Supplement to "Money and monetary policy in Israel during the last decade"*

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Abstract

This online appendix presents the two theoretical models (Model 1 and Model 2), discusses the calibration procedure, provides additional results (on-impact impulse response functions and variance decompositions) and the composition of the FCI (Financial Condition Index) used in Benchimol (2016).

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1 The models

The two New Keynesian DSGE models further developed in Benchimol (2011), and tested for the Eurozone in Benchimol and Fourçans (2017), are used herein. The baseline model assumes separability between consumption and money, meaning that the latter becomes irrelevant in the system (Model 1), while the other assumes non-separability between consumption and money holdings (Model 2). Model 1, the baseline model, is described in Galí (2015) and Benchimol (2014), while Model 2, the model with money, is detailed in Benchimol and Fourçans (2012).

Irrespective of the model used, we assume a representative infinitely lived household seeking to maximize

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} \beta^k U_{t+k} \right]$$

where $U_t$ is the period utility function and $\beta < 1$ is the discount factor.

The household decides how to allocate its consumption expenditure among different goods. This requires that the consumption index $C_t$ be maximized for any given level of expenditure. Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_t C_t + M_t + Q_t B_t \leq B_{t-1} + W_t N_t + M_{t-1}$$

for $t = 0, 1, 2..., \text{ where } P_t$ is an aggregate price index, $M_t$ is the quantity of nominal money holdings at time $t$, $B_t$ is the quantity of one-period nominally riskless discount bonds purchased in period $t$ and maturing in period $t+1$ (each bond pays one unit of money at maturity and its price is $Q_t = \exp (-i_t)$ where $i_t$ is the short-term nominal rate), $W_t$ is the nominal wage, and $N_t$ is the hours of work (or the measure of household members employed).

The above sequence of period budget constraints is supplemented with a solvency condition, such as $\forall t \lim_{n \to \infty} \mathbb{E}_t [B_n] \geq 0$, in order to avoid Ponzi-type schemes.

Preferences are measured through a CES-MIU utility function. Under the assumption of a period utility given by

$$U_t = \frac{1}{1-\sigma} \left( (1-b) C_t^{1-\nu} + b e^i \left( \frac{M_t}{P_t} \right)^{1-\nu} \right)^{\frac{1-\sigma}{1-\nu}} - \frac{\chi}{1+\eta} N_t^{1+\eta}$$

consumption, labor, money, and bond holdings are chosen to maximize Eq. 1 subject to the budget constraint (Eq. 2) and the solvency condition. $\sigma$ is
the coefficient of the relative risk aversion of households (or the inverse of the intertemporal elasticity of substitution), $\eta$ is the inverse of the elasticity of work effort with respect to the real wage, and $\nu$ is the inverse of the elasticity of money holdings with respect to the interest rate, which can be seen as a non-separability parameter. Further, $b \geq 0$ and $\chi > 0$ are positive scale parameters, and $\varepsilon^m_t$ represents the money demand shock.$^1$

This utility function depends positively on the consumption of goods, $C_t$, positively on real money balances, $M_t/P_t$, and negatively on labor, $N_t$.

By assuming that capital plays a rather minor role in the business cycle (Backus et al., 1992), we do not include a capital accumulation process in the model, as in Galí (2015).

We assume a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good but uses an identical technology with the following production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$  \hspace{1cm} (4)

where $A_t = \exp(\varepsilon^a_t)$ is the level of technology assumed to be common to all firms and to evolve exogenously over time, $\varepsilon^a_t$ is the technology shock, and $\alpha$ is the measure of decreasing returns.

All firms face an identical isoelastic demand schedule and take the aggregate price level $P_t$ and aggregate consumption index $C_t$ as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and a staggered price setting. At any time $t$, only a proportion $1 - \theta$ of firms, with $0 < \theta < 1$, can reset their prices optimally, while the remaining firms keep their prices unchanged.

Our study focuses on the following two cases:

- One standard model without money ($b = 0$), Model 1, which does not include money.$^2$ Even though households gain utility from holding money, real money balances become irrelevant in explaining the model dynamics, disappearing from the system without any effect on the other variables.

- A second model assuming non-separability between consumption and money ($b > 0$), Model 2, where the marginal rate of substitution be-

$^1$We do not consider a preference shock in order to have the same number of structural shocks as historical variables (four). However, considering a preference shock would neither change our results nor reduce the role of the money demand shock on the dynamics (Benchimol, 2011).

$^2$This model is equivalent, in terms of results, to assuming separability between consumption and money. Theoretically, this assumption adds to Model 1 ($b = 0$) a money demand equation that does not affect the rest of the model. This work was also conducted with such a model, leading to the results presented in the following sections.

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between current and future consumption depends on current and future real money balances. Accordingly, holding money and consuming during the period is linked.

1.1 Model 1

This model is constituted by four equations with four variables, namely flexible-price output ($\hat{y}_t^f$), inflation ($\hat{\pi}_t$), output ($\hat{y}_t$), and the nominal interest rate ($\hat{i}_t$), and three structural shocks, a markup shock, $\varepsilon^\pi_t$, a monetary policy shock, $\varepsilon^i_t$, and a technology shock, $\varepsilon^a_t$, which are assumed to follow a first-order AR process with an i.i.d.-normal error term such as $\varepsilon^k_t = \rho_k \varepsilon^k_{t-1} + \omega_{k,t}$, where $\omega_{k,t} \sim N(0; \sigma_k)$ for $k = \{p, i, a\}$. The lowercase superscript (′) denotes log-linearized (around the steady state) variables:

\[
\hat{y}_t^f = \delta_a \varepsilon^a_t - \delta_c \]

\[
\hat{\pi}_t = \beta E_t [\hat{\pi}_{t+1}] + \delta_{y,t} \left( \hat{y}_t - \hat{y}_t^f \right) \]

\[
\hat{y}_t = E_t [\hat{y}_{t+1}] - \sigma^{-1} (\hat{i}_t - E_t [\hat{\pi}_{t+1}]) \]

\[
\hat{i}_t = (1 - \lambda_i) \left( \lambda_a (\hat{\pi}_t - \bar{\pi}_a) + \lambda_x (\hat{y}_t - \hat{y}_t^f) \right) + \lambda_i \hat{i}_{t-1} + \varepsilon^i_t \]

where

\[
\delta_a = \frac{1+\eta}{\sigma(1-\alpha)+\eta+\alpha} \quad \delta_c = \frac{(1-\alpha)}{\sigma(1-\alpha)+\eta+\alpha} \ln \left( \frac{\varepsilon}{\bar{z}-1} \right) \]

\[
\delta_{y,t} = \frac{1-\varepsilon}{\sigma(1-\alpha)+\eta+\alpha} \frac{(1+\varepsilon-1)\varepsilon^p_t}{1+(\varepsilon-1)(\alpha+\varepsilon^p_t)} \]

This baseline model is similar to that in Galí (2015) and does not include money in the utility function, production function, or Taylor rule. The monetary policy rule is an ad-hoc reaction function that depends on the monetary authority.

For simplification, our models are presented in a closed economy and the target variables are the inflation rate and output gap (Model 1). This approach rests on the fact that a real or nominal exchange rate effect exists

\footnote{Contrary to Benchimol and Fourçans (2012), our interest rate shock, $\varepsilon^i_t$, is assumed to follow an i.i.d.-normal error term such as $\varepsilon^i_t = \omega_{i,t}$, where $\omega_{i,t} \sim N(0; \sigma_i)$, in order to concur with previous works on Israeli DSGE models.}

\footnote{Benchimol (2015) shows that despite money being introduced into the production function, money plays no role in the economy under unconstrained returns-to-scale in a New Keynesian DSGE framework.}
indirectly (Ball, 1998; Taylor, 2001; Batini et al., 2003). Because the monetary policy rule is the result of the optimization of the central bank’s loss function, the latter based on the model of a closed economy, we choose to test a Taylor rule without any reference to the exchange rate\(^5\) in Model 1 as well as in Model 2.

1.2 Model 2

This model, as developed by Benchimol and Fourçans (2012), leads to six equations with six macro-level variables (and one more shock than the baseline model\(^6\)): flexible-price output (\(\hat{y}_t\)), flexible-price real money balances (\(\hat{m}_t\)), inflation (\(\hat{\pi}_t\)), output (\(\hat{y}_t\)), the nominal interest rate (\(i_t\)), and real money balances (\(\hat{c}_{mp}\)), such that

\[\hat{y}_t = v_{fa} y_{fa,t} + v_{m} \hat{m}_t - v_{c} + v_{sm} \varepsilon_{sm}^m\] (9)

\[\hat{m}_t = v_{ym} E_t \left[\hat{y}_{t+1}\right] + v_{ym} \hat{y}_t + \frac{1}{\nu} \varepsilon_{m}^m\] (10)

\[\hat{\pi}_t = \beta E_t \left[\hat{\pi}_{t+1}\right] + \kappa_{xt} \left(\hat{y}_t - \hat{y}_t\right) + \kappa_{mt} \left(\hat{m}_t - \hat{m}_t\right)\] (11)

\[\hat{y}_t = E_t [\hat{y}_{t+1}] - \kappa_{r} (i_t - E_t [\hat{\pi}_{t+1}]) + \kappa_{mp} E_t [\Delta \hat{m}_{t+1}] + \kappa_{sm} E_t [\Delta \varepsilon_{sm}^m]\] (12)

\[\hat{m}_t = \hat{y}_t - \kappa_i i_t + \frac{1}{\nu} \varepsilon_{m}^m\] (13)

\(^5\)Following certain shocks, the inclusion of the exchange rate in the Taylor rule can significantly change the inflation and output dynamics (Caraiani, 2013). An augmented Taylor rule with an exchange rate-related variable could be useful for the central banks in emerging economies (Filardo et al., 2011), but may be less so for a country such as Israel during and after the GFC. Other variables are frequently included in monetary policy rules, such as money growth, the real exchange rate, and/or deviation of the real exchange rate from an equilibrium level. However, it seems as though none of these transformations turn out to be significant in Israel (Yazgan and Yilmazkuday, 2007). In addition, it has become increasingly clear over time that the inflation target is the key monetary policy objective. The exchange rate is more assimilated to an indicator variable set by the market, while the exchange rate pass-through to prices will reduce over time as inflation target credibility and exchange rate flexibility improve (Leiderman and Bar-Or, 2002).

\(^6\)Contrary to Benchimol and Fourçans (2012), our money demand shock, \(\varepsilon_{m}^m\), is assumed to follow an \(i.i.d.-\)normal error term such as \(\varepsilon_{m}^m = \omega_{m,t}\), where \(\omega_{m,t} \sim N(0; \sigma_m)\), rather than conferring to the money shock a by-definition stronger role in the dynamics.
\[
\hat{t}_t = (1 - \lambda_2) \left( \lambda_\pi (\hat{\pi}_t - t_\pi) + \lambda_x \left( \hat{y}_t - \hat{y}_t' \right) \right) + \lambda_t \hat{t}_{t-1} + \varepsilon_t^i
\]

where

\[
\begin{align*}
\kappa_f &= \frac{1}{\nu - a_1(\nu - \sigma)} \\
\kappa_{mp} &= \frac{\nu - a_1(\nu - \sigma)}{\nu - a_1(\nu - \sigma)(1 - \nu)} \\
\kappa_{sm} &= -\frac{(\nu - a_1(\nu - \sigma)(1 - \nu))}{(\nu - a_1(\nu - \sigma)(1 - \nu))} \\
\kappa_i &= a_2/\nu \\
a_1 &= \frac{1}{1 + (b/(1-b))^{1/c(1-\beta)^{(1-\gamma)/\nu}}} \\
a_2 &= \exp(1/\beta - 1)
\end{align*}
\]

Another difference with Israeli dynamic models is that Benchimol and Fourçans (2012) adopt a Taylor rule that includes the money gap in its objective, whereas we do not (we assume \(\lambda_m = 0\)). Yet, such a rule can be derived from the optimization program of the central bank as a social planner (Woodford, 2003) and for this theoretical reason, we could include in our Taylor rule a money gap variable.\(^7\)

As can be seen, money enters explicitly in the equations that determine output (current output, Eq. 12, and its flexible-price counterpart, Eq. 9) and inflation (Eq. 11), because consumption and money holdings are linked in the agent’s utility function (non-separability assumption), and \(Y_t = C_t\) at equilibrium. Further, this is also related to households’ trade-off between holding money or consuming it.

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\(^7\)Eqs. 8 and 14 do not include natural interest rate variables. However, we also tested these models by including a natural interest rate à la Galí (2015) and this approach leads to similar results. We also tested Model 2 with a money gap variable in the monetary policy rule, again providing the same results as in the main analysis, reinforcing the role of money on output.
2 Parameters

Table 1 summarizes and describes the parameters used in the two models. More information about the calibration of these parameters for the estimation procedure is provided in Section 3.

$\beta$  Intertemporal deterministic discount factor.
$\alpha$  Share of worked hours in the production process.
$\theta$  Probability of firms that keep their prices unchanged (Calvo, 1983).
$\nu$  Inverse elasticity of substitution between consumption and real money balances.*
$\sigma$  Inverse intertemporal elasticity of substitution, which is also, in our framework, the coefficient of relative risk aversion.
$b$  Relative weight of real money balances in utility.*
$\eta$  Inverse Frisch elasticity of labor supply.
$\varepsilon$  Elasticity of substitution between individual goods.
$\lambda_i$  Interest rate smoothing.
$\lambda_\pi$  Inflation coefficient in the monetary policy rule.
$\lambda_x$  Output gap coefficient in the monetary policy rule.
$t_\pi$  Inflation target.
$\rho_a$  Persistence of the technology shock.
$\rho_p$  Persistence of the preference shock.
$\sigma_a$  Standard error of the technology shock.
$\sigma_p$  Standard error of the preference shock.
$\sigma_i$  Standard error of the monetary policy shock.
$\sigma_m$  Standard error of the money shock.*

* Parameter equal to zero in Model 1.

Table 1: Description of the parameters

3 Calibration

Following standard conventions, we calibrate beta distributions for parameters that fall between zero and one, inverse gamma distributions for parameters that need to be constrained to be greater than zero, and normal distributions in other cases. The parameters of both models are calibrated identically, as summarized in Table 2.
The calibration of $\sigma$ is inspired by the approaches taken by Rabanal and Rubio-Ramírez (2005) and by Casares (2007). In line with the risk aversion parameters chosen by these authors (2.5 and 1.5, respectively), we consider that $\sigma = 2$ corresponds to standard risk aversion. We thus adopt the same priors in both models with the same risk aversion calibration.

As in Smets and Wouters (2003), the standard errors of innovations are assumed to follow inverse gamma distributions and we choose a beta distribution for the shock persistence parameters (as well as for the backward component of the Taylor rule), which should be less than one. The calibration of $\alpha$, $\beta$, $\theta$, $\eta$, and $\varepsilon$ comes from Galí (2015) and the recent Israeli literature on DSGE models (Argov, 2012; Argov et al., 2012). The smoothed Taylor rule ($\lambda_i$, $\lambda_\pi$, and $\lambda_x$), as well as their standard errors, are calibrated following Argov et al. (2012). In order to take into consideration possible changes in the behavior of the central bank, we assign a higher standard error to the inflation Taylor rule’s coefficients ($\lambda_\pi$), while $\nu$, the non-separability parameter, must be greater than one. In addition, $\kappa_i$ (Eq. 13) must be greater than one, as this parameter depends on the elasticity of money substitution with respect to the cost of holding money balances, as explained in Söderström (2005); while still informative, this prior distribution is dispersed enough to allow for a wide range of possible and realistic values.

### Table 2: Priors summary

<table>
<thead>
<tr>
<th>Priors</th>
<th>Law</th>
<th>Mean</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$B$</td>
<td>0.33</td>
<td>0.05</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$B$</td>
<td>0.60</td>
<td>0.05</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$N$</td>
<td>1.25</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$N$</td>
<td>2.00</td>
<td>0.10</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$N$</td>
<td>2.00</td>
<td>0.10</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>$N$</td>
<td>6.00</td>
<td>0.10</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>$B$</td>
<td>0.70</td>
<td>0.10</td>
</tr>
<tr>
<td>$\lambda_\pi$</td>
<td>$N$</td>
<td>2.50</td>
<td>0.20</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>$N$</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>$N$</td>
<td>0.92</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note: $N$ stands for the normal distribution, $B$ for the beta distribution, and $I$ for the inverse gamma distribution.
to be considered (i.e. $\sigma > v > 1$). The relative weight of real money balances in utility ($b$) is calibrated to 0.25, as in Benchimol and Fourçans (2012), and the inflation target parameter $t_{\pi}$ is calibrated to 0.92, corresponding to an annual target of 3.68%, which is the mean of the announced targets during our study period.

The calibration of the shock persistence parameters and standard errors of innovations follow Smets and Wouters (2003) and Argov et al. (2012). However, we do not assume that the interest rate shock follows an AR(1) process, as in Argov et al. (2012), in addition to the money shock, mainly because we do not want to confer it a prior strength. All the standard errors of these shocks are assumed to be distributed according to inverse gamma distributions, with prior means of 0.04. The latter law ensures that these parameters have positive support. The AR parameters are all assumed to follow beta distributions. Further, the persistence parameters of the technology and price markup shocks are centered on 0.7 and 0.3, respectively, in line with Argov et al. (2012). We also take a common standard error of 0.1 for the shock persistence parameters, as in Smets and Wouters (2003).

Finally, we run these estimations and simulations with the above calibration, but assume the following monetary policy rule for Model 2:

$$\hat{i}_t = (1 - \lambda_i) \left( \lambda_x (\hat{\pi}_t - \pi_t) + \lambda_x (\hat{y}_t - \check{y}_t) + \lambda_m (\hat{m}_{\pi_t} - \check{m}_{\pi_t}) \right) + \lambda_i \hat{i}_{t-1} + \epsilon_i^t$$

where $\lambda_m$ was estimated with a prior mean of zero, a normal law, and a standard deviation of 0.1. The results are close to those obtained without considering a money gap variable in the monetary policy rule (Eq. 14). Note that under this configuration, $\lambda_m$ is significantly positive throughout the study period ($0.08 \leq \lambda_m \leq 0.12$).

### 4 On-impact impulse responses

The on-impact responses represent the first-period response of the variables to shocks (first-period impulse response function).
Figure 1: On-impact responses of inflation and output over time to a one percent standard deviation shock
Figure 2: On-impact responses of the interest rate and real money over time to a one percent standard deviation shock
Figure 3: On-impact responses of flexible-price output and flexible-price real money over time to a one percent standard deviation shock
5 Other variance decompositions

Variance decomposition is the decomposition of a variable’s variance with respect to the contribution of the shocks to the model. Short-run variance decomposition corresponds to the first-period variance decomposition of the variable, whereas long-run variance decomposition corresponds to the infinite period variance decomposition.

Figure 4: Long- and short-run variance decompositions (%)

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6 Debt and banks in the FCI

**Debt**
- Israeli government bonds options implied volatility
- Corporate bonds liquidity index
- Spread between yield on 10-year unindexed bonds and 3-month makam
- Spread between Israel and US CDS
- Spread between corporate (4 to 7 years of maturity) and government bonds
- Spread between 5-year Israeli government bonds and US government bonds
- Spread between AA-rated and BBB-rated corporate bonds
- Spread between banks’ credit and deposits interest rates
- Volume of corporate bonds issued by companies to total debt balance
- One-year inflation expectations
- Outstanding credit to business sector
- Percentage of firms reporting hard constraints in credit constraints
- Net balance of firms reporting an increase in the volume of credit
- Leverage of non-financial firms

**Banks**
- Spread between financial institutions bonds and government bonds
- Ratio of mortgages to GDP (household leverage)
- Ratio of nonperforming loans to total bank credit
- Leverage of financial firms
- Tier-one capital adequacy ratio
- Banks’ return on equity

Table 3: Debt and banks sectors in the FCI (Michelson and Suhoy, 2014)
References


