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Foreign debt, crowding out and capital flight

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The modern theory of international debt with potential repudiation is extended to the case of a small two sectors economy facing terms of trade uncertainty. My model explains the phenomenon of simultaneous foreign borrowing and investments at home and abroad. I show that foreign debt rationing is a necessary condition for such an occurrence, and that capital flight can accelerate when the world interest rate falls while the credit ceiling necessarily increases. Moreover, capital controls can potentially lead to increased flight.

The magnitude of capital flight out of some heavily indebted LDCs has been staggering in the past decade. During the period 1976–84, the five largest debtor countries have increased their foreign debt by an estimated $243 billion. Simultaneously, private capital outflows out of those countries are estimated at $133 billion for the same period.[1] As a result, capital formation during the last part of the borrowing boom has been much smaller than anticipated. This has raised serious doubts about the capacity of foreign debt to finance economic growth.

This paper develops a model that simultaneously incorporates domestic investment, foreign borrowing with default risk, and capital flight in a framework where both the borrower and the lender actions are derived from optimizing behavior. The model offers a new perspective on the interactions between investment and financing decisions in LDCs. In particular, it rationalizes the common experience of a number of LDCs that financed domestic investment in part with foreign debt while simultaneously investing a part of their financial resources abroad.

The model features absorption capacity as the principal culprit for the declining rates of growth. An important premise of the analysis is that the supply of productive projects that can be generated by a given socio-economic structure during a given span of time is limited.

Following the modern literature on country debt, the model also recognizes that a country can repudiate its international loans when it is in its interest to do so. This occurs when the cost imposed by default is perceived to be smaller than the benefit of not repaying. In these models, the cost of default is taken to be a penalty imposed

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by the lender, as the loss of a fixed proportion of future income (Sachs, 1983), the exclusion from the option of future borrowing (Eaton and Gersovitz, 1981), or the loss of the access to international trade (Gersovitz, 1984; Diwan and Donnenfeld, 1986; Bulow and Rogoff, 1986). In this paper, I will use this last penalty as a default deterrent.[2]

The analysis is concerned with the typical case of an LDC that has under-developed resources that can be exported at uncertain prices. In this context, international bank debt can reduce risk. When commodity prices fall below some cut-off level, the borrowing country will exercise its option to default because the cost of default—a denial of the possibility to export—is small in that event. As a result, it is shown that risk averse capital poor countries prefer to finance domestic investment with foreign debt rather than with domestic savings because the former provides a valuable insurance service. Simultaneously, domestic capital gets invested abroad in order to keep the marginal return of capital from falling below the world interest rate. Thus, in the model, an injection of capital from abroad does not necessarily lead to economic growth. Rather, the absorption capacity of the economy also matters.

The above discussion does not explain why some domestic capital gets invested domestically. In fact, if foreign debt was forthcoming in any amounts, the totality of domestic investment would be financed by foreign loans, and all the free domestic capital would flee abroad. However, default considerations place limits on the total amounts of external finance that a country can use. When the costs of repudiation are less directly related to the size of the outstanding debt than the benefits of it, lenders will ration credit.[3] When the availability of productive domestic projects requires a level of investment that exceeds the debt ceiling, domestic residents will find it in their interest to invest some of their funds domestically.

The model shows that lower world interest rates and domestic capital controls can lead to a larger foreign credit ceiling because the incentives to default are decreased. However, capital flight can either increase or decrease because, offsetting the reduction in the return to funds placed abroad that tends to discourage the flight of domestic capital, there is also a reduction in the rate of return on domestic investment caused by the crowding out effect of larger foreign borrowings.

The analysis in the paper differs from previous work on capital flight. Khan and Al Haque (1985) also model the phenomenon as a consequence of an asymmetric risk of expropriation facing domestic and foreign investors. However, their specific assumption is that foreign debt is guaranteed by the debtor's government and thus safe from default risk while domestic investments by residents face expropriation risk. This gives LDC residents the incentives to place their own funds in riskless saving accounts abroad and foreigners have a comparative advantage at lending for domestic investments. Moreover, residents invest some of their funds domestically when foreign investments are characterized by increasing transaction costs. The model presented here complements this analysis since it shows how bidirectional capital flows and investments at home by residents can arise without transaction costs or expropriation risk on domestic investments, when the default risk facing the foreign debt is taken into consideration.[4]

The paper is organized as follows: Section I sets a simple model of a two sectors capital poor economy and Section II analyzes the default decision. Section III derives the supply of debt as a function of expected capital flight. Section IV examines the debtor portfolio problem and derives the demands for foreign credit and for saving abroad. In Section V, the rational expectation Nash solutions of the game are characterized. Section VI analyzes the effects of capital control policies and of a change in the world interest rate on a class of equilibria. The concluding section discusses various extensions of the basic model.

I. The Analytical Framework

The model of a capital-poor LDC (the home country) is embedded in a simple version of a two sectors (agriculture and industry), two periods model of a small economy facing terms of trade uncertainty. Investment and international borrowing occur in $t=0$, while consumption and possibly trade take place in $t=1$. At the beginning of $t=1$, the terms of trade are revealed and the country has to choose between (i) repaying its international loans and engaging in international trade, and (ii) repudiating its debt and curtailing its exports.

The agricultural sector uses land ($\bar{L}$), and some of its product is exported if international trade takes place. The industrial import competing sector uses machines that are either inherited ($\bar{M}$) or imported. The country is also endowed with some financial capital ($\bar{K}$) that has to be allocated in $t=0$ between flight capital (S) that earns the world risk free return R and the purchase of new machines ($\bar{K}-S$).[5] Moreover, the productive capacity of the industrial sector can also be increased through foreign borrowing (K) at a cost of R^B.[6] The output produced in $t=1$ is given by:

$$\langle 1\rangle \qquad A = a(L),$$

$$\langle 2\rangle \qquad I = i(\bar{M}+K+\bar{K}-S),$$

where A, $a(.)$, I, $i(.)$ represent the output and production functions in the agricultural and industrial sectors respectively. We require:

$$\langle 3\rangle \qquad K \geqslant 0,$$

$$\langle 4\rangle \qquad S \geqslant 0,$$

$$\langle 5\rangle \qquad S \leqslant \bar{K}.$$

Let p represent the relative price of the agricultural good in terms of the industrial good. The international price is denoted by p^T and is assumed to be uncertain in $t=0$. This uncertainty is described by a distribution $F(p)$ and a density $f(p)$ that are continuous on a range $[\underline{p}^T, \bar{p}^T]$. The autarkic price is represented by p^A.[7]

Finally, denote by (X_1, X_2) the country's consumption vector and by Y its income. The country's preferences for consumption are represented by a direct utility $U(X_1, X_2)$ and by an indirect utility function $V(Y,p)$. We will need the following assumptions:

$$\langle A1\rangle \qquad p^A < \underline{p}^T;$$

$$\langle A2\rangle \qquad i(.) \text{ is concave};$$

⟨A3⟩ $U(.,.)$ is convex in its arguments,

$V(Y,p)$ is concave in Y (*i.e.*, the country is risk averse with respect to variations in income),

$V_{Yp}=0$ (*i.e.*, the country's marginal utility of income is independent of prices);[8]

$\langle A4\rangle \qquad i'(\bar{M}+\bar{K}) > R.$

Presumably, if investment in industry is large enough, the autarkic price of the agricultural product could be high enough to make the country a net importer of that product when its international price is low enough. However, ⟨A1⟩ requires that no such trade reversal occurs. ⟨A2⟩ and ⟨A3⟩ are classical technical assumptions. ⟨A4⟩ defines a capital poor country.

II. The Default Decision

At the beginning of $t=1$, output has been produced and the terms of trade are revealed. The country has to decide whether to repay its loan and trade freely or whether, despite the costs involved, it is better off defaulting.[9] In this model, the cost of the penalty imposed by the lender is taken to be the loss of the gains from trade. It is therefore useful to think of the default decision in terms of cost and benefit. The benefit from defaulting consists of the expropriated foreign capital. The benefit will exceed the cost when the gains from trade are small, that is when the international price p^T is small and close to p^A, the marginal rate of substitution in the autarkic situation.

Formally, the country will choose to default on its international debt when

$$\langle 6\rangle \qquad V(Y^T, p^T) < U(X_1^A, X_2^A),$$

where

$$\langle 7\rangle \qquad Y^T = p^T A + I + SR - KR^B,$$

$$\langle 8\rangle \qquad X_1^A = A,$$

$$\langle 9\rangle \qquad X_2^A = I + SR.$$

Equation ⟨7⟩ defines income in the trading regime. It is composed of the value of production evaluated at the world prices, plus the wealth saved abroad, minus the repayment of the international loan. Equations ⟨8⟩ and ⟨9⟩ define the consumption vector in the default (or autarkic) regime: the consumption of good 1 must be equal to the production of that good. Good 2 consumption exceeds domestic production by the amount of capital saved abroad since good 2 is more expensive domestically in autarky.

II.A. Terms of Trade

Intuitively, the country defaults when the gains from trade are too small to offset the benefit of expropriating foreign capital. This occurs when the terms of trade are low enough. Inspection of equation ⟨6⟩ confirms this intuition: The RHS of the inequality is independent of p^T. The LHS, however, increases with p^T reflecting increasingly beneficial terms of trade. Therefore, default occurs for p^T low enough.

Denote by $\hat{p}$ the smallest international price for which debt repayment is the dominant strategy and by $\hat{Y}$ the trade income evaluated at $\hat{p}$.

Definition 1. $\hat{p}$ solves $V(\hat{Y},\hat{p}) = U(X_1^A, X_2^A)$ and default occurs for $p^T < \hat{p}$. When the country defaults, it trades off a lower consumption of good 2 in exchange for a larger consumption of good 1. Consequently, good 2 is more scarce in the default regime than in the trade regimes. An important implication for our analysis is that the marginal utility of good 2 (or equivalently, of income) must be larger in the default regime than in the trade regimes.[10]

Let

$$Y^A = p^A X_1^A + X_2^A$$

and

$$p^A = \frac{\partial U(X_1^A, X_2^A)}{\partial X_1^A} \bigg/ \frac{\partial U(X_1^A, X_2^A)}{\partial X_2^A},$$

then

Lemma 1.

$$\hat{V}_Y^T \equiv \frac{\partial V(\hat{Y}, \hat{p})}{\partial Y} < \frac{\partial V(Y^A, p^A)}{\partial Y} \equiv V_Y^A,$$

and V_Y^T decreases in p^T on $[\hat{p}, \bar{p}^T]$.

Proof. The equation in definition 1 can be rewritten as:

$$V(\hat{Y}^T, \hat{p}^T) = V(Y^A, p^A).$$

But $V(.)$ decreases in price when holding income constant. Since $p^A < \hat{p}$ by $\langle A1 \rangle$, it must be that $\hat{Y}^T > Y^A$. Consequently, $\hat{V}_Y^T < V_Y^A$, since V_y decreases in income but is unaffected by prices by assumption $\langle A3 \rangle$. Finally, Y^T increases in p^T. Thus, V_Y^T decreases in p^T.

II.B. Other Variables Affecting Default

In $t = 1$, the country has SR of its assets placed abroad and it owes its foreign lenders an amount KR^B. How is the incentive to default related to the magnitude of these variables? In other words, how does $\hat{p}$, the smallest international price consistent with debt repayment, behave in K, S, R and R^B? Note that the question is relevant since the level of these variables is determined in $t = 0$, while the default decision is taken in $t = 1$. Thus, we can treat them as exogenous with respect to the default decision.

First, consider the effect of the country's foreign indebtedness on its incentive to default. As K and/or R^B increase, the benefit increases since a larger amount of foreign capital is expropriated. The cost of default, however, decreases at the margin in the amount of borrowed capital because an injection of capital increases the domestic production of the industrial (imported) good, and, thus, leads to smaller gains from trade. In sum, as KR^B increases, the incentive to default also increases because the cost decreases while the benefit increases. Thus, $\hat{p}$ the smallest terms of trade consistent with debt repayment goes up.

Next, consider the effect of the amount of assets placed abroad, S, on the incentive to default. The key here is the availability of the industrial good in both regimes: (i) An increase in assets abroad increases the quantity of that good that can be purchased in the absence of any export. This tends to decrease the cost of the default penalty and to encourage default. (ii) On the other hand, an increase in flight capital reduces the availability of domestic capital in the industrial sector and consequently, the domestic production of the import competing good falls. This makes the country more dependent on international trade and, thus, less likely to default. The net effect of flight capital on the default incentive is therefore ambiguous and it depends on the relative rates at which: (i) good 2 is accumulated abroad (R) and, (ii) the domestic production of good 2 is curtailed at the margin (i'). When the former exceeds the latter, the net effect of capital flight is to encourage default. However, when $i' = R$, the extent of flight capital has marginally no effect on the incentive to default.

Finally, consider the effect of a change in R, the rate of return on assets placed abroad. From the above discussion, it is apparent that as R increases, good 2 is accumulated abroad at a higher rate and, thus, the country becomes less dependent on export revenues and more prone to default. Formally:

Lemma 2. $\hat{p}$ increases in K, R and in R^B. Moreover, $\hat{p}$ increases (decreases) in S when $R > i'$ ($R < i'$).
Proof. Differentiate the equation in definition 1 with respect to the relevant variable. Rearrange using Roy's identity to get:

$$\frac{\partial \hat{p}}{\partial K} = \frac{i'(V_Y^A - \hat{V}_Y^T) + R^B \hat{V}_Y^T}{\hat{V}_Y^T (A - \hat{X}_1^T)} \geqslant 0,$$

$$\frac{\partial \hat{p}}{\partial R^B} = \frac{K}{(A - \hat{X}_1^T)} \geqslant 0,$$

$$\frac{\partial \hat{p}}{\partial S} = \frac{(R - i')(V_Y^A - \hat{V}_Y^T)}{\hat{V}_Y^T (A - \hat{X}_1^T)} \gtreqless 0,$$

$$\frac{\partial \hat{p}}{\partial R} = \frac{S(V_Y^A - \hat{V}_Y^T)}{\hat{V}_Y^T (A - \hat{X}_1^T)} \geqslant 0.$$

V_Y^A is that marginal utility of income evaluated at the autarkic allocation (X_1^A, X_2^A) defined in equations ⟨8⟩ and ⟨9⟩. $\hat{V}_Y^T$ is evaluated at $(\hat{X}_1, \hat{X}_2)$, the consumption vector that maximizes utility in the trade regime at the international price of $\hat{p}$ and under the budget constraint in ⟨7⟩. The derivatives are signed using the fact that $A > \hat{X}_1^T$ from ⟨A1⟩ and that $V_Y^A > \hat{V}_Y^T$ from Lemma 1.

III. The Supply of Foreign Debt

We will assume that foreign banks act competitively in setting the interest rate that they charge on their foreign loans but that they cooperate in the setting and the distribution among themselves of the credit ceiling and in the enforcement of sanctions in case of a default. Credible sanctions against the borrowing country are needed in order to deter default. Since debt contracts give the lender a lien on the debtor's exports that can be enforced in its own legal system, it appears reasonable to assume that it will be in the *ex-post* interest of a defaulting country to refrain from exporting its own goods.[11]

However, the threat of sanctions does not eliminate the possibility of default when the cost and benefit of default are uncertain. The probability of default as perceived in $t = 0$ depends on the level of $\hat{p}$ and is represented by $F(\hat{p})$. As the size of the loan increases, so does $\hat{p}$ and, therefore, $F(\hat{p})$. Hence, the borrower would default with certainty if K was large enough. These considerations imply that foreign debt must be rationed.[12]

Assuming that all banks are risk neutral, they accept to extend loans in $t=0$ at an interest rate R^B that compensates for the default risk. We have:

$$\langle 10\rangle \qquad R^B[1-F(\hat{p})] = R.$$

Consider the shape of the aggregate supply curve for loans $K^S(.)$. To supply a larger debt, the lender increases R^B in order to be compensated for the additional default risk. However, since this also increases $\hat{p}$ and the probability of default $F(\hat{p})$, the proedure is feasible only when this additional increase in the incentives to default is not too large. Formally, differentiate $\langle 10\rangle$ with respect to K and R^B. Solving, we have:

$$\langle 11\rangle \qquad \frac{\partial K^S}{\partial R^B} = \frac{(1-F(\hat{p})) - R^B f(\hat{p})\, \partial\hat{p}/\partial R^B}{R^B f(\hat{p})\, \partial\hat{p}/\partial K}.$$

It is easy to verify that around $K=0$, the expression in $\langle 11\rangle$ is positive. Thus, initially, as K increases so does R^B. However, at some large $\hat{K}$, the numerator in $\langle 11\rangle$ can become non-positive: the increase in R^B increases $F(\hat{p})$ so much that, overall, K must be decreased for $\langle 10\rangle$ to hold. The credit ceiling is defined as the largest size loan that is profitable.

Definition 2. $\hat{K}$ solves $\partial K^S/\partial R^B = 0$.

The supply of debt can be represented by a function $K^s(R^B,.)$. The supply increases in R^B on the range $[0, \hat{R}^B]$. There is a discontinuity at $\hat{R}^B$ at which point the supply becomes infinitely elastic.

We turn our attention now to the relationship between the supply of debt and the expected level of capital flight S^e. When $i' < R$, an increase in flight capital increases the incentive to default (Lemma 1). Thus, the lender reacts by lowering the credit ceiling and by increasing the interest rate he charges. However, when $i' = R$, the incentive to default and, therefore, the supply of debt are unaffected by marginal changes in flight capital.

Lemma 2. $\mathrm{Sign}(\partial K^s/\partial S^e) = \mathrm{Sign}(i' - R)$.

Proof. Apply the implicit function theorem to $\langle 10\rangle$. Solving and rearranging, we get:

$$\frac{\partial K^s}{\partial S^e} = -\frac{\partial\hat{p}/\partial S}{\partial\hat{p}/\partial K} = \frac{(i'-R)(V_Y^A - \hat{V}_Y^T)}{i'(V_Y^A - \hat{V}_Y^T) + R^B \hat{V}_Y^T},$$

which has the same sign as $(i'-R)$ since $V_Y^A > \hat{V}_Y^T$.

IV. Investment and Portfolio Decisions

We are now in a position to analyze the country's decisions in $t=0$ when it has to (i) decide how to allocate its free financial resources between capital flight S and investment in the industrial sector $(\bar{K}-S)$ and (ii) determine its demand for foreign debt K. The exogenous variables that affect those decisions are R the risk free world interest rate, R^B the cost of foreign debt, $\hat{K}^e$ the expected level of the foreign debt ceiling and $\hat{p}^e$, the expectation of the lowest international price consistent with debt repayment.

A solution to the country's allocational program in $t=0$ consists of two functions $K^D(R, R^B, \hat{K}^e, \hat{p}^e)$ and $S^D(R, R^B, \hat{K}^e, \hat{p}^e)$. Note that we are assuming that domestic investors take $\hat{p}^e$ as given and that they do not perceive the effect of their individual actions on $\hat{p}$. However, we will later require that $\hat{p}^e$ be a rational and fulfilled expectation in equilibrium.

There are three facets to the country's allocation problem: (i) how much to invest in the industrial sector; (ii) how to finance this investment; and (iii) how to stabilize future utility levels.

In a world with perfect loan enforcement, there are no instruments that can be used to stabilize future utility which simply fluctuates with the terms of trade. The country would invest until the marginal return in its industrial sector equals the world interest rate R. Since it does not own enough financial resources (assumption ⟨A4⟩, it would fill the gap with foreign debt.[13] In this situation, there would be no capital flight.

The introduction of default considerations changes the country's program considerably because foreign debt can be used as a hedge against low realizations of the terms of trade. By increasing its foreign debt and the probability of default, the country can redistribute resources from the good to the bad states of nature: in the good states (high prices), the country repays the principal and a large interest payment that compensates the banks for the large default risk; but in the bad states, the country defaults on its international obligations, thus pushing its utility level above what is achievable with trade at relatively unfavorable terms. In this context, the investment, financing and stabilization problems become interconnected.

To see this, consider the following marginal portfolio changes from the allocation that obtains in the non-risky debt situation described above. The country can replace some of its resources invested in industry by an equal amount of foreign debt so as to keep investment fixed; simultaneously it can save those freed resources abroad. The expected cost of such a strategy is equal to $[R - R^B F(\hat{p}^e)]$ at the margin. With rational expectations and risk neutral competitive lenders, this expression is equal to zero, indicating that this implicit insurance is fair. Since risk averse agents insure fully when the insurance is offered at fair terms, we would expect the country to substitute domestic capital with foreign capital until it is fully insured.[14]

There are, however, two constraints that might not allow the country to insure fully by using the above procedure:

1. The country can run out of free financial capital before reaching full insurance. In this case, further coverage has to be purchased at unfair odds. This is because further coverage requires more foreign borrowing and overinvestment since the marginal return on investment i' will have to fall below R, the risk free world interest rate.
2. Foreign debt can be rationed in a way that precludes full insurance at the fair odds. In this case, no instruments can provide further coverage.

It will be shown in the next section that, in equilibrium, one of those two constraints must be binding. In essence, foreign debt cannot be used to provide a complete insurance at fair odds because a completely insured borrower has incentives to default too often.

To formalize the above discussion, consider the country's portfolio problem in $t=0$:

$$\langle 12 \rangle \qquad \underset{K,S}{\text{Max}}\, EU = F(\hat{p}^e)U(X_1^A, X_2^A) + \int_{\hat{p}^e}^{\bar{p}^T} V(Y^T, p^T) f(p^T) dp^T$$

subject to ⟨3⟩, ⟨4⟩, ⟨5⟩ and to

$$\langle 13\rangle \qquad K \leqslant \hat{K}^e.$$

The first-order conditions yield after slight manipulations:

$$\langle 14a\rangle \qquad \frac{\partial EU}{\partial K} = i'EV_Y - R^B EV_Y^T \begin{cases} > 0 & \text{if } K^D = \hat{K}^e \\ = 0 & \text{if } \; 0 < K^D < \hat{K}^e \\ < 0 & \text{if } K^D = 0, \end{cases}$$

$$\langle 14b\rangle \qquad \frac{\partial EU}{\partial S} = (R - i')EV_Y \qquad \begin{cases} > 0 & \text{if } S^D = \bar{K} \\ = 0 & \text{if } \; 0 < S^D < \bar{K} \\ < 0 & \text{if } S^D = 0, \end{cases}$$

where

$$EV_Y^T \equiv \int_{\hat{p}^e}^{\bar{p}} V_Y^T f(p^T) dp^T,$$

and

$$EV_Y = F(\hat{p}^e) V_Y^A + EV_y^T.$$

Equation ⟨14a⟩ states that the demand for foreign debt K^D is an interior solution when the expected value across all states of the marginal return on investment is equal to the expected cost of foreign debt. Equation ⟨14b⟩, in turn, indicates that the supply of flight capital S^D is an interior solution when capital earns the same marginal return domestically and abroad.

V. Equilibrium

In this model, the lender determines in $t=0$ a reaction function that maps his expectation of flight capital into a supply of debt function. Simultaneously, the borrower determines two reaction functions that map his expectations of $\hat{p}$ and of the foreign debt ceiling into a demand function for foreign loans and a supply function of flight capital.

We define an equilibrium in $t=0$ in the spirit of a sequential Nash equilibrium. An equilibrium consists of an array $[K^*, S^*, R^{B*}, \hat{p}^*]$ that is consistent with both the lender and the borrower optimality conditions ⟨10⟩, ⟨11⟩, ⟨14a⟩, and ⟨14b⟩ as well as with correct expectations about the other player's actions and about the course of events in the future for each possible realization of the terms of trade. Formally, we have

Definition 3. $[K^*, S^*, R^{B*}, \hat{p}^*]$ is an equilibrium if:

(a) $\qquad \hat{p}^e = \hat{p}^* \quad$ and $\quad \hat{K}^e = \hat{K}(S^*)$,

(b) $\qquad (S^*, K^*) \quad$ solve ⟨12⟩,

(c) $\qquad S^e = S^*$,

(d) $\qquad (K^*, R^{B*}) \quad$ solve ⟨10⟩ and $\hat{K}$ solves ⟨11⟩.

Condition (a) requires the country's expectations to be rational, while (b) requires it to be optimizing. Condition (c) requires the lender's expectations to be rational, and (d) the lender's actions to be maximizing. Note that we have assumed that the borrower does not consider the effect of capital flight on the supply of foreign debt. Thus, instead of being a strategic variable, flight capital is assumed to react to the environment. This is a more realistic depiction of reality, since flight capital originates from agents that are too small to perceive the effect of their actions on aggregate variables. However, the lender is assumed to recognize at the outset the relationship between flight capital and the probability of future defaults.

The model possesses a continuum of equilibria, the characteristics of which depend on the value of the exogenous variables. In some equilibria, there is unidirectional capital flow while in others, there are bidirectional flows. However, it is possible to show that equilibria that are characterized by bidirectional capital movement occur only when (i) either the foreign debt credit ceiling is binding, or (ii) the home country does not invest any of its funds domestically. Consequently, an equilibrium allocation with bidirectional capital movements and domestic investment by residents must also be such that foreign credit is rationed. These results are developed in the next two propositions and are then discussed.

Proposition 1.
In equilibrium, either $S^* = \bar{K}$ or $K^* = \hat{K}$ must necessarily hold.
Proof.
(a) First, we need to show that in equilibrium, $EV_Y > E[V_Y^T | p^T \geqslant \hat{p}]$ must hold. V_Y^T decreases because it decreases in Y^T, and Y^T decreases in p^T. Thus, V_Y^T takes its minimum value at $\hat{p}$ and its maximum value at $\bar{p}^T$. Moreover, in equilibrium, $\hat{V}_Y^T < V_Y^A$ by Lemma 1. Thus:

$$E[V_Y^T | p^T \geqslant \hat{p}] < \hat{V}_Y^T < V_Y^A;$$

but

$$EV_Y \equiv E[V_Y^T | p^T \geqslant \hat{p}](1 - F(\hat{p})) + V_Y^A(F(\hat{p})) > E[V_Y^T | p^T \geqslant \hat{p}].$$

(b) Suppose $S^* \neq \bar{K}$. Then the expression in ⟨14b⟩ must be non-positive, implying that in equilibrium, $i' \geqslant R$ must hold. Now focus on ⟨14a⟩ into which ⟨10⟩ has been substituted:

$$i'EV_Y - \frac{REV_Y^T}{(1 - F(\hat{p}))} = i'EV_Y - RE[V_Y^T | p^T \geqslant \hat{p}].$$

The above expression must be positive, given the result in (a) and $i' \geqslant R$. Thus, $K^* = \hat{K}$.
(c) Supose that $K^* \neq \hat{K}$. Then, the expression in ⟨14a⟩ must be non-positive. Together with the result in (a), this implies that necessarily, $i' < R$. Plug in ⟨14b⟩ to get $\partial EU / \partial S > 0$. Thus $S^* = \bar{K}$.

To understand why an interior solution of the country's program is not consistent with an equilibrium, recall that an interior solution requires the equalization of the marginal utility of income in the autarkic and the trade regimes. But this requires a higher utility level in autarky than with trade at $\hat{p}$ (Lemma 1). Thus, the country would prefer to default at $\hat{p}$ and, therefore, $\hat{p}$ cannot be the equilibrium 'cut-off' international price.

We now turn to a description of the equilibria that are not ruled out by proposition 1. Consider first the class of equilibria for which foreign lending is given by the credit ceiling (*i.e.*, $K^* = \bar{K}$). Either (i) $i'(\bar{M} + \bar{K} + K^*) > R$, and there is no capital flight, since investment at home is more profitable than saving abroad; or (ii) $i'(\bar{M} + \bar{K} + K^* - S^*) = R$ for some $S^* < \bar{K}$, and flight capital is an interior optimum. In this case, the country saves some of its financial resources abroad in order to keep the marginal return on investment from falling below the world's interest rate; or (iii) $i'(\bar{M} + K^*) < R$ with $S^* = \bar{K}$. In this case, all the free domestic resources get invested abroad, but this is not sufficient to raise the domestic marginal return of capital above the world interest rate. There is overinvestment financed with foreign debt, a package that constitutes an unfair insurance at the margin. This can well be an acceptable situation for a risk averse country with high levels of irreversible investments and no available stabilization instrument besides foreign debt.

Now consider the savings constrained equilibria (*i.e.*, $S^* = \bar{K}$). All the free financial resources are invested abroad and foreign debt is contracted, since $i'(\bar{M}) > R$ by ⟨A4⟩. Foreign debt is either an interior solution ($K^* < \hat{K}$), in which case $i'(\bar{M} + \hat{K}) < R$ must hold (since otherwise the country will demand more debt). Or foreign debt is rationed with $i'(\bar{M} + \hat{K}) \gtreqless R$.

Hence, in equilibrium, there is only one case involving domestic investment by residents and bidirectional capital flows. In this class of debt constrained equilibria, the country borrows all the foreign debt that is forthcoming ($\hat{K}$) in an attempt to increase its utility levels in the bad states of nature. Foreign capital is invested in the industrial sector which remains more profitable at the margin than foreign saving. Consequently, the country invests some of its free financial resources domestically until the marginal productivity of capital reaches the level of the world interest rate. The remaining resources flee abroad because of a lack of additional productive projects at home. In sum, some domestic capital is crowded out by foreign debt which is contracted because of its insurance characteristics.

Proposition 2. Debt-constrained equilibrium.
If $0 < S^* < \bar{K}$ then necessarily $K^* = \hat{K}$ and $i' = R$ hold.

A debt-constrained equilibrium is depicted in Figure 1 shown on the next page. The equilibrium pair (S^*, K^*) represents the intersection of the lender and the borrower reaction functions. Because $i' = R$ in equilibrium, the credit ceiling is not dependent on the level of flight capital (Lemma 2). On the other hand, the country reacts to an increase in foreign debt by an increase in capital flight of an equal magnitude, once investment in industry becomes unprofitable. Thus, at the margin, the propensity of domestic capital to flee is one.

VI. Interest Rate Changes in a Debt-Constrained Equilibrium

Intuitively, we see that capital flight increases with R, the return on saving abroad, because saving abroad becomes more profitable. However, this need not be the case in a debt-constrained equilibrium. This is because the credit ceiling $\hat{K}$ decreases with R. Consequently, there is an additional secondary effect that counters the direct effect: as $\hat{K}$ decreases, the marginal productivity at home increases in the industrial sector, thus offsetting the incentive for saving abroad.

Proposition 3. In a debt-constrained equilibrium, $[\partial\hat{K}/\partial R] < 0$, but sign$[\partial S/\partial R]$ is ambiguous.
Proof. The FOC in ⟨14b⟩, ⟨10⟩, and definition 2 determines the equilibrium

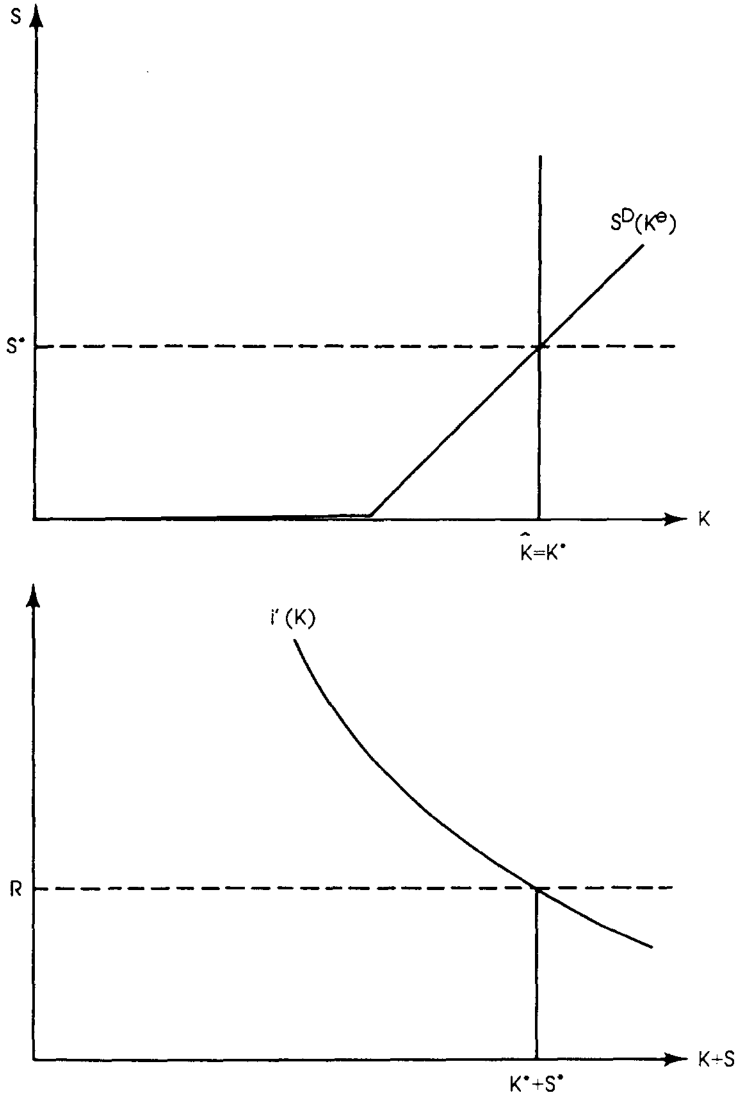

FIGURE 1. Determination of (K^*, S^*) in a debt-constrained equilibrium.

(K^*, S^*) in this case. Differentiating totally ⟨14b⟩ and ⟨10⟩ with respect to R, K, and S, solving using Cramer's rule, and rearranging using proposition 1, we get:

$$\frac{dK}{dR} = \frac{H_s G_R - H_R G_s}{H_k G_s - H_s G_k} \quad \text{and} \quad \frac{dS}{dR} = \frac{H_k G_R - H_R G_k}{H_s G_k - H_k G_s},$$

where $H = R - (1-F)R^B$ and $G = (R - i')EV_Y$.

Evaluating the partial derivatives at (K^*, S^*), we get:

$$H_R = fR^B[\partial\hat{p}/\partial R] > 0, \quad H_k = fR^B[\partial\hat{p}/\partial K] > 0, \quad H_s = fR^B[\partial\hat{p}/\partial S] = 0,$$
$$G_R = EV_Y > 0, \quad G_k = -i''EV_Y > 0, \quad G_s = i''EV_Y < 0,$$

Therefore, we see that:

$$\partial K/\partial R = -\frac{H_R}{H_K} = -[\partial \hat{p}/\partial R]/[\partial \hat{p}/\partial K] < 0, \qquad \text{using lemma 2,}$$

and

$$\partial S/\partial R = [1/i''] - [\partial \hat{p}/\partial R]/[\partial \hat{p}/\partial K] \gtreqless 0.$$

Intuitively, we see that the country defaults more often as R increases because it gets a higher return in autarky, where income is highly valued and it has to pay a higher interest cost in the trading regime where the marginal utility of income is low. Thus, the net cost of default decreases. Similarly, when R decreases, the incentives for default decrease and the credit ceiling increases for each level of capital flight.

The result suggests that a reason why debt reschedulings and 'forced lending' have been relatively successful between 1983 and 1986 may lie in the declining world interest rate, and the higher implied foreign debt ceilings.[15] It also helps understand how this has simultaneously led to some growth in LDC's industrial production and to an acceleration of capital flight.[16] Interestingly, proposition 3 suggests that capital flight should increase when i'' is large, that is, when crowding out considerations are important.

Proposition 3 also suggests that as world interest rates start rising, credit ceilings will decrease and the rescheduling process will be unable to generate as much new money.

In a similar exercise, the model can be used to predict the effects of capital control policies (whose heralded purpose often is the reduction of capital flight). Consider the effects of a small tax on the return of savings abroad or of any policy that reduces the return on foreign savings. This includes credible commitments to make it illegal and costly to repatriate capital in the future and restrictions on the domestic operations of foreign financial institutions that aim at increasing transaction and information costs for foreign investments by residents.

What is important in this exercise is to remember that there are two elements that contribute to the flight of capital: the attractiveness of the return on capital abroad and the return on domestic investment. Clearly, the direct effect of capital controls is to lower $S^*(\hat{K})$. However, foreign lenders are now willing to lend more because the incentives for default are decreased. Since domestic residents are always willing to absorb additional foreign loans, domestic capital gets crowded out. Thus, the total effect of capital controls on capital flight is indeterminate. However, the production of the industrial good clearly increases.

Proposition 4. Let $R^S \neq R$ be the return on saving abroad.

Then,

$$\partial \hat{K}/\partial R^S < 0, \qquad \partial S/\partial S^K \gtrless 0, \qquad \text{and} \qquad \partial i/\partial R^S < 0.$$

Proof. First, we note that the equilibrium concept must be slightly changed with R^S replacing R in equation ⟨14b⟩. The first part is shown as in proposition 3 with R being held constant and R^S being affected by government policy, and with:

$$\frac{\partial \hat{p}}{\partial R^S} = \frac{(V_Y^A - \hat{V}_Y^T)S}{(A_1 - \hat{x}_1)}.$$

Finally, $\partial i/\partial R^S = i'\,\partial \hat{K}/\partial R^S - i'\,\partial S/\partial R^S = i'/i'' < 0$.

VII. Concluding Remarks

I have argued that LDC residents place some of their funds abroad because they prefer to borrow from abroad to finance domestic investment. However, when foreign debt is rationed, this procedure may leave the marginal return on domestic investment above the return on capital placed abroad. In this case, some domestic capital will be invested locally.

The argument hinged on the fact that all residents were assumed to anticipate that they would share the wealth transfer associated with a default according to their share of the foreign debt. In reality, this is unlikely to be the case and the distribution of the gain may be independent of the identity of the borrowers. This introduces a wedge between the social and the private valuation of foreign debt, and it may lead to international borrowings that are below the social optimum. Such considerations may well explain the predominant role of the public sector of LDCs in foreign borrowing transactions. In such circumstances, it is not surprising that it is private capital that flees since it is crowded out by the public sector growth. This scenario is quite consistent with the experience of the large Latin and South American debtors where private capital flight coexisted with government deficits financed by external debts.

The representative consumer assumption plays also another important simplifying role in the model with respect to the risk sharing of the domestic risks. In the absence of an income redistribution policy, if producers in the import competing sector were distinct from those in the export sector for example, industrialists would always gain and cultivators would always lose in case of an imposed autarky. This additional risk can clearly be shared in an economy with a well developed stock market. However, in the presence of thin and inefficient markets for risk sharing, there exist additional incentives for domestic capital to get invested abroad in instruments that provide the risk reduction services that are not offered domestically. This may well explain some of the cross sectional variations in capital flight, with countries with developed stock markets, such as Brazil, experiencing less flight relative to their foreign debt, than countries with poor markets, such as Argentina and Venezuela.

Finally, the model suggests that capital flight is the effect rather than the cause of the scarcity of valuable investment opportunities in LDCs. This relative scarcity should not be surprising given the huge amounts of foreign debt that flowed to LDCs in a very short span of time. In the future, LDC residents' funds placed abroad might well become a very important source of financing as new productive projects get generated in those countries.

Notes

1. The concept and measurement of capital flight is discussed and surveyed in Cumby and Levich (1987). An estimate of the change in foreign indebtedness and of the change in the size of assets held abroad during the period 1976–84 for the five largest debtor countries is given in the

following table:

	(billion of $)	
	Change in debt	Capital flight
Brazil	79.3	17.3
Mexico	79.4	53.4
Argentina	36.3	25.0
Venezuela	28.7	29.6
Philippines	19.4	3.7

Source: Cumby and Levich (1987).

2. I do not, however, impose the condition that the threat be believable in a perfect equilibrium sense.
3. The fact that a risk of default was recognized when the loans were made is illustrated by the spread over Libor that was charged. For example in 1983, those spreads were $2\frac{1}{8}$, $2\frac{1}{4}$ and $1\frac{5}{8}$ per cent respectively for Brazil, Mexico and Argentina.
4. Eaton (1987) extends the Khan and Al Haque (1985) model by relating the risk of expropriation to the existence of publicly guaranteed foreign debt with no country risk. In his model, capital flight is generated by an attempt to escape the tax that the government is expected to raise in the future in order to finance the nationalization of the defaulting domestic concerns. Dornbusch (1985), Ize and Ortiz (1987) and Rivera-Batiz (1987) analyze capital flight in a macroeconomic context. In these papers, the inability of the government to finance expenditure leads to a real reduction of internal debt service, a form of taxation of domestic capital.
5. For simplicity, I assume that the stock of financial capital is an exogenous variable. Thus, the saving process is not endogenized in the model.
6. Thus, we assume that all the debt is used for investment purpose and that none is consumed.
7. This is a marginal rate of substitution rather than a market price since no trade occurs in this situation.
8. In particular, it can be shown that this condition holds when the income elasticity of demand is equal to the coefficient of relative risk aversion.
9. With some loss of generality, the two period model can be used to capture the intuition of an infinite horizon model. In this interpretation, a second period value represents the expected present value of the variable from the second period until infinity. For an example of this technique, see Frenkel and Razin (1986).
10. Note that this result hinges on the assumption about the independence of the marginal utility of income and the price level.
11. Bulow and Rogoff (1986) assume a similar cost for default. For a general discussion on the costs of defaulting, see Kaletsky (1985). On the organization of the lenders in a coalition, see Swodoba (1985).
12. Rationing results in the context of unenforceable foreign debt have been derived in various environments involving different assumptions about the cost and benefit of a default. In particular, see Aizenman (1986), Diwan and Donnenfeld (1986), Eaton and Gersovitz (1981), Gersovitz (1984), and Sachs (1983).
13. In this case, $R = R^B$ will hold since the probability of default is zero.
14. Which in this context means until its marginal utility of income in the default regime is equal to the expected value of its marginal utility of income with debt repayment, the expectation being taken conditionally over the set of all non-default prices $[\hat{p}, \bar{p}^T]$.
15. Forced lending is usually perceived as an attempt by lenders to keep the repayment option alive, as in Krugman (1985). In this context, as the world interest rate declines the value of the repayment option increases. Thus, forced lending is increased.
16. For example, during 1983, as interest rates decreased from a height of 13.3 per cent to less than 10 per cent (on US government short-term debt), the change in indebtedness and in capital held

abroad were estimated to be:

	$ billion	
	Change in debt	Capital flight
Brazil	4.5	1.1
Mexico	7.9	11.7
Argentina	2.4	2.1
Venezuela	0.5	0.4
Philippines	−0.3	−0.8

Source: Cumby and Levich (1987).

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