2} + \frac{1}{4}(x_f^L + m)^{-(1/2)} - \frac{(1-\tau)}{(1-2\tau)}\right] = 0, \quad (25)$$

where

$$x_f^L + m = (p_s^L)^{4/3} = \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H - 1}\right)^{4/3}.$$

7. Note that this would no longer be true if the government could choose fiscal policy *and* control the investment allocation. See Jeanne and Korinek (2013).

3.5. Comparison of Fiscal Policies

How do the fiscal policies for the three cases—unanticipated, commitment, and noncommitment—differ? We already know that the commitment level of m is higher than the noncommitment level. If we compare the noncommitment level with the unanticipated level we see that the first-order conditions (16) (with equality) and (25) are the same. However, since there is crowding out when m is anticipated the level of x_f^L is lower in (25) than in (16). It follows that the value of m satisfying (25) will be higher than that satisfying (16): the noncommitment level is higher than the unanticipated level. The intuition is that when fiscal policy is not anticipated the desirable crowding out effects do not occur and so it is less attractive.

Thus the ranking of fiscal policies from high to low is: commitment, noncommitment, unanticipated.

4. An Extension

So far, we have considered borrowers who can breach any promise to pay future labor income by “disappearing”. In practice, some specialist agents may be able to keep track of borrowers and force them to repay their debts. Specifically, suppose that all payments for secretarial and financial services take place through check transfers and that a bank is able to seize these before they are cashed for consumption. In this way labor income becomes contractible. However, assume that the bank cannot force anyone to work. That is, all the bank can do is to ensure that someone who defaults has zero consumption from their labor income. As a result, uncollateralized lending against future labor income is possible, but there is a repayment constraint. Each worker can borrow up to the point at which ex post he is indifferent between working and repaying the loan and doing nothing and defaulting.

To see how our analysis generalizes, return to the NP equilibrium in Section 2.4. Suppose that the economy has reached period 2 in state i. Let a financial advisor borrow b^i_f, promising to pay that amount out of her period 3 labor income. (The bank finances its lending by accepting deposits from secretaries. Note that, since a state i Arrow security has price 1 in periods 2 and 3, the market rate of interest between periods 2 and 3 is zero.) Given that the financial advisor can use the loan to buy secretarial services that have a price less than 1 he will want to borrow the maximum possible. However, the bank needs to make sure that it will be repaid. In period 3 a financial advisor exerts $(1/2)(p^i_f)^2$ of effort, receiving in exchange a payment $(p^i_f)^2$. His net utility is $(1/2)(p^i_f)^2$. Thus, the maximum he can borrow is $(1/2)(p^i_f)^2$. If he borrowed more he would prefer not to work in period 3, default, and consume nothing (except for any nonlabor income). Given that a financial advisor will borrow $(1/2)(p^i_f)^2$, then (*) is replaced by

$$\max \quad \pi\left[\frac{x_f^H+\frac{1}{2}\left(p_f^H\right)^2}{p_s^H}\right]+(1-\pi)\left[\frac{x_f^L+\frac{1}{2}\left(p_f^L\right)^2}{p_s^L}\right], \tag{26}$$

$$\text{subj. to:} \quad q^H x_f^H + q^L x_f^L \leq e,\ x_f^{H,L} \geq 0.$$

Then (**), (1)–(5), (8) and (9) stay the same while equations (6)–(7) are replaced, respectively, by

$$p_s^H \leq 1\text{: if } p_s^H < 1, \text{ then } \frac{x_f^H+\frac{1}{2}\left(p_f^H\right)^2}{p_s^H} = \frac{p_s^H}{p_f^H};\text{ if } p_s^H = 1,$$
$$\text{then } x_f^H + \frac{1}{2}\left(p_f^H\right)^2 \geq \frac{1}{p_f^H}, \tag{27}$$

and

$$p_s^L \leq 1 : \text{ if } p_s^L < 1, \text{ then } \frac{x_f^L+\frac{1}{2}\left(p_f^L\right)^2}{p_s^L} = \frac{p_s^L}{p_f^L};\text{ if } p_s^L = 1,$$
$$\text{then } x_f^L + \frac{1}{2}\left(p_f^L\right)^2 \geq \frac{1}{p_f^L}. \tag{28}$$

Then (26), (**), (1)–(5), (27)–(28), (8)–(9) characterize the new equilibrium. Propositions 1 and 2 generalize to this version of the model with some natural changes. For example, the condition for the first-best to be achieved becomes $2eR^L \geq 1/2$. The competitive equilibrium will still typically deliver an overinvestment in safe assets. Details are along the lines of Proposition 4 of Hart and Zingales (2011).

5. Conclusions

We have built a simple framework to analyze the role of fiscal policy in attenuating the impact of aggregate shocks on private investment choices and aggregate output. We have shown that the mere lack of pledgeability of labor income, even in the presence of complete markets for securities, makes the competitive equilibrium constrained inefficient. The market will invest too much in producing safe securities and will dedicate too few resources towards risky investments. This result survives even if we make labor income contractible, as long as creditors cannot force anyone to work.

In our simple model, a fiscal policy following a big negative shock can increase not only ex-post output more than one-to-one (fiscal multiplier), but also ex-ante welfare.

We have supposed that the government is able to target directly consumers who are in need of liquidity. If we were to drop this assumption, an interesting set of problems would arise. Would it be cheaper for the government to bail out financial intermediaries rather than to hand out money to consumers randomly? If so, how would this benefit trade off against the potential moral hazard problem financial intermediaries face when they expect to be bailed out in major downturns? We plan to analyze these and other issues in future work.[8]

Appendix: Proofs

Proof of Lemma 1. Suppose $p_f^H < 1$ and $p_s^H < 1$. Then, by equations (8) and (6), $x_s^H + x_f^H = (p_f^H)^2 < 1$, which contradicts equation (23).

Now suppose that $p_f^H < 1$ and $p_s^H = 1$. Then, by equation (8), $x_s^H = (p_f^H)^2 - (1/p_f^H) < 0$, which is impossible. Hence $p_f^H = 1$.

To prove that $p_s^H = 1$, assume the contrary: $p_s^H < 1$. We first show that $x_f^H \geq x_f^L$. Suppose not: $x_f^H < x_f^L$. Then $x_f^L > 0$. From the first-order conditions for (*),

$$\frac{\pi}{p_s^H q^H} \leq \frac{1-\pi}{p_s^L q^L}. \tag{A.1}$$

That is, the utility rate of return on the low-state Arrow security for financial advisors must be at least as high as that on the high-state Arrow security. We also know that there is more output in the high state, so, if $x_f^H < x_f^L$, secretaries must be buying the high-state security, which means that it must give them an attractive return, or, from their first-order condition,

$$\frac{\pi}{q^H} \geq \frac{1-\pi}{p_f^L q^L}, \tag{A.2}$$

where we are using the fact that $p_f^H = 1$.

Putting (A.1) and (A.2) together yields

$$\frac{p_s^L}{p_s^H} \leq p_f^L. \tag{A.3}$$

If $p_s^L = 1$, (A.3) implies $p_s^H = 1$, which we have supposed not to be the case. Hence $p_s^L < 1$. Then we have, from equations (6) and (7), that $(p_s^H)^2 = x_f^H$ and

8. See Hart and Zingales (2014) for an application of the model.

$(p_s^L)^2 = x_f^L p_f^L$. Therefore (A.3) becomes

$$\frac{\left(x_f^L\right)^{1/2}}{\left(x_f^H\right)^{1/2}} \leq \left(p_f^L\right)^{1/2} \leq 1,$$

or $x_f^H \geq x_f^L$, which is a contradiction.

Hence, $x_f^H \geq x_f^L$. Since a financial advisor's utility is increasing in x_f^L and x_f^H, a financial advisor's budget constraint will hold with equality. Thus $q^H x_f^H + q^L x_f^L = e$, which implies $(q^H + q^L)x_f^H \geq e$. Hence, by equation (1), $x_f^H \geq e \geq 1$, implying $p_s^H = 1$ by equation (6).

Proof of Proposition 1.1. To achieve the first-best the supply of secretarial and financial services must be 1 in each state. Since the supply of financial advice is given by p_f^H, p_f^L in (8), (9), it follows that $p_f^H = p_f^L = 1$. The supply of secretarial services is given by (p_s^H/p_f^H), (p_s^L/p_f^L) in equations (6) and (7), and, substituting $p_f^H = p_f^L = 1$, we obtain $p_s^H = p_s^L = 1$. Hence, again from equations (6) and (7), $x_f^H \geq 1$ and $x_f^L \geq 1$.

In the first-best all wheat is invested in the high-yield project: $y^s = 0$ and $y^r = 2e$. Therefore, from (4), $2eR^L = x_f^L + x_s^L \geq x_f^L \geq 1$. Hence, $2eR^L \geq 1$ is a necessary condition.

To prove sufficiency consider a candidate equilibrium where the prices of financial and secretarial services equal 1 in both states, $q^H = (\pi/\bar{R})$, $q^L = (1-\pi)/\bar{R}$, all wheat is invested in the high-yield project, and the financial advisors buy at least one unit of each Arrow security. Since $q^H + q^L < 1 \leq e$, they can afford to do so. Financial advisors and secretaries satisfy their first-order conditions and firms maximize profit. Hence this is indeed a competitive equilibrium.

Proof of Propositions 1.2 and 1.3. The proof of Proposition 1.1 shows that we achieve the first-best if $2eR^L \geq 1$. Consider the case $2eR^L < 1$. We know from Lemma 1 that $p_f^H = p_s^H = 1$. We show first that financial advisors will hold both securities. Given $p_s^H = 1$, equation (6) implies $x_f^H > 0$. Suppose $x_f^L = 0$. This is inconsistent with $p_s^L = 1$ in equation (7). But if $p_s^L < 1$, then, from equation (7), $x_f^L = 0$ implies $p_s^L = 0$. This in turn implies that the marginal return on the low-state Arrow security for a financial advisor is infinite, which means that the first-order condition in (*) cannot hold. Therefore, $x_f^L > 0$.

Since financial advisors hold both securities, we have

$$\frac{\pi}{q^H} = \frac{1-\pi}{p_s^L q^L}. \tag{A.4}$$

Let us assume first that both technologies are used. Then equations (1) and (2) hold as an equality and $q^L = (R^H - 1)/(R^H - R^L)$ and $q^H = (1 - R^L)/(R^H - R^L)$. Therefore

$$p_s^L = \frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1} < 1, \tag{A.5}$$

since $\bar{R} > 1$.

It is easy to see that $p_s^L < p_f^L$. This is clear from (A.5) if $p_f^L = 1$. Suppose $p_f^L < 1$. Then equation (9) implies

$$\left(p_f^L\right)^2 = x_s^L + \left((p_s^L)^2/p_f^L\right) \geq \left((p_s^L)^2/p_f^L\right).$$

Hence, $p_f^L \geq (p_s^L)^{2/3} > p_s^L$ since $p_s^L < 1$. This proves $p_s^L < p_f^L$. It follows that the rate of return on the low security is strictly less than that on the high security for secretaries. So secretaries will not hold the low security (from the first-order condition for (**)): $x_s^L = 0$.

From equation (7),

$$p_f^L x_f^L = \left(p_s^L\right)^2, \tag{A.6}$$

from which it follows, since $p_s^L < 1$ and $p_s^L < p_f^L$, that $x_f^L < 1$. But then equation (9), in combination with $x_s^L = 0$, implies $p_f^L < 1$. Hence, again from equation (9),

$$x_f^L = \left(p_f^L\right)^2. \tag{A.7}$$

Combining (A.6) and (A.7) we have

$$x_f^L = \left(p_s^L\right)^{4/3}. \tag{A.8}$$

Hence

$$p_f^L = \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{2/3}, x_f^L = \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{4/3}. \tag{A.9}$$

If the solution $x_f^L = [((1-\pi)/\pi))((1-R^L)/(R^H-1))]^{4/3} > 2eR^L$, then this candidate equilibrium is feasible. Note that both technologies are used: since $x_f^L > 2eR^L$ the riskless technology must be used and since, by Lemma 1 and (A.7), $x_f^H \geq 1 > x_f^L = x_f^L + x_s^L$ the risky technology must also be used. Also trade of financial and secretarial services is inefficient since p_s^L, p_f^L are both less than 1.

If $x_f^L = [((1-\pi)/\pi))((1-R^L)/(R^H-1))]^{4/3} \leq 2eR^L$, then we solve instead for an equilibrium with $x_f^L = 2eR^L$, $x_s^L = 0$. In this case, there will be no investment in the storage technology (thus $y^s = 0$) and the market clearing condition for securities, equations (3) and (4), simplifies to

$$x_f^H + x_s^H = 2eR^H,$$
$$x_f^L + x_s^L = 2eR^L.$$

Equations (7) and (9) become

$$p_f^L = \left(2eR^L\right)^{1/2},$$
$$p_s^L = \left(2eR^L\right)^{3/4},$$

which are below 1 since $2eR^L < 1$. Financial advisors hold both Arrow securities since $p_s^H, p_s^L > 0$ and so we must have

$$\frac{\pi}{q^H} = \frac{1-\pi}{p_s^L q^L} = \frac{1-\pi}{\left(2eR^L\right)^{3/4} q^L}.$$

This, together with equation (2) with equality, determines q^H and q^L. Thus, in this equilibrium investment is efficient, but the level of trading is not.

Proof that Insurance Markets have no Role. We demonstrate that there is no role for markets that open before period 1 and pay wheat in period 1 contingent on an agent's type (assumed verifiable). For simplicity, let us focus on Case 3 of Proposition 1, where there is inefficiency in both investment and trade. (The result applies to the other cases of Proposition 1 as well.)

Insurance redistributes endowments between those who sell before they buy and those who buy before they sell. That is, insurance entails a financial advisor receiving a transfer θ, $-e \leq \theta \leq e$, from a secretary. (We expect $\theta > 0$ but in principle we could have $\theta < 0$.) Note that much of the proof of Lemma 1 applies for any θ: in particular, $p_f^H = 1$ and $x_f^H \geq x_f^L$. It is easy to rule out the case where $\theta = -e$, that is, financial advisors have no wealth ex post. To see this note that by the financial advisor budget constraint we would have $x_f^H = x_f^L = 0$; hence $x_s^H, x_s^L > 0$ by equations (3)–(5) and so $p_f^H, p_f^L > 0$ from equations (8) and (9); but then, by equations (6) and (7), $p_s^H = p_s^L = 0$; this implies that the marginal utility of wealth for a financial advisor is infinite and so an agent would prefer an insurance contract that gives him positive wealth if he is a financial advisor, that is $\theta > -e$.

So we know that a financial advisor will have positive wealth. In combination with $x_f^H \geq x_f^L$, this implies $x_f^H > 0$. Note that the rest of the proof of Lemma 1 applies as long as $x_f^H \geq 1$. Suppose $x_f^H < 1$. It follows from (3) and (5) that $x_f^L > 0$. In other words, financial advisors and secretaries both buy the high-state Arrow security. But then from (*) and (**) the marginal utility of wealth of financial advisors equals π/q_H while the marginal utility of wealth of secretaries is $\pi/q_H p_s^H$. There are two cases: $p_s^H < 1$ and $p_s^H = 1$. If $p_s^H < 1$ the marginal utility of wealth of financial advisors is higher, implying that each financial advisor will want the maximum redistribution of wealth. Thus in this case a necessary condition for equilibrium is $\theta = e$. But then a financial advisor's budget constraint in combination with $x_f^H \geq x_f^L$ implies $x_f^H \geq 1$, whereas we have assumed $x_f^H < 1$. This is a contradiction. Hence $p_s^H = 1$. But then we must have $x_f^H \geq 1$ from equation (6), which is again a contradiction.

We have established that $x_f^H \geq 1$, which implies $p_s^H = 1$ by equation (6). We may conclude that all of Lemma 1 holds: $p_f^H = p_s^H = 1$, $x_f^H \geq 1$.

Now turn to the low state. Consider the proof of Proposition 1.3. Nothing in the logic of the proof depends on the relative amounts of the financial advisor's or secretary's endowments, namely θ. In particular, the values of $p_s^L, p_f^L, x_s^L, x_f^L, q_H, q_L$ are pinned down by the conditions that both technologies are used and that financial advisors hold both securities. Furthermore, secretaries prefer the high-state security to the low-state one. It follows that the marginal utility of wealth of financial advisors, π/q_H, equals that of secretaries, $\pi/(q_H p_s^H)$ (since $p_s^H = 1$). Thus, it is an equilibrium for no agent to purchase insurance: $\theta = 0$. (There may be other equilibria but all are equivalent to one in which there is no insurance. If $\theta \neq 0$ financial advisors use their additional wealth to buy more high-state Arrow securities.)

Proof of Proposition 3. We know from the proof of Proposition 1.3 that the competitive equilibrium in the case $m = 0$ has the following features:

Financial advisors hold both securities, and so

$$\frac{\pi}{q^H} = \frac{1-\pi}{p_s^L q^L}; \tag{A.10}$$

both technologies are used and so

$$q^L = \frac{R^H - 1}{R^H - R^L} \text{ and } q^H = \frac{1 - R^L}{R^H - R^L}; \tag{A.11}$$

$$x_s^L = 0,\ p_s^L = \frac{1-\pi}{\pi}\frac{1-R^L}{R^H - 1} < 1. \tag{A.12}$$

Furthermore, it is easy to adapt the proof of Proposition 1.3 to show that (A.10)–(A.12) will hold in a competitive equilibrium when the government injects m, for m close to zero. The market clearing conditions (9) and (17) can be written as

$$\frac{x_f^L + m}{p_s^L} = \frac{p_s^L}{p_f^L}, \tag{A.13}$$

$$\frac{x_f^L + m}{p_f^L} = p_f^L, \tag{A.14}$$

which imply

$$p_f^L = (p_s^L)^{2/3}, x_f^L + m = (p_s^L)^{4/3}. \tag{A.15}$$

According to (A.12), the government injection of m does not affect p_s^L. Hence, from (A.15), p_f^L stays the same and so does $x_f^L + m$. We may conclude that there is a 100% crowding out of x_f^L by m.

The market clearing conditions, equations (3)–(5), imply

$$y^s = \frac{x_f^L - 2eR^L}{1 - R^L}, \tag{A.16}$$

$$y^r = \frac{2e - x_f^L}{R^L - 1}. \tag{A.17}$$

Hence, a fall in x_f^L leads to a decrease in y^s, an increase in y^r, and an increase in $x_f^H + x_s^H$.

In summary, we have

$$\frac{dx_f^L}{dm} = -1, \quad \frac{dy^s}{dm} = -\frac{1}{1 - R^L}, \quad \frac{dy^r}{dm} = \frac{1}{1 - R^L}. \tag{A.18}$$

Also from equation (14),

$$\frac{d\tau}{dm} = \frac{c}{2(1 - 2\tau)}. \tag{A.19}$$

To ascertain the effect of m on welfare we differentiate equation (18) with respect to m to obtain

$$\begin{aligned} \frac{dW}{dm} &= \pi \left[\frac{dy^s}{dm} + \frac{dy^r}{dm} R^H \right] + (1 - \pi) \left[-\frac{2}{c}(1 - \tau) \frac{d\tau}{dm} \right] \\ &= \pi \left[\frac{R^H - 1}{1 - R^L} \right] - (1 - \pi) \frac{(1 - \tau)}{(1 - 2\tau)}. \end{aligned} \tag{A.20}$$

Hence, the optimal choice of m is characterized by the first-order condition:

$$\frac{\pi}{(1 - \pi)} \left[\frac{R^H - 1}{1 - R^L} \right] \leq \frac{(1 - \tau)}{(1 - 2\tau)} \tag{A.21}$$

with equality if $m > 0$. Note that the left-hand side of (A.21) is strictly bigger than 1 given that the expected return of the risky asset strictly exceeds 1. Thus, (A.21) cannot be satisfied at $\tau = 0$. We may conclude that a government hand-out ($m > 0$) in the low state is welfare improving.

References

Allen, Franklin and Douglas Gale (2004). "Financial Intermediaries and Markets." *Econometrica*, 72, 1023–1061.

Barro, Robert J. (2006). "Rare Disasters and Asset Markets in the Twentieth Century." *Quarterly Journal of Economics*, 121, 823–866.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas (2008). "An Equilibrium Model of 'Global Imbalances' and Low Interest Rates." *American Economic Review*, 98, 358–393.

Caballero, Ricardo J. and Arvind Krishnamurthy (2009). "Global Imbalances and Financial Fragility." *American Economic Review Papers and Proceedings*, 99, 584–588.

Cocco, João F., Francisco J. Gomez, and Pascal J. Maenhout (2005). "Consumption and Portfolio Choice over the Life Cycle." *Review of Financial Studies*, 18, 491–533.

Dávila, Julio, Jay. H. Hong, Per Krusell, and José-Victor Ríos-Rull (2012). "Constrained Efficiency in the Neoclassical Growth Model with Uninsurable Idiosyncratic Shocks." *Econometrica*, 80, 2431–2467.

Debortoli, Davide and Ricardo Nunes (2013). "Lack of Commitment and the Level of Debt." *Journal of the European Economic Association*, 11, 1053–1078.

Farhi, Emmanuel, Mikhail Golosov, and Aleh Tsyvinski (2009). "A Theory of Liquidity and Regulation of Financial Intermediation." *Review of Economic Studies*, 76, 973–992.

Geanakoplos, John and Herakles Polemarchakis (1986). "Existence, Regularity and Constrained Suboptimality of Competitive Allocations When the Asset Market Is Incomplete." In *Essays in Honor of Kenneth Arrow*, edited by W. Heller, R. Starr, and D. Starrett. Cambridge University Press, Vol. 3, pp. 65–95.

Gromb, Denis and Dimitri Vayanos (2002). "Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs." *Journal of Financial Economics*, 66, 361–407.

Hart, Oliver (1975). "On the Optimality of Equilibrium when the Market Structure Is Incomplete." *Journal of Economic Theory*, 11, 418–443.

Hart, Oliver and Luigi Zingales (2011). "Inefficient Provision of Liquidity." NBER Working Paper No. 17299.

Hart, Oliver and Luigi Zingales (2014). "Banks Are Where The Liquidity Is." NBER Working Paper No. 20207.

He, Zhiguo and Péter Kondor (2012). "Inefficient Investment Waves." NBER Working Paper No. 18217.

Heaton, John and Deborah Lucas (1997). "Market Frictions, Savings Behavior, and Portfolio Choice." *Macroeconomic Dynamics*, 1, 76–101.

Holmstrom, Bengt and Jean Tirole (1998). "Private and Public Supply of Liquidity." *Journal of Political Economy*, 106, 1–40.

Holmstrom, Bengt and Jean Tirole (2011). "*Inside and Outside Liquidity*." MIT Press.

Jevons, W. Stanley (1876). "*Money and the Mechanism of Exchange*." D. Appleton and Co., New York.

Jeanne, Olivier and Anton Korinek (2013). "Macroprudential Regulation Versus Mopping Up After The Crash." NBER Working Paper No. 18675.

Kahle, Kathleen M. and René M. Stulz, (2013). "Access to Capital, Investment, and the Financial Crisis." *Journal of Financial Economics*, 110, 280–299.

Kehoe, Timothy J. and David K. Levine (1993). "Debt-Constrained Asset Markets." *Review of Economic Studies*, 60, 865–888.

Kiyotaki, Nobuhiro and Randall Wright (1989). "On Money as a Medium of Exchange." *Journal of Political Economy*, 97, 927–954.

Korinek, Anton (2012). "Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses." Working paper, University of Maryland.

Kydland, Finn and Edward C. Prescott (1977). "Rules Rather than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy*, 85, 473–492.

Lorenzoni, Guido (2008). "Inefficient Credit Booms." *Review of Economic Studies*, 75, 809–833.
Mian, Atif and Amir Sufi (2009). "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis." *Quarterly Journal of Economics*, 124, 1449–1496.
Mian, A. and Amir Sufi (2012). "What Explains High Unemployment? The Aggregate Demand Channel." Working paper, University of Chicago.
Song, Zheng, Kjetil Storesletten, and F. Zilibotti (2012). "Rotten Parents and Disciplined Children: A Politico-economic Theory of Public Expenditure and Debt." *Econometrica*, 80, 2785–2803.
Woodford, Michael (1990). "Debt as Private Liquidity." *American Economic Review*, 80, 382–388.
Zeldes, Stephen P. (1989). "Consumption and Liquidity Constraints: An Empirical Investigation." *Journal of Political Economy*, 97, 305–346.

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Online Appendix for "Liquidity and inefficient investment"