Measuring Ex-Ante Welfare in Insurance Markets

Nathaniel Hendren*

June, 2017

Abstract

Estimates of willingness to pay in adversely selected markets tend to understate the ex-ante (or utilitarian) willingness to pay for insurance. This paper derives an 'ex-ante' willingness to pay curve that measures welfare from behind the veil of ignorance. I provide a strategy to estimate this curve using market choices and costs, combined with a measure of risk aversion. In examples motivated by previous literature in health insurance, an ex-ante welfare perspective can generate different conclusions about the optimal size of insurance markets, the welfare cost of adverse selection, and the desirability of mandates relative to competitive markets.

1 Introduction

Since Akerlof (1970), it has been recognized that insurance market equilibriums may be inefficient: individuals who are willing to pay the cost they would impose on the insurance company may not obtain insurance. This is because equilibrium prices must reflect the average cost of the insured, not the cost of the marginal enrollee. Motivated by this, there is a large and growing literature estimating the difference between observed willingness to pay and costs to measure the size of this lost market surplus from efficient trades that go unmet in equilibrium. This market surplus measure is then often used to discuss the welfare impact of government policy changes, such as insurance mandates (Einav et al. (2010)).

However, insurance has value from insuring against the realization of risk. Adverse selection occurs when a portion of this risk is already known at the time of contracting. This suggests that the same knowledge of future risk that can generate adverse selection also makes market surplus a potentially misguided measure of welfare. In particular, the average observed willingness to pay for insurance will generally be less than the ex-ante willingness to pay for insurance (Hirshleifer (1971)). Although market surplus measures an interesting question of the value of trades that go unmet in equilibrium, it can be potentially misleading as a tool for evaluating optimal policies such as mandates and subsidies. Indeed, allocations that maximizes welfare from behind the veil of ignorance generally do not maximize market surplus, and thus involve “deadweight loss”.

*Harvard University, nhendren@fas.harvard.edu. I am very grateful to Raj Chetty, David Cutler, Liran Einav, Amy Finkelstein, Mark Shepard, and Mike Whinston for helpful comments and discussions.
This goal of this paper is to (i) characterize the distinction between market surplus and ex-ante welfare by deriving an ex-ante willingness to pay curve within the framework of Einav et al. (2010), (ii) characterize when one can estimate this curve using market willingness to pay and cost curves combined with a measure of risk aversion, and (iii) illustrate how the ex-ante welfare perspective can lead to different conclusions (relative to market surplus) about the optimality of government subsidies and mandates using examples motivated by estimates in Einav et al. (2010).

Before discussing these contributions, it is useful to start with an example. This will help not only to define the problem, but also illustrate this paper’s proposed solution. Related literature will be discussed along the way, and I return to a discussion of the key lessons in the conclusion.

2 Stylized Example

Suppose individuals have $30 dollars but face a risk of losing $m dollars, where $m$ is uniformly distributed between 0 and 10. Assume individuals have no particular knowledge of their risk, and let $D^{Ex-ante}$ denote their willingness to pay (or “demand”) for insurance. This willingness to pay will be given by the solution to:

$$u(30 - D^{Ex-ante}) = E[u(30 - m)]$$

where $E[u(30 - m)] = \frac{1}{10} \int_0^{10} u(30 - m) \, dm$ is the expected utility if uninsured. If individuals have a utility function with a constant coefficient of relative risk aversion of 3 (i.e. $u(c) = \frac{1}{1-\sigma} c^{1-\sigma}$ and $\sigma = 3$), it is straightforward to compute that they are willing to pay $D^{Ex-ante} = 5.50$ for insurance against $m$. This insurance policy would cost the insurer $E[m] = 5$, so that the individuals are willing to pay a markup of 0.50 over actuarially fair insurance.

**Figure 1: Example Willingness to Pay and Cost Curves**

A. Before Information Revealed

B. After Information Revealed

---

2
Figure 1, Panel A, illustrates this scenario using the demand and cost curve framework formalized in Einav et al. (2010). The horizontal axis enumerates the population in descending order by their willingness to pay for insurance (using an index \( s \in [0, 1] \)), and the vertical axis reflects prices and costs in the market. Each individual is willing to pay $5.50 for insurance, generating a flat willingness to pay, or demand, curve of \( D(s) = 5.50 \). Because no one knows anything about their particular cost, each individual imposes a cost of $5 on the insurance company, generating a flat cost curve of \( AC(s) = 5 \). If a competitive market were to open up in this setting, one would expect everyone \( (s^{CE} = 100\%) \) to purchase insurance at a price of $5. This allocation would generate \( W^{Ex-Ante} = 0.50 \) of welfare, as reflected by the integral between the demand and cost curve.

Now, what happens if some information is revealed at the time individuals decide whether to purchase insurance? For simplicity, consider the extreme case that individuals have fully learned their cost. Willingness to pay will equal individuals’ known costs, \( D(s) = m(s) \). Those who learn they will lose $10 will be willing to pay $10 for “insurance” against their loss; individuals who learn they will lose $0 will be willing to pay nothing. The uniform distribution of risks generates a linear demand curve falling from $10 at \( s = 0 \) to $0 at \( s = 1 \). Let \( MC(s) \) denote the cost imposed on the insurer by the marginal type \( s \) that is willing to pay \( D(s) \). An enrollee that is willing to pay \( D(s) \) for insurance imposes a cost of \( D(s) = MC(s) \) on the insurer. Therefore, this demand curve lies everywhere on top of the cost curve of the marginal types, as illustrated in Panel B.

If an insurer were to try to sell insurance, they would need to set prices to cover the average cost of those who purchase insurance. But, the average cost of those choosing to purchase insurance, \( AC(s) \), lies everywhere above the demand curve. Hence, the market would fully unravel: the unique competitive equilibrium would involve no one obtaining any insurance, \( s^{CE} = 0\% \).

What is the welfare cost of this market unraveling? From a market surplus perspective, there is no welfare loss. Because the demand curve equals the marginal cost curve, there are no valuable foregone trades. This reflects an extreme case of a more general phenomenon: the market demand curve does not embody the value of insurance against the realization of risk prior to the measurement of demand. This idea was arguably first developed in the classic work of Hirshleifer (1971).

The traditional approach to capture the ex-ante value of insurance without observing willingness to pay from behind the veil of ignorance would specify economic primitives including a utility function and information set, and then use observed outcome data (e.g. consumption or proxies for consumption) to infer the ex-ante value of insurance. Intuitively, if one knows the utility function, \( u \), and the cross-sectional distribution of the risk, \( m \), then one can use this information to compute \( D^{Ex-Ante} \) in equation (1). For recent implementations of this approach, see Handel et al. (2015), Section IV of Einav et al. (2016), or Finkelstein et al. (2016).

The goal of this paper is to estimate \( D^{Ex-Ante} \) without knowledge of the full distribution of primitives (e.g. \( u \) and \( m \)). Instead, I develop a sufficient-statistics approach that builds a new “ex-ante” demand curve, \( D^{Ex-Ante}(s) \), into the framework of Einav et al. (2010). This curve will measure the willingness to pay an individual has from behind the veil of ignorance to insure a
To illustrate the derivation of the ex-ante demand curve, let \( p_I \) denote the price of insurance and \( p_U \) denote the price of being uninsured (so that \( p_I - p_U \) is the marginal price of obtaining insurance). Assume for simplicity that these prices are set subject to an aggregate resource constraint that requires the total amount of money collected to equal the total cost of the insured, \( sp_I + (1 - s) p_U = sAC(s) \). Imposing this resource constraint is not essential, but as in Einav et al. (2016) it simplifies the setting because one does not need to consider any welfare impact on other parties such as firms or other payers of government subsidies/taxes used to generate insurance take-up. One can now ask: What is the value of expanding the size of the insurance market from \( s \) to \( s + ds \)?

To begin, suppose that the government has set prices such that a fraction \( s = 0.5 \) of the population chooses to purchase insurance, as illustrated in Figure 2, Panel A.\(^2\) Expanding the size of the insurance market lowers the marginal price of insurance, \( p_I - p_U \), by \( D'(s) \). The resource constraint implies that the price faced by the uninsured increases by \( dp_U = -sD'(s) ds \). Conversely, the insured prices must decrease by \( dp_I = (1 - s) D'(s) ds \). These price changes induce a transfer from the uninsured to the insured, as indicated by the blue arrow in Figure 2, Panel B.

From a market surplus perspective, these price changes have no welfare impact – they’re just transfers. But, from behind the veil of ignorance, these transfers have value to the extent to which the marginal utilities of income differ for the insured and uninsured. If the marginal utility of income is higher (lower) for the insured than uninsured, then lowering (raising) the price of insurance has a first order welfare benefit. Accounting for these difference in marginal utilities of income between the insured and uninsured is precisely what will be required for constructing the ex-ante demand curve.

From behind the veil of ignorance, there is a chance \( s \) of being insured. Hence, the ex-ante welfare impact from lower insurance prices is given by \( s(1-s) D'(s) E[u_c|Insured] ds \), where \( E[u_c|Insured] \) is the average marginal utility of income for the fraction \( s \) of the market that is insured. Conversely, there is a chance \( 1 - s \) of being uninsured, so that the welfare impact from higher prices \( p_U \) is given by \( -(1-s) s D'(s) E[u_c|Uninsured] ds \), where \( E[u_c|Uninsured] \) is the average marginal utility of income for the fraction \( 1 - s \) of the market that is uninsured (for notational simplicity, I suppress the dependence of these marginal utilities on \( s, p_I \), and \( p_U \)). Summing these two effects and normalizing by the average marginal utility of income of the insured to generate a money-metric utility measure yields the ex-ante value of expanding the size of the insurance market.

---

\(^1\)Appendix 21 illustrates how the approach can be conducted conditional on observable information, \( X \), if one wishes not to incorporate any redistributive value across individuals with different values of \( X \). In this sense, the primary aim of this paper is not simply to argue for incorporating the redistributive value of insurance; rather it is about using a welfare metric that is stable with respect to the amount of information that happens to be revealed at the time demand is measured. As illustrated in Figure 1, market surplus does not have this stability.

\(^2\)Obtaining \( s = 0.5 \) would require individuals who purchase insurance to pay \( p_I = \$6.25 \) and those who do not purchase insurance to pay \( p_U = \$1.25 \), so that the marginal price of insurance is \( p_I - p_U = \$5 \) and the aggregate resource constraint holds.
market by $ds$:

$$EA(s) = s(1-s)D'(s) \frac{E[u_c|\text{Insured}] - E[u_c|\text{Uninsured}]}{E[u_c|\text{Insured}]}$$

(2)

The first term, $s(1-s)D'(s)$, can loosely be interpreted as the size of the transfer depicted by the blue arrow in Figure 2, Panel B. Steeper slopes of demand imply greater price changes (and thus larger transfers) one moves from $s$ to $s + ds$ of the market being insured. The second term, $\frac{E[u_c|\text{Insured}] - E[u_c|\text{Uninsured}]}{E[u_c|\text{Insured}]}$, is the percentage difference in marginal utilities between the insured and uninsured population. Weighting the transfer by the difference in marginal utilities recovers the ex-ante value of insurance. To the extent to which those who have learned they have high demand for insurance at the time demand is measured (e.g. because they have a medical condition) also have a higher marginal utilities of income, transfers from the uninsured to the insured increase welfare from behind the veil of ignorance. In general, it is also possible that those who are uninsured have a higher marginal utility of income than the insured. This could be the case if the reason for not obtaining coverage is liquidity constraints, so that those choosing to forego insurance have a higher return to other forms of spending. This is ruled out in the simple example presented in this introduction, but will be considered in the more general model in the next Section.

Given $EA(s)$ in equation (2), I define the ex-ante demand curve, $D^{Ex-Ante}(s)$, as

$$D^{Ex-Ante}(s) = D(s) + EA(s)$$

(3)

From behind the veil of ignorance, individuals are willing to pay $D^{Ex-Ante}(s)$ to have prices set such that a fraction $s$ of the market is insured. In this sense, the observed willingness to pay curve, $D(s)$, combined with the ex-ante component, $EA(s)$, in equation (3) characterize the ex-ante willingness to pay for a larger (or smaller) insurance market. In particular, the ex-ante willingness to pay for everyone to be insured, $D^{Ex-Ante}(1)$, is equal to $D^{Ex-ante}$ in equation (1). In this sense, the ex-ante demand curve provides a general method of recovering the ex-ante willingness to pay for insurance using the market demand curve, $D(s)$, combined with percentage the difference in marginal utilities of income between the insured and uninsured. The derivation of the ex-ante demand curve in equation (3) and (2) is the first main result of the paper, which will be provided in a more general setting in Section 3.

3This result is akin to the Baily-Chetty condition in optimal unemployment insurance that measures the value of more generous social insurance using the marginal utility of the beneficiaries (e.g. unemployed) relative to non-beneficiaries (e.g. employed). (Baily (1978); Chetty (2006a)). Here, the beneficiaries of lower insurance prices are those who choose to purchase insurance.

4The derivation in Equation (2) could be applied to the demand for any good – not just insurance. The key question is whether the decision to purchase insurance reveals something about the marginal utility of income. In the context of insurance markets, one naturally expects information to be revealed over time. This makes it particularly reasonable to expect that the marginal utility of income is different for those who choose to purchase versus those who forego insurance. In this sense, the distinction between market demand and ex-ante demand is likely to be particularly important in insurance contexts with adverse selection.

5More precisely, this is true up to an approximation error resulting from the fact that the marginal utility of the insured, $E[u_c|\text{Insured}]$, varies with market size $s$, $s$. 

5
A remaining constraint to estimating $D^{Ex-Ante}(s)$ is that one does not readily observe the differences in marginal utilities between the insured and uninsured. The second main result of the paper builds on the literature on optimal unemployment insurance (e.g. Baily (1978); Chetty (2006a)) by providing conditions under which one can approximate the difference in marginal utilities between the insured versus uninsured using Taylor expansions of the marginal utility function combined with (i) the market demand curve, $D(s)$, and (ii) an estimate of risk aversion.

To see how, return to the example above. The marginal utility of the insured is given by $u_c(30 - p_I)$, where $u_c$ is the marginal utility function (e.g. $u_c(c) = c^{-\sigma}$ if $u(c)$ is constant relative risk aversion). The marginal utility of the uninsured facing known loss $m(s)$ is given by $u_c(30 - p_U - m(s))$. Using the identity $D(s) = m(s)$, the average marginal utility of the uninsured when prices are such that a fraction $s$ of the market purchases insurance is given by $E[u_c(30 - p_U - D(s'))|s' \geq s]$. To construct an approximation for the difference in marginal util-
ities between the insured and uninsured, I take a first order Taylor approximation to the marginal utility function of the insured expanded around the marginal utility of the insured, $30 - p_I$. This yields

$$u_c (30 - p_U - D (s')) \approx u_c (30 - p_I) + u_{cc} (30 - p_I) \left[ (30 - p_U - D (s')) - (30 - p_I) \right]$$

$$\approx u_c (30 - p_I) + u_{cc} (30 - p_I) \left[ p_I - p_U - D (s') \right]$$

where $D (s')$ is the willingness to pay for the uninsured type $s'$ (where $s' \leq s$) and $p_I - p_U = D (s)$ is the equilibrium price of insurance when a fraction $s$ purchases. Therefore, the average marginal utility of the uninsured is approximately given by $u_c + u_{cc} [D (s) - E [D (s') | s \geq s']$, where $u_c$ and $u_{cc}$ are evaluated at $30 - p_I$. Therefore, the percentage difference between the marginal utility of insured and uninsured is given by

$$\frac{E [u_c | \text{Insured}] - E [u_c | \text{Uninsured}]}{E [u_c | \text{Insured}]} \approx \frac{-u_{cc}}{u_c} (D (s) - E [D (s') | s \geq s'])$$ (4)

where $-\frac{u_{cc}}{u_c}$ is the coefficient of absolute risk aversion (evaluated at the insured’s level of consumption, $30 - p_I$) and $D (s) - E [D (s') | s \geq s']$ is the difference between the willingness to pay of the average uninsured person and the price, $D (s) = p_I (s) - p_U (s)$, when a fraction $s$ of the market is insured. The steeper the demand curve, the greater the expected difference in marginal utilities between the insured and uninsured.

To estimate the ex-ante demand curve, the researcher needs one key additional piece of information: risk aversion. This can either be imported from another setting, or one can estimate it internally using the willingness to pay for insurance against remaining risk, as will be shown in Section 4.2. In the example, the coefficient of relative risk aversion is 3, so that the coefficient of absolute risk aversion is approximately $3/25$, where 25 is the average consumption in the population. Combining, this suggests the ex-ante value of insurance from expanding the market when exactly 50% have insurance is $EA (0.5) = 0.5 * 0.5 * (-10) * (3/25) * (5 - 2.5) = 0.75$. From behind the veil of ignorance, individuals are willing to pay an additional $0.75 to expand the size of the insurance market from 50% to 51% insured relative to what would be indicated by their demand curve (which equals $D (0.5) = 5$). This calculation is illustrated in Figure 2, Panel D.

Figure 2, Panel D calculates $EA (s)$ for all values of $s \in [0, 1]$ using the formula given by equation (4). Adding this ex-ante value to the market demand curve yields the “ex-ante demand curve”, $D^{Ex-Ante} (s) = D (s) + EA (s)$, depicted by the solid red line in Figure 2, Panel D. Integrating the ex-ante demand curve from a situation with no one obtaining insurance ($s = 0$) to full insurance

---

6For example, in a CARA-Normal model the coefficient of absolute risk aversion is equal to twice the ratio of the markup individuals are willing to pay for insurance relative to the variance reduction in out of pocket expenses it provides. Section 4.2 provides a more general characterization for more general utility functions and risk distributions. In this simple example, there is no remaining risk that drives insurance demand. As a result, willingness to pay does not reveal anything about risk aversion; but in more realistic empirical applications one in principle can estimate this risk aversion coefficient internally.
\( (s = 1) \) yields an integral of
\[
\int_0^1 D^{Ex-Ante} (s) = 5.50
\]
From behind the veil of ignorance, individuals are willing to pay $5.50 for a full insurance \((s = 1)\) allocation. Despite the approximations along the way, integrating the ex-ante demand curve recovers this ex-ante willingness to pay, \(D^{Ex-Ante}\) in equation (1).\(^7\) The cost of insuring everyone remains $5 so that individuals are willing to pay a $0.50 markup over the cost of the insurance for full insurance. This is the second main result of the paper: it provides formal conditions under which one recover the ex-ante willingness to pay for insurance using the market demand and cost curves, combined with an estimate of risk aversion.

The model in this section is of course highly stylized. For example, there is no moral hazard, no preference heterogeneity, and the model assumed all information about costs was revealed at the time of making the insurance decision. In this sense, it is not readily applicable to most empirical settings, such as the one considered in Einav et al. (2010). The next section steps back and derives the ex-ante demand curve in a more general setting.

### 3 General Model

This section sets up a general model that will derive the demand and cost curves of the Einav et al. (2010) from a utility function that will be used to measure ex-ante (utilitarian) welfare. While the language will generally refer to a health insurance context, it is straightforward to amend the model to capture other insurance settings, such as unemployment insurance. Proposition 1 will establish that the basic formula in equation (2) of the stylized example will continue to hold with a modification to the transfer term to account for the fact that individuals may not fully value their insurance payments at their costs. As in the stylized example, the key additional piece of information that is required to construct the ex-ante demand curve is the difference in marginal utilities of consumption for the insured relative to the uninsured. Section 4 will discuss the general conditions under which the approximation in equation 4 can be used to recover the difference in marginal utilities.

#### 3.1 Setup

Individuals face uncertainty over a future shock, captured by a random variable \(\theta\). After learning \(\theta\), individuals choose their non-medical consumption, \(c\), and medical spending, \(m\). Individuals have a utility function over these choices, \(u(c, m; \theta)\), that is affected by the shock, \(\theta\), which forms the source of uncertainty for the individuals. In addition to affecting their utility, the shock can also affect the individual’s income, \(y(\theta)\).

\(^7\)The approximation error from the Taylor expansion yields a difference that is noticeable only in the third decimal place.
There exists an insurance contract at price \( p_I \) that allows individuals to obtain medical services at cost \( x (m; \theta) \) yielding the budget constraint
\[
c^I (\theta) + x (m^I (\theta); \theta) + p_I \leq y (\theta)
\]
where \( y (\theta) \) is the individual’s income. Conversely, uninsured individuals must pay the full price\(^8\) of \( m \), yielding a budget constraint
\[
c^U (\theta) + m^U (\theta) + p_U \leq y (\theta)
\]
where \( p_U \) is a penalty or tax paid by individuals that are uninsured. Let \( \{ c^I (\theta), m^I (\theta) \} \) denote the choice of consumption and medical spending of a type \( \theta \) if she is insured, and \( \{ c^U (\theta), m^U (\theta) \} \) if she is uninsured.\(^9\)

At the time individuals make the decision to be insured or uninsured, individuals may know something about their particular type \( \theta \), which I denote by a signal \( \tilde{s} \in [0, 1] \). For simplicity, I follow Einav et al. (2010) and assume that only the relative price of insurance, \( p_I - p_U \), affects demand. Appendix B provides a general statement of Proposition 1 when demand is affected differentially by increases of \( p_U \) as opposed to decreases in \( p_I \).

Given \( \tilde{s} \), let \( D (\tilde{s}) \) denote the marginal price that a type \( \tilde{s} \) is willing to pay for insurance. This solves
\[
E \left[ u \left( y (\theta) - x (m^I (\theta); \theta) - D (\tilde{s}) - p_U, m^I (\theta); \theta \right) \bigg| \tilde{s} \right] = E \left[ u \left( y (\theta) - m^U (\theta) - p_U, m^U (\theta); \theta \right) \bigg| \tilde{s} \right]
\]
so that all \( \tilde{s} \) such that \( p_I - p_U \leq D (\tilde{s}) \) will choose to purchase insurance, whereas types \( \tilde{s} \) for which \( D (\tilde{s}) > p_I - p_U \) will choose to remain uninsured and pay the penalty \( p_U \). Without loss of generality, assume that \( \tilde{s} \) is ordered so that demand, \( D (\tilde{s}) \), is decreasing in \( \tilde{s} \).

Following Einav et al. (2010), define the average cost of insurance when a fraction \( s \) of the market owns insurance, \( AC (s) \), where
\[
AC (s) = E \left[ m^I (\theta) - x (m^I (\theta); \theta) \bigg| \tilde{s} \leq s \right]
\]
Let \( MC (s) \) characterize how this average cost changes as the size of the market expands, \( MC (s) = \frac{d}{ds} [sAC (s)] \), where \( sAC (s) \) is the total cost of the insured. Given the assumption that individuals’ choices are not affected by prices \( p_U \) and \( p_I \)\(^{10}\), this marginal cost is the net difference between

---

\(^8\)It is straightforward to generalize the model to allow for partial insurance, or multiple insurance contract choices.

\(^9\)The notation \( m^I (\theta) \) implies that \( m^I (\theta) \) is not a function of the price, \( p_I \). In principle, the choice of \( m^I (\theta) \) could depend on \( p_I \); for example, if insurance is cheaper, individuals may make riskier choices that increase health costs later on. For now, I adopt the common assumption that \( m^I (\theta) \) does not depend on \( p_I \), but it is straightforward to relax this assumption: if \( m^I \) depends on \( p_I \), then the cost to an insurer of raising/lowering their prices would also include a component from the impact of these price changes on the costs of their insured pool.

Similarly, I make the simplifying assumption that \( m^U (\theta) \) does not depend on \( p_U \). However, in contrast to the assumption that \( m^I (\theta) \) does not depend on \( p_I \), this assumption is without loss of generality because of the envelope theorem: \( m^U (\theta) \) is fully paid by the individual so that behavioral responses of \( m^U \) do not affect welfare measures of either the individual or other parties.

\(^{10}\)More generally, if prices do affect the cost to the insurer, this marginal cost function contains an additional term
expenditures and out-of-pocket spending for the marginal type, $s$:

$$MC\left(s\right) = E\left[ m^I\left(\theta\right) - x\left(m^I\left(\theta\right); \theta\right) | \bar{s} = s \right] \tag{7}$$

Finally, let $p_I\left(s\right)$ and $p_U\left(s\right)$ denote the prices of insurance and remaining uninsured when a fraction $s$ of the market owns insurance. By definition, these prices must be consistent with the definition of willingness to pay,

$$D\left(s\right) = p_I\left(s\right) - p_U\left(s\right) \tag{8}$$

I also assume for simplicity that the prices satisfy the resource constraint so that the total amount of resources collected from the insured and uninsured equals the total cost of the insured\(^\text{11}\):

$$sp_I\left(s\right) + \left(1 - s\right)p_U\left(s\right) = sAC\left(s\right) \tag{9}$$

The prices $p_I\left(s\right)$ and $p_U\left(s\right)$ are then defined implicitly as solutions to equations (9) and (8).

### 3.2 Ex-Ante Welfare

Ex-ante welfare is given by expected utility from behind the veil of ignorance. Let $W\left(s\right)$ denote this expected utility when prices are such that a fraction $s$ of the market owns insurance. This is given by

$$W\left(s\right) = \int_0^s E\left[ u\left(y\left(\theta\right) - p_I\left(s\right), m^I\left(\theta\right)\right) | \bar{s} \right] d\bar{s} + \int_s^1 E\left[ u\left(y\left(\theta\right) - m^U\left(\theta\right) - p_U\left(s\right), m^U\left(\theta\right); \theta\right) | \bar{s} \right] d\bar{s} \tag{10}$$

The first term integrates over those who, after learning $\bar{s}$, choose to be insured. The second term integrates over those who choose to be uninsured. The aim of this paper is to develop normative comparisons of policies such as subsidies and mandates that affect the size of the insurance market, $s$, that correspond to rankings in ex-ante expected utility, $W\left(s\right)$.

If one observed or estimated the utility function, one could directly measure $W\left(s\right)$. This would be analogous to the approach to measuring welfare taken by Finkelstein et al. (2016) and Handel et al. (2015). Here, I instead follow the “sufficient statistics” approach of Einav et al. (2010) and build a measure of $W\left(s\right)$ from the willingness to pay and cost curves.

Following Einav et al. (2010), I use the willingness to pay function, $D\left(s\right)$, to capture the impact on the utility of the uninsured,

$$E\left[u\left(y\left(\theta\right) - m^U\left(\theta\right) - p_U, m^U\left(\theta\right); \theta\right) | \bar{s} \right] = E\left[u\left(y\left(\theta\right) - D\left(\bar{s}\right) - p_U, m^I\left(\theta\right); \theta\right) | \bar{s} \right].$$

This yields an expression for $W\left(s\right)$ that does not require keeping track of the uninsured utility:

$$W\left(s\right) = \int_0^s E\left[u\left(y\left(\theta\right) - p_I\left(s\right), m^I\left(\theta\right)\right) | \bar{s} \right] d\bar{s} + \int_s^1 E\left[u\left(y\left(\theta\right) - D\left(\bar{s}\right) - p_U, m^I\left(\theta\right); \theta\right) | \bar{s} \right] d\bar{s}$$

reflecting the net cost of those behavioral responses on the insurance company.

\(^{11}\)It is straightforward to extend the model to the case when there are other parties who benefit from insurance subsidies changes in the price of insurance. In this case, one would still weight the transfers by the marginal utilities as in equation (11) below, but one would also need to account for the incidence of the price changes on these other parties (e.g. insurance companies). By imposing the resource constraint within the market population, the analysis can abstract from these third parties.
Now, consider a hypothetical policy change that expands the size of the market through increasing \( p_U \) and decreasing \( p_I \). For any \( s \), these prices, \( p_I(s) \) and \( p_U(s) \), must satisfy the resource constraint (Equation (9)) and are consistent with demand (Equation (8)). The marginal welfare impact of expanding the size of the market through this policy is given by

\[
W'(s) = -sp'_I(s) E[y(\theta) - p_I(s), m_I(\theta)] |\bar{s} \leq s]
\]

(11)

\[
-(1-s)p'_U(s) E[y(\theta) - D(\bar{s}) - p_U(s), m_I(\theta); \theta] |\bar{s} \geq s]
\]

(12)

The first term captures the welfare increase from lower prices for the insured \( (p'_I < 0) \). From behind the veil of ignorance, this price reduction of \( p'_I \) occurs with chance \( s \) and is valued using the marginal utility of income of the insured, \( E[y(\theta) - p_I(s), m_I(\theta)] |\bar{s} \leq s] \). The second term captures the welfare cost of having higher prices faced by the uninsured \( (p'_U > 0) \). This price increase occurs with a chance \( 1-s \) and is valued using the marginal utility of income of the uninsured, \( E[y(\theta) - D(\bar{s}) - p_U(s), m_I(\theta); \theta] |\bar{s} \geq s] \). Combining equation (11) with the resource constraint yields the main result.

**Proposition 1.** The marginal welfare impact of expanding the size of the insurance market from \( s^* \) to \( s^* + ds \) is given by

\[
W'(s^*) = \frac{E[y(\theta) - p_I(s), m_I(\theta); \theta] |s \leq s^*]}{D_{Ex-Ante(s)}} \approx \frac{D(s^*) + EA(s^*) - MC(s^*)}{\beta(s^*)}
\]

(13)

where \( EA(s^*) \) is the additional ex-ante value of expanding the size of the insurance market,

\[
EA(s^*) = (1 - s^*) \left( MC(s^*) - D(s^*) - s^*D'(s^*) \right) \beta(s^*)
\]

(14)

and \( \beta(s) \) is the percentage difference in marginal utilities of income for the insured relative to the uninsured,

\[
\beta(s) = \frac{E[y(\theta) - p_I(s), m_I(\theta); \theta] |s \leq s] - E[y(\theta) - D(\bar{s}) - p_U(s), m_I(\theta); \theta] |\bar{s} \geq s]}{E[y(\theta) - p_I(s), m_I(\theta); \theta] |s \leq s]}
\]

(15)

**Proof.** See Appendix C

Equation 11 shows that the marginal welfare of expanding the size of the insurance market is given by the sum of \( D(s) + EA(s) - MC(s) \). The term \( D(s) - MC(s) \) is the familiar market surplus term: expanding the size of the insurance market increases ex-ante welfare to the extent to which individuals are willing to pay more than their costs for insurance. But, in addition to this, \( EA(s) \) captures the ex-ante value of expanding the size of the market through its impact on insurance prices. Expanding the insurance market induces a transfer from uninsured to insured of size \((1 - s^*) (MC(s^*) - D(s^*) - s^*D'(s^*)) \). Note that this term reduces to the transfer in equation (2) when demand equals marginal cost, \( D(s) = MC(s) \), as in the stylized example. Moving
financial resources from the uninsured to the insured increases ex-ante welfare to the extent to which the marginal utility of income is higher for the insured than the uninsured, which is captured by the term $\beta(s^*)$.

The sign of $\beta(s^*)$  In most canonical models of insurance, one would expect $\beta(s^*) > 0$. For example, in the stylized example in Section 2, those who choose to purchase insurance expect to face a higher financial loss than those who remain uninsured. This means that the consumption levels of the insured are lower than those of the uninsured. Given concavity of the utility function, it would imply that the marginal utilities of the insured are higher than the uninsured, so that $\beta(s^*) > 0$.

In principle, it is also possible to have $\beta(s^*) < 0$. For example, $\theta$ could reflect a liquidity or income shock to $y(\theta)$ so that the primary driver of the decision to purchase insurance is not a higher expected cost, but rather an income shock. If the uninsured are foregoing insurance purchase because of this liquidity shock (and the cost of insurance is not too high), then it is feasible that those who forego insurance have a higher marginal utility of income than those who purchased, $\beta(s^*) < 0$. In this case, expanding the size of the insurance market will transfer resources from the liquidity constrained to those who are less constrained, which would suggest that $EA(s^*) < 0$. Going forward, most of the discussion will consider the benchmark case where $\beta(s^*) > 0$. But, this highlights the value of future work on the determinants of the insurance purchase and the implications for $\beta(s^*)$.

3.3 Implications for Welfare Analysis

Optimal Size of the Insurance Market  The ex-ante welfare perspective can lead to different conclusions about government intervention in insurance markets. To begin, consider the optimal size of the insurance market. From behind the veil of ignorance, the size of the market that maximizes ex-ante expected utility will set $W'(s^{EA}) = 0$, so that\(^\text{12}\):

\[
D^{Ex-Ante}(s^{EA}) \equiv D(s^{EA}) + EA(s^{EA}) = MC(s^{EA})
\tag{16}
\]

This equates the ex-ante demand to expand the size of the insurance market, $D^{Ex-Ante}(s) \equiv D(s) + EA(s)$, equal to the cost of enrolling the marginal entrant, $MC(s)$.

For comparison, one can also calculate the size of the market, $s^{MS}$, that maximizes market surplus, as proposed by Einav et al. (2010). This will maximize the area between the market demand and cost curves, so that the surplus maximizing size of the market will set market demand equal to the cost of the marginal enrollee:

\[
D(s^{MS}) = MC(s^{MS})
\]

\(^{12}\)If these equations do not have unique solutions, the optimal allocations are those involving the largest fraction of the market insured.
From behind the veil of ignorance, there is a marginal welfare gain of $EA(s)$ from expanding the insurance market beyond the size of the market that sets $D(s) = MC(s)$. As long as the insured have higher marginal utility than the uninsured so that $\beta(s) > 0$, the size of the insurance market that maximizes ex-ante welfare is larger than the size of the market that maximizes market surplus. In this sense, the optimal size of the insurance market from behind the veil of ignorance necessarily will involve marginal deadweight loss in the language of Einav et al. (2010), $MC(s) > D(s)$. As long as the insured have higher marginal utility than the uninsured so that $s > 0$, the size of the market that maximizes ex-ante welfare is larger than the size of the market that maximizes market surplus. In this sense, the optimal size of the insurance market from behind the veil of ignorance necessarily will involve marginal deadweight loss in the language of Einav et al. (2010), $MC(s) > D(s)$. Re-arranging the terms in equation (16), the ex-ante optimal size of the market equates marginal deadweight loss to the ex-ante welfare gain from expanding the size of the insurance market, $EA(s) = MC(s) - D(s)$. From behind the veil of ignorance, individuals are willing to incur a loss of market surplus to the extent to which it helps facilitate ex-ante risk protection.

**Quantifying Inefficiencies: Competitive Markets** In addition to using the ex-ante demand curve to calculate the optimal size of the insurance market, it also quantifies the welfare loss from inefficient allocations that differ from $s^{EA}$. For example, competitive markets may lead to an inefficient provision of insurance because prices must reflect average enrollee, not the costs imposed by the marginal enrollee (Akerlof (1970)). Let $s^{CE}$ denote the largest size of the market that satisfies $D(s^{CE}) = AC(s^{CE})$. The welfare loss from behind the veil of ignorance of having a market allocation of $s^{CE}$ instead of $s^{EA}$ is given by integrating the difference between $D^{Ex-Ante}(s)$ and $MC(s)$ between $s^{CE}$ and $s^{EA}$:

$$W^{CE} \approx \int_{s^{CE}}^{s^{EA}} (D^{Ex-Ante}(s) - MC(s)) \, ds$$

In contrast, one can also compare this to the welfare cost from adverse selection as measured by market surplus, as in Einav et al. (2010). This is given by integrating the difference between $D(s)$ and $MC(s)$ between $s^{CE}$ and the size of the market that maximizes market surplus, $s^{MS}$:

$$M^{CE} = \int_{s^{CE}}^{s^{MS}} [D(s) - MC(s)] \, ds$$

It is straightforward to show that as long as the cost curves slope down (i.e. there is adverse selection) and $\beta(s) > 0$, then $W^{CE} > M^{CE}$.

**Quantifying Inefficiencies: Mandates versus Markets** In addition to calculating the inefficiency of competitive allocations, $s^{CE}$, one can also evaluate the desirability of government interventions such as mandates. If the optimal size of the market is not full insurance, then mandates (or sufficiently high subsidies) generate a welfare loss relative. Integrating between the ex-ante demand curve and the cost curve from $s^{EA}$ to 1 quantifies the size of this welfare loss

$$W^{M mandate} = -\int_{s^{EA}}^{1} (D^{Ex-Ante}(s) - MC(s)) \, ds$$

13
In this sense, one can ask which is better: a competitive market allocation, \( s^{CE} \), or a mandate \( s = 1 \)? The answer to this question depends on the sign of \( W^{CE} - W^{Mandate} \). On the one hand, increasing the size of the market from \( s^{CE} \) to \( s^{EA} \) increases ex-ante welfare by \( W^{CE} \); but further expanding the market to \( s = 1 \) leads to a welfare loss of \( W^{Mandate} \).

A similar analysis can be conducted using market surplus, as in Einav et al. (2010). Here, one can form the lost market surplus from expanding the size of the market from the surplus maximizing size, \( s^{MS} \), to \( s = 1 \):

\[
M^{Mandate} = -\int_{s^{MS}}^{1} (D^{Ex-Ante}(s) - MC(s)) \, ds
\]

As shown in Einav et al. (2010), whether a mandate delivers greater market surplus relative to a competitive market depends on \( M^{CE} - M^{Mandate} \). If cost curves slope down and \( \beta(s) > 0 \), then it is straightforward to see that the welfare cost of a mandate is smaller from behind the veil of ignorance relative to what is suggested by lost market surplus \( W^{Mandate} < M^{Mandate} \). As a result, there may be cases in which government mandates yield to higher ex-ante welfare but lower market surplus than competitive markets. I return to this again in Section 5.

4 Implementation Using Market Demand and Cost Curves

While the analysis above provides theoretical statements about government policies from an ex-ante welfare perspective, empirically constructing ex-ante willingness to pay curve requires an estimate of the difference in marginal utilities between insured and uninsured, \( \beta(s) \). This section provides conditions under which one can write \( \beta(s) \) as a function of market level demand curves combined with a measure of risk aversion as in equation 4 in the stylized example of Section 2. This estimate of risk aversion can either be imported from external settings (e.g. a coefficient of relative risk aversion of 3 or absolute risk aversion of \( 5 \times 10^4 \)), or it can be estimated internally using the relationship between the markup individuals are willing to pay for the variance reduction offered by the insurance.

These assumptions are not without loss of generality, but are common in the literature on optimal unemployment insurance (Baily (1978); Chetty (2006a)) and many of these assumptions are satisfied in the structural models generally used to estimate the marginal WTP for insurance (Handel et al. (2015)). In this sense, it provides a benchmark method to infer whether an ex-ante welfare perspective can lead to different welfare conclusions. Section 4.3 shows how relaxing these assumptions is possible if one observes additional data elements.

4.1 Derivation

A Taylor expansion of the utility function illustrates the potential sources of differences in marginal utilities between insured and uninsured. Recall that if a fraction \( s \) of the market is insured, then all types \( \theta \) who have willingness to pay \( D(\bar{s}) \geq p_I - p_U = D(s) \) will purchase insurance. Let \( \bar{y} = E[y(\theta) | \bar{s} \leq s] \) denote the average income of the insured. And, let \( \bar{c} = \bar{y} - p_I \) denote the
average consumption of the insured. To help illustrate the role of preference heterogeneity, assume \( \theta \) is a uni-dimensional index, \( \theta \in \mathbb{R} \), and assume that the utility function, \( u(c, m; \theta) \), is continuously differentiable with respect to \( \theta \). To a first order Taylor approximation, the marginal utility of the insured, \( E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \hat{s} \leq s \right] \), is given by the marginal utility of the average type, \( u_c(c, \bar{m}, \bar{\theta}) \), where \( \bar{\theta} = E[\theta | \hat{s} \leq s] \) denotes the average type of the insured.\(^{13}\)

How does this compare to the marginal utility of an uninsured type, \( \theta \), for whom \( D(\hat{s}) < p_I - p_U = D(s) \)? A Taylor expansion shows that the difference between \( u_c(c, \bar{m}, \bar{\theta}) \) and marginal utility of an uninsured type is given by

\[
\begin{align*}
\Delta u_c &= u_c(y(\theta) - D(\hat{s}) - p_U, m^I(\theta); \theta) - u_c(c, \bar{m}, \bar{\theta}) \\
&= u_{cc}(y(\theta) - D(\hat{s}) - p_U - (\hat{y} - p_I)) + u_{cm}(m^I(\theta) - \bar{m}) + u_{c\theta}(\theta - \bar{\theta})
\end{align*}
\]

where \( u_{cc}, u_{cm}, \) and \( u_{c\theta} \) are all evaluated at \((c, \bar{m}, \bar{\theta})\). The difference in average marginal utilities between the insured and uninsured is driven by three sources: differences in consumption, \( y(\theta) - D(\hat{s}) - p_U - (\hat{y} - p_I) \), medical spending, \( m^I(\theta) - \bar{m} \), and differences in preferences, \( \theta - \bar{\theta} \).

While the model in Section 3 is general in that it can allow for arbitrary heterogeneity and complementarities in the utility function, one can impose assumptions to reduce the empirical estimation requirements. To begin, it is certainly conceivable that spending more on medical care can increase the marginal utility of consumption, \( u_{cm} \). But, absent clear evidence on this, a natural benchmark is to assume that the marginal utility of consumption does not depend on the level of medical spending.

**Assumption 1. (No Complementarities/Substitutabilities between c and m)** The marginal utility function, \( u_c(c, m; \theta) \), does not depend on \( m \).

This assumption implies that \( u_{cm}(m^I(\theta) - \bar{m}) = 0 \). It would be satisfied if, for example, preferences over \( c \) and \( m \) are additively separable (e.g. \( u(c, m; \theta) = a(c; \theta) + b(m; \theta) \) for some functions \( a \) and \( b \)).

Next, consider the last term in equation (17). It again is certainly possible that differences in preference realizations or health shocks, \( \theta \), lead to differences in insurance demand. For example, those who choose to purchase insurance may have higher risk aversion (and thus higher values of \( u_{cc} \)). But, it is not immediately clear whether there is systematic differences between the insured

\(^{13}\)To see this, note that one can write \( u_c(y(\theta) - p_I(s), m^I(\theta); \theta) \) as

\[
\Delta u_c = u_{cc}(y(\theta) - p_I - \bar{c}) + u_{cm}(m^I(\theta) - \bar{m}) + u_{c\theta}(\theta - \bar{\theta})
\]

where subscripts denote derivatives and \( u_{cc}, u_{cm}, \) and \( u_{c\theta} \) are evaluated at \((\hat{c}, \bar{m}, \bar{\theta})\). Hence,

\[
E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \hat{s} \leq s \right] \approx E\left[u_{cc}(y(\theta) - p_I - \bar{c}) + u_{cm}(m^I(\theta) - \bar{m}) + u_{c\theta}(\theta - \bar{\theta}) | \hat{s} \leq s \right]
\]

which equals zero by the definition of \( \hat{c}, \bar{m}, \) and \( \bar{\theta} \).

\(^{14}\)Note that the term \( u_{cm} \) holds fixed the level of \( \theta \). This means that \( u_{cm} \) is not about whether sicker or healthier have higher marginal utilities of consumption but rather conditional on a health state, does higher medical spending increase or decrease the marginal utility of consumption.
and uninsured in their preferences, \( \theta \), that would generate differences in marginal utilities, \( u_c \).

Thus, a benchmark assumption is to rule out a role of preference heterogeneity in generating differences in marginal utilities of consumption.

**Assumption 2.** (Common Preferences) The marginal utility function, \( u_c(c, m; \theta) \), does not depend on \( \theta \)

This assumption implies that \( u_{c\theta}(\theta - \bar{\theta}) = 0 \) so that the last term in equation (17) is equal to zero.

Under Assumptions 1-2, the difference in marginal utilities depends only on the difference in consumption, \( y(\theta) - D(\bar{s}) - p_U - (\bar{y} - p_I) \), and the curvature of the utility function, \( u_{cc} \). Moreover, the difference in consumption results from two components. First, there is a difference between the willingness to pay of the uninsured type, \( D(\bar{s}) \), and the price of insurance, \( p_I - p_U \). Second, there is a potential difference between income, \( y(\theta) \), and the income of the average insured, \( \bar{y} \). The third assumption rules out differences in marginal utilities driven by income differences.

**Assumption 3.** (No Liquidity / Income Differences) Income does not systematically vary between insured and uninsured, \( \bar{y} = E[y(\theta) | \bar{s} \leq s] = E[y(\theta) | \bar{s} > s] \).

Assumption 2 rules out liquidity effects as a primary source of variation in demand for insurance. As discussed in Section 4.3, one can incorporate liquidity effects if one is able to observe the average income levels of the insured and uninsured.

Assumption 3 implies that the difference in demand between the marginal insured type, \( D(s^*) = p_I - p_U \), and the average uninsured type, \( E[D(s) | s \geq s^*] \), drives differences in consumption between the insured and uninsured. Combined with Assumptions 1-2, this allows one to infer \( \beta(s^*) \) solely from the curvature of the utility function

**Proposition 2.** Suppose Assumptions 1-3 hold. Then,

\[
\beta(s^*) \approx \gamma(s^*) (D(s^*) - E[D(s) | s \geq s^*]) \tag{18}
\]

where the \( \approx \) denotes a first-order Taylor approximation and \( \gamma(s^*) = \frac{u_{cc}(c, \bar{m}; \theta)}{u_c(c, \bar{m}; \theta)} \) is the coefficient of absolute risk aversion. The ex-ante component of willingness to pay is given by

\[
EA(s^*) \approx (1 - s^*) \left( MC(s^*) - D(s^*) - s^* D'(s^*) \right) (s^*) (D(s^*) - E[D(s) | s \geq s^*]) \tag{19}
\]

so that it is identified from the demand and cost curves, combined with a coefficient of absolute risk aversion, \( \gamma \).

\[^{15}\text{As discussed in Section 4.3, one potential source of a relationship between \( \theta \) and \( u_c \) would be if health status affects the marginal utility of income as in Finkelstein et al. (2013).}\]

\[^{16}\text{This representation is analogous to the Baily-Chetty condition that characterizes the difference in the marginal utility of unemployed and employed using their difference in consumption multiplied by a coefficient of curvature of the utility function (Baily (1978); Chetty (2006b)).}\]

\[^{17}\text{For example, in the example in Section 2, the willingness to pay of the average uninsured person is less than the price of insurance, \( p_I - p_U \), capturing the fact that the uninsured have a lower expected losses.}\]"
Proof. Imposing Assumptions 1-3 to equation (17) and dividing by the marginal utility of income of the insured, \( u_c(\tilde{c}, \tilde{m}; \tilde{\theta}) \) yields the result. 

Because the demand and cost curves are the same requirements to measure market surplus as in the Einav et al. (2010) setting, equation (19) provides a benchmark method to ascertain whether there is an important distinction between ex-ante and observed demand with minimal additional data requirements.

### 4.2 Measuring Risk Aversion

In addition to the demand and cost curves in the Einav et al. (2010) framework, measuring ex-ante willingness requires an estimate of risk aversion, \( \gamma(s) \). This can imported from another setting (as will be done in Section 5 below). Or, one can consider a range of risk aversion parameters to understand the size of the potential difference offered by an ex-ante welfare perspective.

However, it is important to note that in principle the the risk aversion coefficient can be inferred within the demand and cost curve setup. One can estimate a risk aversion coefficient by comparing individual’s willingness to pay for insurance to the reduction in variance of expenditures that is provided by the insurance product. For example, it is well-known that if preferences have a constant absolute risk aversion and the risk of medical expenditures is normally distributed (i.e., a “CARA-Normal” model), then the markup individual’s are willing to pay for insurance by is given by the variance reduction offered by the insurance multiplied by \( \frac{\gamma(s)}{2} \).

More generally, one can consider a second-order Taylor approximation to equation (5) that characterizes willingness to pay, \( D(\tilde{s}) \). Let \( p(\tilde{s}) = \frac{\partial \tilde{x}}{\partial \tilde{m}} \) denote the price of additional medical spending when insured. If one assumes that \( u_{cm} = 0 \) (Assumption 1) and adds an additional assumption that \( u_{mm} = 0 \), then the it is straightforward to show\(^{18}\) that the coefficient of absolute risk aversion is given by:

\[
\gamma(\tilde{s}) = \frac{2D(\tilde{s}) - MC(\tilde{s}) + (1 - p(\tilde{s}))E[m^I - m^U | \tilde{s}]}{V} \tag{20}
\]

where \( D(\tilde{s}) - MC(\tilde{s}) \) is the markup individuals of type \( \tilde{s} \) are willing to pay above the cost they impose on the insurer, \( V \) is approximately the reduction in variance of consumption offered by the

\(^{18}\)To see this, suppress notation w.r.t. \( \theta \) and condition all expectations on \( \tilde{s} \). Let \( (\tilde{c}, \tilde{m}, \tilde{\theta}) \) denote the average bundle of an \( \tilde{s} \) type. Taking a Taylor expansion to the utility function around this bundle in equation (5) yields:

\[
\begin{align*}
&u_c(\tilde{s}) \left( E[y - x^I - D(\tilde{s}) - pu - \tilde{c}] \right) + \frac{1}{2} u_{cc} \left( \left[ E[y - x^I - D(\tilde{s}) - pu - \tilde{c}] \right]^2 \right) + u_m E[m^I - \tilde{m}] = \quad u_c(\tilde{s}) \left( E[y - pu - \tilde{c}] \right) + \frac{1}{2} u_{cc} \left( \left[ E[y - pu - \tilde{c}] \right]^2 \right) + \\
&u_c \left( \left( E[y - x^I - D(\tilde{s}) - pu - \tilde{c}] \right) - E[y - pu - \tilde{c}] \right) = \quad \frac{1}{2} u_{cc} \left( E[y - x^I - D(\tilde{s}) - pu - \tilde{c}] \right)^2 \quad - E[y - pu - \tilde{c}] \\
&\quad D(\tilde{s}) - \left( E[m^I - x^I] \right) = \quad \frac{\gamma(\tilde{s})}{2} V + \left( \frac{u_m}{u_c} - 1 \right) E[m^I - m^U] \\
&\quad \gamma(\tilde{s}) = \quad \frac{2}{V} \left( \frac{D(\tilde{s}) - MC(\tilde{s}) + (1 - \frac{u_m}{u_c})E[m^I - m^U]}{V} \right)
\end{align*}
\]

where \( MC(\tilde{s}) = E[m^I - x^I | \tilde{s}] \) is the cost to the insurer of enrolling the type \( \tilde{s} \).
insurance:

\[ V = E \left[ (y - x^U - p_U) \cdot (\hat{c})^2 \right] - E \left[ (y - x^I - D(\hat{s}) - p_U) \cdot \hat{c}^2 \right] \]

and \((1 - p(\hat{s})) \cdot E \left[ m^I - m^U | \hat{s} \right]\) is a correction term to account for moral hazard. \(E \left[ m^I (\theta) - m^U (\theta) | \hat{s} \right]\) is the causal effect of insurance on medical spending to a type \(\theta\). If \(p(\hat{s}) < 1\), some of this additional cost that is imposed on the insurer will not be fully valued by the individual.

In this sense, one needs to observe two additional pieces of information in order to generate an internal measure of risk aversion, \(\gamma(\hat{s})\): (1) the impact of insurance on medical spending for type \(\hat{s}\), \(E \left[ m^I (\theta) - m^U (\theta) | \hat{s} \right]\) and (2) the impact of insurance on the variance of consumption, \(V(\hat{s})\). In this sense, one need not necessarily rely on an external measure of risk aversion, but can instead infer risk aversion from individuals revealed willingness to pay to reduce their variance in consumption.

4.3 Violations of Assumptions

Assumptions 1-3 provide a method to estimate \(EA(s)\) using the market demand and cost curves combined with an estimate of risk aversion, \(\gamma(s)\). Here, I show how one can relax Assumptions 1-3, but doing so introduces additional empirical requirements.

**Assumption 1** To begin, consider Assumption 1. Although this assumption is satisfied in many models of insurance that do not allow for \(m\) to be a separate argument of the utility function, Assumption 1 is violated if consumption of medical spending is a substitute (or complement) to consumption. In this case, \(u_{cm} \neq 0\) and \(\beta(s^*)\) can be written as:

\[
\beta(s^*) \approx \gamma(D(s^*) - E[D(s)|s \geq s^*]) + \frac{u_{cm}}{u_c} (E[m|s \geq s] - \bar{m})
\]

where \(\frac{u_{cm}}{u_c} = \frac{u_{cm}(c,\bar{m},\theta)}{u_c(c,\bar{m},\theta)}\) measures how the marginal utility of consumption varies with the level of medical spending (holding \(c\) and \(\theta\) constant). This complementarity/substitutability of the utility function could be estimated with exogenous variation in both income and prices of medical spending, \(m\). In particular, \(\frac{u_{cm}}{u_c}\) would govern how individuals’ budget allocation between \(c\) and \(m\) varies if one faces higher prices for \(m\) but is compensated with an equivalent increase in income.

**Assumption 2** Assumption 2 would be violated if those who purchase insurance have a different marginal utility of consumption even if they have the same level of consumption. In this case, \(u_{c\theta} \neq 0\) so that

\[
\beta(s^*) = \gamma(D(s^*) - E[D(s)|s \geq s^*]) + \frac{u_{c\theta}}{u_c} (E[\theta|s \geq s] - \bar{\theta})
\]

One potential reason for \(u_{c\theta} \neq 0\) would be if the marginal utility of consumption depended on health status. If sicker people have lower marginal utilities of income (as in Finkelstein et al. (2013)), and
the sick are more likely to purchase insurance, then those who purchase insurance may have lower marginal utilities of income than those who choose not to purchase insurance.

However, one might have thought that heterogeneous risk aversion could generate a violation of Assumption 2. For example, one would expect that the insured might have higher risk aversion than the uninsured. But, this does not necessarily lead to a violation of Assumption 2: it is not necessary that those with greater curvature in the utility function (second derivatives of \( u \)) also have greater (or lower) marginal utilities (first derivatives of \( u \)). For example, if individuals have CRRA preferences, \( u(c) = \frac{1}{1-\sigma}c^{1-\sigma} \) with heterogeneous \( \sigma \), then those with higher \( \sigma \) would have a higher preference for insurance but would have a lower marginal utility of consumption, \( c^{-\sigma} \), so that \( u_{c\theta} < 0 \). But, if individuals have CRRA preferences, \( u(c) = k\theta c^{1-\theta} \), this utility function exhibits the same willingness to pay for insurance for a type \( \theta \) but will have \( u_c = k\theta (1-\theta) c^{-\theta} \), which will be increasing in \( \theta \) for sufficiently large \( k \).

Nonetheless, if individuals have preference heterogeneity, the key question is whether those who choose to purchase insurance have preference that tend to generate higher (or lower) marginal utilities of consumption. To the extent to which this is the case, one can modify the equation for \( \beta(s*) \) by incorporating an estimate of the size of this preference heterogeneity.

**Assumption 3**  Heterogeneous income realizations, \( y(\theta) \), could be a driver of insurance demand, as noted in the end of Section 3.2. If incomes differ between the insured and uninsured, then one can estimate a modified formula for \( \beta(s*) \) as

\[
\beta(s*) = \gamma \left( D(s*) - E[D(s) | s \geq s^*] \right) + E[y(\theta) | \bar{s} \leq s] - E[y(\theta) | \bar{s} > s]
\]

where \( E[y(\theta) | \bar{s} \leq s] - E[y(\theta) | \bar{s} > s] \) is the difference in incomes between the insured and uninsured. If the insured have higher incomes than the uninsured, then the benchmark formula for \( \beta(s*) \) in equation (18) will understate the ex-ante willingness to pay for insurance. However, if one can estimate this difference in average incomes, one can readily modify the ex-ante demand curve to account for this heterogeneity.

**Additional Assumptions**  Beyond Assumption 1-3, it is also useful to discuss the a couple other assumptions that are embedded into the model. First, the Taylor expansion illustrates that the approximation of marginal utilities relies on there being only three potential sources of differences: consumption, medical expenditure, and preference heterogeneity. If there were additional arguments of the utility function, this would need to be incorporated into the calculation. On example of this is dynamics. If there were multiple arguments to consumption and individuals who are insured can spread their payment of insurance across multiple periods, then consumption will not be given by \( y(\theta) - p_I \), but rather \( y(\theta) - \frac{1}{T} p_I \) where \( T \) is the number of periods one can smooth. It is straightforward to show that this implies \( \beta(s*) = \frac{1}{T} \gamma E[D(s*) - D(s) | s \geq s^*] \). Intuitively, only \( 1/T \) of the willingness to pay actually comes from consumption in any given period. In this case, the difference in marginal utilities between the insured and uninsured types will be smaller than is
suggested by the difference in willingness to pay, $E[D(s^*) - D(s) | s \geq s^*]$. However, the difference in marginal utilities between insured and uninsured remains the key ingredient for measuring ex-ante willingness to pay.

Finally, this section has relied heavily on Taylor approximations to obtain a measure for $\beta(s^*)$. To the extent to which utility is quadratic over consumption, the approximation will be exact. But, more generally one would also have a term involving the coefficient of relative prudence, $-\frac{u_{ccc}}{u_c}$, as in Chetty (2006a) for the case of unemployment insurance. Plausible estimates of these higher order utility parameters is rare; incorporating them into the analysis generally involves assuming a functional form for the utility function. For this reason, I proceed using the first-order Taylor approximation for $\beta(s^*)$ in equation (18).

5 Illustration using Three Examples

I illustrate the construction of the ex-ante willingness to pay curves using estimates from Einav et al. (2010). Einav et al. (2010) use variation in prices across business units of Alcoa to estimate demand and cost curves for a more generous health insurance policy relative to a less generous policy. The annual prices and average costs are roughly $400-500, roughly an order of magnitude smaller than the costs that would be associated with a full insurance policy, consistent with this being a top-up decision, not a decision to purchase insurance versus forego insurance altogether.

I will consider three variants of the estimates in Einav et al. (2010). To motivate these scenarios, it is helpful to illustrate how the size of the insurable risk affects the divergence between ex-ante and market demand, $EA(s)$. Equation (19) shows that the size of $EA(s)$ is increasing in the square of the size of the demand and cost curves. To see this, note that replacing $MC(s)$ and $D(s)$ with $aMC(s)$ and $aD(s)$ yields $a^2EA(s)$. This suggests that the distinction between observed demand, $D(s)$, and the ex-ante demand curve, $D(s) + EA(s)$, is increasing in the square of the size of the insurable risk. Thus, the ex-ante welfare perspective diverges more from market surplus in cases when the risk represents a larger fraction of an individual’s consumption.

Because of this, and the fact that the Einav et al. (2010) setting considers a fairly small risk, I consider two additional variations of the estimates in Einav et al. (2010) in addition to their estimates. In particular, I consider a “Medium Risk” scenario that scales all the demand and cost curves by a factor of four. And, I consider a “Large Risk” scenario that scales demand and cost curces by a factor of ten. This large risk scenario leads to prices and costs that are closer to those faced in a market for a full health insurance policy, as opposed to a top-up policy.

In practice, whether or not individuals are able to save may be less of a concern if one uses “internal” measures of risk aversion from equation (20). The value of $\gamma$ that is identified in equation (20) is the willingness to pay for a reduction in variance in out-of-pocket expenditures. If individuals are able to save, then the full variance reduction offered by insurance is actually smaller than $\text{var}(m^r(\theta)) - \text{var}(x(m^r(\theta); \theta))$. In this sense, the $\gamma$ estimated in equation (20) reflects the willingness to pay for a reduction in out of pocket spending, analogous to the willingness to pay for a reduction in the price of insurance. In this sense, ex-ante willingness to pay estimates that rely on internal measures of risk aversion are robust to cases in which the individuals are able to save.

For example, Chetty (2006a) exploits the assumption that individuals have constant relative risk aversion over consumption so that the coefficient of relative prudence is equal to the coefficient of relative risk aversion minus one.
**Background: Demand and Cost Curves from Einav et al. (2010)** Figure 3, Panel A presents the demand and cost curve estimates from Einav et al. (2010). The solid black line presents the estimated demand curve. A $100 increase in the price of insurance leads to a 7pp reduction in the fraction of the market purchasing the more generous policy. Higher prices also lead to a higher average cost of the insured: A $100 increase in price that reduces the market size by 7pp leads to a higher average cost of the insured population of roughly $15.5, as illustrated by the long-dashed blue line. Taking the derivative of total costs yields the marginal cost curve, illustrated by the dashed red line.

A competitive equilibrium in this environment would result in roughly $s^{CE} = 61.7\%$ of the market purchasing the more generous policy, reflected by the intersection between the average cost curve and the demand curve in Panel A. This occurs with a price of $D(s^{CE}) = AC(s^{CE}) = $463.5. But, those that are indifferent to purchasing insurance at a price of $463.5$ on average impose a cost on the insurance company, $MC(s^{CE})$, that is less than their willingness to pay. Aggregating across these potential trades for which demand is above marginal cost, the aggregate lost surplus from adverse selection is $M^{CE} = $9.57. To place this in perspective, this is roughly 3.3% of the size of the market of $286 (61.7\% of $463.5). In this sense, the welfare cost from adverse selection is small.

**Calculation of Ex-Ante Willingness to Pay** While the demand curve suggests individuals at $s^{CE} = 0.617$ are willing to pay $463.5$ for insurance, how much are people willing to pay for a larger insurance market from behind the veil of ignorance? Constructing the ex-ante willingness to pay curve requires an estimate of risk aversion. For this, I take an estimate of $\gamma = 5 \times 10^{-4}$, similar to the mean estimate in Handel et al. (2015) who estimate preferences using choices over employer-provided health plans. As discussed above, an alternative approach would be to calculate the level of risk aversion that is internally consistent with the Einav et al. (2010) demand estimates using equation (20). While this option would likely be preferred from a theoretical standpoint, in practice Einav et al. (2010) do not provide an estimate of the impact of insurance on the variance of medical spending, $V$, or the moral hazard impact of the insurance, $E[mI(\theta) - mU(\theta)|s]$.

Figure 3 Panel B presents the ex-ante demand curve, $D(s) + EA(s)$ in red. To calculate $EA(s)$, I use the formula in equation (19). Note that the linearity of demand implies $E[D(s^*) - D(s)|s \geq s^*] = D'(1 - s^*)$, so that

$$EA(s^*) = (1 - s^*) (MC(s^*) - D(s^*) - s^* D') \gamma D'(1 - s^*) / 2$$

For example, at $s^* = 61.7\%$ (the competitive equilibrium value), the ex-ante component of demand

---

21One could infer a rough approximation for $V$ using the implied variance of the bins in Figure IV of Einav et al. (2010); this yields $V = 22,687$. Combined with the additional assumption of no moral hazard, this would suggest $\gamma \approx 2.463 \times \frac{22687}{286^2} = .01$, which is much larger than most estimates of risk aversion. However, much of this calculation rests on an accurate calculation of the marginal cost curve, which is statistically imprecise in the Einav et al. (2010) setting. Although they find robust evidence that the average cost curve slopes down, the statistical imprecision in the exact level of demand and marginal cost provides an additional rationale for importing the risk aversion coefficient.
is given by

\[ EA(0.617) = (1 - 0.617) (MC(0.617) - D(0.617) - 0.617D') \gamma D' \frac{(1 - 0.617)}{2} \]

Plugging in \( MC(0.617) = 325.88 \), \( D(0.617) = 463.5 \), and \( D' = -1435.97 \), along with \( \gamma = 5 \times 10^{-4} \), yields

\[ EA(0.617) = 39.4 \]

as reflected by the difference between the observed demand curve, \( D(s) \), and the ex-ante demand curve, \( D(s) + EA(s) \), in Figure 3, Panel B. From behind the veil of ignorance, individuals are willing to pay $502.9, as opposed to $463.5 to expand the size of the market when 61.7% of the market is insured.

The ex-ante demand curve intersects the cost curve when a fraction \( s^{EA} = 77.7\% \) of the market is insured, fairly similar to the size of the market that maximizes market surplus, \( s^{MS} = 75.6\% \). Integrating between the ex-ante demand curve and cost curve between \( s^{CE} \) and \( s^{EA} \) yields an ex-ante welfare cost of adverse selection of \( W^{Ex-Ante} = 14.25 \). Put differently, from an ex-ante perspective prior to learning their demand for insurance in this market, individuals would be willing to pay $14.25 to have an optimally priced insurance market in which 77.7% of the population is insured. This suggests that market surplus captures the two thirds (67%) of the ex-ante welfare cost of adverse selection, \( \frac{M^{CE}}{W^{Ex-Ante}} \approx 0.67 \).
Medium Risk: Scaling Demand and Cost Curves from Einav et al. (2010) by 4x  The top-up insurance setting of Einav et al. (2010) considers an insurance product that insures against a relatively small fraction of total medical expenses. The next two examples illustrate how the size of the insurable risk affects the results. First, I consider a 4x scaling of the demand and cost curves in Einav et al. (2010). This scaling has no impact on the fraction of the market that is insured in a competitive equilibrium, \( s^{CE} = 61.7\% \) or on the size of the market that maximizes market surplus, \( s^{MS} = 75.6\% \). However, it does significantly change the ex-ante demand curve. From behind the veil of ignorance, the optimal size of the insurance market is \( s^{EA} = 81.6\% \), and the ex-ante welfare cost of adverse selection in a competitive equilibrium is \( W^{Ex-Ante} = $120.62 \), which is more than three times as large as the lost market surplus of \( M^{CE} = $38.26 \). In this case, lost market surplus captures only 32% of the ex-ante welfare cost of adverse selection.
Large Risk: Scaling Demand and Cost Curves from Einav et al. (2010) by 10x  

Next, I consider a “large risk” example that scales the demand and cost curves in Einav et al. (2010) by a factor of 10. This yields average costs around $5,000, which more closely approximates the costs associated with the extensive margin purchases of health insurance. As shown in Panel D of Figure 3, there is a larger divergence between ex-ante and observed demand. Ex-ante welfare is maximized when a fraction $s^{Ex-Ante} = 85.6\%$ of the market is insured, and the ex-ante welfare cost from adverse selection is $W^{Ex-Ante} = 427$. This contrasts with the lost surplus from adverse selection of $M^{CE} = 77$, which captures only 18% of the total ex-ante welfare cost of adverse selection. In short, for large risks, the ex-ante welfare perspective can generate significantly different conclusions about the optimal size of the insurance market and the welfare cost from competitive equilibrium distortions from adverse selection.

Markets versus Mandates  

In addition to the welfare cost of adverse selection, one can also evaluate the desirability of government intervention such as mandates. In particular, one can ask: Do government insurance mandates deliver higher welfare than competitive insurance markets?

Consider the medium risk scenario for simplicity. Figure 4 presents the observed and ex-ante demand curve, along with the marginal cost curve. It also highlights the shaded regions defined in 3.3. Panel A presents the observed demand curve, along with $M^{Mandate}$ and $M^{CE}$; Panel B presents the ex-ante demand curve, along with $W^{Mandate}$ and $W^{CE}$. To calculate the (ex-ante) welfare impact of a mandate relative to a competitive equilibrium, one can integrate between the demand (or ex-ante demand) curve and marginal cost curve between $s^{CE}$ and 1. From both a market surplus and ex-ante welfare perspective, expanding the size of insurance market starting at $s^{CE}$ increases welfare. However, expanding all the way to 100% coverage involves covering some individuals who value their insurance less than costs.

In this medium risk example, a full mandate reduces market surplus: Expanding the market to $s^{MS}$ increases market surplus by $38.26$, but expanding from $s^{MS}$ to 100% reduces market surplus by $117.84$, as shown in Panel A of Figure 4. Mandates generate a net loss of surplus of $79.58$ relative to competitive markets.

However, from behind the veil of ignorance mandates deliver higher ex-ante welfare than the competitive market allocation. Expanding the market from $s^{CE}$ to $s^{EA}$ increases ex-ante welfare by $120.62$, and further expanding to 100% coverage reduces ex-ante welfare by $94.70$. From behind the veil of ignorance, individuals would be willing to forego $25.92$ of consumption in all states of the world in order to have an insurance mandate instead of a competitive equilibrium, despite the fact doing so lowers total market surplus. In this sense, an ex-ante welfare perspective can lead to different conclusions about the desirability of insurance mandates.
6 Conclusion

This paper develops an ex-ante demand curve that can be used in the Einav et al. (2010) framework to quantify the welfare impact of government policies using a welfare frame from behind the veil of ignorance. Broadly, the analysis reveals several results. First, policies that are optimal from behind the veil of ignorance may not maximize market surplus. For example, the ex-ante optimal size of the insurance market does not involve setting willingness to pay equal to cost, \( D(s) = MC(s) \); but rather it involves setting the additional ex-ante value of risk protection from increasing the size of the insurance market equal to the marginal lost surplus: \( EA(s) = MC(s) - D(s) \). Ex-ante optimal policies may involve deadweight loss.

Second, the size of ex-ante adjustment, \( EA(s) \), depends on the square of the size of the insurable risk. Multiplying demand and cost curves by a factor \( a > 0 \) yields an ex-ante adjustment of \( a^2 EA(s) \), as shown in Section 4 and illustrated by the range of examples with different risk sizes in Section 5. This means that the distinction between ex-ante willingness to pay and the willingness to pay curve observed in the market is likely to be much more important for risks that comprise a larger fraction of one’s wealth or consumption (conversely, these distinctions can perhaps be ignored when considering smaller risks).

Finally, the analysis highlights the importance of future work to understand the difference in marginal utilities between the insured and uninsured. Although the approach in Section 4 provides a benchmark implementation building on the literature on optimal social insurance, it certainly relies on assumptions that could ideally be relaxed. Indeed, violations of these assumptions could be important. For example, if those who choose to forego insurance under the Affordable Care Act are doing so because they have a negative income shock that prevents them from being able to afford insurance, it could be the case that the insured actually have a lower marginal utility of
income than the uninsured \((\beta(s) < 0)\). In this case, Assumption 3 in Section 4 would be violated; however the general ex-ante demand curve in Section 3 remains valid. If \(\beta(s) < 0\), then increasing the mandate penalty and increasing insurance subsidies to expand the size of the insurance may actually deliver lower welfare than is suggested by market surplus.

**References**


Online Appendix: Not For Publication

A Insurance versus Redistribution

The formula in Proposition 1 calculates welfare from behind a single veil of ignorance – i.e. before any information about \( \theta \) is known. The approach provided here can also be amended to facilitate welfare analysis after some observable information, \( X \), has been revealed about \( \theta \). For example, perhaps one does not wish to incorporate the value of insurance to the extent to which it redistributes across those with different incomes or health conditions.

To capture this, one can make adjustments to the baseline formula for \( EA(s) \) by conditioning on the observable characteristics, \( X = x \). To see how this can work, suppose prices, \( p_U \) and \( p_I \), are charged uniformly to people with different values of \( X \) and that a fraction \( s \) of the market purchases insurance.\(^{22}\)

Let \( s_x \) denote the fraction of the population with characteristics \( X = x \) that are uninsured. (note that \( s = E_X[s_x] \) is the total fraction of the market insured). Next, let \( \beta(s, x) \) denote the difference in marginal utilities between the insured and uninsured given by a generalized version of equation (15):

\[
\beta(s, x) = \frac{E[u_c(y(\theta) - p_I(s), m^I(\theta); \theta)|s \leq s_x, X = x] - E[u_c(y(\theta) - D(\hat{s}) - p_U(s), m^I(\theta); \theta)|s \geq s_x, X = x]}{E[u_c(y(\theta) - p_I(s), m^I(\theta); \theta)|s \leq s_x, X = x]}
\]

Now, note that the aggregate impact on \( p_U \) of expanding the size of the insurance market is determined by the aggregate resource constraint, and hence we continue to have \( p'_U(s) = MC(s) - D(s) - s \frac{\partial D}{\partial s} \), where \( \frac{\partial D}{\partial s} \) is the slope of the aggregate demand curve (across all \( X \)). Combining, the ex-ante welfare value of expanding the insurance market for those with characteristics \( X = x \) is given by

\[
EA(s, x) = (1 - s_x) \left( MC(s) - D(s) - s \frac{\partial D}{\partial s} \right) \beta(s_x, x) \tag{21}
\]

and aggregating across all values of \( X \) using equal weights on those with different \( X \) characteristics yields an ex-ante welfare value of \( E_X[EA(s, X)] \). This approach aggregates welfare from behind a set of “veils of ignorance” – one for each value of \( X \). In the limiting case where \( X \) incorporates all information about \( s \), then there is no difference in marginal utilities across \( s \) conditional on \( X \), \( \beta(s_x, x) = 0 \). Hence, there would be no additional ex-ante value to the insurance (\( EA(s, x) = 0 \)). This is simply another way of saying that market surplus treats all sources of differences in demand as redistribution as opposed to having potential insurance value.

\(^{22}\)If prices, \( p_U \) and \( p_I \), are charged differentially to those with different \( X \) characteristics, then one can simply conduct welfare analysis by conditioning on \( X \) everywhere in Proposition 1.
B Case when $\frac{\partial D}{\partial p_U} \neq 1$

Let $D(\tilde{s}, p_U)$ denote the price that a type $\tilde{s}$ is willing to pay for insurance when facing a price $p_U$ of being uninsured. This solves

$$E \left[ u \left( y(\theta) - x \left( m^I(\theta); \theta \right) - D(\tilde{s}, p_U), m^I(\theta); \theta \right) | \tilde{s} \right] = E \left[ u \left( y(\theta) - m^U(\theta) - p_U, m^U(\theta); \theta \right) | \tilde{s} \right]$$

Here, I re-state the main proposition for this general case.

**Proposition.** The marginal welfare impact of expanding the size of the insurance market from $s^*$ to $s^* + ds$ is given by

$$\frac{V'(s^*)}{E[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | s \leq s^*]} \approx p_I(s^*) - p_U(s^*) + EA(s^*) - MC(s^*) \tag{22}$$

where $EA(s^*)$ is the ex-ante value of expanding the size of the insurance market,

$$EA(s^*) = \frac{1 - s^*}{1 + s^* \left( \frac{\partial D}{\partial p_U} - 1 \right)} \left( MC(s^*) - (p_I(s^*) - p_U(s^*)) - s^* \frac{\partial D}{\partial s} \right) \beta(s^*) \tag{23}$$

and $\beta(s)$ is the percentage difference in marginal utilities of income for the insured relative to the uninsured,

$$\beta(s) = \frac{E \left[ u_c \left( y(\theta) - p_I(s), m^I(\theta); \theta \right) | \tilde{s} \leq s \right] - E \left[ u_c \left( y(\theta) - D(\tilde{s}, p_U(s)), m^I(\theta); \theta \right) | \tilde{s} \geq s \right]}{E \left[ u_c \left( y(\theta) - p_I(s), m^I(\theta); \theta \right) | \tilde{s} \leq s \right]} \tag{24}$$

C Proof of Proposition 1

This Appendix walks through the proof of Proposition 1. I consider the general case in Appendix B that allows for take-up to depend separately on $p_I$ and $p_U$, and use the results to consider the subcase when take-up is only a function of $p_I - p_U$.

Let $p_I(s)$ and $p_U(s)$ satisfy the resource constraint (9) and the constraint, $p_I(s) = D(s, p_U(s))$ when fraction $s$ of the market purchasing insurance when facing those prices. Ex-ante welfare, $V(s)$, is given by

$$V(s) = \int_0^s E \left[ u \left( y(\theta) - p_I(s), m^I(\theta); \theta \right) | \tilde{s} \right] d\tilde{s} + \int_s^1 E \left[ u \left( y(\theta) - m^U(\theta) - p_U(s), m^U(\theta); \theta \right) | \tilde{s} \right] d\tilde{s}$$

Substituting the demand function $E \left[ u \left( y(\theta) - m^U(\theta) - p_U, m^U(\theta); \theta \right) | \tilde{s} \right] = E \left[ u \left( c - D(\tilde{s}, p_U(s)), m^I(\theta); \theta \right) | \tilde{s} \right]$ yields:

$$V(s) = \int_0^s E \left[ u \left( y(\theta) - p_I(s), m^I(\theta); \theta \right) | \tilde{s} \right] d\tilde{s} + \int_s^1 E \left[ u \left( y(\theta) - D(\tilde{s}, p_U(s)), m^I(\theta); \theta \right) | \tilde{s} \right] d\tilde{s}$$
so the marginal welfare impact is given by

\[ V'(s) = -sp_I(s) E\left[u'(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right] - (1 - s) p'_U(s) E\left[\frac{\partial D(\tilde{\theta}, p_U(s))}{\partial p_U} u_c(y(\theta) - D(\tilde{s}, p_U(s)), m^I(\theta); \theta) | \tilde{s} \geq s\right] \]

Now,

\[ sp_I(s) + (1 - s) p_U(s) = sAC(s) \]

so that

\[ p_I(s) + s \frac{dp_I}{ds} + (1 - s) \frac{dp_U}{ds} - p_U(s) = MC(s) \]

or

\[ sp'_I(s) + (1 - s) p'_U(s) = MC(s) - p_I(s) + p_U(s) \]

or

\[ sp'_I(s) + (1 - s) p'_U(s) = MDWL(s) \]

where \(-MDWL(s) = p_I(s) - p_U(s) + MC(s)\). If there is sufficiently high DWL from expanding the insurance market, both \(p_I\) and \(p_U\) will go up. But, if there is sufficiently high surplus, the resource constraint will imply that both prices must go down. For intermediate ranges of DWL, one expects the price of insurance to go down and the price of being uninsured to go up.

So, adding and subtracting \((1 - s) p'_U(s) E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right]\) and then dividing by \(E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right]\) yields

\[
\frac{V'(s)}{E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right]} = - \left[sp'_I(s) + (1 - s) p'_U(s)\right] + (1 - s) p'_U(s) \times \left(\frac{E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right]}{E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right]} - E\left[\frac{\partial D(\tilde{s}, p_U(s))}{\partial p_U} u_c(y(\theta) - D(\tilde{s}, p_U(s)), m^I(\theta); \theta) | \tilde{s} \geq s\right]\right) \]

or

\[
\frac{V'(s)}{E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right]} = -MDWL(s) + (1 - s) p'_U(s) \times \left(\frac{E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right]}{E\left[u_c(y(\theta) - p_I(s), m^I(\theta); \theta) | \tilde{s} \leq s\right]} - E\left[\frac{\partial D(\tilde{s}, p_U(s))}{\partial p_U} u_c(y(\theta) - D(\tilde{s}, p_U(s)), m^I(\theta); \theta) | \tilde{s} \geq s\right]\right) \]

where \(p'_U(s) > 0\). So, at an optimum where marginal utilities are higher amongst the insured, we have a positive MDWL as long as \(\frac{\partial D(\tilde{s}, p_U(s))}{\partial p_U} \approx 1\).

What is \(\frac{\partial D(\tilde{s}, p_U(s))}{\partial p_U}\)? Higher penalties should increase the WTP people have for insurance, \(\frac{\partial D}{\partial p_U} > 0\). We have \(D(s, p_U(s)) = p_I(s)\) so that

\[ \frac{\partial D}{\partial s} + \frac{\partial D}{\partial p_U} p'_U(s) = p'_I(s) \]

or

\[ \frac{\partial D}{\partial s} = p'_I(s) - \frac{\partial D}{\partial p_U} p'_U(s) \]

so that as \(s\) increases, it influences demand through the differences in prices, \(p_I\) and \(p_U\). If \(\frac{\partial D}{\partial p_U} \approx 1\),
then $\frac{\partial D}{\partial s} = p_U' - p_U':$ the increase in the size of the market can come from two sources: lower prices or increase penalties. Note $E \left[ u \left( y(\theta) - m^U(\theta) - p_U, m^U(\theta); \theta \right) \mid \bar{s} \right] = E \left[ u \left( y(\theta) - D(s, p_U), m^I(\theta); \theta \right) \mid \bar{s} \right]$ so that differentiating with respect to $p_U$:

$$E \left[ u_c \left( y(\theta) - m^U(\theta) - p_U, m^U(\theta); \theta \right) \mid \bar{s} \right] = E \left[ \frac{\partial D}{\partial p_U} u_c \left( y(\theta) - D(s, p_U), m^I(\theta); \theta \right) \mid \bar{s} \right]$$

Note that $D$ only depends on $s$ (not $\theta$) so that we can solve

$$\frac{\partial D}{\partial p_U} = \frac{E \left[ u_c \left( y(\theta) - m^U(\theta) - p_U, m^U(\theta); \theta \right) \mid \bar{s} \right]}{E \left[ u_c \left( y(\theta) - D(s, p_U), m^I(\theta); \theta \right) \mid \bar{s} \right]}$$

so that marginal utility when insured is lower than marginal utility when uninsured if and only if $\frac{\partial D}{\partial p_U} > 1$.

Now, how can we approximate the difference in marginal utilities? We can take an approximation that assumes (a) $\frac{\partial D(s, p_U(s))}{\partial p_U}$ is roughly constant in $s$ and (b) $u_c$ is locally linear in $s$. Then, Taylor expanding $u_c$ we have

$$\frac{\partial D(s, p_U(s))}{\partial p_U} u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) \approx \frac{\partial D(s, p_U(s))}{\partial p_U} \left[ u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) - u_c \frac{\partial D}{\partial s} (s - \bar{s}) \right]$$

Taking expectations,

$$E \left[ \frac{\partial D(s, p_U(s))}{\partial p_U} u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) \mid \bar{s} \geq s \right] \approx E \left[ \frac{\partial D(s, p_U(s))}{\partial p_U} \left[ u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) - u_c \frac{\partial D}{\partial s} (s - \bar{s}) \right] \mid \bar{s} \geq s \right]$$

where $\frac{\partial D}{\partial p_U}$ is the difference of the impact of raising the price of insurance versus lowering the mandate penalty on demand and $\frac{\partial D}{\partial s}$ is the slope of the demand curve holding $p_U$ constant. In practice, we might assume $\frac{\partial D}{\partial p_U} = 1$, that marginal utilities are constant for the insured (who all pay the same price). In this case,

$$E \left[ \frac{\partial D(s, p_U(s))}{\partial p_U} u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) \mid \bar{s} \geq s \right] \approx E \left[ u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) - u_c \frac{\partial D}{\partial s} (s - \bar{s}) \mid \bar{s} \geq s \right]$$

$$E \left[ \frac{\partial D(s, p_U(s))}{\partial p_U} u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) \mid \bar{s} \geq s \right] \approx E \left[ u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) - u_c \frac{\partial D}{\partial s} (s - \bar{s}) \mid \bar{s} \geq s \right]$$

So, the difference between insured and uninsured marginal utilities is given by

$$\left( \frac{E \left[ u_c \left( y(\theta) - p_I(s), m^I(\theta); \theta \right) \mid \bar{s} \leq s \right] - E \left[ \frac{\partial D(s, p_U(s))}{\partial p_U} u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) \mid \bar{s} \geq s \right]}{E \left[ u_c \left( y(\theta) - p_I(s), m^I(\theta); \theta \right) \mid \bar{s} \leq s \right] - E \left[ u_c \left( y(\theta) - D(s, p_U(s)), m^I(\theta); \theta \right) \mid \bar{s} \geq s \right]} \right) \approx \frac{u_c}{u_c} \left[ D(s) - D(\bar{s}) \right] \mid \bar{s} \geq s$$

So, the welfare impact is approximately

$$V'(s) = -MDWL + (1 - s)p_U(s) \left[ -\frac{u_c}{u_c} E \left[ D(s) - D(\bar{s}) \mid \bar{s} \geq s \right] \right]$$

(25)
Finally, note that one can express $-p_U'(s)$ as follows. The derivative of the resource constraint with respect to $s$ equals the difference between the marginal price of insurance and marginal cost of insuring the type $s$: $-sp_I'(s) - (1 - s)p_U'(s) = (p_I(s) - p_U(s)) - MC(s)$. Note that

$$sp_I(s) + (1 - s)p_U(s) = sAC(s)$$

so that

$$p_I(s) + sp_I'(s) + (1 - s)p_U'(s) - p_U(s) = MC(s)$$

or

$$sp_I'(s) + (1 - s)p_U'(s) = MC(s) - p_I(s) + p_U(s)$$

Moreover, differentiating equation (26) yields $p_I'(s) = \frac{\partial D}{\partial s} + \frac{\partial D}{\partial p_U} p_U'(s)$. To see this, note that:

$$-sp_I'(s) - (1 - s)p_U'(s) = (p_I(s) - p_U(s)) - MC(s)$$

$$-p_U'(s) + s \left( \frac{\partial D}{\partial p_U} - 1 \right) = (p_I(s) - p_U(s)) - MC(s) + s \frac{\partial D}{\partial s}$$

$$-p_U'(s) = \frac{1}{1 + s \left( \frac{\partial D}{\partial p_U} - 1 \right)} \left( (p_I(s) - p_U(s)) - MC(s) + s \frac{\partial D}{\partial s} \right)$$

Combining with equation (25) yields the desired result. Under the additional approximation that $\frac{\partial D}{\partial p_U} = 1$, and replacing $D(s) = p_I(s) - p_U(s)$, this yields

$$V'(s) = D(s) - MC(s) + (1 - s) \left( D(s) - MC(s) + s \frac{\partial D}{\partial s} \right) \left[ \frac{-\mu c}{\mu c} E[D(s) - D(s) | \bar{s} \geq s] \right]$$

which concludes the proof.