Matching, Sorting, and the Distributional Effects of International Trade*

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Abstract

We study the distributional consequences of trade in a world with two industries and two heterogeneous factors of production. Productivity in each production unit reflects the ability of the manager and the abilities of the workers, with complementarity between the two. We begin by examining the forces that govern the sorting of worker and manager types to industries, and the matching of workers and managers within industries. We then consider how changes in relative output prices generated by changes in the trading environment affect sorting, matching, and the distributions of wages and salaries. We distinguish three mechanisms that govern the effects of trade on income distribution: trade increases demand for all types of the factor used intensively in the export sector; trade benefits those types of a factor that have a comparative advantage in the export sector; and trade induces a re-matching of workers and managers within both sectors, which benefits the more able types of the factor that achieves improved matches.

Keywords: heterogeneous labor, matching, sorting, productivity, wage distribution, international trade.

JEL Classification: F11, F16

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1 Introduction

How does international trade affect a country's income distribution? This age-old question has been the subject of a voluminous theoretical literature dating back at least to Ohlin (1933), Haberler (1936), Viner (1937), and of course Stolper and Samuelson (1941). But, until recently, research has focused almost exclusively on the relative earnings of a small number of aggregate (or homogeneous) factors of production. One can think of this research as addressing the determinants of "between-occupation" or "between-skill-group" distribution. There has also been a "between-industry" component to this line of inquiry, as reflected in the work by Jones (1971), Mayer (1974) and Mussa (1974) on models with "sector-specific" factors of production.

However, between-occupation and between-industry wage variation tell only part of the inequality story. Research using individual-level data finds that *within* occupation-and-industry wage variation or *within* skill-group-and-industry variation contributes at least as much as does between-group variation to the overall level of earnings inequality in the United States, Germany, Sweden, and Brazil.[1] Moreover, changes in within-group distributions account for a significant portion of the recent trends in wage inequality. The evidence of substantial within-group dispersion suggests the need for a richer theoretical framework that incorporates factor heterogeneity in order to help us understand more fully the effects of globalization on income distribution.

In this paper, we introduce factor heterogeneity into a multi-factor model of resource allocation in order to study the distributional effects of international trade in finer detail. As in the familiar Hecksher-Ohlin model, we assume that output is produced by the combined efforts of two factors (or occupations), which we call "workers" and "managers." These factors are employed in two competitive industries. But here, the inelastic supply of each factor comprises a continuum of different types. Firms form production units that bring together a manager of some type with a group of workers. There are diminishing marginal returns to adding a greater *number* of workers to a team with a given manager, as in the standard model. Meanwhile, the productivity of a unit depends on the type of the manager and the types of the various workers. Firms must choose not only how many workers and managers to hire, but also what types to employ. Industries may differ both in their factor intensities (as reflected in the diminishing returns to workers per manager) and in the functions that relate productivity to types.

Our model builds not only on Heckscher and Ohlin, but also on Lucas (1978). Lucas assumed that a firm's productivity depends on the ability of its manager (or "entrepreneur"), but that agents are equally productive qua workers. His analysis focused on the sizes of production units as a function of the types of their managers, but he could not address the composition of these units in terms of manager-worker combinations. Eeckhout and Kircher (2012) extended Lucas's approach to allow for heterogeneity of both factors. Like Lucas, they modeled only a single good-producing

[1] See, for example, Card et al. (2013) for Germany, Akerman et al. (2013) for Sweden, Helpman (2015) et al. for Brazil, Mouw and Kalleberg (2010) for the United States, and others.

industry and so they could not study the effects of relative output prices on factor rewards. But they contributed a key result that we borrow here, namely a condition for positive assortative matching of workers and managers. Like them, we posit the existence of complementarity between worker ability and manager ability in determining the productivity of production units in each industry. When these complementarities are strong enough, they imply that firms in an industry will combine better managers with better workers.[2]

In general equilibrium models with homogeneous factors of production, resource allocation can be fully described by the quantities of every input hired into each sector. With heterogeneous factors, the assignment of different types must also be considered. In such a setting, two important aspects of resource allocation that affect income distribution concern the *sorting* of heterogeneous managers and workers to industries and the *matching* of managers and workers in production units within each one. Sorting that is guided by comparative advantage generates endogenous sector specificity, which partly links workers' and managers' rewards to the prices of the goods they produce. Endogenous matching creates an additional channel—absent from previous, multi-sector trade models—through which changes in relative prices can affect the distribution of factor rewards. If the complementarities between manager and worker ability levels are strong enough to determine the composition of the production teams that form in general equilibrium, then changes in relative prices typically induce rematching of managers and workers in each industry. We will be interested in describing the rematching that results from an improvement in a country's terms of trade and in deriving the implications of such changes in the trade environment for within occupation-and-industry income inequality.[3] The role of this channel in shaping within occupation-and-industry income inequality is a novel contribution of our study.

We are not the first to study the implications of sorting and matching for income distribution. However, previous authors have considered the two forces only in isolation. For example, Ohnsorge and Trefler (2007) and Costinot and Vogel (2010) studied the links between trade and income distribution in an assignment model with heterogeneous workers and many sectors, but with a linear production function. In this setting, workers sort to sectors, but do not match with any other factors.[4] Yeaple (2005) and Sampson (2014) allow for matching between heterogeneous workers and firms that have access to different technologies. These authors too adopt a linear production function, but since their firms produce differentiated products in a world of monopolistic competition, the hiring of additional labor generates decreasing returns in terms of revenue, and so they can analyze the sizes of production units. Our model incorporates the forces found in these earlier papers, but also identifies a novel and important interplay between matching and sorting; changes in relative prices generate shifts in the extensive margins of factor sorting, which alter the

[2] See Garicano and Hubbard (2012) for direct evidence of positive assortative matching between managers and workers in the U.S. legal services industry and Fox (2009) for indirect evidence of such matching across a range of U.S. and Swedish industries.

[3] Krishna et al. (2014) report evidence of an endogenous reassignment of workers to firms following the Brazilian trade reform of 1991. They conclude based on this evidence that "[e]ndogenous matching of workers with firms is thus crucial in determining wage outcomes for workers in open economies" (p.252).

[4] See Ruffin (1988) for an antecedent of this approach.

composition of types in each industry and so force a rematching of factor types.

In Section 2, we lay out our general equilibrium model of competitive resource allocation with two heterogeneous factors of production. As already mentioned, the model extends the familiar Heckscher-Ohlin framework to allow for a continuum of types of both factors. In each of the two industries, the productivity of a production unit that includes a manager and some endogenously-chosen number of workers is an increasing, log-supermodular function of the "ability" of the manager and the ability levels of the associated workers. We take the relative output price as exogenous, but use it to represent the country's trading environment.

Section 3 derives the equilibrium conditions for profit-maximization, factor-market clearing, and wage and salary determination. We discuss the equilibrium sorting of workers and managers to industries, first for a case in which productivity is a constant-elasticity function of the ability of the manager and the abilities of the workers, and then for a case with stronger complementarities, namely when productivity is a *strictly* log-supermodular function of the types. In either case, sorting by each factor is guided by a cross-industry comparison of the ratio of the elasticity of productivity with respect to ability to the elasticity of output with respect to factor quantity. When complementarities are strong, the elasticities of productivity with respect to ability reflect the matches that take place, and so the sorting by each factor depends on the choices made by the other factor. After describing the sorting conditions, we define a threshold equilibrium as one in which sorting of each factor is fully described by a single cutoff such that all workers with ability above the cutoff are employed in one industry and the remainder are employed in the other, and similarly for managers. We provide sufficient conditions for the existence of a threshold equilibrium, first allowing for the possibility that high-ability workers and managers might not sort to the same sector, but then focusing on an equilibrium with positive assortative matching across industries.

After characterizing in Section 4 the matches that form between exogenously given sets of worker and manager types and discussing how exogenous expansion of these sets induces rematching that has clear implications for income inequality, we turn in Section 5 to the main task at hand. Here we ask, how do changes in the trading environment affect earnings inequality between occupations, between industries, and within occupation and industry. We begin again with the case of constant-elasticity (or Cobb-Douglas) productivity functions, which generates results that are instructive even if unrealistic. We show that in this environment, an increase in the relative price of a country's export good generates between-occupation redistribution that is reminiscent of the Stolper-Samuelson theorem and between-industry redistribution that is reminiscent of the Ricardo-Viner model with sector-specific factors, but it has no affect on within occupation-and-industry inequality. The complementarities between managers and workers are not strong enough in the Cobb-Douglas case to determine a unique pattern of matching, and the relative productivities of different factor types in an industry are independent of the matches that take place. With the stronger complementarities that are present when the productivity functions are *strictly* log supermodular, the matching pattern in general equilibrium is uniquely determined. Then endogenous rematching generates predictable changes in within occupation-and-industry income distributions.

In Section 5.2.1, we take on the case that probably is most empirically relevant, namely one in which the most able workers and the most able managers sort to the same industry. We show that if factor intensities are similar in the two industries, a change in relative prices must increase within-occupation-and-industry inequality for one factor and reduce it for the other. If, instead, factor intensities differ substantially across sectors, then a richer set of outcomes is possible. For example, an increase in the relative price of the worker-intensive good can raise within-industry inequality among workers in the labor-intensive industry while reducing within-industry inequality among those in the other industry.

Finally, in Section 5.2.2, we consider the distributional effects of price changes when the best workers and the best managers sort to different sectors. In this case, an increase in the relative price of the good produced by the low-ability workers attracts to the industry marginal workers who are more able and marginal managers who are less able than those who are employed there initially. This results in match downgrading for all workers initially in the expanding sector and for those who remain in the contacting sector, which in turn spells a narrowing of within-occupation-and-industry inequality. The outcome for managers is just the opposite.

As the results highlighted in the last two paragraphs illustrate, our framework yields interesting results concerning the impact of trade on earnings inequality. Section 6 offers some concluding remarks, including a discussion of some empirical implications of our theory.

2 The Economic Environment

We study an economy that produces and trades two goods. This is a "Heckscher-Ohlin economy"—with two factors of production that we call "managers" and "workers"—except that there are many "types" of each factor. The inelastic supplies of the heterogeneous workers are represented by a density function $\bar{L}\phi_L(q_L)$, where $\bar{L}$ is the measure of workers in the economy and $\phi_L(q_L)$ is a probability density function (pdf) over worker types, q_L. Similarly, the economy is endowed with a density $\bar{H}\phi_H(q_H)$ of managers of type q_H, where $\bar{H}$ is the measure of managers and $\phi_H(q_H)$ is the pdf for manager types. We take $\phi_L(q_L)$ and $\phi_H(q_H)$ both to be continuous and strictly positive on their respective bounded supports, $S_L = [q_{L\min}, q_{L\max}]$ and $S_H = [q_{H\min}, q_{H\max}]$.

We treat factor endowments as exogenous in order to connect our analysis with previous studies of trade and factor prices in the spirit of Jones (1965, 1971), Mayer (1974), and Mussa (1974). It might also be interesting to allow for occupational choice, as in Lucas (1978), or human capital accumulation, as in Findlay and Kierzkowski (1983). Of course, other interpretations of the two factors also are possible. For example, if the factors are "labor" and "capital," one presumably would want to allow a choice of investment in machines of different types, as in Acemoglu (1998).

Competitive firms can enter freely into either industry. We describe the technology in industry i in terms of the output that can be produced by a manager of some type q_H when combined with workers of various types. The manager has a fixed endowment of time that she allocates among the workers under her control. The productivity of each worker increases with the attention devoted by the manager, albeit with diminishing returns. In this setting, it generically is optimal for the firm to form production units that combine a given manager with an (endogenous) number of workers of a common type.[5] Therefore, to save on notation, we describe the technology in sector i in terms of the amount of potential output x_i that can be produced by a unit with one manager of type q_H and ℓ workers of *common* type q_L, namely

$$x_i = \psi_i (q_H, q_L) \ell^{\gamma_i}, \ \ 0 < \gamma_i < 1, \ \ \text{for } i = 1, 2. \tag{1}$$

Here, $\psi_i (q_H, q_L)$ reflects the productivity of the unit and γ_i is a parameter that captures the diminishing returns to the size of the unit that results from an increase in the manager's span of control. Since we allow γ_1 to differ from γ_2, firms in different industries might find it optimal to combine a manager with different numbers of workers. This gives rise to a possible difference in factor intensities, as in the traditional Heckscher-Ohlin theory.[6] The new element is the productivity term $\psi_i (q_H, q_L)$, which is a function of the factor types. We assume that there exists an ordering of each factor type such that any change in the index affects productivity in the same direction in both industries. Without further loss of generality, then, we can choose the order so that $\psi_i (q_H, q_L)$ is strictly increasing in each of its arguments for $i = 1, 2$. Under this labeling convention, we refer to q_H and q_L as the "ability" of the manager and of the associated workers, respectively.

Importantly, we posit the existence of a complementarity between the ability levels of the manager and the workers that are employed together in a production unit. More able workers are more productive than less able workers no matter who is their manager, but the more able workers are assumed to be relatively more productive compared to their less able counterparts when they are combined with a more able manager rather than a less able manager.[7] Formally, we assume throughout that $\psi_i (q_H, q_L)$ is strictly increasing and twice continuously differentiable and we adopt

Assumption 1 $\psi_i (q_H, q_L)$ is log supermodular for $i = 1, 2$.

Log supermodularity implies that $\psi_i (q''_H, q''_L) / \psi_i (q''_H, q'_L) \geq \psi_i (q'_H, q''_L) / \psi_i (q'_H, q'_L)$ for any $q''_H > q'_H$ and $q''_L > q'_L$. Notice that we allow the industries to differ in the strength of the complementarities between factors, which along with the differences in factor intensities, plays an important role in determining the sorting of types to the two industries.

Much of our analysis will be carried out with a slightly stronger version of our assumption about complementarities, namely

[5] The optimality of combining a given type of manager with workers of a common type arises in other contexts in which the manager has a span of control besides the particular description we offer here; see Eeckhout and Kircher (2012). They show that the key assumption for this result is that there is no teamwork or synergy between workers in the firm, who interact only insofar as they compete for the manager's time and attention.

[6] The assumption of a power function for labor—i.e., that the technologies are Cobb-Douglas in factor *quantities*—is made for expositional convenience; many of our results do not require this assumption, so long as there are no "factor intensity reversals."

[7] See, for example, Garicano and Hubbard (2012), who study assignment patterns in the U.S. legal services industry. They find that the more able partners (managers) team with the more able associates (workers) and argue that their data are best explained by the existence of complementarity between the managers' and workers' skill or ability.

Assumption 1′ $\psi_i(q_H, q_L)$ is *strictly* log supermodular for $i = 1, 2$.

In this case, the weak inequality described in the previous paragraph becomes a strong inequality.

We take all factor markets to be perfectly competitive and frictionless. That is, any firm can hire managers and workers of any type at salaries $r(q_H)$ and wages $w(q_L)$ that it takes as given. There is no imperfect information about individuals' abilities, no search costs of any sort, and no unemployment. Adding frictions to the formation of production units might be an interesting extension of our model.[8]

As in other models with perfect competition, the impact of the trading environment on local factor prices is conveyed via relative output prices. For example, the opening of trade from autarky generates an increase in the relative price of a country's export good. So does a subsequent improvement in its terms of trade. An import tariff raises the relative domestic price of a country's import good, except under the conditions of the so-called Metzler paradox (Metzler, 1949). The relative domestic prices in turn determine the equilibrium wage schedule $w(q_L)$ and the salary schedule $r(q_H)$. Accordingly, we can study the effects of changes in the trading environment on the earnings distribution by considering the comparative static changes in the wage and salary schedules that result from an arbitrary change in relative prices.

3 Sorting and Matching of Managers and Workers

In this section, we lay out the conditions for profit maximization and factor-market clearing, taking output prices as given. These conditions determine *inter alia* the sorting of the different types of workers and managers to the two industries, the matching of workers and managers in production units within each sector, and the equilibrium schedules of wages and salaries. We will characterize the patterns of sorting and matching that can arise in equilibrium and describe some properties of the earnings schedules. Discussion of the responses of wages and salaries to changes in relative prices is deferred until Section 5 below.

Consider a firm in sector i that employs a manager of type q_H. The firm must choose the type of workers q_L and the number of workers ℓ to combine with this manager, given the output price and the wage schedule. The firm's profit, gross of salary payment to the manager, is given by

$$\pi_i(\ell, q_L; q_H) = p_i \psi_i(q_H, q_L)\, \ell^{\gamma_i} - w(q_L)\, \ell \ ,$$

where p_i is the price of good i and $w(q_L)$ is the competitive wage paid to workers with ability q_L. The first-order condition with respect to ℓ yields the conditional labor demand,

$$\ell(q_L; q_H) = \left[\frac{\gamma_i p_i \psi_i(q_H, q_L)}{w(q_L)}\right]^{\frac{1}{1-\gamma_i}} \ , \tag{2}$$

[8] In our working paper, Grossman et al. (2013), we allow for directed search by workers in an environment with search frictions and unemployment. In that setting, many results have a similar flavor to those derived here, but trade affects the distribution of employment across workers of different ability, as well as the distribution of wages.

which is the number of workers the firm would hire if it were to employ a manager with ability q_H, choose workers of type q_L, and face the wage schedule $w(q_L)$.

Next, we substitute $\ell(q_L; q_H)$ into the expression for $\pi_i(\ell, q_L; q_H)$ and compute the first-order condition with respect to q_L. This yields the firm's optimal choice of worker type, given the type of its manager and taking into account the corresponding size of the optimal production unit. The first-order condition can be written as

$$\frac{\varepsilon_{iL}(q_H, q_L)}{\gamma_i} = \varepsilon_w(q_L) \ , \ \ i = 1, 2, \tag{3}$$

where $\varepsilon_{iL}(q_H, q_L) \equiv q_L [\partial\psi_i(q_H, q_L)/\partial q_L] / \psi_i(q_H, q_L)$ is the elasticity of productivity in sector i with respect to worker ability and $\varepsilon_w(q_L) \equiv q_L [\partial w(q_L)/\partial q_L] / w(q_L)$ is the elasticity of the wage schedule. Evidently, the firm sets the ratio of the elasticity of output with respect to worker ability to the elasticity of output with respect to worker quantity equal to the elasticity of the wage schedule.[9] The optimal choice of ability reflects the fact that the firm has two ways to expand output, either by hiring *better* workers or by hiring *more* workers. The rate at which wages rise with ability dictates the appropriate trade-off between the two.

Under Assumption $1'$, strict log supermodularity of the productivity function, there is—in equilibrium—a unique value q_L that solves (3) for every q_H (see below). In this case we denote by $q_L = m_i(q_H)$ the solution to (3) in sector i. For the economy as a whole, the matching function $m(q_H)$ consists of $m_1(q_H)$ for $q_H \in Q_{H1}$ and $m_2(q_H)$ for $q_H \in Q_{H2}$, where Q_{Hi} is the set of managers that is hired in equilibrium in sector i. Alternatively, when the productivity function is log supermodular but not strictly log supermodular, a firm may be indifferent between some type of workers given the ability of its manager, as in the Cobb-Douglas case discussed below.

Who are the managers that actually are hired into sector i in equilibrium? Were a firm to hire a manager with ability q_H and pay her the market salary, $r(q_H)$, its net profit would be $\Pi_i(q_H) = \tilde{\pi}_i(q_H) - r(q_H)$, where $\tilde{\pi}_i(q_H) \equiv \max_{\{\ell, q_L\}} \pi_i(\ell, q_L; q_H)$ is achieved by choosing ℓ and q_L according to (2) and (3). In a competitive equilibrium, every firm operating in sector i breaks even, which implies that $\Pi_i(q_H) = 0$ for all $q_H \in Q_{Hi}$. Moreover, the firms in sector j should not be able to make strictly positive profits by hiring the managers that sort into sector i, or else they would hire these managers instead. This implies $\Pi_j(q_H) \leq 0$ for all $q_H \in Q_{Hi}$, $j \neq i$. We will return to these zero-profit and optimality conditions below.

[9] This condition is analogous to the ones in Costinot and Vogel (2010) and Sampson (2014), except that those papers have $\gamma_i = 1$, because workers are the only factor of production and output is linear in labor quantity. A second, heterogeneous factor of production—such as we have introduced here—is necessary to generate re-matching within sectors, which in turn is needed to explain changes in within-occupation-and-industry wage distribution. Eeckhout and Kircher (2012) show that, given equilibrium wage functions, this condition is necessary and sufficient for the choice of q_L.

3.1 Matching and Sorting with Cobb-Douglas Productivity

It is instructive to begin first with a special case in which productivity is a constant elasticity function of the ability of the manager and that of the worker. For this case, we can write

$$\psi_i\left(q_H, q_L\right) = q_H^{\beta_i} q_L^{\alpha_i} \text{ for } i = 1, 2;\ \alpha_i, \beta_i > 0. \tag{4}$$

For obvious reasons, we shall refer to this as the case of "Cobb-Douglas productivity."

The productivity function in (4) has several special properties that are important in this context. First, the function is log supermodular, but *not strictly* so; it satisfies Assumption 1 but not Assumption 1′. Second, the elasticity of output with respect to worker ability, $\varepsilon_{iL}\left(q_H, q_L\right)$, is a constant α_i and independent of both q_H and q_L. We can define analogously the elasticity of productivity with respect to manager ability, $\varepsilon_{iH}\left(q_H, q_L\right) \equiv q_H\left[\partial\psi_i\left(q_H, q_L\right)/\partial q_H\right]/\psi_i\left(q_H, q_L\right)$. This too is a constant, equal to β_i, in the case of Cobb-Douglas productivity.

With $\varepsilon_{iL} = \alpha_i$, the first-order condition (3) for a firm's interior choice of worker type in sector i requires that $\varepsilon_w\left(q_L\right) = \alpha_i/\gamma_i$. However, with an arbitrary wage schedule, this condition will only be satisfied by a finite number (possibly only one) of values of q_L. Facing such an arbitrary schedule, all firms active in an industry would hire one of these finite number of types. Such choices would not be consistent with full employment of the continuum of worker types that sorts to the industry. We conclude that, as a requirement for full employment, the wage schedule must have a constant elasticity α_1/γ_1 for the range of workers hired into sector 1 and it must have a constant elasticity α_2/γ_2 for the range of workers hired into sector 2. In other words,

$$w\left(q_L\right) = w_i q_L^{\alpha_i/\gamma_i} \text{ for all } q_L \in Q_{Li},\ i = 1, 2, \tag{5}$$

for some constants, w_1 and w_2, where Q_{Li} is the set of workers hired in sector i. The wage schedule (5) makes any firm operating in industry i indifferent between the potential employees in Q_{Li} *no matter what is the type of its manager.* It follows that *matching* of workers and managers is not well determined for the case of Cobb-Douglas productivity; any matches between workers in Q_{Li} and managers in Q_{Hi} can be consistent with equilibrium, provided that the numbers in all production units are given by (2) and that the factor-market clearing conditions are satisfied.

Which workers are employed in each industry? Consider Figure 1, which depicts the qualitative features of the equilibrium wage schedule when $s_L \equiv \alpha_1/\gamma_1 - \alpha_2/\gamma_2 > 0$. Once the "wage anchors", w_1 and w_2, have been determined in the general equilibrium, the solid curve in the figure represents the wage schedule that satisfies (5). The broken curves show what wages for different types of workers would have to be in order to make the firms in an industry indifferent between hiring these types and the types that are actually employed in equilibrium. The fact that $\alpha_1/\gamma_1 > \alpha_2/\gamma_2$ implies that the solid curve lies above the broken curve for industry 2 to the right of the point of intersection, q_L^*, and that the solid curve lies above the broken curve for industry 1 to the left of the intersection point. In equilibrium, the firms in industry 1 are willing to hire any workers with ability above q_L^*, but not those with ability below this level. Meanwhile, firms in

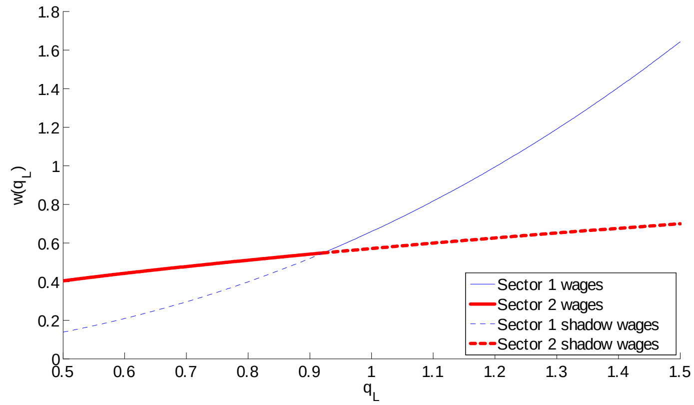

Figure 1: Wage Schedule

industry 2 are willing to hire any workers with ability below q_L^*, but not those with ability above this level. Evidently, those with ability above q_L^* sort to industry 1 and those with ability below q_L^* sort to industry 2, and the marginal workers with ability equal to q_L^* are paid the same wage in either sector. Sorting of workers is guided by s_L, the cross-industry comparison of the ratio of the elasticity of productivity with respect to ability to the elasticity of output with respect to quantity.

What about the managers? In the online appendix we show that the zero-profit condition, $\Pi_i(q_H) = 0$ for all $q_H \in Q_{Hi}$, together with (2), (3) and (5), imply that

$$r(q_H) = r_i q_H^{\beta_i/(1-\gamma_1)} \text{ for all } q_H \in Q_{Hi},\ i = 1, 2, \tag{6}$$

where r_i is a constant analogous to w_i.[10] Then the condition that $\Pi_j(q_H) \leq 0$ for all $q_H \in Q_{Hi}$, $j \neq i$, (i.e., that firms do not want to hire the managers employed in the opposite sector) dictates the sorting pattern for managers: If $s_H \equiv \beta_1/(1-\gamma_1) - \beta_2/(1-\gamma_2) > 0$, then managers with ability above some cutoff q_H^* sort to sector 1 and those with ability below q_H^* sort to sector 2; otherwise, the sorting pattern is just the opposite. Notice that the sorting pattern for managers can be understood similarly to that for workers. Constant returns to scale implies that the elasticity of output with respect to the number of managers in sector i is $1 - \gamma_i$. So, manager sorting also is guided by a cross-industry comparison of the ratio of the elasticity of productivity with respect to ability to the elasticity of output with respect to number.

The case of Cobb-Douglas productivity generates what we will call a *threshold equilibrium*; the equilibrium sorting pattern is characterized by a pair of boundary points, q_L^* and q_H^*, such that all workers with ability above q_L^* sort to some sector while those with ability below q_L^* sort to the other, and similarly all managers with ability above q_H^* sort to some sector while those with ability below q_H^* sort to the other.[11] We note for future reference that there are two possible types

[10] The constants, w_1 and w_2, are determined along with q_L^* by a pair of labor-market clearing conditions for the two sectors (which are provided in the appendix) and the requirement that the wage function is continuous at q_L^*; i.e., $w_1(q_L^*)^{\alpha_1/\gamma_1} = w_2(q_L^*)^{\alpha_2/\gamma_2}$. Given w_1 and w_2, the salary anchors r_1 and r_2 are readily calculated.

[11] For some prices, there may be complete specialization in one sector or the other, in which case $q_L^* = q_{L\min}$ or $q_L^* = q_{L\max}$ and $q_H^* = q_{H\min}$ or $q_H^* = q_{H\max}$. In such cases, marginal changes in prices have no effect on the

of threshold equilibrium that can emerge. If s_L and s_H share the same sign, then the most able workers and the most able managers sort to the *same* sector. We refer to this below as an HH/LL equilibrium, to convey that the "high types" of both factors sort together, as do the "low types." Alternatively, if s_L and s_H are opposite in sign, then the more able managers sort to the same sector as the less able workers. We will refer to such an outcome as an HL/LH equilibrium. In the online appendix we examine compensation patterns in Brazil and find that average wages of workers and average salaries of managers are highly correlated across industries, suggesting that the HH/LL equilibrium may be the more empirically relevant of the two.

3.2 Matching and Sorting with Strictly Log Supermodular Productivity

Armed with an understanding of the knife-edge case of Cobb-Douglas productivity, we turn to a setting with stronger complementarities between manager and worker abilities that arises under Assumption $1'$, which is our central case of interest.

When the productivity function $\psi_i(q_H, q_L)$ is strictly log supermodular, the arguments presented in Eeckhout and Kircher (2012) imply that $m_i(q_H)$ is an increasing function, so that there is positive assortative matching (PAM) in each industry. That is, among the workers and managers that sort to a given industry, the better workers are teamed with the better managers. This is true, because the productivity of a group of more able workers relative to that of a group of less able workers is higher when the groups are combined with a more able manager compared to when they are combined with a less able manager. As we shall see, the equilibrium may or may not exhibit PAM for the economy as a whole. We have (see the online appendix for proof):

Proposition 1 *Suppose that Assumption $1'$ holds. Then: (i) $m_i(q_H)$ is a strictly increasing function for $q_H \in Q_{Hi}$, $i = 1, 2$; (ii) the graph*

$$M_i = [\{q_H, q_L\} \mid q_L = m_i(q_H) \;\; \textit{for all } q_H \in Q_{Hi}]$$

consists of a union of connected and closed sets M_i^n, $n \in N_i$ (i.e., $M_i = \cup_{n \in N_i} M_i^n$), such that $m_i(q_H)$ is continuous in each set M_i^n.

The second part of the proposition implies that the equilibrium allocation sets Q_{Li} and Q_{Hi} are unions of closed intervals. A threshold equilibrium is the special case in which each Q_{Fi} for $F = H, L$ and $i = 1, 2$, is a single interval. The equilibrium matching function for the economy, which we have denoted by $m(q_H)$, comprises $m_1(q_H)$ for $q_H \in Q_{H1}$ and $m_2(q_H)$ for $q_H \in Q_{H2}$.

We prove in the online appendix that the equilibrium wage schedule is differentiable almost everywhere (i.e., except possibly for types that are indifferent between the industries). Then, using the notation for the matching function, we can rewrite (3) as

$$\frac{\varepsilon_{iL}[q_H, m(q_H)]}{\gamma_i} = \varepsilon_w[m(q_H)] \quad \text{for all } \{q_H, m(q_H)\} \in M_i^{n,int},\, n \in N_i,\, i = 1, 2, \tag{7}$$

equilibrium, and so they are uninteresting for our purposes. We do not consider them any further.

where $M_i^{n,int}$ is the set M_i^n excluding its endpoints.[12] This way of expressing a firm's optimal choice of workers emphasizes that the elasticity of productivity with respect to worker ability depends upon the matches between workers and managers that actually form in equilibrium. These matches in turn reflect the sorting patterns of workers and managers to industries.

Using (2) and (3), the zero-profit condition $\Pi_i(q_H) = 0$ for all $q_H \in Q_{Hi}$ can be written now as

$$r(q_H) = \gamma_i^{\frac{\gamma_i}{1-\gamma_i}} (1-\gamma_i) p_i^{\frac{1}{1-\gamma_i}} \psi_i [q_H, m(q_H)]^{\frac{1}{1-\gamma_i}} w[m(q_H)]^{-\frac{\gamma_i}{1-\gamma_i}} \quad \text{for all } q_H \in Q_{Hi},\ i = 1,2. \qquad (8)$$

This equation and (7) imply that

$$\frac{\varepsilon_{iH}[q_H, m(q_H)]}{1-\gamma_i} = \varepsilon_r(q_H) \text{ for all } \{q_H, m(q_H)\} \in M_i^{n,int},\ i = 1,2, \qquad (9)$$

where $\varepsilon_r(q_H) \equiv q_H [\partial r(q_H)/\partial q_H]/r(q_H)$. Notice the similarity with (7); profit maximization and zero profits ensure that the ratio of the elasticity of productivity with respect to manager ability to the elasticity of output with respect to manager quantity is equal, in equilibrium, to the elasticity of the salary schedule. But, as with workers, the elasticity of productivity with respect to (manager) ability depends on the matches that occur in equilibrium.

Equations (7) and (9) comprise a pair of differential equations that relate the matching function, the wage schedule and the salary schedule. A third such equation can be derived from the requirements for factor-market clearing. To this end, consider any connected set of managers $[q_{Ha}, q_H]$ that sorts to industry i and the set of workers $q_L \in [m(q_{Ha}), m(q_H)]$ with whom these managers are matched in equilibrium. A profit-maximizing firm in sector i that employs a manager with ability q_H and workers of ability q_L hires $\gamma_i r(q_H)/(1-\gamma_i) w(q_L)$ workers per manager. Since the matching function is everywhere increasing, it follows that

$$\bar{H} \int_{q_{Ha}}^{q_H} \frac{\gamma_i r(q)}{(1-\gamma_i) w[m(q)]} \phi_H(q)\, dq = \bar{L} \int_{m(q_{Ha})}^{m(q_H)} \phi_L[m(q)]\, dq\ ,$$

where the left-hand side is the measure of workers hired collectively by all firms operating in sector i that employ managers with ability between q_{Ha} and q_H and the right-hand side is the measure of workers available to be teamed with those managers. Since the left-hand side is differentiable in q_H as long as q_H is not indifferent between industries, this equation implies that the matching function $m(q_H)$ also is differentiable at such points. That being the case, we can differentiate the labor-market clearing condition with respect to q_H to derive a differential equation for the matching function, namely

$$\bar{H} \frac{\gamma_i r(q_H)}{(1-\gamma_i) w[m(q_H)]} \phi_H(q_H) = \bar{L} \phi_L[m(q_H)]\, m'(q_H) \text{ for all } \{q_H, m(q_H)\} \in M_i^{n,int}\ . \qquad (10)$$

This condition states that the workers demanded by a (small) set of managers with ability in a small

[12] Due to the continuity of $m_i(q_H)$ in M_i^n, the set M_i^n is a one-dimensional submanifold of the two-dimensional plane and it has two end points.

range around q_H equals the density of workers in the economy that match with these managers.

At last, we are in a position to characterize an equilibrium allocation for the economy, given prices. Such an allocation is fully described by a quadruple of sets, Q_{iF} for $F = H, L$ and $i = 1, 2$, a continuous wage schedule $w(q_L)$, a continuous salary schedule $r(q_H)$ and a piecewise continuous matching function $m(q_H)$ that satisfy the differential equations (7), (9) and (10) and that yield zero profits per (8) for any active sector (and non-positive profits for any inactive sector).

The sorting patterns can be quite complex. We wish to identify conditions that ensure a simple pattern—namely, a *threshold equilibrium*—which is the pattern that emerges in an economy with Cobb-Douglas productivity functions. To motivate our next proposition, recall Figure 1. The figure shows the wage function and shadow wage functions that result with Cobb-Douglas productivity. The firms in industry 1 can outbid those in industry 2 for workers with $q_L > q_L^*$, because the ratio of the elasticity of productivity with respect to worker ability to the elasticity of output with respect to number of workers is higher there. Similarly, industry 2 is willing to pay the most to workers with $q_L < q_L^*$, because $\varepsilon_{2L}/\gamma_2 < \varepsilon_{1L}/\gamma_1$. The wage and shadow-wage functions reflect these (constant) elasticity ratios at each point in the ability distribution.

The wage and shadow-wage functions also reflect these elasticity ratios when the productivity functions are strictly log supermodular; see (3). A potential complication arises, however, because a worker's elasticity ratio depends upon the ability of the manager with whom he is matched, which in turn depends upon the sorting incentives that confront the managers. But suppose that the elasticity ratio in industry 1 is higher than that in industry 2, even if in the former case the workers of some ability level are teamed with the economy's least able manager and in the latter case they are teamed with the economy's most able manager. Considering the complementarity between worker and manager ability levels, the elasticity ratio in industry 1 for a given worker then must be higher than that in industry 2 for the matches that actually take place, no matter what they happen to be. These circumstances ensure the existence of a cutoff ability level for workers q_L^* such that firms in industry 1 are willing to pay more than industry 2 for workers with $q_L > q_L^*$, and the opposite is true for workers with $q_L < q_L^*$. In the online appendix, we formally prove

Proposition 2 *Suppose that Assumption 1′ holds and that*

$$\frac{\varepsilon_{iL}(q_{H\min}, q_L)}{\gamma_i} > \frac{\varepsilon_{jL}(q_{H\max}, q_L)}{\gamma_j}$$

for all $q_L \in S_L$, $\quad i \neq j$, $\quad i, j = 1, 2$. *Then, in any competitive equilibrium with employment of workers in both sectors, the more able workers with* $q_L > q_L^*$ *are employed in sector* i *and the less able workers with* $q_L < q_L^*$ *are employed in sector* j*, for some* $q_L^* \in S_L$.

We have seen for the case of Cobb-Douglas productivity that an analogous condition that compares elasticity ratios across sectors guides the sorting of managers. Specifically, whichever industry has the higher ratio of the elasticity of productivity with respect to manager ability to the elasticity of output with respect to manager quantity attracts the more able managers. Again, with a general, strictly log supermodular productivity function the sorting incentives for the other factor (workers, in this case) can complicate this comparison of elasticity ratios. But, in analogy to Proposition 2, they will not do so if the forces attracting the more able managers to sort to a sector would remain active even if the match there were consummated with the economy's least able workers and the match in the other sector were consummated with the economy's most able workers. We record

Proposition 3 *Suppose that Assumption 1′ holds and that*

$$\frac{\varepsilon_{iH}\left(q_H, q_{L\min}\right)}{1-\gamma_i} > \frac{\varepsilon_{jH}\left(q_H, q_{L\max}\right)}{1-\gamma_j}$$

for all $q_H \in S_H, \quad i \neq j, \quad i,j = 1,2$*. Then, in any competitive equilibrium with employment of managers in both sectors, the more able managers with* $q_H > q_H^*$ *are employed in sector* i *and the less able managers with* $q_H < q_H^*$ *are employed in sector* j*, for some* $q_H^* \in S_H$*.*

Clearly, if the inequality in Proposition 2 holds for some i and j and the inequality in Proposition 3 also holds for some i' and j', then the outcome is a threshold equilibrium. As with the case of Cobb-Douglas productivity, such an equilibrium can take one of two forms. If $i = i'$ and $j = j'$, then the more able workers sort to the same sector as the more able managers, which characterizes an HH/LL equilibrium. Alternatively, if $i = j'$ and $j = i'$, then the more able workers sort to the opposite sector from the more able managers, which defines an HL/LH equilibrium.

It is possible to provide a weaker sufficient condition for the existence of a threshold equilibrium of the HH/LL variety. If the most able managers sort to some industry i, this can only strengthen the incentives for the most able workers to sort there as well, considering the complementarities between factor types. Similarly, if the most able workers sort to industry i, this will strengthen the incentives for the most able managers to do so as well. This reasoning motivates the following proposition (proven in the online appendix).

Proposition 4 *Suppose that Assumption 1′ holds. If*

$$\frac{\varepsilon_{iL}\left(q_H, q_L\right)}{\gamma_i} > \frac{\varepsilon_{jL}\left(q_H, q_L\right)}{\gamma_j} \quad \text{for all } q_H \in S_H, \quad q_L \in S_L,$$

and

$$\frac{\varepsilon_{iH}\left(q_H, q_L\right)}{1-\gamma_i} > \frac{\varepsilon_{jH}\left(q_H, q_L\right)}{1-\gamma_j} \quad \text{for all } q_H \in S_H, \quad q_L \in S_L,$$

for $i \neq j, \quad i,j = 1,2$*, then in any competitive equilibrium with employment of managers and workers in both sectors, the more able managers with* $q_H > q_H^*$ *and the more able workers with* $q_L > q_L^*$ *are employed in sector* i*, while the less able managers with* $q_H < q_H^*$ *and the less able workers with* $q_L < q_L^*$ *are employed in sector* j*, for some* $q_H^* \in S_H$ *and some* $q_L^* \in S_L$*.*

The difference in the antecedents in Propositions 2 and 3 on the one hand and in Proposition 4 on the other is that, in the former, we compare the elasticity ratio for each factor when it is combined with the least able type of the other factor in one sector versus the most able type in the other sector, whereas in the latter we compare the elasticity ratios for common partners in the two sectors. The difference arises, because an HH/LL equilibrium has PAM within *and across* industries, whereas an HL/LH equilibrium has PAM only within industries. In an HL/LH equilibrium, an able manager in sector i might be tempted to move to sector j despite a generally greater responsiveness of productivity to ability in i, because the better workers have incentive to sort to j, and with log supermodularity of $\psi_j(\cdot)$, the able manager stands to gain most from this superior match. In contrast, in an HH/LL equilibrium, the able manager in sector i would find less able workers to match with were she to move to sector j, so the temptation to switch sectors in order to upgrade partners is not present.

We have derived sufficient conditions for the existence of a threshold equilibrium in which the allocation set for each factor and industry comprises a single, connected interval. These conditions are not necessary, however, because the matches available to types that are quite different from the marginal type might not overturn their comparative advantage in one sector or the other. Nonetheless, not all parameter configurations give rise to equilibria with such a simple sorting pattern. An example of a more complex sorting pattern and the parameters that underlie it is provided in Lim (2015). In that example, the most able and least able workers sort to sector 1 while an intermediate interval of worker types sort to sector 2. The firms in sector 1 hire the economy's most able managers whereas those in sector 2 hire those with ability below some threshold level. The matching function $m(q_H)$ is piecewise continuous and exhibits PAM within each industry. But the example illustrates a "sorting reversal" for workers that arises because the elasticity ratio for labor is higher in sector 1 when worker ability is low or high, but higher in sector 2 for a middle range of abilities. Of course, other sorting patterns besides the one depicted in this example also are possible.

4 Matching and Earnings within Groups

Before we turn to the effects of changes in the trade environment on the distributions of wages and salaries, it will prove useful to examine in some detail the implications of our equilibrium conditions for the particular matches that form among a group of workers and a group of managers that happen to be combined in equilibrium, and for the distributions of wages and salaries in the two groups. To this end, consider a group of managers comprising all those with ability in the interval $Q_H = [q_{Ha}, q_{Hb}]$ and a group of workers comprising all those with ability in the interval $Q_L = [q_{La}, q_{Lb}]$. Suppose these two groups happen to sort to some industry i in a competitive equilibrium and that, collectively, the managers and workers in these two groups happen to be matched together, exhaustively. We are interested in the properties of the solution to the system of differential equations comprising (7), (9) and (10) along with the zero-profit condition, (8), and the two boundary conditions, $q_{La} = m_i(q_{Ha})$ and $q_{Lb} = m_i(q_{Hb})$. Throughout this section, we assume the existence of strong complementarities between worker and manager types; i.e., we take

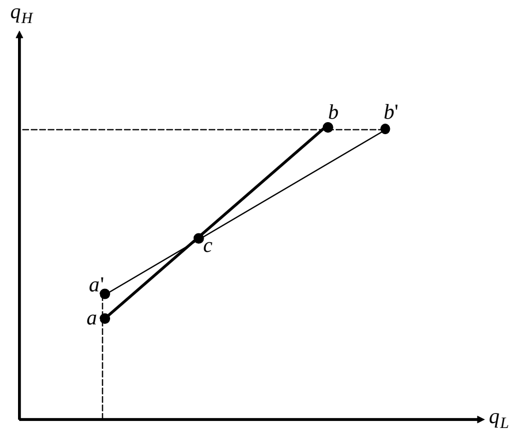

Figure 2: Shift in the matching function when q_L rises at the top and q_H rises at the bottom.

productivity to be a strictly log supermodular function of the two ability levels.

In the appendix, we prove that the solution has several notable properties. First, if the price p_i were to rise without any change in the composition of the two groups, then the matches between particular members of the groups would remain unchanged and all wages and salaries would rise by the same proportion as the output price. Second, if the number of managers in Q_H were to increase by some proportion h relative to the number of workers in Q_L, without any change in the relative densities of the different types, then the wages of all workers in Q_L would rise by the proportion $(1-\gamma_i)\,h$, while the salaries of all types in Q_H would fall by the proportion $\gamma_i h$. Again, there would be no change in the matching between manager and worker types.

Now suppose that one or both of the groups were to expand or contract on the extensive margin without any change in the composition of types among the original members of the two groups. That is, suppose that Q_H were to change to $Q'_H = [q'_{Ha}, q'_{Hb}]$ and Q_L were to change to $Q'_L = [q'_{La}, q'_{Lb}]$, but with no change in $\bar{L}\phi_L(q_L)$ or $\bar{H}\phi_H(q_H)$. We find (see Lemma 2 in the appendix) that the matching functions that apply before and after the change can intersect at most once. Moreover (see Lemma 6), if such an intersection exists, the situation with the steeper matching function at the point of intersection also has lower wages and higher salaries for all ability levels of workers and managers that are common to the two settings. This reflects the associated changes in the sizes of the production units; a steeper matching function implies that each manager is teamed with a larger group of workers, which enhances the marginal product of the manager and reduces the marginal product of the workers at any given ability level of either factor.

These points can be seen more clearly with the aid of Figure 2, which exhibits two (inverse) matching functions: one by the thick curve between points a and b, the other by the thin curve between points a' and b'. The difference in the two matching functions reflects a difference in boundary points; in the figure, $q'_{Ha} > q_{Ha}$ and $q'_{Lb} > q_{Lb}$. Due to PAM, both curves slope upwards. Although for general boundary changes the two curves need not intersect (one can be everywhere above the other), continuity of the matching functions implies that for the situation depicted in the figure the two curves must intersect at least once. However, by Lemma 2 in the appendix, the two curves can have at most one point in common, so there can be no points of intersection besides c. Since the thin matching function is steeper at c (the inverse matching function is flatter), Lemma 6 implies that managerial salaries are higher for managers with $q_H \in [q'_{Ha}, q'_{Hb}]$ while wages fall for all workers with $q_L \in [q_{La}, q_{Lb}]$.

A special case arises when only one boundary point changes. If, for example, q_{Lb} increases to $q_{Lb'}$ while Q_H does not change, then the point (q_{La}, q_{Ha}) is common to the two matching curves. The slope of the thin matching function must be greater at the single point of intersection than the that of the thick matching function. Therefore salaries rise for all mangers in Q_H and wages fall for all workers in Q_L.

The adjustment in matching that is illustrated in Figure 2 also has implications for within-group inequality. Consider the wage distribution among workers in Q_L. The differential equation (7) implies that

$$\ln w_i(q_{Lz'}) - \ln w_i(q_{Lz}) = \int_{q_{Lz}}^{q_{Lz'}} \frac{\psi_{iL}[\mu_i(x), x]}{\gamma_i \psi_i[\mu_i(x), x]} dx, \quad \text{for all } q_{Lz}, q_{Lz'} \in Q_L \ , \tag{11}$$

where $\mu_i(\cdot)$ is the inverse of $m_i(\cdot)$. If follows that, if all workers with abilities between some q_{Lz} and $q_{Lz'}$ are teamed with less able managers than before, the wage of type $q_{Lz'}$ declines relative to that of type q_{Lz}. The downgrading of managers is detrimental to both of these workers, but the complementarity between factor types means that it is especially so to the more able of the pair. Specifically, strict log supermodularity of $\psi_i(q_H, q_L)$ implies that $\psi_{iL}(q_H, q_L)/\psi_i(q_H, q_L)$ is a strictly increasing function of q_H. It follows that a rematching of a group of workers with less able managers, as depicted in Figure 2 for workers with ability to the right of point c, generates a narrowing of wage inequality within this group. And a rematching of a group of workers with more able managers, as depicted in Figure 2 for workers with ability to the left of point c, generates a widening of wage inequality within this group.[13] By a similar argument (and using the differential equation (9) for salaries), the rematching depicted in Figure 2 generates a spread in the salary distribution for managers in Q'_H with abilities above point c and a narrowing in the salary distribution for managers in Q'_H with abilities below point c. We therefore have:

Proposition 5 *If Assumption 1′ holds, then whenever matches improve for a group of workers employed in some sector, they deteriorate for the managers with whom they were paired, and vice versa. As a result, whenever matches either improve or deteriorate for* all *workers in a sector, within-occupation-and-industry inequality among workers and managers shift in opposite directions.*

This is a testable implication of our model. It finds some (weak) support in the Brazilian data presented in the online appendix, where we show that changes in inequality among Brazilian workers and managers are negatively correlated across industries, albeit insignificantly so.

[13] Costinot and Vogel (2010) and Sampson (2014) find similar results for wage inequality when workers downgrade their matches with firms that differ in technological sophistication.

5 The Effects of Trade on Earnings Inequality

We come finally to the main concern of our analysis: How does trade affect the distribution of earnings within and between occupations and industries? We study the effects of trade by examining comparative statics with respect to output prices. In a world of competitive industries, an opening of trade induces an increase in the relative price of a country's export good. An expansion of trade opportunities does likewise. So too does a reduction in a country's import barriers, except under conditions for the Metzler paradox. So, we can study the effects of trade without introducing details of other countries simply by investigating how output prices feed through to factor markets.[14]

To preview what lies ahead, we will identify and describe three forces that are at work in this setting. Two are familiar and one is new. First, whenever $\gamma_1 \neq \gamma_2$, our model features factor intensity differences across industries. As is well known from the Stolper-Samuelson theorem, this consideration introduces an effect of trade on *between-occupation distribution*; an increase in the relative price of a good tends to increase demand for all types of the factor used intensively in producing that good, while reducing the demand for all types of the other factor. Second, our model incorporates factor heterogeneity that, whenever $\psi_1(\cdot) \neq \psi_2(\cdot)$, generates comparative advantage for certain types of each factor in one industry or the other. This feature introduces an effect of trade on *between-industry distribution*; an increase in the relative price of a good tends to increase the rewards for all types of both occupations that enjoy a comparative advantage in producing that good, and to reduce the returns to types that hold a comparative disadvantage in doing so. This effect is familiar from the Ricardo-Viner model with sector specificity. Finally, whenever $\psi_i(q_H, q_L)$ exhibits strict log supermodularity, our model determines the matches that form between managers and workers in each industry. This feature introduces an effect of trade on *within-group (occupation-and-industry) distribution.*

As shown in Akerman et al. (2013), the within occupation-and-industry variation in wages accounts for 59% of the variance of log wages in Sweden in 2001 and 66% of the change in this measure of inequality between 2001 and 2007 (see their Table 2). Moreover, it also explains 83% of the residual wage inequality in 2001 and 79% of the change in residual wage inequality between 2001 and 2007 (see their Table 3), with residual wage inequality accounting for 70% of wage inequality in Sweden in 2001 and 87% of the change in wage inequality between 2001 an 2007. A comparably large role of wage variation within occupation-and-industry is reported in Helpman et al. (2015) for Brazil. We identify a channel through which trade impacts within occupation-and-industry wage inequality that may contribute to explaining these features of the data.

[14] In our working paper, Grossman et al. (2013), we link the pattern of trade to cross-country differences in quantities and distributions of the two factors. Thus, we treat the price change that results from an opening of trade as an endogenous reflection of factor-endowment differences. Here, we take the price changes as exogenous in order to focus attention on the distributional implications of changes in the trade environment.

5.1 Wages and Salaries with Cobb-Douglas Productivity

As before, it is instructive to begin with the case in which productivity in each sector is only weakly log supermodular. We revisit an economy with Cobb-Douglas productivity as described in (4).

Recall from Section 3.1 that, with Cobb-Douglas productivity in each sector, the sorting of factors to sectors is guided by a cross-industry comparison of the ratio of the elasticity of productivity with respect to a factor's ability to the elasticity of output with respect to factor quantity. That is, when $\alpha_i/\gamma_i > \alpha_j/\gamma_j$, higher ability confers a comparative advantage among workers for employment in industry i, while when $\beta_{i'}/(1-\gamma_{i'}) > \beta_{j'}/\left(1-\gamma_{j'}\right)$, higher ability confers a comparative advantage among managers for employment in industry i'.

It is clear from (5) and (6) that trade has no effect on within-group inequality in these circumstances. The relative wage of any two workers with ability levels q_{La} and q_{Lb} that are both employed in the same sector i before and after any change in the trading environment is fully determined by their relative ability levels; i.e., $w(q_{La})/w(q_{Lb}) = (q_{La}/q_{Lb})^{\alpha_i/\gamma_i}$. Similarly, the relative salary of any two managers with ability levels q_{Ha} and q_{Hb} that are employed in sector i prior to and subsequent to a change in the trading environment is fully determined by their relative abilities. Evidently, the complementarity between factor types must be strong enough to induce meaningful rematching, or else relative wages within any occupation-and-industry group will be fixed by technological considerations and unaffected by trade.

The effects of trade on between-occupation and between-industry inequality are derived in the online appendix. Here we briefly report certain limiting cases. Suppose, for example, that $\gamma_i \approx \gamma_j$; i.e., there are only small cross-industry differences in factor intensity. In this case, if $\alpha_j > \alpha_i$, high-ability workers have a comparative advantage in sector j relative to sector i, and *vice versa* for low-ability workers. Then, if the relative price of good j increases, this changes the between-industry distribution, favoring those (high-ability workers) employed in sector j relative to those (low-ability workers) employed in sector i. Every worker ultimately employed in sector j gains relative to every worker ultimately employed in sector i. An analogous explanation applies to the changes in the between-industry distribution of managerial salaries when $\beta_j > \beta_i$.

If, in addition to $\gamma_i \approx \gamma_j$ and $\alpha_j > \alpha_i$, we have $\beta_i \approx \beta j$, then the workers have industry specificity, but the managers do not (or only slightly so). As in the classic Ricardo-Viner model (e.g., Jones, 1971), we find that when the relative price of good j rises, the real incomes of all (high-ability) workers who start in industry j increase while the real incomes of all (low-ability) workers who remain in industry i fall. In contrast, trade has a qualitatively similar impact on all manager types; their real salaries rise in terms of good i but fall in terms of good j.

Now suppose that $\gamma_j > \gamma_i$, whereas $\alpha_i/\gamma_i \approx \alpha_j/\gamma_j$ and $\beta_i/(1-\gamma_i) \approx \beta_j/\left(1-\gamma_j\right)$. With this constellation of parameters, the forces that give certain types of each factor a comparative advantage in one sector or the other are muted. No matter what sorting pattern emerges, the predominant effect of trade will be on the between-occupation distribution. In particular, since sector j makes relatively intensive use of workers and sector i makes relatively intensive use of managers, an increase the relative price of good j raises wages of workers relative to salaries of managers. Indeed, we can go further to say—as an extension of the Stolper-Samuelson theorem—that when $\gamma_j > \gamma_i$, an increase in the relative price of good j raises the real income of every type of worker and reduces the real income of every type of manager.[15]

In less extreme cases, the Stolper-Samuelson and Ricardo-Viner forces coexist. We find that the worker types with comparative advantage in industry i always gain relative to those with comparative advantage in industry j when the relative price of good i rises. Similarly, the manager types that sort to industry i gain relative to those that sort to industry j. Whether a group of workers or a group of managers benefits absolutely, and not just relatively, from a change in the trade environment depends on the direction and strength of the Stolper-Samuelson forces; for example, all workers may gain from an increase in p_i/p_j if industry i is much more labor intensive than industry j, whereas only some may gain if the difference in factor intensity is smaller, and all may lose if the factor-intensity ranking runs in the opposite direction.[16]

The results described here are interesting and will help us to understand those that follow. But the Cobb-Douglas case does not permit trade to affect within occupation-and-industry earnings inequality. Yet Helpman et al. (2015) show, for example, that within-group variation accounted for a majority of the overall change in Brazilian wage inequality that occurred during the period that spanned the trade liberalization of 1991 (see also our online appendix for a discussion of this evidence). To allow for changes in within-group inequality, we must re-introduce Assumption 1′.

5.2 Wages and Salaries with Strictly Log Supermodular Productivity

We henceforth assume that productivity in each sector is a strictly log supermodular function of the ability of the manager and the abilities of the workers; i.e., we adopt Assumption 1′. We shall limit our attention to threshold equilibria; i.e., those that can be characterized by a pair of cutoff points, q_L^* and q_H^*, such that all workers with ability above the cutoff sort to one industry and all those with ability below the cutoff sort to the other, and similarly for managers.

In the online appendix, we prove a general result that applies to all threshold equilibria. Consider the effects of a change in the relative price of some good j on output levels and factor allocation. Not surprisingly, an increase in p_j/p_i induces a rise in the aggregate output of good j and a decline in the aggregate output of good i. In principle, this could be accomplished by a reallocation of only one factor from industry i to industry j. In fact, however, this does not happen; when p_j/p_i rises, the numbers of workers and managers employed in sector j both expand, while the numbers employed in sector i contract, through changes in q_L^* and q_H^*.

Recall that two types of threshold equilibria can arise in our model, an HH/LL equilibrium in which the more able types of both factors sort to the same industry and an HL/LH equilibrium in

[15] If we instead assume that $\gamma_j > \gamma_i$ and $\alpha_i/\gamma_i > \alpha_j/\gamma_j$, while $\beta_i/(1-\gamma_i) \approx \beta_j/(1-\gamma_j)$, then the Stolper-Samuelson forces reinforce the positive effects of an increase in the relative price of good j on the low-ability workers while offsetting the negative effects of this price change on the high-ability workers. In such circumstances, the real incomes of the least able workers must rise, whereas those of the most able workers can rise or fall.

[16] These findings are reminiscent of those described by Mussa (1982) for an economy with "imperfect factor mobility" and by Grossman (1983) for an economy with "partially mobile capital."

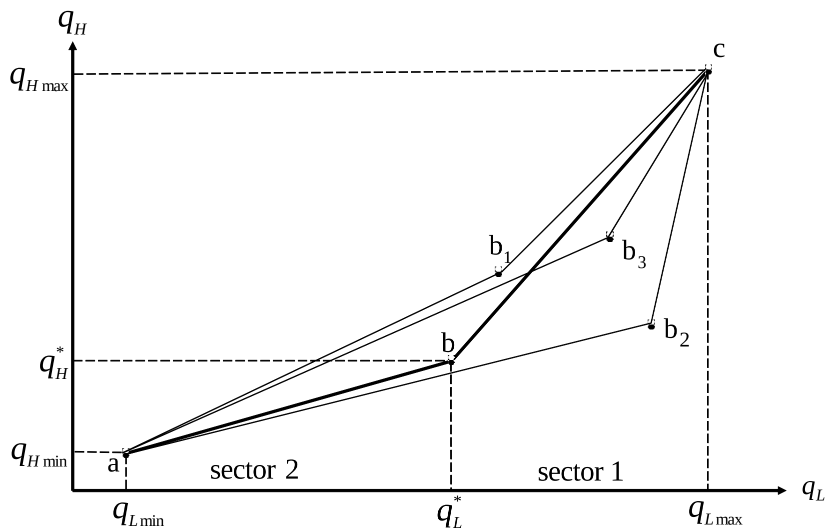

Figure 3: Effects of a rise in p_2/p_1 on matching: HH/LL equilibrium

which the more able managers sort to the same industry as the less able workers. Only the former type of equilibrium exhibits economy-wide PAM. In the online appendix we report a strongly positive correlation across industries between the average wage paid to Brazilian workers and the average salary paid to managers. This suggests that, at least in Brazil, positive assortative matching is an economy-wide phenomenon. Accordingly, we focus most of our attention on the HH/LL equilibrium. Inasmuch as the HL/LH may be relevant in other contexts, we briefly discuss some interesting features of such equilibria in Section 5.2.2 below.

5.2.1 Inequality in an HH/LL Equilibrium

In Figure 3, the thick curve *abc* represents the qualitative features of the inverse matching function in an initial HH/LL equilibrium in which the most able types of both factors sort to industry 1. The curve is upward sloping along its entire length, reflecting PAM within and across sectors. Now suppose that the relative price of good 2 rises, inducing a reallocation of resources to industry 2. From our earlier discussion, we know that both q_L^* and q_H^* must increase, which means that point b shifts up and to the right. The figure depicts three conceivable locations for the new threshold, at b_1, b_2 and b_3. Lim (2015) provides numerical examples of each such possibility.

If the new threshold falls at a point such as b_1, the outcome implies match upgrades for all workers and match downgrades for all managers. If, instead, the new threshold falls at a point such as b_2, the managers see their matches improve, while workers see their's deteriorate. Finally, if the new threshold point is b_3, matches improve for low-ability workers and deteriorate for high-ability workers, and the opposite for managers.

To understand when each outcome may occur and its implications for inequality, we suppose first that relative factor intensities are the same in the two industries; i.e., $\gamma_1 = \gamma_2$. In such circumstances, the two sectoral matching functions $m_1(q_H)$ and $m_2(q_H)$ of an HH/LL equilibrium must shift in the same direction in response to any small changes in the relative price p_2/p_1 (see the online appendix). Although we have not been able to prove that the same must occur for large price changes, neither could we find any numerical counterexamples. It seems that with $\gamma_1 = \gamma_2$, the threshold must shift to a point like b_1 or b_2, with matches either improving for all workers and deteriorating for all managers, or *vice versa*; see Lim (2015) for further discussion.

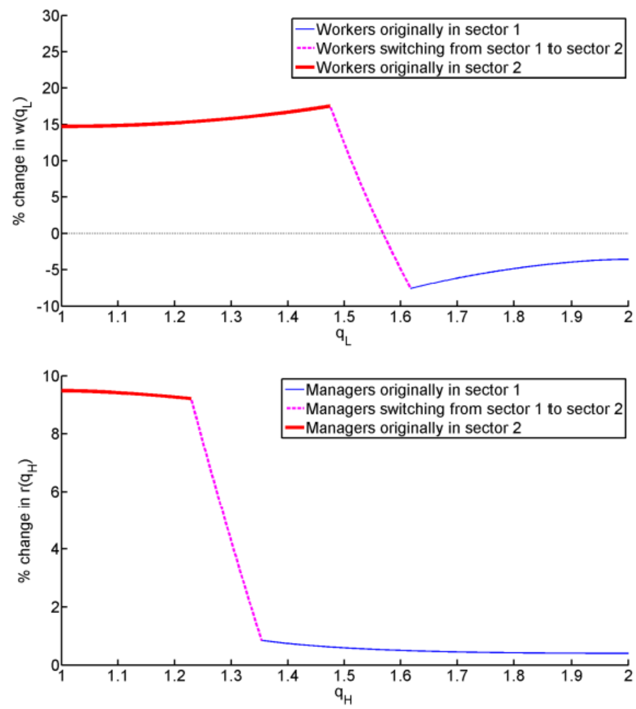

Figure 4: Effects of a 10% increase in p_2 on wages and salaries in an HH/LL equilibrium without Stolper-Samuelson effects

Figure 4 depicts an example of the wage and salary effects that result from an increase in the relative price of the good produced by the economy's least able workers and managers when factor intensities are the same in both industries ($\gamma_1 = \gamma_2$) and when workers' matches improve and managers' matches deteriorate everywhere. The example depicts a case such as in Figure 3, when the threshold shifts to a point such as b_1. The parameter values that underlie this example are provided in Lim (2015). Notice that the improved matching for workers implies a ubiquitous increase in within occupation-and-industry wage inequality; in each sector, the more able workers gain relative to the less able workers. Meanwhile, wages rise in the low-paying industry 2 relative to those in the high-paying industry 1. An economy-wide measure of wage inequality will reflect a balancing of these offsetting forces. At the same time, managerial salaries become more equal both within industries, across industries, and for the economy as a whole. Clearly, factor specificity explains the cross-industry redistribution, while rematching in the presence of factor complementarities explains the within-industry effects.

We can also deduce the implications for real incomes in this case. The inverse matching function becomes steeper at point a in Figure 3 when the threshold shifts from b to b_1. By Lemmas 1 and

6 in the appendix, this implies that the real income of the economy's least able worker must rise. *A fortiori*, real wages must rise for all workers who were initially employed in sector 2. Notice in Figure 4 that the proportional wage hikes for all workers initially in sector 2 exceed 10%, which is the percentage increase in the price of good 2 that is reflected in this example. Meanwhile, the inverse matching function becomes flatter at point c, which implies a fall in the real income for the economy's most able worker and, *a fortiori*, for all workers who remain employed in industry 1 after the price change. Indeed, in Figure 4 , nominal wages (in terms of good 1) fall for all workers initially in sector 1. In these circumstances, the salaries of the least able managers must fall in terms of good 2, while the salaries of the most able managers must rise in terms of good 1.[17] It follows that real incomes may increase (or decrease) for some (or all) of the managers, depending on the composition of their consumption baskets.

To summarize our findings for an HH/LL equilibrium with equal factor intensities, we have

Proposition 6 *Suppose that Assumption 1′ holds, that $\gamma_1 = \gamma_2$, and that that there exists a threshold equilibrium with an HH/LL sorting pattern in which the more able types sort to some sector i, $i \in \{1,2\}$. Then a small increase in the relative price p_j/p_i: (i) improves matches for all types of one factor F and deteriorates matches for all types of the other factor K, $F, K \in \{H, L\}$, $K \neq F$; (ii) raises within occupation-and-industry income inequality for factor F and reduces it for factor K in both sectors; (iii) reduces between-industry inequality for both factors; and (iv) raises real earnings of all types of factor F that are initially employed in the expanding sector and reduces them for all types of factor F that remain employed in the contracting sector.*

The effects on within occupation-and-industry inequality, on between-industry inequality, and on real incomes described in Proposition 6 do not require that factor intensities be the same in the two industries. They arise anytime the matching functions shift in the same direction in both sectors. However, such shifts in the sectoral matching functions are more likely to occur when the factor-intensity difference is small.

We next consider opposing shifts in the two industry matching functions that can occur when factor-intensity differences are substantial. In Figure 3, we illustrated a case in which the threshold shifts to point b_3, such that the inverse matching function shifts up in industry 2 and down in industry 1. Alternatively (but not shown in the figure), the inverse matching function for sector 1 might be flatter than that for sector 2, and the former might shift up while the latter shifts down. In the online appendix, we prove that the inverse matching function for some sector i is steeper than that for sector j at a point of intersection if and only if $\gamma_i > \gamma_j$. Moreover, if matches improve for workers in one sector and deteriorate for those in the other, the upgrading always occurs in the labor-intensive industry.

Figure 5 depicts outcomes for another example described in Lim (2015). In this example, the relative price rises in a labor-intensive sector that also happens to attract the economy's least able workers and managers. The figure shows that an increase in the relative price of good 2

[17] These statements follow from the fact that the inverse matching function becomes steeper at a but flatter at c.

(the good produced by the low-ability types) generates a spread of the wage distribution in the former sector and a contraction in the latter. Between-industry wage inequality narrows thanks to the relative gains for the low-ability workers who have a comparative advantage in the expanding sector. Meanwhile, salary inequality narrows among managers in the expanding sector, widens among those who remain in the contracting sector, and diminishes between industries.

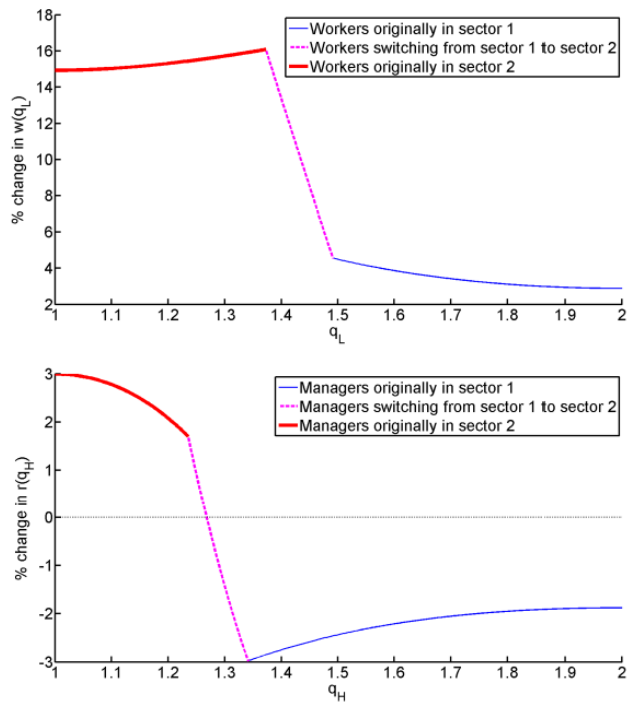

Figure 5: Effects of a 10% increase in p_2 on wages and salaries in an HH/LL equilibrium with opposite shifts in sectoral matching functions and moderate Stolper-Samuelson forces

In this example, all workers initially employed in sector 2 enjoy real incomes gains; their wages rise proportionately more than the price increase of 10%. This is always true when the labor-intensive sector employs the least able workers and the relative price of the labor-intensive good rises, because Lemmas 1 and 6 in the appendix ensure that the real wage in terms of good 2 increases for the worker with ability $q_{L\,\min}$ and other workers initially employed in the industry fare even better. In the example, the wages of workers who remain in sector 1 increase less than in proportion to the rise in p_2, but a stronger Stolper-Samuelson force could generate real income gains for these workers as well. Meanwhile, managers who remain in sector 1 see a decline in their real salaries inasmuch as the Stolper-Samuelson force and the Ricardo-Viner force push in the same direction. The decline in real income for the managers of type $q_{H\,\max}$ is ensured by Lemma 6, and the other managers who remain in the industry lose ground relative to this type.

Our next proposition summarizes our findings for an HH/LL equilibrium in which the sectoral matching functions shift in opposite directions in the two industries. This outcome requires that the factor intensities differ sufficiently across the two sectors.

Proposition 7 *Suppose that Assumption 1′ holds and that there exists a threshold equilibrium with an HH/LL sorting pattern. If a change in relative price improves matches for factor $F \in \{H, L\}$ in one sector but not the other, then the matches must improve in the sector that uses factor F intensively. This generates an increase in within occupation-and-industry inequality for types of factor F employed in the F-intensive sector and a reduction in within occupation-and-industry inequality for types employed in the other sector. Between industry inequality declines for both factors if and only if the relative price rises for the good produced by the less able types. If industry i uses factor F relatively intensively and p_i/p_j rises, then real incomes increase for all types of factor F initially employed in industry i and fall for all types of factor K, $K \neq F$, that remain employed in industry j, $j \neq i$.*

The outcomes described in Proposition 7 and illustrated in Figure 5 are broadly consistent with the data for Brazil before and after its major trade reform in 1991. As we report in the online appendix, changes in relative prices from 1986 to 1994 are positively correlated with changes in within-industry inequality among workers and negatively correlated with changes in within-industry inequality among managers.

5.2.2 Inequality in an HL/LH Equilibrium

In Figure 6, the solid curves cd and ab depict the qualitative features of the inverse matching function in an HL/LH equilibrium in which industry 2 attracts the more able managers and the less able workers. Each segment is upward sloping, representing the PAM that occurs within each sector. But, as the figure shows, PAM does not apply to the economy as a whole.

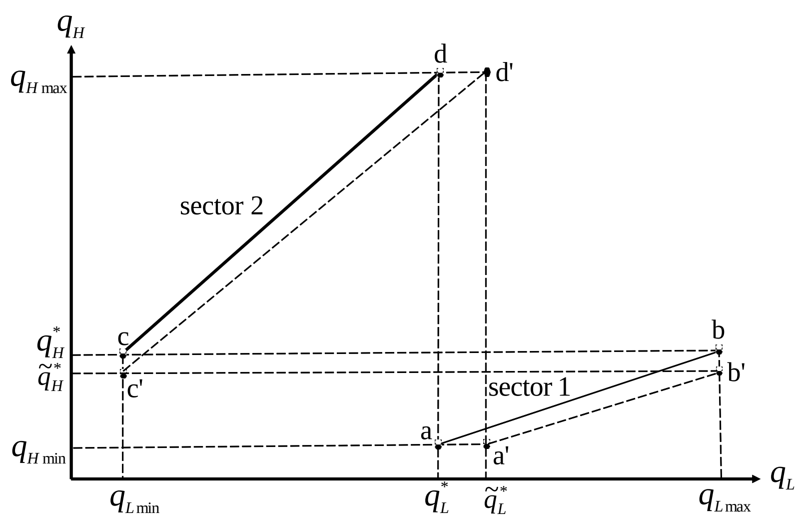

Figure 6: Effects of a rise in p_2/p_1 on matching: HL/LH equilibrium

Now suppose that the relative price of good 2 rises. As we have noted, the allocations to industry 2 of both workers and managers must expand on the extensive margin. In other words,

q_L^* rises to a point like $\tilde{q}_L^*$, while q_H^* falls to a point like $\tilde{q}_H^*$. Accordingly, the new boundary points for industry 1 move to a' and b', whereas those for industry 2 become c' and d'.

The *ex post* inverse matching function for industry 1 connects a' with b'. By Lemma 2 in the appendix, it cannot cross ab more than once. Evidently, the new curve for industry 1 must lie everywhere below the initial curve, as drawn. By similar reasoning, the new inverse matching function for industry 2 also lies everywhere below the old curve; it must connect c' and d' and it cannot cross cd twice. Thus, every worker initially employed in industry 2 or ultimately employed in industry 1 matches with a less able manager than before. Correspondingly, all managers who initially were employed in industry 2 or who remain employed in industry 1 are matched with more able workers than before.

This rematching again has implications for within occupation-and-industry income inequality. The downward shift in the inverse matching function for industry 2 implies, by (11), that the relative wage of any worker rises relative to that of another, more able worker in the same industry. This means that the wage schedule among workers in industry 2 tilts in favor of those at the bottom end of the industry pay scale. The same is true among workers that remain employed in sector 1 subsequent to the contraction of that industry. Within occupation-and-industry inequality declines for this group of workers as well. Moreover, wage inequality declines in the set of workers that switches industries.[18]

Figure 7 shows the wage and salary effects for another parameterized example from Lim (2015). The figure displays the qualitative features described in the previous paragraph. In particular, the plot of proportional wage changes against q_L in the top panel is downward sloping along its entire length. Wage inequality narrows in both sectors and in the economy as a whole. The results for managerial salaries are analogous, but opposite, as depicted in the bottom panel of figure.

We summarize our findings about the effects of relative price movements on wage and salary inequality in an HL/LH equilibrium in the following proposition:

Proposition 8 *Suppose that Assumption 1′ holds and that there exists a threshold equilibrium with an HL/LH sorting pattern in which the low-ability types of factor F and the high-ability types of factor K sort to industry i for all relative prices in some connected interval, $K \in \{H, L\}$, $K \neq F$ and $i \in \{1, 2\}$. Then any increase in the relative price of good i within this interval raises within occupation-and-industry income inequality and overall income inequality among types of factor K and reduces within occupation-and-industry income inequality and overall income inequality among types of factor F.*

Notice that Proposition 8 makes no reference to the factor intensities in the two sectors.

While Proposition 8 speaks to inequality within occupations, it says nothing about redistribution between occupations, nor about the effects of trade on the (absolute) real income levels of any

[18] Consider two workers, with abilities q_{Lc} and q_{Ld} that both switch industries, with $q_{Lc} > q_{Ld}$. By (7), the elasticity of the wage schedule $\varepsilon_w(q_L)$ is determined, *ex post*, by the elasticity ratio for the expanding industry i; whereas beforehand it was determined by the elasticity ratio for the contracting industry j. The condition for the sorting of high-abiilty workers to sector i implies that the former elasticity ratio is higher. Accordingly, the wage elasticity falls among this group of workers.

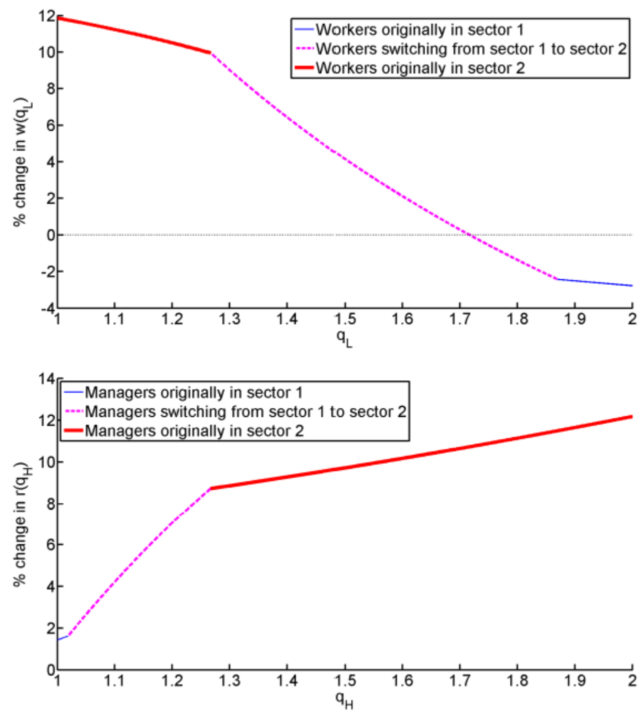

Figure 7: Effects of a 10% increase in p_2 on wages and salaries in an HL/LH equilibrium without Stolper-Samuelson forces

groups. For this we turn to numerical simulations whose details are reported in Lim (2015). When differences in factor intensities are small, Stolper-Samuelson forces are negligible. Then our findings are consistent with the intuition of the Ricardo-Viner model. When the relative price of the good produced by the low-ability workers and the high-ability manager rises, the highest-ability manager and the lowest-ability worker both see their real incomes rise. These individuals are the ones with the strongest comparative advantage in the expanding sector. In general, incomes of those (workers or managers) who are initially employed in the expanding sector rise substantially relative to those of their occupational counterparts that remain employed in the contracting sector. In the example depicted in Figure 7, all workers who remain in industry 1 suffer real wage losses. The managers in industry 1, on the other hand, see small nominal salary gains, and so their real incomes might rise if their expenditures are sufficiently biased toward the good they produce. In any case, this example highlights the between-industry redistribution that results from specificity of the different factor types.

In an economy with a substantial difference in factor intensities, on the other hand, the Stolper-Samuelson effect becomes relevant. If, for example, industry 2 is significantly more worker-intensive than industry 1 workers of all types may see a rise in real wage, while all managers may suffer real income losses. Of course, the workers employed in industry 2 fare better than their counterparts in industry 1, since their types confer a comparative advantage in producing good 2. Similarly, the very able managers employed in industry 2 experience smaller real income losses than their less able counterparts. Proposition 8 prescribes a ubiquitous increase in salary inequality and a ubiquitous fall in wage inequality. A host of other configurations can emerge, but all can be understood similarly with reference to the relevant factor intensities that generate between-occupation redistribution and the sector-specificities that generate between-industry redistribution; see Lim (2015) for further examples.

6 Concluding Remarks

We have developed a framework that can be used to study the effects of trade on income inequality. Our model features two industries, two factors of production, and perfect competition, in keeping with a familiar setting from neoclassical trade theory. Indeed, we have chosen this economic environment so that we might draw on a deep understanding of the distributional effects of trade in the Heckscher-Ohlin and Ricardo-Viner models. To the standard set-up, we have added heterogeneous types of each of the two factors of production. With this simple extension, our model is capable of generating rich predictions about the effects of trade on within occupation-and-industry earnings inequality. Such effects seem to be important in the data, yet are beyond the reach of much of the existing literature.

Redistribution within occupations and industries occurs in response to relative price changes whenever technologies exhibit strong complementarities between the types of the various factors that are employed together in a production unit. We have assumed that productivity in each unit is a log supermodular function of the ability of the manager and the ability levels of the workers and we have allowed for cross-sectoral differences in factor intensity as well as differences in the complementarities between worker and manager types.

The effects of trade on income distribution are mediated by relative output prices. Accordingly, we have studied how changes in prices affect the equilibrium wage and salary schedules. We have focused on threshold equilibria in which all of the more able workers sort to one industry while all of the less able workers sort to the other, and similarly for managers.

Our analysis can provide guidance to the empirical researcher. It points to the importance of distinguishing employees by occupation and industry when studying the effects of trade on income inequality. As we know from the classic papers in neoclassical trade theory, the distributional effects of changes in the trade environment can differ for managers versus workers and for employees in an export industry versus those in an import-competing industry. To this we have added the effects of trade on within occupation-and-industry inequality, and we have derived novel predictions about this type of inequality that have clear empirical implications.

A broad implication of this theory is that, whenever a change in the terms of trade induces improved matches in an industry for some factor, the within occupation-and-industry earnings inequality in the group increases. A downgrading of matches for some factor in some industry in turn generates a decline in within-group inequality. These predictions reflect the assumption that the more able (and better paid) types benefit relatively more from the upgrading of their partners than do their less able counterparts. This is a keystone for more specific results that link changes in inequality to structural features of the economic environment. When changes in matching are observable, this prediction of the model can be tested directly. Otherwise the empirical analysis can build on other relationships predicted by the theory.

One specific prediction of our model is that, when the more able managers and the more able workers sort to the same industry, as appears to be true in the Brazilian data reported in the online appendix, and when the difference in factor intensities across industries is not too large, a change in the terms of trade increases the within occupation-and-industry earnings inequality for one factor of production and reduces it for the other. In this case of economy-wide PAM and small factor-intensity differences, matches necessarily improve for one factor and deteriorate for the other in both sectors of the economy.

But we have also found that, in an equilibrium in which the more able types of both factors sort to the same industry, a shift in the terms of trade may induce match upgrades for some factor in one sector and downgrades in the other sector. This outcome requires a sufficiently large difference across industry in factor intensities. When it occurs, the within occupation-and-industry earnings inequality of each factor widens in one industry and narrows in the other. The theory also predicts in which industry a factor's within-group earnings inequality should increase: that should happen in the industry that uses the factor intensively.

When the more able managers sort to the same sector as the less able workers, the within occupation-and-industry earnings inequality of each factor moves in the same direction in every industry in response to a change in the terms of trade. Inequality widens for the factor whose most able types are employed in the industry that has experienced a relative price hike and declines for the factor whose most able types are employed in the other industry. If instances of negative PAM across sectors can be found in some economies, these predictions about the relationship between price movements and inequality changes should be readily testable.

Our approach to introducing factor heterogeneity could also be applied to other trade models. For example, it would be straightforward to incorporate matching of heterogeneous types of multiple factors in a setting à la Sampson (2014) with monopolistic competition and fixed costs of exporting. Or one could do so in a model of horizontal foreign direct investment, to study the formation of international production teams, as in Antràs et al. (2006). We think it would be particularly interesting to introduce search frictions to capture possible impediments to the perfect matching of worker and manager types. In such a setting, one could ask how globalization impacts the formation of production teams and thereby the productivity of firms.

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Appendix

Lemmas and Proofs for Section 4

Assume that Assumption 1' holds and suppose that some sector employs workers and managers whose abilities form the intervals $I_L = [q_{La}, q_{Lb}]$ and $I_H = [q_{Ha}, q_{Hb}]$, respectively. To simplify notation, we drop the sectoral index i and we consider the following industry equilibrium conditions corresponding to (8) - (10) for one particular sector:

$$r(q_H) = \bar{\gamma} p^{\frac{1}{1-\gamma}} \psi[q_H, m(q_H)]^{\frac{1}{1-\gamma}} w[m(q_H)]^{-\frac{\gamma}{1-\gamma}}, \quad \bar{\gamma} = \gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma) \tag{12}$$

$$\frac{\psi_L[q_H, m(q_H)]}{\gamma\psi[q_H, m(q_H)]} = \frac{w'[m(q_H)]}{w[m(q_H)]}, \tag{13}$$

$$\bar{H}\frac{\gamma r(q_H)}{(1-\gamma)w[m(q_H)]}\phi_H(q_H) = \bar{L}\phi_L[m(q_H)]m'(q_H), \tag{14}$$

and the boundary conditions,

$$\begin{aligned} m(q_{Hz}) &= q_{Lz}, \quad z = a, b; \\ q_{Lb} &> q_{La} > 0, \quad q_{Hb} > q_{Ha} > 0. \end{aligned} \tag{15}$$

Equation (12) is taken from (8), (13) is taken from (7) and (14) is taken from (10). We seek to characterize the solution for the three functions, $w(\cdot)$, $r(\cdot)$ and $m(\cdot)$.

We use (12) and (13) to obtain

$$\ln r(q_H) - \ln r(q_{H0}) = \int_{q_{H0}}^{q_H} \frac{\psi_H[x, m(x)]}{(1-\gamma)\psi[x, m(x)]}dx, \quad \text{for } q_H, q_{H0} \in I_H, \tag{16}$$

$$\ln w(q_L) - \ln w(q_{L0}) = \int_{q_{L0}}^{q_L} \frac{\psi_L[\mu(x), x]}{\gamma\psi[\mu(x), x]}dx, \quad \text{for } q_L, q_{L0} \in I_L, \tag{17}$$

where $\mu(\cdot)$ is the inverse of $m(\cdot)$. We substitute (12) into (14) to obtain

$$\begin{aligned} \frac{1}{1-\gamma}\ln w[m(q_H)] &= \frac{1}{1-\gamma}\ln\gamma + \ln\left(\frac{\bar{H}}{\bar{L}}\right) + \frac{1}{1-\gamma}\ln p \\ &+ \frac{1}{1-\gamma}\ln\psi[q_H, m(q_H)] + \log\phi_H(q_H) - \log\phi_L[m(q_H)] - \log m'(q_H). \end{aligned} \tag{18}$$

The differential equations (13) and (18) together with the boundary conditions (15) uniquely determine the solution of $w(\cdot)$ and $m(\cdot)$ when the productivity function $\psi(\cdot)$ is twice continuously differentiable and strictly log supermodular and the density functions $\phi_F(\cdot)$, $F = H, L$, are continuously differentiable.

By differentiating (18) and substituting (13) into the result, we generate a second-order differential equation for the matching function,

$$\frac{m''(q_H)}{m'(q_H)}=\frac{\psi_H[q_H,m(q_H)]}{(1-\gamma)\psi[q_H,m(q_H)]}-\frac{\psi_L[q_H,m(q_H)]m'(q_H)}{\gamma\psi[q_H,m(q_H)]}+\frac{\phi_H'(q_H)}{\phi_H(q_H)}-\frac{\phi_L'[m(q_H)]m'(q_H)}{\phi_L[m(q_H)]}. \quad (19)$$

Given boundary conditions $m(q_{Ha})=q_{La}$, $m'(q_{Ha})=t_a>0$, this differential equation has a unique solution, which may or may not satisfy the boundary condition $m(q_{Hb})=q_{Lb}$ in (15). The solution to the original matching problem is found by identifying a value t_a that yields a solution to (19) that satisfies (15). Note that this solution depends neither on the price p nor on the factor endowments $\bar{H}$ and $\bar{L}$. Therefore, changes in these variables do no affect the matching function, but they change all wages and salaries proportionately, as can be seen from (18), and (12). Using hats to denote proportional changes, e.g., $\hat{p}=dp/p$, we have

Lemma 1 *(i) The matching function $m(\cdot)$ does not depend on $\left(p,\bar{H},\bar{L}\right)$. (ii) An increase in the price p, $\hat{p}>0$, raises the wage and salary schedules proportionately by $\hat{p}$. (iii) An increase in $\bar{H}/\bar{L}$ such that $\hat{H}-\hat{L}=\hat{\eta}>0$ raises the wage schedule proportionately by $(1-\gamma)\hat{\eta}$ and reduces the salary schedule proportionately by $\gamma\hat{\eta}$.*

We now prove several lemmas that are used in the main analysis.

Lemma 2 *Let $[m_{\varkappa}(q_H),w_{\varkappa}(q_L)]$ and $[m_{\varrho}(q_H),w_{\varrho}(q_L)]$ be solutions to the differential equations (13) and (18), each for different boundary conditions (15), such that $m_{\varkappa}(q_0)=m_{\varrho}(q_0)=q_{L0}$ and $m_{\varrho}'(q_0)>m_{\varkappa}'(q_0)$ for $q_0\in S_{H\varkappa}\cap S_{H\varrho}$. Then $m_{\varrho}(q_H)>m_{\varkappa}(q_H)$ for all $q_H>q_0$ and $m_{\varrho}(q_H)<m_{\varkappa}(q_H)$ for all $q_H<q_{H0}$ in the overlapping range of abilities.*

Proof. Consider $q_H>q_{H0}$ and suppose that, contrary to the claim, there exists a $q_{H1}>q_{H0}$ such that $m_{\varrho}(q_{H1})\leq m_{\varkappa}(q_{H1})$. Then differentiability of $m_{\iota}(\cdot)$, $\iota=\varkappa,\varrho$, implies that there exists $q_{H2}>q_{H0}$ such that $m_{\varrho}(q_{H2})=m_{\varkappa}(q_{H2})$, $m_{\varrho}(q_H)>m_{\varkappa}(q_H)$ for all $q_H\in(q_{H0},q_{H2})$ and $m_{\varrho}'(q_{H2})<m_{\varkappa}'(q_{H2})$. This also implies $\mu_{\varrho}(x)<\mu_{\varkappa}(x)$ for all $x\in(m_{\varrho}(q_{H0}),m_{\varrho}(q_{H2}))$, where $\mu_{\iota}(\cdot)$ is the inverse of $m_{\iota}(\cdot)$. Under these conditions (18) implies $w_{\varrho}[m_{\varrho}(q_{H0})]<w_{\varkappa}[m_{\varrho}(q_{H0})]$ and $w_{\varrho}[m_{\varrho}(q_{H2})]>w_{\varkappa}[m_{\varrho}(q_{H2})]$, and therefore

$$w_{\varkappa}[m_{\varrho}(q_{H2})]-w_{\varkappa}[m_{\varrho}(q_{H0})]<w_{\varrho}[m_{\varrho}(q_{H2})]-w_{\varrho}[m_{\varrho}(q_{H0})].$$

On the other hand, (17) implies

$$\ln w_{\iota}[m_{\varrho}(q_{H2})]-\ln w_{\iota}[m_{\varrho}(q_{H0})]=\int_{m_{\varrho}(q_{H0})}^{m_{\varrho}(q_{H2})}\frac{\psi_L[\mu_{\iota}(x),x]}{\gamma\psi[\mu_{\iota}(x),x]}dx,\quad \iota=\varkappa,\varrho.$$

Together with the previous inequality, this gives

$$\int_{m_{\varrho}(q_{H0})}^{m_{\varrho}(q_{H2})}\frac{\psi_L[\mu_{\varkappa}(x),x]}{\psi[\mu_{\varkappa}(x),x]}dx<\int_{m_{\varrho}(q_{H0})}^{m_{\varrho}(q_{H2})}\frac{\psi_L\left[\mu_{\varrho}(x),x\right]}{\psi\left[\mu_{\varrho}(x),x\right]}dx.$$

Note, however, that strict log supermodularity of $\psi(\cdot)$ and $\mu_\varrho(x) < \mu_\varkappa(x)$ for all $x \in (m_\varrho(q_{H0}), m_\varrho(q_{H2}))$ imply the reverse inequality, a contradiction. It follows that $m_\varrho(q_H) > m_\varkappa(q_H)$ for all $q_H > q_{H0}$. A similar argument shows that $m_\varrho(q_H) < m_\varkappa(q_H)$ for all $q_H < q_{H0}$. ■

We next show how the matching function and wage function respond to the boundary conditions. First consider the shift of the equilibrium matching function in response to a rise in q_{Lb}, which shifts the boundary point (q_{Hb}, q_{Lb}) but not (q_{Ha}, q_{La}) in (15). It then follows from Lemma 2 that the old and new matching functions intersect only at q_{Ha}. Therefore an increase in q_{Lb} increases the ability of workers matched with every manager except for the least able manager. Other shifts in the boundary points can be analyzed in similar fashion to establish

Lemma 3 *(i) $dm(q_H)/dq_{La} > 0$ for all $q_H < q_{Hb}$ and $d\mu(q_L)/dq_{La} < 0$ for all $q_L < q_{Lb}$; (ii) $dm(q_H)/dq_{Lb} > 0$ for all $q_H > q_{Ha}$ and $d\mu(q_L)/dq_{Lb} < 0$ for all $q_L > q_{La}$; (iii) $d\mu(q_L)/dq_{Ha} > 0$ for all $q_L < q_{Lb}$ and $dm(q_H)/dq_{Ha} < 0$ for all $q_H < q_{Hb}$; and (iv) $d\mu(q_L)/dq_{Hb} > 0$ for all $q_L > q_{La}$ and $dm(q_H)/dq_{Hb} < 0$ for all $q_H > q_{Ha}$.*

Next consider changes in a boundary (q_{Hz}, q_{Lz}), $z = a, b$. For concreteness, suppose that (q_{Hb}, q_{Lb}) changes. Then the new and old matching functions coincide at the other boundary point, (q_{Ha}, q_{La}), which does not change. In this case, Lemma 2 implies that either the two matching functions coincide in the overlapping range of abilities or one is above the other everywhere except for at (q_{Ha}, q_{La}). A similar argument applies to changes in (q_{Ha}, q_{La}). We thus have:

Lemma 4 *In response to a shift in a single boundary (q_{Hz}, q_{Lz}), $z = a, b$, either the new matching functions coincide with the old matching function in the overlapping range of abilities or one matching function is above the other everywhere except for at the opposite boundary point.*

We next discuss the impact of boundaries on wages and salaries. We focus on wages, but note that if a shift in boundaries raises the wage of workers with ability q_L then it must reduce the salary of managers teamed with these workers. This can be seen from (12) by noting that a change in boundaries has no impact on $r(\cdot)$ through an induced shift in the matching function due to the first-order condition (13) (a version of the Envelope Theorem). Therefore the change in salary $r(q_H)$ is driven by the change in wages of workers matched with managers of ability q_H. We record this result in

Lemma 5 *Suppose that the boundaries (q_{Hz}, q_{Lz}), $z = a, b$, change and that, as a result, $w(q_L)$ rises for some q_L such that q_L and $q_H = m^{-1}(q_L)$ are in the overlapping range of abilities of the old and new boundaries. Then $r(q_H)$ declines.*

For the subsequent analysis the following lemma is useful:

Lemma 6 *Let $[m_\varkappa(q_H), w_\varkappa(q_L)]$ and $[m_\varrho(q_H), w_\varrho(q_L)]$ be solutions to (13) and (18), each for different boundary conditions (15), such that $m_\varkappa(q_{H0}) = m_\varrho(q_{H0}) = q_{L0}$ and $m'_\varrho(q_{H0}) > m'_\varkappa(q_{H0})$ for some $q_{H0} \in S_{L\varkappa} \cap S_{L\varrho}$, and let $r_\varrho(q_H)$ and $r_\varkappa(q_H)$ be the corresponding solutions to (12). Then $w_\varrho(q_L) < w_\varkappa(q_L)$ and $r_\varrho(q_H) > r_\varkappa(q_H)$ in the overlapping range of abilities.*

Proof. From Lemma 2, we know that $m_\varrho(q_H) > m_\varkappa(q_H)$ for all $q_H > q_{H0}$ and $m_\varrho(q_H) < m_\varkappa(q_H)$ for all $q_H < q_{H0}$ in the overlapping range of abilities and $\mu_\varrho(x) < \mu_\varkappa(x)$ for all $x > q_{L0}$ and $\mu_\varrho(x) > \mu_\varkappa(x)$ for all $x < q_{L0}$ in the overlapping range of abilities. Moreover, $m'_\varrho(q_{H0}) > m'_\varkappa(q_{H0})$ and (18) imply

$$\ln w_\varkappa(q_{L0}) > \ln w_\varrho(q_{L0})$$

while (17) implies

$$\ln w_\iota(q_L) - \ln w_\iota(q_{L0}) = \int_{q_{L0}}^{q_L} \frac{\psi_L[\mu_\iota(x), x]}{\gamma\psi[\mu_\iota(x), x]} dx, \quad \iota = \varkappa, \varrho.$$

Together, these inequalities imply

$$\begin{aligned} \ln w_\varkappa(q_L) - \ln w_\varrho(q_L) &> \int_{q_{L0}}^{q_L} \frac{\psi_L[\mu_\varkappa(x), x]}{\gamma\psi[\mu_\varkappa(x), x]} dx - \int_{q_{L0}}^{q_L} \frac{\psi_L\left[\mu_\varrho(x), x\right]}{\gamma\psi\left[\mu_\varrho(x), x\right]} dx \\ &= \int_{q_L}^{q_{L0}} \frac{\psi_L\left[\mu_\varrho(x), x\right]}{\gamma\psi\left[\mu_\varrho(x), x\right]} dx - \int_{q_L}^{q_{L0}} \frac{\psi_L[\mu_\varkappa(x), x]}{\gamma\psi[\mu_\varkappa(x), x]} dx. \end{aligned}$$

For $q_L > q_{L0}$, the right-hand side of the first line is positive by the strict log supermodularity of the productivity function and $\mu_\varrho(x) < \mu_\varkappa(x)$ for all $x > q_{L0}$, and the second line is positive for $q_L < q_{L0}$ by the strict log supermodularity of the productivity function and $\mu_\varrho(x) > \mu_\varkappa(x)$ for all $x < q_{L0}$. It follows that $w_\varkappa(q_L) > w_\varrho(q_L)$ for all q_L in the overlapping range of abilities. A similar argument establishes that $r_\varkappa(q_H) < r_\varrho(q_H)$ for all q_H in the overlapping range of abilities. ■

This lemma, together with Lemma 4, have straightforward implications for the impact of boundary points on the wage and salary functions.

Corollary 1 *Suppose that the lower boundary (q_{Ha}, q_{La}) changes and the matching function shifts upwards as a result. Then salaries decline and wages rise in the overlapping range of abilities. The converse holds when the matching function shifts downwards.*

Corollary 2 *Suppose that the upper boundary (q_{Hb}, q_{Lb}) changes and the matching function shifts upwards as a result. Then salaries rise and wages decline in the overlapping range of abilities. The converse holds when the matching function shifts downwards.*

From (17) we also see that a change in boundaries that shifts upwards the matching function reduces wage inequality, because for every two ability levels the ratio of the wage of a high-ability worker to the wage of a low-ability worker declines for all types in between. For salaries it is the opposite, as one can see from (16). We therefore have

Lemma 7 *Suppose that the matching function shifts upwards in response to a shift in the boundaries (15). Then wage inequality narrows and salary inequality widens. The opposite is true when the matching function shifts downwards.*