1. Introduction

Detecting newly appearing and disappearing pigmented skin lesions (PSL) is important for early detection of malignant melanoma. Dermatologists advocate total-body photography and track corresponding PSLs across the time. A computer vision system for PSL tracking improves the matching process. PSL tracking can be formulated as graph matching.

Problem: When PSLs are close to each other, previous methods are prone to mis-correspond a PSL with nearby ones. These matching errors become obvious when one examines how groups of nearby PSLs are ordered.

Our contribution: We proposed a new set of Jacobian based graph descriptors that enforce topology preservation into the graph matching problem.

2. PSL Matching Formulation

We represent PSLs configuration of the images by graphs \( G(V,E,C) \) and \( G'(V',E',C') \) (\( V \): nodes, \( E \): edges, and \( C \): cliques).

<table>
<thead>
<tr>
<th>Problem</th>
<th>Unknown</th>
<th>Objective function</th>
<th>Weights and terms</th>
<th>Optimization method</th>
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<tr>
<td>PSL matching</td>
<td>X matching matrix</td>
<td>( E(X) = w^t_i \phi_i(X,P,P') )</td>
<td>( w_{ij} = \frac{\phi_j(X)}{w^t_i \phi_i(X)} ) ( \phi_i(X) = \frac{d \phi_i}{d X} )</td>
<td>Tensor based [PAMI11]</td>
</tr>
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</table>

- \( \phi_i \) is mapped to \( V'_i \) and \( X'_i \) is 0 otherwise.
- \( \phi_i(X, V, V') = \sum_{ij} X_{ij} \text{abs}(V_i - V'_j) \), \( \{ \phi_w, \phi_v, V, V' \} \in \mathbb{R}^n \) (i.e. encourages similar features).
- \( \phi(X, E, C) = \sum_{ij} X_{ij} \text{abs}(E_{ij} - E'_{ij}) \), \( \{ \phi_w, \phi_v, E, E' \} \in \mathbb{R}^k \).
- \( \phi(X, C, C') = \sum_{ijk} X_{ijk} \text{abs}(C_{ijk} - C'_{ijk}) \), \( \{ \phi_w, \phi_v, C, C' \} \in \mathbb{R}^l \).
- The parameters \( \{ \mathbb{R}^n, \mathbb{R}^k, \mathbb{R}^l \} \) dimensions depend on the unary, binary, and ternary descriptors in our formulations.

3. Jacobian based Features

We borrow and adapt the topology preservation idea in image registration to the PSL matching problem.

**Topology preservation in image registration (rectilinear grids):** It has been shown that the deformed grid preserves topology if its Jacobian determinants are positive everywhere on the grid.

Visualisation of the forward and backward neighboring points of \((x, y)\). For example, \( J^f \) is computed based on the forward-forward (ff) partial derivatives of the deformation \( h \) considering ff neighbors of \((x, y)\), which are \((x, y+1)\) and \((x+1, y)\).

\[
J^f(x, y) = \left\{ f^f_i(x, y), f^f_j(x, y), f^f_k(x, y) \right\}.
\]

\[
f^f_i(x, y) = f(x, y+1) - f(x, y), \quad f^f_j(x, y) = f(x, y+1) - f(x, y), \quad f^f_k(x, y) = f(x, y) - f(x, y-1).
\]

**Topology preservation for graphs of PSLs:** We compute a variant of these Jacobian matrices for a set of corresponding cliques (triplets) between the two graphs.

4. Results

We evaluated our method on 194 images (97 pairs) of human back digital color images.

To validate our assumption that correct matchings preserve topology, we computed the Jacobian matrices of the non-corresponding and corresponding triplets of the two graphs, respectively. \( x\) and \( y \) axes represent \( \text{det}(J(ijk)) \) and \( \text{det}(J(i'j'k')) \) of \( \{i, j, k\} \) and \( \{i', j', k'\} \) triplets of the graphs with incorrect and correct mappings.

We set the nodes/edges/cliques' compatibilities in our objective function as the differences between \( \text{det}(J(ijk)) \) and \( \text{det}(J(i'j'k')) \).

The PSL matching error is defined as \( \Delta_M(\hat{X}, X) = \| \hat{X} - X \|_2 \), where \( X \) and \( \hat{X} \) are the ground truth and the estimated matchings, respectively. Note how incorporating the proposed node descriptor improves the matching accuracy.