

# Supplement to "Money and monetary policy in the Eurozone: an empirical analysis during crises"

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## Abstract

We provide a short description of the two theoretical models (Model 1 and Model 2) used in [Benchimol and Fourçans \(2017\)](#). We also provide tables summarizing the mean of posterior means and standard deviations for each micro and macro parameters, for each model, and over our three crisis periods.

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# 1 The models

Both models consist of households that supply labor, purchase goods for consumption, and hold bonds; and firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their price during each period. Households and firms behave optimally, that is, households maximize the expected present value of utility, and firms maximize profits. There is also a central bank controlling the nominal rate of interest. These models are inspired by [Smets and Wouters \(2007\)](#), [Galí \(2008\)](#), and [Walsh \(2017\)](#).

## 1.1 The baseline separable model

The following New Keynesian DSGE model is mainly inspired by [Galí \(2008\)](#) and serves as a baseline model (Model 1).

### 1.1.1 Households

We assume a representative infinitely-lived household, seeking to maximize

$$E_t \left[ \sum_{k=0}^{\infty} \beta^k U_{t+k} \right] \quad (1)$$

where  $U_t$  is the period utility function and  $\beta < 1$  is the discount factor.

The household decides how to allocate its consumption expenditures among the different goods. This requires that the consumption index  $C_t$  be maximized for any given level of expenditure. Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_t C_t + M_t + Q_t B_t \leq B_{t-1} + W_t N_t + M_{t-1} \quad (2)$$

for  $t = 0, 1, 2, \dots$ . Here,  $P_t$  is an aggregate price index,  $M_t$  is the quantity of money holdings at time  $t$ ,  $B_t$  is the quantity of one-period nominally riskless discount bonds purchased in period  $t$  and maturing in period  $t+1$  (each bond pays one unit of money at maturity and its price is  $Q_t$ , where  $i_t = -\ln Q_t$  is the short-term nominal rate),  $W_t$  is the nominal wage, and  $N_t$  is hours of work (or the measure of household members employed).

The above sequence of period budget constraints is supplemented with a solvency condition, such as  $\forall t \lim_{n \rightarrow \infty} E_t [B_n] \geq 0$ , in order to avoid Ponzi-type

schemes. Preferences are measured with a common time-separable utility function. Under the assumption of a period utility given by

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi N_t^{1+\eta}}{1+\eta} \quad (3)$$

where consumption, labor, and bond holdings are chosen to maximize Eq. 1, subject to the budget constraint Eq. 2 and the solvency condition.  $\sigma$  is the coefficient of relative risk aversion of households (or the inverse of the intertemporal elasticity of substitution),  $\eta$  is the inverse of the elasticity of work effort with respect to the real wage, and  $\chi$  is a positive scale parameter.<sup>1</sup>

### 1.1.2 Firms

Backus et al. (1992) have shown that capital appears to play a rather minor role in the business cycle. Therefore, to simplify the analysis and focus on the role of money, we do not include a capital accumulation process in the model, as in Galí (2008).

We assume a continuum of firms indexed by  $i \in [0, 1]$ . Each firm produces a differentiated good but uses an identical technology with the following production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad (5)$$

where  $A_t = \exp(\varepsilon_t^a)$  is the level of technology assumed to be common to all firms and to evolve exogenously over time,  $\varepsilon_t^a$  is the technology shock, and  $\alpha$  is the measure of decreasing returns.

All firms face an identical isoelastic demand schedule and take the aggregate price level  $P_t$  and aggregate consumption index  $C_t$  as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time  $t$ , only a fraction  $1 - \theta$  of firms, with  $0 < \theta < 1$ , can reset their prices optimally, whereas the remaining firms index their prices to lagged inflation.

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<sup>1</sup>As in Benchimol (2014), Eq. 3 could be replaced by a time-separable MIU function such as

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma e^{\varepsilon_t^m}}{1-\vartheta} \left( \frac{M_t}{P_t} \right)^{1-\vartheta} - \frac{\chi N_t^{1+\eta}}{1+\eta}, \quad (4)$$

where  $\vartheta$  is the inverse of the elasticity of money holdings with respect to the interest rate,  $\varepsilon_t^m$  is a money shock accounting for changes in households' money holdings, and  $\gamma$  is a positive scale parameter. This specification makes money, and thus  $\varepsilon_t^m$ , irrelevant to the rest of the system (Smets and Wouters, 2007).

### 1.1.3 Central bank

Finally, the model is closed by adding the following monetary policy smoothed Taylor-type reaction function:

$$\hat{i}_t = (1 - \lambda_i) \left( \lambda_\pi (\hat{\pi}_t - \pi^*) + \lambda_x (\hat{y}_t - \hat{y}_t^f) \right) + \lambda_i \hat{i}_{t-1} + \varepsilon_t^i \quad (6)$$

where  $\lambda_\pi$  and  $\lambda_x$  are policy coefficients reflecting the weight on inflation and the output gap, respectively; whereas the parameter  $0 < \lambda_i < 1$  captures the degree of interest rate smoothing.  $\varepsilon_t^i$  is an exogenous *ad hoc* shock that accounts for fluctuations in the nominal interest rate,  $\pi^*$  is the inflation target, and the lowercase superscript ( $\hat{\cdot}$ ) denotes log-linearized (around the steady state) variables.

### 1.1.4 Solution

The solution of this model leads to four equations with four variables, namely, flexible-price output ( $\hat{y}_t^f$ ), inflation ( $\hat{\pi}_t$ ), output ( $\hat{y}_t$ ), and nominal interest rate ( $\hat{i}_t$ ); and three structural shocks that are assumed to follow a first-order autoregressive process with an *i.i.d.*-normal error term such as  $\varepsilon_t^k = \rho_k \varepsilon_{t-1}^k + \omega_{k,t}$ , where  $\omega_{k,t} \sim N(0; \sigma_k)$  for  $k = \{p, i, a\}$ .

$$\hat{y}_t^f = \frac{1 + \eta}{\sigma(1 - \alpha) + \eta + \alpha} \varepsilon_t^a - \frac{(1 - \alpha) \ln\left(\frac{\varepsilon}{\varepsilon - 1}\right)}{\sigma(1 - \alpha) + \eta + \alpha} \quad (7)$$

$$\hat{\pi}_t = \beta E_t [\hat{\pi}_{t+1}] + \frac{(1 - \theta) \left(\frac{1}{\theta} - \beta\right) (\sigma(1 - \alpha) + \eta + \alpha) (1 + (\varepsilon - 1) \varepsilon_t^p)}{1 + (\varepsilon - 1) (\varepsilon_t^p + \alpha)} (\hat{y}_t - \hat{y}_t^f) \quad (8)$$

$$\hat{y}_t = E_t [\hat{y}_{t+1}] - \sigma^{-1} (\hat{i}_t - E_t [\hat{\pi}_{t+1}]) \quad (9)$$

$$\hat{i}_t = (1 - \lambda_i) \left( \lambda_\pi (\hat{\pi}_t - \pi^*) + \lambda_x (\hat{y}_t - \hat{y}_t^f) \right) + \lambda_i \hat{i}_{t-1} + \varepsilon_t^i \quad (10)$$

where  $\varepsilon_t^p$  is the price markup shock,  $\varepsilon_t^i$  is the monetary policy shock, and  $\varepsilon_t^a$  is the technology shock.

This baseline model is close to Galí (2008) and does not include money in the utility function, the production function, or the Taylor rule.<sup>2</sup>

<sup>2</sup>Considering a time-separable MIU function (Eq. 4) simply defines money demand as a function of output, interest rate and its corresponding micro-founded shock ( $\varepsilon_t^m$ ). Because money does not appear in Eq. 7 to Eq. 10, money becomes irrelevant to the rest of the system.

## 1.2 The non-separable model

Money could be introduced in the utility function (MIU) either in a separable or non-separable manner. In the case where money is included in a separable manner, even though households gain utility from holding money, real money balances become irrelevant in explaining the dynamics of the model. Hence, our strategy is to introduce money with a non-separability assumption between consumption and real money balances (Model 2). In this case, the marginal rate of substitution between current and future consumption depends on current and future real money balances. Therefore, there is a link between holding money and consumption during the period.

As in the previous model, the representative infinitely-lived household seeks to maximize Eq. 1 with the period utility function  $U_t$ , such as

$$U_t = \frac{1}{1-\sigma} \left( (1-b) C_t^{1-\nu} + b e^{\varepsilon_t^m} \left( \frac{M_t}{P_t} \right)^{1-\nu} \right)^{\frac{1-\sigma}{1-\nu}} - \frac{\chi}{1+\eta} N_t^{1+\eta} \quad (11)$$

where consumption, labor, money, and bond holdings are chosen to maximize Eq. 1, subject to the same budget constraint and the same solvency condition as in the baseline model. This constant elasticity of substitution (CES) utility function depends positively on the consumption of goods,  $C_t$ , positively on real money balances,  $M_t/P_t$ , and negatively on labor,  $N_t$ , as in the baseline model.  $\nu$  is the inverse of the elasticity of money holdings with respect to interest rate and can be seen as a *non-separability* parameter.  $b$  and  $\chi$  are positive scale parameters. We use the same production function as in the baseline model.

In addition, a money variable appears in the monetary policy rule due to the optimization program of the central bank with respect to the inflation and output equations that include money (Woodford, 2003). Generally, in the literature, money is introduced through a money growth variable (Ireland, 2003; Andrés et al., 2006, 2009; Canova and Menz, 2011; Barthélemy et al., 2011). Benchimol and Fourçans (2012) introduce a *money-gap* variable and show that, at least in the Eurozone, it is empirically more significant than other measures of the money variable. We, therefore, use also a *money-gap* variable in our Taylor rule.<sup>3</sup>

The model leads to six equations with six macro variables, namely, flexible-price output ( $\hat{y}_t^f$ ), flexible-price real money balances ( $\widehat{mp}_t^f$ ), inflation ( $\hat{\pi}_t$ ),

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<sup>3</sup>A money-gap variable in the monetary policy reaction function is also used in Benchimol (2011) and Benchimol (2015). Benchimol (2016) uses a standard Taylor (1993) rule (without money) leading to similar results.

output ( $\hat{y}_t$ ), nominal interest rate ( $\hat{i}_t$ ), and real money balances ( $\widehat{mp}_t$ ), such that

$$\hat{y}_t^f = v_a^y \varepsilon_t^a + v_m^y \widehat{mp}_t^f - v_c^y + v_{sm}^y \varepsilon_t^m \quad (12)$$

$$\widehat{mp}_t^f = v_{y+1}^m E_t [\hat{y}_{t+1}^f] + v_y^m \hat{y}_t^f + \frac{1}{\nu} \varepsilon_t^m \quad (13)$$

$$\hat{\pi}_t = \beta E_t [\hat{\pi}_{t+1}] + \kappa_{x,t} (\hat{y}_t - \hat{y}_t^f) + \kappa_{m,t} (\widehat{mp}_t - \widehat{mp}_t^f) \quad (14)$$

$$\begin{aligned} \hat{y}_t = & E_t [\hat{y}_{t+1}] - \kappa_r (\hat{i}_t - E_t [\hat{\pi}_{t+1}]) \\ & + \kappa_{mp} E_t [\Delta \widehat{mp}_{t+1}] + \kappa_{sm} E_t [\Delta \varepsilon_{t+1}^m] \end{aligned} \quad (15)$$

$$\widehat{mp}_t = \hat{y}_t - \kappa_i \hat{i}_t + \frac{1}{\nu} \varepsilon_t^m \quad (16)$$

$$\hat{i}_t = (1 - \lambda_i) \left( \lambda_\pi (\hat{\pi}_t - \pi^*) + \lambda_x (\hat{y}_t - \hat{y}_t^f) + \lambda_m (\widehat{mp}_t - \widehat{mp}_t^f) \right) + \lambda_i \hat{i}_{t-1} + \varepsilon_t^i \quad (17)$$

where

$$\begin{aligned} v_a^y &= \frac{1+\eta}{(\nu-a_1(\nu-\sigma))(1-\alpha)+\eta+\alpha} & \kappa_r &= \frac{1}{\nu-a_1(\nu-\sigma)} \\ v_m^y &= \frac{(1-\alpha)(\nu-\sigma)(1-a_1)}{(\nu-a_1(\nu-\sigma))(1-\alpha)+\eta+\alpha} & \kappa_{mp} &= \frac{(\sigma-\nu)(1-a_1)}{\nu-a_1(\nu-\sigma)} \\ v_c^y &= \frac{(1-\alpha)}{(\nu-a_1(\nu-\sigma))(1-\alpha)+\eta+\alpha} \ln \left( \frac{\varepsilon}{\varepsilon-1} \right) & \kappa_{sm} &= -\frac{(1-a_1)(\nu-\sigma)}{(\nu-a_1(\nu-\sigma))(1-\nu)} \\ v_{sm}^y &= \frac{(1-\alpha)(\nu-\sigma)(1-a_1)}{((\nu-a_1(\nu-\sigma))(1-\alpha)+\eta+\alpha)(1-\nu)} & \kappa_i &= a_2/\nu \\ v_y^m &= 1 + \frac{a_2}{\nu} (\nu - a_1 (\nu - \sigma)) & a_1 &= \frac{1}{1+(b/(1-b))^{1/\nu}(1-\beta)^{(\nu-1)/\nu}} \\ v_{y+1}^m &= -\frac{a_2}{\nu} (\nu - a_1 (\nu - \sigma)) & a_2 &= \frac{1}{\exp(1/\beta)-1} \end{aligned}$$

$$\kappa_{x,t} = (\nu - a_1 (\nu - \sigma) + \frac{\eta+\alpha}{1-\alpha}) \frac{(1-\alpha)(\frac{1}{\theta}-\beta)(1-\theta)(1+(\varepsilon-1)\varepsilon_t^p)}{1+(\alpha+\varepsilon_t^p)(\varepsilon-1)}$$

$$\kappa_{m,t} = (\sigma - \nu) (1 - a_1) \frac{(1-\alpha)(\frac{1}{\theta}-\beta)(1-\theta)(1+(\varepsilon-1)\varepsilon_t^p)}{1+(\alpha+\varepsilon_t^p)(\varepsilon-1)}.$$

As can be seen, money enters explicitly in the equations that determine output (current output, Eq. 15, and its flexible-price counterpart, Eq. 12) and inflation (Eq. 14). Money enters these equations because consumption and money are linked in the agent's utility function and  $Y_t = C_t$  at equilibrium.

Notice that if  $\sigma = \nu$ , Eq. 11 becomes a standard separable utility function, where the direct influence of real money balances on output, inflation, and flexible-price output disappears.

## 2 Parameters sensitivity over time

### 2.1 European exchange rate mechanism crisis

Mean of posterior means and standard errors of structural and non-structural parameters over the ERM crisis.

	Model 1		Model 2		Model 2	
	Mean	Std	Mean	Std	Mean	Std
$\theta$	0.68	0.0077	0.64	0.0099	$\nu_a^y$	0.854 0.0023
$\alpha$	0.32	0.0031	0.30	0.0050	$\nu_m^y$	-0.049 0.0039
$\sigma$	2.49	0.0194	1.65	0.0248	$\nu_c^y$	0.055 0.0005
$\pi^*$	1.93	0.0033	1.90	0.0043	$\nu_{sm}^y$	0.132 0.0105
$\nu$			1.37	0.0024	$\kappa_{mp}$	0.109 0.0086
$\lambda_\rho$	0.43	0.0218	0.46	0.0156	$\kappa_{sm}$	0.297 0.0236
$\lambda_\pi$	1.70	0.0429	1.78	0.0409	$\kappa_i$	0.419 0.0007
$\lambda_x$	1.04	0.0648	0.83	0.0405	$1/\nu$	0.732 0.0013
$\lambda_{mp}$			0.66	0.0417		
$\rho_a$	0.98	0.0017	0.97	0.0030		
$\rho_p$	0.95	0.0039	0.96	0.0038		
$\rho_i$	0.38	0.0138	0.35	0.0113		
$\rho_m$			0.77	0.0266		

Table 1: Means of posterior means and standard errors of micro and macro-parameters over the ERM crisis

## 2.2 Dot-com crisis

Mean of posterior means and standard errors of structural and non-structural parameters over the Dot-com crisis.

	Model 1		Model 2			Model 2	
	Mean	Std	Mean	Std		Mean	Std
$\theta$	0.67	0.0062	0.62	0.0057	$\nu_a^y$	0.850	0.0043
$\alpha$	0.31	0.0026	0.29	0.0039	$\nu_m^y$	-0.049	0.0063
$\sigma$	2.55	0.0182	1.66	0.0400	$\nu_c^y$	0.055	0.0005
$\pi^*$	1.91	0.0022	1.88	0.0032	$\nu_{sm}^y$	0.132	0.0164
$\nu$			1.37	0.0020	$\kappa_{mp}$	0.108	0.0135
$\lambda_\rho$	0.42	0.0064	0.45	0.0070	$\kappa_{sm}$	0.291	0.0353
$\lambda_\pi$	1.79	0.0221	1.87	0.0291	$\kappa_i$	0.417	0.0006
$\lambda_x$	1.03	0.0199	0.82	0.0121	$1/\nu$	0.729	0.0011
$\lambda_{mp}$			0.63	0.0116			
$\rho_a$	0.98	0.0021	0.97	0.0024			
$\rho_p$	0.96	0.0011	0.97	0.0009			
$\rho_i$	0.37	0.0100	0.34	0.0097			
$\rho_m$			0.79	0.0510			

Table 2: Means of posterior means and standard errors of micro and macro-parameters over the Dot-com crisis

## 2.3 Global Financial Crisis

Mean of posterior means and standard errors of structural and non-structural parameters over the Global Financial Crisis.

	Model 1		Model 2		Model 2	
	Mean	Std	Mean	Std	Mean	Std
$\theta$	0.67	0.0156	0.63	0.0198	$v_a^y$	0.821 0.0043
$\alpha$	0.30	0.0088	0.29	0.0078	$v_m^y$	-0.095 0.0091
$\sigma$	2.50	0.0352	1.94	0.0574	$v_c^y$	0.053 0.0008
$\pi^*$	1.90	0.0039	1.88	0.0046	$v_{sm}^y$	0.260 0.0292
$\nu$			1.37	0.0076	$\kappa_{mp}$	0.202 0.0192
$\lambda_\rho$	0.44	0.0146	0.47	0.0138	$\kappa_{sm}$	0.552 0.0623
$\lambda_\pi$	1.75	0.0406	1.83	0.0400	$\kappa_i$	0.419 0.0023
$\lambda_x$	0.98	0.0335	0.80	0.0290	$1/\nu$	0.732 0.0041
$\lambda_{mp}$			0.60	0.0312		
$\rho_a$	0.98	0.0026	0.98	0.0034		
$\rho_p$	0.97	0.0025	0.97	0.0033		
$\rho_i$	0.43	0.0224	0.41	0.0175		
$\rho_m$			0.91	0.0117		

Table 3: Means of posterior means and standard errors of micro and macro-parameters over the GFC

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