The Productivity Slowdown and the Declining Labor Share: A Neoclassical Exploration*

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Abstract

We explore the possibility that a global productivity slowdown is partly responsible for the widespread decline in the labor share of national income. In a neoclassical growth model with endogenous human capital accumulation à la Ben Porath (1967) and capital-skill complementarity à la Grossman et al. (2017), the steady-state labor share is positively correlated with the rates of capital-augmenting and labor-augmenting technological progress. We calibrate the key parameters describing the balanced growth path to U.S. data for the early postwar period and find that a one percentage point slowdown in the growth rate of per capita income can account for about one half of the observed decline in global labor shares.

Keywords: neoclassical growth, balanced growth, technological progress, capital-skill complementarity, labor share, capital share

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1 Introduction

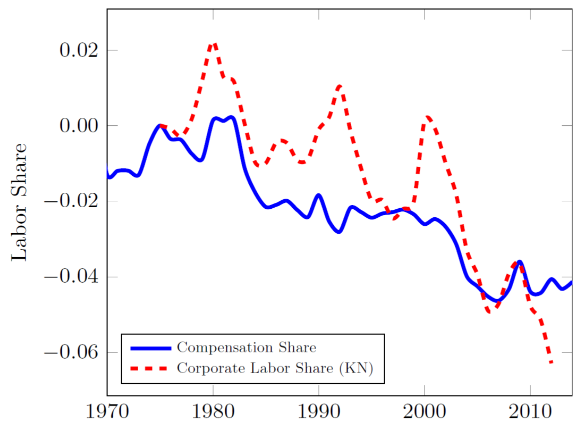
The labor share in national income has fallen dramatically in recent years. Meanwhile, measured productivity growth has slowed in many countries. In this paper, we explore the possibility that these two, seemingly-unrelated phenomena might in fact be connected by a process of neoclassical growth with endogenous human capital accumulation and capital-skill complementarity.

We will not wade into the debate about the exact timing or magnitude of the decline in the labor share, which already reflects a great deal of careful data work.[1] But to set the stage for our later discussion, we employ international data on labor compensation from the Penn World Tables and on compensation in the corporate sector from Karabarbounis and Neiman (2014) and regress the reported labor share in 125 and 66 countries, respectively, on country and year fixed effects. Figure 1 plots the time effects, namely the common component in world trends. It appears from the figure that the global decline in the labor share began sometime around 1980 and amounts to a total of between four and seven percentage points.

Several explanations have been offered for this decline. Karabarbounis and Neiman (2014) and Piketty (2014) propose variants of what Rognlie (2015) terms an "accumulation view." Piketty argues that, for a variety of reasons, aggregate savings have risen globally relative to national incomes, which has generated an increase in capital-to-output ratios. Karabarbounis and Neiman call attention to a drop in the price of investment goods relative to consumer goods, which may have led to increased capital accumulation and thereby a change in the capital share. As Rognlie (2015), Lawrence (2015), and Oberfield and Raval (2015) point out, these explanations for the fall in the labor share require an aggregate elasticity of substitution between

[1] The decline in the labor share has been documented by many researchers, including Elsby et al. (2013), Karabarbounis and Neiman (2014), Bridgman (2017), Rognlie (2015), Lawrence (2015), Koh et al. (2016), Barkai (2016), Kehrig and Vincent (2017), and others. The precise magnitude of the drop is disputed and the starting date difficult to pinpoint, for a number of reasons. Elsby et al. (2013) and Karabarbounis and Neiman (2014) outline the difficulties associated with attributing self-employment income to either capital or labor. Barkai (2016) discusses the evolution of the profit share, which he distinguishes from payments to capital and labor. Bridgman (2017) and Rognlie (2015) note the distinction between gross and net capital shares, and the problems that arise in measuring depreciation, especially for intangible assets. Koh et al. (2016) focus on obstacles to assessing the returns to intellectual property. Despite these many caveats, a consensus has emerged that the labor share in the United States has sustained a substantial and prolonged decline on the order of five or six percentage points. Karabarbounis and Neiman (2014) and Piketty and Zucman (2014) emphasize that the tilt in the income distribution is not peculiar to the United States, but rather is a global phenomenon.

Figure 1: Global Labor Share



Sources: Compensation Share from Penn World Tables 9.0 and Corporate Labor Share from Karabarbounis and Neiman (2014). Compensation includes only wages and salaries of employees. Each series plots time fixed effects from a regression of labor shares on time fixed effects and country fixed effects, weighted by real GDP. The compensation is calculated using 125 countries, and the corporate labor share series is calculated using 66 countries. OPEC countries are excluded from both.

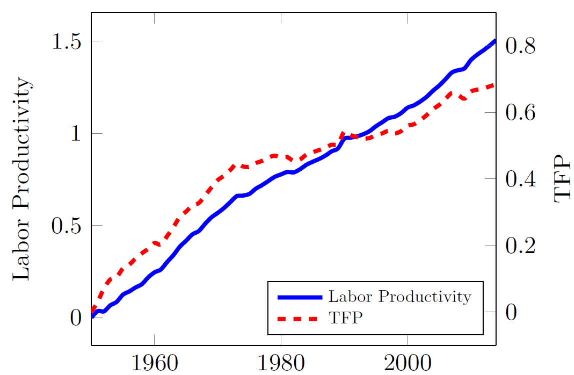
capital and labor in excess of one, which seems at odds with the preponderance of empirical evidence. Oberfield and Raval suggest, instead, a once-off shift in the bias of technology in favor of capital. Elsby et al. (2013) point to the expansion of offshoring as a possible source of the income shifts. Acemoglu and Restrepo (2018) suggest that automation of tasks previously performed by labor can cause a permanent reduction in the labor share. Meanwhile, Autor et al. (2017) and Kehrig and Vincent (2017) ascribe the fall in the labor share to rising industry concentration and the growing dominance of "superstar firms." Relatedly, Barkai (2016) and De Loecker and Eeckhout (2017) point to gains in the profit share that reflect increased markups.

Without denying the possible relevance of some of these considerations in various countries, we propose a novel and potentially complementary explanation. We suggest a conceptual link between the declining labor share and the apparent slowdown in productivity growth that has occurred over roughly the same period.[2] Figure 2 plots the common component from regressions

[2] The trends in world productivity growth are even more controversial. Researchers have debated the magnitude of the slowdown, whether it is a cyclical or secular phenomenon, and what is the inception date (if any) of the long-run decline. Gordon (2010, 2012, 2016) has argued most forcefully that productivity growth slowed permanently in the United States beginning in the 1970s and that the average annual growth rate of total factor productivity (TFP) in the last four decades has been at least one percentage point slower than in the preceding five decades. Fernald (2014) reports slower growth in TFP and labor productivity from 1973 to 1995 than in the preceding 25 years, followed by a decade of exceptional growth performance, and then a return to the earlier,

of labor productivity growth and total factor productivity growth (TFP) on country and year fixed effects, again using data from the Penn World Tables. The figure indicates a slowdown of about one percentage point per year in average TFP growth since the mid-1970's, in line with Gordon's (2010, 2012, 2016) estimates for the United States. We will argue below that a productivity slowdown generates a decline in the steady-state schooling-adjusted effective capital-to-labor ratio in a setting of neoclassical growth with endogenous schooling choices and a certain form of capital-skill complementarity. When the aggregate elasticity of substitution between labor and capital (holding education constant) is less than one, such a decline implies a shift in the long-run distribution of national income toward capital and away from labor. Our numerical calculations suggest that a productivity slowdown of one percentage point per year could account for perhaps half (or even more) of the observed changes in income shares.

Figure 2: Global Productivity



Source: Penn World Tables 9.0. Each series plots time fixed effects from a regression of log productivity on time fixed effects and country fixed effects, weighted by real GDP. The labor productivity series is calculated using 166 countries, and the TFP series is calculated using 107 countries. OPEC countries are excluded from both.

In what follows, we extend a standard neoclassical growth model to incorporate endogenous human capital accumulation. The economy comprises overlapping generations of family members that procreate and perish with constant probabilities. Newborns begin life without

slower rate of progress during the Great Recession and beyond. Jorgenson et al. (2014) concur that high rates of productivity growth during the period from 1995 to 2005 were transitory and exceptional, and that trend productivity growth probably has slowed since then. Some, like Mokyr (2014), Feldstein (2017), and Brynjolfsson and McAfee (2011, 2014) contend that efforts to measure productivity growth are hampered by enormous difficulties in gauging output quality and the value of new products. They see substantial underestimation of the recent record of productivity growth due to mismeasurement. Bryne et al. (2016) and Syverson (2017) dispute these claims.

human capital, but accumulate skills by devoting all their time to education. Once individuals choose to enter the labor force, they divide their time optimally between working and learning. Meanwhile, competitive firms allocate capital to workers as a function of their skill levels. The output of a worker together with the capital allocated to her is increasing in human capital. Optimal savings finance additions to the capital stock, which depreciates at a constant rate.

Growth is sustained by exogenous technological progress. Technical progress in our model takes three forms. *Labor-augmenting technical progress* raises the productivity of all workers proportionally, independent of their skill level or their capital usage. *Disembodied capital-augmenting technical progress* raises the productivity of all capital used in production irrespective of the number or type of workers that operate the machines. New machines embody *investment specific technical progress.* On a balanced growth path associated with constant rates of technical progress, capital, consumption and output grow at constant rates and the factor shares in national income are constant.

We specify a class of aggregate production functions that is like the one we described in Grossman et al. (2017). Production functions in this class exhibit constant returns to scale in the two physical inputs, capital and labor time. We impose parameter restrictions to ensure that the marginal product of human capital is everywhere positive and that the elasticity of substitution between capital and labor (for a worker of any given skill level) is less than one. Critically, human capital enters the production function in a manner that is akin to capital-using technical progress. That is, as skill levels grow, firms find it optimal to substitute capital for raw labor at any given factor prices. This specification of the technology reflects an assumed capital-skill complementarity, a feature of the aggregate production function that was first hypothesized by Griliches (1969) and corroborated by many researchers since. As we showed in our earlier paper, the combination of a technology in the specified class of production functions and the opportunity for endogenous schooling allows for the existence of a balanced growth path even in the presence of capital-augmenting technical progress and an elasticity of substitution between capital and labor that is less than one.[3]

[3]That is, the Uzawa Growth Theorem (Uzawa, 1961) does not apply in circumstances where human capital accumulates endogenously and capital and skills are complementary.

Although we draw on ideas from our earlier paper, there is an important distinction between the two. In Grossman et al. (2017), we assumed that agents have "short lives"; they are born at one instant and die in the next. This assumption simplified our analysis, as it eliminated intertemporal considerations from the process of human capital accumulation. Each new generation allocated a fraction of its time to work and the rest to schooling as a function of the prevailing wage. The interest rate played no role in decisions about schooling, nor did the path of subsequent wages. In the environment we consider here with finite lifespans and overlapping generations, agents must be forward looking. They invest more in education when the real interest rate is low and when wages are rising more quickly. It turns out that this difference is critical for determining income shares. With fleeting lives, factor shares depend only on parameters of the instantaneous aggregate production function. In contrast, when lives are finite and education decisions are forward looking, the growth process itself influences the distribution of income. It is this mechanism linking the growth process to the steady-state factor shares that is the focus of the current paper.

Our model admits simple analytical expressions for the long-run factor shares. If we assume—in keeping with the empirical evidence—that the elasticity of intertemporal substitution is less than one, then the labor share in national income is an increasing function of the rates of capital-augmenting and labor-augmenting technological progress. Therefore, a productivity slowdown of any sort results in a decline in the steady-state labor share. The mechanism operates through the forward-looking schooling choices. When growth slows, the real interest rate falls, which leads individuals to target a higher level of education for a given level of the capital stock. Inasmuch as skills are capital using, this reduces the effective capital to labor ratio in the typical firm, which in turn redistributes income from labor to capital, given an elasticity of substitution less than one. Moreover, if the growth slowdown is due in any part to a decline in the rate of capital-augmenting technical progress, the increase in targeted education for any effective capital-to-labor ratio takes place alongside a deceleration of schooling gains.

How important is this redistributive channel quantitatively? To answer this question, we take parameters to match the average birth rate, the average death rate, the rate of labor productivity growth, the internal rate of return on schooling, and the factor shares of the pre-slowdown era in the United States, as well as a conservative estimate of the elasticity of intertemporal substitution. One key parameter remains, which can be expressed either in terms of the composition of technical progress in the pre-slowdown steady state or as a measure of the capital-skill complementarity in the aggregate production function. We are cautious about this parameter, because Diamond and McFadden (1965) and Diamond et al. (1978) tell us that it cannot be identified from time series data on inputs and outputs, while our formula tells us that it plays a central role in our quantitative analysis. We consider a range of alternatives, including some derived from estimation of the cross-industry and cross-regional relationships implied by our model. In all of the alternatives we consider, a one percentage point slowdown in secular growth implies a substantial redistribution of income shares from labor to capital, representing between one half and all of the observed shift in factor shares in the recent global experience.

Our analysis provides potential resolutions to several puzzles in the recent literature. First, as first pointed out by Karabarbounis and Neiman (2014) and corroborated by the regressions reported in Figure 1, the fall in the labor share is largely a global phenomena. Yet, several of the explanations that have been offered rely on institutional features of the U.S. economy. To the extent that the productivity slowdown also is widespread, as Figure 2 suggests, our model predicts global shifts in income shares. Second, whereas researchers such as Growiec et al. (2018) find that the labor share is strongly pro-cyclical at medium-run horizons in time series data, those like Kehrig and Vincent (2017) and Autor et al. (2017) find the opposite (negative) correlation between the growth rate and the labor share in cross-sections of firms or industries. Our model reconciles such findings: in the cross-section, the interest rate is common, so the mechanism that we emphasize cannot operate; in the time series, the interest rate responds to changes in the growth process. Third, our model offers an explanation for the different estimates of the elasticity of substitution between capital and labor in studies like Karabarbounis and Neiman (2014) and Glover and Short (2017) that use cross-country differences in the movement of investment-good prices to identify the elasticity of substitution from the heterogeneous changes in factor shares and those like Antràs (2004), Klump et al. (2007) and Oberfield and Raval (2015) that rely for identification on variation in capital-labor ratios or in the relative price of capital and labor. A key difference is that the latter authors control for differences in human capital or education, whereas the former do not.[4] If the data were generated by our model, the strategy used by Antràs (2004), Klump et al. (2007) or Oberfield and Raval (2015) would uncover a long run elasticity of substitution between capital and labor less than one, because their approaches control explicitly or implicitly for changes in schooling. In contrast, the strategy used by Karabarbounis and Neiman (2014) or Glover and Short (2017) would generate a unitary elasticity of substitution, because their estimates include the induced change in educational attainment that results from a change in the relative price of capital.[5]

The remainder of the paper is organized as follows. In Section 2, we develop our neoclassical growth model with perpetual youth and endogenous human capital accumulation, drawing on Blanchard (1985) for the former and Ben Porath (1967) for the latter. Section 3 characterizes the balanced growth path. In Section 4, we discuss the analytical relationship between rates of capital-augmenting and labor-augmenting technological progress and the long-run factor shares. Section 5 presents our quantitative exploration of how a one percentage point slowdown in the trend growth rate might affect the distribution of income between capital and labor. Section 6 concludes.

2 A Neoclassical Growth Model with Endogenous Education

In this section we develop a simple neoclassical, overlapping-generations (OLG) model with exogenous capital-augmenting and labor-augmenting technological progress, endogenous cap-

[4] Antràs (2004) and Klump et al. (2007) use a measure of efficiency units of labor with constant quality by augmenting hours with a measure of changes in the quality of the workforce due to increases in schooling and experience. Oberfield and Raval (2015) use a measure of local wages that is computed as a residual after controlling for schooling and experience.

[5] Karabarbounis and Neiman (2014) find an elasticity of substitution greater than one. But when Glover and Short (2017) re-estimate their regressions while controlling for the growth rate of consumption (but still without controlling for cross-country differences in schooling), they find an aggregate elasticity of substitution that is not statistically different from one.

ital accumulation à la Ramsey (1928), Cass (1965), and Koopmans (1965), and endogenous schooling choices à la Ben Porath (1967). Our model features perpetual youth, as in Blanchard (1985), and capital-skill complementarity, as in Grossman et al. (2017). The economy admits a unique balanced growth path despite ongoing capital-augmenting technical progress and an assumed elasticity of substitution between capital and labor of less than one. We use the model in Section 5 below to explore the long-run implications for factor shares of a once-and-for-all slowdown in productivity growth.

The economy is populated by a unit mass of identical family dynasties.[6] The representative dynasty comprises a continuum of individuals of mass N_t at time t. Each living individual generates a new member of her dynasty with a constant, instantaneous probability λdt in a period of length dt and faces a constant instantaneous probability of demise νdt in that same period, with $\lambda > 0, \nu \geq 0$. With these constant hazard rates of birth and death, the size of a dynasty at time t is given by

$$N_t = e^{(\lambda-\nu)(t-t_0)} N_{t_0}.$$

Each newborn enters the world devoid of human capital. An individual is endowed at each instant with a unit of time that she can divide arbitrarily between *working* and *learning*. Work yields a wage at time t that reflects the state of technology and the size of the aggregate capital stock, as well as the individual's accumulated human capital, h_t. Learning occurs at full-time school or in continuing education. An individual who works a fraction ℓ_t of her time at t and devotes the remaining fraction $1 - \ell_t$ of her time to education accumulates human capital according to

$$\dot{h}_t = 1 - \ell_t. \tag{1}$$

The time constraint implies $\ell_t \in [0, 1]$.

[6] We assume here that families maximize dynastic utility, including the discounted well-being of unborn generations. The qualitative results would be much the same in a Yaari (1965) economy with (negative) life insurance and no bequests, as developed in Blanchard and Fischer (1989, ch.3).

The representative family maximizes at time t_0 its dynastic utility,

$$U_{t_0} = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} N_t \frac{c_t^{1-\eta} - 1}{1-\eta} dt \ ,$$

subject to an intertemporal budget constraint, where c_t is per capita consumption by family members at time t, η is the inverse of the elasticity of intertemporal substitution, and ρ is the subjective discount rate. As usual, the Euler equation implies

$$\frac{\dot{c}_t}{c_t} = \frac{\iota_t - \rho}{\eta} \ , \tag{2}$$

where ι_t is the real interest rate in terms of consumption goods at time t. To limit the number of cases and to conform with widespread empirical evidence, we assume henceforth that $\eta > 1$.[7] We also assume that the discount rate is sufficiently large to render dynastic utility finite.[8]

Firms hire capital and workers to produce a single, homogeneous final good. Consider a firm that employs K units of capital and L workers, each of whom has the same human capital, h. With the technology available at time t, such a firm can produce

$$Y = F\left(A_t K, B_t L, h\right)$$

units of output, where A_t represents the state of disembodied, capital-augmenting technology, B_t is the state of labor-augmenting technology, and $F(\cdot)$ is homogeneous of degree one in its first two arguments; i.e., there are constant returns to scale in the two physical inputs.[9] Following Grossman et al. (2017), we assume that $F(\cdot)$ falls within a particular class of production functions and we impose some parameter restrictions. Specifically, we adopt

[7] See, for example, Hall (1988), Campbell (2003) and Yogo (2004) for estimates using macro data, and Attanasio and Weber (1993) and Vissing and Jorgenson (2002) for estimates using micro data.

[8] In particular, we require

$$\rho > \lambda - \nu + (1-\eta)\, g_y \tag{3}$$

where g_y is the growth rate of per capita income along the balanced growth path. We will express g_y in terms of the fundamental paramaters below.

[9] With constant returns to scale, the total output of a firm that hires a variety of workers with heterogenous levels of human capital is simply the sum of the amounts produced by the various groups that have a common h using the capital that is optimally allocated to them.

Assumption 1 *The production function can be written as* $F(A_tK, B_tL, h) = \tilde{F}\left(e^{-ah}A_tK, e^{bh}B_tL\right)$, *with* $a > 0, b > \lambda \geq 0$, *where*

(i) $\tilde{F}(\cdot)$ *is homogeneous of degree one in* A_tK *and* B_tL*;*

(ii) $f(k) \equiv \tilde{F}(k, 1)$ *is strictly increasing, twice differentiable, and strictly concave for all* k*;*

(iii) $\sigma_{KL} \equiv F_K F_L / F F_{KL} < 1$ *for all* K, L*, and* h*; and*

(iv) $\lim_{k\to 0} kf'(k)/f(k) < b/(a+b)$.

The functional-form assumption makes schooling akin to capital-using (or labor-saving) technical progress; i.e., an increase in human capital raises the demand for capital relative to that for raw labor at the initial factor prices. As we discussed in Grossman et al. (2017) this assumption about technology provides for the existence of a balanced growth path in the presence of capital-augmenting technical progress, even when the elasticity of substitution between capital and labor is strictly less than one. Assumption 1.iii imposes this restriction on the elasticity of substitution, which is in keeping with the preponderance of empirical evidence. Moreover, with our functional form, $\sigma_{KL} < 1$ implies that $d(F_h/F_L)/dK > 0$; i.e., that capital accumulation boosts the marginal product of human capital relative to the marginal product of raw labor, a form of capital-skill complementarity that also is consistent with empirical research (see Goldin and Katz, 2007). Assumption 1.iv ensures that the marginal product of human capital is positive for all K L, and h.[10]

Final output can be used either for consumption or investment. A unit of output produces one unit of the consumption good or q_t units of the investment good at time t, where growth in q_t captures investment-specific technological change, as in Greenwood et al. (1997). Thus,

$$Y_t = C_t + I_t/q_t$$

[10] An alternative but formally equivalent way to express the class of production functions specified by Assumption 1 is

$$F(A_tK, B_tL, h) = (B_tL)^{1-\beta}\, \mathcal{F}\left(A_tK, e^{bh/\beta}B_tL\right)^{\beta},$$

with $\beta = b/(a+b)$..

and

$$\dot{K}_t = I_t - \delta K_t \ ,$$

where C_t and K_t are aggregate consumption and the aggregate capital stock, respectively, I_t is gross investment, and δ is the constant rate of capital depreciation.

Technology evolves exogenously in our model. Let $\gamma_L = \dot{B}/B$ be the constant rate of labor-augmenting technological progress, $g_A = \dot{A}/A$ the constant rate of *disembodied* capital-augmenting progress, and $g_q = \dot{q}/q$ the constant rate of *embodied* (or investment-specific) technological progress. Define $\gamma_K \equiv g_A + g_q$ as the *total* rate of capital-augmenting technological progress. We are interested in the relationship between γ_K and γ_L and the steady-state capital and labor shares, θ and $1-\theta$, respectively.

3 Characterizing the Balanced Growth Path

In order to solve for a balanced growth path (BGP), we impose some parameter restrictions.

Assumption 2 *The parameters of the economy satisfy*

(i) $a > \gamma_K$*;*

(ii) $\lim_{k\to 0} \frac{kf'(k)}{f(k)} > \frac{\Omega}{1+\Omega} > \lim_{k\to\infty} \frac{kf'(k)}{f(k)}$ *, where* $\Omega \equiv \frac{b-\lambda}{a} - \frac{(\eta-1)\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right)+\rho-(\lambda-\nu)}{a-\gamma_K}$*;*

(iii) $(\eta - 1)\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right) + \rho - (\lambda - \nu) > 0.$

Assumption 2 is needed to ensure the existence of a BGP with finite dynastic utility. It also generates interior choices for the optimal labor supply among those that have completed their full-time schooling.

The competitive firms take the rental rate for capital, R_t, as given. A firm that hires one unit of labor with human capital h at time t combines that labor with $\kappa_t(h)$ units of physical capital, where $\kappa_t(h)$ is determined implicitly by

$$e^{-ah} A_t \tilde{F}_K \left[e^{-ah} A_t \kappa_t(h), e^{bh} B_t\right] = R_t. \tag{4}$$

The worker is paid her marginal product which, with constant returns to scale, is the difference between revenue and capital costs, or

$$W_t(h) = \tilde{F}(\cdot) - e^{-ah} A_t \kappa_t(h) \tilde{F}_K(\cdot) . \tag{5}$$

Individuals use the wage schedule $W_t(h)$, together with their rational expectations of the evolution of wages and the interest rate to make their schooling choices.

Considering that there is a continuum of members of every dynasty and that families maximize dynastic utility, each individual chooses the path of her time allocation $\{\ell_t\}$ to maximize the expected present value of earnings. For an individual born at time τ, the problem is

$$\max \int_{\tau}^{\infty} e^{-\int_{\tau}^{t}(\iota_z+\nu)dz} \ell_t W_t(h_t)\, dt$$

subject to $h_\tau = 0$, $\dot{h}_t = 1 - \ell_t$, and $0 \leq \ell_t \leq 1$. Let μ_t be the co-state variable associated with human-capital accumulation. Then the first-order conditions imply

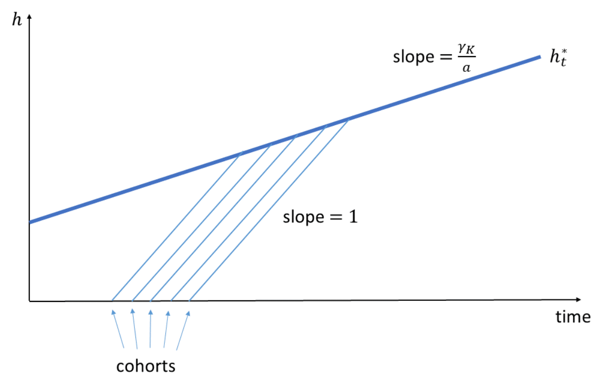
$$\left.\begin{array}{c} W_t(h_t) < \mu_t \\ W_t(h_t) = \mu_t \\ W_t(h_t) > \mu_t \end{array}\right\} \Rightarrow \left\{\begin{array}{c} \ell_t = 0 \\ \ell_t \in [0,1] \\ \ell_t = 1 \end{array}\right. \tag{6}$$

and

$$\dot{\mu}_t = (\iota_t + \nu)\, \mu_t - \ell_t W_t'(h_t) \ . \tag{7}$$

Let us define a balanced growth path (BGP) as a dynamic equilibrium with constant rates of growth of output, consumption, and capital, and with income shares for capital and labor that are constant and strictly positive. We find such a BGP by a process of "guess and verify." We hypothesize that the optimal schooling choices for members of each new cohort entail full-time attendance at school until the students accumulate human capital equal to a time-varying threshold h_t^*, followed by entry into the workforce, albeit with continuing education that keeps individuals' human capital equal to the growing threshold. These schooling strategies, which we indeed find to be optimal given Assumption 2.i, are depicted in Figure 3. Here, the lines with unit slope represent the human capital accumulation of each cohort during the periods that its members are full-time students. Once a cohort's human capital reaches h_t^*, its members devote a fraction γ_K/a of their time to education, just like all others that have finished their full-time schooling. This schooling behavior implies that all workers in the labor force share a common level of human capital $h_t = h_t^*$, irrespective of their birth dates. As a result, firms allocate the same amount of physical capital to all workers. Needless to say, this feature of the model simplifies aggregation across cohorts substantially.

Figure 3: Human Capital Accumulation by Birth Cohort



We further conjecture that the interest rate, ι, is constant along the BGP, as is the division of time between work and education, ℓ, for those that have completed full-time school and entered the workforce. We prove in the appendix the following lemma that describes important features of the balanced growth path:

Lemma 1 *Suppose g_q, g_A and g_B are constants and Assumptions 1 and 2 are satisfied. Then there exists a unique BGP characterized by*

$$\ell = 1 - \frac{\gamma_K}{a} \tag{8}$$

and

$$z_t \equiv \frac{e^{-ah_t^*} A_t K_t}{e^{bh_t^*} B_t L_t} = z^* \text{ for all } t. \tag{9}$$

Here, z_t adjusts the effective capital-labor ratio at time t (i.e., A_tK_t/B_tL_t) for the prevailing level of human capital of those in the workforce, taking into account the different complementarity between human capital and each of the primary factors of production. We henceforth refer to z_t as the *schooling-adjusted effective capital-to-labor ratio.*

Equation (8) implies that the human capital threshold increases linearly with time,

$$\dot{h}^*_t = \frac{\gamma_K}{a}. \tag{10}$$

Let s_τ denote the years in full-time school (or "educational attainment") for the cohort born at time τ. This is the time it takes for them to catch up with the human capital threshold, i.e., $s_\tau = h^*_{\tau+s_\tau}$. But, with the threshold rising according to (10), $h^*_{\tau+s_\tau} = h^*_\tau + s_\tau\gamma_K/a$. Thus, educational attainment also increases linearly with time,

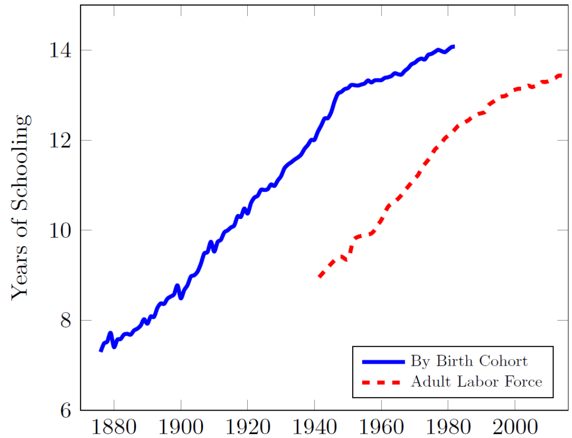
$$\dot{s}_\tau = \frac{\gamma_K}{a-\gamma_K}. \tag{11}$$

Recall that $a > \gamma_K$ by Assumption 2.i. Therefore, educational attainment rises in the steady state if and only if the rate of capital-augmenting technical progress is strictly positive.

Goldin and Katz (2007) have measured educational attainment for cohorts of U.S. workers born between 1876 and 1982. In Figure 4, we plot their data along with the average years of schooling among male workers as reported by Jones (2016).[11] Viewed through the lens of our model, these data are consistent with the presence of ongoing capital-augmenting technical progress. Educational attainment among male workers grew quite linearly for more than sixty years, which is consistent with the evolution of s_τ predicted by (11). Then, for cohorts born after about 1947, the yearly rise in average educational attainment declined. In the model, such a decline would require a slowdown in the rate of capital-augmenting technical progress around the time that these workers entered the labor force.

[11] We thank Lawrence Katz for furnishing us with several additional years of data beyond those reported in Goldin and Katz (2007).

Figure 4: U.S. Education by Birth Cohort and among Adult Labor Force



Sources: Years of schooling by birth cohort from Goldin and Katz (2007) and additional data from Lawrence Katz. Years of Schooling of adult labor force from Jones (2016). Each series includes only males.

Lemma 1 states that the schooling-adjusted effective capital-labor ratio converges to a constant value, z^*, in the long run. This is the key to balanced growth in the presence of capital-augmenting technological progress and an elasticity of substitution between capital and labor less than one. As capital accumulates and becomes more productive, the capital share in national income would tend to fall when $\sigma_{KL} < 1$. However, the capital-skill complementarity implies an increase in the return to schooling. The extra schooling is capital-using, which puts upward pressure on the capital share. With the functional form in Assumption 1, the offsetting forces just balance, and the capital share remains constant.[12]

Why then is it optimal for active workers to upgrade their human capital continuously so as to keep z_t constant? For an interior choice of $\ell \in (0,1)$, the present value of extra human capital, μ_t, must equal the opportunity cost of investment, which is the instantaneous wage, $W_t(h_t)$; see equation (6). But then if ι, ℓ, and g_W are constant, as conjectured, (7) implies

$$\frac{\ell W_t'(h_t^*)}{\iota + \nu - g_W} = W_t(h_t^*) \ . \tag{12}$$

[12] Put differently, (10) implies that $e^{-ah_t^*} A_t q_t$ is constant along the balanced growth path. So, the induced investment in human capital is just what is needed to offset the exogenous improvement in capital productivity.

But Assumption 1 delivers

$$\frac{W_t'(h_t)}{W_t(h_t)} = b - a\frac{\theta[z_t(h_t)]}{1-\theta[z_t(h_t)]}, \tag{13}$$

where $\theta(z_t) \equiv z_t f'(z_t)/f(z_t)$ is the capital share. Notice that the capital share depends only on the schooling-adjusted effective capital-to-labor ratio. So, a choice of h_t^* that keeps z_t constant also keeps $W_t'(h_t^*)/W_t(h_t^*)$ constant, which is consistent with (12).[13]

Using the optimal allocation of time to school and work, we can now calculate the (constant) growth rates of the labor force, wages, and output per capita, along with the constant interest rate and capital share. The aggregate labor force at time t is the product of the fraction of time that the typical worker devotes to gainful employment and the mass of the surviving population that has completed the phase of full-time schooling. The measure of individuals that were born at τ and that are still alive at time t is $\lambda N_\tau e^{-\nu(t-\tau)} = \lambda N_t e^{-(\lambda-\nu)(t-\tau)} e^{-\nu(t-\tau)} = \lambda N_t e^{-\lambda(t-\tau)}$. All those who were born at or before $t - h_t^*$ have already entered the labor force. Therefore,

$$\begin{aligned} L_t &= \left(1-\frac{\gamma_K}{a}\right)\int_{-\infty}^{t-h_t^*} \lambda N_t e^{-\lambda(t-\tau)} d\tau \\ &= \left(1-\frac{\gamma_K}{a}\right) N_t e^{-\lambda h_t^*}. \end{aligned} \tag{14}$$

It follows from (14) that labor-force participation, L_t/N_t, changes at the rate $g_L - g_N = -\lambda\gamma_K/a < 0$. Declining labor-force participation mirrors the increasing educational attainment, which requires a longer initial stay in school for each new cohort.

Next we derive the growth rate of wages. Compensation grows thanks to ongoing technological progress, as well as ongoing investments in physical and human capital. Using (4) and (5), we calculate that, along a BGP, the wage paid to each worker in the labor force (who has

[13] Note that for (13) to be satisfied with a constant value of z_t, we need a sufficiently large range for $zf'(z)/f(z)$. We show in the appendix that Assumption 2.ii guarantees the existence of a solution to (13).

growing human capital of h_t^*) increases at the rate[14]

$$g_W = \gamma_L + \frac{b}{a}\gamma_K.$$

Since factor shares are constant along the BGP, aggregate output is proportional to labor income, so the growth rate of output per capita can be expressed as

$$\begin{aligned} g_y &= g_W + g_L - g_N \\ &= \gamma_L + \frac{b-\lambda}{a}\gamma_K. \end{aligned}$$

Combining this expression with Assumption 2.iii implies that the finite utility restriction (3) holds on the BGP. Also, per capita consumption is proportional to per capita output, so (2) gives the long-run interest rate as

$$\begin{aligned} \iota &= \rho + \eta g_y \\ &= \rho + \eta\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right). \end{aligned} \tag{15}$$

Finally, we come to the steady-state factor shares. In the steady state, (12) and (13) imply

$$\gamma_L + \frac{b}{a}\gamma_K = \iota + \nu - \left(1 - \frac{\gamma_K}{a}\right)\left(b - a\frac{\theta}{1-\theta}\right)$$

[14]We substitute for the arguments of $\tilde{F}(\cdot)$ and $\tilde{F}_K(\cdot)$ using $z = e^{-(a+b)h_t^*}A_t\kappa_t(h_t^*)/B_t$ and note that z is constant along a balanced growth path. The no-arbitrage condition for capital accumulation implies that $R_t q_t - \dot{q}_t/q_t - \delta = \iota_t$, and thus, on a BGP with a constant interest rate and a constant rate of investment-specific technical progress, $\dot{R}_t/R_t = -g_q$. Totally differentiating (4) and (5) with z constant implies

$$-g_q = g_A - a\dot{h}_t^*$$

and

$$\frac{\dot{W}_t}{W_t} = \gamma_L + b\dot{h}_t^*,$$

from which it follows that

$$\frac{\dot{W}_t}{W_t} = \gamma_L + \frac{b}{a}\gamma_K.$$

or

$$\frac{\theta}{1-\theta} = \frac{b + \gamma_L - (\iota + \nu)}{a - \gamma_K}. \tag{16}$$

Next we substitute for the long-run interest rate, using (15), which gives us a relationship between the long-run capital share and the primitive parameters of the economy, namely

$$\frac{\theta}{1-\theta} = \frac{b-\lambda}{a} - \frac{(\eta - 1)\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right) - \lambda + \nu + \rho}{a - \gamma_K}. \tag{17}$$

We summarize our characterization of the long-run equilibrium as follows:

Proposition 1 *Suppose the aggregate production function obeys Assumption 1, the parameters satisfy Assumption 2 and g_q, g_A and g_B are constant. Then on the unique balanced growth path new cohorts are full-time students until their human capital reaches a threshold h_t^* that grows linearly with time. Once a cohort enters the labor force, its members devote a constant fraction $\ell = 1 - \gamma_K/a$ of their time to work and the remaining time to ongoing education. Wages grow at constant rate $\gamma_L + (b/a)\,\gamma_K$ and per capita income grows at constant rate $\gamma_L + (b-\lambda)\,\gamma_K/a$. The long-run real interest rate is given by (15) and the long-run capital share is determined by (17).*

4 Technological Progress and Factor Shares

We are ready to discuss the relationship between the parameters γ_L and γ_K that describe the rate and nature of technological progress and the long-run distribution of national income between capital and labor. Let us begin with (16), which expresses θ as a function of γ_K and γ_L, taking the real interest rate as given. If, for example, the aggregate economy comprises a continuum of small regional economies or similar industries that face a common interest rate due to nationwide asset trade, then (16) would describe the cross-sectional relationship between growth rates of output and factor shares. From this equation, it is clear that θ would be *positively* correlated with both γ_K and γ_L *in the cross section*; regions and industries with faster rates of capital or labor-augmenting technological progress would have higher shares of their income paid to capital in an economy with a uniform interest rate.

But in a closed economy (or a global economy), the real interest rate is endogenous and responds to changes in the growth process. Equation (17) informs us about the long-run relationship between factor shares and rates of technological progress. Recall our assumption that $\eta > 1$, i.e., that the elasticity of intertemporal substitution is less than one. By differentiating the expression on the-right hand side of (17) and making use of Assumption 2.iii, which ensures finite dynastic utility, we establish our key result:

Proposition 2 *When $\eta > 1$, a decrease in γ_K or γ_L reduces the long-run labor share, $1 - \theta$.*

In other words, a productivity slowdown—no matter whether it is caused by a slowdown in the pace of labor-augmenting technological progress, the pace of disembodied capital-augmenting technological progress, or the pace of investment-specific technological progress—will shift the distribution of national income from labor to capital. Our model predicts a *negative* correlation between the growth rate and the capital share in (low-frequency) *time series* data.

What accounts for this shift in factor shares? Note first from (15) that, in response to an exogenous shock to the growth process, the interest rate moves in the same direction as the growth rate of per capita income. Moreover, with $\eta > 1$, the response of the former is greater than that of the latter. Thus, a productivity slowdown that causes g_y to fall will cause $\iota - g_y$ to fall as well. On a BGP, wages grow almost at the same rate as per capita income, so $\iota - g_W$ also falls. This term appears of course in the expression for the optimal human capital threshold (12); whereas a decline in the growth rate of wages makes staying in school less desirable, a decline in the interest rate makes extended schooling more palatable. In the long run, the latter effect dominates, so by a combination of (12) and (13), z^* eventually falls. In other words, we find that the long-run schooling-adjusted effective capital-to-labor ratio declines in response to secular stagnation, once proper adjustment is made for the optimal response of targeted human capital and the greater complementarity of schooling with physical capital than with raw labor. Finally, with an elasticity of substitution between capital and labor of less than one, a decline in the schooling-adjusted effective capital-labor ratio spells a rise in the capital share and a corresponding decline in the labor share.

For clarity, we need to distinguish the effect of a productivity slowdown on z^*—the long-run schooling-adjusted effective capital-to-labor ratio—from its effect on the *rate of increase* of educational attainment, as recorded in equations (10) and (11). Since a productivity slowdown generates a decline in the real interest rate that exceeds the decline in the growth rate of wages, it causes students and workers to target a higher *level* of human capital, given the capital-to-labor ratio adjusted only for technology prevailing at the time. This is true regardless of whether the productivity slowdown can be traced to a deceleration of capital-augmenting technical progress, labor-augmenting technical progress, or a combination of the two. The induced movement of the schooling-adjusted effective capital-to-labor ratio generates a shift in factor shares in a setting with a non-unitary elasticity of substitution between capital and labor. Meanwhile, the long-run *annual increase* in educational attainment may *decline or remain the same*; it declines if the productivity slowdown has as its source (at least partly) a deceleration of capital-augmenting technological progress, whereas it remains the same if the productivity slowdown owes entirely to a deceleration of labor-augmenting technological progress.

To reconcile a fall in z^*—which requires a *higher* target for human capital at a point in time, given the capital stock, the labor force, and the state of technology—with a slowdown in human capital accumulation, we need to consider the system dynamics for all of the economic variables, not only in the new steady state, but also in the transition between steady states. Our model is poorly suited for examining realistic transition dynamics, because we have not included any adjustment costs for investing or disinvesting in physical or human capital. Nonetheless, we have simulated the dynamic response to a reduction in the rate of capital-augmenting technical progress using the parameters that we describe in Section 5, in order to see how h_t responds in the short and long run. We show in the appendix a pair of transition paths corresponding to declines in γ_K and γ_L. We find that, following an unanticipated, once-and-for-all drop in γ_K, human capital h_t briefly jumps above the path it was following in the initial steady state, but quickly dips below the original path due to the decline in $\dot{h}_t$ and $\dot{s}_t$. In other words, except for a brief initial period, the level of human capital is lower at every moment in time than it would have been had productivity growth not suffered the negative shock, even though $z_t = e^{-(a+b)h_t^*}(A_t K_t / B_t L_t)$ also sinks below its initial path. The labor share rises slightly immediately following the shock, but quickly declines to its lower, long-run level. We do not take these dynamics too seriously, but report them only to suggest that there is no necessary inconsistency between a decline in z^* and the declines in $\dot{s}_t$ and $\dot{h}_t$ that are reported by Goldin and Katz (2007) and Jones (2016), respectively.

Before leaving this section, it is worth emphasizing the contrast between the mechanism that we propose here to account for the declining labor share and an explanation that relies on the canonical neoclassical theory of the functional distribution of income, as first elucidated by John Hicks (1932) and Joan Robinson (1933). The canonical approach begins with a constant-returns-to-scale, two-factor, production function, $F(AK, BL)$. Then, if factors are paid their marginal products, the change in the ratio of factor shares is given by

$$d\ln\left(\frac{\theta}{1-\theta}\right) = \frac{\sigma_{KL}-1}{\sigma_{KL}}\left(d\ln\frac{K}{L} + d\ln\frac{A}{B}\right) .$$

Suppose $\sigma_{KL} < 1$, as suggested by a preponderance of the evidence. Then, an increase in the capital share requires either a fall in the level of the capital-labor ratio or a decline in the level of A/B; i.e., a shift in technology that augments the productivity of labor relative to the productivity of capital. But Karabarbounis and Neiman (2014) point to a global increase in the capital-labor ratio, and Piketty and Zucman (2013) similarly point to an increase in the capital-output ratio. So both are led to argue that σ_{KL} exceeds one in order to square these observations with the decline in the labor share. Their claims in this vein have not found widespread acceptance.

In our model with endogenous education and capital-skill complementarities, it is not the *levels* of the technology parameters that determine the factor shares, but rather the *rates* of technological change. Moreover, the factor bias of technical change does not play a critical role in our story. Both a fall in the rate of labor-augmenting technological change and a fall in the rate of capital-augmenting technological change will result in a long-run decline in the labor

Table 1: Targeted Moments and Parameters

Parameter/Moment		**Value**
Birth rate	λ	2.16%
Death rate	ν	0.95%
IRR on schooling	ι	10%
Capital share	θ	0.35
Growth in labor productivity	$\gamma_L + \frac{b}{a}\gamma_K$	0.024
Increase in schooling	$\dot{s}_\mathcal{T} = \frac{\gamma_K}{a-\gamma_K}$	0.088
Intertemporal elasticity of substitution	$\frac{1}{\eta}$	0.5

share, because both have qualitatively similar effects on the optimal length of time in school.[15] We will find in the next section that the quantitative effects of a productivity slowdown also are similar no matter what form that slowdown takes.

5 A Quantitative Exploration

In this section, we assess the decline in the labor share that our model suggests would result from a one percentage point reduction in trend growth of per capita income. We rely on the empirical literature to set some of our parameters and choose others to match moments from the U.S. historical experience. However, we have no firm basis for specifying the magnitude of the capital-skill complementarity that is reflected in the parameter a in the production function, $\tilde{F}\left(e^{-ah}A_tK, e^{bh}B_tL\right)$. Given our other moments, this parameter would be pinned down if we knew the bias of technical progress in the pre-slowdown period. But this bit of historical information also proves elusive. To complete our exercise, we pursue two different approaches. In Section 5.1, we introduce plausible but *ad hoc* assumptions about the bias in technical progress along the initial BGP. In Section 5.2, we attempt to estimate the parameter a using cross-sectional data on U.S. regions and industries. In Section 5.3, we discuss the sensitivity of our conclusions to our parameter choices.

[15] Factor shares are slightly more sensitive to γ_K than to γ_L, because a decline in the rate of capital-augmenting technological progress expands the long-run labor supply per worker, ℓ, whereas a decline in labor-augmenting technological progress does not. An increase in ℓ also enhances the incentive for full-time students to delay their entry into the labor force, per (12) and (13), beyond the effect of the decline in $\iota - g_W$.

Table 1 tabulates the parameters that we have specified and the moments that we have targeted throughout our quantitative exercise. The birth rate and death rate are averages for the United States for the period 1950-1970, as reported by the United Nations Population Division. The internal rate of return on schooling is a central estimate from a large literature on returns to investments in education; see, for example, the reviews by Card (1999) and Heckman et al. (2006). The capital share in the United States fluctuated narrowly around 35% in the period from 1950 to 1980, and perhaps beyond.[16] Labor productivity in the nonfarm business sector grew in the United States at an average compound rate of 2.4% per annum from 1950 to 1980, according to the Federal Reserve Economic Data (FRED). Goldin and Katz (2007) report a fairly steady, average annual increase in educational attainment for each new birth cohort of 0.88 years per decade for those born between 1880 and 1945.

The intertemporal elasticity of substitution plays an important role in our quantitative analysis, as we shall see below. Estimates derived from macroeconomic data typically range from 0 to 0.3; see, for example, Hall (1988), Campbell (2003), and Yogo (2004). Estimates that make use of micro (household) data often are larger and vary more widely; for example, those reported by Attanasio and Weber (1993) and Vissing-Jørgensen (2002) range from 0.5 to 1.0. Attanasio and Weber (1993) and Guvenen (2006) have shown how such differences between estimates using macro and micro data can arise due to liquidity constraints that may limit the participation of many households in equity markets. The estimates using macro data capture better the response of aggregate consumption growth to interest rates, which is the pertinent margin in our model. Still, our estimate of the response of factor shares to changes in productivity growth is quite sensitive to this crucial parameter. So as not to overstate this response, we take a central and conservative estimate of the elasticity of intertemporal substitution of $1/\eta = 0.5$.

Finally, we need a value for the parameter a or, alternatively, values for the parameters γ_K and γ_L that describe the bias in technical progress in the initial steady state. Alas, the Diamond-

[16] As we discussed in Section 1, there are various ways to measure factor shares, which vary especially with the treatment of entrepreneurial income. Our target of 0.35 for the capital share is central in the literature, and our bottom-line results are not very sensitive to small changes in this value.

McFadden "Impossibility Theorem" tells us that we cannot identify these parameters from time series data. In the next section, we explore the implications of some *ad hoc* assumptions about γ_K and γ_L, and also about which one declined during the most recent period of slower growth. In the succeeding section, we employ cross-sectional data for U.S. regions and industries in a crude attempt to estimate a directly.

5.1 *Ad Hoc* Assumptions about the Bias in Technical Progress

We have assumed that the model economy was evolving along an initial BGP, with labor productivity expanding at 2.4% per annum, before experiencing a once-and-for-all slowdown in productivity growth. In this section, we explore the implications of plausible but *ad hoc* assumptions about the bias of technical change in the pre-slowdown period. One simple assumption maintains that technical change in this period was factor neutral, so that $\gamma_K = \gamma_L$ before the slowdown. A second simple assumption is that the observed average decline in investment goods prices of 2% per year represents the full extent of investment-specific technical change, and that the disembodied technological progress was factor neutral ($g_A = \gamma_L$). We also need to know the form taken by the productivity slowdown. We entertain two alternative assumptions. At one extreme, we posit that only capital-augmenting technical progress decelerated, enough so to generate a one percentage point per year decline in labor-productivity growth. At the other extreme, we posit that only labor-augmenting technical progress slowed, again so as to generate a one percentage point decline in labor-productivity growth.

Table 2 shows the implications of these alternative assumptions. The top panel assumes equal rates of labor and capital augmenting technical progress of 1.1% per annum in the pre-slowdown period. Then a reduction in the rate of labor-augmenting progress that generates a one percent decline in the growth of labor productivity shifts about 3.3% of national income from labor to capital relative to the baseline, whereas a reduction in the rate of capital-augmenting technical progress shifts about 4.1% of national income. Only the latter type of productivity slowdown can generate a deceleration of educational attainment in our model, such as has been observed in the data. With these parameters, the annual increase in schooling slows to

Table 2: Response of Capital Share to Productivity Slowdown: Ad Hoc Examples

$\gamma_K = \gamma_L = 1.1\% \Rightarrow a = 0.132$						
	γ_K	γ_L	Growth in per capita Income	Annual Increase in Schooling	Interest Rate	Capital Share
Baseline	1.1%	1.1%	2.2%	0.09	10.0%	0.35
$\gamma_L \downarrow$	1.1%	0.1%	1.2%	0.09	8.0%	0.383
$\gamma_K \downarrow$	0.3%	1.1%	1.4%	0.02	8.3%	0.39

$g_q = 2.0\%$, $g_A = \gamma_L = 0.4\% \Rightarrow a = 0.293$						
	γ_K	γ_L	Growth in per capita Income	Annual Increase in Schooling	Interest Rate	Capital Share
Baseline	2.4%	0.4%	2.2%	0.09	10.0%	0.35
$\gamma_L \downarrow$	2.4%	-0.6%	1.2%	0.09	8.0%	0.365
$\gamma_K \downarrow$	1.2%	0.4%	1.3%	0.04	8.2%	0.368

0.20 years per decade. Note that Goldin and Katz (2007) report an annual increase of 0.16 per decade for the cohorts born after 1947. The bottom panel in Table 2 takes instead a baseline for capital-augmenting technical progress of 2.4% growth per year, comprising disembodied annual gains of 0.4% and investment-specific technical progress of 2.0% per year. In this calculation, the rate of labor-augmenting technical progress also is 0.4%. We see that the implied shifts in income shares that result from a productivity slowdown are somewhat smaller in this case, totalling 1.5% of national income if all of the slowdown is due to a reduction in γ_L and 1.8% in the opposite extreme, when it is only capital-augmenting technical progress that slows.

Our *ad hoc* assumptions about the bias in technical progress along the initial BGP allow us to back out the key parameters in the production function that reflect the strength of the capital-skill complementarity. Given the parameter values listed in Table 1, we must have $a = 0.132$ and $b = 0.164$ in order for the steady-state of the model with neutral technical progress to reproduce the aforementioned moments in the U.S. data. If technical progress instead was biased toward capital, as assumed in the bottom panel, then the implied parameters of the aggregate production function are $a = 0.293$ and $b = 0.251$. We have no real basis to judge the plausibility of these alternative parameter values, so we proceed now to conduct some crude estimation.

5.2 Estimation of Production-Function Parameters using Cross-Sectional Data

Our estimation strategy makes use of data from the BEA Regional Accounts for different states and industries in the United States. To perform this estimation, we must assume that industries in different states participate in an integrated national capital market and thus face a common interest rate. We also assume, somewhat heroically, that different industries share the same production-function parameter, a. We allow the technology parameter b to vary by industry and we let the rates of capital and labor-augmenting technological progress differ by industry and state.

We begin with equations (12) and (13), which together imply

$$\left(b - a\frac{\theta}{1-\theta}\right) l = \iota + \nu - g_W \ .$$

This equation relates the ratio of the factor shares to the growth rate of wages and the interest rate. Define $\bar{\theta}$ as the average labor compensation across industry and states, and $\bar{\ell}$ as the average fraction of time devoted to work.[17] Taking a first-order approximation around $\left(\bar{\theta}, \bar{\ell}\right)$ and using $\ell = 1 - \dot{h}$, we find

$$\left(b - a\frac{\bar{\theta}}{1-\bar{\theta}}\right)\left(1 - \dot{h}\right) - a\frac{\bar{l}}{\left(1-\bar{\theta}\right)^2}\left(\theta - \bar{\theta}\right) = \iota + \nu - g_W.$$

This relationship provides the basis for our cross-sectional estimation. Let j denote an industry and s a state. We perform the regression

$$1 - \theta_{js} = \alpha_s + \alpha_j + \xi_j \dot{h}_{js} + \beta g_{Wjs} + \varepsilon_{js} \ , \tag{18}$$

where $1 - \theta_{js}$ is the average labor share in a regional industry during a specified period, g_{Wjs} is the growth rate of wages in that state-industry pair, and $\dot{h}_{js}$ is the average annual increase in

[17] We estimate $\bar{\ell}$ by first computing average human capital accumulation $\bar{\dot{h}}$ as the annualized change in the average schooling among those working in a state-industry between 1970 and 2000 in the Census data and then applying $\bar{\ell} = 1 - \bar{\dot{h}}$.

years of schooling among those that work in the state-industry.[18] The regression includes state and industry fixed effects, and allows the coefficient on human capital accumulation to vary by industry to reflect the assumption that b may vary by industry. The estimated coefficient of interest is $\hat{\beta}$, because it allows us to identify a according to

$$\hat{a} = -\frac{\left(1-\bar{\theta}\right)^2}{\bar{l}\hat{\beta}} \ .$$

In order to carry out the estimation of (18), we compute the labor share in each industry-state pair from the BEA Regional Accounts by dividing total compensation by value added.[19] We compute the growth rate of wages using total compensation divided by employment as our measure of the wage. Finally, we derive a measure of schooling among workers in a state-industry from data reported in the decennial census.

Before proceeding, we highlight several estimation issues. First, our data on wages are replete with measurement error, due both to the high rate of imputation in the BEA accounts[20] and to the fact that our wage data are reported as compensation per worker, rather than compensation per hour, so that, for example, we cannot distinguish part-time from full-time workers in our employment measure. Not only does the measurement error introduce attenuation bias as usual, but if the extent of the measurement error has been increasing over time—as documented by White (2014) and inferred by Bils and Klenow (2017)—then our data construction would mechanically induce a positive correlation between labor shares and wage growth. To address this concern, we compute a second measure of wages from the decennial census by taking the average wage reported by those living in a given state and working in a given industry. While these wages also are measured imperfectly, the measurement error is quite different in nature: wages reported in the census may not reflect the actual wages paid, they do not include benefits, and the sample of workers is not designed to be representative at the state-industry level. We believe that the measurement error for state-industry wages derived from the census

[18] We show in the appendix that the average schooling among those in the workforce is $h_t^* - \frac{\dot{s}}{\lambda}$ where $\dot{s} = \frac{\gamma_K}{a-\gamma_K}$.

[19] Note that this measure excludes proprietors' income.

[20] See White et al. (2017).

is likely uncorrelated with that in the BEA wage data, inasmuch as the two have very different sources. Accordingly, we can use either of the wages series as an instrument for the other.

Second, we note that industry definitions changed from SIC classification to NAICS classification after 1997. To guard against the risk of confounding real changes with changes due to reclassification, we elect to compute the regressions separately for the 1970-1997 period using the SIC classification and for the 2000-2012 period using the NAICS classification.

Third, we find that our estimates are sensitive to the time horizon we use for calculating wage changes and average factor shares. In particular, the longer is the window, the smaller is our estimate of a. This could well be explained by a slower adjustment of schooling choices to underlying productivity trends than is suggested by our model. We believe that the estimates that make use of longer horizons better capture the long-run changes that would be found in the aggregate data. Accordingly, we choose to use the largest time windows that our data allow.

Table 3 presents our estimates for 1970-1997 using SIC industry classifications and for 2000-2012 using NAICS industry classifications, respectively.[21] We consistently find an inverse relationship between the average labor share in the state-industry and the average rate of wage growth, as would be predicted by our model. Our preferred estimate appears in the first column of Panel A, where we have used the BEA measure of wage growth, which implies $a = 0.19$.

In Table 4, we repeat the exercise of simulating the effects of a one percentage point slowdown in annual labor-productivity growth. In this case, the values of γ_K and γ_L in the baseline calibration are those needed for the model to match the annual increase in schooling, the capital share, the rate of return on education, and the growth rate of labor productivity in the pre-slowdown period. Again, we simulate the slowdown in labor-productivity growth as being the result of either a deceleration of capital-augmenting technological progress or of labor-augmenting technological progress.

We find that, for the range of values of a suggested by our estimation using state and industry data, a one percentage point slowdown in trend productivity growth can account for

[21] In all regressions, we have omitted observations for state-industry cells that include fewer than 100 workers. In the appendix, we report a variety of additional regression results, which we have computed as a check on robustness.

Table 3: Wage Growth and Labor Shares

Panel A: SIC Classification, 1970-1997	(1)	(2)	(3)	(4)
	BEA	BEA	Census	Census
Wage Growth	-2.17***	-4.45**	-2.29**	-7.38***
	(0.52)	(1.54)	(0.83)	(1.70)
Instrument	—	Census	—	BEA
Implied value of a	0.19	0.093	0.18	0.056
Observations	1065	1065	1065	1065

Panel B: NAICS Classification, 2000-2012	(1)	(2)	(3)	(4)
	BEA	BEA	Census	Census
Wage Growth	-1.05***	-3.50*	-0.71*	-5.15***
	(0.28)	(1.43)	(0.28)	(1.50)
Instrument	—	Census	—	BEA
Implied value of a	0.35	0.10	0.52	0.071
Observations	1107	1107	1107	1107

Robust standard errors in parentheses
$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Notes: Value Added, Labor Compensation and Employment for each state-industry-year from the BEA Regional Accounts; Wages and Years of Schooling calculated from the Census for 1970, 1980, 1990, 2000 and from the ACS for 2000-2012. Labor share is average across years of ratio of labor compensation to value added. State-industry-year cells with fewer than 100 observations are excluded from sample. Columns 1 and 2 use wage growth computed from BEA as independent variable, whereas columns 3 and 4 use wage growth from Census. Columns 1 and 3 estimated by OLS, columns 2 and 4 by IV. Each regression includes state fixed effects, industry fixed effects, and trends in years of state-industry schooling with coefficients that vary by industry.

a sizeable shift in income from labor to capital. With the parameters reflected in the table, the capital share rises between two and three percentage points.

Table 4 shows that the rate of increase of schooling can slow considerably if the decline in productivity growth is caused by a fall in γ_K; or it may not change at all if the cause was a drop in γ_L. In fact, the long-run change in the rate of increase of schooling is the natural moment that would reveal the nature of the productivity slowdown.

5.3 Sensitivity of Results to Various Parameters

In a range of simulations, we find that the productivity slowdown might be responsible for a half or more of the decline in the labor share in national income. How sensitive is this conclusion

Table 4: Response of Capital Share to Productivity Slowdown: Estimates of Capital-Schooling Complementarity using Cross-Sectional Data

Central Estimate of a: $a = 0.19$						
	γ_K	γ_L	Growth in per capita Income	Annual Increase in Schooling	Interest Rate	Capital Share
Baseline	1.5%	0.8%	2.2%	0.09	10.0%	0.35
$\gamma_L \downarrow$	1.5%	-0.2%	1.2%	0.09	8.0%	0.373
$\gamma_K \downarrow$	0.6%	0.8%	1.3%	0.03	8.2%	0.378

to our parameter choices?

From equation (17), we see that

$$d\left(\frac{\theta}{1-\theta}\right) = \frac{1-\eta}{a-\gamma_K} dg_y + \frac{(1-\eta)\, g_y + \lambda - \nu - \rho}{(a-\gamma_K)^2} d\gamma_K \ . \tag{19}$$

Consider the first term on the right-hand side of (19). We have assumed a one percentage point decline in annual labor productivity growth, which fixes $dg_{Y/L} = dg_W$. Moreover, (10) states that $\dot{h} = \gamma_K / a$, and $\dot{h} = 0.088$ in the data. Therefore, $a - \gamma_K \approx a$. It follows that the shift in income associated with the first term is mostly governed by two parameters, a and η. We have considered a range of possibilities for a. And we have taken a conservative estimate of the elasticity of intertemporal substitution of 0.5, which yields a value of $\eta = 2$. A smaller value of the elasticity of intertemporal substitution would imply a larger value of η and therefore a greater sensitivity of relative factor shares to a change in the per capita growth rate. The second term in (10) applies only if the productivity slowdown was generated by a decline in the rate of capital-augmenting technical progress. This term is positive (for $d\gamma_K < 0$) by (3), which is the parameter restriction needed to ensure finite dynastic utility; so it only strengthens the forces in our model. We conclude that, once we admit a reasonable amount of capital-skill complementarity (as captured by the parameter a) a productivity slowdown can account for a substantial redistribution of income from labor to capital for all plausible values of the other parameters.

6 Concluding Remarks

In this paper, we have proposed a novel explanation for part of the decline in the labor share that has taken place in recent years. We added human capital accumulation à la Ben Porath (1967) to a plain-vanilla neoclassical growth model. In this setting, if human capital is more complementary with physical capital than with raw labor and if the elasticity of substitution between physical capital and labor is less than one (holding constant the level of schooling), then the rate of labor productivity growth and the share of labor in national income will be positively correlated across steady states. Accordingly, a slowdown in productivity growth—such as has apparently occurred in the recent period—can lead to a shift in the functional distribution of income away from labor and toward capital. The mechanism requires a fall in the real interest rate, which has also been part of the recent experience. When the interest rate falls relative to the growth rate of wages, individuals target a higher level of human capital for any given size of the capital stock and state of technology. When human capital is more complementary to physical capital than to raw labor, the elevated human capital target implies a greater relative demand for capital. With an elasticity of substitution between capital and labor less than one, the shift in relative factor demands generates a rise in the capital share at the expense of labor. Moreover, if the productivity slowdown is associated with a deceleration of declining investment-good prices or with a fall in the rate of disembodied capital-augmenting technical progress, then the model predicts a slowdown in the annual expansion of educational attainment, which also matches the data in recent economic history.

Our story has additional attractive features. First, unlike several of the other explanations for the decline in the labor share, ours does not rely on considerations that are specific to the United States. The shift in aggregate factor shares has been seen in the data for many countries, especially among the advanced countries. The productivity slowdown also has been a common phenomenon, at least in the OECD countries. Real interest rates have fallen globally. And educational gains have slowed in many advanced countries. Our growth model, which we developed for a closed economy, can be interpreted as applying to a global economy comprising (at least) the technologically-advanced countries. A productivity slowdown that is common to these leading-edge economies should generate a decline in the interest rate and, in the presence of capital-skill complementarity, a widespread fall in labor shares.

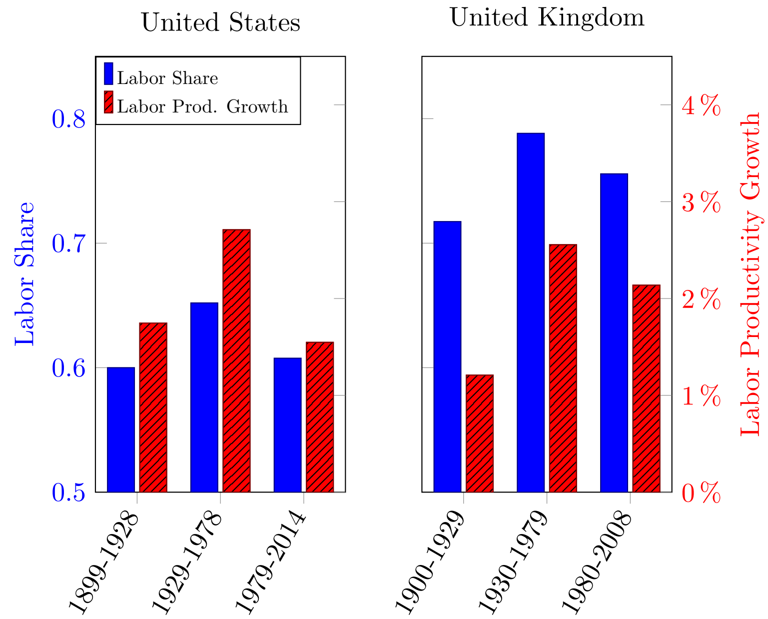
Second, our model can help to reconcile the different conclusions about the size of the elasticity of substitution between capital and labor found in, for example, Antràs (2004), Klump et al. (2007), and Oberfield and Raval (2015) on the one hand, and Karabarbounis and Neiman (2014) and Glover and Short (2017), on the other. The former studies estimate the elasticity of substitution between capital and hours holding human capital fixed; i.e., after controlling for schooling. They find an elasticity of substitution (which is σ_{KL}, in our notation) less than one. The latter studies, in contrast, use cross-country differences in the movement of investment-good prices to identify the elasticity of substitution from heterogeneous changes in factor shares. Inherently, their method incorporates into the elasticity estimate the adjustment in schooling that takes place in response to investment-good prices. Our model implies that factor shares will be *insensitive* to the price of investment goods after controlling for changes in the growth rate of wages and in the real interest rate, which amounts to a unitary aggregate elasticity of substitution after allowing for human-capital adjustment.

Third, our mechanism is quite consistent with some recent empirical findings on the effects of financial deregulation on the demand for higher education and on the labor share. Sun and Yannelis (2016) and Leblebicioğlu and Weinberger (2017) both use the staggered deregulation of the banking industry across states of the United States as a natural experiment to study the effects of credit market frictions. Sun and Yannelis find that college enrollment and completion increase significantly in response to an expansion in available household credit after banking deregulation. Leblebicioğlu and Weinberger find that state-wide labor shares declined in response to local banking deregulation.[22] Both of these findings are exactly what would be predicted by our model if deregulation generates a decline in the real interest rate facing

[22]In the canonical neoclassical theory, an exogenous decline in the relative price of capital goods and an exogenous fall in the real interest rate have qualitatively similar impacts on factor shares, because both reduce the rental rate of capital. In our model, these shocks affect factor shares differently due to the human capital response. A decline in the relative price of capital leaves factor shares unchanged after the subsequent adjustment of schooling, whereas a decline in the real interest rate reduces the labor share, because it increases the desired level of human capital relative to the effective capital-labor ratio.

households when they make their schooling decisions.

Figure 5: Historical Labor Share



Sources: US labor share for 1899-1948 from Kuznets (1952) and for 1949-2014 from BLS; UK labor share from Clark (2010), US labor productivity from Robert Gordon, UK Labor Productivity from Bank of England, "A Millennium of Macroeconomic Data," (available at http://www.bankofengland.co.uk/research/Pages/datasets/default.aspx).

Finally, we have focused in this paper on exploring a potential explanation for recent trends in the labor share. But it is possible that our story holds broader sway in economic history.[23] Figure 5 shows the evolution of the labor share in the United States and the United Kingdom since the beginning of the twentieth century and the evolution of labor productivity in each country over the same period. Evidently, these two variables have been temporally correlated throughout modern history. For example, the period from 1900 until approximately 1930 was a period of slow productivity growth in the United States and United Kingdom. It was also a period of an historically low labor share. When productivity growth subsequently accelerated, the labor share rose in tandem. While we are cautious about drawing firm conclusions from

[23]Interestingly, Franck and Galor (2017) provide evidence that investments in the steam engine were complementary to human capital in early post-industrial France. The authors find that regions that used steam engines more intensively had more teachers, a greater share of children attending primary school, a greater fraction of apprentices in the population, a greater share of literate conscripts, and greater public outlays for primary schools.

such casual observations, it is possible that productivity growth and the functional distribution of income have been linked for quite some time.

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Appendix for "The Productivity Slowdown and the Declining Labor Share: A Neoclassical Exploration"

by

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Proofs from Section 3

Proof of Lemma 1

The first step in solving for the BGP is to show that there exists a threshold human capital level h_t^* such that at time t all individuals with human capital below h_t^* devote all their time to schooling and all individuals with human capital above h_t^* work full-time.

Consider an individual with human capital h_t at time t and labor supply path ℓ_τ for $\tau \geq t$. Let $\tilde{\ell}_\tau$ be an alternative labor supply path defined by

$$\tilde{\ell}_\tau = \begin{cases} \ell_\tau + \epsilon, & \tau \in [t, t+\Delta], \\ \ell_\tau - \epsilon, & \tau \in (t+\Delta, t+2\Delta], \\ \ell_\tau, & \tau > t+2\Delta. \end{cases}$$

where $\epsilon \in \mathbb{R}$ and $\Delta > 0$. The individual's human capital under labor supply path $\tilde{\ell}_\tau$ is given by

$$\tilde{h}_\tau = \begin{cases} h_\tau - \epsilon(\tau - t), & \tau \in [t, t+\Delta], \\ h_\tau - \epsilon\left(t + 2\Delta - \tau\right), & \tau \in [t+\Delta, t+2\Delta], \\ h_\tau, & \tau \geq t+2\Delta. \end{cases}$$

Note that human capital is unaffected outside the interval $(t, t+2\Delta)$.

Let S be the difference between the individual's expected present value of earnings under $\tilde{\ell}_\tau$ and under ℓ_τ. We have

$$
\begin{aligned}
S &= \int_t^{t+2\Delta} e^{-\int_t^\tau (\iota_s+\nu)ds} \left[\tilde{\ell}_\tau W_\tau(\tilde{h}_\tau) - \ell_\tau W_\tau\left(h_\tau\right)\right] d\tau, \\
&= \int_t^{t+\Delta} e^{-\int_t^\tau (\iota_s+\nu)ds} \left\{\ell_\tau \left(W_\tau\left[h_\tau - \epsilon(\tau - t)\right] - W_\tau\left[h_\tau\right]\right) + \epsilon W_\tau\left[h_\tau - \epsilon(\tau - t)\right]\right\} d\tau \\
&+ \int_{t+\Delta}^{t+2\Delta} e^{-\int_t^\tau (\iota_s+\nu)ds} \left\{\ell_\tau \left(W_\tau\left[h_\tau - \epsilon\left(t + 2\Delta - \tau\right)\right] - W_\tau\left[h_\tau\right]\right) - \epsilon W_\tau\left[h_\tau - \epsilon\left(t + 2\Delta - \tau\right)\right]\right\} d\tau,
\end{aligned}
$$

where the second equality uses the expressions for $\tilde{\ell}_\tau$ and $\tilde{h}_\tau$ above. Expressing the functions in the integrands in terms of Taylor series around t, computing the integrals and dropping terms that are $o(\Delta^2)$ implies that for Δ close to zero we have

$$
S \approx \epsilon\Delta^2 \left[(\iota_t + \nu)W_t(h_t) - W_t'(h_t) - \frac{\partial W_t(h_t)}{\partial t}\right]. \tag{20}
$$

The intuition for this expression is as follows. If $\epsilon > 0$, choosing $\tilde{\ell}_\tau$ instead of ℓ_τ means increasing labor supply today and reducing labor supply tomorrow. The benefit of this change is $(\iota_t+\nu)W_t(h_t)$, which gives the increase in the expected present value of earnings from bringing forward the date at which labor income is received. The costs of working more today and less tomorrow are: $W_t'(h_t)$, which captures the reduction in earnings resulting from the individual having lower human capital tomorrow, and; $\frac{\partial W_t(h_t)}{\partial t}$, which reflects the increase in wages over time.

Equation (13) gives $W_t'\left(h_t\right)$. By differentiating the wage function (5) we also obtain

$$
\frac{\partial W_t(h_t)}{\partial t} = \left\{\gamma_L + \left(g_A - \frac{\dot{R}_t}{R_t}\right) \frac{\theta\left[z_t\left(h_t\right)\right]}{1 - \theta\left[z_t\left(h_t\right)\right]}\right\} W_t(h_t),
$$

where $\theta(z) \equiv z f'(z)/f(z)$ and $z_t(h) \equiv e^{-(a+b)h} \frac{A_t \kappa_t(h)}{B_t}$. Substituting these expressions into (20) yields

$$
S \approx \epsilon\Delta^2 W_t(h_t) \left[\iota_t + \nu - b - \gamma_L + \left(a + \frac{\dot{R}_t}{R_t} - g_A\right) \frac{\theta\left[z_t\left(h_t\right)\right]}{1 - \theta\left[z_t\left(h_t\right)\right]}\right]. \tag{21}
$$

Assumption 1 implies $\theta(z)$ is strictly decreasing in z as shown in Grossman et al. (2017).

Rearranging equation (4) which characterizes optimal capital use we also have

$$R_t = e^{-ah_t} A_t f' \left[z_t(h_t) \right], \tag{22}$$

and differentiating this expression with respect to h_t implies z_t is strictly decreasing in h_t. It follows that $\theta \left[z_t \left(h_t \right) \right] / (1 - \theta \left[z_t \left(h_t \right) \right])$ is strictly increasing in h_t.

Assume $a + \dot{R}_t / R_t - g_A > 0$ and that there exists a finite, strictly positive h_t^* such that

$$\iota_t + \nu - b - \gamma_L + \left(a + \frac{\dot{R}_t}{R_t} - g_A \right) \frac{\theta \left[z_t \left(h_t^* \right) \right]}{1 - \theta \left[z_t \left(h_t^* \right) \right]} = 0, \tag{23}$$

implying the right hand side of (21) equals zero. We will prove below that these assumptions hold on a BGP. Given $a + \dot{R}_t / R_t - g_A > 0$, the right hand side of equation (21) is strictly increasing in h_t if and only if $\epsilon > 0$. Consequently, individuals with human capital below h_t^* have a strictly higher expected present value of earnings under labor supply path $\tilde{\ell}_\tau$ than under ℓ_τ whenever $\epsilon < 0$. Likewise, individuals with human capital above h_t^* have a strictly higher expected present value of earnings under $\tilde{\ell}_\tau$ than ℓ_τ whenever $\epsilon > 0$. Low human capital individuals prefer to study today and work tomorrow, while the opposite is true for high human capital individuals. Since labor supply is bounded on the interval $[0, 1]$ it follows that the individual's optimal labor supply is given by $\ell_t = 0$ if $h_t < h_t^*$ and $\ell_t = 1$ if $h_t > h_t^*$.

Now, consider a BGP. Recall that we define a BGP as a dynamic equilibrium with constant rates of growth of output, consumption and capital and constant factor shares of income. The Euler equation (2) implies the real interest rate ι_t is constant on a BGP. The real interest rate must also equal the return from purchasing the investment good which gives the no-arbitrage condition

$$\iota_t = q_t R_t - \delta - g_q. \tag{24}$$

The no-arbitrage condition implies that on a BGP $\dot{R}_t / R_t = -g_q$. Therefore, $a + \dot{R}_t / R_t - g_A = a - \gamma_K$ which is strictly positive by Assumption 2.i. It follows that $a + \dot{R}_t / R_t - g_A > 0$ on a

BGP as assumed above.

Equation (23) implies the human capital threshold for entering the workforce h_t^* satisfies

$$\frac{\theta\left[z_t\left(h_t^*\right)\right]}{1-\theta\left[z_t\left(h_t^*\right)\right]}=\frac{b+\gamma_L-(\iota+\nu)}{a-\gamma_K}, \tag{25}$$

showing that $z_t(h_t^*)=z^*$ must be constant on a BGP which proves equation (9) in Lemma 1. Differentiating (22) with respect to time while holding $z_t(h_t^*)$ constant yields

$$\dot{h}_t^*=\frac{\gamma_K}{a}.$$

Therefore, in order for their human capital to increase at the same rate as h_t^*, individuals at the threshold human capital level must choose labor supply $\ell=1-\gamma_K/a$ as claimed in equation (8) of Lemma 1.

At time t any individuals with human capital above h_t^* work full-time and do not increase their human capital. Consequently, on a BGP it is not possible for individuals to have human capital above h_t^* since h_t^* is growing over time. Given this observation, the remaining properties of the unique BGP can be derived as in the discussion following Lemma 1 in the paper. In particular, equation (15) gives the real interest rate on the BGP and substituting (15) into (25) gives (17) which defines the BGP value of θ and, therefore, also of z^* and h_t^*. Assumption 2.ii guarantees there exists a finite, strictly positive h_t^* that solves equation (17). This completes the proof of Lemma 1.

Stability of the BGP

The economy's behavior away from the BGP depends upon whether or not any individuals work full-time. We will solve for the transition dynamics under the assumption that departures from the BGP are sufficiently small that nobody chooses to work full-time. This means there will always be a threshold human capital level h_t^* such that individuals with human capital below h_t^* are in full-time education, individuals with human capital equal to h_t^* have an interior labor supply choice and there are no individuals with human capital above h_t^*.

The transition dynamics can be expressed in terms of four variables

$$\tilde{z}_t \equiv e^{-(a+b)h_t^*} \frac{A_t K_t}{B_t L_t}, \tag{26}$$

$$\tilde{c}_t \equiv e^{-g_c t} c_t = e^{-\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right)t} c_t, \tag{27}$$

$$\tilde{h}_t \equiv h_t^* - \frac{\gamma_K}{a} t, \tag{28}$$

$$\tilde{K}_t \equiv e^{-g_K t} K_t = e^{-\left[g_q + \gamma_L + \frac{b-\lambda}{a}\gamma_K + \lambda - \nu\right]t} K_t. \tag{29}$$

where g_c and g_K denote the BGP growth rates of consumption per capita and the capital stock, respectively. Note that $g_K = g_q + g_c + \lambda - \nu$ and that capital market clearing requires $\tilde{z}_t = z_t(h_t^*)$ in equilibrium. $\tilde{z}_t$ and $\tilde{c}_t$ are jump variables, while $\tilde{h}_t$ and $\tilde{K}_t$ are the economy's two state variables. All four variables are stationary on the BGP.

There are four differential equations that characterize the transition dynamics: the human capital accumulation equation (1), the Euler equation (2), the capital accumulation equation and the human capital threshold equation (23) which defines h_t^*. We will express these four equations in terms of $\tilde{z}_t$, $\tilde{c}_t$, $\tilde{h}_t$ and $\tilde{K}_t$.

The labor force L_t is given by $L_t = e^{-\lambda h_t^*} \ell_t N_t$. Substituting this expression into the human capital accumulation equation (1) and using (26), (28) and (29) implies

$$\dot{\tilde{h}}_t = 1 - \frac{\gamma_K}{a} - \frac{A_0}{B_0 N_0} \frac{\tilde{K}_t}{\tilde{z}_t} e^{(\lambda - a - b)\tilde{h}_t}. \tag{30}$$

Substituting the optimal capital use equation (22) into the no-arbitrage condition (24) and imposing capital market clearing gives

$$\iota_t = q_0 A_0 e^{-a\tilde{h}_t} f'(\tilde{z}_t) - \delta - g_q. \tag{31}$$

Using this expression in the Euler equation (2) and applying the definitions in (26)-(28) gives

$$\dot{\tilde{c}}_t = \left[q_0 A_0 e^{-a\tilde{h}_t} f'(\tilde{z}_t) - \delta - g_q - \rho - \eta g_c \right] \frac{\tilde{c}_t}{\eta}. \tag{32}$$

The capital accumulation equation is

$$\dot{K}_t = q_t \left(Y_t - N_t c_t \right) - \delta K_t.$$

Using $Y_t = e^{bh_t^*} B_t L_t f(\tilde{z}_t)$ together with (26)-(29) we can rewrite this as

$$\dot{\tilde{K}}_t = \left[\frac{q_0 A_0 e^{-a\tilde{h}_t} f'(\tilde{z}_t)}{\theta(\tilde{z}_t)} - q_0 N_0 \frac{\tilde{c}_t}{\tilde{K}_t} - \delta - g_K \right] \tilde{K}_t. \tag{33}$$

Finally, we turn to the human capital threshold equation (23). Imposing capital market clearing in the optimal capital use equation (22) and then differentiating with respect to time gives

$$\frac{\dot{R}_t}{R_t} = -a\dot{h}_t^* + g_A + \frac{f''(\tilde{z}_t)}{f'(\tilde{z}_t)} \dot{\tilde{z}}_t.$$

We also have

$$\sigma_{KL}(\tilde{z}_t) = -\left[1 - \theta(\tilde{z}_t) \right] \frac{f'(\tilde{z}_t)}{\tilde{z}_t f''(\tilde{z}_t)}.$$

Substituting these expressions into (23) and using (1), (26), (28), (29) and (31) yields

$$\dot{\tilde{z}}_t = \left[-b - \gamma_L + \nu - \delta - g_q + q_0 A_0 e^{-a\tilde{h}_t} f'(\tilde{z}_t) + a \frac{A_0}{B_0 N_0} \frac{\tilde{K}_t}{\tilde{z}_t} e^{(\lambda - a - b)\tilde{h}_t} \frac{\theta(\tilde{z}_t)}{1 - \theta(\tilde{z}_t)} \right] \frac{\tilde{z}_t \sigma_{KL}(\tilde{z}_t)}{\theta(\tilde{z}_t)}. \tag{34}$$

Equations (30), (32), (33) and (34) determine the transition dynamics.

Setting $\dot{\tilde{h}}_t = \dot{\tilde{c}}_t = \dot{\tilde{K}}_t = \dot{\tilde{z}}_t = 0$ implies that on the BGP the stationary values of these four variables satisfy

$$
\begin{aligned}
\frac{A_0}{B_0 N_0}\frac{\tilde{K}^*}{\tilde{z}^*}e^{(\lambda-a-b)\tilde{h}^*} &= 1-\frac{\gamma_K}{a}, \\
q_0 A_0 e^{-a\tilde{h}^*} f'(\tilde{z}^*) &= \delta + g_q + \rho + \eta g_c, \\
\frac{q_0 A_0 e^{-a\tilde{h}^*} f'(\tilde{z}^*)}{\theta(\tilde{z}^*)} - q_0 N_0 \frac{\tilde{c}^*}{\tilde{K}^*} &= \delta + g_K, \\
\frac{\theta(\tilde{z}^*)}{1-\theta(\tilde{z}^*)} &= \frac{1}{a-\gamma_K}\left[b+\gamma_L-\nu-\eta g_c-\rho\right].
\end{aligned}
$$

Linearizing (30), (32), (33) and (34) about the stationary steady state therefore gives

$$
\dot{\tilde{h}}_t = \frac{A_0}{B_0 N_0}\frac{1}{\tilde{z}^*}e^{(\lambda-a-b)\tilde{h}^*}\left[-(\lambda-a-b)\tilde{K}^*\left(\tilde{h}_t-\tilde{h}^*\right)-\left(\tilde{K}_t-\tilde{K}^*\right)+\frac{\tilde{K}^*}{\tilde{z}^*}\left(\tilde{z}_t-\tilde{z}^*\right)\right],
$$

$$
\dot{\tilde{c}}_t = q_0 A_0 e^{-a\tilde{h}^*}\frac{\tilde{c}^*}{\eta}\left[-af'(\tilde{z}^*)\left(\tilde{h}_t-\tilde{h}^*\right)+f''(\tilde{z}^*)\left(\tilde{z}_t-\tilde{z}^*\right)\right],
$$

$$
\begin{aligned}
\dot{\tilde{K}}_t = &-aq_0 A_0 e^{-ah^*}\tilde{K}^*\frac{f(\tilde{z}^*)}{\tilde{z}^*}\left(\tilde{h}_t-\tilde{h}^*\right) - q_0 N_0\left(\tilde{c}_t-\tilde{c}^*\right) \\
&+ q_0 N_0\frac{\tilde{c}^*}{\tilde{K}^*}\left(\tilde{K}_t-\tilde{K}^*\right) - q_0 A_0 e^{-ah^*}\tilde{K}^*\frac{f(\tilde{z}^*)-\tilde{z}^* f'(\tilde{z}^*)}{(\tilde{z}^*)^2}\left(\tilde{z}_t-\tilde{z}^*\right),
\end{aligned}
$$

$$
\begin{aligned}
\dot{\tilde{z}}_t = &-a\left[q_0 A_0 e^{-a\tilde{h}^*}f(\tilde{z}^*)\sigma(\tilde{z}^*)-(\lambda-a-b)\frac{A_0}{B_0 N_0}\tilde{K}^* e^{(\lambda-a-b)\tilde{h}^*}\frac{\sigma(\tilde{z}^*)}{1-\theta(\tilde{z}^*)}\right]\left(\tilde{h}_t-\tilde{h}^*\right) \\
&+ a\frac{A_0}{B_0 N_0}e^{(\lambda-a-b)\tilde{h}^*}\frac{\sigma(\tilde{z}^*)}{1-\theta(\tilde{z}^*)}\left(\tilde{K}_t-\tilde{K}^*\right) - a\frac{A_0}{B_0 N_0}\frac{\tilde{K}^*}{\tilde{z}^*}e^{(\lambda-a-b)\tilde{h}^*}\frac{\sigma(\tilde{z}^*)}{1-\theta(\tilde{z}^*)}\left(\tilde{z}_t-\tilde{z}^*\right) \\
&+ \left[-q_0 A_0 e^{-a\tilde{h}^*}\frac{f(\tilde{z}^*)-\tilde{z}^* f'(\tilde{z}^*)}{\tilde{z}^*} + a\frac{A_0}{B_0 N_0}\tilde{K}^* e^{(\lambda-a-b)\tilde{h}^*}\frac{\theta'(\tilde{z}^*)\sigma(\tilde{z}^*)}{\theta(\tilde{z}^*)\left[1-\theta(\tilde{z}^*)\right]^2}\right]\left(\tilde{z}_t-\tilde{z}^*\right).
\end{aligned}
$$

This system of linear first order differential equations can be written as

$$\begin{pmatrix} \dot{\tilde{h}}_t \\ \dot{\tilde{c}}_t \\ \dot{\tilde{K}}_t \\ \dot{\tilde{z}}_t \end{pmatrix} = \begin{pmatrix} \alpha_{11} & 0 & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & 0 & 0 & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & 0 & \alpha_{43} & \alpha_{44} \end{pmatrix} \begin{pmatrix} \tilde{h}_t - \tilde{h}^* \\ \tilde{c}_t - \tilde{c}^* \\ \tilde{K}_t - \tilde{K}^* \\ \tilde{z}_t - \tilde{z}^* \end{pmatrix} \tag{35}$$

and using the definitions of $\tilde{h}^*$, $\tilde{c}^*$, $\tilde{K}^*$ and $\tilde{z}^*$ to simplify we obtain

$$\begin{aligned}
\alpha_{11} &= -(\lambda - a - b)\left(1 - \frac{\gamma_K}{a}\right), \\
\alpha_{13} &= -\frac{1}{\tilde{K}^*}\left(1 - \frac{\gamma_K}{a}\right), \\
\alpha_{14} &= \frac{1}{\tilde{z}^*}\left(1 - \frac{\gamma_K}{a}\right), \\
\alpha_{21} &= -a\Lambda\frac{\tilde{c}^*}{\eta}, \\
\alpha_{24} &= -\Lambda\frac{1-\theta(\tilde{z}^*)}{\sigma(\tilde{z}^*)\tilde{z}^*}\frac{\tilde{c}^*}{\eta}, \\
\alpha_{31} &= -a\Lambda\frac{\tilde{K}^*}{\theta(\tilde{z}^*)}, \\
\alpha_{32} &= -q_0N_0, \\
\alpha_{33} &= q_0N_0\frac{\tilde{c}^*}{\tilde{K}^*} \\
\alpha_{34} &= -\Lambda\frac{\tilde{K}^*}{\tilde{z}^*}\frac{1-\theta(\tilde{z}^*)}{\theta(\tilde{z}^*)}, \\
\alpha_{41} &= -a\frac{\tilde{z}^*\sigma(\tilde{z}^*)}{\theta(\tilde{z}^*)}\left[\Lambda - (\lambda - a - b)\left(1 - \frac{\gamma_K}{a}\right)\frac{\theta(\tilde{z}^*)}{1-\theta(\tilde{z}^*)}\right], \\
\alpha_{43} &= (a - \gamma_K)\frac{\tilde{z}^*}{\tilde{K}^*}\frac{\sigma(\tilde{z}^*)}{1-\theta(\tilde{z}^*)}, \\
\alpha_{44} &= -\Lambda\frac{1-\theta(\tilde{z}^*)}{\theta(\tilde{z}^*)} - (a - \gamma_K)\frac{1}{1-\theta(\tilde{z}^*)}.
\end{aligned}$$

where

$$\Lambda = \delta + g_q + \rho + \eta g_c.$$

The BGP is locally saddle-path stable if the matrix of α coefficients has two eigenvalues with negative real parts. We have not been able to characterize the sign of the eigenvalues analytically, but, by imposing a functional form restriction on $f(z)$ we can check the stability of the BGP numerically. We assume $f(z) = (1+z^{\alpha})^{\frac{1}{\alpha}\frac{b}{a+b}}$ where α is calibrated to ensure the elasticity of substitution between capital and labor $\sigma_{KL} = 0.6$. Under this assumption there exists a locally saddle-path stable BGP for all the parameter configurations used in Section 5.

Proofs and Transition Dynamics from Section 4

Proof of Proposition 2

Differentiating equation (17) with respect to γ_K yields

$$\frac{1}{(1-\theta)^2}\frac{\partial\theta}{\partial\gamma_K} = -\frac{\eta-1}{a-\gamma_K}\frac{b-\lambda}{a} - \frac{(\eta-1)(\gamma_L + \frac{b-\lambda}{a}\gamma_K) - \lambda + \nu + \rho}{(a-\gamma_K)^2}.$$

The first term on the right hand side is negative when $\eta > 1$ since Assumption 1 imposes $b > \lambda$. The second term on the right hand side is negative by Assumption 2.iii which guarantees finite utility on the BGP. It follows that an increase in γ_K reduces θ or, equivalently, that a reduction in γ_K reduces labor's share of income.

Differentiating equation (17) with respect to γ_L yields

$$\frac{1}{(1-\theta)^2}\frac{\partial\theta}{\partial\gamma_L} = -\frac{\eta-1}{a-\gamma_K},$$

which is negative if and only if $\eta > 1$. Thus, a reduction in γ_L increases θ and lowers labor's share of income.

Transition Dynamics

To simulate the transition dynamics following an unanticipated permanent decline in labor productivity growth we use the linearized system of differential equations given by (35). Let ζ_1 and ζ_2 be the negative eigenvalues of the matrix of α coefficients with corresponding eigenvectors $\mathbf{x_1}$ and $\mathbf{x_2}$. Assuming the shock occurs at time $t = 0$, we have that for $t > 0$ the deviation of the system from the new stationary steady state is given by

$$\begin{pmatrix} \tilde{h}_t - \tilde{h}^* \\ \tilde{c}_t - \tilde{c}^* \\ \tilde{K}_t - \tilde{K}^* \\ \tilde{z}_t - \tilde{z}^* \end{pmatrix} = D_1 e^{\zeta_1 t}\mathbf{x_1} + D_2 e^{\zeta_2 t}\mathbf{x_2}$$

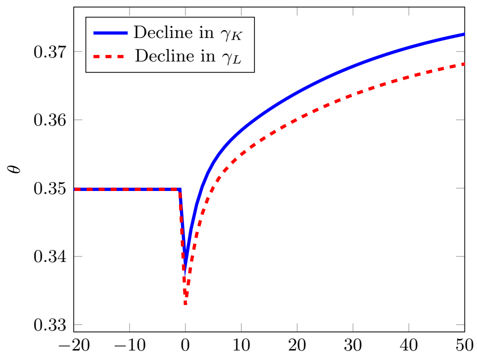
where the constants D_1 and D_2 are chosen to ensure that the state variables $\tilde{h}_t$ and $\tilde{K}_t$ are continuous at time zero.

Figure 6 uses this expression to plot the transition dynamics for the calibration in Table 4 with $a = 0.19$. In addition to matching the moments described in Section 5, we assume $f(z) = (1 + z^{\alpha})^{\frac{1}{\alpha}\frac{b}{a+b}}$ where α is calibrated to ensure the elasticity of substitution between capital and labor $\sigma_{KL} = 0.6$ on the initial BGP and we set $\delta = 0.1$, $N_0 = 1$ and $q_0 = A_0 = B_0 = 2$.

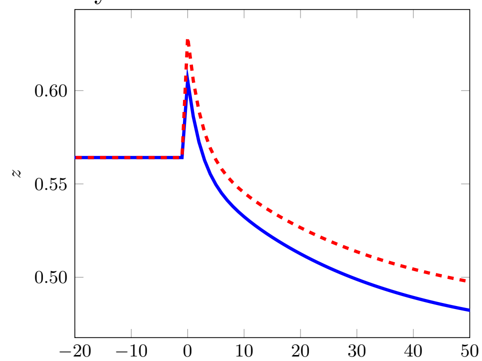
Figure 6 shows the responses of the capital share θ, the schooling-adjusted effective capital-to-labor ratio z, human capital h^* and the log of the capital-to-output ratio $\log K/(qY)$. Each panel shows two plots. The blue solid line corresponds to a decline in γ_K resulting from a fall in g_A. The red dashed line corresponds to a decline in γ_L. Each decline is chosen to generate a one percentage point per annum decline in labor productivity growth. In panel (c) the dotted lone shows the path of human capital on the initial BGP.

Figure 6 shows a long-run decline in the rate of increase of human capital accumulation if and only if the shock reflects a decrease in the rate of capital-augmenting technical progress. In this case, human capital briefly rises above the path associated with the initial steady state, but soon falls below that path. In other words, despite the decline in the schooling-adjusted capital-to-labor ratio shown in panel (b), the schooling target is soon below what it would have been absent the slowdown in capital-augmenting technical progress. Panel (d) shows a rise in the capital-output ratio, in keeping with what we find for the common global component when we regress capital-output ratios from the Penn World Table and country and time fixed effects.

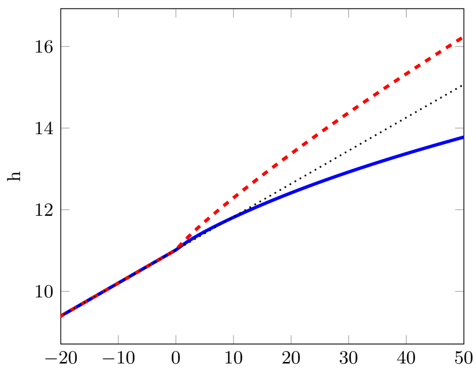
Figure 6: Transition Dynamics



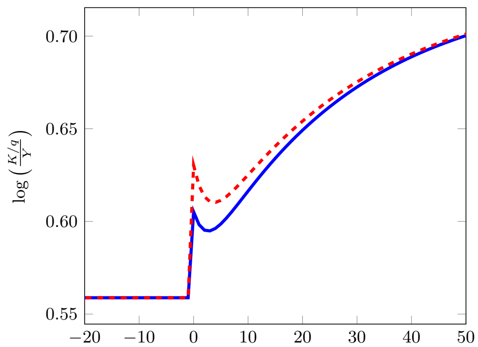
(a) Capital Share



(b) Schooling-Adjusted K-L Ratio



(c) Human Capital



(d) Log Capital-Output Ratio

Proofs from Section 5

We show here that the average schooling among those in the workforce is $h_t^* + \frac{\dot{s}}{\lambda+\nu}$ where $\dot{s} = \frac{\gamma_K}{a-\gamma_K}$.

Following the discussion in Section 3, the measure of individuals born at τ that are alive at t is $\lambda N_t e^{-\lambda(t-\tau)}$. Thus the average schooling among those in the workforce is

$$\bar{s}^{workforce} = \frac{\int_{-\infty}^{t-h_t^*} s_\tau \lambda N_t e^{-\lambda(t-\tau)} d\tau}{\int_{-\infty}^{t-h_t^*} \lambda N_t e^{-\lambda(t-\tau)} d\tau}$$

Using the change of variables $u = (t - h_t^*) - \tau$, this is

$$\bar{s}^{workforce} = \frac{N_t e^{-\lambda h_t^*} \int_0^\infty s_{t-h_t^*-u} \lambda e^{-\lambda u} du}{N_t e^{-\lambda h_t^*} \int_0^\infty \lambda e^{-\lambda u} du} = \int_0^\infty s_{t-h_t^*-u} \lambda e^{-\lambda u} du$$

Finally, for cohort born at $\tau < t - h_t$, we have that $s_{t-h_t^*-u} = s_{t-h_t^*} - u\dot{s} = h_t^* - u\dot{s}$, this is

$$\begin{aligned} \bar{s}^{workforce} &= \int_0^\infty \left[h_t^* - u\dot{s}\right] \lambda e^{-\lambda u} du \\ &= h_t^* - \frac{\dot{s}}{\lambda} \end{aligned}$$

Additional Cross-Section Specifications from Section 5

This section gives more detail about the relationship between wage growth and labor shares across states and industries. The top panel in Table 5 shows that the relationship between these variables is weaker when one considers a shorter time span. The baseline regression reported in Panel A of Table 3 related average labor shares to wage growth over the period 1970 to 1997. Panel A of Table 5 shows the results of regressions that use shorter time spans, namely 1970-1990 and 1980-1997. For the most part, the estimated relationship between the variables is weaker. One possible explanation might be that schooling decisions are based on expectations of wage growth, and wage growth that is longer lasting is more likely to be incorporated into expectations.

We also find that our estimates are sensitive to whether or not we include all of the industry-state observations in the sample. Our baseline estimation excluded industry-states for which there were fewer than 100 observations in the Census sample that we could use to calculate average wages and average years of schooling. Panel B shows that, for the most part, the estimated relationship between labor share and wage growth is less strong when we do not exclude these observations. One explanation for this finding might be that individuals who are employed in a large industry in some state view themselves as likely to remain in that industry throughout their working life, or at least for a long time. Then, the expected wage growth in that industry would be the main driver of their schooling decision. In contrast, those that work in small industries may consider wage growth in other state sectors when making their school choice, if they anticipate that they are quite likely to change industries at some point in their career.

Table 5: Wage Growth and Labor Shares, Alternative Specifications

(a) Panel A: Short Time Span

	(1)	(2)	(3)	(4)
SIC Classification, 1970-1990				
	BEA	BEA	Census	Census
Wage Growth	-2.21***	-4.38**	-1.70**	-7.61***
	(0.48)	(1.34)	(0.56)	(1.76)
Instrument	—	Census	—	BEA
Implied value of a	0.20	0.10	0.26	0.057
Observations	1060	1060	1060	1060
SIC Classification, 1980-1997				
	(1)	(2)	(3)	(4)
	BEA	BEA	Census	Census
Wage Growth	-0.63*	-2.92***	-1.60***	-2.68*
	(0.29)	(0.86)	(0.46)	(1.25)
Instrument	—	Census	—	BEA
Implied value of a	0.65	0.14	0.26	0.15
Observations	1706	1706	1706	1706

(b) Panel B: Including All Observations

	(1)	(2)	(3)	(4)
SIC Classification, 1970-1997				
	BEA	BEA	Census	Census
Wage Growth	-1.42***	-5.18**	-1.21***	-6.93***
	(0.35)	(1.74)	(0.37)	(1.98)
Instrument	—	Census	—	BEA
Implied value of a	0.29	0.080	0.34	0.060
Observations	1818	1818	1818	1818
NAICS Classification, 2000-2012				
	(1)	(2)	(3)	(4)
	BEA	BEA	Census	Census
Wage Growth	-0.56**	-1.97	-0.12	-3.89
	(0.19)	(2.10)	(0.13)	(2.00)
Instrument	—	Census	—	BEA
Implied value of a	0.65	0.19	2.96	0.094
Observations	2079	2079	2079	2079

Robust standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Value Added, Labor Compensation and Employment for each state-industry-year from the BEA Regional Accounts; Wages and Years of Schooling calculated from the Census for 1970, 1980, 1990, 2000 and from the ACS for 2000-2012. Labor share is average across years of ratio of labor compensation to value added. In Panel A, any state-industry-year for which there were fewer than 100 observations in the Census/ACS was dropped. In Panels B, all observations were kept. Columns 1 and 2 use wage growth computed from BEA as independent variable, whereas columns 3 and 4 use wage growth from Census. Columns 1 and 3 estimated by OLS, columns 2 and 4 by IV. Each regression includes state fixed effects, industry fixed effects, and trends in years of state-industry schooling with coefficients that vary by industry.