Electoral Competition with Fake News*

Gene M. Grossman
Princeton University

Elhanan Helpman
Harvard University

Current Draft: September 2022

Abstract

Misinformation pervades political competition. We introduce opportunities for political candidates and their media supporters to spread fake news about the policy environment and perhaps about parties' positions into a familiar model of electoral competition. In the baseline model with full information, the parties' positions converge to those that maximize aggregate welfare. When parties can broadcast fake news to audiences that disproportionately include their partisans, policy divergence and suboptimal outcomes can result. We study a sequence of models that impose progressively tighter constraints on false reporting and characterize situations that lead to divergence and a polarized electorate.

Keywords: policy formation, probabilistic voting, misinformation, polarization, fake news

JEL Classification: D78

*We thank Guido Tabellini and the late Alberto Alesina for helpful discussions and Benjamin Niswonger for research assistance. Grossman thanks CREI for its hospitality.

1 Introduction

> Misinformation on the part of the public makes for bad lawmaking on the part of the government.
>
> Keohane (2010)

Do Facts Matter?, Hochschild and Einstein (2015) ask in the title of their monograph on the role of misinformation in American politics that opens with the above quote. Do unwise policies result when politicians and the media spread false information and the citizenry accepts it unquestioningly when deciding how to vote? The authors use case studies to argue that "people's willingness to use mistaken factual claims in their voting and public engagement is ... dangerous to a democratic polity." (2015, p.14)

Here we address a similar question with game-theoretic tools. We take the well-known probabilistic voting model of Lindbeck and Weibull (1987), in which parties and candidates have no policy preferences, as our baseline environment. In this setting, there are two political parties that differ in exogenous ways and a population of heterogeneous voters that evaluates them differently. The parties compete by staking positions on a "pliable" policy issue that voters consider along with their assessments of the parties' fundamentals. In an equilibrium with accurately informed voters, the election delivers pliable policies that maximize aggregate welfare. But what if voters are ill informed and parties can compete by making false claims about the policy environment and the positions supported by themselves and their rivals? Under what circumstances will the potential for spreading "fake news" distort the parties' positions away from those that are socially desirable? Will the parties make competing claims that polarize the electorate or will they broadcast similar announcements? We ask these questions in a sequence of models with increasingly tight constraints on the scope for false reporting. First, we give the parties free rein to make claims both about a parameter that affects the desirability of alternative policies (i.e., the "state of the world") and about their and their rival's positions on the matter. The parties reach different audiences and have a greater chance of being heard by their own partisans. Next, we restrict the parties to announce their own position accurately, while still allowing false claims about the state of the world and about the rival's intentions. Finally, we suppose that voters learn both parties' actual positions, but still may be misled about the attractiveness of alternative policy options. In each case, we ask whether the parties converge or diverge in their positions and announcements and whether the fake news distorts the ultimate policy outcome.

While misinformation has long been a tool in political competition, recent trends have heightened concern about the spread of misleading or "fake" news. Social media and other internet outlets enable politicians and their allies to reach ever-larger, targeted audiences. Guess et al. (2018) estimate that one in four Americans visited a fake news website in the six weeks before the 2016 U.S. presidential election—where fake news is defined as the most extreme form of misleading information inasmuch as its content is intentionally and verifiably false. Allcott and Gentzkow (2017) document the increasing role that social media play as a source of political information and argue that "people who get news from Facebook (or other social media) are less likely to receive evidence about the true state of the world that would counter an ideologically aligned but false story" (p.221). Moreover, these authors and Silverman and Singer-Vine (2016) report survey evidence that many voters have difficulty distinguishing real and fake news and that many believe the false claims they encounter. Guess et al. (2018) provide evidence of selective exposure to fake news sources: Republican voters were more likely to receive news from pro-Trump sources than Democratic voters. This characteristic of the information technology features prominently in our modeling of the parties' strategic use of fake news.

We contribute to a small literature on the role of imperfect information in electoral competition. Early contributions by Baron (1994) and Grossman and Helpman (1996) feature informed and uninformed voters, with the latter responding mechanically to campaign spending financed by interest groups. Glaeser et al. (2005) explain "extremism" in policy positions in a model in which individuals vote only if the expected benefit exceeds an idiosyncratic voting cost and potential voters are more likely to learn the position of their affiliated party than the position of the nonaffiliated party. In this paper, we study formally the strategic use of misinformation in political competition.[1]

A larger literature addresses the role of the media in providing information to voters. Although we do not consider the media to be the only source of false and misleading information—especially now that politicians and political parties can readily use social media to make direct contact with voters—partisan media certainly do play a role in informing and misinforming the electorate.

[1] Misinformation plays a role not only in democratic politics, but also (or more so) in autocracries. See, for example, Martinez (2022), who estimates the overstatement of econonomic growth by autocratic rulers.

Strömberg (2015) and Prior (2013) provide excellent surveys of the theoretical and empirical literatures, respectively, on the role of the media in electoral politics.[2] Strömberg (2015) distinguishes models in which the media provides accurate information to uninformed voters from models that feature media bias. As examples of the former, Strömberg (2004) studies situations in which the media informs voters about candidates' policy positions, whereas Snyder and Strömberg (2010) consider information about the quality of government services that voters use to assess the expected competence of the incumbent. In both papers, and others like them, the information provided by the media serves to improve the selection of candidates, enhance political accountability, and raise expected welfare. In contrast, Bernhardt et al. (2008) and Chan and Suen (2008) model the political ramifications of biased news coverage. They begin with Zaller's (1992, p. 313) observation that the media lead voters to "hold opinions that they would not hold if aware of the best available information and analysis." Bernhardt et al. (2008) focus on information suppression that occurs when news outlets withhold negative information about the competence of the candidate they prefer. They show how this can lead to the election of inferior candidates. Chan and Suen (2008) model candidate endorsements by partisan newspapers. In their setting, voters may be confused by countervailing endorsements, but if readers choose their news sources based on their ideological positions, the partisan guidance can eliminate voter mistakes. In both of these settings featuring fully rational voters, the authors conclude that ideological media reinforces prior beliefs and polarizes the electorate.

Our model differs from these precedents inasmuch as we consider a more active role for the politicians in choosing policy positions and we model information that concerns both the positions taken and the conditions that determine the optimal policy response.[3] Perhaps the paper most similar to ours is a recent one by Wolton (2019). He posits two politicians with policy preferences and media outlets that also rank alternative policy outcomes. An incumbent chooses policy in the first period with knowledge of the state of the world and to reflect her own type, be it moderate

[2] A recent paper by Peisakhin and Rozenas (2018) provides fresh evidence that the media can impact voting behavior even when the source is conspicuously biased. They show that Russian television programming persuaded voters to adopt more pro-Russian attitudes and boosted overall support for pro-Russian parties in the 2014 Ukranian election.

[3] Matějka and Tabellini (2021) take a complementary approach. In a model of probabilistic voting similar to ours, they introduce rationally-inattentive voters who allocate costly efforts to absorb political news. Their voters actively choose the noiseness of the signals they receive, whereas ours are passive in their news acquistion. However, their candidates do not intentionally distort the signals they send, whereas strategic manipulation of information is the central focus of our analysis.

or extreme. The single, rational voter seeks to infer the incumbent's type from her action and also from (possibly) slanted reports released by the media. The reports by a biased media will impact the policy choices by the incumbent when electoral incentives are sufficiently strong. Compared to a world with unbiased media, the presence of a biased media can generate a better first-period policy outcome, but harm the electorate's choice between the candidates. Key differences between Wolton (2019) and this paper include our supposition that voters are not able or willing to make sophisticated, Bayesian inferences and our treatment of a large and heterogeneous electorate.

Two of our findings bear particular emphasis. First, we find that the effect of fake news on policy choices depends on the scope for misrepresentation—in a somewhat counterintuitive way. When parties can misrepresent broadly, including false reports about their own intentions, they can "speak" separately to their two audiences. To those that obtain their information from biased sources, the parties misrepresent both the state of the policy environment and their own intended response. This leaves them free to choose an optimal response (in view of the actual state of the world) to attract votes from those that are fully informed. In contrast, when the scope for misrepresentation is more narrow—extending only to reports about the policy environment and possibly to claims about a rival's intentions, but not to false allegations about own positions—then the parties must speak to two audiences at once. To attract votes from the well-informed, they prefer an appropriate response to the true policy environment. But to appeal to those to whom they have spread false information, they lean to policies that would be appropriate for the fictitious states. In short, policy choices need not be more distorted when the scope for misrepresentation is greater.

Second, we find that a polarized electorate can result when the fraction of misinformed voters is intermediate, but not when it is extremely high or extremely low. In the extreme cases, the parties have incentive to woo the same group of voters, be they the well-informed or the misinformed. The announcements that are strategically advantageous to the two parties are similar in these cases. In contrast, when both well-informed and uninformed voters are well represented in the voting population, the parties' best responses might diverge, especially when voters' sourcing habits are asymmetric. The party that attracts a greater share of listeners among its uninformed partisans adopts an electoral strategy that relies heavily on fake news. It issues an extreme report about the state of the world and a position that is appropriate for that false state. In these circumstances, the rival party might respond by appealing more to the informed voters. With a position far from that of its rival, it may lead its uninformed audience to believe something very different about the state of the world.

2 A Model of Electoral Competition with Fake News

Two political parties, L and R, vie for support. The parties differ in some exogenously-given ways that appeal differently to the heterogeneous voters. Voters also care about a "pliable" policy issue that will be contested in the election and about which the parties hold no preferences. The parties use their positions and broadcasts on this issue instrumentally to woo voters. Individuals may have access to disparate information and hold different beliefs about the intentions of the two parties with regard to the pliable policy and about the state of the policy environment.

Specifically, consider a voter i who believes that party L would carry out the pliable policy m_i^L, that party R would carry out the pliable policy m_i^R, and that the state of world is characterized by the parameter θ_i, a scalar that impacts her assessment of the alternative policy options. This voter casts her ballot for party L if and only if

$$u\left(m_i^L, \theta_i\right) - u\left(m_i^R, \theta_i\right) \geq \eta_i,$$

where η_i reflects her relative preference for party R based on the fundamental differences between the parties. In the unit mass of voters, the preference parameter η_i is drawn from a well-behaved cumulative distribution function, $F(\eta)$, with a support that includes both positive and negative values.

The utility function $u(m, \theta)$ is increasing in θ, concave in m, twice continuously differentiable, and supermodular.[4] Moreover, the value of m that maximizes $u(m, \theta)$ is finite for all θ in the feasible range, $\left[\underline{\theta}, \overline{\theta}\right]$. It follows that the value of m that maximizes $u(m, \theta)$ is increasing in θ and lies in the bounded range $[\underline{m}, \overline{m}]$, where $\underline{m} \equiv \arg\max_m u(m, \underline{\theta})$ and $\overline{m} \equiv \arg\max_m \left(m, \overline{\theta}\right)$. We assume that $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$, $m_i^L \in [\underline{m}, \overline{m}]$ and $m_i^R \in [\underline{m}, \overline{m}]$ for all i; i.e., voters believe that θ falls within the feasible range and that the position of each party corresponds to one that is optimal for

[4] The assumption that $u(m, \theta)$ is increasing in θ is little more than an ordering convention that assigns higher indexes to those states that voters prefer. The substantive assumption is that of supermodularity, which implies $u_{m\theta}(\theta, m) > 0$; i.e., higher values of the policy are especially vaulable in higher-indexed states.

some feasible state. We also assume that for any feasible combination of θ_i, m_i^L, and m_i^R, there exist values of η_i in the support of $F(\eta)$ such that i prefers R and values of η_i such that i prefers L.

Voters access news from disparate sources. A fraction λ^I of the populace receives accurate information.[5] These voters know the true value of θ, which is $\theta^I \in (\underline{\theta}, \overline{\theta})$, and the true positions and intentions of the parties, m^L and m^R. The remaining voters form their impressions based on reports from biased sources. These sources may include partisan media outlets or announcements (e.g., "tweets") issued directly by the politicians themselves. The messages assert the state of the policy environment and possibly the positions of one or both of the parties. These assertions might bear no relationship to the truth. We refer to such misinformation broadly as "fake news."

Voters choose their news source non-strategically, but differ in their listening habits based on their ideological proclivities. In particular, a voter who prefers party R on ideological grounds and who does not have access to reliable information is more likely to tune in to a media source that is partial to party R than is another voter who fundamentally prefers party L. Letting $\pi(\eta)$ denote the probability that a voter with $\eta_i = \eta$ who accesses fake news hears the reports of a source that is aligned with party R, we assume $\pi'(\eta) > 0$. The uninformed voters take what they hear at face value; if their partisan information source reports, for example, that $\theta = \tilde{\theta}$, then they use this value in assessing the (perceived) policy positions.

Two aspects of the assumed voter behavior bear further discussion. First, we treat the uninformed voters as naïve or, one might even say, "gullible." This is in keeping with one prominent strand of the political science literature that traces back at least to the influential report by Berelson et al. (1954). The report concludes (p.308) that "[t]he democratic citizen is expected to be well informed about political affairs. He is supposed to know what the issues are, what their history is, what the relevant facts are, what alternatives are proposed, what the party stands for, what the likely consequences are. By such standards, the voter falls short." This assessment is shared by many subsequent authors.[6] A failure to collect or process information need not be a sign of

[5] In our calculations, we take the population of informed voters to be representative of the overall electorate. It would make no difference to our conclusions, however, if the probability of an individual being fully informed varied with her political leanings; i.e., if λ_i^I were a function of η_i. The parties have similar incentives to cater to the fully informed with their choices of pliable policies no matter what is the composition of this group. We would need only to calculate the density of voters with every partisan leanings in the *group of uninformed voters*, instead of the density in the population as a whole.

[6] For example, Campbell et al. (1960, p. 170) conclude that "many people know the existence of few if any of

"irrationality" inasmuch as information acquisition is costly and voters may realize their voting behavior only trivially affects the expected policy outcome.[7] Of course, a lack of information at the individual level need not imply poor decision-making by the electorate as a whole, especially if voters are numerous and draw sophisticated inferences from the noisy signals they manage to observe. But systematic and correlated errors in the signals impedes information aggregation in many situations. Krosnick and Brannon (1993) uncover significant differences between informed and uninformed citizens in their susceptibility to priming. Bartels (1996) finds, in his study of six U.S. presidential elections, that vote probabilities differ substantially and systematically from what would be expected of fully informed voters, and that these systematic errors, although diluted, are not eliminated by aggregation. Our assumption of unquestioning acceptance of biased news is, of course, an extreme one, but the studies we cited in the introduction provide evidence that fake news has had powerful effects on followers' beliefs and the extreme assumption allows us to capture this reality in a simple if stylized way.

Second, we treat individuals as entirely passive in their choice of media. The available evidence suggests that voters do gravitate to outlets that share their political leanings, but that other selection criterion, such as entertainment value and the usefulness of local news, also play a role (see Prior, 2015, p.109-110).[8] We capture this reality in reduced form by $\pi(\eta)$, whereby an individual's partisanship is positively correlated with her listening and reading behavior, but not perfectly so. We note that our model imposes little structure on the function $\pi(\eta)$, except that it is increasing and, sometimes, that does not satisfy $F'(\eta)\pi(\eta) = F\prime(-\eta)[1-\pi(-\eta)]$ for all η. When $F'(\eta)\pi(\eta) = F\prime(-\eta)[1-\pi(-\eta)]$ for all η, the number of voters with a preference for party R of size η that receives their news from an R-leaning outlet matches the number with this same size of preference for party L that obtains their news from an L-leaning outlet. We will sometimes

the major issues of policy." More recently, Bernhardt et al. (2008) cite evidence from the Harris Poll that many Americans held mistaken beliefs about the facts surrounding the Iraq war. Stantcheva (2020) documents widespread factual errors and faulty reasoning by Americans surveyed about tax policy, health policy, trade policy, and monetary policy.

[7] Caplin (2008) developed a theory to explain votes for candidates and parties that support sub-optimal policies with an appeal to the tiny probability that an individual voter will be pivotal in a large election. Caplin's voters are willing to indulge a biased world view, because the cost of doing so is so small.

[8] Coe et al. (2008), Hollander (2008), Jamieson and Cappella (2008), Garrett (2009), Stroud (2011), and Martin and Yurukoglu (2017) all report evidence of partisan selectivity in choice of cable television networks. Gentzhow and Shapiro (2010) find evidence of sorting by partisan leaning in newspaper readership and Gentzkow and Shaprio (2011) finds similarly for online media usage. Iyengar and Hahn (2009) tested for selectivity experimentally, by exposing partisans to news headlines with a random medium logo attached. Participants were more likely to want to read a story when the partisan leanings of the alleged source matched their own.

be interested in functions that do not exhibit this form of strong symmetry. Among the functions that satisfy our requirements for $\pi(\eta)$ are ones that closely approximate a situation in which each voter seeks out the source that aligns most closely with her own partisan predisposition.[9]

It is straightforward to construct examples that fit our framework based on recent policy controversies. For example, the policy m might represent the number of *immigrants* that are admitted into a country, while θ affects (inversely) the social and economic cost of absorbing immigrants. Then $u_{m\theta} > 0$ applies if the optimal number of immigrants increases as the cost of absorption falls (see appendix). News outlets might exaggerate the cost of immigration in one direction or the other, while perhaps also misrepresenting the parties' positions on the matter. Or the policy m might represent the size of a *tariff* on imports, while θ represents (inversely) the induced foreign price. The optimal tariff is greater when exporters "pay for the tariff"; i.e., when the induced foreign price is low. In this case, the media might exaggerate the pass-through of tariffs to domestic prices and possibly the parties' openness to trade.[10]

2.1 The Parties' Objectives and Actions

Each party (or its media surrogates) seeks to maximize its share of the aggregate vote.[11] In the first two stages of the game, the parties adopt positions m^R and m^L on the pliable policy issue. In the main body of the paper, we assume that the parties make these choices sequentially, with the incumbent party R moving first, and the challenger party L responding subsequently. This specification facilitates our analysis in Section 5 of the case when the parties must report the policy positions truthfully, because (as we show in the appendix) no pure-strategy equilibrium exists for the simultaneous move game in this setting. We maintain the assumption of sequential choices throughout the main text for reasons of consistency. However, this assumption has no bearing on our qualitative results in Sections 3 and 4, as we show in the appendix by characterizing the

[9] See the online appendix for an example of such a $\pi(\eta)$ function.

[10] In our model, voters' preferences concerning the pliable party are assumed to be independent of their partisan leanings. This assumption might be violated if the pliable policy has a clear, left-right orientation. For example, voters that favor gun control and abortion rights might also tend to favor progressive taxation, even though these policies are distinct and unrelated. Our model fits best for policies that do not fit readily on a left-right spectrum, such as trade protection or pork-barrel spending projects.

[11] Some research has addressed the objectives of the media and the reasons for media bias. Anderson and McLaren (2012) study owners' political motives whereas Baron (2006) considers journalists' interest in career advancement. Maximizing readership or profits may also induce bias, as in Mullainathan and Shleifer (2005) and Gentzkow and Shapiro (2010), where media have incentives to slant coverage toward their audiences' predispositions.

outcomes of games with simultaneous choices of m^R and m^L.

In the third stage of the game, each party issues its claims about the policy positions that have been adopted. Party J alleges its own position to be m^{JJ} and that of its rival to be $m^{J\widetilde{J}}$, where $\widetilde{J}$ denotes the party that is not J. The parties might feel constrained in these latter pronouncements, either because voters have ready access to accurate information of this sort or because the parties perceive a reputational cost from misrepresenting their own positions. In order to understand how such considerations affect the prospects for polarization and suboptimal policies, we proceed to analyze three cases with increasingly tighter reporting constraints. First, we allow party J to claim any position $m^{JJ} \in [\underline{m}, \overline{m}]$ as its own and any $m^{J\widetilde{J}} \in [\underline{m}, \overline{m}]$ as its rival's intention. Then, we suppose that each party must report its own position accurately, but can misrepresent that of its rival. Finally, we constrain all reports of policy positions to be truthful, while still allowing misrepresentation of the policy environment. In any case, we assume that the parties simultaneously make claims about policy environment in a final stage of the game. Party J reports the state of the world as θ^J, for $J = L, R$, with free rein to announce any $\theta^J \in \left[\underline{\theta}, \overline{\theta}\right]$.

Given the announcements that have been made (whether some are constrained to be truthful or not), we can compute the parties' implied vote shares. The support for party L comprises (i) those among the fully-informed that prefer m^L to m^R with the knowledge that $\theta = \theta^I$, (ii) those among the uninformed voters that obtain their news from a source that favors party L and that prefer m^{LL} to m^{LR} under the (possibly mistaken) belief that $\theta = \theta^L$, and (iii) those among the uninformed voters that obtain their news from a source that favors party R and that prefer m^{RL} to m^{RR} under the belief that $\theta = \theta^R$. Summing these components, we have

$$\begin{aligned} s^L = {} & \lambda^I F\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] \\ & + \left(1 - \lambda^I\right) \int_{-\infty}^{u\left(m^{LL}, \theta^L\right) - u\left(m^{LR}, \theta^L\right)} \left[1 - \pi\left(\eta\right)\right] dF\left(\eta\right) \\ & + \left(1 - \lambda^I\right) \int_{-\infty}^{u\left(m^{RL}, \theta^R\right) - u\left(m^{RR}, \theta^R\right)} \pi\left(\eta\right) dF\left(\eta\right) \end{aligned} \tag{1}$$

where s^L is the vote share for party L and $s^R = 1 - s^L$ is the remaining vote share.[12] Here, the first

[12] Note that s^L is deterministic in this setting with a continuum of voters. As is well known, it would be straightforward to add a valence shock reflecting the uncertain popularity of each party at a moment in time in order to make s^L random and thus leave the parties with a real electoral contest. With these types of valence shocks, we would assume that parties maximize their probabilities of capturing more than half of the votes. These yields maximization

term on the right-hand side of (1) gives the fraction of the λ^I knowledgeable voters that prefers m^L to m^R in (true) state θ^I, in view of the baseline preferences for the two parties. The second term gives the fraction of the $1-\lambda^I$ uninformed voters that, with probability $1-\pi(\eta)$, obtain their reports from an L-leaning source and decide to vote for party L in view of their induced belief that $m^L = m^{LL}$, $m^R = m^{LR}$, and $\theta = \theta^L$. The third term gives the fraction of the $1-\lambda^I$ uninformed voters that, with probability $\pi(\eta)$, obtain their news from an R-leaning source and decide nonetheless to vote for party L after hearing that $m^L = m^{RL}$, $m^R = m^{RR}$, and $\theta = \theta^R$.

2.2 The Full-Information Benchmark

As a benchmark, we recall the outcome of electoral competition with complete and accurate information; see, for example, Lindbeck and Weibull (1987). The full-information benchmark is captured in our model by the special case with $\lambda^I = 1$. In this case, each party J has a dominant strategy to adopt the policy that appeals most to the fully-informed voters; i.e., $m^J = m^I \equiv \arg\max_m u\left(m, \theta^I\right)$, for $J = L, R$. With full information, the positions converge to the policy that maximizes the representative voter's welfare given the true state of the world.

2.3 Reports about the Policy Environment

As we have indicated, the parties or their media representatives report about the state of the policy environment in the final stage of the political game.[13] The incentives to report about θ at this stage are common to the settings with and without fake news about policy positions. We consider these incentives now.

To this end, suppose that the parties have staked the positions m^L and m^R, and that the audience for the broadcasts by party J believe these positions to be m^{JJ} and $m^{J\tilde{J}}$ for $J = L, R$. Given these beliefs, the announcement about θ by party J can only affect voting by those that receive their news from the J-leaning source. Party J makes itself most appealing to this group of

of s^L and $1-s^L$, respectively, by parties L and R, as Lindbeck and Weibull (1987) have shown.

[13] It makes no difference whether the parties issue their reports about the state of the world after their announcements about the policy positions or at the same time as their announcements about these positions. A party's report about the positions fully determines its report about the state of the world and neither report is influenced by the rival's announcements.

voters by claiming that $\theta = \theta^J$, where

$$\theta^J = \arg \max_{\theta \in [\underline{\theta}, \overline{\theta}]} u\left(m^{JJ}, \theta\right) - u\left(m^{J\widetilde{J}}, \theta\right) \ , J = L, R.$$

The supermodularity of $u(\cdot)$ then implies

$$\theta^J = \begin{cases} \overline{\theta} & \text{for } m^{JJ} > m^{J\widetilde{J}} \\ \underline{\theta} & \text{for } m^{JJ} < m^{J\widetilde{J}} \end{cases} , J = L, R. \tag{2}$$

Evidently, both parties issue *extreme pronouncements* about the state of the world. If the subscribers to party J's broadcasts believe that party J will invoke a policy greater than that of its rival, the party wants its audience to believe that high values of m are maximally beneficial to their utility. If the audience believes the opposite to be true about the ordering of the parties' positions, then it wants its listeners to believe that low values of m are best. When talk is cheap, exaggeration reins supreme.

3 Unconstrained Reporting of Policy Positions

In this section, we suppose that parties (or their media allies) can claim whatever they like about the two policy positions when broadcasting to their respective audiences. Those that hear the news reported by party R or its surrogates will vote based on a comparison of $u\left(m^{RR}, \theta^R\right)$ and $u\left(m^{RL}, \theta^R\right)$. Clearly, this comparison is independent of the position actually taken by party L and the news broadcast by that party. Similarly, those that receive their news from party L compare $u\left(m^{LL}, \theta^L\right)$ to $u\left(m^{LR}, \theta^L\right)$, which is independent of the position and announcements of party R. Evidently, the outcome is the same whether the parties stake their positions sequentially (as we assume in the main text) or simultaneously (as we also consider in the appendix).

When broadcasting its news, party J seeks to maximize its appeal to those that hear and accept its claims, while also trying to make its rival look maximally repugnant. With these objectives in mind, party J faces a choice between two extremes. It might announce its own position to be the one most preferred when the state of the world is as high as possible while claiming that the rival intends an abhorrent policy at the opposite end of the spectrum, or it might announce its own position to be the one most preferred when the state of the world is as low as possible while claiming that its rival supports the policy at the uppermost end of the spectrum. Among these alternatives, it chooses the one that creates the greatest gap in audience perceptions. Note that both parties face the same incentives in this regard: if $u\left(\overline{m},\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)>u\left(\underline{m},\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)$, then each maximizes the perception gap by claiming the highest possible value of θ along with its own alleged support for the policy that goes along with that state and the ill-advised intentions of the rival to enact the lowest credible policy level; otherwise they each choose the opposite extreme announcements. Thus

$$\left(\theta^J,m^{JJ},m^{J\tilde{J}}\right)=\begin{cases}\left(\overline{\theta},\overline{m},\underline{m}\right) & \text{for } u\left(\overline{m},\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)>u\left(\underline{m},\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\\ \left(\underline{\theta},\underline{m},\overline{m}\right) & \text{for } u\left(\overline{m},\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)<u\left(\underline{m},\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\end{cases},\ \text{for } J=L,R.$$

What positions do the parties adopt in the initial stages of the electoral game? In the current setting, these choices affect only the behavior of knowledgeable voters. Given the prior choice of m^R, party L chooses m^L to maximize appeal among those who understand the state of the world to be θ^I, i.e., to maximize $u\left(m^L,\theta^I\right)-u\left(m^R,\theta^I\right)$. It has a dominant strategy to choose m^I. Anticipating this response, party R perceives the same dominant strategy. In short, the parties converge on the pliable policy that is socially optimal, much as in the setting with complete and accurate information.

We summarize in

Proposition 1 *Suppose that the parties choose their actual positions sequentially and that each party is subsequently unconstrained in its reports about the state of the world, its own policy position, and that of its rival. Then the policy positions converge to those that maximize utility given the actual state of the world ($m^L=m^R=m^I$) and the announcements converge to whichever extreme offers the greatest perception gap; either $\left(\theta^L,m^{LL},m^{LR}\right)=\left(\theta^R,m^{RR},m^{RL}\right)=\left(\overline{\theta},\overline{m},\underline{m}\right)$ or $\left(\theta^L,m^{LL},m^{LR}\right)=\left(\theta^R,m^{RR},m^{RL}\right)=\left(\underline{\theta},\underline{m},\overline{m}\right)$.*

4 Accurate Reporting of Own Positions

Now suppose that each party feels compelled to report its own intentions accurately, while it takes leeway in misrepresenting the position of its rival. As we observed in Section 2.3, the parties should

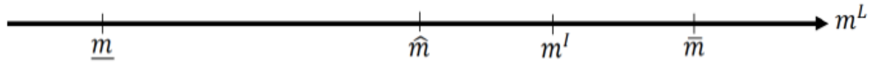


Figure 1: Choosing m^L with Accurate Reporting of Own Position

expect that reports about the state of the world to be extreme. If $m^{J\tilde{J}} < m^J$, party J will announce that $\theta = \overline{\theta}$, whereas if $m^{J\tilde{J}} > m^J$, it will announce that $\theta = \underline{\theta}$. Anticipating such an announcement, party J no longer wishes to choose the utility-maximizing policy position, m^I. On the one hand, such a position would capture the greatest support among knowledgeable voters, who recognize its optimality in view of the objective conditions of the policy environment. On the other hand, such a position would not appeal to those who are (mis)led to believe that θ is extreme. When choosing its position, each party trades off the marginal appeal to knowledgeable voters against the marginal attraction to the audience for its fake news. Given the sequential ordering of the moves, the incumbent party R must also take into account the effect that its choice will have on the response by party L.

To identify the parties' equilibrium choices, we must first anticipate what false report each will issue about the other's position after m^R and m^L have been chosen. Suppose party J has adopted a position of m^J. If m^J is close to $\overline{m}$, party J makes itself maximally attractive to its audience by reporting $m^{J\widetilde{J}} = \underline{m}$ and later that $\theta = \overline{\theta}$. Alternatively, if m^J is close to $\underline{m}$ the party makes itself seem attractive by reporting $m^{J\widetilde{J}} = \overline{m}$ and then that $\theta = \underline{\theta}$. In general, there is an intermediate value of m, say $\hat{m}$, such that if and only if $m^J > \hat{m}$, party J prefers to announce $m^{J\widetilde{J}} = \underline{m}$ and $\theta^J = \overline{\theta}$ to the alternative of announcing $m^{J\widetilde{J}} = \overline{m}$ and $\theta^J = \underline{\theta}$. The tipping point, $\hat{m}$, is defined by

$$u\left(\hat{m}, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right) = u\left(\hat{m}, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right). \tag{3}$$

Notice that the fake news reports might concur or clash. If both parties have adopted positions on the same side of $\hat{m}$, both will issue the same (false) report about their rival's stance and follow with the same biased report about the policy environment. Alternatively, if the two parties adopt positions on opposite sides of $\hat{m}$, then the parties will issue divergent reports about their rival's intentions and then opposite extreme reports about the state of the world.

We turn to the parties' policy positions, beginning with the challenger party L, which takes its rival's position m^R as pre-committed.[14] As we indicated before, the party faces a trade-off between appealing to informed voters and catering to the audience for its fake news. Consider Figure 1, which shows the extreme policy positions, $\underline{m}$ and $\bar{m}$, as well as the cutoff $\hat{m}$ and the policy m^I that maximizes welfare when $\theta = \theta^I$. For illustrative purposes, we have placed m^I to the right of $\hat{m}$, but analogous arguments would apply in the reverse case. Suppose party L contemplates taking *any* position between $\hat{m}$ and m^I. In the event, a rightward shift of its position would appeal to both informed voters, who prefer a policy of m^I, and to the uninformed voters that hear their news from an L-leaning source, who are misled to think they want $\bar{m}$. The rightward shift captures votes from both groups, without sacrificing any support among voters that hear their reports from party R. Therefore, the optimal response to any m^R cannot fall in the range $\left[\hat{m}, m^I\right]$. Nor can the optimal response be at the extreme where $m^L = \bar{m}$, because a leftward deviation from such a position wins support from some informed voters while sacrificing only very few votes among the uninformed.[15] Evidently, s^L reaches a (local) maximum somewhere between m^I and $\bar{m}$.

This local maximum is one candidate for party L's best response to m^R. Another possibility lies to the left of $\hat{m}$. If party L reduces its position below $\hat{m}$, it gains votes among the uninformed who get their news from an L-leaning source and who will be told, in such circumstances, that $\theta = \underline{\theta}$. But a move leftward from $\hat{m}$ would cost votes among the informed, who would see the shift as unattractive in view of their favorite policy of m^I. If the former effect dominates, then a leftward shift from $\hat{m}$ would win the party votes relative to announcement of $m^L = \hat{m}$. In this case, a second local maximum of s^L would exist between $\underline{m}$ and $\hat{m}$. It would not be optimal, however, for party L to stake an extreme position at $\underline{m}$, because a small increase from there would win support among the informed while losing only a negligible number of votes among the uninformed.

Irrespective of whether the best response by party L is to announce a policy strictly between m^I and $\bar{m}$ or one strictly between $\underline{m}$ and $\hat{m}$, the position must satisfy a first-order condition that balances marginal gains and losses from a small deviation. Using the first line of (1), we see that a small deviation of dm^L from any m^L alters s^L by $\lambda^I f\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] u_m\left(m^L, \theta^I\right) dm^L$ via the influence on informed voters, where $f(\eta) \equiv F'(\eta)$ is the density of voters with a predis-

[14] In the appendix, we consider the game in which the parties stake their positions simultaneoulsy. Note that the simultaneous-move and sequential-move games are the same when $F(\eta)$ is uniform, because in that case (as we shall see) each party has a dominant strategy in its choice of position on the pliable issue.

[15] Since $u\left(m, \bar{\theta}\right)$ is maximized at $\bar{m}$, $u_m\left(\bar{m}, \bar{\theta}\right) = 0$. Although a small leftward shift in m from $\bar{m}$ is unattractive to those that belief $\theta = \bar{\theta}$, the perceived loss in utility is very small and therefore so is the resulting shift in votes.

position to party R of η. Here, $\lambda^I f\left[u\left(m^L,\theta^I\right)-u\left(m^R,\theta^I\right)\right]$ gives the number of informed voters on the margin between voting for L and voting for R when the positions are m^L and m^R, and $u_m\left(m^L,\theta^I\right)dm^L$ reflects the assessment of the policy change among those that know the state to be θ^I. As for the impact on uninformed voters that get their news from a source aligned with party L, the marginal effect can be calculated using the second line of (1). The party reports its own intentions accurately as $m^{LL}=m^L$ while misrepresenting the rival's position as m^{LR} and the state of the world as θ^L. A small change in m^L increases the party's vote share by $\left(1-\lambda^I\right)f\left[u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)\right]\left\{1-\pi\left[u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)\right]\right\}u_m\left(m^L,\theta^L\right)dm^L$, considering the fraction of uninformed voters that are nearly indifferent between their two choices. Here, $\left(1-\lambda^I\right)f\left[u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)\right]$ is the number of swing voters who are indifferent between the parties when they believe that R will invoke a policy m^{LR} in the state θ^L, while $1-\pi\left[u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)\right]$ is the fraction of such voters that receive their news from an L-leaning source. The vote-maximizing position for party L balances the marginal gains and losses among the two groups of voters, and thus satisfies the first-order condition,

$$\frac{\partial s^L\left(m^L,m^R\right)}{\partial m^L}=\lambda^I f\left[u\left(m^L,\theta^I\right)-u\left(m^R,\theta^I\right)\right]u_m\left(m^L,\theta^I\right)+$$
$$\left(1-\lambda^I\right)f\left[u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)\right]\left\{1-\pi\left[u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)\right]\right\}u_m\left(m^L,\theta^L\right)=0\ . \quad (4)$$

Notice that the optimal choice of m^L depends on m^R, unless partisan preferences are uniformly distributed so that $f(\eta)$ is a constant. This is because the number of swing voters that are ready to switch sides if they see a more favorable choice of m^L depends on m^R.

Now consider the problem facing party R, which moves by choosing m^R to minimize s^L, anticipating in the process the best response by party L. For a marginal change in m^R, we have

$$\frac{ds^L\left(m^L,m^R\right)}{dm^L}=\frac{\partial s^L\left(m^L,m^R\right)}{\partial m^R}+\frac{\partial s^L\left(m^L,m^R\right)}{\partial m^L}\frac{dm^L}{dm^R}. \quad (5)$$

Note, however, that (4) implies that the second term in (5) vanishes, so the first-order condition for party R,

$$\frac{\partial s^L\left(m^L,m^R\right)}{\partial m^R}=\lambda^I f\left[u\left(m^R,\theta^I\right)-u\left(m^L,\theta^I\right)\right]u_m\left(m^R,\theta^I\right)+$$
$$\left(1-\lambda^I\right)f\left[u\left(m^{RL},\theta^R\right)-u\left(m^R,\theta^R\right)\right]\pi\left[u\left(m^{RL},\theta^R\right)-u\left(m^R,\theta^R\right)\right]u_m\left(m^R,\theta^R\right)=0\ , \quad (6)$$

applies locally at its optimal policy choice, m^R. Equation (4) might have a unique solution, which will be true if $s^L\left(m,m^R\right)$ is increasing in m to the left of $\hat{m}$. In case (4) has more than one solution for a given m^R, the local first-order condition must hold for each such solution. Moreover, $\max_m s^L\left(m,m^R\right)$ is a continuous function of m^R. It follows by arguments similar to those that apply for party L, neither $m^R=\underline{m}$, nor $m^R=\overline{m}$, nor $m^R=\hat{m}$ can be an optimal choice for party R. It follows that $m^R\in(\underline{m},\hat{m})\cup(\hat{m},\overline{m})$. In short, m^J is interior, and either $m^J\in(\underline{m},\hat{m})$ or $m^J\in(\hat{m},\overline{m})$ for both $J=L$ and $J=R$.

Will the parties' positions converge, as they do with a fully-informed electorate or with unconstrained fake news? This question boils down to whether (4) and (6) can both be satisfied for a common value of m. Note that $m^L=m^R$ implies that both positions fall on the same side of $\hat{m}$, so that $\theta^L=\theta^R$ and $m^{LR}=m^{RL}$; i.e., when the actual positions coincide, the parties' reports about the state of the world and their rival's intentions will converge as well. With this understanding in mind, we see that (4) and (6) *cannot* both be satisfied for a common value of m if $f(\eta)[1-\pi(\eta)]\neq f(-\eta)\pi(-\eta)$ for all η.[16] Policy divergence emerges whenever the parties face asymmetric incentives to cater to their respective audiences. If $\pi(\eta)\neq 1-\pi(-\eta)$, it means that the fraction of voters who favor party R by an amount η that receive their news from an R-leaning source differs from the fraction of voters who favor L by that same amount who obtain their news from an L-leaning source. If $f(\eta)\neq f(-\eta)$, it means that the density of voters that leans to party R by an amount η differs from the density that leans to party L by that amount. If either of these two situations arises, then the parties will face different trade-offs when staking their policy positions unless the two sources of asymmetry happen to just offset one another. In

[16] If the parties adopt similar positions, the first terms in equations (4) and (6) are the same. Also, since $m^L=m^R$ implies $\theta^L=\theta^R$, $u_m\left(m^L,\theta^L\right)=u_m\left(m^R,\theta^R\right)$ at any common value of m. But $u_m\left(m^L,\theta^L\right)$ is weighted in (4) by $\left(1-\lambda^I\right)f\left[u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)\right]\left\{1-\pi\left[u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)\right]\right\}$, whereas $u_m\left(m^R,\theta^R\right)$ is weighted in (6) by $\left(1-\lambda^I\right)f\left[u\left(m^{RL},\theta^R\right)-u\left(m^R,\theta^R\right)\right]\pi\left[u\left(m^{RL},\theta^R\right)-u\left(m^R,\theta^R\right)\right]$. If $f(\eta)[1-\pi(\eta)]$ differs from $f(-\eta)\pi(-\eta)$ for all η, the weights cannot be the same for a common value of m and thus the two first-order conditions cannot both be satisfied with $m^L=m^R$. See the appendix for more details.

short, any asymmetry in the numbers of partisan voters in the parties' respective news audiences breeds divergent policy positions.

Now suppose that $f(\eta) = f(-\eta)$ and $\pi(-\eta) = 1 - \pi(\eta)$ for all η in the support of $F(\eta)$. In this case, party L can guarantee itself a share s^* of the votes by mimicking its rival's prior policy choice, where from (1), $s^* = \lambda F(0) + (1-\lambda)\int_{-\infty}^{\infty} [1 - \pi(\eta)] f(\eta)\, d\eta$.[17] In the appendix we show that there exists an $\tilde{m}$ that is one of the (at most, two) values of m that satisfies both local first-order conditions, (4) and (6), such that if party R chooses $m^R = \tilde{m}$, party L will follow suite. Since $m^R = \tilde{m}$ secures a fraction $1 - s^*$ of the votes for party R, and the party can fare no better than this, $\left(m^L, m^R\right) = (\tilde{m}, \tilde{m})$ constitutes an equilibrium in the sequential game.

Next, we are interested in the conditions for polarization; i.e., conditions under which the parties' equilibrium positions fall on opposite sides of $\hat{m}$ so that their extreme reports about the policy environment generate antithetical views of the world. The optimal strategy for each party clearly depends on the fraction of knowledgeable voters. When λ^I is very small, for example, $u\left(m, \theta^I\right)$ plays little role in a party' choice of platform. Instead, the parties cater to their respective audiences by adopting positions close to $\underline{m}$ or close to $\overline{m}$, according to whether $u(\underline{m}, \underline{\theta}) - u(\overline{m}, \underline{\theta})$ is larger or smaller than $u\left(\overline{m}, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)$. In this case, the parties choose nearly the same positions, both on the same side of $\hat{m}$. Consequently, they both broadcast the same false news reports. At the other extreme, when λ^I is close to one, both parties cater mostly to well-informed voters. In this case, both stake positions close to m^I. If $m^I > \hat{m}$, then each party J reports $\theta^J = \overline{\theta}$ and $m^{J\tilde{J}} = \underline{m}$, whereas if $m^I < \hat{m}$, each reports $\theta^J = \underline{\theta}$ and $m^{J\tilde{J}} = \overline{m}$. Again, the fake news reports coincide. It follows that polarization can occur only for *intermediate values* of λ^I.

Figure 2 illustrates an outcome with polarization. In this example, partisanship is uniformly distributed between some $\eta_{\min}$ and $\eta_{\max}$. Recall that a uniform distribution with constant $f(\eta) = 1/(\eta_{\max} - \eta_{\min})$ implies that each party has a dominant strategy; its vote-maximizing platform does not vary with the position of its rival. The figure is drawn for a case with $\lambda^I = 0.4$, a setting in which sixty percent of the electorate is susceptible to fake news. We take $u(m, \theta) = m - m^2/2\theta$, $\underline{\theta} = 1$, $\bar{\theta} = 2$ and $\theta^I = 1 + \sqrt{3}/2 \approx 1.87$, which implies that $\underline{m} = 1$, $\bar{m} = 2$, $m^I = 1 + \sqrt{3}/2$, and $\hat{m} = \sqrt{3} \approx 1.73$.[18] A party J that chooses $m^J > \sqrt{3}$ will report $\theta^J = 2$ and $m^{J\tilde{J}} = 1$, whereas one

[17] The second term in s^*, a constant, is just the sum of the last two terms in (1) when $f(\eta)[1 - \pi(\eta)] = f(-\eta)\pi(-\eta)$ for all η and the parties' policies and announcements are the same; see the appendix for further details.

[18] With $u(m, \theta) = m - m^2/2\theta$, utility is maximized by setting $m = \theta$. The value of $\hat{m} = \sqrt{3}$ follows from solving

that sets $m^J < \sqrt{3}$ will report $\theta^J = 1$ and $m^{J\tilde{J}} = 2$.[19]

The figure shows two functions, $V^L\left(m^L\right)$ and $V^R\left(m^R\right)$, such that the vote share s^L of party L is linearly related to the the difference between the two (see the appendix for details). Since, with a uniform distribution for η, m^R does not affect V^L and m^L does not affect V^R, each party J chooses its m^J to maximize V^J.

As we observed in our discussion of Figure 1, party L must gain by shifting its position from $\hat{m}$ toward (and beyond) m^I; $V^L\left(m^L\right)$ is increasing to the right of $\hat{m}$. Such a shift captures support among the informed voters and also gains votes among the uninformed, because the induced announcement of $\theta^L = 2$ and $m^{LR} = 1$ makes the party more attractive to all falsely informed voters that receive their news from an L-leaning source. For the parameters depicted in the figure, Party L also gains by shifting its position to the left from $\hat{m}$. Although such a move costs votes among the informed, the gain among the uninformed more than compensates in this case. The vote share of party L in the left panel of Figure 2 reaches a local minimum at $\hat{m} = \sqrt{3}$. The same logic applies for party R, and the right panel shows that $V^R\left(m^R\right)$ also reaches a local minimum at $\hat{m}$.

When $\hat{m}$ represents a local minimum, each party has two local maxima, one to the left of $\hat{m}$ and one to the right. In our example, the global maximum for party L occurs for $m^L \approx 1.3 < \hat{m}$, whereas that for party R occurs at $m^R \approx 1.9 > \hat{m}$. When the parties stake these positions, party L reports $\theta = 2$ while party R reports $\theta = 1$, thereby polarizing the electorate. The different incentives facing the two parties arise from the asymmetry of the $\pi(\eta)$ function. In the example of Figure 2, $\pi(\eta) < 1 - \pi(-\eta)$ for all η. Therefore, an uninformed voter with a given pre-disposition η toward party R is less likely to receive its news from a source biased toward that party than is a comparable voter with a leaning of the same amount toward party L to hear from an L-leaning source. This makes party R relatively more intent on capturing informed votes than party L. The former party chooses a position much closer to $m^I = 1 + \sqrt{3}/2$, whereas party L is willing to depart substantially from that optimal policy to gain more from its fake news.

We summarize the findings in this section as follows.

Proposition 2 *Suppose that the parties choose their policy positions sequentially and that each party is subsequently unconstrained in its reports about the position of its rival and the state of the*

(3) in this case.

[19] In constructing this example, we also assume $\pi(\eta) = e^{\eta}/\left(e^{\eta} + 5\right)$. See the appendix for more details.

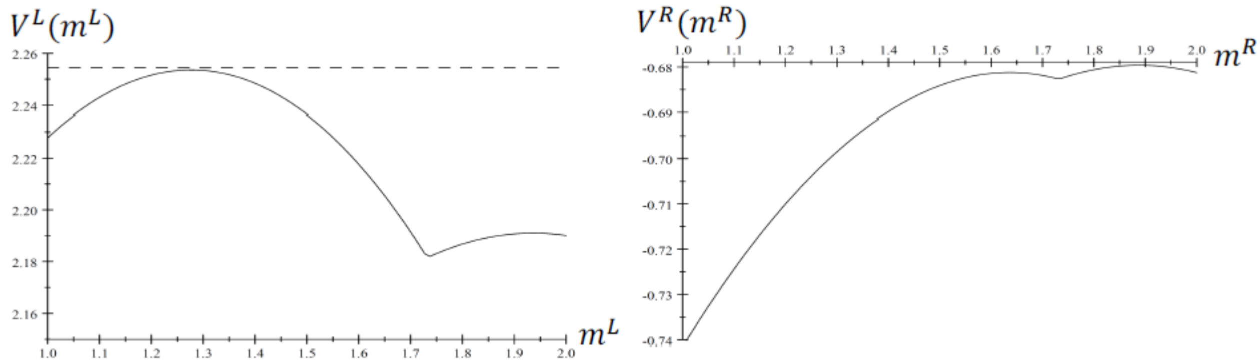


Figure 2: Accurate Reporting of Own Position: Policy Divergence

world, but must report its own position accurately. Then,if $f(\eta) = f(-\eta)$ *and* $\pi(\eta) = 1 - \pi(-\eta)$ *for all* η*, an equilibrium exists with convergent policies; i.e.,* $m^L = m^R = \tilde{m}$*. In contrast, if* $f(\eta)\pi(\eta) \neq f(-\eta)[1 - \pi(-\eta)]$ *for all* η*, any equilibrium has divergent policies; i.e.,* $m^L \neq m^R$*. The announcements about the state of the world and the rival party's policy position converge if* $\min\{m^L, m^R\} > \hat{m}$ *or* $\max\{m^L, m^R\} < \hat{m}$ *and diverge otherwise. Polarized announcements can arise for intermediate values of* λ^I*, but not for* λ^I *near zero or near one.*

5 Complete Information about Policy Positions

In this section, we focus on a setting in which false reporting is confined to claims about the state of the policy environment. First, the incumbent party R chooses its position, m^R. Then, the challenger party L does likewise. These positions become common knowledge to the entire electorate. In the final stage, the parties simultaneously report their fake news about the state of the world, θ. Knowledgeable voters compare $u(m^L, \theta^I)$ to $u(m^R, \theta^I)$. Uninformed voters that access their news from a source aligned with party J compare $u(m^L, \theta^J)$ to $u(m^R, \theta^J)$, for $J = L, R$.

We begin, as usual, with the final stage of the game. Given the chosen positions, m^L and m^R, the parties issue reports about the state of the world. Each party wishes to render itself maximally attractive to its audience. By arguments that are familiar by now, party J reports $\theta^J = \overline{\theta}$ if $m^J > m^{\tilde{J}}$ and $\theta^J = \underline{\theta}$ if $m^J < m^{\tilde{J}}$, for $J = L, R$.

Now consider the choice of position by party L, the challenger in this case.[20] As before, the

[20] Obviously, the identity of the challenger is arbitrary, and the label L and the designation of "challenger" have no substantive meaning. We simply label as R whichever party moves first; our results distinguish only the first and second movers.

party trades off the appeal to knowledgeable voters of a policy close to m^I versus the appeal to misinformed voters of a policy closer to one of the extremes. The challenger might choose a policy above m^R, anticipating the ensuing fake-news reports of $\theta^L = \overline{\theta}$ and $\theta^R = \underline{\theta}$. Among these, the party's optimal choice is the one that maximizes

$$s^L_{above} = \lambda^I F\left[u\left(m, \theta^I\right) - u\left(m^R, \theta^I\right)\right] + \\ \left(1-\lambda^I\right)\left\{\int_{-\infty}^{u\left(m,\overline{\theta}\right)-u\left(m^R,\overline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta + \int_{-\infty}^{u\left(m,\underline{\theta}\right)-u\left(m^R,\underline{\theta}\right)} \pi\left(\eta\right) f\left(\eta\right) d\eta\right\},$$

which we denote by m^L_{above}. Alternatively, it might choose a policy below m^R, anticipating in this case that $\theta^L = \underline{\theta}$ and $\theta^R = \overline{\theta}$. The best choice among these is the one that maximizes

$$s^L_{below} = \lambda^I F\left[u\left(m, \theta^I\right) - u\left(m^R, \theta^I\right)\right] + \\ \left(1-\lambda^I\right)\left\{\int_{-\infty}^{u\left(m,\underline{\theta}\right)-u\left(m^R,\underline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta + \int_{-\infty}^{u\left(m,\overline{\theta}\right)-u\left(m^R,\overline{\theta}\right)} \pi\left(\eta\right) f\left(\eta\right) d\eta\right\},$$

which we denote by m^L_{below}. The party's best response to m^R is the one that yields the greater vote share among these two alternatives. We write the best response as $m^L\left(m^R\right)$ and the resulting vote share as $s^L\left(m^R\right)$.

In the first stage, the incumbent party R chooses m^R, anticipating the reaction to its choice and recognizing that the electoral competition is a zero-sum game. Therefore, the incumbent maximizes its own vote share by setting

$$m^R \in \underset{m \in [\underline{m}, \overline{m}]}{\arg\min}\; s^L\left(m\right).$$

To characterize the equilibrium outcomes, we observe first that the challenger party L always can invoke a strategy of matching the incumbent's position, which then ensures the party a fraction $F\left(0\right)$ of the votes. Clearly, no equilibrium outcome can give party L less than this vote share; i.e., $s^L\left(m\right) \geq F\left(0\right)$. It follows that party R can do no better than the fraction $1 - F\left(0\right)$ of the votes. If R can find a position that induces L to match, this option must be an equilibrium strategy for the incumbent.

We now argue that policy convergence never is an equilibrium outcome when $\pi\left(0\right) < 1/2$; i.e., when an uninformed voter who is indifferent between the parties on ideological grounds is more likely to tune in to broadcasts by the challenger than to those by the incumbent. To this end, we conjecture the existence of an equilibrium with $m^R = m^L = \tilde{m}$, and then show that, with $\pi(0) < 1/2$, party L can profitably deviate to win more than the share $F(0)$ of the votes.

If party L deviates from matching m^R and instead sets $m^L = m^R + \varepsilon$, $\varepsilon > 0$, it will induce party R to report $\theta^R = \underline{\theta}$, while its own subsequent report will be $\theta^L = \overline{\theta}$. The change in votes for a small ε is

$$ds_+^L = \lambda^I f(0)\, u_m\left(\tilde{m}, \theta^I\right) \varepsilon + \left(1 - \lambda^I\right) f(0) \left\{\pi(0)\, u_m\left(\tilde{m}, \underline{\theta}\right) + [1 - \pi(0)]\, u_m\left(\tilde{m}, \overline{\theta}\right)\right\} \varepsilon.$$

If, instead, the party deviates to $m^L = m^R - \varepsilon$, $\varepsilon > 0$, it will induce party R to report $\theta^R = \overline{\theta}$, while its own subsequent report will be $\theta^L = \underline{\theta}$. The vote change that results from this small deviation is

$$ds_-^L = -\lambda^I f(0)\, u_m\left(\tilde{m}, \theta^I\right) \varepsilon - \left(1 - \lambda^I\right) f(0) \left\{\pi(0)\, u_m\left(\tilde{m}, \overline{\theta}\right) + [1 - \pi(0)]\, u_m\left(\tilde{m}, \underline{\theta}\right)\right\} \varepsilon.$$

Summing these two, we have

$$ds_+^L + ds_-^L = \left(1 - \lambda^I\right) f(0) \left[1 - 2\pi(0)\right] \left[u_m\left(\tilde{m}, \overline{\theta}\right) - u_m\left(\tilde{m}, \underline{\theta}\right)\right] \varepsilon\ .$$

But the supermodularity of $u(\cdot)$ implies $u_m\left(\tilde{m}, \overline{\theta}\right) > u_m\left(\tilde{m}, \underline{\theta}\right)$; i.e., the marginal value of an increase in m is greater in the highest state of the world than in the lowest state. Then, if $\pi(0) < 1/2$, $ds_+^L + ds_-^L > 0$, which implies that at least one of these deviations increases the vote share for party L. We have thus established

Proposition 3 *Suppose that the parties choose their positions sequentially, with the incumbent R choosing first and the challenger L choosing second. These positions subsequently become known to all voters, but uninformed voters learn about the state of the world from a biased source. If $\pi(0) < 1/2$, the equilibrium policies diverge $\left(m^L \neq m^R\right)$ and there is polarization of fake news; if $m^J > m^{\tilde{J}}$, $\theta^J = \overline{\theta}$ and $\theta^{\tilde{J}} = \underline{\theta}$, for $J = L, R$. In the equilibrium, $s^L > F(0)$; i.e., the challenger reaps a benefit from moving second.*

Intuitively, when $\pi(0) < 1/2$, a voter with no partisan leanings is more likely to hear the fake news reported by the challenger party L. The second mover eschews the option to mimic the

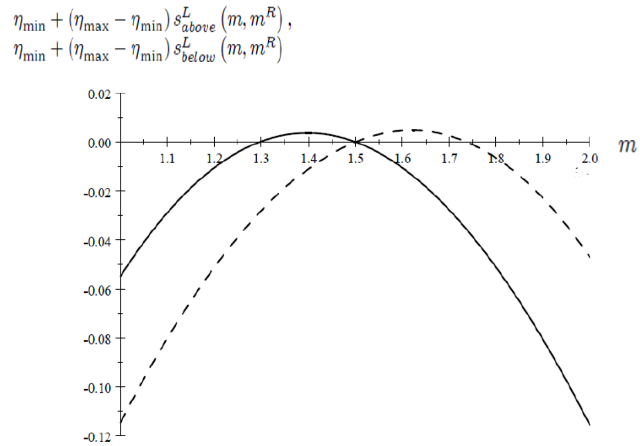


Figure 3: Complete Information about Policy Positions: Policy Convergence

incumbent, because a better option for this party is to shift its position away from that of its rival in one direction or the other. There is bound to be some direction that improves the second-mover's electoral prospects inasmuch as the anticipated extreme reporting of the state of the world leaves the swing voters especially sensitive to policy differences between the candidates.

In contrast, when $\pi(0) \geq 1/2$, it is the first-moving incumbent that reaches a majority of the uninformed swing voters. Then the incumbent will always be able to choose a position that leaves the challenger with no better option than to match. In Figure 3, we illustrate such an outcome. The figure uses the same setting and parameters as in Figure 2.[21] The solid curve in Figure 3 depicts a positive, linear displacement of $s^L_{above}(m, m^R)$, while the dashed curve depicts a positive, linear displacement of $s^L_{below}(m, m^R)$, both drawn for $m^R = 1.5$. Party L can choose any point on the solid curve to the right of $m^R = 1.5$ or any point on the dotted curve to the left of this point. Clearly, $m^L = 1.5$ represents the best response by party L to $m^R = 1.5$. Therefore, $m^R = m^L = 1.5$ represent a pair of equilibrium platforms. Note that the equilibrium policy differs from the optimal policy, $m^I \approx 1.87$.

In the appendix we show that, when $\pi(0) = 1/2$, the equilibrium outcome is unique. The incumbent adopts the unique position that induces matching by the challenger and thereby captures a fraction $F(0)$ of the votes. When $\pi(0) > 1/2$, the asymmetry in audiences gives the incumbent more latitude in choosing its position. In this case, there exists a range of values of m that party

[21] Again, we take $f(\eta) = 1/(\eta_{\max} - \eta_{\min})$, $\pi(\eta) = e^{\eta}/(e^{\eta} + 5)$ and $u(m, \theta) = m - m^2/2\theta$, with $\lambda^I = 0.4$, $\underline{\theta} = 1$, $\bar{\theta} = 2$ and $\theta^I = 1 + \sqrt{3}/2$.

R can choose such that the challenger party L will follow suit. We thus have

Proposition 4 *Suppose that the parties choose their positions sequentially, with the incumbent R choosing first and the challenger L choosing second. These positions subsequently become known to all voters, but uninformed voters learn about the state of the world from a biased source. If $\pi(0) \geq 1/2$, the equilibrium policies converge, with $m^L = m^R = \tilde{m}$, as do the fake news reports $(\theta^L = \theta^R)$. If $\pi(0) = 1/2$, the equilibrium policy outcome is unique, with $m^L = m^R = m^I$. If $\pi(0) > 1/2$, there exists a continuum of common positions, all yielding $s^L = F(0)$ and $s^R = 1 - F(0)$. These multiple equilibria differ in their implications for voter welfare.*

6 Conclusions

We have introduced strategic misinformation into an otherwise standard model of electoral competition. As a benchmark, our model predicts policy convergence and welfare maximization when voters are fully informed. More generally, we assume that some voters have access to accurate information while others rely on biased sources to form their understanding of the policy environment and perhaps the parties' policy positions. Among these uninformed voters, those that are partisan to some party are more likely to receive their information from a source that serves the interests of that party. We find circumstances in which fake news has real effects: the spread of such news may cause parties' policy positions to diverge and both may depart from the policy levels that are socially desirable. Such outcomes are most likely when each party or its media representative feels compelled to report accurately about its own position; then the parties face a trade-off in choosing their position between appealing to those who are well informed about the state of the world and those that will be misled to believe that the state is extreme in one direction or the other.

Our analysis is highly stylized and represents only a simple first step. Most importantly, the voters in our model are passive; they do not choose their information sources to achieve any particular objectives and they accept uncritically whatever it is that they hear. Further progress could perhaps be made by introducing some behavioral motives for voters' listening and reading habits and by allowing for some (limited) sophistication in their interpretation of the news. A more active role for the media would also be desirable, be they motivated by profits, partisanship, or career concerns. The salience of misinformation in modern day politics and our demonstration that fake news can matter for policy outcomes makes this a ripe topic for further research.

References

[1] **Allcott, Hunt, and Matthew Gentzkow**. 2017. "Social Media and Fake News in 2016 Election." *Journal of Economic Perspectives* 31 (2): 211-6.

[2] **Anderson, Simon and John E. McLaren**. 2012. "Media Mergers and Media Bias with Rational Consumers." *Journal of the European Economic Association* 10 (4): 831-59.

[3] **Baron, David P**. 1994. "Electoral Competition with Informed and Uninformed Voters." *American Political Science Review* 88 (1): 33-47.

[4] **Baron, David P.** 2006. "Persistent Media Bias." **Journal of Public Economics** 90 (1-2): 1-36.

[5] **Bartels, Larry M.** 1996. "Uninformed Votes: Information Effects in Presidential Elections." *American Political Science Review* 40 (1): 194-230.

[6] **Berelson, Bernard R., Lazarsfeld, Paul F., and William N. McPhee**. 1954. *Voting: A Study of Opinion Formation in a Presidential Campaig*n. Chicago: University of Chicago Press.

[7] **Bernhardt, Dan, Krasa, Stefan, and Mattias Polburn**. 2008. "Political Polarization and the Electoral Effects of Media Bias." *Journal of Public Economics* 98 (5-6): 1092-104.

[8] **Campbell, Angus, Converse, Philip E., Miller, Warren E. and Donald E. Stokes**. 1960. T*he American Voter*. New York: John Wiley and Sons.

[9] **Caplan, Bryan**. 2008. *The Myth of the Rational Voter: Why Democracies Choose Bad Policies*. Princeton: Princeton University Press.

[10] **Chan, Jimmy, and Wing Suen**. 2008. "A Spatial Theory of News Consumption and Electoral Competition." *Review of Economic Studies* 75 (3): 699-728.

[11] **Coe, Kevin, Tewksbury, David, Bond, Bradley J., Drogos, Kristin L., Porter, Robert W., Yahn, Ashley and Yuanyuan Zhang**. "Hostile News: Partisan Use and Perceptions of Cable News Programming. *Journal of Communications* 58 (2): 201-19.

[12] **Garrett, R. Kelly**. 2009. "Politically Motivated Reinforcement Seeking: Reframing the Selective Exposure Debate." *Journal of Communications* 59 (4): 676-99.

[13] **Gentzkow, Matthew and Jesse M. Shapiro**. 2010. "What Drives Media Slant? Evidence from U.S. Daily Newspapers." *Econometrica* 78 (1): 35-71.

[14] **Gentzkow, Matthew and Jesse M. Shapiro**. 2011. "Ideological Segregation Online and Offline." *Quarterly Journal of Economics* 126 (4): 1799-839.

[15] **Glaeser, Edward L., Ponzetto, Giacomo A.M., and Jesse M. Shapiro**. 2005. "Strategic Extremism: Why Republicans and Democrats Divide on Religious Values." *The Quarterly Journal of Economics* 120 (4): 1283-330.

[16] **Grossman, Gene M. and Elhanan Helpman**. 1996. "Electoral Competition and Special Interest Politics." *Review of Economic Studies* 63 (2): 265–286.

[17] **Guess, Andrew, Nyhan, Brendan, and Jason Reifler**. 2018. "Selective Exposure to Misinformation: Evidence from the Consumption of Fake News during the 2016 U.S. Presidential Campaign." European Research Council Working Paper.

[18] **Hochschild, Jennifer L., and Katherine Levine Einstein**. 2015. *Do Facts Matter: Information and Misinformation in American Politics*. Norman, OK: University of Oklahoma Press.

[19] **Hollander, Barry A**. 2008. "Tuning Out or Tuning Elsewhere: Partisanship, Polarization, and Media Migration from 1998 to 2006." *Journal of Mass Communication Quarterly* 85 (1): 23-40.

[20] **Iyengar, Shanto, and Kyu S. Hahn**. 2009. "Red Media, Blue Media: Evidence of Ideological Selectivity in Media Use." *Journal of Communication* 59 (1): 19-39.

[21] **Jamieson, Kathleen Hall and Joseph N. Cappella**. 2008. *Echo Chamber: Rush Limbaugh and the Conservative Media Establishment*. New York: Oxford University Press.

[22] **Keohane, Joe. 2010**. "Imaginary Fiends: Crime in America Keeps Going Down. Why does the Public Refuse to Believe It?" *The Boston Globe*, February 14.

[23] **Krosnick, Jon A., and Laura A. Brannon**. 1993. "The Impact of the Gulf War on the Ingredients of Presidential Evaluations: Multidimensional Effects of Political Involvement." *American Political Science Review* 87 (4): 963-75.

[24] **Lindbeck, Assar and Jörgen W. Weibull**. 1987. "Balanced-budget Redistribution as the Outcome of Political Competition." *Public Choice* 52 (3): 273-97.

[25] **Martin, Gregory J. and Ali Yurugoklu**. 2017. "Bias in Cable News: Persuasion and Polarization." *American Economic Review* 107 (9): 2565-99.

[26] **Martinez, Luis R.** 2022. "How Much Should we Trust the Dictator's GDP Growth Estimates." *Journal of Political Economy*, forthcoming.

[27] **Matějka, Filip and Guido Tabellini**. 2021. "Electoral Competition with Rationally Inattentive Voters." *Journal of the European Economic Association* 19 (3): 1899-1935.

[28] **Mullainathan, Sendhil and Andrei Shleifer**. 2005. "The Market for News." *American Economic Review* 95 (4): 1031-53.

[29] **Peisakhin, Leonid and Arturas Rozensas**. 2018."Electoral Effects of Biased Media: Russian Television in Ukraine." *American Journal of Political Science* 62 (3): 535-50.

[30] **Prior, Markus**. 2013. "Media and Political Polarization." *Annual Review of Political Science* 16: 101-27.

[31] **Silverman, Craig and Jeremy Singer-Vine**. 2016. "Most Americans Who See Fake News Believe It, New Survey Says." *BuzzFeed News*, December 6.

[32] **Snyder Jr., James N. and David Strömberg**. 2010. "Press Coverage and Accountability." *Journal of Political Economy* 118 (2): 355-408.

[33] **Stantcheva, Stefanie**. 2020. "Understanding Economic Policies: What Do People Know and Learn." Harvard University Working Paper.

[34] **Strömberg, David**. 2004. "Mass Media Competition, Political Competition, and Public Policy." *Review of Economic Studies* 71 (1): 265-84.

[35] **Strömberg, David.** 2015. "Media and Politics." *Annual Review of Economics* 7: 173-205.

[36] **Stroud, Natalie Jomini**. 2011. *Niche News: The Politics of News Choice*. New York: Oxford University Press.

[37] **Wolton, Stephane**. 2019. "Are Biased Media Bad for Democracy?" *American Journal of Political Science* 63 (3): 548-62.

[38] **Zaller, John**. 1992. *The Nature and Origins of Mass Opinion*. Cambridge UK: Cambridge University Press.

Online Appendix

for

Electoral Competition with Fake News

Gene M. Grossman and Elhanan Helpman

In this appendix, we provide further details supporting the arguments made in the main text.

Appendix for Section 2

Recall that the utility function $u(m,\theta)$ is increasing in θ, concave in m, twice continuously differentiable, and supermodular. The latter implies $u_{m\theta}(m,\theta) > 0$. In Section 2 of the paper, we mention an example in which m is the number of immigrants and θ is inversely related to the cost of absorbing immigrants. The details of the example are as follows. Output is produced according to a constant-returns-to scale-technology, $f(n,m) = (n^{\alpha} + m^{\alpha})^{1/\alpha}$, $\alpha \in (0,1)$, where n is the number of domestic workers, normalized so that $n = 1$. Assuming that immigrants are paid a competitive wage, they generate surplus income for domestic residents of $b(m) = f(1,m) - f_m(1,m)\,m$. The function $b(m)$ is increasing and concave. Let $c(m)/\theta$ be the cost of absorbing m immigrants, where $c(\cdot)$ is increasing and convex and θ is a cost shifter. Then $u(m,\theta) \equiv b(m) - c(m)/\theta$, with $u_{\theta}(m,\theta) > 0$ and $u_{m\theta}(m,\theta) > 0$. That is, $u(\cdot)$ is supermodular. The optimal number of immigrants, m^I, satisfies

$$u_m\left(m^I,\theta^I\right) = 0.$$

It follows that the optimal number of immigrants is increasing in θ^I.

Appendix for Section 3

Recall that in the game of this section the choice of $\left(\theta^J, m^{JJ}, m^{J\tilde{J}}\right)$ for given values of m^J, $J = L, R$ satisfies

$$\left(\theta^J, m^{JJ}, m^{J\tilde{J}}\right) = \begin{cases} \left(\overline{\theta}, \overline{m}, \underline{m}\right) & \text{for } u\left(\overline{m},\overline{\theta}\right) - u\left(\underline{m},\overline{\theta}\right) > u\left(\underline{m},\underline{\theta}\right) - u\left(\overline{m},\underline{\theta}\right) \\ \left(\underline{\theta}, \underline{m}, \overline{m}\right) & \text{for } u\left(\overline{m},\overline{\theta}\right) - u\left(\underline{m},\overline{\theta}\right) < u\left(\underline{m},\underline{\theta}\right) - u\left(\overline{m},\underline{\theta}\right) \end{cases}, \text{ for } J = L, R.$$

That is, in the last stage of the game the choice of announcement of the state of the world, θ^J, of ones own strategy, m^{JJ}, and the strategy of the rival, $m^{J\tilde{J}}$, is independent of the chosen positions in the first two stages of the game. For this reason the same $\left(\theta^J, m^{JJ}, m^{J\tilde{J}}\right)$ will be chosen in the last stage of the game when the positions m^L and m^R are chosen simultaneously in a first stage; i.e., when the first stage is a simultaneous-move game. It is then evident from (1) that in this type of game party L maximizes its vote share by maximizing $u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)$ in the first stage while party R maximizes its vote share by minimizing $u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)$ in the first stage. Under these circumstances every party has a dominant strategy $m^J = m^I = \arg\max_m u\left(m, \theta^I\right)$, $J = L, R$, which is the same as the equilibrium strategy in the sequential-move game. It follows that the equilibrium outcomes are the same in the simultaneous-move game as in the sequential-move game.

Appendix for Section 4

In Section 4 of the paper, we discuss the case in which each party feels compelled to report accurately about its own intentions, but is free to misrepresent the state of the world and the position of its rival. In this section of the appendix, we provide the technical details. We begin by reproducing equation (1) with accurate reporting of own intentions, i.e., $m^{LL} = m^L$:

$$\begin{aligned} s^L\left(m^L, m^R\right) &= \lambda^I F\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] \\ &+ \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} \left[1-\pi\left(\eta\right)\right] dF\left(\eta\right) \\ &+ \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^{RL},\theta^R\right)-u\left(m^R,\theta^R\right)} \pi\left(\eta\right) dF\left(\eta\right). \end{aligned} \tag{7}$$

From equation (2) in the main text, which outlines the optimal announcement about the policy environment, we have for this case

$$\theta^J = \begin{cases} \overline{\theta} & \text{for } m^J > m^{J\tilde{J}} \\ \underline{\theta} & \text{for } m^J < m^{J\tilde{J}} \end{cases}, \; J = L, R. \tag{8}$$

At the third stage of the game, when the positions m^L and m^R are given, party L wishes to maximize $u\left(m^L, \theta^L\right) - u\left(m^{LR}, \theta^L\right)$ while R wishes to minimize $u\left(m^{RL}, \theta^R\right) - u\left(m^R, \theta^R\right)$. Therefore, the best response for party J is to choose either $\theta^J = \overline{\theta}$ and $m^{J\tilde{J}} = \underline{m}$, which is optimal if

$$u\left(m^J,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)>u\left(m^J,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\ ,$$

or else $\theta^J=\underline{\theta}$ and $m^{J\widetilde{J}}=\overline{m}$, which is optimal if the inequality runs in the opposite direction. Supermodularity of $u(m,\theta)$ implies that $u\left(m,\overline{\theta}\right)-u\left(m,\underline{\theta}\right)$ is increasing in m. In addition,

$$u\left(m^J,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)>u\left(m^J,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\ \text{for}\ m^J=\overline{m},$$

$$u\left(m^J,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)<u\left(m^J,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\ \text{for}\ m^J=\underline{m}.$$

Therefore, there exists an $\hat{m}\in(\underline{m},\overline{m})$ that satisfies

$$u\left(\hat{m},\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)=u\left(\hat{m},\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right). \tag{9}$$

Accordingly, $\left\{\theta^J,m^{J\widetilde{J}}\right\}=\left\{\overline{\theta},\underline{m}\right\}$ is the best strategy when $m^J>\hat{m}$ and $\left\{\theta^J,m^{J\widetilde{J}}\right\}=\left\{\underline{\theta},\overline{m}\right\}$ is the best strategy when $m^J<\hat{m}$. For $m^J=\hat{m}$, party J is indifferent between the two strategies. Note that party J has a dominant strategy in the second stage of the game. This finding is summarized in

Lemma 1 *For given choices of m^L and m^R in the first two stages of the game and truthful reporting of own positions, party J has a dominant strategy in the third stage of the game that is independent of its rival's play. This strategy is given by*

$$\left\{\theta^J,m^{J\widetilde{J}}\right\}=\begin{cases}\left\{\overline{\theta},\underline{m}\right\} & \text{for } m^J>\hat{m},\\ \left\{\underline{\theta},\overline{m}\right\} & \quad m^L<\hat{m},\end{cases}\qquad J=L,R,$$

where $\hat{m}$ is implicitly defined in (9). For $m^J=\hat{m}$ party J is indifferent between $\left\{\overline{\theta},\underline{m}\right\}$ and $\left\{\underline{\theta},\overline{m}\right\}$.

Using (7), (8) and Lemma 1, we obtain the partial derivatives of the vote share function

$s^L\left(m^L, m^R\right)$:

$$\begin{aligned} & \frac{\partial s^L\left(m^L, m^R\right)}{dm^L} \qquad (10) \\ = \; & \lambda^I f\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] u_m\left(m^L, \theta^I\right) \\ & + \begin{cases} \left(1-\lambda^I\right)\left\{1-\pi\left[u\left(m^L, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)\right]\right\} f\left[u\left(m^L, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)\right] u_m\left(m^L, \overline{\theta}\right) & \text{for } m^L > \hat{m}, \\ \left(1-\lambda^I\right)\left\{1-\pi\left[u\left(m^L, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)\right]\right\} f\left[u\left(m^L, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)\right] u_m\left(m^L, \underline{\theta}\right) & \text{for } m^L < \hat{m}, \end{cases} \end{aligned}$$

$$\begin{aligned} & \frac{\partial s^L\left(m^L, m^R\right)}{dm^R} \qquad (11) \\ = \; & -\lambda^I f\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] u_m\left(m^R, \theta^I\right) \\ & - \begin{cases} \left(1-\lambda^I\right)\pi\left[u\left(\underline{m}, \overline{\theta}\right) - u\left(m^R, \overline{\theta}\right)\right] f\left[u\left(\underline{m}, \overline{\theta}\right) - u\left(m^R, \overline{\theta}\right)\right] u_m\left(m^R, \overline{\theta}\right) & \text{for } m^R > \hat{m}, \\ \left(1-\lambda^I\right)\pi\left[u\left(\overline{m}, \underline{\theta}\right) - u\left(m^R, \underline{\theta}\right)\right] f\left[u\left(\overline{m}, \underline{\theta}\right) - u\left(m^R, \underline{\theta}\right)\right] u_m\left(m^R, \underline{\theta}\right) & \text{for } m^R < \hat{m}, \end{cases} \end{aligned}$$

where $f(\eta) = F'(\eta)$ is the density function associated with the distribution $F(\eta)$.

Now consider the second stage of the game. Party L, the follower, seeks in this stage to maximize $s^L\left(m^L, m^R\right)$ given m^R. Note that L's optimal strategy satisfies $\underline{m} < m^L < \overline{m}$, because $\partial s^L\left(m^L, m^R\right)/\partial m^L$ is positive at $m^L = \underline{m}$ and negative at $m^L = \overline{m}$ for all $m^R \in [\underline{m}, \overline{m}]$. In addition, $m^L = \hat{m}$ is not an optimal strategy, because for $m^L = \hat{m}$ to be an optimal strategy the following inequalities would have to be satisfied:

$$\lim_{m \nearrow \hat{m}} \frac{\partial s^L\left(m^L, m^R\right)}{dm^L} \geq 0 \geq \lim_{m \searrow \hat{m}} \frac{\partial s^L\left(m^L, m^R\right)}{dm^L}.$$

Using (10), these inequalities are satisfied if and only if

$$\begin{aligned} & \lambda^I f\left[u\left(\hat{m}, \theta^I\right) - u\left(m^R, \theta^I\right)\right] u_m\left(\hat{m}, \theta^I\right) \\ & + \left(1-\lambda^I\right)\left\{1-\pi\left[u\left(\hat{m}, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)\right]\right\} f\left[u\left(\hat{m}, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)\right] u_m\left(\hat{m}, \underline{\theta}\right) \\ \geq \; & 0 \geq \lambda^I f\left[u\left(\hat{m}, \theta^I\right) - u\left(m^R, \theta^I\right)\right] u_m\left(\hat{m}, \theta^I\right) \\ & + \left(1-\lambda^I\right)\left\{1-\pi\left[u\left(\hat{m}, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)\right]\right\} f\left[u\left(\hat{m}, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)\right] u_m\left(\hat{m}, \overline{\theta}\right). \end{aligned}$$

However, from the definition of $\hat{m}$ in (9) and the supermodularity of $u(m,\theta)$, we have

$$u_m(\hat{m},\underline{\theta}) < 0 \text{ and } u_m\left(\hat{m},\overline{\theta}\right) > 0,$$

which implies a violation of the required inequalities. Therefore, $\hat{m}$ is not a best response to m^R for any value of m^R. We conclude that the best response of L satisfies

$$\frac{\partial s^L\left(m^L,m^R\right)}{dm^L} = 0, \tag{12}$$

and $m^L \in (\underline{m},\hat{m}) \cup (\hat{m},\overline{m})$, where the left-hand side of this equation is given by (10). Equation (12) can have more than one solution for m^L, each one being a best response of L.

Party R, the leader, moves in the first stage of the game, seeking to minimize $s^L\left(m^L,m^R\right)$, and accounting for the dependence of m^L on its choice of m^R. Since (12) is satisfied for every choice of $m^R \in [\underline{m},\overline{m}]$, it follows that—anticipating the response of L's policy to m^R—the marginal change in $s^L\left(m^L,m^R\right)$ in response of an increase in m^R is

$$\frac{\partial s^L\left(m^L,m^R\right)}{dm^R} + \frac{\partial s^L\left(m^L,m^R\right)}{dm^L}\frac{dm^L}{dm^R} = \frac{\partial s^L\left(m^L,m^R\right)}{dm^R},$$

which is given by (11). In cases in which (12) has more than one solution of m^L for a given m^R, this holds for each one of these solutions, and $\max_{m^L} s^L\left(m^L,m^R\right)$ is a continuous function of m^R. In these circumstances, by arguments similar to the arguments about m^L above, neither $m^R = \underline{m}$ nor $m^R = \overline{m}$ nor $m^R = \hat{m}$ are best plays by party R. It follows that $m^R \in (\underline{m},\hat{m}) \cup (\hat{m},\overline{m})$ and it satisfies

$$\frac{\partial s^L\left(m^L,m^R\right)}{dm^R} = 0. \tag{13}$$

In short, m^J is interior, and either $m^J \in (\underline{m},\hat{m})$ or $m^J \in (\hat{m},\overline{m})$ for $J = L,R$, and therefore the equilibrium strategies satisfy (12) and (13). It is evident from this characterization that an equilibrium of a game in which parties L and R move simultaneous in stage one of the game, choosing m^J, $J = L,R$, and choosing θ^J and $m^{J\tilde{J}}$ in the second stage, is also characterized by (12) and (13). And since $\max_{m^L} s^L\left(m^L,m^R\right)$ is a continuous function of m^R and $m^R \in [\underline{m},\overline{m}]$, there always exists an equilibrium for this game.

Now suppose that there exists an $\tilde{m} \in (\underline{m}, \hat{m}) \cup (\hat{m}, \overline{m})$, such that $m^L = m^R = \tilde{m}$ are equilibrium strategies; i.e., there exists an equilibrium with platform convergence at $\tilde{m}$. Then, the following first-order conditions have to satisfied:

$$\frac{\partial s^L(\tilde{m}, \tilde{m})}{dm^L} = \frac{\partial s^L(\tilde{m}, \tilde{m})}{dm^R} = 0,$$

which, using (10)-(11), are satisfied if and only if either $\tilde{m} > \hat{m}$ and

$$\begin{gathered} \lambda^I f(0) u_m\left(\tilde{m}, \theta^I\right) + \\ \left(1-\lambda^I\right)\left\{1 - \pi\left[u\left(\tilde{m}, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)\right]\right\} f\left[u\left(\tilde{m}, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)\right] u_m\left(\tilde{m}, \overline{\theta}\right) = 0, \end{gathered} \tag{14}$$

$$\begin{gathered} -\lambda^I f(0) u_m\left(\tilde{m}, \theta^I\right) - \\ \left(1-\lambda^I\right) \pi\left[u\left(\underline{m}, \overline{\theta}\right) - u\left(\tilde{m}, \overline{\theta}\right)\right] f\left[u\left(\underline{m}, \overline{\theta}\right) - u\left(\tilde{m}, \overline{\theta}\right)\right] u_m\left(\tilde{m}, \overline{\theta}\right) = 0, \end{gathered} \tag{15}$$

or $\tilde{m} < \hat{m}$ and

$$\begin{gathered} \lambda^I f(0) u_m\left(\tilde{m}, \theta^I\right) + \\ \left(1-\lambda^I\right)\left\{1 - \pi\left[u(\tilde{m}, \underline{\theta}) - u(\overline{m}, \underline{\theta})\right]\right\} f\left[u(\tilde{m}, \underline{\theta}) - u(\overline{m}, \underline{\theta})\right] u_m(\tilde{m}, \underline{\theta}) = 0, \end{gathered} \tag{16}$$

$$\begin{gathered} -\lambda^I f(0) u_m\left(\tilde{m}, \theta^I\right) - \\ \left(1-\lambda^I\right) \pi\left[u(\overline{m}, \underline{\theta}) - u(\tilde{m}, \underline{\theta})\right] f\left[u(\overline{m}, \underline{\theta}) - u(\tilde{m}, \underline{\theta})\right] u_m(\tilde{m}, \underline{\theta}) = 0. \end{gathered} \tag{17}$$

In the former case, i.e., $\tilde{m} > \hat{m}$, the two first-order conditions imply

$$\left\{1 - \pi\left[u\left(\tilde{m}, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)\right]\right\} f\left[u\left(\tilde{m}, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)\right] = \pi\left[u\left(\underline{m}, \overline{\theta}\right) - u\left(\tilde{m}, \overline{\theta}\right)\right] f\left[u\left(\underline{m}, \overline{\theta}\right) - u\left(\tilde{m}, \overline{\theta}\right)\right],$$

whereas in the latter case they imply

$$\left\{1 - \pi\left[u(\tilde{m}, \underline{\theta}) - u(\overline{m}, \underline{\theta})\right]\right\} f\left[u(\tilde{m}, \underline{\theta}) - u(\overline{m}, \underline{\theta})\right] = \pi\left[u(\overline{m}, \underline{\theta}) - u(\tilde{m}, \underline{\theta})\right] f\left[u(\overline{m}, \underline{\theta}) - u(\tilde{m}, \underline{\theta})\right].$$

Evidently, neither of the last two conditions can be satisfied when $\pi(\eta) f(\eta) \neq [1-\pi(-\eta)] f(-\eta)$ for all η in the support of $F(\eta)$, in which case there exists no equilibrium with platform convergence.

Next, suppose that

$$\pi(\eta) = 1 - \pi(-\eta) \quad \text{and } f(\eta) = f(-\eta) \text{ for all } \eta \text{ in the support of } F(\eta). \tag{18}$$

Then there exists an equilibrium with convergence in policy positions. To see why, we first show that in this case the follower obtains a vote share s^* by mimicking the leader's policy, for all $m^R \in [\underline{m}, \overline{m}]$. Therefore, if there exists a policy $m^R = \tilde{m}$ such that the best response of the follower includes $m^L = \tilde{m}$, then $\tilde{m}$ is an equilibrium play of R. Second, we show that such an $\tilde{m} \in (\underline{m}, \overline{m})$ exists.

To characterize s^*, we use (7) together with (18) to obtain

$$\begin{aligned}
s^L\left(m^R, m^R\right) &= \lambda F(0) + \int_{-\infty}^{u\left(m^R,\bar{\theta}\right)-u\left(\underline{m},\bar{\theta}\right)} [1-\pi(\eta)] f(\eta)\, d\eta + \int_{-\infty}^{u\left(\underline{m},\bar{\theta}\right)-u\left(m^R,\bar{\theta}\right)} \pi(\eta) f(\eta)\, d\eta \\
&= \lambda F(0) + \int_{-\infty}^{u\left(m^R,\bar{\theta}\right)-u\left(\underline{m},\bar{\theta}\right)} [1-\pi(\eta)] f(\eta)\, d\eta + \int_{-\infty}^{u\left(\underline{m},\bar{\theta}\right)-u\left(m^R,\bar{\theta}\right)} [1-\pi(-\eta)] f(-\eta)\, d\eta \\
&= \lambda F(0) + \int_{-\infty}^{u\left(m^R,\bar{\theta}\right)-u\left(\underline{m},\bar{\theta}\right)} [1-\pi(\eta)] f(\eta)\, d\eta - \int_{+\infty}^{u\left(m^R,\bar{\theta}\right)-u\left(\underline{m},\bar{\theta}\right)} [1-\pi(\eta)] f(\eta)\, d\eta \\
&= \lambda F(0) + \int_{-\infty}^{u\left(m^R,\bar{\theta}\right)-u\left(\underline{m},\bar{\theta}\right)} [1-\pi(\eta)] f(\eta)\, d\eta + \int_{u\left(m^R,\bar{\theta}\right)-u\left(\underline{m},\bar{\theta}\right)}^{+\infty} [1-\pi(\eta)] f(\eta)\, d\eta \\
&= \lambda F(0) + \int_{-\infty}^{\infty} [1-\pi(\eta)] f(\eta)\, d\eta.
\end{aligned}$$

Namely,

$$s^L\left(m^R, m^R\right) = s^* := \lambda F(0) + \int_{-\infty}^{\infty} [1-\pi(\eta)] f(\eta)\, d\eta \quad \text{for all } m^R \in [\underline{m}, \overline{m}].$$

To show existence of an $\tilde{m} \in (\underline{m}, \overline{m})$ such that $m^R = m^L = \tilde{m}$ are equilibrium strategies, we first develop a useful expression of the share function $s^L(\cdot)$ under condition (18). Using $\pi(\eta) f(\eta) =$

$[1-\pi(-\eta)]\, f(-\eta)$,

$$
\begin{aligned}
s^L &= \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] \\
&\quad + \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, dF(\eta) + \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^{RL},\theta^R\right)-u\left(m^R,\theta^R\right)} [1-\pi(-\eta)]\, f(-\eta)\, d\eta \\
&= \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] \\
&\quad + \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, dF(\eta) - \left(1-\lambda^I\right)\int_{\infty}^{u\left(m^R,\theta^R\right)-u\left(m^{RL},\theta^R\right)} [1-\pi(\eta)]\, f(\eta)\, d\eta \\
&= \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] \\
&\quad + \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, dF(\eta) + \left(1-\lambda^I\right)\int_{u\left(m^R,\theta^R\right)-u\left(m^{RL},\theta^R\right)}^{\infty} [1-\pi(\eta)]\, f(\eta)\, d\eta.
\end{aligned}
$$

That is:

$$
\begin{aligned}
s^L &= \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] \\
&\quad + \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, dF(\eta) + \left(1-\lambda^I\right)\int_{u\left(m^R,\theta^R\right)-u\left(m^{RL},\theta^R\right)}^{\infty} [1-\pi(\eta)]\, f(\eta)\, d\eta \\
&= \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] \\
&\quad + \left(1-\lambda^I\right)\int_{-\infty}^{+\infty} [1-\pi(\eta)]\, dF(\eta) + \left(1-\lambda^I\right)\int_{u\left(m^R,\theta^R\right)-u\left(m^{RL},\theta^R\right)}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, f(\eta)\, d\eta
\end{aligned}
$$

or

$$
\begin{aligned}
s^L &= \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] \\
&\quad + \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, dF(\eta) + \left(1-\lambda^I\right)\int_{u\left(m^R,\theta^R\right)-u\left(m^{RL},\theta^R\right)}^{\infty} [1-\pi(\eta)]\, f(\eta)\, d\eta \\
&= s^* + \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] - \lambda^I F(0) + \left(1-\lambda^I\right)\int_{u\left(m^R,\theta^R\right)-u\left(m^{RL},\theta^R\right)}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, f(\eta)\, d\eta.
\end{aligned}
$$

Let

$$
S := \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] + \left(1-\lambda^I\right)\int_{u\left(m^R,\theta^R\right)-u\left(m^{RL},\theta^R\right)}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, f(\eta)\, d\eta.
$$

Then party L maximizes S while party R minimizes S. Using Lemma 1, we therefore obtain S as a function of m^L and m^R:

$$\begin{aligned} S\left(m^L, m^R\right) &: = \lambda^I F\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] \\ &+ \left(1-\lambda^I\right)\begin{cases} \int_{u\left(m^R,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)}^{u\left(m^L,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta & \text{for } m^L > \hat{m} \text{ and } m^R > \hat{m} \\ \int_{u\left(m^R,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)}^{u\left(m^L,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta & \text{for } m^L > \hat{m} \text{ and } m^R < \hat{m} \\ \int_{u\left(m^R,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)}^{u\left(m^L,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta & \text{for } m^L < \hat{m} \text{ and } m^R > \hat{m} \\ \int_{u\left(m^R,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)}^{u\left(m^L,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta & \text{for } m^L < \hat{m} \text{ and } m^R < \hat{m} \end{cases} . \end{aligned}$$

Consider

$$\frac{\partial S\left(m^L, m^R\right)}{\partial m^L} := \lambda^I f\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] u_m\left(m^L, \theta^I\right)$$

$$+ \left(1-\lambda^I\right)\begin{cases} \left[1-\pi\left(u\left(m^L,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)\right)\right] f\left(u\left(m^L,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)\right) u_m\left(m^L,\overline{\theta}\right) & \text{for } m^L > \hat{m} \text{ and } m^R > \hat{m} \\ \left[1-\pi\left(u\left(m^L,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)\right)\right] f\left(u\left(m^L,\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)\right) u_m\left(m^L,\overline{\theta}\right) & \text{for } m^L > \hat{m} \text{ and } m^R < \hat{m} \\ \left[1-\pi\left(u\left(m^L,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\right)\right] f\left(u\left(m^L,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\right) u_m\left(m^L,\underline{\theta}\right) & \text{for } m^L < \hat{m} \text{ and } m^R > \hat{m} \\ \left[1-\pi\left(u\left(m^L,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\right)\right] f\left(u\left(m^L,\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\right) u_m\left(m^L,\underline{\theta}\right) & \text{for } m^L < \hat{m} \text{ and } m^R < \hat{m} \end{cases} .$$

For $m^L = m^R = \tilde{m}$ this derivative equals zero if and only if

$$\begin{aligned} 0 &= \lambda^I f\left(0\right) u_m\left(\tilde{m}, \theta^I\right) \qquad (19) \\ &= \left(1-\lambda^I\right)\begin{cases} \left[1-\pi\left(u\left(\tilde{m},\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)\right)\right] f\left(u\left(\tilde{m},\overline{\theta}\right)-u\left(\underline{m},\overline{\theta}\right)\right) u_m\left(\tilde{m},\overline{\theta}\right) & \text{for } \tilde{m} > \hat{m} \\ \left[1-\pi\left(u\left(\tilde{m},\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\right)\right] f\left(u\left(\tilde{m},\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\right) u_m\left(\tilde{m},\underline{\theta}\right) & \text{for } \tilde{m} < \hat{m} \end{cases} . \end{aligned}$$

We now provide an analysis of existence for the case $m^I \in (\hat{m}, \overline{m})$; a similar analysis applies to the case $m^I \in (\underline{m}, \hat{m})$. For $m^I \in (\hat{m}, \overline{m})$ there is a solution $\tilde{m} \in \left(m^I, \overline{m}\right)$, because in this case the right-hand side of (19) is positive for all $\tilde{m} \in (\hat{m}, m^I]$ and negative for $\tilde{m}$ close to $\overline{m}$. Moreover, in this case there is a solution $\tilde{m} \in (\underline{m}, \hat{m})$ if and only if

$$0 > \lambda^I f\left(0\right) u_m\left(\hat{m}, \theta^I\right) + \left(1-\lambda^I\right)\left\{\left[1-\pi\left(u\left(\hat{m},\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\right)\right] f\left(u\left(\hat{m},\underline{\theta}\right)-u\left(\overline{m},\underline{\theta}\right)\right) u_m\left(\hat{m},\underline{\theta}\right)\right\},$$

because right-hand side of (19) is positive for $\tilde{m}$ close to $\underline{m}$. This inequality holds for m^I close to $\hat{m}$. If there is no solution $\tilde{m} < \hat{m}$, then the unique solution $\tilde{m} \in \left(m^I, \overline{m}\right)$ is an equilibrium, in the sense that the best response of L to $m^R = \tilde{m}$ is $m^L = \tilde{m}$, because in this case $S\left(m^L, \tilde{m}\right)$ is increasing for all $m^L < \tilde{m}$. In this equilibrium

$$S\left(\tilde{m}, \tilde{m}\right) = \lambda^I F\left(0\right).$$

Now suppose that there exist solutions $\tilde{m}_u > m^I$ and $\tilde{m}_l < \hat{m}$ to (19). For $m^L = m^R = \tilde{m}_u$ to be an equilibrium, L should not gain by switching from $\tilde{m}_u$ to $\tilde{m}_l$, and therefore the following has to be satisfied:

$$\begin{aligned}
\lambda^I F\left(0\right) &\geq \lambda^I F\left[u\left(\tilde{m}_l, \theta^I\right) - u\left(\tilde{m}_u, \theta^I\right)\right] + \left(1-\lambda^I\right)\int_{u\left(\tilde{m}_u, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)}^{u\left(\tilde{m}_l, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta \\
&\geq \lambda^I F\left(0\right) - \lambda^I \int_{\tilde{m}_l}^{\tilde{m}_u} f\left[u\left(\tilde{m}_l, \theta^I\right) - u\left(m, \theta^I\right)\right] u_m\left(m, \theta^I\right) dm \\
&\quad + \left(1-\lambda^I\right)\int_{u\left(\tilde{m}_u, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)}^{u\left(\tilde{m}_l, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta,
\end{aligned}$$

while for $m^L = m^R = \tilde{m}_l$ to be an equilibrium L should not gain by switching to $\tilde{m}_u$, and therefore the following has to be satisfied:

$$\begin{aligned}
\lambda^I F\left(0\right) &\geq \lambda^I F\left[u\left(\tilde{m}_u, \theta^I\right) - u\left(\tilde{m}_l, \theta^I\right)\right] + \left(1-\lambda^I\right)\int_{u\left(\tilde{m}_l, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)}^{u\left(\tilde{m}_u, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta \\
&\geq \lambda^I F\left(0\right) + \lambda^I \int_{\tilde{m}_l}^{\tilde{m}_u} f\left[u\left(m, \theta^I\right) - u\left(\tilde{m}_l, \theta^I\right)\right] u_m\left(m, \theta^I\right) dm \\
&\quad + \left(1-\lambda^I\right)\int_{u\left(\tilde{m}_l, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)}^{u\left(\tilde{m}_u, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta.
\end{aligned}$$

The first of these conditions can be expressed as

$$-\lambda^I \int_{\tilde{m}_l}^{\tilde{m}_u} f\left[u\left(\tilde{m}_l, \theta^I\right) - u\left(m, \theta^I\right)\right] u_m\left(m, \theta^I\right) dm - \left(1-\lambda^I\right)\int_{u\left(\tilde{m}_l, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)}^{u\left(\tilde{m}_u, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta \leq 0, \tag{20}$$

while the second can be expressed as

$$\lambda^I \int_{\tilde{m}_l}^{\tilde{m}_u} f\left[u\left(m, \theta^I\right) - u\left(\tilde{m}_l, \theta^I\right)\right] u_m\left(m, \theta^I\right) dm + \left(1-\lambda^I\right)\int_{u\left(\tilde{m}_l, \underline{\theta}\right) - u\left(\overline{m}, \underline{\theta}\right)}^{u\left(\tilde{m}_u, \overline{\theta}\right) - u\left(\underline{m}, \overline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] f\left(\eta\right) d\eta \leq 0. \tag{21}$$

Next note that with a symmetric density that satisfied $f\left(\eta\right) = f\left(-\eta\right)$, at least one of these conditions holds and only one can be satisfied with strict inequality. If (20) holds with strict inequality, than the unique equilibrium is $\tilde{m}_u$, and if (21) holds with strict inequality, than $\tilde{m}_l$ is the unique equilibrium. If both inequalities equal zero, then both $\tilde{m}_l$ and $\tilde{m}_u$ are equilibria of the sequential game.

Next consider the special case of a uniform distribution function,

$$F(\eta) = \frac{\eta - \eta_{\min}}{\eta_{\max} - \eta_{\min}}. \tag{22}$$

We assume that the support of this distribution is broad enough that positive fractions of voters favor each party no matter what are the beliefs about the pliable policy environment and the parties' positions. In this case, (7) can be re-written as

$$\begin{aligned}(\eta_{\max} - \eta_{\min})\, s^L = {} & \lambda^I \left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right) - \eta_{\min}\right] \\ & + \left(1-\lambda^I\right)\left\{\int_{\eta_{\min}}^{u\left(m^L,\theta^L\right)-u\left(m^{LR},\theta^L\right)} [1-\pi(\eta)]\, d\eta + \int_{\eta_{\min}}^{u\left(m^{RL},\theta^R\right)-u\left(m^R,\theta^R\right)} \pi(\eta)\, d\eta\right\}.\end{aligned}$$

Using (8) and Lemma 1, this implies that the vote share function $s^L\left(m^L, m^R\right)$ satisfies

$$\lambda^I \eta_{\min} + (\eta_{\max} - \eta_{\min})\, s^L\left(m^L, m^R\right) \equiv V^L\left(m^L\right) - V^R\left(m^R\right),$$

where

$$V^L(m) = \lambda^I u\left(m, \theta^I\right) + \left(1-\lambda^I\right)\begin{cases} \int_{\eta_{\min}}^{u(m,\underline{\theta})-u(\overline{m},\underline{\theta})} [1-\pi(\eta)]\, d\eta & \text{for } m \le \hat{m} \\ \int_{\eta_{\min}}^{u(m,\overline{\theta})-u(\underline{m},\overline{\theta})} [1-\pi(\eta)]\, d\eta & \text{for } m \ge \hat{m} \end{cases}$$

and

$$V^R(m) = \lambda^I u\left(m, \theta^I\right) - \left(1-\lambda^I\right)\begin{cases} \int_{\eta_{\min}}^{u(\overline{m},\underline{\theta})-u(m,\underline{\theta})} \pi(\eta)\, d\eta & \text{for } m \le \hat{m} \\ \int_{\eta_{\min}}^{u(\underline{m},\overline{\theta})-u(m,\overline{\theta})} \pi(\eta)\, d\eta & \text{for } m \ge \hat{m} \end{cases}.$$

Note that $V^L(\cdot)$ is a function of m^L but not m^R, whereas $V^R(\cdot)$ is a function of m^R but not m^L. It follows that L chooses m^L to maximize $V^L\left(m^L\right)$ while R chooses m^R to maximize $V^R\left(m^R\right)$. It is evident that, in this case, each party has a dominant strategy and therefore the solution to the simultaneous-play game in $\left\{m^L, m^R\right\}$ is the same as the solution to a game with sequential play, independently of which party moves first.

The following example using the uniform distribution in (22) illustrates the possibility of an equilibrium with divergent policies and polarization in fake news.

Example

Suppose $\underline{\theta} = 1$ and $\overline{\theta} = 2$. Let the utility function be given by

$$u\left(m, \theta\right) = m - \frac{1}{2\theta} m^2.$$

These imply $\underline{m} = 1$ and $\overline{m} = 2$. In addition,

$$\begin{aligned} u\left(\underline{m}, \underline{\theta}\right) &= \frac{1}{2};\ u\left(\overline{m}, \underline{\theta}\right) = 0; \\ u\left(\overline{m}, \overline{\theta}\right) &= 1;\ u\left(\underline{m}, \overline{\theta}\right) = \frac{3}{4}; \\ \hat{m} &= \sqrt{3} \simeq 1.73. \end{aligned}$$

Now assume that $\lambda^I = 0.4$, $\theta^I = \left(\sqrt{3} + 2\right)/2 \simeq 1.87$, and $\pi\left(\cdot\right)$ takes the form

$$\pi\left(\eta\right) = \frac{e^\eta}{e^\eta + 5}.$$

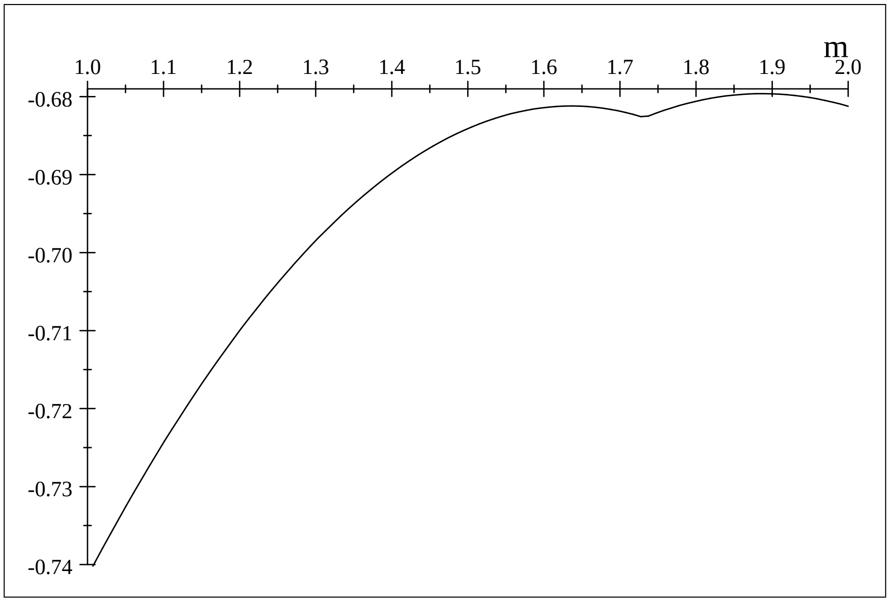
Using these properties and values, we obtain

$$V^L\left(m\right) - \left(1 - \lambda^I\right)\left[\ln\left(e^{\eta_{\min}} + 5\right) - \eta_{\min}\right] =$$
$$\lambda^I\left(m - \frac{1}{2\theta^I} m^2\right) + \left(1 - \lambda^I\right)\begin{cases} m - \frac{1}{2}m^2 - \ln\left(e^{m - \frac{1}{2}m^2} + 5\right) & \text{for } m \leq \hat{m}, \\ m - \frac{1}{4}m^2 - \frac{3}{4} - \ln\left(e^{m - \frac{1}{4}m^2 - \frac{3}{4}} + 5\right) & \text{for } m \geq \hat{m}, \end{cases},$$

$$V^R\left(m\right) = \lambda^I\left(m - \frac{1}{2\theta^I} m^2\right) - \left(1 - \lambda^I\right)\begin{cases} \ln\left(e^{-m + \frac{1}{2}m^2} + 5\right) & \text{for } m \leq \hat{m}, \\ \ln\left(e^{\frac{3}{4} - m + \frac{1}{4}m^2 -} + 5\right) & \text{for } m \geq \hat{m}. \end{cases}$$

The first figure below plots $V^R(m)$, which takes the form

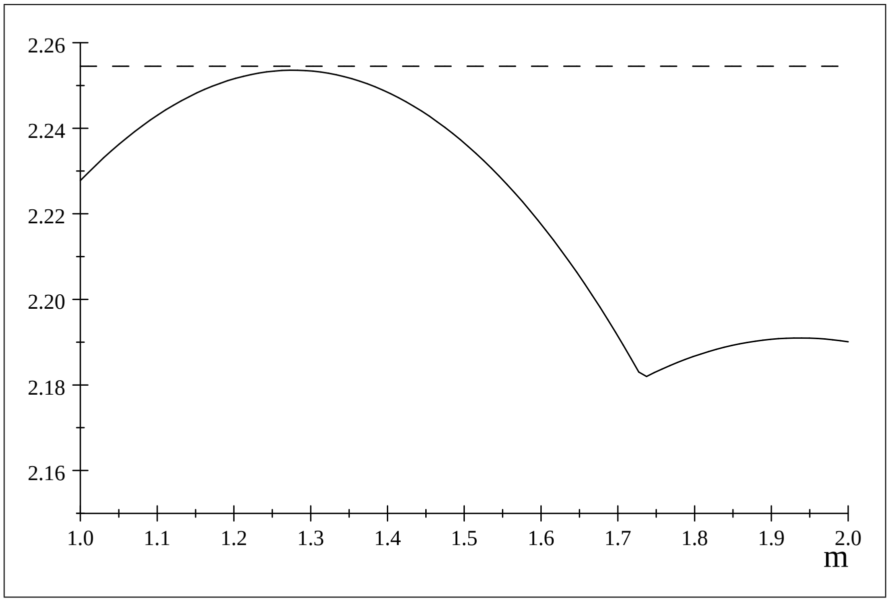
$$V^R(m) = \max\{0.4\left(m - \frac{1}{\sqrt{3}+2}m^2\right) - (1-0.4)\ln\left(\exp\left(-\left(m - m^2/2\right)\right) + 5\right),$$
$$0.4\left(m - \frac{1}{\sqrt{3}+2}m^2\right) - (1-0.4)\ln\left(\exp\left(3/4 - \left(m - m^2/4\right)\right) + 5\right)\}.$$



In this case, the optimal (dominant) strategy for party R is $m^R \approx 1.9 > \hat{m}$. The next figure plots

$$V^L(m) = 2.771\,6$$
$$+\max\{0.4\left(m - \frac{1}{\sqrt{3}+2}m^2\right) + (1-0.4)\left(m - \frac{1}{2}m^2 - \ln\left(\exp\left(m - \frac{1}{2}m^2\right) + 5\right)\right),$$
$$0.4\left(m - \frac{1}{\sqrt{3}+2}m^2\right) + (1-0.4)\left(m - \frac{1}{4}m^2 - \frac{3}{4} - \ln\left(\exp\left(m - \frac{1}{4}m^2 - \frac{3}{4}\right) + 5\right)\right)\},$$

where assuming $\eta_{\min} = -3$ yields $\left(1-\lambda^I\right)\left[\ln\left(e^{\eta_{\min}} + 5\right) - \eta_{\min}\right] \simeq 2.7716$.



In this case, the optimal (dominant) strategy for party L is $m^L \simeq 1.3 < \hat{m}$. It follows that, in equilibrium, the parties choose positions on opposite sides of $\hat{m}$. The different incentives for the two parties reflect the asymmetry in the probability function, $\pi(\eta)$. This function satisfies

$$\pi(\eta) + \pi(-\eta) = \frac{e^{\eta}}{e^{\eta}+5} + \frac{e^{-\eta}}{e^{-\eta}+5} = \frac{2+5\left(e^{\eta}+e^{-\eta}\right)}{26+5\left(e^{\eta}+e^{-\eta}\right)} < 1,$$

and therefore $\pi(\eta) < 1 - \pi(-\eta)$ for all η. Thus, a voter with a given leaning η toward party R is less likely to hear news from a source biased toward party R than is a voter of comparable leaning $-\eta$ toward party L likely to hear news from a source biased toward party L.

Appendix for Section 5

In Section 5 of the paper, we discuss the case in which the media cannot misrepresent the policy positions adopted by either of the parties. Their fake news concerns only the state of the policy environment. In this case, the vote-share function (7) becomes

$$\begin{aligned} s^L\left(m^L, m^R\right) &= \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] \\ &\quad + \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L,\theta^L\right)-u\left(m^R,\theta^L\right)} \left[1-\pi(\eta)\right] dF(\eta) \\ &\quad + \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L,\theta^R\right)-u\left(m^R,\theta^R\right)} \pi(\eta)\, dF(\eta). \end{aligned}$$

From equation (2) in the main text,

$$\theta^J = \begin{cases} \overline{\theta} & \text{for } m^J > m^{\tilde{J}} \\ \underline{\theta} & \text{for } m^J < m^{\tilde{J}} \end{cases}, \; J = L, R. \tag{23}$$

We therefore have

$$\begin{aligned} s^L_{above}\left(m^L, m^R\right) &= \lambda^I F\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] \\ &+ \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L, \overline{\theta}\right) - u\left(m^R, \overline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] dF\left(\eta\right) \\ &+ \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L, \underline{\theta}\right) - u\left(m^R, \underline{\theta}\right)} \pi\left(\eta\right) dF\left(\eta\right), \end{aligned} \tag{24}$$

$$\begin{aligned} s^L_{below}\left(m^L, m^R\right) &= \lambda^I F\left[u\left(m^L, \theta^I\right) - u\left(m^R, \theta^I\right)\right] \\ &+ \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L, \underline{\theta}\right) - u\left(m^R, \underline{\theta}\right)} \left[1-\pi\left(\eta\right)\right] dF\left(\eta\right) \\ &+ \left(1-\lambda^I\right)\int_{-\infty}^{u\left(m^L, \overline{\theta}\right) - u\left(m^R, \overline{\theta}\right)} \pi\left(\eta\right) dF\left(\eta\right), \end{aligned} \tag{25}$$

where $s^L_{above}\left(m^L, m^R\right)$ is the vote share of party L conditional on $m^L \geq m^R$ and $s^L_{below}\left(m^L, m^R\right)$ is the vote share conditional on $m^L \leq m^R$. Again, party L seeks to maximize s^L while party R seeks to minimize it.

Simultaneous Moves

We first consider the game in which the parties move simultaneously in the first stage. First, we show that there exists no pure-strategy equilibrium with policy convergence. The argument proceeds as follows. Suppose that there exists an $\tilde{m} \in [\underline{m}, \overline{m}]$ such that $m^L = m^R = \tilde{m}$ is an equilibrium in the simultaneous-move game of the first stage. Then $s^L = s^L_0$, where

$$s^L_0 = \lambda^I F\left(0\right) + \left(1-\lambda^I\right)\left\{\int_{-\infty}^{0} \left[1-\pi\left(\eta\right)\right] dF\left(\eta\right) + \int_{-\infty}^{0} \pi\left(\eta\right) dF\left(\eta\right)\right\}. \tag{26}$$

Now consider a deviation by party L to $m^L = \tilde{m} + \varepsilon$, ε small and positive. If $\tilde{m}$ is the equilibrium strategy for party L it has to be the case that s^L_0 is larger or equal to the vote share that party L

would obtain following such a deviation. Using (24) and (26), this implies

$$\begin{aligned}\left(1-\lambda^I\right)\left\{\int_{-\infty}^{0}\left[1-\pi\left(\eta\right)\right]dF\left(\eta\right)+\int_{-\infty}^{0}\pi\left(\eta\right)dF\left(\eta\right)\right\} \quad \geq \quad & \lambda^I F\left[u\left(\tilde{m}+\varepsilon,\theta^I\right)-u\left(\tilde{m},\theta^I\right)\right] \\ & +\left(1-\lambda^I\right)\int_{-\infty}^{u\left(\tilde{m}+\varepsilon,\overline{\theta}\right)-u\left(\tilde{m},\overline{\theta}\right)}\left[1-\pi\left(\eta\right)\right]dF\left(\eta\right) \\ & +\left(1-\lambda^I\right)\int_{-\infty}^{u\left(\tilde{m}+\varepsilon,\underline{\theta}\right)-u\left(\tilde{m},\underline{\theta}\right)}\pi\left(\eta\right)dF\left(\eta\right).\end{aligned}$$

Using a first-order approximation to the right-hand side of this inequality, we obtain

$$0\geq\lambda^I u_m\left(\tilde{m},\theta^I\right)+\left(1-\lambda^I\right)\left\{\left[1-\pi\left(0\right)\right]u_m\left(\tilde{m},\overline{\theta}\right)+\pi\left(0\right)u_m\left(\tilde{m},\underline{\theta}\right)\right\}.$$

A similar analysis for a downward deviation by party L, to $m^L=\tilde{m}-\varepsilon$, ε small and positive, implies

$$0\geq-\lambda^I u_m\left(\tilde{m},\theta^I\right)-\left(1-\lambda^I\right)\left\{\left[1-\pi\left(0\right)\right]u_m\left(\tilde{m},\underline{\theta}\right)+\pi\left(0\right)u_m\left(\tilde{m},\overline{\theta}\right)\right\}.$$

Likewise, a similar analysis of the small deviations available to party R, which seeks to minimize s^L, implies that

$$0\leq-\lambda^I u_m\left(\tilde{m},\theta^I\right)-\left(1-\lambda^I\right)\left\{\left[1-\pi\left(0\right)\right]u_m\left(\tilde{m},\underline{\theta}\right)+\pi\left(0\right)u_m\left(\tilde{m},\overline{\theta}\right)\right\}$$

and

$$0\leq\lambda^I u_m\left(\tilde{m},\theta^I\right)+\left(1-\lambda^I\right)\left\{\left[1-\pi\left(0\right)\right]u_m\left(\tilde{m},\overline{\theta}\right)+\pi\left(0\right)u_m\left(\tilde{m},\underline{\theta}\right)\right\}.$$

The four inequalities together imply

$$0=\lambda^I u_m\left(\tilde{m},\theta^I\right)+\left(1-\lambda^I\right)\left\{\left[1-\pi\left(0\right)\right]u_m\left(\tilde{m},\underline{\theta}\right)+\pi\left(0\right)u_m\left(\tilde{m},\overline{\theta}\right)\right\}$$

and

$$0=\lambda^I u_m\left(\tilde{m},\theta^I\right)+\left(1-\lambda^I\right)\left\{\left[1-\pi\left(0\right)\right]u_m\left(\tilde{m},\overline{\theta}\right)+\pi\left(0\right)u_m\left(\tilde{m},\underline{\theta}\right)\right\},$$

and thus

$$\left(1-\lambda^I\right)\left[1-2\pi\left(0\right)\right]\left[u_m\left(\tilde{m},\overline{\theta}\right)-u_m\left(\tilde{m},\underline{\theta}\right)\right]=0.$$

Supermodularity implies $u_m(\tilde{m},\overline{\theta}) > u_m(\tilde{m},\underline{\theta})$. It follows that, for $\lambda^I < 1$, this condition can be satisfied if and only if $\pi(0) = 1/2$. We therefore conclude that whenever some individuals are ill-informed and $\pi(0) \neq 1/2$, there exists no pure-strategy equilibrium with platform convergence.

We next examine the conditions for a pure-strategy equilibrium with divergent policies. Without loss of generality, we consider the case in which $m^L > m^R$. In this case, party L maximizes $s^L_{above}(m^L, m^R)$ and its optimal choice of policy satisfies

$$\begin{aligned}
&\lambda^I f\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] u_m\left(m^L,\theta^I\right) \\
&\quad + \left(1-\lambda^I\right)\left\{1-\pi\left[u\left(m^L,\overline{\theta}\right) - u\left(m^R,\overline{\theta}\right)\right]\right\} f\left[u\left(m^L,\overline{\theta}\right) - u\left(m^R,\overline{\theta}\right)\right] u_m\left(m^L,\overline{\theta}\right) \\
&\qquad + \left(1-\lambda^I\right)\pi\left[u\left(m^L,\underline{\theta}\right) - u\left(m^R,\underline{\theta}\right)\right] f\left[u\left(m^L,\underline{\theta}\right) - u\left(m^R,\underline{\theta}\right)\right] u_m\left(m^L,\underline{\theta}\right) = 0.
\end{aligned}$$

Also, since R minimizes s^L, its best response satisfies

$$\begin{aligned}
&-\lambda^I f\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] u_m\left(m^R,\theta^I\right) \\
&\quad - \left(1-\lambda^I\right)\left\{1-\pi\left[u\left(m^L,\overline{\theta}\right) - u\left(m^R,\overline{\theta}\right)\right]\right\} f\left[u\left(m^L,\overline{\theta}\right) - u\left(m^R,\overline{\theta}\right)\right] u_m\left(m^R,\overline{\theta}\right) \\
&\qquad - \left(1-\lambda^I\right)\pi\left[u\left(m^L,\underline{\theta}\right) - u\left(m^R,\underline{\theta}\right)\right] f\left[u\left(m^L,\underline{\theta}\right) - u\left(m^R,\underline{\theta}\right)\right] u_m\left(m^R,\underline{\theta}\right) = 0.
\end{aligned}$$

Moreover, since each party can mimic the policy of its rival, the resulting vote share s^L cannot be larger than or smaller than the share that what obtains when $m^L = m^R$. This implies

$$\begin{aligned}
s^L_0 &= \lambda^I F\left[u\left(m^L,\theta^I\right) - u\left(m^R,\theta^I\right)\right] \\
&\quad + \left(1-\lambda^I\right)\left\{\int_{-\infty}^{u\left(m^L,\overline{\theta}\right)\ -u\left(m^R,\overline{\theta}\right)} \left[1-\pi(\eta)\right] dF(\eta) + \int_{-\infty}^{u\left(m^L,\underline{\theta}\right)\ -u\left(m^R,\underline{\theta}\right)} \pi(\eta)\, dF(\eta)\right\}.
\end{aligned}$$

The last three equations constitute a nonlinear system that would need to be satisfied by the two variables, m^L and m^R, with $m^L > m^R$. Generically, no such solution exists. We therefore conclude

Lemma 2 *Let the policy choices m^J, $J = L, R$, be known to all voters, let the fraction of informed voters be smaller than one, i.e., $\lambda^I < 1$, and let $\pi(0) \neq 1/2$. Then, if both parties move simultaneously in the first stage of the game, there exists (generically) no equilibrium in pure strategies.*

We have not been able to derive interesting insights about the properties of mixed-strategy equilibria for this game.

Sequential Moves

We now consider the game in which an incumbent, say party R, moves before the challenger when choosing its position in the initial stage of the game. To characterize the best response for the challenger, party L, observe that this party obtains the vote share $s^L_{above}\left(m^L, m^R\right)$ if it chooses $m^L > m^R$ and the share $s^L_{below}\left(m^L, m^R\right)$ if it chooses $m^L < m^R$, where $s^L_{above}\left(m^L, m^R\right)$ and $s^L_{below}\left(m^L, m^R\right)$ are given in (24) and (25), respectively. The largest vote share that party L can secure in response to m^R is

$$s^L_{\max}\left(m^R\right) = \max\left\{\max_{m\in[\underline{m},m^R]} s^L_{below}\left(m, m^R\right), \max_{m\in[m^R,\overline{m}]} s^L_{above}\left(m, m^R\right)\right\}. \tag{27}$$

Under these conditions, the equilibrium strategy of party R is

$$m^R = \arg\min_{m\in[\underline{m},\overline{m}]} s^L_{\max}\left(m^R\right). \tag{28}$$

In this sequential game, the existence of an equilibrium in pure strategies is assured.

Next, note that party L always has the option to choose $m^L = m^R$, in which case its vote share would be s^L_0, defined in (26). For this reason, the equilibrium vote share satisfies $s^L \geq s^L_0$ and if there exists an m^R such that $s^L_{\max}\left(m^R\right) = s^L_0$, this m^R must be an equilibrium play for party R.

We now show that there exists no equilibrium with policy convergence when $\lambda^I < 1$ and $\pi(0) < 1/2$. The argument proceeds as follows. Suppose there were to exist an $\tilde{m}$ such that $m^R = \tilde{m} \in (\underline{m}, \overline{m})$ and $m^L = \tilde{m}$ are equilibrium plays for the incumbent and the challenger, respectively. Then for $\varepsilon > 0$,

$$s^L_{above}(\tilde{m}+\varepsilon, \tilde{m}) \leq s^L_0 = s^L_{above}(\tilde{m}, \tilde{m}),$$

$$s^L_{below}(\tilde{m}-\varepsilon, \tilde{m}) \leq s^L_0 = s^L_{below}(\tilde{m}, \tilde{m}).$$

Using (24) and (25), these inequalities imply

$$0 \geq \lambda^I u_m\left(\tilde{m}, \theta^I\right) + \left(1-\lambda^I\right)\left\{[1-\pi(0)]\, u_m\left(\tilde{m}, \overline{\theta}\right) + \pi(0)\, u_m\left(\tilde{m}, \underline{\theta}\right)\right\}, \tag{29}$$

$$0 \geq -\lambda^I u_m\left(\tilde{m},\theta^I\right) - \left(1-\lambda^I\right)\left\{\left[1-\pi\left(0\right)\right]u_m\left(\tilde{m},\underline{\theta}\right) + \pi\left(0\right)u_m\left(\tilde{m},\overline{\theta}\right)\right\}. \tag{30}$$

Summing the two inequalities yields

$$0 \geq \left(1-\lambda^I\right)\left[1-2\pi\left(0\right)\right]\left[u_m\left(\tilde{m},\overline{\theta}\right) - u_m\left(\tilde{m},\underline{\theta}\right)\right].$$

The supermodularity of $u\left(m,\theta\right)$ implies $u_m\left(\tilde{m},\overline{\theta}\right) > u_m\left(\tilde{m},\underline{\theta}\right)$. Therefore, this last inequality must be violated whenever $\lambda^I < 1$ and $\pi\left(0\right) < 1/2$. It is also easy to verify that neither $\tilde{m} = \underline{m}$ nor $\tilde{m} = \overline{m}$ can arise in equilibrium. For example, suppose that $\tilde{m} = \underline{m}$. Then only a deviation to $m^L = \underline{m} + \varepsilon$ is possible, in which case only (29) has to be satisfied, which becomes

$$0 \geq \lambda^I u_m\left(\underline{m},\theta^I\right) + \left(1-\lambda^I\right)\left\{\left[1-\pi\left(0\right)\right]u_m\left(\underline{m},\overline{\theta}\right) + \pi\left(0\right)u_m\left(\underline{m},\underline{\theta}\right)\right\}.$$

But since $u_m\left(\underline{m},\theta^I\right) > 0$, $u_m\left(\underline{m},\overline{\theta}\right) > 0$ and $u_m\left(\underline{m},\underline{\theta}\right) = 0$, this inequality is violated. A similar argument establishes that $\tilde{m} = \overline{m}$ violates (30). We have proven

Lemma 3 *Suppose the parties play sequentially in the first stage of the game and that their choices m^J, $J = L, R$, become known to all voters. Let party R be the first mover and party L the second mover and let the fraction of informed voters be strictly smaller than one, i.e., $\lambda^I < 1$, with $\pi\left(0\right) < 1/2$. Then, there does not exist any equilibrium with platform convergence; i.e. $m^R \neq m^L$.*

We now provide an example that has $\pi\left(\eta\right) > 1 - \pi\left(-\eta\right)$ for all η, so that $\pi\left(0\right) > 1/2$. In this example, there does exist an equilibrium with $m^L = m^R$.

Example

Suppose $\underline{\theta} = 1$ and $\overline{\theta} = 2$. Let the utility function be given by

$$u\left(m,\theta\right) = m - \frac{1}{2\theta}m^2.$$

These imply $\underline{m} = 1$ and $\overline{m} = 2$. Moreover,

$$\begin{aligned} u\left(\underline{m},\underline{\theta}\right) &= \frac{1}{2};\ u\left(\overline{m},\underline{\theta}\right) = 0; \\ u\left(\overline{m},\overline{\theta}\right) &= 1;\ u\left(\underline{m},\overline{\theta}\right) = \frac{3}{4}. \end{aligned}$$

Now assume that $\lambda^I = 0.4$, $\theta^I = \left(\sqrt{3}+2\right)/2 \simeq 1.87$ and that the probability function takes the form

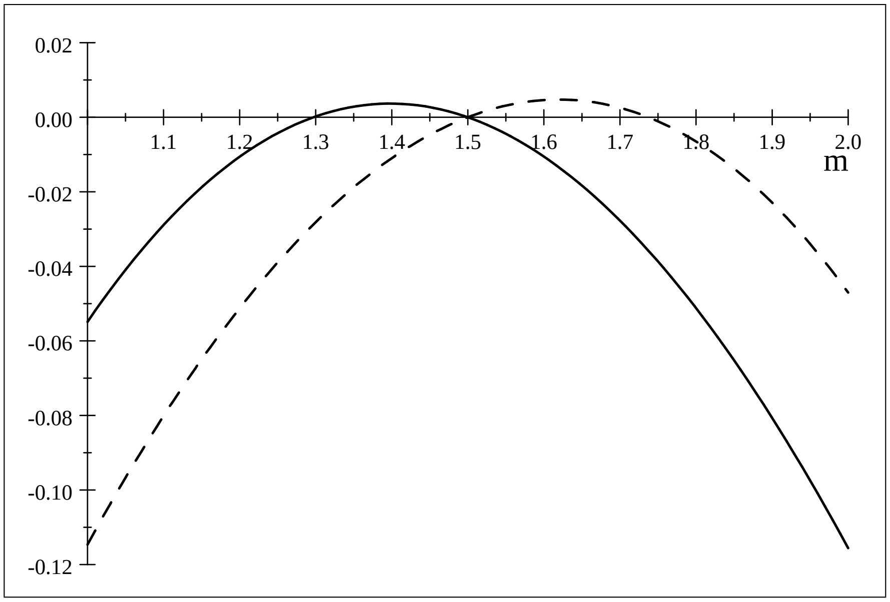
$$\pi(\eta) = \frac{e^{\eta}}{e^{\eta}+0.5}.$$

This probability function implies $\pi(\eta) > 1 - \pi(-\eta)$ for all η and therefore $\pi(0) > 1/2$. Finally, assume that $F(\eta)$ is uniform as in (22). Using these properties and values, we obtain

$$\begin{aligned}
&\eta_{\min} + (\eta_{\max} - \eta_{\min})\, s^L_{above}\left(m, m^R\right) \\
&\equiv 0.4\left\{m - \frac{1}{(\sqrt{3}+2)}m^2 - \left[m^R - \frac{1}{(\sqrt{3}+2)}\left(m^R\right)^2\right]\right\} \\
&+ 0.6\ln\left(e^{m-\frac{1}{2}m^2-\left[m-\frac{1}{2}\left(m^R\right)^2\right]} + 0.5\right) \\
&+ 0.6\left\{\left[m - \frac{1}{4}m^2 - \left[m^R - \frac{1}{4}\left(m^R\right)^2\right] - \ln\left(e^{m-\frac{1}{4}m^2-\left[m^R-\frac{1}{4}\left(m^R\right)^2\right]} + 0.5\right)\right]\right\},
\end{aligned}$$

$$\begin{aligned}
&\eta_{\min} + (\eta_{\max} - \eta_{\min})\, s^L_{below}\left(m, m^R\right) \\
&\equiv 0.4\left\{m - \frac{1}{(\sqrt{3}+2)}m^2 - \left[m^R - \frac{1}{(\sqrt{3}+2)}\left(m^R\right)^2\right]\right\} \\
&+ 0.6\ln\left(e^{m-\frac{1}{4}m^2-\left[m^R-\frac{1}{4}\left(m^R\right)^2\right]} + 0.5\right) \\
&+ 0.6\left\{\left[m - \frac{1}{2}m^2 - \left[m^R - \frac{1}{2}\left(m^R\right)^2\right] - \ln\left(e^{m-\frac{1}{2}m^2-\left[m^R-\frac{1}{2}\left(m^R\right)^2\right]} + 0.5\right)\right]\right\},
\end{aligned}$$

In Figure 3, reproduced below, the dashed curve plots $\eta_{\min} + (\eta_{\max} - \eta_{\min})\, s^L_{below}$ while the the solid curve plots $\eta_{\min} + (\eta_{\max} - \eta_{\min})\, s^L_{above}$ for $m^R = 1.5$. As is clear from the figure, $m^L = m^R$ is the best response of party L to $m^R = 1.5$. That is, the example has an equilibrium with convergent platforms.



We next show that, for $\pi(0) = 1/2$, there exists a *unique* equilibrium that is characterized by policy convergence. First note that $m^L = m^R$ is never an equilibrium if and only if for all feasible m^R either

$$\lim_{m \searrow m^R} \frac{\partial s^L_{above}\left(m, m^R\right)}{\partial m} > 0$$

or

$$\lim_{m \nearrow m^R} \frac{\partial s^L_{below}\left(m, m^R\right)}{\partial m} < 0.$$

However, for $\pi(0) = 1/2$,

$$\lim_{m \searrow m^R} \frac{\partial s^L_{above}\left(m, m^R\right)}{\partial m} = \lambda^I f(0)\, u_m\left(m^R, \theta^I\right) + \frac{1}{2}\left(1-\lambda^I\right) f(0)\left[u_m\left(m^R, \overline{\theta}\right) + u_m\left(m^R, \underline{\theta}\right)\right],$$

$$\lim_{m \nearrow m^R} \frac{\partial s^L_{below}\left(m, m^R\right)}{\partial m} = \lambda^I f(0)\, u_m\left(m^R, \theta^I\right) + \frac{1}{2}\left(1-\lambda^I\right) f(0)\left[u_m\left(m^R, \underline{\theta}\right) + u_m\left(m^R, \overline{\theta}\right)\right],$$

and therefore

$$\lim_{m \searrow m^R} \frac{\partial s^L_{above}\left(m, m^R\right)}{\partial m} = \lim_{m \nearrow m^R} \frac{\partial s^L_{below}\left(m, m^R\right)}{\partial m} \quad \text{for all } m^R \in \left(\underline{m}, \overline{m}\right).$$

If there exists an $m_0 \in \left(\underline{m}, \overline{m}\right)$ that satisfies

$$\lambda^I u_m\left(m_0, \theta^I\right) + \frac{1}{2}\left(1-\lambda^I\right)\left[u_m\left(m_0, \overline{\theta}\right) + u_m\left(m_0, \underline{\theta}\right)\right] = 0, \tag{31}$$

then party R's optimal strategy is $m^R = m_0$ and party L's best response is $m^L = m^R$, because these plays yield $s^R = 1 - F(0)$ and party R can never fare better than this.

We now show that continuity of $u_m(m, \theta)$ ensures the existence of such an m_0. Define the function

$$\mathcal{M}\left(m, \theta^I, \underline{\theta}, \overline{\theta}\right) \equiv \lambda^I u_m\left(m, \theta^I\right) + \frac{1}{2}\left(1-\lambda^I\right)\left[u_m\left(m, \overline{\theta}\right) + u_m\left(m, \underline{\theta}\right)\right].$$

This function satisfies

$$\mathcal{M}\left(\underline{m}, \theta^I, \underline{\theta}, \overline{\theta}\right) = \lambda^I u_m\left(\underline{m}, \theta^I\right) + \frac{1}{2}\left(1-\lambda^I\right) u_m\left(\underline{m}, \overline{\theta}\right) > 0$$

and

$$\mathcal{M}\left(\overline{m}, \theta^I, \underline{\theta}, \overline{\theta}\right) = \lambda^I u_m\left(\overline{m}, \theta^I\right) + \frac{1}{2}\left(1-\lambda^I\right) u_m\left(\overline{m}, \underline{\theta}\right) < 0.$$

By continuity, there exists an $m_0 \in (\underline{m}, \overline{m})$ such that $\mathcal{M}\left(m_0, \theta^I, \underline{\theta}, \overline{\theta}\right) = 0$. Moreover, the strict concavity of $u(m, \theta)$ in m implies that m_0 is unique.

Next, consider the case with $\pi(0) > 1/2$. We then have

$$\begin{aligned}
\lim_{m \searrow m^R} \frac{\partial s^L_{above}\left(m, m^R\right)}{\partial m} &= \mathcal{M}_{above}\left(m^R, \theta^I, \underline{\theta}, \overline{\theta}\right) \\
&\equiv \lambda^I f(0)\, u_m\left(m^R, \theta^I\right) + \left(1-\lambda^I\right)\left\{\left[1-\pi(0)\right] f(0)\, u_m\left(m^R, \overline{\theta}\right) + \pi(0) f(0)\, u_m\left(m^R, \underline{\theta}\right)\right\}
\end{aligned}$$

and

$$\begin{aligned}
\lim_{m \nearrow m^R} \frac{\partial s^L_{below}\left(m, m^R\right)}{\partial m} &= \mathcal{M}_{below}\left(m^R, \theta^I, \underline{\theta}, \overline{\theta}\right) \\
&\equiv \lambda^I f(0)\, u_m\left(m^R, \theta^I\right) + \left(1-\lambda^I\right)\left\{\left[1-\pi(0)\right] f(0)\, u_m\left(m^R, \underline{\theta}\right) + \pi(0) f(0)\, u_m\left(m^R, \overline{\theta}\right)\right\}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \mathcal{M}_{above}\left(m^R, \theta^I, \underline{\theta}, \overline{\theta}\right) - \mathcal{M}_{below}\left(m^R, \theta^I, \underline{\theta}, \overline{\theta}\right) \\
= \; & \left[1-2\pi(0)\right]\left[u_m\left(m^R, \overline{\theta}\right) - u_m\left(m^R, \underline{\theta}\right)\right] f(0) < 0 \text{ for all } m^R \in [\underline{m}, \overline{m}].
\end{aligned}$$

Further, define $m_{0,a}$ as the value of m that satisfies

$$\mathcal{M}_{above}\left(m_{0,a}, \theta^I, \underline{\theta}, \overline{\theta}\right) = 0, \tag{32}$$

and $m_{0,b}$ as the value of m that satisfies

$$\mathcal{M}_{below}\left(m_{0,b}, \theta^I, \underline{\theta}, \overline{\theta}\right) = 0. \tag{33}$$

The same argument we used to establish the existence and uniqueness of $m_0 \in (\underline{m}, \overline{m})$ can now be used to establish the existence and uniqueness of $m_{0,a}$ and $m_{0,b}$. Since both $\mathcal{M}_{above}\left(m, \theta^I, \underline{\theta}, \overline{\theta}\right)$ and $\mathcal{M}_{below}\left(m, \theta^I, \underline{\theta}, \overline{\theta}\right)$ are decreasing in m and $\mathcal{M}_{above}\left(m, \theta^I, \underline{\theta}, \overline{\theta}\right) > \mathcal{M}_{below}\left(m, \theta^I, \underline{\theta}, \overline{\theta}\right)$ for all $m \in [\underline{m}, \overline{m}]$, it follows that $m_{0,b} > m_{0,a}$. This implies that, for every $m^R \in (m_{0,a}, m_{0,b})$,

$$\lim_{m \searrow m^R} \frac{\partial s^L_{above}\left(m, m^R\right)}{\partial m} < 0$$

and

$$\lim_{m \nearrow m^R} \frac{\partial s^L_{below}\left(m, m^R\right)}{\partial m} > 0.$$

In this case, the best response by party L to $m^R \in (m_{0,a}, m_{0,b})$ is to choose $m^L = m^R$, leading to $s^L = F(0)$ and $s^R = 1 - F(0)$. For this reason every $m^R \in (m_{0,a}, m_{0,b})$ is an vote-maximizing strategy for party R. As $\pi(0)$ declines, the interval $(m_{0,a}, m_{0,b})$ shrinks and, in the limit, as $\pi(0) \searrow 1/2$, $(m_{0,a}, m_{0,b}) \longrightarrow (m_0, m_0)$.

We summarize these finding in

Lemma 4 *Consider the sequential game in which party R moves first and party L moves second, and both media truthfully report the two policy positions. Then if $\pi(0) = 1/2$, the equilibrium is unique and characterized by policy convergence with $m^L = m^R = m_0$, where m_0 satisfies (31). If $\pi(0) > 1/2$, there exist $m_{0,a}, m_{0,b} \in (\underline{m}, \overline{m})$, $m_{0,a} < m_{0,b}$, where $m_{0,a}$ satisfies (32) and $m_{0,b}$ satisfies (33), such that every $m^R \in (m_{0,a}, m_{0,b})$ is an optimal strategy for party R and the best response by party L is $m^L = m^R$.*

For some purposes, we might want to evaluate alternative equilibria using the welfare criterion $u\left(m, \theta^I\right)$, the utility of the typical voter when accurately informed. Using this criterion, the continuum of equilibria that arises when $\pi(0) > 1/2$ can be normatively ranked.