Labor Market Rigidities, Trade and Unemployment*

Elhanan Helpman
Harvard University and CIFAR

Oleg Itskhoki
Harvard University

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Abstract

We study a two-country two-sector model of international trade in which one sector produces homogeneous products while the other produces differentiated products. The differentiated-product industry has firm heterogeneity, monopolistic competition, search and matching in its labor market, and wage bargaining. Some of the workers searching for jobs end up being unemployed. Countries are similar except for frictions in their labor markets, which include efficiency of matching, cost of vacancies, firing costs, and unemployment benefits. We study the interaction of labor market rigidities and trade impediments in shaping welfare, trade flows, productivity, and unemployment. We show that both countries gain from trade but that the flexible country—which has lower labor market frictions—gains proportionately more. A flexible labor market confers comparative advantage; the flexible country exports differentiated products on net. A country benefits from lowering frictions in its labor market, but this harms the country's trade partner. And the simultaneous proportional lowering of labor market frictions in both countries benefits both of them. The model generates rich patterns of unemployment. In particular, better labor market institutions do not ensure lower unemployment, and unemployment and welfare can both rise in response to a policy change or falling trade costs.

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1 Introduction

International trade and international capital flows link national economies. Although such links are considered to be beneficial for the most part, they produce an interdependence that occasionally has harmful effects. In particular, shocks that emanate in one country may negatively impact trade partners. On the trade side, links through terms-of-trade movements have been widely studied, and it is now well understood that, say, capital accumulation or technological change can worsen a trade partner's terms of trade and reduce its welfare. On the macro side, the transmission of real business cycles has been widely studied, such as the impact of technology shocks in one country on income fluctuations in its trade partners.

Although a large literature addresses the relationship between trade and unemployment, we fall short of understanding how these links depend on labor market institutions. There is growing awareness that institutions affect comparative advantage and trade flows. Levchenko (2007), Nunn (2007) and Costinot (2006) provide evidence on the impact of legal institutions, while Cuñat and Melitz (2007) and Chor (2006) provide evidence on the impact of labor market institutions.

Indeed, measures of labor market flexibility developed by Botero et al. (2004) differ greatly across countries.[1] The rigidity of employment index, which is an average of three other indexes—difficulty of hiring, difficulty of firing, and rigidity of hours—shows wide variation in its range between zero and one hundred (where higher values represent larger rigidities). Importantly, countries with very different development levels may have similar labor market rigidities. For example, Chad, Morocco and Spain have indexes of 60, 63 and 63, respectively, which are about twice the average for the OECD countries (which is 33.3) and higher than the average for sub-Saharan Africa. The United States has the lowest index, equal to zero, while Australia has an index of three and New Zealand has an index of seven, all significantly below the OECD average. Yet some of the much poorer countries also have very flexible labor markets, e.g., both Uganda and Togo have an index of seven.

We develop in this paper a two-country model of international trade in order to study the effects of labor market frictions on trade flows, productivity, welfare and unemployment. We are particularly interested in the impact of a country's labor market rigidities on its trade partner, and the differential impact of lower trade impediments on countries with different labor market institutions. Blanchard and Wolfers (2000) emphasize the need to allow for interactions between shocks and differences in labor market institutions in order to explain the evolution of unemployment in European economies. They show that these interactions are empirically important. On the other side, Nickell et al. (2002) emphasize changes over time in labor market institutions as important determinants of the evolution of unemployment in OECD countries. While these studies use rich data on labor market institutions, our theoretical model parametrizes labor market rigidities in a

[1] Their original data has been updated by the World Bank and is now available at http://www.doingbusiness.org/ExploreTopics/EmployingWorkers/. The numbers reported in the text come from this site, downloaded on May 20, 2007. It is important to note that other measures of labor market characteristics are available for OECD countries; see Nickell (1997) and Blanchard and Wolfers (2000).

simple way, which can be related to a variety of labor market features, such as the cost of vacancies, the efficiency of matching in labor markets, firing costs and unemployment insurance. We focus the analysis on search and matching frictions in Sections 2-6, and show in Section 7 how the results generalize to economies with firing costs and unemployment benefits. We show, however, that even the simpler search and matching frictions generate rich patterns of unemployment in response to both variation across countries in labor market rigidities and changes in trade impediments.

The literature on trade and unemployment is large and varied. One strand of this literature considers economies with minimum wages, of which Brecher (1974) represents an early contribution.[2] Another approach, due to Matusz (1986), uses implicit contracts. A third approach, exemplified by Copland (1989), incorporates efficiency wages into trade models.[3] Yet another line of research uses fair wages. Agell and Lundborg (1995) and Kreickemeier and Nelson (2006) illustrate this approach. The final approach uses search and matching in labor markets. While two early studies extended the two-sector model of Jones (1965) to economies with this type of labor market friction,[4] Davidson, Martin and Matusz (1999) provide a particularly valuable analysis of international trade with labor markets that are characterized by Diamond-Mortensen-Pissarides-type search and matching frictions.[5] In their model differences in labor market frictions, both across sectors and across countries, generate Ricardian-type comparative advantage.[6]

Our two-sector model incorporates Diamond-Mortensen-Pissarides-type frictions into a sector that produces differentiated products; another sector manufactures homogeneous goods under constant returns to scale. In the differentiated-product sector heterogeneous firms compete monopolistically, as in Melitz (2003). These firms exercise market power in the product market on the one hand, and bargain with workers over wages on the other.[7] As in models with home market effects, it is costly to trade differentiated products. Moreover, there are fixed and variable trade costs. While we conduct most of the analysis under the assumption that there is full employment in the homogeneous sector, we show in Section 6 how the results generalize to economies with unemployment in that sector.[8]

We develop the model in stages. The next section describes demand, product markets, labor markets, and the determinants of wages and profits. In the following section, Section 3, we discuss the structure of equilibrium, focusing on the case in which both countries are incompletely

[2] His approach has been extended by Davis (1998) to study how wages are determined when two countries trade with each other, one with and one without a minimum wage.

[3] See also Brecher (1992) and Hoon (2001).

[4] See Davidson, Martin and Matusz (1988) and Hosios (1990).

[5] See Pissarides (2000) for the theory of search and matching in labor markets.

[6] More work has followed this line of inquiry than the other approaches mentioned in the text. Recent examples include Davidson and Matusz (2006a, 2006b) and Moore and Ranjan (2005).

[7] A surge of papers has incorporated labor market frictions into models with heterogeneous firms. Egger and Kreickemeier (2006) examine trade liberalization in an environment with fair wages and Davis and Harrigan (2007) examine trade liberalization in an environment with efficiency wages; both papers focus on the wage dispersion of identical workers across heterogeneous firms in symmetric countries. Mitra and Ranjan (2007) examine offshoring in an environment with search and matching and Felbermayr, Prat and Schmerer (2008) study trade in a one-sector model with search and matching and symmetric countries.

[8] There we also show how the results generalize to economies with variable trade costs in the homogeneous sector.

specialized, and—as in Melitz (2003)—only a fraction of firms export in the differentiated-product industries and some entrants exit those industries. This is followed by an analysis of the impact of labor market frictions on trade, welfare, and productivity in Section 4. There we also study the differential impact of lower trade impediments on countries with different labor market institutions. Importantly, we show that both countries gain from trade in welfare terms and in terms of total factor productivity, independently of trade costs and differences in labor market institutions.[9] However, the country with lower frictions in the labor market gains from trade proportionately more. The lowering of labor market frictions in one country raises its welfare, but it harms the trade partner. Nevertheless, both countries benefit from simultaneous proportional improvements in labor market institutions across the world.

By lower frictions in its labor market a country gains a competitive advantage in the differentiated sector, which is reminiscence of a productivity improvement. As a result, it attracts more firms into this sector while the foreign country attracts fewer firms into this sector. The entry and exit of firms overwhelms the terms of trade movement, leading to welfare gains in the country with improved labor market frictions and welfare losses in its trade partner.

In Section 4 we also show that labor market flexibility is a source of comparative advantage. The flexible country has a larger fraction of exporting firms and it exports differentiated products on net. Moreover, the share of intra-industry trade is smaller and the total volume of trade is larger the larger are the differences in labor market rigidities. We also show that welfare and productivity are higher in the more flexible country.

In Section 5 we take up unemployment. We show that the relationship between unemployment and labor market rigidities is hump-shaped when the countries are symmetric. An improvement in labor market institutions decreases the sectoral rate of unemployment and induces more workers to search for jobs in the differentiated-product sector, which has the higher sectoral rate of unemployment. These two effects impact unemployment in opposite directions, with the latter dominating in highly rigid labor markets and the former dominating in highly flexible labor markets. As a result, unemployment initially increases and then decreases as labor market institutions improve, starting from high levels of rigidity. We also show that if a single country improves its labor market institutions this reduces unemployment in the country's trading partner, by inducing a labor reallocation from the differentiated-product sector to the homogeneous-product sector.

We also show that lowering trade impediments can increase unemployment in one or both countries, despite its positive welfare effects, and that the interaction between trade impediments and labor market rigidities produces rich patterns of unemployment. Specifically, differences in rates of unemployment do not necessarily reflect differences in labor market institutions; the flexible country can have higher or lower unemployment, depending on the height of trade impediments and the levels of labor market frictions.

[9] We also show that the combination of variable trade costs and differences in labor market institutions have to satisfy a certain condition for the equilibrium to have incomplete specialization in both countries. However, the welfare results extend to cases with partial or full specialization. Moreover, we show in Section 7 how these welfare results have to be qualified in the presence of unemployment benefits.

The unemployment results depend on certain structural features of the model, while the welfare, productivity, and trade pattern results are less sensitive to these characteristics. In particular, the impact of trade liberalization on unemployment as a function of differences in labor market rigidities, depends on the assumption that there are labor market rigidities in the differentiated-product sector but not in the homogeneous-product sector. Under these circumstances trade liberalization induces an expansion of activity in the sector with the higher *sectoral* rate of unemployment. These results are generalized, in Section 6, by introducing unemployment into the homogeneous-product sector. In particular, as long as the sectoral rate of unemployment is higher in the differentiated sector, the results do not change. But in cases in which it is higher in the homogeneous sector, the response of unemployment to shocks changes.

In Section 7 we add firing costs and unemployment benefits to the menu of labor market frictions, and we discuss conditions under which the previous results remain valid, as well as how the results change when these conditions are not satisfied. In addition, we analyze the impact of reforms in firing costs and unemployment benefits on resource allocation, unemployment and welfare of the reforming country and its trade partner. The last section summarizes some of the main insights from this analysis.

2 Preliminaries

We develop in this section the building blocks of our analytical model. They consist of a demand structure, technologies, product and labor market structures, and determinants of wages and profits. After describing these ingredients in some detail, we discuss in the next two sections general equilibrium interactions in a two-country world. In order to focus on labor market rigidities, we assume that the two countries are identical except for labor market frictions. This means that the demand structure and the technologies are the same in both countries. They can differ in the size of their labor endowment, but this difference is not consequential for the type of equilibrium we discuss in the main text.

2.1 Preferences and Demand

Every country has a representative agent who consumes a homogeneous product q_0 and a continuum of brands of a differentiated product whose real consumption index is Q. The real consumption index of the differentiated product is a constant elasticity of substitution aggregator:

$$Q = \left[\int_{\omega\in\Omega} q(\omega)^{\beta} d\omega\right]^{\frac{1}{\beta}}, \quad 0 < \beta < 1, \tag{1}$$

where $q(\omega)$ represents the consumption of variety ω, Ω represents the set of varieties available for consumption, and β is a parameter that controls the elasticity of substitution between brands.[10]

[10] Alternatively, we could interpret Q to be a homogeneous final product and the $q(\omega)$s to be intermediate inputs.

Consumer preferences between the homogeneous product, q_0, and the real consumption index of the differentiated product, Q, are represented by the quasi-linear utility function[11]

$$\mathbb{U} = q_0 + \frac{1}{\zeta} Q^{\zeta}, \quad 0 < \zeta < \beta.$$

The restriction $\zeta < \beta$ ensures that varieties are better substitutes for each other than for the outside good q_0.[12] We also assume that the consumer has a large enough income level to always consume positive quantities of the outside good, in which case it is convenient to choose the outside good as numeraire, so that its price equals one, i.e., $p_0 = 1$. Under the circumstances $p(\omega)$, the price of brand ω, and P, the price index of the brands, are measured relative to the price of the homogeneous product.[13]

The utility function $\mathbb{U}$ implies that a consumer with spending E who faces the price index P for the differentiated product chooses $Q = P^{-1/(1-\zeta)}$ and $q_0 = E - P^{-\zeta/(1-\zeta)}$.[14] As a result, the demand function for brand ω can be expressed as

$$q(\omega) = Q^{-\frac{\beta-\zeta}{1-\beta}} p(\omega)^{-\frac{1}{1-\beta}} \tag{2}$$

and the *indirect* utility function as

$$\mathbb{V} = E + \frac{1-\zeta}{\zeta} P^{-\frac{\zeta}{1-\zeta}} = E + \frac{1-\zeta}{\zeta} Q^{\zeta}. \tag{3}$$

As usual, the indirect utility function is increasing in spending and declining in price. A higher price index P reduces the demand for Q, and—holding expenditure E constant—reduces welfare. This decline in welfare results from the fact that consumer surplus, $(1-\zeta) P^{-\zeta/(1-\zeta)}/\zeta = (1-\zeta) Q^{\zeta}/\zeta$, declines as P rises and Q falls. In what follows, we characterize equilibrium values of Q, from which we infer welfare levels.

Note from (2) that the demand for every variety decreases in its own price and in the real consumption index Q. The latter results from the fact that higher Q implies that the differentiated-product market is more competitive, because the price index P is lower, and P is lower either because prices of competing brands are lower or because there is more variety available in the market.

[11] Alternatively, we could use a homothetic utility function in q_0 and Q; see Appendix for a discussion of this case.

[12] This model can be analyzed without the restriction $\zeta > 0$. This assumption, however, allows us to avoid discussing alternative special cases and brings out some of the interesting results in a clear way.

[13] The price index P is given by

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{-\frac{\beta}{1-\beta}} d\omega \right]^{-\frac{1-\beta}{\beta}}.$$

[14] The assumption that consumer spending on the outside good is positive is equivalent to assuming $E > P^{-\zeta/(1-\zeta)}$. Since $\zeta > 0$, the demand for Q is elastic and total spending PQ rises when P falls.

2.2 Technologies and Market Structure

All goods are produced with labor, which is the only factor of production. The homogeneous product requires one unit of labor per unit output and the market for this product is competitive. When h_0 workers are employed in the production of the homogeneous product, its output level equals h_0.

The market for brands of the differentiated product is monopolistically competitive. A firm that seeks to supply a brand ω bears an entry cost f_e in terms of the homogeneous good, which covers the technology cost and the cost of setting up shop in the industry. After bearing this cost, the firm learns how productive its technology is, as measured by θ; a θ-firm requires $1/\theta$ workers per unit output. In other words, if a θ-firm employs h workers it produces θh units of output. Before entry the firm expects θ to be drawn from a known cumulative distribution $G_\theta(\theta)$.

After entry the firm has to bear a fixed production cost f_d in terms of the homogeneous good; without it no manufacturing is possible. Following Melitz (2003), we assume that the differentiated-product sector bears a fixed cost of exporting f_x in terms of the homogeneous product. In addition, it bears a variable cost of exporting of the melting-iceberg type: $\tau > 1$ units have to be exported for one unit to arrive in the foreign country. As is common in models with home market effects, we assume that there are no trade frictions in the homogeneous-product sector.[15]

We label the two countries A and B. If a country-j firm, $j = A, B$, with productivity θ hires h_j workers and chooses to serve only the domestic market, then (2) implies that its revenue equals

$$R_j = Q_j^{-(\beta-\zeta)} \Theta^{1-\beta} h_j^{\beta},$$

where $\Theta \equiv \theta^{\beta/(1-\beta)}$ is a transformed measure of productivity that is more convenient for our analysis. If, instead, this firm chooses also to export, then it has to allocate output θh_j across the domestic and foreign market; $\theta h_j = q_{dj} + q_{xj}$, where q_{dj} represents the quantity allocated to the domestic market and q_{xj} represents the quantity allocated to the export market. From (2) these quantities have to satisfy

$$q_{dj} = Q_j^{-\frac{\beta-\zeta}{1-\beta}} p_{dj}^{-\frac{1}{1-\beta}} \quad \text{and} \quad q_{xj} = \tau Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}} \left(\tau p_{xj}\right)^{-\frac{1}{1-\beta}}.$$

In this specification $(-j)$ is the index of the country other than j, while p_{dj} and p_{xj} are producer prices of home and foreign sales, respectively. Note that when exports are priced at p_{xj}, consumers in the foreign country pay an effective price of τp_{xj} due to the variable export costs. Under the circumstances they demand $Q_{(-j)}^{-(\beta-\zeta)/(1-\beta)} \left(\tau p_{xj}\right)^{-1/(1-\beta)}$ consumption units. To deliver these consumption units the supplier has to manufacture q_{xj} units, as shown above. Such a producer maximizes total revenue when marginal revenues are equalized across markets. In the case of constant elasticity of demand functions this requires equalization of producer prices. The resulting

[15] See, however, Section 6 for a discussion of the impact of trade frictions in the homogeneous sector.

total revenue is then

$$R_j = \left[Q_j^{-\frac{\beta-\zeta}{1-\beta}} + \tau^{-\frac{\beta}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}} \right]^{1-\beta} \Theta^{1-\beta} h_j^{\beta}.$$

The revenue function of such a firm can therefore be represented by

$$R_j(\Theta) = \left[Q_j^{-\frac{\beta-\zeta}{1-\beta}} + I_j(\Theta)\, \tau^{-\frac{\beta}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}} \right]^{1-\beta} \Theta^{1-\beta} h_j^{\beta}, \tag{4}$$

where $I_j(\Theta)$ is an indicator variable that equals one if the firm exports and zero otherwise.

2.3 Wages and Profits

There are no labor frictions in the homogeneous-product sector, which means that workers can be replaced there at no cost.[16] As a result, the labor market is competitive and all manufacturers pay the same wages. Moreover, since the market for the final good is also competitive and the value of the marginal product of labor equals one, the wage rate in this industry equals one, i.e., $w_0 = p_0 = 1$.

Unlike the homogeneous-product sector, labor market frictions exist in the differentiated-product industry. In particular, firms in this industry face hiring costs of labor. A Θ-firm from country j that seeks to employ h_j workers bears the hiring cost $b_j h_j$ in terms of the homogeneous good, where b_j is exogenous to the firm, but it depends on labor market conditions to be discussed below. It follows that a worker cannot be replaced without cost. Under these circumstances, a worker inside the firm is not interchangeable with a worker outside the firm, and workers have bargaining power after being hired. Workers exploit this bargaining power in the wage determination process.

We assume that the h_j workers and the firm engage in strategic wage bargaining with equal weights in the manner suggested by Stole and Zwiebel (1996a,b). This leads to the distribution of revenue $R_j(\Theta)$ according to Shapley values. The revenue function (4) than implies that the firm gets a fraction $1/(1+\beta)$ of the revenue and the workers get a fraction $\beta/(1+\beta)$.[17] This result is derived under the assumption that at the bargaining stage a worker's outside option is unemployment, and the value of unemployment is normalized to zero (because there are no unemployment benefits and the model is static). In Section 7 we show how equilibrium wages are determined when unemployment benefits are positive.

Anticipating the outcome of this bargaining game, a Θ-firm that wants to stay in the industry chooses an employment level that maximizes profits. That is, it solves the following problem:

$$\pi_j(\Theta) \equiv \max_{\substack{I_j \in \{0,1\},\\ h_j \geq 0.}} \left\{ \frac{1}{1+\beta} \left[Q_j^{-\frac{\beta-\zeta}{1-\beta}} + I_j \tau^{-\frac{\beta}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}} \right]^{1-\beta} \Theta^{1-\beta} h_j^{\beta} - b_j h_j - f_d - I_j f_x \right\}. \tag{5}$$

[16] See, however, Section 6 for an analysis of frictions in this market.

[17] See Section 7 for an explicit derivation of a more general result, which allows for different levels of bargaining power of workers relative to the firm. Note that the workers' share of revenue is decreasing in the concavity of the revenue function (i.e., increasing in β). A more concave revenue function implies that the loss of a marginal worker results in a smaller reduction in revenue, which reduces the worker's bargaining power.

The solution to this problem implies that the employment level of a Θ-firm in country j can be decomposed into[18]

$$h_j(\Theta) = h_{dj}(\Theta) + I_j(\Theta)\, h_{xj}(\Theta),$$

where $h_{dj}(\Theta)$ represents employment for domestic sales, $h_{xj}(\Theta)$ represents employment for export sales, and

$$\left.\begin{aligned} h_{dj}(\Theta) &= \phi_1^{\frac{1}{\beta}} b_j^{-\frac{1}{1-\beta}} Q_j^{-\frac{\beta-\zeta}{1-\beta}} \Theta, \\ h_{xj}(\Theta) &= \phi_1^{\frac{1}{\beta}} b_j^{-\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}} \Theta, \end{aligned}\right\} \tag{6}$$

where

$$\phi_1 = \left(\frac{\beta}{1+\beta}\right)^{\frac{\beta}{1-\beta}}.$$

Moreover, a country-j firm with productivity Θ pays wages[19]

$$w_j(\Theta) = b_j, \tag{7}$$

and its operating profits are

$$\pi_j(\Theta) = \pi_{dj}(\Theta) + I_j(\Theta)\, \pi_{xj}(\Theta),$$

where $\pi_{dj}(\Theta)$ represents operating profits from domestic sales, $\pi_{xj}(\Theta)$ represents operating profits from export sales, and

$$\left.\begin{aligned} \pi_{dj}(\Theta) &= \phi_1\phi_2 b_j^{-\frac{\beta}{1-\beta}} Q_j^{-\frac{\beta-\zeta}{1-\beta}} \Theta - f_d, \\ \pi_{xj}(\Theta) &= \phi_1\phi_2 b_j^{-\frac{\beta}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}} \Theta - f_x, \end{aligned}\right\} \tag{8}$$

where

$$\phi_2 = \frac{1-\beta}{1+\beta}.$$

Note that higher labor market rigidity, reflected in a higher b_j, reduces proportionately gross operating profits (i.e., not accounting for fixed costs) in the domestic and foreign market. Therefore, an increase in b_j is similar to a proportional reduction in the productivity of all country j's firms.

The profit functions in (8) imply that exporting is profitable if and only if $\pi_{xj}(\Theta) \geq 0$, i.e.,

[18] This convenient decomposition is possible only when output and hiring costs are proportional to employment h_j.

[19] Recall that the wage rate equals the fraction $\beta/(1+\beta)$ of revenue divided by h. Using (6) this implies a wage rate equal to b_j, which is independent of the firm's export status. That is, all firms, exporters and nonexporters alike, pay equal wages. Helpman, Itskhoki and Redding (2008a,b) develop a richer model, in which there is unobserved worker heterogeneity in addition to firm heterogeneity, wages are higher in more productive firms, and exporters pay a wage premium. Bernard and Jensen (1995) and Fariñas and Martín-Marcos (2007) provide evidence to the effect that exporting firms pay higher wages.

there exists a cutoff productivity level, Θ_{xj}, defined by

$$\pi_{xj}(\Theta_{xj}) = 0, \tag{9}$$

such that all firms with productivity above this cutoff export (provided they choose to stay in the industry) and all firms with productivity below it do not export. Firms with low productivity that do not export may nevertheless make money from supplying the domestic market. For this to be the case, their productivity has to be at least as high as Θ_{dj}, implicitly defined by

$$\pi_{dj}(\Theta_{dj}) = 0. \tag{10}$$

We shall consider equilibria in which $\Theta_{xj} > \Theta_{dj} > \Theta_{\min} \equiv \theta_{\min}^{\beta/(1-\beta)}$, where $\theta_{\min}$ is the lowest productivity level in the support of the distribution $G_\theta(\theta)$. That is, equilibria in which high-productivity firms profitably export and supply the domestic market, intermediate-productivity firms cannot profitably export but can profitably supply the domestic market, and low-productivity firms cannot make money and exit. Anticipating this outcome, a prospective firm enters the industry only if expected profits from entry are at least as high as the entry cost f_e. Therefore the free-entry condition is

$$\int_{\Theta_{dj}}^{\infty} \pi_{dj}(\Theta)\, dG(\Theta) + \int_{\Theta_{xj}}^{\infty} \pi_{xj}(\Theta)\, dG(\Theta) = f_e, \tag{11}$$

where $G(\Theta)$ is the distribution of Θ induced by $G_\theta(\theta)$. The first integral represents expected profits from domestic sales, while the second integral represents expected profits from foreign sales. In equilibrium expected profits just equal entry costs.

2.4 Labor Market

A country is populated by families. Each family has a fixed supply of L workers, and the family is the representative consumer whose preferences were described in Section 2.1. We assume that there is a continuum of identical families in every country, and the measure of these families equals one in every country.[20]

A family allocates workers to sectors—N_j workers to the differentiated-product sector and $L-N_j$ workers to the homogeneous-product sector—which determines in which sector every worker searches for work. Once committed to a sector, a worker cannot switch sectors. Thus, there is perfect intersectoral mobility *ex ante* and no mobility *ex post*. The homogeneous-product sector has no labor market frictions and every job pays a wage of one. Therefore workers seeking jobs in this sector expect to be employed with probability one and to obtain a wage $w_0 = 1$.

Unlike the homogeneous-product sector, labor market frictions exist in the differentiated-product sector. Some workers seeking jobs in this sector become unemployed. Let H_j be aggregate em-

[20] When preferences are homothetic rather than quasi-linear, the family interpretation is useful but not essential. See the Appendix for a discussion of homothetic preferences, risk aversion, and ex-post inequality.

ployment in the differentiated sector. Then unemployment is positive in country j when $H_j < N_j$, i.e., not all workers searching for jobs find employment. An individual searching for work in the differentiated-product sector expects to find a job with probability $x_j = H_j/N_j$, where x_j measures the degree of tightness in the sector's labor market. Conditional on finding a job an individual expects to be paid a wage $w_j = b_j$ (see (7)). Therefore the expected income from searching for a job in the differentiated sector is $x_j b_j$.

A family allocates workers to sectors so as to maximize the family's aggregate income. As a result, a family chooses $0 < N_j < L_j$ only if

$$x_j b_j = 1. \tag{12}$$

In other words, in an equilibrium with incomplete specialization, the expected wage rate in the differentiated sector just equals the wage rate in the homogeneous sector. This is similar to the indifference between staying in the countryside and migrating to the city in the Harris and Todaro (1970) model.[21] Unemployment is an equilibrium outcome (i.e., $x_j < 1$) when $b_j > 1$, as we assume.[22]

We now interpret the parameter b_j of the cost-of-hiring function $b_j h_j$; this parameter is exogenous to the firm but endogenous to the industry. As we have seen, N_j workers search for work in the differentiated sector and only H_j of them find jobs. Assuming that to attract workers firms have to post vacancies—which are then only partially filled by individuals searching for jobs—implies that b_j depends on the degree of tightness of the labor market, as measured by x_j. This is a standard implication of the Diamond-Mortensen-Pissarides model of search and unemployment (see, for example, Pissarides (2000)). In particular, following Blanchard and Gali (2008), we assume that

$$b_j = a_j x_j^{\alpha}, \quad a_j > 1 \text{ and } \alpha > 0. \tag{13}$$

We consider a_j to be a measure of frictions in the labor market; higher values of a_j can result from higher costs of vacancies or from less efficient matching between workers and firms.[23] We shall say that a country has better labor market institutions if it has a smaller a_j.

Next note that (12) and (13) uniquely determine the hiring cost b_j and tightness in the labor

[21] A similar condition holds in the Amiti and Pissarides (2005) model, which is otherwise quite different from ours.

[22] One can generalize the model to allow wages to vary in the homogeneous-product sector. A simple modification would be the following: Suppose that the homogeneous-product sector uses labor and a sector-specific input under constant returns to scale. Then the wage rate in this sector, w_{0j}, is a decreasing function of labor employment, $L - N_j$. In this event the right-hand side of (12) has to be replaced with $w_{0j}(L - N_j)$, where $w_{0j}(\cdot)$ is a decreasing function. The other equilibrium conditions do not change.

[23] To justify this formulation, let $a_1 V^{\eta} N^{1-\eta}$, $a_1 > 0$, $0 < \eta < 1$, be a matching function, where V represents aggregate vacancies and N represents the number of individuals searching for work. Then $H = a_1 V^{\eta} N^{1-\eta}$, which implies $V/H = a_1^{-1/\eta} x^{(1-\eta)/\eta}$. It follows that a firm that wants to hire h workers needs to post $v = a_1^{-1/\eta} x^{(1-\eta)/\eta} h$ vacancies. Next assume that the cost of posting v vacancies is $a_2 v$ in terms of the homogeneous good, where $a_2 > 0$ is a parameter. Then a firm that wants to hire h workers has to bear the hiring cost $a x^{\alpha} h$, where $a = a_2/a_1^{1/\eta}$ and $\alpha = (1-\eta)/\eta > 0$. That is, a is rising with the cost of posting vacancies, a_2, and declining with the productivity of the matching technology, a_1. See Blanchard and Gali (2008).

market x_j:

$$\left.\begin{aligned} x_j &= a_j^{-\frac{1}{1+\alpha}}, \\ w_j = b_j &= a_j^{\frac{1}{1+\alpha}}. \end{aligned}\right\} \tag{14}$$

It follows that a country's labor market frictions, as measured by a_j, uniquely determine its wage rate and labor market tightness in the differentiated-product sector. Since $a_j > 1$ in every country, this assumption implies that in each one of them the wage rate paid by differentiated-product firms exceeds one and $x_j < 1$, which implies positive unemployment.

Evidently, the model is bloc recursive, in the sense that the equilibrium wage rate and tightness in the labor market are uniquely determined by labor market rigidities. We show in Section 7 that this property also holds with richer labor market institutions, which include firing costs and unemployment benefits. The implication is that labor market frictions determine (b_j, x_j) in country j, and these in turn impact other endogenous variables, such as trade, welfare and unemployment. In other words, (b_j, x_j) provides a sufficient statistic for labor market rigidities. By varying a_j in (14) we can trace out the feasible set of this sufficient statistic when the only source of labor market rigidity is search and matching.[24] Since b_j is uniquely determined by a_j, i.e., $b_j = a_j^{1/(1+\alpha)}$, and $x_j = 1/b_j$ (see (14)), we treat the derived parameter b_j as the measure of labor market rigidity. Specifically, we shall call a country with lower b_j the flexible country and a country with higher b_j the rigid country.

The economy's rate of unemployment is given by $u_j = (N_j - H_j)/L$, which is a function of the number of individuals searching for jobs in the differentiated-product sector and the employment level in this sector. This unemployment rate can be expressed as

$$u_j = \frac{N_j}{L}(1 - x_j), \tag{15}$$

which is a weighted average of the sectoral unemployment rates, where the weights are the fractions of workers seeking jobs in every sector. Since there is full employment in the homogeneous-product sector, this weighted average equals the share of workers seeking jobs in the differentiated sector, N_j/L_j, times the unemployment rate in that sector, $1 - x_j > 0$. It follows that the unemployment rate can rise either because it rises in the differentiated-product sector or because more individuals search for work in the sector with higher unemployment, which is the differentiated sector.

3 Equilibrium Structure

In the main text we focus on equilibria with incomplete specialization, in which every country produces both homogeneous and differentiated products. Equilibria with specialization are discussed in the Appendix. The purpose of this section is to describe properties of incomplete specialization equilibria. As a result, the analysis in this section is somewhat technical, wile most of the substantive results and their intuition are discuss in subsequent sections.

[24] In Section 7 this set is richer as a result of the presence of firing costs and unemployment benefits.

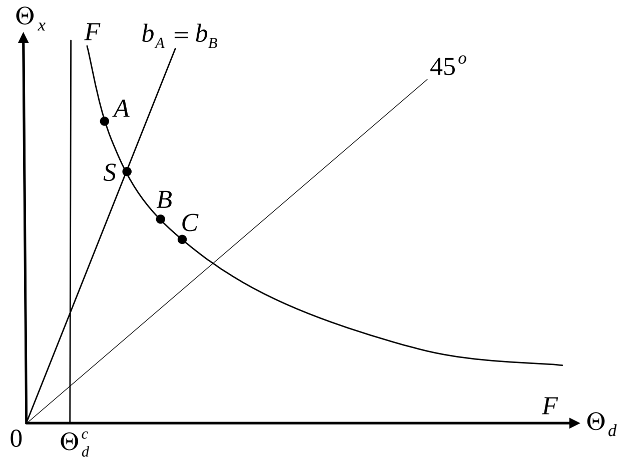

Figure 1: Cutoffs in a trading equilibrium

Equations (8)-(10) yield the following expressions for the domestic market and export cutoffs:

$$\left.\begin{aligned} \Theta_{dj} &= \frac{1}{\phi_1\phi_2} f_d b_j^{\frac{\beta}{1-\beta}} Q_j^{\frac{\beta-\zeta}{1-\beta}}, \\ \Theta_{xj} &= \frac{1}{\phi_1\phi_2} f_x b_j^{\frac{\beta}{1-\beta}} \tau^{\frac{\beta}{1-\beta}} Q_{(-j)}^{\frac{\beta-\zeta}{1-\beta}}. \end{aligned}\right\} \tag{16}$$

Now substitute these expressions into (8) and the resulting profit functions into the free-entry condition (11) to obtain

$$f_d \int_{\Theta_{dj}}^{\infty} \left(\frac{\Theta}{\Theta_{dj}} - 1 \right) dG(\Theta) + f_x \int_{\Theta_{xj}}^{\infty} \left(\frac{\Theta}{\Theta_{xj}} - 1 \right) dG(\Theta) = f_e, \quad j = A, B. \tag{17}$$

This form of the free-entry condition generates a curve in $(\Theta_{dj}, \Theta_{xj})$ space on which every country's cutoffs have to be located, because this curve depends only on the common cost variables and on the common distribution of productivity. Moreover, this curve is downward-sloping, as depicted by FF in Figure 1, and each country has to be located above the 45^o line for the export cutoff to be higher than the domestic cutoff.[25]

Also note that as the export cutoff goes to infinity, the domestic cutoff approaches the cutoff of a closed economy, which is represented by Θ_d^c in the figure. It therefore follows that if the cutoff Θ_d in the closed economy is larger than $\Theta_{\min}$, so is Θ_d in the open economy.[26] Finally note that

[25] Note, from (16), that in a symmetric equilibrium, in which $Q_j = Q_{(-j)}$, the export cutoff is higher if and only if $\tau^{\beta/(1-\beta)} f_x > f_d$, which is the condition required for exporters to be more productive in Melitz (2003). We assume for convenience that this condition is satisfied for all $\tau \geq 1$, in which case $f_x > f_d$.

[26] The autarky production cutoff is the solution to

$$f_d \int_{\Theta_d^c}^{\infty} \left(\frac{\Theta}{\Theta_d^c} - 1 \right) dG(\Theta) = f_e,$$

(16) yields

$$\frac{\Theta_{xj}}{\Theta_{d(-j)}} = \frac{f_x \tau^{\frac{\beta}{1-\beta}}}{f_d} \left[\frac{b_j}{b_{(-j)}}\right]^{\frac{\beta}{1-\beta}}, \quad j = A, B. \tag{18}$$

Equations (17) and (18) can be used for solving the four cutoffs as functions of labor market frictions and cost parameters. As is evident, the cutoffs do not depend on the *levels* of labor market rigidities, only on their relative size. And once the cutoffs have been solved, they can be substituted into (16) to obtain solutions for the real consumption indexes Q_j.

Our primary interest is in the influence of τ, b_A, and b_B on the trading economies. We therefore use (17) and (18) to calculate the impact of these variables on the cutoffs, obtaining

$$\left.\begin{aligned} \hat{\Theta}_{dj} &= \frac{\delta_{xj}}{\Delta}\left[-\left(\delta_{x(-j)} + \delta_{d(-j)}\right)\left(\hat{b}_j - \hat{b}_{(-j)}\right) - \left(\delta_{d(-j)} - \delta_{x(-j)}\right)\hat{\tau}\right], \\ \hat{\Theta}_{xj} &= \frac{\delta_{dj}}{\Delta}\left[\left(\delta_{x(-j)} + \delta_{d(-j)}\right)\left(\hat{b}_j - \hat{b}_{(-j)}\right) + \left(\delta_{d(-j)} - \delta_{x(-j)}\right)\hat{\tau}\right], \end{aligned}\right\} \tag{19}$$

where

$$\delta_{dj} = \frac{f_d}{\Theta_{dj}} \int_{\Theta_{dj}}^{\infty} \Theta dG(\Theta), \quad \delta_{xj} = \frac{f_x}{\Theta_{xj}} \int_{\Theta_{xj}}^{\infty} \Theta dG(\Theta), \quad \Delta = \frac{1-\beta}{\beta}\left(\delta_{dA}\delta_{dB} - \delta_{xA}\delta_{xB}\right).$$

Note that δ_{dj}/ϕ_2 is average revenue per entering firm from domestic sales in country j and δ_{xj}/ϕ_2 is average revenue per entering firm from export sales.[27] Moreover, δ_{dj} equals average gross operating profits (not accounting for fixed costs) per entering firm from domestic sales and δ_{xj} equals average gross operating profits per entering firm from exporting.

It is straightforward to show that $\Delta > 0$.[28] Therefore an increase in a country's relative labor market frictions, say $b_j/b_{(-j)}$, raises the country's export cutoff and reduces its domestic cutoff, in addition to reducing the foreign country's export cutoff and raising the foreign country's domestic cutoff. This establishes

Lemma 1 *Let $b_A > b_B$. Then $\Theta_{dA} < \Theta_{dB}$ and $\Theta_{xA} > \Theta_{xB}$.*

Moreover, an increase in country j's trade cost raises its export cutoff and reduces its domestic

which does not depend on labor market frictions. Note also that $\Theta_d^c > \Theta_{\min}$ if and only if $\left(\bar{\Theta}/\Theta_{\min}\right) > 1 + f_e/f_d$, where $\bar{\Theta}$ is the mean of Θ, which we assume to be satisfied. This results from the fact that the integral on the left-hand side of the above equation is decreasing in Θ_d^c and assumes its largest value of $\left(\bar{\Theta}/\Theta_{\min}\right) - 1$ when $\Theta_d^c = \Theta_{\min}$.

[27] To see this, note that profit maximization (5) implies $\pi_{zj}(\Theta) = \phi_2 R_{zj}(\Theta) - f_z$ for $z = d, x$, where $R_{dj}(\Theta)$ is revenue from domestic sales and $R_{xj}(\Theta)$ is revenue from foreign sales. Then, from the zero profit conditions (9)-(10), we have $R_{zj}(\Theta) = f_z/\phi_2 \cdot \Theta/\Theta_{zj}$. As a result, the average revenue per entering firm equals

$$\int_{\Theta_{zj}}^{\infty} R_{zj}(\Theta) dG(\Theta) = \frac{f_z}{\phi_2 \Theta_{zj}} \int_{\Theta_{zj}}^{\infty} \Theta dG(\Theta) = \frac{\delta_{zj}}{\phi_2}.$$

[28] Proof: To show that $\Delta > 0$, observe that $\Theta_{xj} > \Theta_{dj}$ implies $\delta_{dj}/\delta_{xj} > \left(f_d/\Theta_{dj}\right)/\left(f_x/\Theta_{xj}\right)$ for $j = A, B$. Using these inequalities together with (18) then implies $\delta_{dA}\delta_{dB}/\left(\delta_{xA}\delta_{xB}\right) > \tau^{2\beta/(1-\beta)} > 1$, in which case $\Delta > 0$. Also note that $\delta_{dA}\delta_{dB}/\left(\delta_{xA}\delta_{xB}\right) > \tau^{2\beta/(1-\beta)} > 1$ implies $\delta_{dj} > \delta_{xj}$ in at least one country, and that $\delta_{dj} > \delta_{xj}$ is always satisfied for both countries in a symmetric equilibrium with $b_A = b_B$ and hence by continuity in its vicinity.

cutoff if and only if $\delta_{d(-j)} > \delta_{x(-j)}$. We will shortly show that, indeed, $\delta_{dj} > \delta_{xj}$ in both countries in this type of equilibrium. Therefore an increase in τ raises the export cutoff and reduces the domestic cutoff in both countries.

These insights can be conveniently summarized with the aid of Figure 1. When the two countries have the same labor market rigidities, i.e., $b_A = b_B$, both have cutoffs at point S in the figure, which is the intersection of ray $b_A = b_B$ with FF.[29] If, instead, country A has worse labor market institutions, then A's cutoffs are at point A while B's cutoffs are at point B.[30] The larger the gap in labor market frictions between these countries, the higher A is on the FF curve and the lower B is. In contrast, improvements in the trading environment, which reduce τ, shift down points A and B along the FF curve. These results have important implications for the variation of outcome variables across countries, as well as for the international transmission of shocks, which we discuss below. One immediate implication is that Q_j is higher in the flexible country.[31] Therefore we state:

Lemma 2 *Let $b_A > b_B$. Then $Q_A < Q_B$.*

For our equations to describe an equilibrium with incomplete specialization, it is necessary to ensure positive entry of firms in both countries, i.e., $M_j > 0$ for $j = A, B$, where M_j is the number of firms that enter the differentiated sector in country j. This places restrictions on the permissible difference across countries in labor market rigidities. We now derive implications of these restrictions.

First, recall that $Q_j^{\zeta} = P_j Q_j$ is total spending on differentiated products in country j, and $M_j \delta_{zj}/\phi_2$ is total revenue from domestic sales when $z = d$ and from foreign sales when $z = x$. Since aggregate spending has to equal aggregate revenue, we have[32]

$$Q_j^{\zeta} = M_j \frac{\delta_{dj}}{\phi_2} + M_{(-j)} \frac{\delta_{x(-j)}}{\phi_2}.$$

Having solved for the cutoffs, which uniquely determine the δ_{zj}s, and the real consumption indexes,

[29] The equation of this ray is derived from (16) to be $\Theta_x = \left[\tau^{\beta/(1-\beta)} f_x/f_d\right] \Theta_d$. Therefore its slope is $\tau^{\beta/(1-\beta)} f_x/f_d > 1$.

[30] As a convention, we choose country A to be the rigid country and B to be the flexible country.

[31] Proof: Equation (16) implies $(Q_A/Q_B)^{(\beta-\zeta)/(1-\beta)} = (\Theta_{dA}/\Theta_{dB})(b_B/b_A)^{\beta/(1-\beta)}$. It follows that if, say, B is the flexible country, then $b_B/b_A < 1$, and from the previous analysis, $\Theta_{dA}/\Theta_{dB} < 1$. As a result, $Q_A/Q_B < 1$.

[32] Alternatively, this condition can be derived from (1), the definition of the real consumption index Q_j. A Θ-firm produces an output level $q_{dj}(\Theta) = \Theta^{(1-\beta)/\beta} h_{dj}(\Theta)$ for domestic sales when $\Theta \geq \Theta_{dj}$ and an output level $q_{xj}(\Theta) = \Theta^{(1-\beta)/\beta} h_{xj}(\Theta)$ for export when $\Theta \geq \Theta_{xj}$. Foreign buyers consume only $q_{xj}(\Theta)/\tau$ units of these exportables due to the variable trade costs. Therefore, if a measure M_j of firms have entered the industry in country j, then

$$Q_j = \left[M_j \int_{\Theta_{dj}}^{\infty} \Theta^{1-\beta} h_{dj}(\Theta)^{\beta} dG(\Theta) + M_{(-j)} \tau^{-\beta} \int_{\Theta_{x(-j)}}^{\infty} \Theta^{1-\beta} h_{x(-j)}(\Theta)^{\beta} dG(\Theta) \right]^{\frac{1}{\beta}}.$$

Substituting in the equilibrium values of $h_{dj}(\Theta)$ and $h_{xj}(\Theta)$ from (6) and using (16), yields the equation in the main text.

Q_j, these equations yield the following solution for the number of entrants:

$$M_j = \frac{(1-\beta)\,\phi_2}{\beta\Delta}\left[\delta_{d(-j)}Q_j^{\zeta} - \delta_{x(-j)}Q_{(-j)}^{\zeta}\right]. \tag{20}$$

It is straightforward to show that $\delta_{dA} > \delta_{xA}$ in the rigid country A.[33] Under these circumstances (20) implies $M_B > 0$, because Q_j is larger in the flexible country (see Lemma 2). In addition, (20) implies that a necessary condition for $M_A > 0$ is $\delta_{dB}/\delta_{xB} > (Q_B/Q_A)^{\zeta} > 1$. In other words, in an incomplete specialization equilibrium we have $\delta_{dj} > \delta_{xj}$ for $j = A, B$. Moreover, Lemma 1 implies that δ_{dj} is smaller in the flexible country and δ_{xj} is larger in the flexible country. We therefore have

Lemma 3 *In an equilibrium with incomplete specialization, $\delta_{dj} > \delta_{xj}$ in both countries. Moreover, if $b_A > b_B$, then $\delta_{dA} > \delta_{dB}$ and $\delta_{xA} < \delta_{xB}$.*

This lemma implies that revenues from domestic sales exceeds revenue from exporting, and that revenue from exporting as a share of total revenue is larger in the flexible country.

Equation (20) can also be used to calculate the difference in entry. In particular,

$$M_A - M_B = \frac{(1-\beta)\,\phi_2}{\beta\Delta}\left[(\delta_{dB}+\delta_{xA})\,Q_A^{\zeta} - (\delta_{dA}+\delta_{xB})\,Q_B^{\zeta}\right].$$

Therefore, we have[34]

Lemma 4 *Let $b_A > b_B$. Then $M_A < M_B$.*

We show in the Appendix that for every $\tau > 1$ there exists a unique threshold $\bar{b}(\tau) > 1$, such that $M_j > 0$ for $j = A, B$ if and only if $b_A/b_B < \bar{b}(\tau)$, where A is the rigid country. When $b_A/b_B \geq \bar{b}(\tau)$, the rigid country specializes in homogeneous goods. Evidently, $\bar{b}(\tau)$ provides an upper bound on differences in labor market frictions that support equilibria with production of differentiated products in both countries.[35]

Next consider the determinants of the number of workers searching for jobs in the differentiated sector, N_j, and aggregate employment in that sector, H_j. On the one hand, the wage bill in the differentiated sector, w_jH_j, equals N_j, because the wage rate is $w_j = b_j = 1/x_j$ (see (14)) and $x_j = H_j/N_j$ by definition. This implies that aggregate income equals L, where N_j is derived from the differentiated sector and $L - N_j$ is derived from the homogeneous sector. On the other hand,

[33] Since $\Delta > 0$, it is necessary to have $\delta_{dj} > \delta_{xj}$ in at least one country. However, the rigid country A has a higher export cutoff and a lower domestic cutoff. Therefore $\delta_{dA} > \delta_{xA}$ in the rigid country. Moreover, as shown in footnote 28, $\delta_{dB} > \delta_{xB}$ in the flexible country as well, as long as labor market rigidities do not differ much across countries.

[34] Proof: Let B be the flexible country. Then from Lemmas 2 and 3, $\delta_{dB} < \delta_{dA}$, $\delta_{xA} < \delta_{xB}$, and $Q_B > Q_A$, in which case $M_B > M_A$.

[35] Given τ, this limit can be depicted by point C on the FF curve in Figure 1, which is located between the $b_A = b_B$ ray and the 45^o line, such that the equilibrium point of the flexible country, point B, has to be above C for both countries to be incompletely specialized. In the Appendix we also analyze equilibria with specialization when $b_A/b_B \geq \bar{b}(\tau)$.

the wage bill in the differentiated sector equals the fraction $\beta/(1+\beta)$ of revenue (a result from the bargaining game). Therefore

$$N_j = \frac{\beta}{1+\beta} M_j \left(\frac{\delta_{dj}}{\phi_2} + \frac{\delta_{xj}}{\phi_2} \right), \tag{21}$$

where $M_j (\delta_{dj} + \delta_{xj})/\phi_2$ is total revenue in the differentiated sector. It follows that, once the cutoffs and the numbers of firms are known, this equation determines the number of workers searching for jobs in the differentiated-product industry.[36] Having solved for N_j, aggregate employment in the differentiated sector is

$$H_j = x_j N_j. \tag{22}$$

This completes the description of an equilibrium with incomplete specialization.

4 Trade, Welfare and Productivity

In this section we explore channels through which the two countries are interdependent. For this purpose we organize the discussion around two main themes: the impact of a country's labor market frictions on its trade partner, and the differential effects of trade impediments on countries with different labor market institutions.

4.1 Welfare

We are interested in knowing how labor market institutions and trade frictions affect welfare, and in particular their differential effects on the welfare of the flexible and rigid countries. Since aggregate spending E equals aggregate income, and aggregate income equals L (as we explained above), the indirect utility function (3) implies that welfare is higher the larger the real consumption index of differentiated products Q_j is. We have already shown that Q_j is higher in the flexible country, in which case the flexible country is better off than the rigid country.

Now combine the formulas for change in the cutoffs (19) with (16) to obtain

$$\frac{\beta-\zeta}{1-\beta}\hat{Q}_j = \frac{1}{\Delta}\left[-\delta_{d(-j)} (\delta_{xj} + \delta_{dj}) \hat{b}_j + \delta_{xj} \left(\delta_{x(-j)} + \delta_{d(-j)} \right) \hat{b}_{(-j)} - \delta_{xj} \left(\delta_{d(-j)} - \delta_{x(-j)} \right) \hat{\tau} \right]. \tag{23}$$

This equation has a number of implications. First, it shows that an improvement in a country's labor market institutions raises its real consumption index Q_j and therefore its welfare, but it reduces the trade partner's welfare. On the other side, a simultaneous improvement in the labor market institutions of both countries, at a common rate $\hat{b}_A = \hat{b}_B$, raises everybody's welfare.[37]

[36] Using the fact that $H_j = x_j N_j = N_j / b_j$, one can derive (21) directly from the definition of sectoral employment, H_j:

$$H_j = M_j \left[\int_{\Theta_{dj}}^{\infty} h_{dj}(\Theta)\, dG(\Theta) + \int_{\Theta_{xj}}^{\infty} h_{xj}(\Theta)\, dG(\Theta) \right],$$

after substituting in the equilibrium employment levels $h_{dj}(\Theta)$ and $h_{xj}(\Theta)$ from (6), using (16).

[37] This follows from the fact that $-\delta_{d(-j)} (\delta_{xj} + \delta_{dj}) + \delta_{xj} \left(\delta_{x(-j)} + \delta_{d(-j)} \right) = -\beta\Delta/(1-\beta) < 0$.

Second, in view of Lemma 3 (specifically, in view of $\delta_{dj} > \delta_{xj}$ in both countries), a reduction in trade impediments raises welfare in both countries. We summarize these findings in[38]

Proposition 1 *(i) Welfare is higher in the flexible country. (ii) An improvement in labor market institutions in one country raises its welfare and reduces the welfare of its trade partner. (iii) A simultaneous improvement in labor market institutions in both countries, with $\hat{b}_A = \hat{b}_B$, raises welfare in both of them. (iv) A reduction of trade impediments raises welfare in both countries and Q_j rises proportionately more in the flexible country.*

The second part of this proposition is intriguing: it states that a country harms the trade partner by improving its own labor market institutions. Moreover, this happens despite the fact that the trade partner $(-j)$ enjoys better terms of trade as a result of improved labor market institutions in country j, because $(-j)$ pays lower prices for imported varieties from j. The reason for this result is that better labor market institution in country j act like productivity improvements in this country, which makes foreign firms less competitive and therefore *crowds them out* from the differentiated sector. As a result, fewer foreign firms enter the industry, which harms foreign welfare, and this negative welfare effect is larger than the welfare gain from improved terms of trade.[39]

Proposition 1 also establishes that the gains from trade are unequally distributed, with the flexible country gaining more. The reason is that, before trade liberalization, the differentiated-product market is more competitive in the flexible country than in the rigid country (i.e., P is lower in the flexible country). As a result, exporters of brands of the differentiated product gain from foreign-market access more in the flexible country than in the rigid country.[40]

The last part of this proposition establishes that both countries gain from trade, because autarky is attained when $\tau \to \infty$.[41] Moreover, we show in the Appendix that both countries gain from trade when the difference in labor market institutions is large enough to effect an equilibrium in which the rigid country specializes in the production of homogeneous products. We therefore have

Proposition 2 *Both countries gain from trade.*

[38] The very last part of the proposition follows from the fact that (23) implies

$$\frac{\beta-\zeta}{1-\beta}\left[\hat{Q}_j - \hat{Q}_{(-j)}\right] = -\frac{1}{\Delta}\left[\left(\delta_{dj}+\delta_{xj}\right)\left(\delta_{d(-j)}+\delta_{x(-j)}\right)\left(\hat{b}_j - \hat{b}_{(-j)}\right) + \left(\delta_{d(-j)}\delta_{xj} - \delta_{x(-j)}\delta_{dj}\right)\hat{\tau}\right].$$

Under these circumstances $\hat{Q}_j > \hat{Q}_{(-j)}$ in response to $\hat{\tau} < 0$, when $\hat{b}_j < \hat{b}_{(-j)}$ (by Lemma 3) .

[39] Demidova (2006) studies a full employment model with exogenous differences in productivity distributions across countries, and finds that: (a) productivity improvements in one country hurt its trade partner; and (b) falling trade costs benefit disproportionately the more productive country, and may even hurt the less productive country. Our results on labor market frictions are similar to these (except that in our case both countries necessarily gain from falling trade costs), because—not withstanding unemployment—labor market frictions have analogous effects to productivity.

[40] Additional intuition is obtained from Lemma 3, which implies that firms in the flexible country receive a larger fraction of revenue from exporting. As a result, falling trade costs result in a larger increase in profitability of exporting firms in the flexible country, which leads to relatively more entry in this country and to disproportionately higher welfare gains.

[41] The following is a direct proof of the gains-from-trade argument: We have seen that the domestic cutoff is higher in every country in the trading equilibrium than in autarky. The first equation in (16) then implies that Q_j is higher in every country in the trading equilibrium, because this equation also holds in autarky.

This proposition is interesting, because it is well known that gains from trade are not ensured in economies with nonconvexities and distortions.[42] Moreover, in addition to the standard nonconvexities and distortions that exist in models of monopolistic competition, our model contains frictions in labor markets, which makes the gains-from-trade result even more remarkable.

4.2 Trade Structure

From Lemma 1 we know that the country with better labor market institutions has a lower export cutoff and a higher domestic cutoff; therefore it also has a larger fraction of exporting firms in the differentiated-product sector. In addition, we know that exports per entering firm equal δ_{xj}/ϕ_2. Therefore exports of differentiated products from country j to $(-j)$ are

$$X_j = M_j \frac{\delta_{xj}}{\phi_2}.$$

Lemma 3 states that the country with better labor market institutions has a larger δ_{xj} and Lemma 4 states that it has more firms. Therefore the flexible country exports a higher value of differentiated products X_j, which implies that it exports differentiated products on net. As a consequence, the rigid country exports homogeneous goods.

As in the standard Helpman-Krugman model of trade in differentiated products, there is intra-industry trade. We can therefore decompose the volume of trade into intra-industry and intersectoral trade. Let country A be the rigid country and let B be the flexible country. Then, because trade is balanced, the total volume of trade equals $2X_B$ and the volume of intra-industry trade equals $2X_A$. Therefore the share of intra-industry trade equals

$$\frac{X_A}{X_B} = \frac{\delta_{xA} M_A}{\delta_{xB} M_B}.$$

Using (20) this share can be expressed as

$$\frac{X_A}{X_B} = \frac{\dfrac{\delta_{dB}}{\delta_{xB}} - \left(\dfrac{Q_B}{Q_A}\right)^{\zeta}}{\dfrac{\delta_{dA}}{\delta_{xA}} \left(\dfrac{Q_B}{Q_A}\right)^{\zeta} - 1}.$$

Equations (19) and (23) then imply that the share of intra-industry trade is smaller the larger the ratio b_A/b_B is.

The results on trade structure are summarized in

Proposition 3 *(i) A larger fraction of firms export in the flexible country. (ii) The flexible country exports differentiated products on net and imports homogeneous goods. (iii) The share of intra-industry trade is smaller the larger the proportional gap in labor market institutions.*

[42] See Helpman and Krugman (1985).

That is, labor market institutions impact comparative advantage and the share of intra-industry trade in a particular way. Evidently, these are testable implications about trade flows.[43]

4.3 Productivity

In this section we discuss the implications of our model for total factor productivity (TFP). Alternative measures of TFP can be used to characterize the efficiency of production. We choose to focus on one such measure—the employment-weighted average of firm-level productivity—which is commonly used in the literature.[44] In the differentiated-product sector this measure is

$$TFP_j = \frac{M_j}{H_j}\left[\int_{\Theta_{dj}}^{\infty} \Theta^{\frac{1-\beta}{\beta}} h_{dj}(\Theta)dG(\Theta) + \int_{\Theta_{xj}}^{\infty} \Theta^{\frac{1-\beta}{\beta}} h_{xj}(\Theta)dG(\Theta)\right]. \tag{24}$$

Recall that $q_{zj}(\Theta) = \Theta^{(1-\beta)/\beta}h_{zj}(\Theta)$ for $z = d, x$. Therefore, TFP_j equals the output of differentiated products divided by employment in the differentiated-product sector.[45] Note that TFP_j measures productivity in the differentiated-product sector only, rather than in the entire economy, and productivity in the homogeneous-product sector is constant and equal to one. We discuss in the Appendix a productivity measure that accounts for the compositional effects across sectors.

Using (6) and (8)-(10), we can express (24) as

$$TFP_j = \frac{\delta_{dj}\varphi_{dj} + \delta_{xj}\varphi_{xj}}{\delta_{dj} + \delta_{xj}} = \varpi_{dj}\varphi_{dj} + \varpi_{xj}\varphi_{xj}, \tag{25}$$

where $\varpi_{dj} = \delta_{dj}/(\delta_{dj}+\delta_{xj})$ is the share of domestic sales in revenue and ϖ_{xj} is the share of exports, i.e., $\varpi_{xj} = 1 - \varpi_{dj}$, $j = A, B$. Moreover,

$$\varphi_{zj} \equiv \varphi(\Theta_{zj}) = \frac{\int_{\Theta_{zj}}^{\infty} \Theta^{1/\beta} dG(\Theta)}{\int_{\Theta_{zj}}^{\infty} \Theta dG(\Theta)}, \quad z = d, x,$$

where φ_{dj} represents the average productivity of firms that serve the home market and φ_{xj} represents the average productivity of exporting firms. It follows that aggregate productivity equals the weighted average of the productivity of firms that serve the domestic market and the productivity of firms that export, with the revenue shares serving as weights. We show in the Appendix that $\varphi(\cdot)$ is an increasing function. Therefore average productivity is higher among exporters, i.e., $\varphi_{xj} > \varphi_{dj}$.

Expression (25) implies that the cutoffs $\{\Theta_{dj}, \Theta_{xj}\}$ uniquely determine the TFP_js, because ϖ_{zj}

[43] Additionally, under Pareto-distributed productivity, the model also implies that the volume of trade is larger the larger the gap in labor market rigidities is and the smaller the trade impediments are (see Appendix).

[44] This corresponds to the measure analyzed by Melitz (2003) in the appendix. Note that Melitz uses revenue to weight firm productivity levels. However, in equilibrium, revenue is proportional to employment, in which case his and our productivity indexes are the same.

[45] An alternative, and potentially more desirable, measure of productivity, would divide output by the number of workers searching for jobs in the differentiated-product sector, N_j. This measure is always smaller than TFP_j by the factor x_j. It follows that labor market liberalization has an additional positive effect on this measure of productivity as compared to the measure used in the main text.

and φ_{zj} depend only on the cutoffs. Moreover, since the two cutoffs are linked by the free-entry condition (17), TFP_j can be expressed as a function of the domestic cutoff Θ_{dj}. This implies that in a closed economy TFP_j is not responsive to changes in labor market institutions, because Θ_d^c is uniquely determined by the fixed costs of entry and production and the ex ante productivity distribution.

Productivity TFP_j is higher in the trade equilibrium than in autarky, because $\varphi(\Theta_{xj}) > \varphi(\Theta_{dj}) > \varphi(\Theta_d^c)$, and in autarky $\varpi_x^c = 0$. That is, the average productivity of exporters and nonexporters alike is higher in the trade equilibrium than is the average productivity of firms in autarky. In addition, trade reallocates revenue to the exporting firms, which are on average more productive. For both these reasons trade raises TFP_j. We summarize these results in

Proposition 4 *(i) In the closed economy, TFP_j does not depend on the quality of labor market institutions; (ii) TFP_j is higher in any trade equilibrium than in autarky.*

Next recall that in an open economy a reduction of trade costs raises the domestic cutoff and reduces the export cutoff. In addition, an improvement in labor market institutions in country j raises Θ_{dj} and $\Theta_{x(-j)}$ and reduces $\Theta_{d(-j)}$ and Θ_{xj}. Finally, a simultaneous and proportional improvement in labor market institutions in both countries (i.e., $\hat{b}_A = \hat{b}_B < 0$) leaves all these cutoffs unchanged (see (19)).

How do changes in labor market frictions impact productivity? In the case in which both countries' labor market frictions decline by the same factor of proportionality, the answer is simple: the TFP_js do not change. As long as productivity is measured with regard to the number of employed workers rather than the number of workers searching for jobs, measured sectoral productivity levels are not sensitive to the absolute level of frictions in the labor markets; only the *relative* level of these frictions matters. This result points to a shortcoming of this TFP measure. We nevertheless continue the analysis with this measure, because it is commonly used in the literature.

A shock that raises the domestic cutoff Θ_{dj} and reduces the export cutoff Θ_{xj} affects TFP_j through three channels. First, the reallocation of revenue from firms that serve the home market to exporters raises the weight on the productivity of exporters, ϖ_{xj}, which raises in turn TFP_j. Second, some least-efficient firms exit the industry, thereby raising the average productivity of firms that sell only in the home market, φ_{dj}, which raises TFP_j. Finally, some firms with productivity below Θ_{xj} begin to export, thereby reducing the average productivity of exporters, φ_{xj}, which reduces TFP_j.[46]

The presence of the third effect, which goes against the first two, does not enable us to sign the impact of single-country labor market reforms on productivity; in general, productivity may increase or decrease. The sharp result for the comparison of autarky to trade derives from the fact that, in a move from autarky to trade, the third effect is nil. In the Appendix, we provide sufficient conditions for productivity to be monotonically rising with Θ_{dj}, and therefore declining

[46] Formally, this decomposition can be represented as $\widehat{TFP}_j = \hat{\varpi}_{xj}(\varphi_{xj} - \varphi_{dj}) + (1 - \varpi_{xj})\hat{\varphi}_{dj} + \varpi_{xj}\hat{\varphi}_{xj}$ with $\hat{\varpi}_{xj} > 0$, $\hat{\varphi}_{dj} > 0$ and $\hat{\varphi}_{xj} < 0$.

with b_j and τ and rising with $b_{(-j)}$. In this section, however, we limit our discussion to the case of Pareto-distributed productivity draws, which yields sharp predictions.

Under the assumption of Pareto-distributed productivity, that is, $G(\Theta) = 1 - (\Theta_{\min}/\Theta)^k$ for $\Theta \geq \Theta_{\min}$, (25) yields[47]

$$\widehat{TFP}_j = \frac{\delta_{dj}\left(\varphi_{xj} - \varphi_{dj}\right)\left(k - \frac{1-\beta}{\beta}\right)}{\delta_{dj}\varphi_{dj} + \delta_{xj}\varphi_{xj}}\hat{\Theta}_{dj}, \tag{26}$$

where $k > 1/\beta$ is required for TFP_j to be finite, and we therefore assume that it holds. As a result, TFP_j is higher the higher Θ_{dj} is (and the lower Θ_{xj} is). It follows that productivity is higher in the flexible country, and an improvement in a country's labor market institutions raises its productivity and reduces the productivity of its trade partner. An implication of this result is that the gap in productivity between the flexible and rigid countries is increasing in b_A/b_B, the relative quality of their labor market institutions. These results are summarized in

Proposition 5 *Let Θ be Pareto-distributed with a shape parameter $k > 1/\beta$. Then: (i) TFP_j is higher in the flexible country; (ii) an improvement in labor market institutions in country j raises TFP_j and reduces $TFP_{(-j)}$; (iii) a reduction of trade costs raises TFP_j in both countries.*

In other words, total factor productivity is higher in the flexible country, and while a reduction of labor market frictions in any country raises its own total factor productivity, this hurts the total factor productivity of the country's trade partner.

5 Unemployment

Before discussing the variation of unemployment across countries with different labor market institutions in Sections 5.2 and 5.3, we first examine the determinants of unemployment in a world of symmetric countries.

5.1 Symmetric Countries

We study in this section countries with $b_A = b_B = b$, in order to understand how changes in the common level of labor market frictions and the common level of variable trade costs affect unemployment. In such equilibria, the cutoffs Θ_d and Θ_x, the consumption index Q, the number of entrants M, the number of individuals searching for jobs in the differentiated-product sector N, the number of workers employed in that sector H, and the rate of unemployment u are the same in both countries. We therefore drop the country index j for convenience. From Section 3 we know that two symmetric economies are at the same point on the FF curve in Figure 1 (point S), the location of this point is invariant to the common level of labor market frictions, and this point is higher the larger τ is. Moreover, (23) implies that Q is lower the higher are either b or τ, and a

[47] In this calculation, an increase in Θ_{dj} is accompanied by a decrease in Θ_{xj} in order to satisfy the free-entry condition (17). See Appendix for derivation of this equation.

Figure 2: Unemployment in a world of symmetric countries

lower value of Q leads to lower welfare. In other words, higher frictions in trade or labor markets reduce welfare.

In order to assess the impact of labor market rigidities on unemployment, we need to know their quantitative impact on Q. For this reason we use (23) to obtain

$$\hat{Q} = -\frac{\beta}{\beta - \zeta}\left(\hat{b} + \frac{\delta_x}{\delta_d + \delta_x}\hat{\tau}\right).$$

Next combine (20) and (21) to obtain $N = \beta Q^{\zeta} / (1 + \beta)$, which together with the previous equation yields

$$\hat{N} = -\frac{\beta\zeta}{\beta - \zeta}\left(\hat{b} + \frac{\delta_x}{\delta_d + \delta_x}\hat{\tau}\right).$$

Finally, from (14) and the unemployment equation (15), we have $\hat{u} = \hat{N} + \hat{b} / (b - 1)$, which together with the formula for $\hat{N}$ implies

$$\hat{u} = \left(\frac{1}{b-1} - \frac{\beta\zeta}{\beta - \zeta}\right)\hat{b} - \frac{\beta\zeta}{\beta - \zeta}\frac{\delta_x}{\delta_d + \delta_x}\hat{\tau}.$$

It is evident from this formula that better labor market institutions (lower b) reduce unemployment if and only if

$$b < 1 + \frac{\beta - \zeta}{\beta\zeta},$$

i.e., if and only if labor market frictions are low to begin with. If these frictions are high, however, and the above inequality is reversed, then improvements in labor market institutions raise the rate of unemployment. In fact, the relationship between b and the rate of unemployment has an inverted U shape as depicted in Figure 2. To understand this result, note that changes in labor market frictions impact unemployment through two channels: the rate of unemployment in the differentiated sector

$1-x$, and the fraction of people searching for jobs in this sector N/L. Improvements in labor market institutions raise x and thereby reduce the rate of unemployment. On the other hand, improvements in labor market institutions attract more workers to the differentiated-product sector and thereby raise the rate of unemployment. The former dominates when labor market frictions are low, while the latter dominates when labor market frictions are high.[48]

Now consider changes in trade impediments. As the formula for change in the rate of unemployment shows, a lower trade cost τ *raises* the rate of unemployment, independently of the common level of frictions in labor markets or the initial level of trade frictions.[49] Since the lowering of trade costs raises welfare, this means that welfare and unemployment respond in opposite directions to changes in trade costs. And since reducing trade impediments does not affect tightness in labor markets, the rise in unemployment is a consequence of an increase in N and H by the same factor of proportionality.

We summarize the main findings of this section in

Proposition 6 *In a symmetric world economy: (i) improvements in labor market institutions, common to both countries, reduce unemployment if and only if frictions in the labor markets are low and satisfy $b < 1 + (\beta - \zeta)/\beta\zeta$; and (ii) reductions in trade impediments raise unemployment.*

An intriguing result is that lower trade barriers raise unemployment. To understand the intuition behind this result, observe that the lowering of trade impediments makes exporting more profitable in the differentiated-product sector, without affecting tightness in its labor market. As a result, more firms choose to export in this industry and exporters choose to export larger volumes. In addition, domestic firms that do not serve foreign markets become less profitable, which leads to more exit of low-productivity firms. On account of these changes labor demand rises. To accommodate this demand, more individuals search for jobs in the differentiated-product industry. Under these circumstances, the sectoral unemployment rates remain the same, but the economy's unemployment rises because more workers choose to attach themselves to the high-wage sector, which has the higher rate of unemployment.

Also note that unemployment can increase or decrease when welfare rises. That is, depending on the nature of the disturbance and the initial institutional environment, unemployment and welfare can move in the same or in opposite directions. For this reason changes in unemployment do not reflect changes in welfare. This results from the standard property of search and matching models, in which unemployment is a productive activity; it enables workers to be employed in both low-wage and high-wage activities. Under these circumstances an expansion of the high-wage\high-unemployment sector results in higher unemployment, but may also raise welfare. In this type of environment, other statistics—such as total employment in the high-wage sector H—better proxy for welfare than the rate of unemployment.

[48] It can also be shown that in the symmetric case lower frictions in labor markets lead to increased entry of firms M, an increase in N proportionately to M, and a more than proportional increase in employment H.

[49] The effect of a reduction in trade costs on unemployment is larger the larger is the share of trade in the sector's revenue, i.e., the larger is $\delta_{xj}/(\delta_{dj} + \delta_{xj})$. When the economies are nearly closed, this effect is very small.

5.2 Small Asymmetries

Consider a world in which country B has the better labor market institutions, so that $b_A > b_B$. Then the labor market is tighter in the flexible country B, and the unemployment rate is lower in its differentiated-product sector. The question is whether the country's overall unemployment rate is also lower? The reason this may not be the case is that more individuals might be searching for jobs in the high-unemployment sector in the country with lower labor market frictions. We answer this question below for the case in which labor rigidities do not vary much across countries. In the next section we discuss global comparisons for the case in which productivity is distributed Pareto.

Suppose that we start from a symmetric equilibrium with $b_A = b_B$. As a result, the two countries look alike in all respects. Next suppose that labor market rigidities rise in country A but do not change in country B, so that $\hat{b}_A > 0$ and $\hat{b}_B = 0$. Then we can use (19) and (23) to calculate the response of the cutoffs and the real consumption index in each of these countries, evaluated at the initially symmetric equilibrium, and we can combine these results with the other equilibrium conditions to derive the proportional change in the number of individuals seeking jobs in the differentiated-product sectors of both countries. The technical details are provided in the Appendix, where we show that

$$\hat{N}_A = -\Psi_{NA}\ \hat{b}_A,$$

$$\hat{N}_B = \Psi_{NB}\ \hat{b}_A,$$

where the coefficients Ψ_{Nj} are determined by the initial equilibrium, $\Psi_{NA} > \beta\zeta/(\beta-\zeta)$, $\Psi_{NB} > 0$, and where $\Psi_{NA} \to \beta\zeta/(\beta-\zeta)$ and $\Psi_{NB} \to 0$ as $\tau \to \infty$. Evidently, an increase in labor market frictions in country A reduces the number of individuals searching for jobs in A's differentiated-product sector and increases the number of individuals searching for jobs in country B. Under these circumstances, (15) yields

$$\hat{u}_A = -\left(\Psi_{NA} - \frac{1}{b-1}\right)\hat{b}_A,$$

$$\hat{u}_B = \Psi_{NB}\ \hat{b}_A.$$

The implication is that the deterioration of labor market institutions in A raises unemployment in B, while unemployment rises in A if and only if $b < 1 + 1/\Psi_{NA}$, i.e., if and only if the frictions in the labor markets are low to begin with; otherwise the rate of unemployment declines in A. Since $\Psi_{NA} > \beta\zeta/(\beta-\zeta)$, the open economy A would require even lower labor market frictions than a comparable closed economy for a deterioration in its labor market institutions to raise its unemployment. Moreover, since

$$\hat{u}_A - \hat{u}_B = -\left(\Psi_{NA} + \Psi_{NB} - \frac{1}{b-1}\right)\hat{b}_A,$$

country A has the higher rate of unemployment after a deterioration in its labor market institutions if and only if

$$b < 1 + \frac{1}{\Psi_{NA} + \Psi_{NB}},$$

or if and only if the initial level of frictions in the labor market is rather low. If the initial level of frictions in the labor markets is high, thereby violating this inequality, then country A has the lower rate of unemployment.

These results are summarized in

Proposition 7 *In the vicinity of a symmetric equilibrium: (i) the flexible country has a lower rate of unemployment if and only if the level of friction in both labor markets are low, i.e., if and only if* $b < 1 + 1/(\Psi_{NA} + \Psi_{NB})$*; and (ii) an improvement in a country's labor market institutions reduces the rate of unemployment in its trade partner, yet it reduces home unemployment if and only if the initial level of friction in both labor markets are low, i.e., if and only if* $b < 1 + 1/\Psi_{NA}$.

It is evident from this proposition that a country's level of unemployment depends not only on its own labor market institutions but also on those of its trade partner. Moreover, better domestic labor market institutions do not guarantee lower unemployment relative to the trade partner, unless the frictions in both labor markets are low. As a result, one cannot infer differences in labor market institutions from observations of unemployment rates.

To understand the intuition behind these results, first note that an improvement in a country's labor market institutions affects its unemployment rate through two channels: on the one hand, the country's labor market becomes tighter, which reduces the unemployment rate in its differentiated-product sector; on the other hand, more workers search for jobs in the differentiated-product sector. As a result of these opposing effects, the overall rate of unemployment declines when the first channel dominates and rises when the second channel dominates. The first channel dominates when the frictions in the labor markets are small, while the second channel dominates when these frictions are large.

An interesting implication of Proposition 7 is that improvements in a country's labor market institutions reduces the rate of unemployment in its trade partner. This results from the fact that a reduction of frictions in the labor market of country j makes j more competitive in the differentiated-product industry. As a result, the demand shifts from brands of country $(-j)$ to brands of j. In response, the differentiated-product sector contracts in country $(-j)$, which means that fewer people search there for jobs in this industry. Since the labor market frictions do not change in country $(-j)$, the rate of unemployment in its differentiated-product sector does not change either. It therefore follows that the overall rate of unemployment declines in $(-j)$ because fewer workers search there for jobs and the fraction of those who find employment does not change.

5.3 Large Asymmetries

The results reported so far were derived for either symmetric countries or for a world with small asymmetries; we have not been able to derive general analytical results about unemployment for countries with large differences in labor market rigidities. To make progress on this issue, we therefore turn to the case of Pareto-distributed productivity levels, and after deriving one analytical result we resort to simulations. The following numerical examples are interesting, because they show how unemployment rates compare across countries when labor frictions differ substantially across countries, and they show how the degree of similarity in labor market institutions interacts with trade frictions in shaping unemployment rates.

For the purpose of the simulations we assume that productivity is distributed Pareto. Therefore the distribution function is

$$G(\Theta) = 1 - \left(\frac{\Theta_{\min}}{\Theta}\right)^k, \text{ for } \Theta \geq \Theta_{\min} \text{ and } k > 2.$$

As is well known, the shape parameter k controls the dispersion of Θ, with smaller values of k representing more dispersion. It has to be larger than two for the variance of productivity to be finite. We show in the Appendix how the equilibrium conditions are simplified when productivity is distributed Pareto, and these equations are used to generate our numerical examples. One convenient implication of the Pareto assumption is that condition (11) implies $\delta_{dj} + \delta_{xj} = kf_e$, and therefore aggregate revenue in the differentiated sector is independent of labor market frictions and is the same in both countries.

Combining (20) and (21), we obtain the following expression for global revenues generated in the differentiated sector:

$$Q_A^{\zeta} + Q_B^{\zeta} = \frac{1}{\phi_2}\left[M_A(\delta_{dA} + \delta_{xA}) + M_B(\delta_{dB} + \delta_{xB})\right] = \frac{1+\beta}{\beta}(N_A + N_B).$$

Therefore, whenever $Q_A^{\zeta} + Q_B^{\zeta}$ rises, the world-wide allocation of workers to the differentiated sector, $N_A + N_B$, must also increase. Moreover, under the Pareto assumption

$$Q_A^{\zeta} + Q_B^{\zeta} = \frac{kf_e}{\phi_2}(M_A + M_B),$$

so that the total number of entrants into the differentiated sector must also increase. Finally, under the Pareto assumption equation (21) implies

$$\frac{N_j}{M_j} = \frac{\beta}{1-\beta} kf_e.$$

That is, the number of workers searching for jobs in the differentiated sector relative to the number of firms is the same in both countries and independent of trade or labor market frictions.

Next note that Proposition 1 establishes that a reduction in trade costs raises Q_j in both countries. Therefore, the above discussion implies that a reduction in trade costs increases $N_A + N_B$ and $M_A + M_B$ by the same factor of proportionality. In the Appendix we also show that N_A/N_B declines with reductions in τ when A is the rigid country, i.e., $b_A > b_B$. This then implies that N_B, the number of job-seekers in the differentiated sector of the flexible country B, necessarily increases.

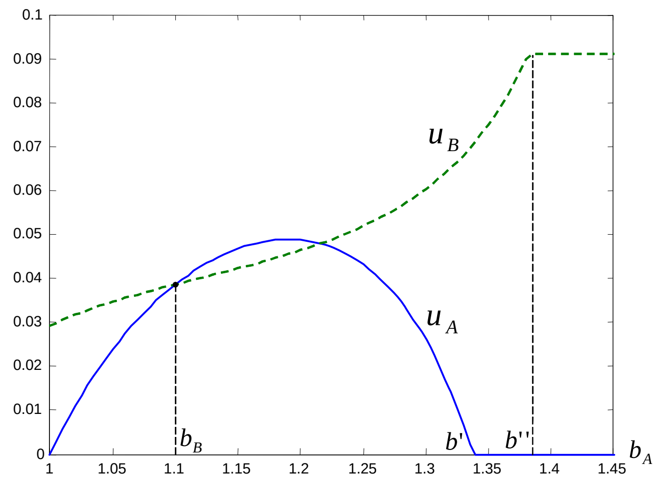

Figure 3: Unemployment as a function of b_A when b_B is low ($b_B = 1.1$ and $\tau = 1.1$)

Since a fall in τ does not affect sectoral labor market tightness, we conclude that a reduction in trade costs increases unemployment in the flexible country. This proves

Proposition 8 *Let productivity be distributed Pareto. Then a reduction in trade impediments raises unemployment in the flexible country.*

The intuition behind this result is the following. Lower trade impediments increase the global size of the differentiated sector, which features increasing returns to scale and love of variety. As a result, the country with more flexible labor markets, which has a competitive edge in this sector, becomes more specialized in differentiated products. That is, the number of entering firms, employment, and the number of job-seekers in the differentiated sector, all increase in the flexible country. This compositional shift leads to a higher rate of unemployment in this country. As we show, however, numerically below, the unemployment rate may increase or decrease in the rigid country.

We now proceed with the numerical examples. Figure 3 depicts the response of unemployment rates to variation in country A's labor market frictions, b_A; the rising broken-line curve represents country B and the hump-shaped solid-line curve represents country A.[50] Country B has $b_B = 1.1$, and therefore the two countries have the same rate of unemployment when $b_A = 1.1$. As b_A rises, country A becomes more rigid. This raises initially the rate of unemployment in both countries, but the flexible country's rate of unemployment remains lower for a while. At some point, however, the rate of unemployment reaches a peak in the rigid country A, and it falls for further increases in b_A. As a result, the two rates of unemployment become equal again, after which further increases in rigidity in country A raise the rate of unemployment in the flexible country and reduce it in the rigid country, so that the rate of unemployment is higher in the flexible country thereafter. The mechanism that operates here is that once the labor market frictions become high enough in

[50] In Figures 3-4 we use the following parameters: $f_x = 3$, $f_d = 1$, $f_e = 0.5$, $k = 2.5$, $\beta = 0.75$, $\zeta = 0.5$ and $L = 0.1$.

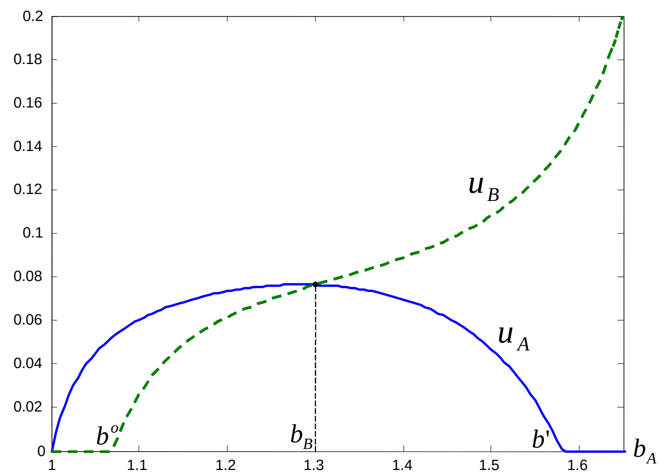

Figure 4: Unemployment as a function of b_A when b_B is high ($b_B = 1.3$ and $\tau = 1.1$)

country A, the contraction of the differentiated-product sector leads to overall lower unemployment in the rigid country despite the fact that its sectoral unemployment is high. When b_A is very high the sectoral unemployment rate is very high, but no individuals search for jobs in this sector, as a result of which there is no unemployment at all. This explains the hump in A's curve. Note that in the range in which the rate of unemployment falls in country A the rate of unemployment keeps rising in country B. The reason is that there is no change in market tightness in country B and its differentiated-product sector becomes more competitive the more rigid the labor market becomes in A. As a result the differentiated-product sector attracts more and more workers in country B, which raises its rate of unemployment. The monotonic impact of country A's labor market rigidities on the unemployment rate in B holds globally, and not only around the symmetric equilibrium.[51]

Figure 4 is similar to Figure 3, except that now the level of labor market frictions in country B is higher, i.e., $b_B = 1.3$, and therefore the two curves intersect at $b_A = 1.3$. Moreover, starting with a symmetric world that has these higher labor market rigidities, increases in b_A always raise unemployment in B and reduce unemployment in A. As a result, the rigid country has lower unemployment independently of the difference in labor market institutions.

A comparison between Figures 3 and 4 demonstrates the importance of the overall level of labor market rigidities for unemployment outcomes. When labor market frictions are high, the flexible country always has a higher rate of unemployment. Moreover, the rates of unemployment in the two countries move in opposite directions as labor market institutions change in either of the countries. In contrast, when labor market rigidities are low and the differences in labor market institutions are not large, the rate of unemployment is lower in the flexible country and the rates of unemployment in both countries co-move in response to the changes in labor market institutions.

The next three figures depict variations in unemployment in response to trade frictions, in the

[51] In Figures 3-4, country A specializes in the homogeneous good when $b_A \geq b'$; similarly, in Figure 4, country B specializes in the homogeneous good when $b_A \leq b^o$.

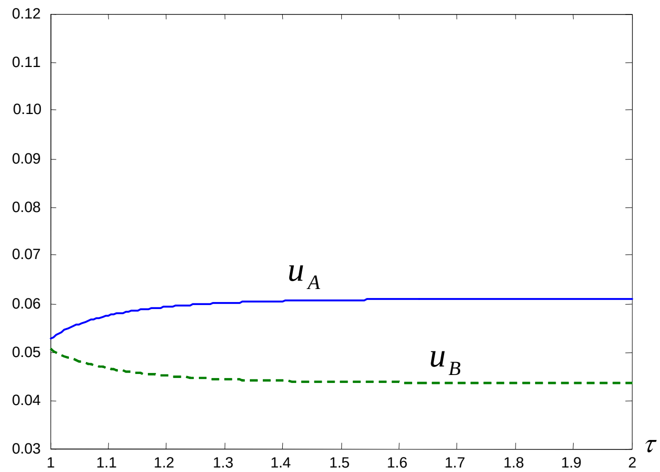

Figure 5: Unemployment as a function of τ when b_A and b_B are low ($b_A = 1.2$ and $b_B = 1.12$)

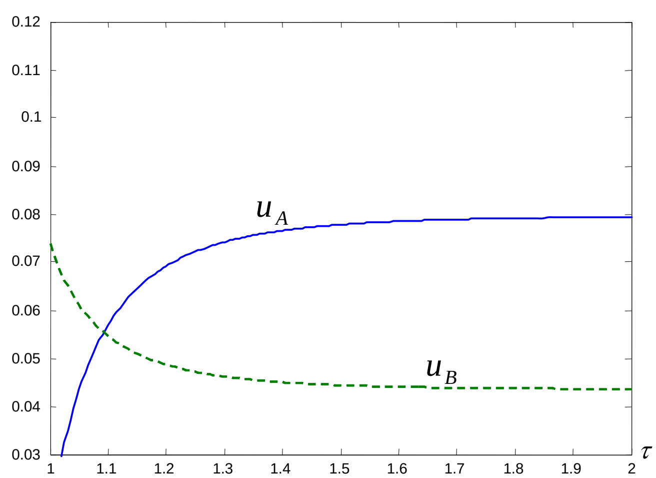

Figure 6: Unemployment as a function of τ when b_A is high and b_B is low ($b_A = 1.35$ and $b_B = 1.12$)

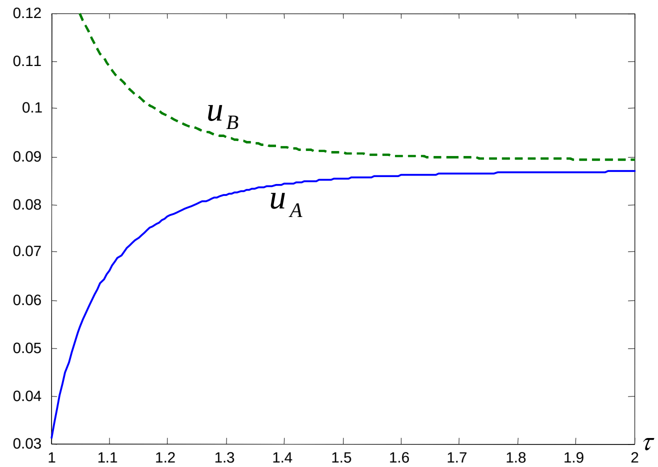

Figure 7: Unemployment as a function of τ when b_A and b_B are high ($b_A = 1.9$ and $b_B = 1.6$)

form of variable trade costs τ: Figure 5 for the case of low frictions in labor markets, Figure 6 for the case in which frictions are low in the flexible country but high in the rigid country, and Figure 7 for the case in which frictions are high in both countries.[52] In all three cases unemployment rises in the flexible country (as predicted by Proposition 8) and falls in the rigid country when trade frictions decline.[53] Nevertheless, the rate of unemployment is not necessarily higher in the rigid country. In particular, unemployment is always higher in the rigid country when frictions in labor markets are low in both countries, yet unemployment is always higher in the flexible country when frictions in labor markets are high in both countries. In between, when labor market frictions are low in the flexible country and high in the rigid country, the relative rate of unemployment depends on trade impediments; it is lower in the rigid country when the trade frictions are low and lower in the flexible country when the trade frictions are high. This shows that labor market frictions interact with trade impediments in shaping unemployment.

6 Unemployment and Trade Impediments in the Homogeneous Sector

In this section we introduce unemployment and trade costs into the homogeneous-product sector, in order to examine the sensitivity of our main results to these modifications. The analysis proceeds in two separate parts, because there exist no significant interactions between these frictions.

[52] In Figures 5-7 we use the following parameters: $f_x = 5$, $f_d = 1$, $f_e = 0.5$, $k = 2.5$, $\beta = 0.75$, $\zeta = 0.5$, and $L = 0.1$.

[53] This pattern is not general. As we know, in the symmetric case lower trade impediments raise unemployment in both countries. We have also simulated examples in which the rigid country has a hump in its rate of unemployment as trade frictions vary (this requires $b_A \gtrsim b_B$).

6.1 Unemployment in the Homogeneous Sector

To introduce unemployment into the homogeneous-product sector in a simple way, assume as before that in this sector the product market and the labor market are both competitive, so that output is sold for $p_{0j} = 1$ and every employed worker gets the same wage rate $w_{0j} = 1$. Unlike the previous case, however, now assume that a worker has a marginal product of one with probability x_{0j}, $0 < x_{0j} < 1$, and zero with probability $1 - x_{0j}$, and that this information is revealed only after he attaches himself to the homogeneous sector. If not productive, the worker cannot get a job and he becomes unemployed. If productive, he gets a job and a wage of one. Under these circumstances the sectoral unemployment rate in the homogeneous-product industry is $1 - x_{0j}$.

Except for this modification, all other features of the model remain the same. As a result, the equilibrium wage rate in the differentiated sector of country j is $w_j = b_j$, where $b_j = a_j x_j^{\alpha}$ is the equilibrium hiring cost and x_j is the measure of tightness in the differentiated sector's labor market. Since x_j also equals the probability of finding a job in the differentiated sector, the indifference condition for workers between searching for jobs in the two sectors is $w_{0j}x_{0j} = w_j x_j$, or $x_{0j} = b_j x_j$. Together with $b_j = a_j x_j^{\alpha}$ this condition implies that

$$x_j = \left(\frac{x_{0j}}{a_j}\right)^{\frac{1}{1+\alpha}} \qquad \text{and} \qquad b_j = \left(a_j x_{0j}^{\alpha}\right)^{\frac{1}{1+\alpha}}. \tag{27}$$

Therefore, as before, x_j is decreasing and b_j is increasing in a_j (see (14)).[54] In addition, tightness in the labor market, x_j, and the cost of search and matching, b_j, now depend on x_{0j}. In particular, a higher probability of employment in the homogeneous sector leads to a tighter labor market in the differentiated sector and to higher hiring costs in that sector. As a result, the wage rate rises in the differentiated sector.

Having the solution to (b_j, x_j) from (27), we can next solve for the sectoral allocation of resources, because the equilibrium conditions in the differentiated sector do not change. For example, the zero-profit condition (16) and the free-entry condition (17) remain the same.[55] As a result, all propositions from Section 4 remain valid. In particular, both countries gain from trade: the rigid country imports differentiated products on net, and the flexible country has a more productive differentiated-product sector.

The equilibrium unemployment rate is now given by

$$u_j = (1 - x_{0j})\,\frac{L - N_j}{L} + (1 - x_j)\frac{N_j}{L},$$

where the right-hand side of the equation expresses the rate of unemployment as a weighted average of sectoral rates of unemployment. This formula emphasizes a new feature: an expansion of the workforce that searches for jobs in the differentiated sector raises unemployment if and only if the

[54] To guarantee an interior solution, we now require $a_j > x_{0j}$, which ensures $x_j < 1$. However, this does not imply $b_j > 1$ any more.

[55] Note, however, that now $b_A > b_B$ can result from country A having a higher a_j or a higher x_{0j}.

sectoral rate of unemployment is higher in the differentiated sector, i.e., $1 - x_j > 1 - x_{0j}$. Evidently, this condition was satisfied when there was no unemployment in the homogeneous-product sector, and a number of the results concerning the impact of changes in economic fundamentals on unemployment depend on this feature. It is clear that these results still hold when there is unemployment in the homogeneous sector, as long as the rate of unemployment is higher in the differentiated sector. But once the homogeneous sector has the higher rate of unemployment, the results change.

As an example consider the impact of trade on unemployment when the two countries are symmetric. Proposition 6 states that reductions in trade impediments raise unemployment. The mechanism through which unemployment rises in the case of full employment in the homogeneous sectors, is that reductions in trade frictions expand the differentiated sector and N_j rises. Since changes in trade frictions do not affect tightness in the labor market, the increase in the number of people searching for jobs in the differentiated sector raises unemployment, because more people search for jobs in the sector with the higher sectoral rate of unemployment. The same argument applies when $x_{0j} < 1$, as long as the differentiated sector has the higher rate of unemployment. But once the homogeneous sector has a higher rate of unemployment, a reduction of trade frictions *reduces* unemployment, because now an increase in N_j involves a reallocation of workers from the high- to the low-unemployment industry. More generally, in all cases in which the sectoral composition impacts unemployment, the compositional effect operates in the opposite direction when unemployment is higher in the homogeneous sector than when it is higher in the differentiated sector.

6.2 Trade Impediments in the Outside Sector

We next modify the assumptions about trade costs, and assume that in addition to the trade costs in the differentiated sector there is a variable trade cost in the homogeneous-product sector. The latter cost is of the melting-iceberg type, so that $\tau_0 > 1$ units of the homogeneous good have to be shipped for one unit to arrive in the trade partner country. All other features of the model remain the same as in Sections 2-5, including the assumption of full employment in the homogeneous-product sector.

First note that in the symmetric case there is no intersectoral trade, which means that there is no trade in homogeneous goods; only brands of the differentiated product are traded internationally. Evidently, this equilibrium is not influenced by trade costs in the homogeneous sector, and all our results concerning symmetric countries do not change. We therefore consider, below, asymmetric countries, and for comparison with the previous results, we focus on asymmetries that are large enough to ensure intersectoral trade.[56]

As before, let A be the rigid country, so that $b_A > b_B$. In this event, A exports homogeneous

[56] If $\tau_0 > 1$ and b_A is only slightly larger than b_B, then there is no trade in homogeneous goods, only in differentiated products. See below.

goods and imports differentiated products on net.[57] Now prices of homogeneous goods differ across countries, and we choose the homogeneous good in country A to be the numeraire, i.e., $p_{0A} = 1$. Since country B imports this good and there is a trade cost τ_0, the price of the homogeneous good in country B is $p_{0B} = \tau_0 > 1$. As a result, the wage rate in the homogeneous sector is $w_{0A} = p_{0A} = 1$ in country A and $w_{0B} = p_{0B} = \tau_0 > 1$ in country B.

We continue to measure b_j in units of the homogeneous good. Therefore in the differentiated sector, wages are $w_A = p_{0A} b_A = b_A$ in country A and $w_B = p_{0B} b_B = \tau_0 b_B$ in country B. It follows that for a worker to be indifferent between searching for a job in the homogeneous or differentiated sector the following condition has to be satisfied:

$$p_{0j} b_j x_j = w_{0j}.$$

However, since $w_{0j} = p_{0j}$ in every country, this condition is equivalent to (12). Moreover, since b_j is determined by tightness in the labor market, according to (13), it follows that the hiring cost b_j and labor market tightness x_j depend only on the cost of vacancies and the search and matching technology, as described in (14).

Given the price p_{0j} of the homogeneous good in country j, the demand for variety ω of the differentiated product equals

$$q_j(\omega) = Q_j^{-\frac{\beta-\zeta}{1-\beta}} \left[\frac{p_j(\omega)}{p_{0j}} \right]^{-\frac{1}{1-\beta}}$$

in country j, and this equation replaces the demand function (2). Therefore the indirect utility function becomes $\mathbb{V}_j = E_j/p_{0j} + (1-\zeta)\, Q_j^{\zeta}/\zeta$, and the profit functions from domestic sales and exporting become

$$\begin{aligned}
\pi_{dj}(\Theta) &= \phi_1 \phi_2 b_j^{-\frac{\beta}{1-\beta}} p_{0j} Q_j^{-\frac{\beta-\zeta}{1-\beta}} \Theta - p_{0j} f_d, \\
\pi_{xj}(\Theta) &= \phi_1 \phi_2 \tau^{-\frac{\beta}{1-\beta}} b_j^{-\frac{\beta}{1-\beta}} p_{0j}^{-\frac{\beta}{1-\beta}} p_{0(-j)}^{\frac{1}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}} \Theta - p_{0j} f_x.
\end{aligned}$$

Note that the higher price of homogeneous products in country B, i.e., $p_{0B} = \tau_0 > 1$, has no impact on the profit function from domestic sales of firms in the differentiated sector of country A, but it raises those firms' profits from exporting. Moreover, the higher p_{0B} reduces export profits for differentiated-product firms in country B.

The domestic and export cutoffs are defined by $\pi_{zj}(\Theta_{zj}) = 0$ for $z = d, x$, and the free-entry condition remains (17). The latter results from the fact that entry costs and all fixed costs are paid in domestic units of the homogeneous good. Following the same steps as in Section 3, we can

[57] In what follows, we provide a condition which ensures this outcome.

therefore derive the equilibrium expressions for M_j and N_j, which now read:

$$\phi_2 Q_j^{\zeta} = M_j \delta_{dj} + \frac{p_{0(-j)}}{p_{0j}} M_{-j} \delta_{x(-j)},$$
$$\phi_2 N_j = \phi_1^{\frac{1-\beta}{\beta}} M_j \left(\delta_{dj} + \delta_{xj}\right),$$

where, as before, $\delta_{zj} \equiv f_z/\Theta_{zj} \int_{\Theta_{zj}}^{\infty} \Theta dG(\Theta)$ for $z = d, x$. The interpretation of these conditions does not change. The new term, $p_{0(-j)}/p_{0j}$, just converts units of revenue from foreign sales into units of domestic revenue. Finally, the unemployment rate is still given by (15).

The revenue of country j from exports of differentiated products is $M_j p_{0j} \delta_{xj}/\phi_2$. Noting that $p_{0B}/p_{0A} = \tau_0$, we can therefore express the condition that country B exports the differentiated product on net as[58]

$$M_A \delta_{xA} < \tau_0 M_B \delta_{xB}. \tag{28}$$

We provide in the following lemma a sufficient condition for this inequality (see the Appendix for details):

Lemma 5 *Let*

$$\left(b_A/b_B\right)^{\beta} > \tau_0. \tag{29}$$

Then $\Theta_{dA} < \Theta_{dB}$, $\Theta_{xA} > \Theta_{xB}$, $Q_A < Q_B$ *and* $M_A < M_B$.

Since $\Theta_{xA} > \Theta_{xB}$ implies $\delta_{xA} > \delta_{xB}$, a corollary of this lemma is that (29) implies (28). Intuitively, since a high trade cost τ_0 raises the cost of homogeneous goods in the flexible country, and therefore costs of running businesses in the differentiated sector, the flexible country needs as a result a large enough comparative advantage in differentiated products to export them on net.

We show in the Appendix that there exists an upper bound $\bar{\tau}_0 > (b_A/b_B)^{\beta} > 1$ that can be used to define intervals of transport costs with different equilibrium properties, which are described in the following:

Proposition 9 *Let* $\tau_0 < \bar{\tau}_0$. *Then: (i) there is intersectoral trade; (ii) for* $\tau_0 < (b_A/b_B)^{\beta}$, *all the results from Sections 3-5 hold; (iii) for* $\tau_0 > (b_A/b_B)^{\beta}$, *the rigid country gains more from a reduction in* τ; *and (iv) the flexible country gains and the rigid country looses from a reduction in* τ_0.

We have shown that our main results do not change in the presence of trade costs for homogeneous goods, as long as these costs are not too large relative to the advantage of the flexible country in labor market frictions. Alternatively, when the trade costs of homogeneous goods are large, the rigid country gains more than the flexible country from a reduction of trade costs in the differentiated sector. And in either case, reductions of trade costs in the homogeneous-product

[58] As noted above, we do not consider equilibria without intersectoral trade. For high enough trade costs τ_0, the unique equilibrium has no intersectoral trade, and it satisfies $M_A \delta_{xA} = \left(p_{0B}/p_{0A}\right) M_B \delta_{xB}$, where $p_{0B}/p_{0A} < \tau_0$. This no-net-trade condition then becomes one of the equilibrium conditions, and it allows us to solve for p_{0B}.

sector benefit the flexible country and hurt the rigid country. It follows that—while both countries may be interested in lower trade frictions in the differentiated sector—they have conflicting interests concerning trade frictions in the homogeneous sector.

7 Firing Costs and Unemployment Benefits

Our analysis has focused on search and matching as the main frictions in labor markets. We now introduce firing costs and unemployment benefits as additional sources of labor market rigidities. These labor market policies are widespread and they differ greatly across countries.[59] After introducing these policies into the model, we show that many results are not sensitive to their details, and we explain how other results are modified when firing is costly and the outside option of unemployed workers is unemployment benefits. Moreover, we show how changes in these policies in one country impact the domestic economy and the trade partner.

We now denote by $b_{sj} = a_j x_j^{\alpha}$ the cost of search and matching to an individual firm in the differentiated sector of country j. The interpretation is that such a firm has to bear the cost b_{sj} in order to be matched with a single worker when tightness in the labor market is x_j. With firing costs and unemployment benefits, b_{sj} does not represent all the relevant hiring costs, however, and we need to explicitly derive them. Moreover, while x_j is still defined as the number of matched workers divided by the number of workers searching for jobs, the former does not equal the number of employed workers due to the presence of firing.

Let $0 < b_{uj} < 1$ represent unemployment benefits in country j; this is the income of a worker who searched for a job in the differentiated sector and did not find one.[60] These unemployment benefits are paid by the government, and the government raises taxes in a lump-sum fashion to finance them. The government also pays unemployment benefits to workers that have been fired. Those workers receive a fraction η of b_{uj} in addition to severance pay b_{pj} from the employer. We assume that $b_{pj} + \eta b_{uj} > b_{uj}$, so that a worker prefers to be matched and fired than not to be matched at all.

To model firing costs in a simple way, first assume that after a firm has been matched with a worker the worker is found unsuitable for the job with probability σ, in which case the firm chooses to fire him. The firing costs consist of two components: an administrative cost b_{fj}, which represents waste, and a severance pay b_{pj}, which is a transfer to the worker. Therefore, the firm's firing costs are $b_{fj} + b_{pj}$ per fired worker.

Under these circumstances a firm has to recruit $h/(1-\sigma)$ workers in order to have h employees, and it therefore bears the search and matching cost $b_{sj}h/(1-\sigma)$ in country j. In addition, since it fires a fraction σ of the recruited workers, it bears the firing cost $(b_{fj} + b_{pj})\,\sigma h/(1-\sigma)$. In other words, the effective recruitment cost per worker—which results from search and matching

[59] Governments can also influence search and matching costs by facilitating the flow of information about job vacancies and unemployed workers. Moreover, in some countries there are government agencies that directly assign unemployed workers to firms, and workers need to try these jobs in order to be eligible for unemployment benefits.

[60] We require $b_{uj} < 1$ to ensure that not all workers search for jobs in the differentiated sector.

plus firing—equals

$$b_{rj} = \frac{a_j x_j^\alpha + \sigma (b_{fj} + b_{pj})}{1-\sigma}.$$

Now consider a firm with productivity Θ that has h employees after recruitment and firing. Using (4), its revenue can be expressed as $A_j \Theta^{1-\beta} h^\beta$, where

$$A_j \equiv \left[Q_j^{-\frac{\beta-\zeta}{1-\beta}} + I_j \tau^{-\frac{\beta}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}} \right]^{1-\beta}.$$

We assume that a worker who quits his job gets unemployment benefits b_{uj} and no severance pay. Therefore, if $w_j(\Theta, h)$ is the equilibrium wage rate as a function of productivity and employment, then, following Stole and Zwiebel (1996a,b), equal division of the marginal surplus between the firm and a worker implies that

$$\frac{\partial}{\partial h} \left[A_j \Theta^{1-\beta} h^\beta - w_j(\Theta, h) h \right] = w(\Theta, h) - b_{uj}.$$

The left-hand side represents the marginal gain of the firm from employing the worker, accounting for the fact that his departure will impact the wage rate of the remaining workers. The right-hand side represents the marginal gain of the worker from staying with the firm, which equals the excess of wages over unemployment benefits. This yields a differential equation for the wage function, which has the solution[61]

$$w_j(\Theta, h) = \frac{\beta}{1+\beta} \frac{A_j \Theta^{1-\beta} h^\beta}{h} + \tfrac{1}{2} b_{uj}.$$

Without unemployment benefits workers receive the fraction $\beta/(1+\beta)$ of revenue, while with unemployment benefits they received this same share of revenue plus half of the unemployment benefits. This implies that the firm gets $A_j \Theta^{1-\beta} h^\beta - w_j(\Theta, h) h$, which equals the share $1/(1+\beta)$ of revenue minus half the unemployment benefits. Under the circumstances the firm solves a problem similar to (5), except that now b_j is not equal to the search and matching cost per worker, but rather to the recruitment cost b_{rj} plus half the unemployment benefits. That is

$$b_j = \frac{a_j x_j^\alpha + \sigma (b_{fj} + b_{pj})}{1-\sigma} + \tfrac{1}{2} b_{uj}. \tag{30}$$

[61] It is possible to generalize this game to unequal weights, with, say, λ_j being the relative weight of the firm; the larger λ_j is, the larger the bargaining power of the firm is. Under these circumstances the equilibrium division of surplus is given by

$$\frac{\partial}{\partial h} \left[A_j \Theta^{1-\beta} h^\beta - w_j(\Theta, h) h \right] = \lambda_j \left[w(\Theta, h) - b_{uj} \right],$$

and the solution to the equilibrium wage function is

$$w_j(\Theta, h) = \frac{\beta}{\beta + \lambda_j} \frac{A_j \Theta^{1-\beta} h^\beta}{h} + \frac{\lambda_j}{1+\lambda_j} b_{uj}.$$

Variation in λ_j across countries impacts differences in (b_j, x_j), and can also be viewed as a source of differences in labor market institutions. For brevity, we leave out this source of variation and assume in the main text that $\lambda_j = 1$ in both countries.

The first-order condition of this problem then implies that the equilibrium wage rate is

$$w_j = b_j + \tfrac{1}{2} b_{uj},$$

which is independent of the firm's productivity. That is, all firms pay the same wages, which are now higher in terms of the outside good as a result of unemployment benefits.

As before, families allocate workers across industries to maximize expected income. In the homogeneous sector a worker finds a job with probability one and the job pays a sure wage of one. In the differentiated sector a worker is matched with a firm with probability x_j. Conditional on being matched, he is fired with probability σ and retained with probability $1 - \sigma$. In the former case he gets severance pay b_{pj} plus unemployment benefits ηb_{uj}; in the latter case he gets a wage w_j. Finally, if a worker is not matched with a firm, which happens with probability $1 - x_j$, he gets unemployment benefits b_{uj}. Therefore the expected income of a worker in the differentiated sector is $x_j \left[\sigma \left(b_{pj} + \eta b_{uj}\right) + (1 - \sigma) w_j\right] + (1 - x_j) b_{uj}$. The indifference condition between searching for work in the homogeneous or differentiated sector therefore becomes

$$1 = x_j \left[\sigma \left(b_{pj} + \eta b_{uj}\right) + (1 - \sigma) \left(b_j + \tfrac{1}{2} b_{uj}\right)\right] + (1 - x_j) b_{uj}. \tag{31}$$

Equations (30) and (31) uniquely determine the hiring cost b_j and tightness in the labor market x_j as functions of labor market frictions. They do not have simple closed-form solutions as in (14), but they can be characterized nevertheless. Once we have the vectors (b_j, x_j) for the two countries, they can be used together with the sectoral equilibrium conditions (16), (17), (20) and (21) to determine the equilibrium values of productivity cutoffs, real consumption indexes, numbers of firms, and the number of people searching for jobs in the differentiated sectors. The employment levels can then be solved from $H_j = (1 - \sigma) x_j N_j$.[62] In particular, b_j impacts these variables in the same way it impacted them in the absence of firing costs and unemployment benefits, except for N_j, which now has no simple relation to b_j.

The rate of unemployment equals

$$u_j = \frac{N_j}{L} \left(1 - x_j + x_j \sigma\right), \tag{32}$$

because workers are unemployed either because they are not matched or they are matched and fired. And welfare equals

$$\mathbb{V}_j = L - \left(1 - x_j + x_j \eta \sigma\right) b_{uj} N_j + \frac{1 - \zeta}{\zeta} Q_j^{\zeta}, \tag{33}$$

[62] Note that, with the new interpretation of b_j, all the sectoral equilibrium equations are the same as before, except for (21) and (22), which need to be adjusted for the probability of firing and unemployment benefits. These equations now become:

$$b_j H_j = \frac{\beta}{1+\beta} M_j \left(\frac{\delta_{dj}}{\phi_2} + \frac{\delta_{xj}}{\phi_2}\right),$$
$$H_j = (1 - \sigma) x_j N_j,$$

respectively.

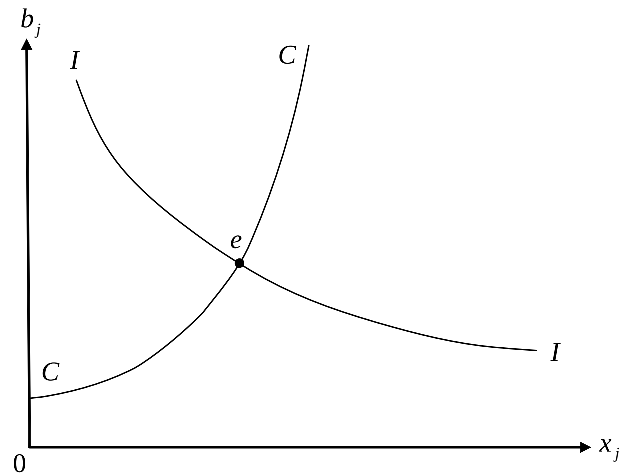

Figure 8: Hiring cost and labor market tightness

where $(1 - x_j + x_j\eta\sigma)\, b_{uj}N_j$ represents taxes needed to pay for the unemployment program, with $(1 - x_j)\, b_{uj}N_j$ representing payments to individuals who were not matched and $x_j\eta\sigma b_{uj}N_j$ representing payments to individuals who were fired.

In order to understand how firing costs and unemployment benefits affect the allocation of resources, we first need to understand how these policies affect the equilibrium hiring costs b_j and tightness measures in the labor markets x_j. For simplicity, we assume that the probability of being fired σ and the unemployment replacement fraction η are constant and the same in both countries. Therefore cross-country differences in these policies can arise either because they have different unemployment benefits b_{uj}, different administrative costs of firing b_{fj}, or different severance pays b_{pj}.[63] As before, differences in the cost of posting vacancies and the efficiency of matching technologies determine differences in a_j across countries. The impact of all these parameters on b_j and x_j can be worked out from the equilibrium conditions (30) and (31), which provide a solution to (b_j, x_j). This solution is depicted in Figure 8. The upward sloping curve CC represents the cost-of-hiring equation (30) while the downward-sloping curve II represents the indifference condition (31). The intersection point e represents the equilibrium (b_j, x_j). The location of these curves is determined by country j's labor market frictions, including labor market policies.

Curve CC slops upward, because an increase in labor market tightness raises the cost of search and matching and therefore the overall hiring cost. The II curve slopes downward for the following reason. First, an increase in b_j raises wages of employed workers, and therefore it raises the expected income on the right-hand side of the indifference condition (31). Second, an increase in the probability of being matched, x_j, also raises expected income on the right-hand side of the

[63] Administrative costs of firing depend on firing procedures, such as how many warnings and at what time spans firms have to issue before firing a workers, what types of hearings have to be held with labor union representatives before a worker is fired, and the like. Some of these costs are determined by labor unions, others by government regulation. Severance pays are also partly determined by labor unions and partly by government regulation.

indifference condition (31), because the conditional expected income from being matched (the term in the square bracket on the right-hand side of (31)) is larger than unemployment benefits.[64] As a result, the II curve slops downward.

Next suppose that A is the rigid country, in the sense that $b_A > b_B$, which means that hiring costs are larger in country A. Unlike the case discussed in previous sections, where differences in hiring costs emanated from differences in a_j across countries, now these differences can arise from labor market policies concerning firing and unemployment. But whatever generates differences in b_j across countries can be summarized in the ranking $b_A > b_B$. Under these circumstances the results from Section 3 concerning the comparison of outcomes in the differentiated sectors of the two countries, do not change. In particular, Lemmas 1-4 still hold. That is, if both countries are incompletely specialized, then the rigid country has a smaller domestic cutoff and a higher export cutoff, it has a lower real consumption index in the differentiated sector, and it has fewer firms in that sector. As a result, all the propositions from Section 4 that do not concern welfare remain valid. For example, Proposition 3 on trade structure and Proposition 5 on productivity do not change. Moreover, all the propositions concerning welfare also remain valid if there are no unemployment benefits, or if b_{uj} is small.

Propositions concerning welfare can change when $b_{uj} > 0$, because in this case the welfare of country j depends not only on the real consumption index Q_j, but also on aggregate unemployment benefits that are financed by taxes (see (33)). Different welfare results can arise in this case because the impact of taxes on welfare can be larger than the impact of real consumption of differentiated products, and the two of them can influence welfare in opposite directions. As an example consider a reduction in trade impediments. This raises the real consumption index Q_j in both countries, which is welfare improving. On the other hand, it may attract more workers to the differentiated sector (N_j may rise). As a result, the unemployment compensation bill may increase, leading to higher taxes that reduce welfare.[65] With high unemployment benefits the latter effect can be larger.

Now consider unemployment in a world of symmetric countries. In this case trade raises unemployment in both of them, because it attracts more workers to the differentiated sectors and does not changed tightness in the labor markets (see (32)). Moreover, every reduction in trade frictions raises unemployment.

In contrast, a reduction in hiring costs b_j has a more nuanced impact on unemployment. On the one hand, in a world of symmetric countries a common decline in b_j raises N_j in both countries, because more workers are attracted to the differentiated sectors. On the other hand, tightness in the labor market x_j may rise or decline, depending on what caused b_j to fall. To understand why, we need to discuss the impact of labor market policies on (b_j, x_j).

An increase in a_j or b_{fj} raises directly firms' labor costs without directly affecting workers'

[64] We assume that being fired provides higher income than unemployment benefits, i.e., $b_{pj} + \eta b_{uj} > b_{uj}$. Therefore we have $\sigma(b_{pj} + \eta b_{uj}) + (1-\sigma)\left(b_j + \frac{1}{2}b_{uj}\right) > \sigma b_{uj} + (1-\sigma)\left(b_j + \frac{1}{2}b_{uj}\right) = b_{uj} + (1-\sigma)\left(b_j - \frac{1}{2}b_{uj}\right) > b_{uj}$, where the last inequality follows from (30).

[65] Note that a decline in τ does not change tightness in the labor markets, so that the cost of the unemployment program $(1 - x_j + x_j\eta\sigma)\, b_{uj} N_j$ has to increase.

payoffs. As a result, it raises the CC curve without affecting the II curve, which implies that b_j rises and x_j falls. Unlike these parameters, an increase in either b_{pj} or b_{uj} raises directly firms' labor costs and also raises directly workers' payoffs. As a result, CC shifts upward while II shifts downward, which implies that x_j declines while b_j may rise or decline.

We show in the Appendix that

$$\frac{db_j}{db_{pj}} < 0 \text{ if and only if } b_{uj} > 1 - \alpha a_j x_j^{1+\alpha}.$$

An example of a case in which $b_{uj} > 1 - \alpha a_j x_j^{1+\alpha}$ is the following. Suppose we start from an equilibrium with no firing costs and no unemployment benefits in country j. Then (30) and (31) yield $x_j = a_j^{-1/(1+\alpha)}$ and $b_j = a_j^{1/(1+\alpha)}/(1-\sigma)$. Therefore $\alpha a_j x_j^{1+\alpha} > 1$ for $a_j > 1$ and $\alpha \geq 1$. Introducing into this country a small severance pay reduces both x_j and b_j.

One interesting implication of this example is that the introduction of severance pay can raise welfare. In the example the severance pay reduces hiring costs and thereby raises the real consumption index Q_j. Moreover, this policy does not affect taxes, as long as there are no unemployment benefits. Therefore welfare rises.[66] A second interesting implication is that the relationship between hiring costs and severance pay may be nonmonotonic. If, as in the example, we start with $\alpha a_j x_j^{1+\alpha} > 1$, initial increases in severance pay reduce hiring costs. But as severance pay rises, tightness in the labor market, x_j, declines. For high enough levels of severance pay x_j is low enough to reverse the inequality to $\alpha a_j x_j^{1+\alpha} < 1$. In this new range, increases in severance pay keep pushing down tightness in the labor market, but now they raise the cost of hiring. Therefore the relationship between b_j and b_{pj} is U shaped.[67]

We also show in the Appendix that

$$\frac{db_j}{db_{uj}} < 0 \text{ if and only if } b_{uj} > 1 - \alpha a_j x_j^{\alpha}\left[2\frac{1-\sigma(1-\eta)x_j}{1-\sigma} - x_j\right].$$

Note that the term in the square brackets is positive for $0 < x_j < 1$. Therefore the above inequality can be satisfied for $b_{uj} < 1$. As an example, consider the case in which $\sigma = 0$ (no firing). In this case (30) and (31) imply

$$b_j = a_j^{\frac{1}{1+\alpha}}(1-b_{uj})^{\frac{\alpha}{1+\alpha}} + \tfrac{1}{2}b_{uj},$$

$$\frac{\mathrm{d}b_j}{\mathrm{d}b_{uj}} = \frac{1}{2} - \frac{\alpha}{1+\alpha}\left(\frac{a_j}{1-b_{uj}}\right)^{\frac{1}{1+\alpha}}.$$

If country j has no unemployment benefits and $\alpha a_j^{1/(1+\alpha)}/(1+\alpha) > 1/2$, i.e., the right-hand side of the second equation is negative for $b_{uj} = 0$, then the introduction of unemployment benefits

[66] Also note that if $\alpha a_j x_j^{\alpha} < 1$ in the initial equilibrium, the introduction of a small severance pay raises the hiring cost and reduces welfare.

[67] The U shape of this relationship is a more general phenomenon. If $b_{uj} > 1 - \alpha a_j x_j^{1+\alpha}$ for some $b_{pj} > 0$, then the same inequality holds for all smaller severance pays. Moreover, since $b_{uj} < 1$ and x_j is declining in b_{pj}, it follows that for large enough values of b_{pj} we have $b_{uj} < 1 - \alpha a_j x_j^{1+\alpha}$. Therefore a necessary and sufficient condition for a U-shaped relationship between b_j and b_{pj} is $b_{uj} > 1 - \alpha a_j x_j^{1+\alpha}$ when $b_{pj} = 0$, because $x_j \to 0$ as $b_{pj} \to \infty$.

reduces j's hiring costs, and the hiring costs are lower the higher the unemployment benefits are.[68] Alternatively, suppose that $\alpha a_j^{1/(1+\alpha)}/(1+\alpha) < 1/2$. Then hiring costs rise with unemployment benefits when unemployment benefits are low. When unemployment benefits are high, however, further increases in unemployment benefits reduce hiring costs ($\mathrm{d}b_j/\mathrm{d}b_{uj}$ becomes negative). Therefore in this case the relationship between hiring costs and unemployment benefits has an inverted U shape.

Our findings on the determinants of hiring costs and tightness in labor markets can be summarized in

Lemma 6 *(i) Increases in a_j, b_{fj}, b_{pj} and b_{uj} all reduce x_j. (ii) Increases in a_j and b_{fj} raise b_j. (iii) Increases in b_{pj} and b_{uj} can increase or reduce b_j. Moreover, the relationship between b_j and b_{pj} can be U shaped, while the relationship between b_j and b_{uj} can have an inverted U shape.*

While the introduction of both severance pay and unemployment benefits can reduce hiring costs, the introduction of severance pay that reduces hiring costs raises welfare in the absence of unemployment benefits, while the introduction of unemployment benefits that reduce hiring costs may reduce welfare. Note that in both cases Q_j rises. Yet, while in the case of severance pay an increase in real consumption of differentiated products guarantees an increase in welfare, in the case of unemployment benefits it does not, because unemployment benefits have to be financed with taxes and taxes reduce welfare. Moreover, if the introduction of unemployment benefits raises hiring costs, then this program is welfare reducing because it leads to higher taxes and lower real consumption of differentiated products.

It remains to consider the impact of these policies on the trade partner. For this purpose all we need to know is their impact on hiring costs. If a reform in labor market policies in country j reduces b_j, then country j becomes more competitive in differentiated products, which leads to a contraction of $Q_{(-j)}$ in the trade partner, as shown in Section 4. Under the circumstances welfare of the trade partner declines if it has small unemployment benefits, because with small unemployment benefits the impact on taxes is small. If, however, the trade partner has large unemployment benefits, the trade partner's welfare may increase, because the contraction of its differentiated sector reduces the tax bill for unemployment benefits.

The general conclusion from this analysis is that our results from previous sections concerning the allocation of resources do not depend on the details of the firing costs and unemployment benefits, only on the (b_j, x_j)s, and the welfare results still hold when unemployment benefits are small. High unemployment benefits can change the welfare results, however, because they impact taxes needed to finance these benefits. Moreover, the presence of firing costs and unemployment benefits introduces policy instruments that can change hiring costs and tightness in labor markets in ways that differ from the impacts of search and matching costs. As a result, they broaden the set of feasible vectors (b_j, x_j) and thereby broaden the set of feasible outcomes.

[68] Under these circumstances unemployment benefits raise unemployment in symmetric countries, because sectoral unemployment rises (x_j declines) and more people search for jobs in the differentiated sector (declines in b_j raise N_j).

8 Concluding Comments

We have studied the interdependence of countries that trade homogeneous and differentiated products with each other, and whose labor markets are characterized by search and matching frictions in the differentiated-product industry. Variation in labor market frictions and the interactions between trade impediments and labor market institutions generate rich patterns of unemployment. For example, better labor market institutions do not ensure lower unemployment, and unemployment and welfare can both rise in response to a policy change.

Contrary to the complex patterns regarding unemployment, the model yields sharp predictions about welfare. In particular, both countries gain from trade, but the gains are unevenly distributed, with the flexible country gaining proportionately more. The latter implies that a country stands to gain more from reforming its labor market when trade frictions are low rather than high. Reducing frictions in the domestic labor market raises the competitiveness of home firms. This improves the foreign country's terms of trade, but also crowds out foreign firms from the differentiated-product sector. As a result welfare rises at home and declines abroad (i.e., the terms-of-trade improvement in the foreign country is overwhelmed by the competitiveness effect). Nevertheless, across-the-board improvements in labor market institutions raise welfare in both countries. These results contrast with the implications of models of pure comparative advantage, in which movements in the terms of trade dominate the outcomes.[69]

We also show that labor market institutions confer comparative advantage, and that differences in these institutions shape trade flows. In particular, the flexible country exports differentiated products on net and imports homogeneous goods. Moreover, the larger the difference in labor market frictions the lower is the share of intra-industry trade. These are testable implications about trade flows and international patterns of specialization.

In addition, we show that trade raises total factor productivity in the differentiated-product sectors of both countries (productivity is constant in the homogeneous good sector). Importantly, however, productivity is higher in the flexible country.

We also show how to integrate firing costs and unemployment benefits into this framework. An interesting implication of these policies is that higher severance pay or higher unemployment benefits may reduce the cost of hiring, by substantially reducing tightness in the labor market. Under these circumstances they improve competitiveness. When this type of improvement in competitiveness is achieved with severance pay, welfare rises. But when this type of improvement in competitiveness is achieved with unemployment benefits, welfare may decline, because unemployment benefits are financed with taxes while severance pay is not. Our paper provides an analytical framework for studying interdependence in the presence of these types of labor market frictions, with interesting implications for trade and unemployment.

[69] See, for example, Brügemann (2003) and Alessandria and Delacroix (2008). The former examines the support for labor market rigidities in a Ricardian model in which the choice of regime impacts comparative advantage. The latter analyzes a two-country model with two goods, in which every country specializes in a different product and governments impose firing taxes. The authors find that a coordinated elimination of these taxes yields welfare gains for both counties, yet no country on its own has an incentive to do it.

Appendix

An alternative specification with homothetic preferences

We consider here an alternative specification of the model, with CRRA-CES preferences instead of quasi-linear preferences used in the main text, leaving the rest of the setup unchanged. The expected utility is $\mathbb{U} = \mathbb{E}\mathcal{C}^{1-\sigma}/(1-\sigma)$, where $\mathbb{E}$ is the expectations operator, $\sigma \in [0,1)$ is the relative risk aversion coefficient and $\mathcal{C}$ is a CES bundle of homogeneous and differentiated goods:

$$\mathcal{C} = \left[\vartheta^{1-\zeta} q_0^{\zeta} + (1-\vartheta)^{1-\zeta} Q^{\zeta}\right]^{1/\zeta}, \quad \zeta < \beta, \quad 0 < \vartheta < 1.$$

The ideal price index associated with this consumption bundle is

$$\mathcal{P} = \left[\vartheta + (1-\vartheta) P^{\frac{-\zeta}{1-\zeta}}\right]^{-\frac{1-\zeta}{\zeta}},$$

where the price of the homogeneous good p_0 is again normalized to one and P is the price of the differentiated product in terms of the homogeneous good.

The demand for homogeneous and differentiated goods is given by

$$q_0 = \vartheta \mathcal{P}^{\zeta/(1-\zeta)} E = \frac{\vartheta E}{\vartheta + (1-\vartheta) P^{\frac{-\zeta}{1-\zeta}}},$$

$$Q = (1-\vartheta)\left(\frac{P}{\mathcal{P}}\right)^{\frac{-1}{1-\zeta}} \frac{E}{\mathcal{P}} = \frac{(1-\vartheta) P^{\frac{-1}{1-\zeta}} E}{\vartheta + (1-\vartheta) P^{\frac{-\zeta}{1-\zeta}}},$$

where E is expenditure in units of the homogeneous good. Using these demand equations, we derive the indirect utility function

$$\mathbb{V} = \frac{1}{1-\sigma}\mathbb{E}\left(\frac{E}{\mathcal{P}}\right)^{1-\sigma}.$$

Since $\mathcal{P}$ is increasing in P, the indirect utility is falling in P for a given $\mathbb{E}E^{1-\sigma}$. Also Q is decreasing in P.

Next, the demand level for differentiated varieties is

$$D \equiv Q P^{\frac{1}{1-\beta}} = \frac{(1-\vartheta) P^{\frac{\beta-\zeta}{(1-\beta)(1-\zeta)}} E}{\vartheta + (1-\vartheta) P^{\frac{-\zeta}{1-\zeta}}},$$

which increases in P given $\beta > \zeta$. It proves useful to introduce the aggregate revenue variable

$$R \equiv PQ = D^{1-\beta} Q^{\beta} = \frac{(1-\vartheta) P^{\frac{-\zeta}{1-\zeta}} E}{\vartheta + (1-\vartheta) P^{\frac{-\zeta}{1-\zeta}}},$$

which, like Q and opposite to D, decreases in P. Note that with homothetic utility, demands and revenues are linear in income, E, which allows for simple aggregation. Specifically, in the expressions above E can be interpreted as aggregate income equal to $E = (L-N) + wxN$.

Most of the remaining derivation of equilibrium conditions remains unchanged, with D replacing $Q^{-(\beta-\zeta)/(1-\beta)}$ in the text. Specifically, after this substitution the free entry condition and zero profit conditions are unchanged, which allows us to solve for equilibrium cutoffs and equilibrium D's in the same manner as in the text. Qualitatively all the relationships still hold, except that now instead of Q as the sufficient statistic for welfare and demand level it is more convenient to express all aggregate variables as functions of P. Additionally, $R = PQ$ replaces Q^ζ in the expressions for M and N. Finally, the indifference condition for workers now includes a risk premium for employment seeking in the sector with labor market frictions:

$$xw^{1-\sigma} = 1,$$

so that $xw > 1$ when $x < 1$ and there is a chance of being unemployed. The equilibrium wage is still equal to $b = ax^\alpha$, which now leads to

$$\begin{aligned} x &= b^{-(1-\sigma)} = a^{\frac{-(1-\sigma)}{1+\alpha(1-\sigma)}}, \\ w &= b = a^{\frac{1}{1+\alpha(1-\sigma)}}. \end{aligned}$$

Since all workers are indifferent between a certain wage of one in the homogeneous-good sector and the wage of w with probability x in the differentiated-good sector, we have at the aggregate level $\mathbb{E}E^{1-\sigma} = L^{1-\sigma}$, where expectation is taken both across states for individual workers (employed/unemployed for those searching in the differentiated sector) and across workers (searching for jobs in the two sectors) and L is the total number (measure) of individual workers in the economy.

Note that with homothetic preferences and $\sigma < 1$, we have dropped the family interpretation. In this case, the structure of demand and indirect utility does not change if the worker becomes unemployed, and aggregation is straightforward with $\mathbb{E}E^{1-\sigma} = L^{1-\sigma}$ and $E = L + (b^\sigma - 1)N$.[70] As a result, this specification can be used to analyze issues such as the ex-post income distribution and winners and losers from policy reforms.

Without showing the explicit derivation (which follows the same steps as in the text), we provide as an illustration a few comparative statics results for the symmetric open economies with homothetic preferences. Specifically, we consider proportional labor market deregulation in both countries. We have $\hat{D} = \beta/(1-\beta)\hat{b}$, so that, as before, P decreases and Q and R increase as b falls. This also implies an increase in welfare.

As before, we can express the total wage bill in the differentiated sector as

$$bH = \frac{\beta}{1+\beta}R = \frac{\beta}{1+\beta}M(\delta_d + \delta_x)/\phi_2,$$

where the δ_zs are average revenues per entering firm as defined in the text. We still have $H = xN$. Using $x = b^{-(1-\sigma)}$, we have the expression for the number of workers searching for a job in the differentiated sector:

$$N = \frac{\beta}{1+\beta}b^{-\sigma}R.$$

Since R is decreasing in b, we have that N decreases in b as before. As a result, there are still two opposing effects on the unemployment rate: $\hat{u} = \hat{b}/(b-1) + \hat{N}$. The change in unemployment rate is again ambiguous: it still falls if initial labor market institutions are flexible enough and increases otherwise. These results are qualitatively the same as those derived in the text under quasi-linear preferences.

Additionally, we can discuss now ex post inequality. A fall in b increases x and reduces w, which both lead to lower ex-post inequality. At the same time it increases N which may increase or reduce inequality depending on the initial size of the differentiated sector. It follows that comparative statics for inequality is

[70] Note that if $\sigma \geq 1$, we still need to recur to the family risk-sharing interpretation in order to avoid an infinite risk premium for employment-seeking in the differentiated product sector with a possibility of being unemployed. Alternatively, the extension of the model with unemployment benefits ensures that the risk premium is finite.

ambiguous in the same way as those for unemployment rate.

Complete and Incomplete Specialization Equilibria

We derive here a limit on b_A/b_B which secures an equilibrium in which both countries are incompletely specialized. Then we discuss equilibria for which this condition is violated and the rigid country specializes in homogeneous products. Throughout we assume for concreteness that A is the rigid country, so that $b_A/b_B \geq 1$. Following the main text, we assume $\zeta > 0$. For brevity, we will analyze only the equilibria with $\Theta_{xB} > \Theta_{dB} > \Theta_{\min}$, so that in the flexible country not all producing firms export and there also are firms that exit. The whole analysis can be carried out in a similar manner when either one of the inequalities fails, and the results are broadly similar. Finally, for concreteness, we assume that $f_x \geq f_d$. This assumption is useful because it guarantees that $\delta_{xB} < \delta_{dB}$ in the type of equilibria that we consider, which avoids the need to discuss different cases. Again, the same analysis can be carried out when $f_x < f_d$ and it yields similar results.

Equation (20) in the text implies that $M_A = 0$ whenever

$$\delta_{dB}\left(\frac{Q_A}{Q_B}\right)^{\zeta} \leq \delta_{xB}.$$

When this condition is satisfied with equality we also find, using (16), that

$$\delta_{dB}\left[\frac{\Theta_{xB}}{\Theta_{dB}}\frac{f_d}{f_x}\tau^{\frac{-\beta}{1-\beta}}\right]^{\zeta\frac{1-\beta}{\beta-\zeta}} = \delta_{xB}. \tag{34}$$

Note that this relationship is a (generally nonlinear) upward-sloping curve in $(\Theta_{dB}, \Theta_{xB})$-space, lying between the 45°-line and $\Theta_{xB} = \Theta_{dB}\tau^{\beta/(1-\beta)}f_x/f_d$ (i.e., the equilibrium condition when $b_A = b_B$).[71]

We can now prove the following

Lemma 7 *Let $\tau > 1$ and $b_A > b_B$. Then there exists a unique $\bar{b}(\tau)$, with $\bar{b}'(\tau) > 0$, such that* (34) *holds for $b_A/b_B = \bar{b}(\tau)$. For $b_A/b_B < \bar{b}(\tau)$, there is incomplete specialization in equilibrium so that $M_A > 0$. For $b_A/b_B \geq \bar{b}(\tau)$, country A specializes in the homogeneous good so that $M_A = 0$.*

Proof: Recall that Θ_{dB} is decreasing and Θ_{xB} is increasing in τ. This implies that δ_{dB}/δ_{xB} is increasing in τ. (19) implies that $\tau^{\frac{-\beta}{1-\beta}}\Theta_{xB}/\Theta_{dB}$ is increasing in τ. Next, Θ_{xB}/Θ_{dB} and δ_{dB}/δ_{xB} are decreasing in b_A/b_B. These considerations, together with (34), imply that $\bar{b}(\tau)$ is unique and increasing in τ whenever it is finite.[72] Finally, Q_A/Q_B is decreasing in b_A/b_B. Therefore, from (20), $M_A > 0$ whenever $b_A/b_B < \bar{b}(\tau)$ and $M_A = 0$ whenever $b_A/b_B \geq \bar{b}(\tau)$.

We consider now equilibria with complete specialization. By Lemma 7 complete specialization in country A occurs whenever $b_A/b_B \geq \bar{b}(\tau)$, so that $M_A = 0$. The equilibrium system in this case consists of five

[71] In the special case of a Pareto distribution it is a ray through the origin.

[72] Note that $\bar{b}(\tau) > 1$ by construction, since $\Theta_{xB} = \Theta_{dB}\tau^{\beta/(1-\beta)}f_x/f_d$ when $b_A = b_B$.

equations jointly determining $\{\Theta_{dB}, \Theta_{xB}, Q_A, Q_B, M_B\}$:

$$\begin{aligned}
\phi_1\phi_2\Theta_{dB} &= f_d b_B^{\frac{\beta}{1-\beta}} Q_B^{\frac{\beta-\zeta}{1-\beta}}, \\
\phi_1\phi_2\Theta_{xB} &= f_x \tau^{\frac{\beta}{1-\beta}} b_B^{\frac{\beta}{1-\beta}} Q_A^{\frac{\beta-\zeta}{1-\beta}}, \\
Q_A^{\beta\frac{1-\zeta}{1-\beta}} &= \phi_1 M_B b_B^{\frac{-\beta}{1-\beta}} \tau^{\frac{-\beta}{1-\beta}} \int_{\Theta_{xB}}^{\infty} \Theta dG(\Theta), \\
Q_B^{\beta\frac{1-\zeta}{1-\beta}} &= \phi_1 M_B b_B^{\frac{-\beta}{1-\beta}} \int_{\Theta_{dB}}^{\infty} \Theta dG(\Theta), \\
f_d \int_{\Theta_{dB}}^{\infty} \left(\frac{\Theta}{\Theta_{dB}} - 1\right) dG(\Theta) + f_x \int_{\Theta_{xB}}^{\infty} &\left(\frac{\Theta}{\Theta_{xB}} - 1\right) dG(\Theta) = f_e
\end{aligned}$$

The first two equations are the cutoff conditions, i.e., counterparts of (16) in the text, but only for country B now since country A does not produce differentiated goods. The third and fourth equations come from the definitions of Q_js, as the equations that precede (20) in the text. The final equation is the free entry condition (17).

The first two equations imply as before

$$\frac{\Theta_{xB}}{\Theta_{dB}} = \tau^{\frac{\beta}{1-\beta}} \frac{f_x}{f_d} \left(\frac{Q_A}{Q_B}\right)^{\frac{\beta-\zeta}{1-\beta}}. \tag{35}$$

Taking the ratios of the conditions for Q_A and Q_B, we have

$$\left(\frac{Q_B}{Q_A}\right)^{\beta\frac{1-\zeta}{1-\beta}} = \tau^{\frac{\beta}{1-\beta}} \frac{f_x}{f_d} \frac{\Theta_{dB}\delta_{dB}}{\Theta_{xB}\delta_{xB}}.$$

The last two results together yield a simple relationship

$$\left(\frac{Q_A}{Q_B}\right)^{\zeta} = \frac{\delta_{xB}}{\delta_{dB}}. \tag{36}$$

We now are ready to prove

Proposition 10 *In equilibrium with specialization ($M_A = 0$), the following comparative statics results hold:*

1. *Θ_{dB}, Θ_{xB}, and (Q_A/Q_B) do not respond to changes in b_B. Q_A and Q_B fall proportionally in response to an increase in b_B; M_B also decreases proportionally with Q_j^{ζ}.*

2. *Θ_{dB} increases and Θ_{xB} decreases in response to a fall in τ. M_B, Q_B, and Q_A/Q_B all increase in response to a fall in τ. Thus, the rigid country benefits more from a reduction in trade impediments.*

Proof: We plug (36) into (35) and take the log-derivative:

$$\hat{\Theta}_{xB} - \hat{\Theta}_{dB} = \frac{\beta}{1-\beta}\hat{\tau} + \frac{\beta-\zeta}{\zeta(1-\beta)}\left(\hat{\delta}_{xB} - \hat{\delta}_{dB}\right),$$

where $\hat{\delta}_{zB} = -\left[f_z \Theta_{zB} G'(\Theta_{zB}) + \delta_{zB}\right]\hat{\Theta}_{zB}$ for $z \in \{d, x\}$. From free entry we have

$$\delta_{dB}\hat{\Theta}_{dB} + \delta_{xB}\hat{\Theta}_{xB} = 0.$$

Combining these results we have

$$\left[\frac{\delta_{dB}+\delta_{xB}}{\delta_{xB}}+2\delta_{dB}+f_d\Theta_{dB}G'(\Theta_{dB})+f_x\Theta_{xB}G'(\Theta_{xB})\frac{\delta_{dB}}{\delta_{xB}}\right]\hat{\Theta}_{dB}=-\frac{\beta}{1-\beta}\hat{\tau},$$

so that Θ_{dB} decreases and Θ_{xB} increases in τ. Also note that neither threshold responds to b_B. This result implies that δ_{dB} increases and δ_{xB} decreases in τ and they do not respond to b_B. This observation together with (36) imply that Q_A/Q_B does not respond to b_B and decreases in τ. Log-differentiation of the cutoff condition for Θ_{dB} results in

$$\frac{\beta-\zeta}{1-\beta}\hat{Q}_B=-\frac{\beta}{1-\beta}\hat{b}_B+\hat{\Theta}_{dB}.$$

Therefore, Q_B and Q_A fall proportionally as b_B increases; Q_A and Q_B both decrease in τ and Q_A does so proportionally more. Finally, the results for M_B follow from the equation for Q_B:

$$\hat{M}_B=\zeta\hat{Q}_B-\hat{\delta}_{dB}.$$

Therefore, M_B falls in τ; it also falls in b_B at the same rate as Q_B^{ζ}.

Proposition 10 emphasizes an important difference between an equilibrium with specialization and an equilibrium with incomplete specialization (see Proposition 1 in the text). In equilibria with specialization, the rigid country is the one that gains proportionately more from a reduction in trade impediments. Moreover, both countries equally gain from improvements in the labor market institutions of the flexible country. The reason is that now there is no competitiveness effect, which was crowding out the firms in the rigid country as trade was becoming less costly or as the labor market of the trade partner was becoming more flexible. Now the only effect is the terms of trade effect, i.e., the reduction in the price level of the differentiated goods when τ or b_B fall. A reduction in trade costs on top of that makes the consumption baskets (i.e., the number of varieties and their quantities consumed) in the two countries more similar which leads to a partial convergence in the welfare differential between the countries. Note, however, that both countries still gain from trade. The reason that the flexible country gains despite the deterioration in its terms of trade is that the additional profits from the foreign market lead to more entry of domestic firms, which enhances welfare.

Derivation of results on productivity for Section 4.3

We first show that $\varphi_{zj}=\varphi(\Theta_{zj})$ is monotonically increasing in Θ_{zj}. The log-derivative of $\varphi(\Theta_{zj})$ is

$$\hat{\varphi}(\Theta_{zj})=\Theta_{zj}G'(\Theta_{zj})\left[\frac{\Theta_{zj}}{\int_{\Theta_{zj}}^{\infty}\Theta dG(\Theta)}-\frac{\Theta_{zj}^{1/\beta}}{\int_{\Theta_{zj}}^{\infty}\Theta^{1/\beta}dG(\Theta)}\right]\hat{\Theta}_{zj}.$$

The term in the square brackets is positive since

$$\frac{\Theta_{zj}^{1/\beta}}{\int_{\Theta_{zj}}^{\infty}\Theta^{1/\beta}\frac{dG(\Theta)}{1-G(\Theta_{zj})}}<\left(\frac{\Theta_{zj}}{\int_{\Theta_{zj}}^{\infty}\Theta\frac{dG(\Theta)}{1-G(\Theta_{zj})}}\right)^{1/\beta}<\frac{\Theta_{zj}}{\int_{\Theta_{zj}}^{\infty}\Theta^{1/\beta}\frac{dG(\Theta)}{1-G(\Theta_{zj})}},$$

where the first inequality follows from Jensen's inequality and the second inequality comes from the fact that $\beta<1$ and $\Theta_{zj}<\int_{\Theta_{zj}}^{\infty}\Theta\frac{dG(\Theta)}{1-G(\Theta_{zj})}$.

Next we provide the general expression for a log-change in aggregate productivity:

$$\widehat{TFP}_j = \left\{ 1 + \frac{\kappa_{dj}}{\varphi_{xj} - \varphi_{dj}} \left[\frac{\kappa_{xj}}{\kappa_{dj}} \left(\Theta_{xj}^{\frac{1-\beta}{\beta}} - TFP_j \right) + \left(TFP_j - \Theta_{dj}^{\frac{1-\beta}{\beta}} \right) \right] \right\} \frac{\delta_{dj}(\varphi_{xj} - \varphi_{dj})}{\delta_{dj}\varphi_{dj} + \delta_{xj}\varphi_{xj}} \hat{\Theta}_{dj}, \tag{37}$$

where

$$\kappa_{zj} \equiv \kappa(\Theta_{zj}) = \frac{f_z \Theta_{zj} G'(\Theta_{zj})}{\delta_{zj}} = \frac{\Theta_{zj} G'(\Theta_{zj})}{\frac{1}{\Theta_{zj}} \int_{\Theta_{zj}}^{\infty} \Theta dG(\Theta)}.$$

A series of sufficient conditions can be suggested for the terms in curly brackets to be positive. Since $TFP_j \geqslant \Theta_{dj}^{(1-\beta)/\beta}$ is always true, it is sufficient to require that

$$TFP_j \leqslant \Theta_{xj}^{(1-\beta)/\beta},$$

which holds for large enough Θ_{xj}, i.e., when the economy is relatively closed. However, this inequality fails to hold when Θ_{xj} approaches Θ_{dj}. If this condition fails, it is sufficient to have

$$\left[TFP_j - \Theta_{dj}^{(1-\beta)/\beta} \right] / \left[TFP_j - \Theta_{xj}^{(1-\beta)/\beta} \right] \geqslant \kappa_{xj} / \kappa_{dj},$$

which is, in particular, satisfied when $\kappa_{dj} \geqslant \kappa_{xj}$. This latter condition is always satisfied if $\kappa(\cdot)$ is a non-increasing function and is equivalent to

$$-\frac{\Theta G''(\Theta)}{G'(\Theta)} \geq 2 + \frac{\Theta G'(\Theta)}{\frac{1}{\Theta} \int_{\Theta}^{\infty} \xi dG(\xi)},$$

that is $G''(\cdot)$ has to be negative and large enough in absolute value. This condition is satisfied for the Pareto distribution since in this case $\kappa(\cdot)$ is constant and, thus, $\kappa_{dj} \equiv \kappa_{xj}$. However, it is not satisfied, for example, for the exponential distribution.

Finally, the necessary and sufficient condition is

$$\left(\kappa_{xj} - \kappa_{dj} \right) TFP_j - \left(\kappa_{xj} \Theta_{xj}^{(1-\beta)/\beta} - \kappa_{dj} \Theta_{dj}^{(1-\beta)/\beta} \right) \leqslant \varphi_{xj} - \varphi_{dj}$$

which is satisfied when

$$\frac{\left(\kappa_{xj} - \kappa_{dj} \right) \left(TFP_j - \Theta_{dj}^{(1-\beta)/\beta} \right)}{\varphi_{xj} - \varphi_{dj}} = \left(\kappa_{xj} - \kappa_{dj} \right) \left[\varpi_{xj} + \frac{\varphi_{dj} - \Theta_{dj}^{(1-\beta)/\beta}}{\varphi_{xj} - \varphi_{dj}} \right] \leqslant 1.$$

This condition also does not hold in general; however, it is certainly satisfied for large enough Θ_{xj}.

Now we provide the derivation of equation (26) under the assumption of Pareto-distributed productivity draws. When Θ is distributed Pareto with the shape parameter $k > 1/\beta$, there is a straightforward way of computing the change in TFP_j. Taking the log derivative of (25), we have

$$\widehat{TFP}_j = \left[\frac{\delta_{dj}\varphi_{dj}\hat{\delta}_{dj} + \delta_{xj}\varphi_{xj}\hat{\delta}_{xj}}{\delta_{dj}\varphi_{dj} + \delta_{xj}\varphi_{xj}} - \frac{\delta_{dj}\hat{\delta}_{dj} + \delta_{xj}\hat{\delta}_{xj}}{\delta_{dj} + \delta_{xj}} \right] + \frac{\delta_{dj}\varphi_{dj}\hat{\varphi}_{dj} + \delta_{xj}\varphi_{xj}\hat{\varphi}_{xj}}{\delta_{dj}\varphi_{dj} + \delta_{xj}\varphi_{xj}}.$$

Under the Pareto assumption, the free-entry condition (17) can be written as $\delta_{dj} + \delta_{xj} = kf_e$, which implies

$\delta_{dj}\hat{\delta}_{dj} + \delta_{xj}\hat{\delta}_{xj} = 0$. We use this to simplify

$$\widehat{TFP}_j = \frac{\delta_{dj}\varphi_{dj}\left(\hat{\delta}_{dj} + \hat{\varphi}_{dj}\right) + \delta_{xj}\varphi_{xj}\left(\hat{\delta}_{xj} + \hat{\varphi}_{xj}\right)}{\delta_{dj}\varphi_{dj}}.$$

Next note that $\delta_{zj} = f_z \frac{k}{k-1}(\Theta_{\min}/\Theta_{zj})^k$ so that $\hat{\delta}_{zj} = -k\hat{\Theta}_{zj}$ and $\varphi_{zj} = \frac{k-1}{k-1/\beta}\Theta_{zj}^{(1-\beta)/\beta}$, implying $\hat{\varphi}_{zj} = (1-\beta)/\beta\hat{\Theta}_{zj}$. Thus, the log-derivative of the free-entry condition can also be written as $\delta_{dj}\hat{\Theta}_{dj} + \delta_{xj}\hat{\Theta}_{xj} = 0$. Therefore,

$$\delta_{dj}\left(\hat{\delta}_{dj} + \hat{\varphi}_{dj}\right) = -\delta_{xj}\left(\hat{\delta}_{xj} + \hat{\varphi}_{xj}\right) = -\left[k - (1-\beta)/\beta\right]\delta_{dj}\hat{\Theta}_{dj}.$$

Using this, we obtain our result (26) in the text

$$\widehat{TFP}_j = \frac{\delta_{dj}\left(\varphi_{xj} - \varphi_{dj}\right)\left[k - (1-\beta)/\beta\right]}{\delta_{dj}\varphi_{dj} + \delta_{xj}\varphi_{xj}}\hat{\Theta}_{dj}.$$

Finally, we discuss an alternative measure of productivity which takes into account the sectoral composition of resource allocation:

$$TFP'_j = \frac{L - N_j}{L} + \frac{N_j}{L}\frac{H_j}{N_j}TFP_j,$$

which is a weighted average of 1 (the productivity in the homogeneous sector) and $TFP''_j \equiv H_j/N_j \cdot TFP_j$ (productivity in the differentiated-product sector). The weights are the respective fractions of the two sectors in the labor force. Note that $\widehat{TFP''_j} = \widehat{TFP}_j - \hat{b}_j$. If $TFP''_j > 1$, an extensive margin increase in the size of the differentiated sector improves productivity. Reduction in trade costs and labor market deregulation additionally shift resources towards the differentiated sector by increasing N_j. These are the additional effects captured by this alternative measure of aggregate productivity.

Derivation of results for the case of small asymmetries in Section 5.2

For $\hat{b}_B = \hat{\tau} = 0$ and $\hat{b}_A > 0$, (19) yield:

$$\hat{\Theta}_{dA} = -\frac{\delta_{xA}}{\Delta}\left(\delta_{xB} + \delta_{dB}\right)\hat{b}_A < 0, \qquad \hat{\Theta}_{xA} = \frac{\delta_{dA}}{\Delta}\left(\delta_{xB} + \delta_{dB}\right)\hat{b}_A > 0,$$

$$\hat{\Theta}_{dB} = \frac{\delta_{xB}}{\Delta}\left(\delta_{xA} + \delta_{dA}\right)\hat{b}_A > 0, \qquad \hat{\Theta}_{xB} = -\frac{\delta_{dB}}{\Delta}\left(\delta_{xA} + \delta_{dA}\right)\hat{b}_A < 0.$$

Using these expressions together with (23) we obtain

$$\frac{\beta - \zeta}{1 - \beta}\hat{Q}_A = -\frac{\delta_{dB}}{\Delta}\left(\delta_{xA} + \delta_{dA}\right)\hat{b}_A < 0,$$
$$\frac{\beta - \zeta}{1 - \beta}\hat{Q}_B = \frac{\delta_{xB}}{\Delta}\left(\delta_{xA} + \delta_{dA}\right)\hat{b}_A > 0.$$

For an initially symmetric equilibrium these expressions become

$$\hat{\Theta}_{dA} = -\frac{\delta_x\beta}{(1-\beta)\left(\delta_d - \delta_x\right)}\hat{b}_A < 0, \qquad \hat{\Theta}_{xA} = \frac{\delta_d\beta}{(1-\beta)\left(\delta_d - \delta_x\right)}\hat{b}_A > 0,$$

$$\hat{\Theta}_{dB} = \frac{\delta_x\beta}{(1-\beta)\left(\delta_d - \delta_x\right)}\hat{b}_A > 0, \qquad \hat{\Theta}_{xB} = -\frac{\delta_d\beta}{(1-\beta)\left(\delta_d - \delta_x\right)}\hat{b}_A < 0$$

and

$$\frac{\beta-\zeta}{1-\beta}\hat{Q}_A = -\frac{\delta_d\beta}{(1-\beta)(\delta_d-\delta_x)}\hat{b}_A < 0,$$
$$\frac{\beta-\zeta}{1-\beta}\hat{Q}_B = \frac{\delta_x\beta}{(1-\beta)(\delta_d-\delta_x)}\hat{b}_A > 0.$$

To examine the impact of labor market institutions on unemployment, consider

$$Q_j^{\beta\frac{1-\zeta}{1-\beta}} = \phi_1\left[M_j b_j^{-\frac{\beta}{1-\beta}}\int_{\Theta_{dj}}^{\infty}\Theta dG(\Theta) + M_{(-j)} b_{(-j)}^{-\frac{\beta}{1-\beta}}\tau^{-\frac{\beta}{1-\beta}}\int_{\Theta_{x(-j)}}^{\infty}\Theta dG(\Theta)\right].$$

$$N_j = \phi_1^{\frac{1}{\beta}} M_j b_j^{-\frac{\beta}{1-\beta}}\left[Q_j^{-\frac{\beta-\zeta}{1-\beta}}\int_{\Theta_{dj}}^{\infty}\Theta dG(\Theta) + \tau^{-\frac{\beta}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}}\int_{\Theta_{xj}}^{\infty}\Theta dG(\Theta)\right].$$

We differentiate these equations, starting from a symmetric equilibrium with $b_A = b_B$, and consider a small increase in b_A. This yields

$$\beta\frac{1-\zeta}{1-\beta}\hat{Q}_j = \sigma_d\left[\hat{M}_j - \frac{\beta}{1-\beta}\hat{b}_j\right] - \mu_d\hat{\Theta}_{dj} + \sigma_x\left[\hat{M}_{(-j)} - \frac{\beta}{1-\beta}\hat{b}_{(-j)}\right] - \mu_x\hat{\Theta}_{x(-j)},$$
$$\hat{N}_j = \hat{M}_j - \frac{\beta}{1-\beta}\hat{b}_j - \frac{\beta-\zeta}{1-\beta}\sigma_d\hat{Q}_j - \mu_d\hat{\Theta}_{dj} - \frac{\beta-\zeta}{1-\beta}\sigma_x\hat{Q}_{(-j)} - \mu_x\hat{\Theta}_{xj},$$

where

$$\sigma_d = \frac{\int_{\Theta_d}^{\infty}\Theta dG(\Theta)}{\int_{\Theta_d}^{\infty}\Theta dG(\Theta) + \tau^{-\frac{\beta}{1-\beta}}\int_{\Theta_x}^{\infty}\Theta dG(\Theta)} \quad \text{and} \quad \sigma_x = 1-\sigma_d < \sigma_d$$

and

$$\mu_z \equiv \sigma_z\frac{f_z}{\delta_z}G'(\Theta_z)\Theta_z, \qquad z\in\{d,x\}.$$

The first two equations can be expressed as

$$\sigma_d\hat{M}_A + \sigma_x\hat{M}_B = \left(-\Psi_A + \tfrac{\beta\sigma_d}{1-\beta}\right)\hat{b}_A,$$
$$\sigma_x\hat{M}_A + \sigma_d\hat{M}_B = \left(\Psi_B + \tfrac{\beta\sigma_x}{1-\beta}\right)\hat{b}_A,$$

where

$$\Psi_A \equiv \frac{\beta}{1-\beta}\frac{1}{\delta_d-\delta_x}\left(\beta\frac{1-\zeta}{\beta-\zeta}\delta_d + \mu_d\delta_x + \mu_x\delta_d\right) > 0,$$
$$\Psi_B \equiv \frac{\beta}{1-\beta}\frac{1}{\delta_d-\delta_x}\left(\beta\frac{1-\zeta}{\beta-\zeta}\delta_x + \mu_d\delta_x + \mu_x\delta_d\right) > 0.$$

This system then yields the solution

$$\hat{M}_A = \left(\frac{\beta}{1-\beta} - \Psi_{MA}\right)\hat{b}_A, \qquad \Psi_{MA} \equiv \frac{\sigma_d\Psi_A + \sigma_x\Psi_B}{\sigma_d-\sigma_x},$$
$$\hat{M}_B = \Psi_{MB}\hat{b}_A, \qquad \Psi_{MB} \equiv \frac{\sigma_x\Psi_A + \sigma_d\Psi_B}{\sigma_d-\sigma_x}.$$

Note that

$$\Psi_{MA}-\frac{\beta}{1-\beta}>\frac{\beta}{1-\beta}\left(\frac{\sigma_d+\sigma_x\frac{\delta_x}{\delta_d}}{\sigma_d-\sigma_x}\frac{\delta_d}{\delta_d-\delta_x}\beta\frac{1-\zeta}{\beta-\zeta}-1\right)>\frac{\beta}{1-\beta}\left(\beta\frac{1-\zeta}{\beta-\zeta}-1\right)=\frac{\beta\zeta}{\beta-\zeta}>0.$$

Therefore, M_A unambiguously decreases in b_A and M_B increases in b_A. Also note that as $\tau\to\infty$, δ_x, μ_x and σ_x all go to 0 and $\sigma_d\to 1$. This implies that $\Psi_{MB}\to 0$ as well and $\Psi_{MA}\to\frac{\beta}{1-\beta}\frac{\beta(1-\zeta)}{\beta-\zeta}$ so that $\hat{M}_B=0$ and $\hat{M}_A=-\frac{\beta\zeta}{\beta-\zeta}\hat{b}_A$, which is exactly the case of the closed economy.

Next we turn to the analysis of N_j, H_j and u_j. Using the condition for N_j above and the results for M_j we have

$$\hat{N}_A=\left[-\Psi_{MA}+\frac{\beta}{1-\beta}\frac{1}{\delta_d-\delta_x}\left\{\sigma_d\delta_d-\sigma_x\delta_x+\mu_d\delta_x-\mu_x\delta_d\right\}\right]\hat{b}_A\equiv-\Psi_{NA}\ \hat{b}_A,$$
$$\hat{N}_B=\left[\Psi_{MB}-\frac{\beta}{1-\beta}\frac{1}{\delta_d-\delta_x}\left\{\sigma_d\delta_x-\sigma_x\delta_d+\mu_d\delta_x-\mu_x\delta_d\right\}\right]\hat{b}_A\equiv\Psi_{NB}\ \hat{b}_A,$$

where the first term in the square brackets comes from the changes in M_j and b_j and the second term comes from the change in the thresholds and outputs. Next note that

$$\Psi_{NA}=\frac{\beta}{1-\beta}\frac{1}{\delta_d-\delta_x}\frac{1}{\sigma_d-\sigma_x}\left[2\sigma_x\mu_d\delta_x+2\sigma_d\mu_x\delta_d+2\sigma_d\sigma_x(\delta_d+\delta_x)+\frac{\zeta(1-\beta)}{\beta-\zeta}(\sigma_d\delta_d+\sigma_x\delta_x)\right]>0,$$
$$\Psi_{NB}=\frac{\beta}{1-\beta}\frac{1}{\delta_d-\delta_x}\frac{1}{\sigma_d-\sigma_x}\left[2\sigma_x\mu_d\delta_x+2\sigma_d\mu_x\delta_d+2\sigma_d\sigma_x(\delta_d+\delta_x)+\frac{\zeta(1-\beta)}{\beta-\zeta}(\sigma_d\delta_x+\sigma_x\delta_d)\right]>0.$$

When $\tau\to\infty$, we have $\Psi_{NA}\to\frac{\beta\zeta}{\beta-\zeta}$ and $\Psi_{NB}\to 0$, and when $\tau<\infty$ we have $\Psi_{NA}>\frac{\beta\zeta}{\beta-\zeta}$.

Total employment in the differentiated sector changes according to:

$$\hat{H}_A=\hat{N}_A+\hat{x}_A=-(1+\Psi_{NA})\hat{b}_A\qquad\text{and}\qquad\hat{H}_B=\hat{N}_B=\Psi_{NB}\ \hat{b}_A$$

and unemployment responds as

$$\hat{u}_A=\hat{N}_A-\frac{x}{1-x}\hat{x}_A=-\left(\Psi_{NA}-\frac{1}{b-1}\right)\hat{b}_A\qquad\text{and}\qquad\hat{u}_B=\hat{N}_B=\Psi_{NB}\ \hat{b}_A.$$

Note that if an economy is effectively open (i.e., there is trade in equilibrium) we have

$$\Psi_{NA}-\frac{1}{b-1}>\frac{\beta\zeta}{\beta-\zeta}-\frac{1}{b-1}.$$

Therefore, unemployment at home will rise in response to increased flexibility in the home labor market in an open economy whenever it does so in a closed economy, but not necessarily the opposite.

Solution of the model under the Pareto assumption for Section 5.3

We characterize here the solution of the model under the assumption that productivity draws Θ are distributed Pareto with the shape parameter $k>2$. That is, $G(\Theta)=1-(\Theta_{\min}/\Theta)^k$ defined for $\Theta\geq\Theta_{\min}$. We use this characterization in Section 5.3 in order to solve numerically for the equilibrium response of unemployment to different shocks. In the end of this appendix we provide some analytical results under Pareto distributed productivity referred to in the text.

Pareto-distributed productivity leads to the following useful functional relationship:

$$\delta_{zj} \equiv \frac{f_z}{\Theta_{zj}} \int_{\Theta_{zj}}^{\infty} \Theta dG(\Theta) = f_z \frac{k}{k-1} \left(\frac{\Theta_{\min}}{\Theta_{zj}} \right)^k, \quad z \in \{d, x\}$$

so that $\hat{\delta}_{zj} = -k\hat{\Theta}_{dj}$. As a result, we can rewrite the free entry condition (17) as

$$f_d \Theta_{dj}^{-k} + f_x \Theta_{xj}^{-k} = (k-1) f_e \Theta_{\min}^{-k} \qquad \Leftrightarrow \qquad \delta_{dj} + \delta_{xj} = k f_e.$$

Manipulating cutoff conditions (16) and the free entry condition above, we can obtain two equations to solve for $\{\Theta_{dj}, \Theta_{xj}\}$. For concreteness, consider $j = A$:

$$\begin{aligned} f_d \Theta_{dA}^{-k} + f_x \Theta_{xA}^{-k} &= (k-1) f_e \Theta_{\min}^{-k}, \\ f_x \left[\tau^{\frac{\beta}{1-\beta}} \frac{f_x}{f_d} \psi^{-\frac{\beta}{1-\beta}} \right]^{-k} \Theta_{dA}^{-k} + f_d \left[\tau^{\frac{\beta}{1-\beta}} \frac{f_x}{f_d} \psi^{\frac{\beta}{1-\beta}} \right]^{k} \Theta_{xA}^{-k} &= (k-1) f_e \Theta_{\min}^{-k}, \end{aligned}$$

where $\psi \equiv b_A / b_B$ is the relative labor market rigidity of country A. This is a linear system in $\{\Theta_{dA}^{-k}, \Theta_{xA}^{-k}\}$ and there are similar conditions for country B, with ψ^{-1} replacing ψ. The solution to this system is given by

$$\begin{aligned} \Theta_{dA}^{-k} &= \frac{f_x}{\Delta_\Theta} \left[\tau^{\frac{\beta k}{1-\beta}} \left(\frac{f_x}{f_d} \right)^{k-1} \psi^{\frac{\beta k}{1-\beta}} - 1 \right], \\ \Theta_{xA}^{-k} &= \frac{f_d}{\Delta_\Theta} \left[1 - \tau^{\frac{-\beta k}{1-\beta}} \left(\frac{f_x}{f_d} \right)^{-(k-1)} \psi^{\frac{\beta k}{1-\beta}} \right], \end{aligned}$$

where

$$\Delta_\Theta = \frac{f_x^2 \Theta_{\min}^k}{(k-1) f_e} \tau^{\frac{-\beta k}{1-\beta}} \left(\frac{f_x}{f_d} \right)^{-k} \psi^{\frac{\beta k}{1-\beta}} \left[\tau^{\frac{2\beta}{1-\beta}} \left(\tau^{\frac{\beta}{1-\beta}} \frac{f_x}{f_d} \right)^{2(k-1)} - 1 \right] > 0.$$

Using this result we can derive a condition on primitive parameters for $\Theta_{dj} < \Theta_{xj}$ to hold in equilibrium:

$$\frac{f_d}{f_d + f_x} \left(\tau^{\frac{\beta}{1-\beta}} \frac{f_x}{f_d} \right)^k + \frac{f_x}{f_d + f_x} \left(\tau^{\frac{\beta}{1-\beta}} \frac{f_x}{f_d} \right)^{-k} > \max \left\{ \psi^{\frac{-\beta k}{1-\beta}}, \psi^{\frac{\beta k}{1-\beta}} \right\}, \tag{38}$$

which is satisfied for large τ and for $\psi \equiv b_A / b_B$ not very different from one. Next note that as $\tau \to \infty$, $\Theta_{xA} \to \infty$ and $\Theta_{dA} \to \left[\frac{f_d}{(k-1) f_e} \right]^{1/k} \Theta_{\min} \equiv \Theta_d^c$. Therefore, the condition for $\Theta_d^c > \Theta_{\min}$ is $k < 1 + f_d / f_e$ which is equivalent to the condition in the text. One can also show that Θ_{dA} decreases in τ in the range $\tau \in (\tau^*, \infty)$ where

$$\tau^* = \tau^*(\psi, f_x / f_d) : \qquad (\tau^*)^{\frac{\beta k}{1-\beta}} = (f_d / f_x)^{k-1} \left(\psi^{\frac{\beta k}{1-\beta}} + \sqrt{\psi^{\frac{2\beta k}{1-\beta}} - 1} \right).$$

The first cutoff condition in (16) allows to solving for Q_j once Θ_{dj} is known; Q_j is also decreasing in τ in the range (τ^*, ∞). It is straightforward to show that Q_j decreases in b_A and increases in b_B. Using the Pareto assumption and the equation for M_j (20), we get

$$M_j = \phi_2 \frac{k-1}{k} \Theta_{\min}^{-k} \frac{f_d Q_j^{\zeta} \Theta_{d(-j)}^{-k} - f_x Q_{(-j)}^{\zeta} \Theta_{x(-j)}^{-k}}{f_d^2 \Theta_{dA}^{-k} \Theta_{dB}^{-k} - f_x^2 \Theta_{xA}^{-k} \Theta_{xB}^{-k}}. \tag{39}$$

The condition for $M_A > 0$ can then be written as

$$\left(\frac{\Theta_{xB}}{\Theta_{dB}}\right)^k \left(\frac{Q_A}{Q_B}\right)^\zeta > 1.$$

One can show that this inequality imposes a restriction on parameters $\{\tau, \psi, f_x/f_d\}$ such that $\tau > \tau^*(\psi, f_x/f_d)$, which implies that Q_j is decreasing in τ whenever there is no complete specialization ($M_j > 0$ for both j). This is consistent with Lemma 1 in the text.

Finally, using the condition for N_j (21), the the optimal employment levels (6) and conditions (16), we get

$$N_j = \phi_1^{\frac{1-\beta}{\beta}} \phi_2^{-1} M_j \left[\delta_{dj} + \delta_{xj}\right] = \phi_1^{\frac{1-\beta}{\beta}} k f_e M_j,$$

that is, under Pareto assumption, N_j is always proportional with M_j. The remaining equilibrium conditions are

$$H_j = N_j/b_j \qquad \text{and} \qquad u_j = (1 - b_j^{-1})N_j/L.$$

We use the equations above to solve for equilibrium comparative statics numerically. Certain analytical results can also be obtained under the Pareto assumption for M_j, N_j and u_j departing from (39).

Remark for Section 4.3 Under the Pareto assumption we can get a simple prediction about the response of trade volume to τ, b_A and b_B. Recall that the total volume of trade (when $b_A > b_B$) equals $2X_B$ where we have

$$X_B = \phi_2^{-1} M_B \delta_{xB} = \frac{\frac{\delta_{dA}}{\delta_{xA}} Q_B^\zeta - Q_A^\zeta}{\frac{\delta_{dA}\delta_{dB}}{\delta_{xA}\delta_{xB}} - 1} = \frac{\frac{f_d}{f_x}\left(\frac{\Theta_{xA}}{\Theta_{dA}}\right)^k Q_B^\zeta - Q_A^\zeta}{\frac{f_d^2}{f_x^2}\left(\frac{\Theta_{xA}\Theta_{xB}}{\Theta_{dA}\Theta_{dB}}\right)^k - 1} = \frac{\frac{f_d}{f_x}\left(\frac{\Theta_{xA}}{\Theta_{dA}}\right)^k Q_B^\zeta - Q_A^\zeta}{\tau^{\frac{2\beta k}{1-\beta}}\left(\frac{f_x}{f_d}\right)^{2(k-1)} - 1}.$$

As b_A increases or b_B falls, the denominator remains unchanged while Θ_{xA}/Θ_{dA} and Q_B increase and Q_A decreases. As a result the volume of trade unambiguously rises. Finally, one can also show that X_B decreases in τ. Substitute the expression for Θ_{xA}/Θ_{dA} (derived from (16)) in the expression for X_B to get

$$X_B = Q_A^\zeta \frac{\tau^{\frac{\beta k}{1-\beta}}\left(\frac{f_x}{f_d}\right)^{k-1}\left(\frac{Q_B}{Q_A}\right)^{\zeta + k\frac{\beta-\zeta}{1-\beta}} - 1}{\tau^{\frac{2\beta k}{1-\beta}}\left(\frac{f_x}{f_d}\right)^{2(k-1)} - 1}.$$

Now note that X_B decreases in τ since Q_A and Q_B/Q_A decrease in τ and $Q_B > Q_A$.

Proof of Proposition 7 In the text we show that $N_A + N_B$ increases as τ falls. We show now that when $b_A > b_B$, N_A/N_B decreases as τ falls, which implies that N_B necessarily increases. Under the Pareto assumption $\delta_{dj} + \delta_{xj} = kf_e$. Therefore, (20) and (21) imply

$$\frac{N_A}{N_B} = \frac{M_A}{M_B} = \frac{\delta_{dB} Q_A^\zeta - \delta_{xB} Q_B^\zeta}{\delta_{dA} Q_B^\zeta - \delta_{xA} Q_A^\zeta} = \frac{1 - \frac{\delta_{xB}}{kf_e}\left[1 + \left(\frac{Q_B}{Q_A}\right)^\zeta\right]}{\frac{\delta_{dA}}{kf_e}\left[1 + \left(\frac{Q_B}{Q_A}\right)^\zeta\right] - 1} < 1,$$

where the last inequality comes from Lemma 4 under the assumption that $b_A > b_B$. From Proposition 1, Q_B/Q_A increases as τ falls. Taking this and the fact that $N_A < N_B$ into account, it is sufficient to show that $d\delta_{xB} - d\delta_{dA} = \delta_{xB}\hat{\delta}_{xB} - \delta_{dA}\hat{\delta}_{dA} > 0$ in response to a fall in τ, to establish that N_A/N_B declines in this case. Under the Pareto-distributed productivity,

$$\delta_{xB}\hat{\delta}_{xB} - \delta_{dA}\hat{\delta}_{dA} = k\left(\delta_{dA}\hat{\Theta}_{dA} - \delta_{xB}\hat{\Theta}_{xB}\right) = k\frac{(\delta_{dB}\delta_{xB} - \delta_{dA}\delta_{xA})(\delta_{dA} - \delta_{xA})}{\Delta}\left(-\hat{\tau}\right) > 0,$$

where the second equality comes from (19) and the inequality is obtained by Lemma 3 and the fact that under the Pareto assumption $\delta_{dA} + \delta_{xA} = \delta_{dB} + \delta_{xB} = kf_e$. This proves that N_B increases as τ falls when $b_A > b_B$. Since changes in τ do not affect labor market tightness, x_B, the only effect on the unemployment rate u_B is through N_B, and hence the unemployment rate in the flexible country increases in response to trade liberalization.

Proof of results for Section 6.2

Consider the equilibrium system in the case when there are trade frictions in the homogeneous-good sector and country B has a more flexible labor market ($b_A > b_B$). As described in the text, the free entry condition is still given by (17) and is the same for both countries. The production cutoff is given by the following zero profit condition:

$$\pi_{dj}(\Theta_{dj}) = p_{0j}\left[\phi_1\phi_2 b_j^{-\frac{\beta}{1-\beta}} Q_j^{-\frac{\beta-\zeta}{1-\beta}}\Theta_{dj} - f_d\right] = 0$$

and hence defines a symmetric condition for both countries independently of p_{0j}. The only asymmetry is in the export cutoff condition:

$$\pi_{xj}(\Theta_{xj}) = p_{0j}\left[\phi_1\phi_2\tau^{-\frac{\beta}{1-\beta}} b_j^{-\frac{\beta}{1-\beta}}\left(p_{0(-j)}/p_{0j}\right)^{\frac{1}{1-\beta}} Q_{(-j)}^{-\frac{\beta-\zeta}{1-\beta}}\Theta_{xj} - f_x\right] = 0.$$

We want to find the value of $p_{0B}/p_{0A} = \tau_0$, for a given ratio $b_A/b_B > 1$, such that $\Theta_{xA} = \Theta_{xB}$. The free entry condition then implies $\Theta_{dA} = \Theta_{dB}$ and the production cutoff condition implies

$$\left(\frac{b_A}{b_B}\right)^{\frac{\beta}{1-\beta}} = \left(\frac{Q_B}{Q_A}\right)^{\frac{\beta-\zeta}{1-\beta}}.$$

Using this, the export cutoff conditions result in

$$\tau_0 = \left(\frac{b_A}{b_B}\right)^{\beta}.$$

Now taking the full derivative of the equilibrium system, we obtain the following comparative statics:

$$\hat{\Theta}_{dA} = -\frac{\delta_{xB}}{\Delta}(\delta_{dB} + \delta_{xB})\left(\hat{b}_A - \hat{b}_B - \hat{\tau}_0/\beta\right),$$
$$\hat{\Theta}_{dB} = -\frac{\delta_{xA}}{\Delta}(\delta_{dA} + \delta_{xA})\left(\hat{b}_B - \hat{b}_A + \hat{\tau}_0/\beta\right).$$

This implies that Θ_{dB}/Θ_{dA} is decreasing in $\tau_0(b_B/b_A)^{\beta}$, and from previous discussions we know that $\Theta_{dA} = \Theta_{dB}$ when $\tau_0\left(b_B/b_A\right)^{\beta} = 1$. This proves that $\Theta_{dA} < \Theta_{dB}$ if and only if $\tau_0 < \left(b_A/b_B\right)^{\beta}$. This also implies $\delta_{dA} > \delta_{dB}$. >From the free entry condition we know that the same restriction is necessary and sufficient for $\Theta_{xA} > \Theta_{xB}$, which also implies $\delta_{xA} < \delta_{xB}$. From the domestic production cutoffs conditions, we then know that this restriction implies $Q_B/Q_A > \left(b_A/b_B\right)^{\beta/(\beta-\zeta)} > 1$, and hence is sufficient but not necessary for $Q_B > Q_A$.

Finally, solving for M_js as in Section 3, we have

$$M_A - M_B = \frac{(1-\beta)\phi_2}{\beta\Delta}\left[(\delta_{dB} + \delta_{xA}/\tau_0)Q_A^\zeta - (\delta_{dA} + \tau_0\delta_{xB})Q_B^\zeta\right].$$

Therefore, the discussion above implies that $\tau_0 < \left(b_A/b_B\right)^\beta$ is also sufficient, but not necessary, for $M_B > M_A$. Recall that the condition for net trade in the differentiated good, $M_A\delta_{xA} < M_B\tau_0\delta_{xB}$, is therefore also satisfied, which implies that it is valid to do the comparative statics on the equilibrium system above when $\tau_0 \leq \left(b_A/b_B\right)^\beta$.

Note that $M_A\delta_{xA} < M_B\tau_0\delta_{xB}$ is satisfied with slack when $\tau_0 = \left(b_A/b_B\right)^\beta$. Therefore, for a given b_A/b_B, τ and other parameters of the model, there exists a $\bar{\tau}_0 > \left(b_A/b_B\right)^\beta$ such that $M_A\delta_{xA} < M_B\tau_0\delta_{xB}$ holds if and only if $\tau_0 < \bar{\tau}_0$.[73]

Whenever $\tau_0 < \bar{\tau}_0$, we have equilibrium with intersectoral trade. In this case, the equilibrium system is as described above. Moreover, the comparative statics results with respect to b_A, b_B and τ are still given by (19) and (23). Since the ranking of δ_{zj}s is preserved when $\tau_0 < \left(b_A/b_B\right)^\beta$, all the results of Section 4 still apply in this case. When, however, $\tau_0 > \left(b_A/b_B\right)^\beta$, we have $\delta_{xA} > \delta_{xB}$ and $\delta_{dA} < \delta_{dB}$, which implies that country A gains more from trade, while other comparative statics results are unchanged.

Finally, note from the comparative statics above that when $\tau_0 < \bar{\tau}_0$, Θ_{dA} is increasing and Θ_{dB} is decreasing in τ_0. Domestic production cutoff conditions then imply that Q_A increases and Q_B decreases in τ_0.

Proof of results for Section 7

We reproduce here equations (30):[74]

$$b = \frac{ax^\alpha + \sigma(b_f + b_p)}{1-\sigma} + \frac{1}{2}b_u,$$

and substitute it into (31) to obtain:

$$x\left[ax^\alpha + \sigma b_f + 2\sigma b_p - \sigma(1-\eta)b_u\right] = 1 - b_u.$$

Note that this last expression implicitly determines x as a function of the labor market characteristics. Taking the full differential, we have

$$\begin{aligned}\left[1 - b_u + \alpha a x^{1+\alpha}\right]\hat{x} = &- ax^{1+\alpha}\hat{a} - \sigma x db_f - 2\sigma x db_p,\\ &- [1 - \sigma(1-\eta)x]db_u,\end{aligned}$$

where as before a hat denotes log-derivative so that $\hat{x} = dx/x$. This immediately implies that x decreases in all labor market parameters, a, b_f, b_p and b_u.[75]

Next, the full differential for b is given by

$$db = \frac{\alpha a x^{1+\alpha}}{1-\sigma}\hat{x} + \frac{1}{1-\sigma}\left[ax^\alpha\hat{a} + \sigma(db_f + db_p)\right] + \frac{1}{2}db_u.$$

[73] We do not prove here that $\bar{\tau}_0$ is finite. However, it is immediate to see that as $b_A/b_B \to 1$, $\bar{\tau}_0 \to 1$, that is there is no intersectoral trade around the symmetric equilibrium with arbitrarily small trade frictions in the homogenous good.

[74] We drop subscript j in this appendix as it causes no confusion.

[75] One can additionally show that x decreases in σ and η.

Note that the direct effect of all labor market parameters on b is positive. However, they all have negative indirect effects on b through labor market tightness, x. Formally, combining the expressions for db and $\hat{x}$, we have:

$$
\begin{aligned}
db = &+ \frac{(1-b_u)ax^{\alpha}}{(1-\sigma)\left[1-b_u+\alpha a x^{1+\alpha}\right]}\hat{a} \\
&+ \frac{\sigma}{1-\sigma}\frac{1-b_u}{1-b_u+\alpha a x^{1+\alpha}}db_f \\
&+ \frac{\sigma}{1-\sigma}\frac{1-b_u-\alpha a x^{1+\alpha}}{1-b_u+\alpha a x^{1+\alpha}}db_p \\
&+ \left[\frac{1}{2}-\frac{\alpha a x^{\alpha}\left[1-\sigma(1-\eta)x\right]}{(1-\sigma)\left[1-b_u+\alpha a x^{1+\alpha}\right]}\right]db_u.
\end{aligned}
$$

Note that b always increases in a and b_f. Yet, it may increase or decrease in b_p and b_u. It decreases in b_p whenever $1-b_u < \alpha a x^{1+\alpha}$, as stated in the text. The condition for b to decrease in b_u is slightly more complex. One way to write it is as given in the text:

$$
1-b_u < \alpha a x^{\alpha}\left[2\frac{1-\sigma(1-\eta)x}{1-\sigma}-x\right].
$$

When $\sigma = 0$, it collapses to

$$
1-b_u < \alpha a x^{\alpha}(2-x).
$$

Moreover, in this case the expression for x becomes $1-b_u = ax^{1+\alpha}$, so that we can rewrite the condition above as

$$
x < \alpha(2-x) \qquad \text{or} \qquad x < 2\alpha/(1+\alpha),
$$

which is equivalent to the expression in the text.

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