Switching Volatility in a Nonlinear Open Economy*

Jonathan Benchimol† and Sergey Ivashchenko‡

October 7, 2019

Abstract

Uncertainty about a regime’s economy can change drastically around a crisis. An imported crisis, such as the global financial crisis (GFC) in the Euro area, shines a spotlight on the effect of foreign shocks. By estimating an open-economy nonlinear dynamic stochastic general equilibrium model for the Euro area and the US including Markov switching volatility shocks, we show that these shocks are significant during the GFC compared to more normal periods. We describe how US shocks from both the real economy and financial markets impacted the Euro area economy, and how bond reallocation occurred between short- and long-term maturities during the GFC. The estimated nonlinearities when domestic and foreign financial markets impact the economy should not be neglected. The nonlinear behavior of market-related variables highlights the importance of higher-order estimation to provide additional interpretations to policymakers.

Keywords: DSGE, Volatility Shocks, Markov Switching, Open Economy, Financial Crisis, Nonlinearities.

JEL Classification: C61, E32, F21, F41.

*This paper does not necessarily reflect the views of the Bank of Israel or the Ministry of Finance of the Russian Federation. We thank Robert Kollmann, John B. Taylor, Yossi Yakhin, and participants at the Bank of Israel Research Department seminar; the 3rd CEPR MMCN Annual Conference; the 5th Applied Macroeconomics Workshop by the Henan University and INFER; and the 49th Money, Macro, and Finance Research Group conferences for their valuable comments.

†Bank of Israel, Jerusalem, Israel. Corresponding author. Phone: +972-2-6552641. Fax: +972-2-6669407. Email: jonathan.benchimol@boi.org.il

‡Russian Academy of Sciences (IREP), Ministry of Finance (FRI), and Saint-Petersburg State University, Saint Petersburg, Russia.
1 Introduction

A widespread consensus in macroeconomics based on the linear new Keynesian model was shaken by the global financial crisis (GFC). These linear closed-economy dynamic stochastic general equilibrium (DSGE) models were silent about sharp variance changes, economic structural breaks, and distribution shifts. Consequently, regime-switching DSGE models have become the natural framework for analyzing macroeconomic dynamics (Maih, 2015).

An economic regime change could be related to a severe domestic or foreign financial crisis. The GFC started in the US and impacted the Euro area (EA), thus changing the global economic environment for these two economies. This switching process and analysis of such an international transition’s volatility are not possible with the standard (linear) closed-economy DSGE models commonly used in the literature. While usual DSGE models cannot reproduce switching volatility effects, linear Markov switching DSGE (MSDSGE) models reproduce these effects only partially.

Linear DSGE models are useful for describing global macroeconomic stylized facts, but not all economic dynamics can be replicated (Smets and Wouters, 2003, 2007), even though central banks frequently use them to assist forecasting and monetary policy decisions as well as to provide a narrative to the public (Edge and Gürkaynak, 2010). A nonlinear model estimated at higher-order solutions is essential for analyzing volatility shocks (Fernández-Villaverde et al., 2011), term structure (Rudebusch and Swanson, 2012), risk premia (Andreasen, 2012), and welfare dynamics (Garín et al., 2016).

Higher-order approximations of DSGE models are key in determining whether changing (switching) volatility is a driving force of business cycle fluctuations (Bloom, 2009). According to Markov processes, the volatility of several shocks can change over time. Furthermore, Markov switching (MS) models provide tractable ways to study agents’ expectation formation about changes in the economy, such as those occurring during a crisis (Foerster et al., 2016).

A vast body of literature has emerged in the two last decades about dynamic open-economy models (Galí and Monacelli, 2005; Adolfson et al., 2007; Justiniano and Preston, 2010). An analysis of the dynamic impacts resulting from regime-switching volatility changes in such a framework is still missing. No study has used MSDSGE models with switching volatility shocks (SVSs). We fill this gap by considering the consequences of SVSs in a two-country MSDSGE model.

One way of influencing the variance of stochastic processes driving the economy requires third-order approximations with the usual perturbation method (Fernández-Villaverde and Rubio-Ramírez, 2013). Although our model is relatively simple, this method would include more than 30 state variables and 10 autoregressive exogenous processes, making the third-order approximation and model estimation very
slow. In addition, this approach suggests a slow drifting of volatility, whereas during crises, high levels of volatility switching are more appropriate. This characteristic is generally captured by MS processes in which a second-order approximation is required to analyze volatility shocks (Andreasen, 2010). For this purpose, we use nonlinear approximation algorithms and filters to estimate our MSDSGE models (Binning and Maih, 2015; Maih, 2015). However, for the various reasons presented in Appendix A, we develop and use a generalization of the quadratic Kalman filter applied to MSDSGE models.\footnote{The MS quadratic Kalman filter (MSQKF) we use is presented in Appendix A.}

As domestic and foreign transmission channels were substantial during the GFC and previous crises (King, 2012; Benchimol and Fourçans, 2017), two relevant transmission channels complete the model. Households can buy or sell domestic or foreign bonds in the long or short term, and their money holdings increase their utility.

The model is estimated using the EA and US quarterly data compiled from 1995Q2 through 2015Q3 under three different specifications: a baseline version without MS; a version allowing MS in technology only; and another more developed version allowing MS in three exogenous processes for each country, that is, technology, home, and foreign monetary policy processes. To the best of our knowledge, this study is the first attempt to introduce long-term interest rates with embedded SVSs into a nonlinear open-economy DSGE model.

This exercise highlights several interesting results and policy implications. Firstly, we show and quantify that the average US and EA responses to shocks are different, especially around 2009Q1, which is also a case from the switching-volatility point of view. These differences essentially come from nonlinearities in economic dynamics, although our results are close to those obtained with a linear open-economy DSGE model (Chin et al., 2015). Secondly, we demonstrate the consequences of SVSs on US and EA economic dynamics. SVSs produce a combination of short-term deflation and long-term inflation effects in line with Kiley (2014) but with some asymmetries between the two economies. We demonstrate that SVSs partially cause financial flows, showing that SVSs significantly affect both the trade-off between short- and long-term bonds and consumption around the crisis. Thirdly, we confirm that SVSs have a stronger impact on US monetary policy compared with the EA monetary policy. The latter result has several policy implications, such as monetary policy uncertainty switches.

Our results suggest that policymakers should use nonlinear models to deal with an open economy and market-related variables, which are subject to more nonlinear dynamics than standard closed-economy variables are. By comparing our models and estimations, we also show that considering a common technology and both domestic and foreign monetary policy switching volatility shocks describes better
the US and EA dynamics.

The remainder of the paper is organized as follows. Section 2 presents the model used for the estimation presented in Section 3. Section 4 presents the results and Section 5 interprets them. Section 6 concludes, and the Appendix presents additional results.

2 The model

Our generic model consists of a symmetric two-country model in which the domestic \((d)\) and foreign \((f)\) households maximize their respective utilities subject to their budget constraints (Section 2.1), firms maximize their respective benefits (Section 2.2), and central banks follow their respective \textit{ad-hoc} Taylor-type rules and budget constraints (Section 2.3). The model’s equilibrium (Section 2.4) and stochastic structure (Section 2.5) are also presented in this section.

2.1 Households

For each country \(i \in \{d, f\}\), we assume a representative infinitely lived household seeking to maximize

\[
E_t \left[ \sum_{t=0}^{\infty} \varepsilon_{i,t-1}^u U_{i,t} \right],
\]

where \(\varepsilon_{i,t-1}^u < 1\) is the exogenous process corresponding to households’ country-specific intertemporal preferences\(^2\), and \(U_{i,t}\) is the households’ country-specific intertemporal utility function, such as

\[
U_{i,t} = \left( \frac{C_{i,t}}{\Psi_{i,t}} - h_i \frac{C_{i,t-1}}{Z_{i,t-1}} \right)^{1-\frac{1}{\gamma_{i,c}}} + \varepsilon_{i,t}^m \left( \frac{M_{i,t}}{Z_{i,t} \Psi_{i,t}} \right)^{1-\frac{1}{\gamma_{i,m}}} - \varepsilon_{i,t}^l \frac{L_{i,t}}{1 + \frac{1}{\sigma_{i,l}}} - \Psi_{i,t},
\]

where \(C_{i,t}\) is the country-specific \textit{Dixit and Stiglitz (1977)} aggregator of households’ purchases of a continuum of differentiated goods produced by firms, \(M_{i,t}\) indicate the country-specific end-of-period households’ nominal money balances, \(Z_t\) is the common level of technology progress,\(^3\) \(P_{i,t}\) is the country-specific \textit{Dixit

---

\(^2\)At time \(t\), households know their intertemporal preferences for \(t + 1\) but have uncertainty about their preferences for the future. Hence, they know their preference multiplier for \(t + 1\). While they know \(\varepsilon_{i,t}^u\) at time \(t\), they do not know \(\varepsilon_{i,t+1}^u\) at time \(t\). Because utilities for \(t + 1\) should be multiplied by \(\varepsilon_{i,t+1}^u\), the current period utilities should be multiplied by \(\varepsilon_{i,t}^u\).

\(^3\)See, among others, Fagan et al. (2005), Schmitt-Grohé and Uribe (2011), and Diebold et al. (2017) for similar detrending. A stochastic trend with drift is suggested by the data—nonzero mean growth rate of macro-variables. Any DSGE model without trends is unrelated to real-world
and Stiglitz (1977) aggregated price index, and $\Psi_{i,t}$ is the country-specific cost function described by Eq. 3. $\sigma_{i,c}$ is the country-specific intertemporal substitution elasticity of habit-adjusted consumption (i.e., inverse of the coefficient of relative risk aversion), $\sigma_{i,m}$ is the country-specific partial interest elasticity of money demand, and $\sigma_{i,l}$ is the country-specific Frisch elasticity of labor supply. $\varepsilon_{i,t}^m$ and $\varepsilon_{i,t}^l$ are the country-specific exogenous processes corresponding to real money holding (liquidity) preferences and the worked hours (disutility of labor) of households, respectively.

The country-specific household’s cost function, $\Psi_{i,t}$, is defined by

$$\Psi_{i,t} = \frac{1}{2} \sum_{j \in \{sr, lr\}} \varphi_{i,j} \left( \frac{B_{i,j,t}}{P_{i,t} C_{i,t-1}} - \mu_{i,j} \right)^2 + \varphi_{i,f} \left( \frac{e_{i,t} B_{i,f,j,t}}{P_{i,t} C_{i,t-1}} - \mu_{i,f} \right)^2,$$

where $\forall k \in \{d, f\}$ and $\forall j \in \{sr, lr\}$, $\varphi_{i,k,j}$ and $\mu_{i,k,j}$ are scale parameters related to the bonds’ rigidity, and $B_{i,k,j,t}$ represents $j$-term $k$-bonds bought by households in country $i$ in period $t$, where $k$ represents the issuing country of the bond and $j$ its maturity, that is, short-term ($sr$) or long-term ($lr$) bonds. $e_{i,t}$ is the country-specific exchange rate relating to the number of domestic currency units available for one unit of foreign currency at time $t$ (i.e., $e_{d,t} = 1/e_{f,t}$).

The market consists of domestic and foreign one-period short- and long-term bonds. Long-term bonds pay country-specific shares ($S_i$) of their current nominal value in each period.5 In practice, $S_i$ defines the bond duration (average time until cash flows are received).

Then, $\forall i \in \{d, f\}$, the country-specific households’ budget constraint, can be statistics and any approximation of a solution in initial terms–without removing trends–will not satisfy the Blanchard and Kahn (1980) conditions–explosive solution. Although the use of several trends is better (Schmitt-Grohé and Uribe, 2011), it requires a much more complicated model.

4When two agents with different intertemporal preferences trade the same security–especially bonds–credit-borrowing constraints are mandatory to avoid agents taking unrealistic positions. Thus, we add a quadratic portfolio adjustment rigidity for each type of bond position in the households utility function, which produces smoothed restrictions. For simplification, we do not modulate such rigidity by restricting negative values. Although our approach is close to the portfolio adjustment costs à la Schmitt-Grohé and Uribe (2003) or price rigidity à la Rotemberg (1982), we assume preference costs–in the utility function–while Schmitt-Grohé and Uribe (2003) assume real costs–in the budget constraint. As it is more likely households feel disutility from its financial position deviations from the steady-state, we do not assume it requires real goods to compensate these deviations.

5A long-term bond with a nominal value of one domestic currency unit produces $S_d$ units of domestic currency in the first period, $S_d (1 - S_d)$ in the second period, $S_d (1 - S_d)^2$ in the third period, and so on. Because inflation-linked bonds are relatively rare and has smaller liquidity in the US and the EA, we price bonds in nominal terms.
expressed as follows:

\[
P_{i,t}C_{i,t} + M_{i,t} + \sum_{j \in \{sr, lr\}} B_{i,i,j,t}Q_{d,j,t} + e_{i,t}B_{i,-i,j,t}Q_{-i,j,t}
\]

\[
= W_{i,t}L_{i,t} + M_{i,t-1} + D_{i,t} + B_{i,i,ar,t-1} + B_{i,i,lr,t-1}((1-S_i)Q_{i,i,lr,t} + S_i)
\]

\[
+ e_{i,t}B_{i,-i,ar,t-1} + e_{i,t}B_{i,-i,lr,t-1}((1-S_{-i})Q_{i,-i,lr,t} + S_{-i}),
\]

where index \(-i\) denotes the other country (i.e., if \(i = d\), then \(-i = f\); if \(i = f\), then \(-i = d\)), and \(Q_{k,j,t} = \exp(-r_{k,j,t})\) denotes the price of the country-specific \((k)\) nominal interest rate at maturity \(j\). \(W_{i,t}\) is the country-specific wage index, and \(D_{i,t}\) represents dividends paid by firms in country \(i\) at time \(t\). Optimality conditions are detailed in the online appendix.

Some DSGE models include a single variable for lump-sum tax and dividends in the budget constraint (Schmitt-Grohé and Uribe, 2011), some others use two separate variables (Smets and Wouters, 2007). For simplification reasons, we do not have a lump-sum tax in our model and only report dividends instead.

Note that money and the money demand shock do not influence the economy in the case of separable (additive) money in the utility function (Galí, 2015). However, the nonexistence of a lump sum tax in our model that controls the bond position changes this mechanism. Our model has no such restrictive lump-sum taxation, which leads to the influence of money (and money demand shock) on the economy.

### 2.2 Firms

The continuum of identical firms, in which each firm produces a differentiated good using identical technology, is represented by the following production function:

\[
Y_{F,i,t}(j) = A_{i,t}L_{i,t}(j),
\]

where \(A_{i,t} = A_{i}Z_{t}\) is the country-specific level of technology, assumed to be common to all firms in country \(i\), and evolving exogenously over time, and \(A_{i}\) is a country-specific total factor productivity scale parameter.

As in Galí (2015), to simplify our analysis, we do not include the capital accumulation process in this model, which appears to play a minor role in the business cycle (Backus et al., 1992) and assume constant return to scale for simplification purposes.\(^6\) The exogenous process \(Z_{t}\) introduces a stochastic trend to the model.

\(^6\)In this simple case, we also do not consider money in the production function. Several examples exploring this particular set-up are available in the literature (Benchimol, 2015; Gorton and He, 2016). Note that given the complexity of our model and empirical exercise, we assume long-term exogenous growth in a model without capital. Further research should analyze the benefits of capital as a factor of production to explain long-term growth.
It allows explaining the nonzero steady-state growth of the economy (Chaudourne et al., 2014; Diebold et al., 2017). Although alternative techniques to introduce a unit root exist (Schmitt-Grohé and Uribe, 2011), they complicate the model. For instance, Smets and Wouters (2007) reconstruct the deterministic component of the trend which affects the model accuracy.

All firms face an identical isoelastic demand schedule and take the country-specific aggregate price level, $P_{i,t}$, and aggregate consumption index, $C_{i,t}$, as given. Following Rotemberg (1982), our model features monopolistic competition and staggered price setting and assumes that a monopolistic firm faces a quadratic cost of adjusting nominal prices measured in terms of the final good given by

$$
\frac{1}{P_{i,t}Z_t} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} D_{i,t+s} - \varphi_{i,p} \left( \frac{P_{i,t+s}(j)}{P_{i,t+s-1}(j)} - 1 \right)^2 P_{i,t+s} Y_{i,t+s} \right],
$$

(6)

where $P_{s,i,t} = \exp (v_i \pi_t + (1 - v_i) \pi_{i,t+s-1})$ represents the country-specific weighted average between country-specific steady-state inflation, $\pi_t$, and country-specific previous inflation, $\pi_{i,t-1}$, in period $t$, where $v_i$ is the country-specific weight, and $\pi_{i,t} = \ln (P_{i,t}/P_{i,t-1})$.

$P_{i,t}(j)$ is the price of goods $j$ in period $t$ from firms in country $i$, $R_{i,t} = \exp (r_{i,t})$ is the short-term nominal interest rate, and $\varphi_{i,p} \geq 0$ is the degree of nominal price rigidity in country $i$. The country-specific adjustment cost, which accounts for the negative effects of price changes on the customer–firm relationship in country $i$, increases in magnitude with the size of the price change and with the overall scale of the country-specific economic activity $Y_{i,t}$.

In each period $t$, the firm’s budget constraint requires

$$
D_{i,t} + W_{i,t} L_{i,t} = P_{i,t}(j) Y_{i,t}(j),
$$

(7)

where $Y_{F,i,t}(j)$ represents the production firms in country $i$ of goods $j$ in period $t$. Firms cannot make any investment (Eq. 7) and distribute all their benefits through dividends (Eq. 6).

The final consumption good is a constant elasticity of substitution composite of domestically produced and imported aggregates of intermediate goods that produces demand for firm output, such as

$$
Y_{F,i,t+s}(j) = \omega_i Y_{i,t+s} \left( \frac{P_{i,t+s}(j)}{P_{i,t+s}(j)} \right)^{\varepsilon_{i,t+s}} + (1 - \omega_i) Y_{-i,t} \left( \frac{e_{i,t+s} P_{-i,t}}{P_{i,t+s}(j)} \right)^{\varepsilon_{i,t+s}},
$$

(8)

where the exogenous process $\varepsilon_{i,t+s}$ represents the country-specific price markup shock (elasticity of demand in country $i$), and parameter $\omega_i$ defines a country-specific preference for local demand.
The aggregate country-specific price level also follows the usual constant elasticity of substitution aggregation, such as

\[ P_{i,t}^{\varepsilon_{i,t}} = \omega_i P_{i,t} (j)^{1-\varepsilon_{i,t}} + (1 - \omega_i) (e_{i,t} P_{-i,t} (j))^{1-\varepsilon_{i,t}}, \]

where the local price index includes domestic and foreign prices as usual in open-economy models.

### 2.3 Central bank

The central banks follow a Taylor (1993)-type rule, such as

\[ R_{i,t} = \varepsilon_{i,t}^r \rho_{i,s} (\hat{\pi}_{i,t} - \pi_{i,s}) + \rho_{i,y} (\hat{y}_{i,t}^\gamma - y_{i,t}^\gamma) + \rho_{i,e} (\hat{e}_{i,t} - e_{i,e}), \]

where \( \varepsilon_{i,t}^r \) captures the country-specific monetary policy shocks, \( \hat{\pi}_{i,t} \) is the country-specific inflation gap expressed as the ratio between country-specific CPI and its corresponding steady-state, \( \hat{y}_{i,t}^\gamma \) is the country-specific output gap expressed as the ratio between country-specific output (normalized by technological progress) and its corresponding steady-state, and \( \hat{e}_{i,t} \) is the country-specific real exchange rate gap expressed as the ratio between the real exchange rate of country \( i \) and its corresponding steady-state.

The parameter \( \rho_{i,s} \) captures interest rate-decision smoothing, and \( \rho_{i,y} \) and \( \rho_{i,e} \) capture weight placed by the monetary authority of country \( i \) on the inflation gap, output gap, and real exchange rate, respectively.

A standard budget constraint applies to debt bought by central banks, such as

\[ \frac{B_{i,g,t}}{R_{i,t}} = B_{i,g,t-1} + M_{i,t} - M_{i,t-1}, \]

where \( B_{i,g,t} \) represents country-specific nominal bonds bought by the local central bank in period \( t \).

In our model, we assume that central banks can buy only short-term bonds, as was the case in the US and EA before the GFC.

### 2.4 Equilibrium

In equilibrium, country-specific demand consists merely of consumption, such as

\[ Y_{i,t} = C_{i,t}, \]

and each bond should be bought, requiring that

\[ B_{i,sr,t} + B_{-i,i,sr,t} + B_{i,g,t} = 0, \]
and

\[ B_{i,i,i,r,t} + B_{-i,i,i,r,t} = 0. \quad (14) \]

Note that the country-specific demand presented in Eq. 12, \( Y_{i,t} \), is different from the country-specific supply presented in the production function (Eq. 5), \( Y_{F,i,t} \). As in Berka et al. (2018) that also have only one source of demand, this simplification (Eq. 12) substantially decreases the number of variables which is key for nonlinear estimation.

### 2.5 Stochastic structure

The exogenous processes we use are defined such as, \( \forall i \in \{ d, f \} \) and \( \forall j \in \{ u, m, l, p, r, y \} \),

\[ \phi_{i,t}^j = \eta_{i,j} \phi_{i,t-1}^j + (1 - \eta_{i,j}) \bar{n}_{i,j} + \xi_{i,j,t} \quad (15) \]

where parameter \( \bar{n}_{i,j} \) defines the country-specific steady-state of exogenous process \( j \), \( \eta_{i,j} \) the country-specific autocorrelation level, and \( \xi_{i,j,t} \) the country \( (i) \) shock-specific \( (j) \) white noise (zero-mean normal distribution).

The demand elasticity exogenous process is defined by \( \phi_{i,t}^p = \varepsilon_{i,t}^p \), the intertemporal preference exogenous process by \( \phi_{i,t}^u = \ln \left( \frac{\varepsilon_{i,t}^u}{\varepsilon_{i,t-1}^u} \right) \), the technology progress by \( \phi_t^l = \ln \left( \frac{Z_t}{Z_{t-1}} \right) \), and other exogenous processes by \( \forall i \in \{ d, f \} \) and \( \forall j \in \{ m, l, r \} \), \( \phi_{i,t}^j = \ln \left( \varepsilon_{i,t}^j \right) \).

A summary of all variables used in the model is available in Appendix B.

### 3 Methodology

In this section, we present the dataset used for the estimations (Section 3.1), as well as the estimation (Section 3.2) and nonlinear impulse response functions’ computation (Section 3.3) methodologies.

#### 3.1 Data

We estimate our model with quarterly EA (domestic) and US (foreign) Organisation for Economic Co-operation and Development (OECD) data from 1995Q2 to 2015Q3. In addition, we use the Euro/Dollar (EUR/USD) exchange rate from the European Central Bank (ECB) and Federal Reserve Economic Data (from the Federal Reserve Bank of St. Louis) for the exchange rate before the creation of the EA (1999). The 11 observed variables are as follows: real gross domestic product (GDP) growth rate (EA and US), GDP deflator (EA and US), ratio of domestic demand to GDP (EA and US), 3-month interbank rate (EA and US), 10-year interest rate (EA and US), and EUR/USD growth rate.
With five country-specific shocks and one joint total factor productivity shock, the number of shocks is equal to the number of observed variables. Our model and empirical investigation include the long-term interest rate, allowing us to capture long-term bond demand/supply effects through their interest rates in both countries. We also capture monetary aggregate dynamics and negative interest rates. The use of the 3-month interbank rate from the OECD database makes the zero lower bound (ZLB) problem less critical as it becomes negative for EMU at several periods. Consequently, although we did not explicitly model unconventional monetary policies, our data are not silent about some unconventional monetary policy effects.

3.2 Estimation

Our switching (two-regime) model is estimated in three different ways with maximum likelihood techniques. First, we estimate a baseline version of our model, without SVSs (without switching). As the productivity shock remains the main source of uncertainty in the business cycle (Bloom et al., 2018), another version is estimated by considering only one SVS in \( Z_t \) (hereafter, 1SVS). A third version considers both productivity and monetary policy switching volatility shocks: \( \varepsilon_{d,t}^r, \varepsilon_{f,t}^r, \) and \( Z_t \) (hereafter, 3SVS). The 3SVS model intends to capture volatility regime switches occurred during the GFC in both the US and EA, as suggested by Mavromatis (2018). Monetary policy and productivity shocks are the main driving forces of the business cycles. Additional SVSs are feasible in theory, but in practice, they require significant additional computing resources and may not change the results or make the model more realistic.

The model solution approximation is computed with an efficient second-order perturbation method developed by Maih (2015). We use the MSQKF filter described in Appendix A, which is an extension of the QKF for Markov-switching case (Ivashchenko, 2014). The switching volatility and the second-order approximation features constitute the nonlinearities of our models. We use the first four quarters as a presample of our three estimations and jackknife bootstrapping for robustness purposes.\(^7\) The estimation results of these three models are provided and compared in Appendix C to show that the 3SVS model, which includes switching volatility in the technology and monetary policy shocks, is the best model to

---

\(^7\)Our table of observations has 11 columns (observables) and 82 rows (periods). We randomly discard four observations from this table and perform maximum likelihood estimation. We repeat this process more than 100 times and receive a robust variance estimation. Note that our methodology—jackknife bootstrapping—is different from prefiltering that is not using likelihood values corresponding to the first four quarters for all variables. We apply the jackknife bootstrapping that suggests discarding of four observations randomly, combining variable and period.
explain current and forecasted aggregate and individual (observable) dynamics.

Note that the share of steady-state inflation indexation ($v_i$) differs across countries and versions of the model. The coefficient for the US is close to that of Smets and Wouters (2007). The version without switching has a larger share of steady-state inflation indexation. The other models could produce smaller estimated values of the $v_i$ parameter, which are close to the 1SVS result for the EA, and even smaller for Canada, which is close to the EA results in the version without switching (Justiniano and Preston, 2010). The share of steady-state inflation indexation for the EA is much smaller. The 3SVS version produces the closest values of corresponding parameters. Thus, volatility switching might have an important influence on inflation persistence, and the share of past inflation indexation ($1 - v_i$) is one of the key elements for inflation persistence.

For the model with variance switching in multiple exogenous shocks, regime 2 has higher variance of $Z_t$. However, in this case, several variances in the second state are smaller.

Fig. 1 presents filtered values of regime 1 probabilities and three selected exogenous processes ($\varepsilon_{d,t}^p$, $\varepsilon_{f,t}^p$, and $Z_t$). Fig. 1 presents $\text{Prob}(r_t = 1)$ conditional on data probability where $\text{Prob}(r_t = 1)$ corresponds to the probability to be in regime one in period $t$.

It appears there are some differences between the filtered values of exogenous processes, but they are moderate. In addition, some differences about state probabilities are visible, but they are linked to the state of model 1SVS, whereas state probabilities of model 3SVS are more reliable. The latter correspond to the actual main crises that occurred during the sample under consideration. The difference between the filtered values of exogenous processes is generally smaller, but it is larger a few years after beginning of the GFC. Economic driving forces are generally not impacted by SVSs, except at some points of time, especially during crises. This is also the case when monetary policy shocks are considered.

### 3.3 Impulse response functions

In order to analyze the response of variables to economic shocks, we compute for each variable its impulse response function (IRF) to each shock. The standard definition, such as presented in Dynare (Adjemian et al., 2011), defines IRF as the expected difference between the trajectory with one shock in a single period one standard deviation higher and the usual trajectory. More precisely, we express this as

$$\text{IRF}_t (x, \xi) = E [x_t | \xi_1 \sim N (\sigma (\xi), \sigma (\xi))] - E [x_t | \xi_1 \sim N (0, \sigma (\xi_1))],$$

where $x_t$ is the value of the variable of interest for which IRF is computed in period $t$, $\xi_1$ is the shock of interest that deviates in period 1, $\sigma (.)$ is the standard error
operator, $E[\cdot]$ is the expectation operator, and $N$ is the normal law.

We generalize this definition in the nonlinear case by making the magnitude and sign of the shock more important. Such a generalization requires the introduction of parameter $s$ in Eq. 16 to determine the number of standard deviations in the shock, such as

$$IRF_{t,s}(x,\xi) = \frac{E[x_t|\xi_1 \sim N(\sigma(\xi_1)s,\sigma(\xi_1))] - E[x_t|\xi_1 \sim N(0,\sigma(\xi_1))]}{s}.$$  (17)

In addition, we compute the IRFs conditional on the state variables’ vector $X_t$.
to show differences between IRFs at different states of the world, such as

$$IRF_{t,s}(x_t, \xi_t | X_0) = \frac{E[x_t|\xi_1 \sim N(\sigma(\xi_1)s, \sigma(\xi_1)); X_0]}{s} - E[x_t|\xi_1 \sim N(0, \sigma(\xi_1)); X_0]$$

where $X_0$ is a vector of state variables before the shock.

The IRF for switching shock is

$$IRF_t(x, v_0, v_1) = E[x_t|r_0 = v_0; r_1 = v_1] - E[x_t|r_0 = v_0],$$

where $r_t$ is the regime variable at time $t$, and $v_0$ and $v_1$ are particular switching values of the regime that are of interest.

In the practical computation of expectations, we use a simulation with the same exogenous shocks for both parts of the IRF equation. We use 50,000 draws for averaging and 100 presample draws for unconditional IRF.\(^8\)

4 Results

In this section, we present the responses of our model after a switching volatility shock (Section 4.1) and a monetary policy shock (Section 4.2). Furthermore, we present and analyze some nonlinearities (Section 4.3). Other results are available upon request. Additional performance measures showing the advantages of volatility switching (3SVS) model compared to the other models are presented in Appendix C.

4.1 Switching volatility shock

Fig. 2 presents the IRFs of SVSs from states 1 to 2 (with higher volatility for $Z_t$) for the 1SVS model. We compute the unconditional IRF and plot the mean IRF and +/- two standard deviations (std) of IRF.

Fig. 2 shows that the regime probability effect disappears without strong persistence (around 10 periods). However, the effect on the model’s variables is much more persistent and differs across countries. Following an SVS, inflation increases in the two countries during the first periods, involving an increase of the US short-term nominal interest rate, while the EA’s short-term nominal interest rate stays stable. The picture changes drastically in later periods when the US’s and EA’s long- and short-term interest rates decrease with inflation rates. Only GDP growth and the exchange rate are stabilized after several periods.

\(^8\)We consider the steady-state as the initial point, and we draw the trajectory for 100 periods. The shock occurs in period 101, and we repeat this 50,000 times.
The US long-term rate decreases more smoothly a few quarters after the shock compared with the EA long-term nominal interest. This difference between the US and EA long-term nominal interest rates can be explained by the different durations of the EA ($s_d = 0.6$) and US ($s_f = 0.06$) long-term bonds.

In addition, monetary policy weights, by generating different short-term interest rates, could explain this phenomenon. The US has a stronger response to inflation and a smaller smoothing coefficient compared with the EA. Consequently, the US’s short-term nominal interest rate decreases with inflation and increases later, while that for the EA increases slightly. This difference in the conduct of monetary policy produces fluctuations in the exchange rate and ratio of domestic demand to GDP.

Fig. 3 provides a more robust picture compared with Fig. 2.

Indeed, the 1SVS model suggests only small differences between regimes (the standard deviations are close), implying a small effect on the economy of switching,
which explains the small values obtained in Fig. 2. However, the 3S VS model suggests much larger differences and a substantial impact of switching shocks on the economy.

Fig. 3 highlights that SVSs impact US inflation and nominal interest rates in both the short and long terms, while the impact on the EA economy is less significant. Such SVSs durably impact long-term US interest rates, but this is not the case for the EA’s long-term interest rates.

Uncertainty around the EA’s short-term nominal interest rate, measured as the gap between -2 std and +2 std around the IRF, is stronger than that around the US’s short-term nominal interest rate.

In addition, demand-to-GDP ratios of the two countries display substantial uncertainty, showing that SVSs in monetary policy shocks of the two countries have important implications for the economy.

In Fig. 3, the economy switches to regime 2, which means a substantial in-
crease in foreign and domestic monetary policy shocks’ volatility and a decrease of the total factor productivity shock volatility. A higher uncertainty means higher interest rates. However, the central bank controls interest rates, buys bonds, and prints money that leads to higher inflation. As the economy is open, domestic changes are substantial, and foreign households buy more domestic bonds. Foreign households work more and sell more goods to the domestic country. Foreign investment in the domestic market makes foreign currency cheaper. Thus, foreign households increase investments and hold more money. As this effect is powerful, foreign inflation decreases, leading to lower foreign interest rates.

Figure 4: Conditional and unconditional IRFs to a switching (to regime 2) volatility shock (1SVS).

The average effect of unconditional SVSs might differ from that of conditional IRFs. For example, Fig. 4 compares unconditional IRF with conditional IRFs for 2009Q1 and 2003Q4. In other words, we use filtered values of the variable vector for corresponding dates as the condition (the initial point for a draw).
These IRFs are different in several aspects. Indeed, the regime probability IRF differs in the configurations shown in Fig. 4 where we compare the best expansion (2003Q4) and worst recession (2009Q1) periods. This could be because the economy is in regime 2 when the shock occurs in the case of the unconditional IRF, while the conditional IRF could be in regime 1 before the shock.

Besides, the 3SVS model highlights significant differences between the crises as well as between the US and EA (Fig. 5).

Figure 5: Conditional and unconditional IRFs to a switching (to regime 2) volatility shock (3SVS).

These differences are more reliable compared to the 1SVS model. For instance, the EA short-run nominal interest rate was not similarly impacted by the switching during good times and bad times (such as the subprime crises) and their corresponding SVSs. Furthermore, significant differences are observed for the ratio of domestic demand to GDP in both countries.
An SVS has a stronger impact on the EA’s demand-to-GDP ratio compared with the US, at least after the Dot-com crisis. In addition, Fig. 5 shows that the EA consumes more, while the US consumes less. At the same time, US inflation and interest rates decrease slightly further compared with the unconditional points and the EA.

The consequences of regime switching are compared for both models in Fig. 6. As expected, the IRFs are significantly different, mainly due to the switch of multiple variances. The response of the 3SVS model is less monotonic, and the magnitudes of the IRFs are different for almost all economic variables.

Figure 6: Unconditional IRFs to a switching (to regime 2) volatility shock for 1SVS and 3SVS models.

Fig. 6 shows that the switch of multiple variances significantly affects the exchange rate as well as US long-run nominal interest rates, while EA short and long-run nominal interest rates are less affected. However, the EA demand-to-GDP ratio is more impacted than the US ratio.
The 3SVS model captures several dynamics that a 1SVS model without switching cannot, such as decreasing short-term US nominal interest rate or oscillating EA inflation.

Fig. 7 shows how the 1SVS model influences financial variables, unconditionally and conditionally, to the reference dates.

Figure 7: Conditional and unconditional IRFs to a switching (to regime 2) volatility shock for financial variables (1SVS)

Although the magnitude of these IRFs is relatively low, some conclusions can be drawn. 1SVS during the subprime, or after the Dot-com crisis, and unconditional to time, are clearly different (Fig. 7). An SVS increases US bonds bought by the EA and US households’ bonds as well as EA bonds bought by the ECB (short run).

Following such an SVS shock, the exchange rate, short- and long-run EA bonds bought by EA households and the Federal Reserve, and money held by US house-
holds decrease.

The main problem in this scenario is that it assumes that the aftermaths of the Dot-com and subprime crises are almost similar, at least in terms of IRFs and the impact of an SVS on financial variables. However, we know this was not the case and that financial transmission channels during these two crises were fundamentally different.

Fig. 8 shows a more coherent picture with significant and reliable differences in the IRF between the situations after the Dot-com crisis and during the subprime crisis.

![Figure 8: Conditional and unconditional IRFs to a switching (to regime 2) volatility shock for financial variables (3SVS)](image)

Figure 8: Conditional and unconditional IRFs to a switching (to regime 2) volatility shock for financial variables (3SVS)

Indeed, 3SVS during the subprime crisis increased US bonds bought by US households and EA bonds bought by the ECB, while this was not the case after the Dot-com crisis or unconditionally.
Such shocks decreased the exchange rate and money held by EA households in all cases, while money held by US households increased.

Fig. 8 shows that the response of short-term US bonds is due to an increase in the Federal Reserve’s bond position, while other agents decrease their bond position. In the EA, the picture is different: the ECB slightly increases its bond position, and both European and US households decrease their EA long-term bond positions.

Then, because US and EA households are selling their US bonds, by the construction of our model, the Federal Reserve has no other choice than to buy them after the Dot-com crisis. Such a result is very close to the reality of the last decade.

Besides, Fig. 7 shows that following such SVSs, both countries’ households hold more money after several periods and sell EA long-term bonds. This result is a direct consequence of the increase of the short-term EA bond position and consumption. US households increase their overall bond position and money holdings, such that EUR returns to the EA and USD returns to the US.

Another interesting result lies in the differences between the 1SVS and 3SVS models. The 1SVS model (Fig. 7) almost does not discriminate between the two conditional IRFs (2003Q4 and 2009Q1,) while the 3SVS model (Fig. 8) differentiates between these two dates, which are economically (and financially) substantially different. Consequently, the 3SVS model could match the stylized financial facts better compared with the 1SVS model (and a fortiori compared with the baseline model without switching).

In terms of the IRF levels, the 3SVS model brings higher volatility to the responses of economic variables, especially for the exchange rate, money holdings, and bonds quantities. Volatility shocks were essential drivers of the GFC crisis and, as we will see hereafter, nonlinearities will also impact economic dynamics.

4.2 Monetary policy shock

Fig. 9 shows the consequences of an EA monetary policy shock for each model. It can be observed that responses are almost similar across models. The US long-term interest rate is lower with the 3SVS model—the price of long-term bonds is higher.

A domestic (EA) monetary policy shock leads to higher inflation in Fig. 9. Hence, the real interest rate increases, leading to lower money position and higher bonds position. The government budget means that it creates additional income for households. It means higher consumption that increases import. Importation growth leads to a cheaper national currency. This will lead to inflation growth and domestic production growth.

However, responses of a US monetary policy shock differ depending on the model (Fig. 10), especially for the demand-to-GDP ratio, long-term interest rates,
and GDP growth in the first quarters. US inflation responses are more pronounced in the 3SVS model compared with the model without SVSs.

Besides, EA and US growth rates are significantly different in the first quarters, showing that the model without switching allows more variability in the first periods to US and EA growth, with different signs at some points of time.

A foreign monetary policy shock leads to lower inflation in Fig. 10. The central bank puts a significant weight to inflation. A lower inflation expectation leads to lower inflation and interest rates, which then motivates households to increase money and decrease bonds. This produces an additional cash flow that is spent on consumption. Additional demand leads to higher imports. This makes national currency relatively cheaper and produces some growth in domestic production.

Interestingly, long-term interest rates have different responses in the US and
Figure 10: Unconditional IRFs to a positive US monetary policy shock (one standard deviation).

EA. While the US long-term nominal interest rate decreases sharply in the 3SVS model, the decrease in the EA long-term nominal interest rate is less pronounced in this model. Without switching, the US long-term nominal interest rate decreases less compared with the 3SVS model, while the EA long-term nominal interest rate increases more compared with the 3SVS model. Thus, we consider that SVSs could provide relevant information for monetary policy decisions.

In line with stylized facts, a symmetric monetary policy shock does not have similar consequences if it is in the EA or the US.
### 4.3 Nonlinearities

Previous impulse response figures considered only a one standard deviation positive shock. However, in a nonlinear world, responses are also nonlinear. How should these nonlinearities be quantified? Fig. 11 to Fig. 14 present unconditional IRFs after a monetary policy shock with different magnitudes, allowing us to assess the importance of these nonlinearities.

Fig. 11 presents IRFs after an EA monetary policy shock according to the 1SVS model.

![Graphs showing IRFs for different magnitudes of a monetary policy shock](image)

**Figure 11:** Unconditional IRFs to a EA monetary policy shock with different magnitudes (1SVS). std. stands for standard deviation.

While +1 std and +3 std are very similar, one crucial nonlinearity resides in -3 std, which is also similar to positive shocks. This nonlinearity is easily understandable mathematically (power 2), avoiding a symmetric response, which
is standard in DSGE models’ IRFs linearized at the first order.

However, this negative EA monetary policy shock (-3 std) has a lower response compared with the other positive shock, even though the direction is similar. Non-linearities could lower the efficiency of monetary policy shocks, which is an important result for monetary authorities using simple linear models to assess economic situations and take monetary policy decisions.

It is interesting to note that nonlinearities are more visible in the economy, and such a picture, as presented in Fig. 11, has a shallow impact (the scale is always between $10^{-3}$ and $10^{-5}$).

Fig. 12 presents IRFs after an EA monetary policy shock according to the 3SVS model.

Figure 12: Unconditional IRFs to a EA monetary policy shock with different magnitudes (3SVS). std. stands for standard deviation.

Unlike Fig. 11, Fig. 12 presents the magnitudes of higher nonlinearities, and
these results are more in line with the literature (An and Schorfheide, 2007), especially for exchange rates (Altavilla and De Grauwe, 2010).

Furthermore, nonlinearities influence EA inflation uncertainty in an interesting way. While the negative monetary policy shock (-3 std) has an important impact on EA inflation in the first periods, it exceeds +1 (+3 std) after several periods, highlighting the nonstandard perspective allowed by nonlinearities.

Besides, Fig. 13 highlights an important result for policymakers. Responses to a US monetary policy shock have different nonlinearities compared with responses to an EA monetary policy shock (Fig. 11). EA inflation and the demand-to-GDP ratios of both the US and EA behave almost non-linearly following a US monetary policy shock (at least in the first periods). In these cases, -3 std and +3 std are asymmetric, whereas in the previous case (EA monetary policy shock), we find small nonlinearities (asymmetries).

Interestingly, +1 std and +3 std US monetary policy shocks do not have the same consequences for EA growth (Fig. 13). This shows that nonlinearities contribute to explaining why strong monetary policy reactions do not have the same consequences as small monetary policy reactions. The same comment applies to the US demand-to-GDP ratio.

Thus, policymakers should analyze economic decisions, including their own, through the spectrum of nonlinear models to optimize the magnitude of their reaction function.

Fig. 14 presents IRFs after a US monetary policy shock according to the 3SVS model.

Nonlinearities are present in the case of a US monetary policy shock (Fig. 14). EA inflation displays the same phenomenon as presented previously (Fig. 12), denoting strong differences between the linear and nonlinear models again.

Moreover, the role of SVSs is significant, especially concerning demand-to-GDP ratios and exchange rates. Overall, nonlinearities and SVSs on monetary policy shocks impact not only the magnitudes of the considered dynamics but also the actual dynamics as well as their orders.

Such a result is fundamental for policymakers and economists willing to model economies around a crisis. Open-economy models are suitable for such nonlinearities (Altavilla and De Grauwe, 2010) and masking nonlinearities by using linear models to analyze such economies could lead to inadequate economic interpretations and policy decisions.

5 Interpretation

Section 2 presents an original model featuring households, firms, and the central banks of two economies, with households able to buy domestic or foreign short-
term bonds. Following Kiley (2014), we show that the short-term nominal interest rate has a more substantial effect on the overall economy than the long-term nominal interest rate and that both short- and long-term interest rates are key determinants of consumption. However, our results also highlight that the EA’s long-term interest rates comove strongly with US long-term rates rather than with short-term rates (Chin et al., 2015). This result is confirmed with the 3SVS model (Fig. 9) in which the US long-term nominal interest rate reacts more strongly to an EA short-term nominal interest rate shock.

Following Chin et al. (2015), we find that US disturbances have an important influence on EA economies (Fig. 13). These results are confirmed by variance decomposition of variables with respect to shocks (Appendix D) and distance correlations between variables (online appendix).
In addition, we find that US money shocks impact EA real variables in the long run, but also EA and US financial markets (Appendix D). We enhance the literature by highlighting new transmission channels of money compared with other studies conducted through linear closed-economy DSGE models with money (Benchimol and Fourçans, 2012, 2017; Benchimol and Qureshi, 2019). Unlike this body of literature, we obtain the role of money in the economy without assuming nonseparability between money and consumption (Benchimol, 2016), nor a cash-in-advance constraint (Feenstra, 1986) or money in the production function (Benchimol, 2015). Money holdings from households and central banks needed to buy bonds involve such a role of money (Eq. 11). Although the additive separable utility function (Eq. 2) excludes real money balances from the IS curve (Jones and Stracca, 2008), money has a role through the money in the utility function and
households’ budget constraints, because of the direct effect (Eq. 3) highlighted by Andrés et al. (2009).

An interesting result regarding inflation’s variance decomposition is that the price-markup shock (demand elasticity shock) plays a critical role in the EA, while in the US, this shock explains only a small share of inflation dynamics, illustrating how EA and US economies behave differently during crises. In the long run, this is explained by the strength of price-markup shocks explaining domestic as well as foreign wage dynamics. This smaller effect of the price markup shock on the inflation rate in the short run can be caused by nonlinear dynamics, which are not present in standard closed-economy models.

Another result relates to the intertemporal preferences shock, showing that it has minor short-term explanatory power, whereas it becomes one of the most important shocks in the long run for explaining some variable dynamics (Appendix D), such as the part of inflation dynamics that are not explained by the price-markup shock. This fact is essential for models that are not modeled without domestic and foreign preference shocks. The absence of such a shock in the literature on open-economy DSGE models could conceal additional dynamics that could complete the economic scenarios developed by policymakers, such as on foreign and domestic bonds or private consumption.

Our nonlinear open-economy DSGE model with several SVSs allows us to enrich the dynamics of interest rate markets for different maturities (Section 4.1). Fig. 2 shows that the US response to inflation is stronger compared with the EA. How can one conciliate this with the stabilization objectives of the Fed and ECB? The official objectives of the Fed are to react to both inflation and output growth or unemployment while the ECB’s one is to mainly react to inflation. However, these official objectives differ from the concrete reaction to these variables. First of all, the existence of an additional component in the Fed’s objectives does not mean a lower response to US inflation. We have shown that the Fed responds substantially more (in absolute values) to the output gap and exchange rate compared to the ECB, which could compensate its response to inflation. As an active central bank, this stronger Fed response to economic changes leads to faster stabilizing effects with more significant interest rate fluctuations compare to the ECB. The smaller ECB responses smoothed interest rates during a more extended (stabilization) period. Both monetary policies correspond to official objectives but have significant differences in the preferences across the components of these objectives. Other explanations relate the weaker response of the ECB to inflation dynamics compared to those of the Fed. Tensions within the ECB Governing Council, a change in the post-GFC inflation target and objectives, the quantitative easing and the ZLB could also explain this lower inflation coefficient compared to the Fed’s.
Including several SVSs could, at least during crises, more accurately explain changes in US and EA inflation as well as changes in US and EA interest rates at different maturities. The possibility of switching in different elements of the economy, such as technology and monetary policy (and not only technology), is essential during crises. It is natural to capture such stylized facts by including several SVSs. Each SVS could capture specific switching volatility that can change the regime of the overall economy for a specific sector. Fig. 6 clearly shows that such modeling is more appropriate for capturing changes in inflation and interest rates compared with a model with only one SVS for technological progress.

In addition, such shocks are important for capturing changes in an open economy; for instance, Fig. 10 shows that after a US monetary policy shock, EA inflation is assumed to decrease just after the shock in the model without SVSs, whereas in reality, this is not the case. Then, models with one or several SVSs could capture the reality more accurately, especially during crisis periods for which macroeconomic and financial variables are not well explained. The transmission channels are explained in Section 4.2.

Last but not least, our models can provide results about nonlinear IRFs, highlighting significant nonlinear behaviors of market-related variables, such as exchange and interest rates. Such dynamics are absent from most policymakers’ models for several reasons (e.g., technical complexity, material limitations, and time and computational costs). However, our policy recommendation resulting from the results of this study is that nonlinear models should be used when dealing with an open economy and market-related variables, which are sometimes subject to highly nonlinear dynamics compared with more standard closed-economy variables.

6 Conclusion

A two-country open-economy MSDSGE model was developed to understand several stylized events that occurred during the GFC, such as how the regime-switching volatility shock impacts between the EA and the US were transmitted to the real and financial variables.

By using second-order approximation and Markov SVS, we showed that SVSs are the main driving force of the shock transmissions during crises. We showed how SVSs affect the US and EA economies and that they involve i) money transfers between economies and ii) interest rate maturity trade-offs that could produce structural changes in the economy. Then, it appears that SVSs impact US and EA consumption in opposite ways.

Price-markup and money shocks behave differently to standard linear models. Due to direct effects (Andrés et al., 2009), the roles of both domestic and for-
eign real money holdings are significant in the long run as well as the short run, especially for bond variables and rate-related variables.

Furthermore, the difference between the average response of SVSs and the response on specific dates illustrates how SVSs are relevant during crises but less so in standard times. Unlike the EU monetary policy, which is less impacted by SVSs, US monetary policy is significantly influenced by SVSs.

The main policy implication relates to the way monetary authorities model the economy, especially in an open-economy world with interlinked financial markets. Our models showed that it is important for policymakers to consider nonlinear models and SVSs during crisis periods (or when uncertainty about a current regime increases). If policymakers continue to use standard linear models and ignore SVSs, they might also ignore some nonlinear dynamics as well as the underlying interactions between financial markets and the economy. SVSs could be one of the promising features that could be included in the next generation of macroeconomic models.

References


33


Appendix

A Markov Switching Quadratic Kalman Filter

This appendix presents the fast-deterministic filter used for the estimation of our nonlinear MSDSGE model. The collapsing rule of the sigma point Kalman filters developed by Binning and Maih (2015) is unusual. This filter family uses variance equal to the weighted average of variance conditional on regimes. Such formula holds for raw moments but does not hold for central moments.\footnote{Let us consider two regimes. The regimes probabilities, \( p(r_t|r_{t+1}) \), are \( p(1|1) = p(2|2) = 0.95 (0.6) \) and \( p(1|2) = p(2|1) = 0.05 (0.4) \). The mean condition on regime, \( x(r_t) \), are \( x(1) = 1 \) and \( x(2) = -1 \), and the variance condition on each regime is 1. Hence, the variance condition on the future regime, \( V(r_{t+1}) \), would be \( V(1) = V(2) = 1.19 (1.96) \), while the formula from Binning and Maih (2015) gives 1 in both cases. This demonstrates that their formula generates substantial errors in the case of regime uncertainty.} Our MSQKF fixes
this property by correcting the formulas for variances.\textsuperscript{10} MSQKF is a Gaussian assumed filter and uses collapsing before forecasting as in Binning and Maih (2015).

Particle filter approaches have the advantage of an unbiased likelihood estimation. However, these approaches produce a stochastic estimation of likelihood, which is a substantial disadvantage. They do not allow standard optimization algorithms to be used. Fixed random draws are required for optimization algorithms with particle filters. However, it influences the main advantage of particle filters. Markov chain Monte Carlo (MCMC) inefficiency increases significantly: the required number of draws should be 10 (from 5 to 400 depending on the number of particles) times higher for the same accuracy of MCMC (Pitt et al., 2012). An additional disadvantage of particle filters is related to their computational costs. They require a large number of particles to be comparable with deterministic filters and are about 100 times slower than deterministic nonlinear filters (Andreasen, 2013; Ivashchenko, 2014; Kollmann, 2015). For all these reasons, we do not use particle filters.

The purpose of a filter in DSGE models is to compute both the model variable vector, $X_t$, density conditional on vectors of the observed variables $Y_1,...,Y_t$ and density and likelihood of the observed variables $Y_1,...,Y_t$. Computing the density means computing the parameters of the density approximation. In some specific cases, this approximation is equal to the density (e.g., normal distribution).

Most of the filters loop the following steps:

1. Computation of the initial density of $X_t$;
2. Computation of the density of $Y_t$ as a function of the density of $X_t$ (see Appendix A.1);
3. Computation of the likelihood of $Y_t$ (see Appendix A.2);
4. Computation of the conditional density of $X_t|Y_t$ (see Appendix A.3);
5. Computation of the density of $X_{t+1}$ as a function of the density of $X_t|Y_t$ (see Appendix A.4); and

Our MSQKF assumes a Gaussian density approximation in step 5, unlike the sigma-point one. The sigma-point one is easier to implement for any type of state-space model. The Gaussian one produces a better quality of filtration when densities are close to Gaussian ones (Ivashchenko, 2014).

\textsuperscript{10}The description, properties, and comparisons of MSQKF are detailed in Ivashchenko (2014, 2016).
The suggested model of the data-generating process is determined by Eq. 20 to Eq. 22 and a discrete MS process for the regime variable, $r_t$, where $X_{\text{state},t}$ is the vector of state variables (a subset of model variable vector $X_t$), and $\varepsilon_t$ and $u_t$ are vectors of independent shocks (model innovations and measurement errors) that have a zero-mean normal distribution. $\delta$ is a constant equal to one and related to the perturbation with respect to uncertainty. The second-order approximation of the MSDSGE model is computed with the RISE toolbox (Maih, 2015).

$$Y_t = HX_t + u_t \quad (20)$$
$$Z_t = [\begin{array}{cc} X_{\text{state},t} & \delta \\ \varepsilon_t \end{array}] \quad (21)$$
$$X_{t+1} = A_{0,r_{t+1}} + A_{1,r_{t+1}}Z_t + A_{2,r_{t+1}}(Z_t \otimes Z_t) \quad (22)$$

where $\otimes$ is the Kronecker product.

The difference from the usual DSGE model second-order approximation is the existence of regime dependence. Each filtering step is described below. The nonlinear filters (including the suggested ones) use some approximations. Computations within the filters that use approximations will be highlighted.

### A.1 Density of $Y_t$ as a function of the density of $X_t$

The initial information for this step is that the density of $X_t$ is a normal mixture. The linear equation for the observed variables, Eq. 20, presents the density of $Y_t$ as a normal mixture with the same probabilities of regimes and the following expectations and variances (conditional on the regime):

$$E_s[Y_t] = E_s[HX_t + u_t] = HE_s[X_t], \quad (23)$$
$$V_s[Y_t] = V_s[HX_t + u_t] = HV_s[X_t]H' + V_s[u_t], \quad (24)$$

where $E_s[.]$ and $V_s[.]$ denote the expectation and variance operators conditional on regime $s$.

### A.2 Likelihood of $Y_t$

The initial information for this step is that the density of $Y_t$ is a normal mixture. This means that the likelihood can be determined as

$$L [Y_t] = \sum_{s=1}^{N_S} p(r_t = s)L [Y_t | r_t = s]$$
$$= \sum_{s=1}^{N_S} p(r_t = s) e^{-\frac{1}{2}(Y_t - E_s[Y_t])'(V_s[Y_t])^{-1}(Y_t - E_s[Y_t])} \frac{1}{(2\pi)^{N_Y} |V_s[Y_t]|^{1/2}}, \quad (25)$$

where $L[.]$ is the likelihood, $N_S$ the number of regimes, and $N_Y$ the number of observed variables.
A.3 Conditional density of $X_t|Y_t$

The initial information for this step is the vector of observation $Y_t$ and that the density of $X_t$ is a normal mixture. The linear Eq. 20 allows a computation conditional on the regime and observation density the same way as the Kalman filter in Eq. 26 to Eq. 28.

$$K_s^t = (V_s [Y_t])^{-1} H V_s [X_t] \quad (26)$$

$$E_s [X_t|Y_t] = E_s [X_t] + K_s (Y_t - E_s [Y_t]) \quad (27)$$

$$V_s [X_t|Y_t] = (I_{N_X} - K_s H) V_s [X_t] (I_{N_X} - K_s H)' \quad (28)$$

$$p (r_t = s|Y_t) = \frac{p (r_t = s; Y_t)}{p (Y_t)} \quad (29)$$

Eq. 29 shows the probability of regime $s$ conditional on the observed variables. $p (Y_t)$ is the likelihood (computed in Appendix A.2), and $p (Y_t|r_t = s)$ has a normal density.

A.4 Density of $X_{t+1}$ as a function of the density of $X_t|Y_t$

The initial information for this step is the density for the vector of model variables $X_t$ (normal mixture).

The first step is the computation of the expectation ($E_{s,1}$) and variance ($V_{s,1}$) of vector $X_t$ conditional on the future state, such as

$$E_{s,1} = E (X_t|r_{t+1} = s) = \sum_{k=1}^{N_r} \frac{p (r_t = k) p (r_{t+1} = s|r_t = k)}{p (r_{t+1} = s)} E_k (X_t), \quad (30)$$

$$V_{s,1} = -E_{s,1} (E_{s,1})' + \sum_{k=1}^{N_r} \frac{p (r_t = k) p (r_{t+1} = s|r_t = k)}{p (r_{t+1} = s)} \left( E_k (X_t) E_k (X_t)' + V_k (X_t) \right). \quad (31)$$

The next step is the approximation (collapsing rule): we believe that the density of vector $X_t$ is a normal mixture with regime probabilities $p (r_{t+1} = s)$ and Gaussian densities with moments $E_{s,1}$ and $V_{s,1}$.

Knowledge of the conditional density of $X_t$ gives us the density of $Z_t$. It allows us to compute the conditional moments of the future vector of variables $X_{t+1}$ ($X_{t+1|r_{t+1}}$ is the future vector of model variables conditional on future regime $r_{t+1}$).

$$Z_{0,t,r_{t+1}} = Z_{t,r_{t+1}} - E_{r_{t+1}} [Z_{t,r_{t+1}}] \quad (32)$$

38
\[X_{t+1,r_{t+1}} = A_{0,r_{t+1}} + A_{1,r_{t+1}} Z_{t,r_{t+1}} + A_{2,r_{t+1}} (Z_{t,r_{t+1}} \otimes Z_{t,r_{t+1}})\]
\[= B_{0,r_{t+1}} + B_{1,r_{t+1}} Z_{0,t,r_{t+1}} + B_{2,r_{t+1}} (Z_{0,t,r_{t+1}} \otimes Z_{0,t,r_{t+1}})\]  
(33)

\[E[X_{t+1,r_{t+1}}] = B_{0,r_{t+1}} + B_{2,r_{t+1}} vec\{V[Z_{t,r_{t+1}}]\} = B_{0,r_{t+1}} + B_{2,r_{t+1}} vec\{V_{r_{t+1}}\}\]  
(34)

\[vec\{V[X_{t+1,r_{t+1}}]\} = (B_{1,r_{t+1}} \otimes B_{1,r_{t+1}}) vec\{V_{r_{t+1}}\}\]
\[+ (B_{2,r_{t+1}} \otimes B_{2,r_{t+1}}) \left( vec\{V_{r_{t+1}}\} \otimes vec\{V_{r_{t+1}}\} + vec\{V_{r_{t+1}}\} \otimes V_{r_{t+1}} \right)\]  
(35)

where \(vec\{\cdot\}\) is the vectorization operator.

Eq. 32 to Eq. 35 are similar to the equations developed in Ivashchenko (2014). The difference is that these formulas became formulas for moments, conditional on the regime.

The last action of this step is an approximation. We suggest that the density of \(X_{t+1}\) is a normal mixture with moments according to Eq. 34 to Eq. 35.

B Summary of variables

Table 1 presents a summary of all variables used in our model. For each variable, the table refers to the equations in which the variable is used.

C Estimation results

Table 2 presents the median absolute errors (MAE) and log predictive score (LPS) for each observed variable to assess the forecast quality of our models and illustrate the importance of switching volatility. We compute the LPS based on Gaussian density, which is suggested by the MS-DGSF model.\(^{11}\)

We also computed these statistics for the 3SVS model in the case that the volatility state is always in regime one and in the case where the volatility state is always in regime two to illustrate the importance of the Markov switching.\(^{12}\)

We show that 3SVS always in regime one is much worse in terms of forecasting compared to 3SVS in regime two. At the same time, 3SVS produces the best density forecasts. The difference in terms of the sum of individual LPSs is relatively small because it does not take into account the correlation between forecasts, which increases the advantage of 3SVS model.

Tables 3 to 5 present the estimation results for each model. Our results are generally in line with the DSGE literature. The persistence of monetary policy

\(^{11}\)In the case of LPS based on Gaussian mixture density, the LPS should equal the log-likelihood divided by the number of periods (for multivariate measure).

\(^{12}\)See Fig. 1 for estimated probabilities.
Variable Description Equations

$B_{i,j,k,t}$ Bonds bought by households $i$ in currency $j$ with maturity $k$ 3, 4, 13, 14

$B_{i,g,t}$ Bonds bought by central bank or government in country $i$ 11, 13

$C_{i,t}$ Consumption of households of country $i$ 2, 3, 4, 12

$D_{i,t}$ Dividends of firms from country $i$ 6, 7

$e_t$ Exchange rate in terms of number of units of domestic currency per 1 unit of foreign 3, 4, 8, 9, 10

$L_{i,t}$ Labor in country $i$ 2, 4, 5, 7

$M_{i,t}$ Money stock in country $i$ 2, 4, 11

$P_{i,t}$ Aggregate price level in country $i$ 2, 3, 4, 6, 7, 8, 9, 10

$P_{i,t}(j)$ Price for goods of firms $j$ in country $i$ 6, 7, 8, 9

$R_{i,k,t}$ Interest rate in currency $i$ with maturity $k$ 4, 6, 10, 11

$W_{i,t}$ Wage in country $i$ 4, 7

$Y_{i,t}$ Demand in country $i$ 6, 8, 10, 12

$Y_{F,i,t}(j)$ Production of firms $j$ in country $i$ 5, 7, 8

$z_{i,t}$ Exogenous process of type $j$ in country $i$ 1, 2, 8, 9, 10, 15

$Z_t$ Exogenous technology process 2, 5, 6, 15

Table 1: Summary of variables used in model’s equations

The coefficient of relative risk aversion is close to unity and lower than that found in the literature (Benchimol, 2014).

We do not compare the models through their respective log-likelihood ratio for several reasons. Firstly, the difference between two log-likelihood values of two different models does not mean that we must disregard the model with the lowest log-likelihood even if the advantage is statistically significant. For instance, the latter model could still be used to perform forecasting under changing environments (Benchimol and Fourçans, 2017, 2019). Secondly, whatever the log-likelihood, it can be argued that the model is designed to capture only specific characteristics of the data. It is an open question as to whether log-likelihood is an adequate measure to evaluate how well the model accounts for particular aspects of the data.

Nevertheless, we report hereafter log-likelihood values and corresponding likelihood tests. The log-likelihood of 0SVS equals 3581.70, 1SVS equals 3593.35, 3SVS equals 3611.67. It means that LR-test of 1SVS vs 0SVS p-value is 3.51e-05, 3SVS vs 1SVS p-value is 1.1e-08 and 3SVS vs 0SVS p-value is 1.25e-11. Consequently, a more flexible model explains significantly better the data. Our estimation of the
<table>
<thead>
<tr>
<th></th>
<th>MAE</th>
<th></th>
<th></th>
<th>LPS</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No SVS</td>
<td>1SVS</td>
<td>3SVS</td>
<td>No SVS</td>
<td>1SVS</td>
<td>3SVS</td>
</tr>
<tr>
<td>EA GDP deflator</td>
<td>0.408%</td>
<td>0.366%</td>
<td>0.384%</td>
<td>3.66</td>
<td>3.70</td>
<td>3.67</td>
</tr>
<tr>
<td>US GDP deflator</td>
<td>0.156%</td>
<td>0.147%</td>
<td>0.150%</td>
<td>4.58</td>
<td>4.60</td>
<td>4.59</td>
</tr>
<tr>
<td>EA 3m rate</td>
<td>0.040%</td>
<td>0.040%</td>
<td>0.041%</td>
<td>5.68</td>
<td>5.68</td>
<td>5.88</td>
</tr>
<tr>
<td>US 3m rate</td>
<td>0.086%</td>
<td>0.085%</td>
<td>0.083%</td>
<td>5.35</td>
<td>5.33</td>
<td>5.36</td>
</tr>
<tr>
<td>EA demand to GDP</td>
<td>0.611%</td>
<td>0.597%</td>
<td>0.596%</td>
<td>3.43</td>
<td>3.40</td>
<td>3.40</td>
</tr>
<tr>
<td>US demand to GDP</td>
<td>0.580%</td>
<td>0.577%</td>
<td>0.587%</td>
<td>3.61</td>
<td>3.63</td>
<td>3.62</td>
</tr>
<tr>
<td>EA GDP growth</td>
<td>0.775%</td>
<td>0.734%</td>
<td>0.741%</td>
<td>3.30</td>
<td>3.32</td>
<td>3.31</td>
</tr>
<tr>
<td>US GDP growth</td>
<td>0.424%</td>
<td>0.448%</td>
<td>0.416%</td>
<td>3.45</td>
<td>3.46</td>
<td>3.48</td>
</tr>
<tr>
<td>EA 10y rate</td>
<td>0.065%</td>
<td>0.067%</td>
<td>0.068%</td>
<td>5.64</td>
<td>5.64</td>
<td>5.64</td>
</tr>
<tr>
<td>US 10y rate</td>
<td>0.062%</td>
<td>0.059%</td>
<td>0.054%</td>
<td>5.26</td>
<td>5.29</td>
<td>5.29</td>
</tr>
</tbody>
</table>

Table 2: Forecasting performance for one-step ahead forecasts.

covariance matrix allows us to construct a Laplace approximation of the marginal likelihood (maximum likelihood estimation is equivalent to a Bayesian one with flat priors).

It should be noted that the results are sensitive to the approximation methodology. We use the RISE function 'solve_accelerate'. If we try to compute approximation without this function—and compute the likelihood—the resulting values would be: 3413.21 (0SVS), 3530.66 (1SVS), and 2515.20(3SVS). This is probably due to the iterative nature of the MS-DSGE solution approximation that converges to a slightly different solution. The sharp likelihood of the nonlinear approximation transforms this small difference into a significant difference in the likelihood. Thus, even small details of the solution algorithm can be crucial in a nonlinear world.
\begin{table}[h]
\centering
\begin{tabular}{cccccccccccc}
\hline
 & Priors & & & Postiors & & & & Priors & & & Postiors \\
 & LB & UB & Mean & Std. & LB & UB & Mean & Std. & LB & UB & Mean & Std. \\
\hline
\(\eta_{d,u}\) & -0.01 & 0.00 & -0.006 & 0.000 & & & & & & & & \\
\(\eta_{d,m}\) & -20.0 & 20.0 & -6.493 & 0.263 & & & & & & & & \\
\(\eta_{d,r}\) & 0.00 & 0.01 & 0.000 & 0.000 & & & & & & & & \\
\(\eta_{d,p}\) & 1.00 & 20.0 & 3.281 & 0.020 & & & & & & & & \\
\(\eta_{f,u}\) & -0.01 & 0.00 & -0.000 & 0.000 & & & & & & & & \\
\(\eta_{f,m}\) & -20.0 & 20.0 & -7.558 & 0.001 & & & & & & & & \\
\(\eta_{f,r}\) & 0.00 & 0.01 & 0.000 & 0.000 & & & & & & & & \\
\(\eta_{f,p}\) & 1.00 & 20.0 & 11.28 & 0.008 & & & & & & & & \\
\(\eta_{y}\) & 0.00 & 0.01 & 0.000 & 0.000 & & & & & & & & \\
\(\eta_{d,u}\) & -1.00 & 1.00 & 0.978 & 0.004 & & & & & & & & \\
\(\eta_{d,d}\) & -1.00 & 1.00 & 0.975 & 0.001 & & & & & & & & \\
\(\eta_{d,m}\) & -1.00 & 1.00 & 0.891 & 0.007 & & & & & & & & \\
\(\eta_{d,r}\) & -1.00 & 1.00 & 0.180 & 0.021 & & & & & & & & \\
\(\eta_{d,p}\) & -1.00 & 1.00 & 0.948 & 0.001 & & & & & & & & \\
\(\eta_{f,u}\) & -1.00 & 1.00 & 0.462 & 0.030 & & & & & & & & \\
\(\eta_{f,m}\) & -1.00 & 1.00 & 0.914 & 0.002 & & & & & & & & \\
\(\eta_{f,r}\) & -1.00 & 1.00 & 0.756 & 0.009 & & & & & & & & \\
\(\eta_{f,p}\) & -1.00 & 1.00 & 0.953 & 0.002 & & & & & & & & \\
\(\eta_{y}\) & -1.00 & 1.00 & 0.960 & 0.001 & & & & & & & & \\
\(\sigma_{\xi_{d,u}}\) & 0.00 & 10.0 & 0.001 & 0.000 & & & & & & & & \\
\(\sigma_{\xi_{d,l}}\) & 0.00 & 10.0 & 0.152 & 0.007 & & & & & & & & \\
\(\sigma_{\xi_{d,m}}\) & 0.00 & 10.0 & 0.124 & 0.004 & & & & & & & & \\
\(\sigma_{\xi_{d,r}}\) & 0.00 & 10.0 & 0.001 & 0.000 & & & & & & & & \\
\(\sigma_{\xi_{d,p}}\) & 0.00 & 10.0 & 1.342 & 0.030 & & & & & & & & \\
\(\sigma_{\xi_{f,u}}\) & 0.00 & 10.0 & 0.007 & 0.001 & & & & & & & & \\
\(\sigma_{\xi_{f,l}}\) & 0.00 & 10.0 & 0.048 & 0.001 & & & & & & & & \\
\(\sigma_{\xi_{f,m}}\) & 0.00 & 10.0 & 0.106 & 0.002 & & & & & & & & \\
\(\sigma_{\xi_{f,r}}\) & 0.00 & 10.0 & 0.002 & 0.000 & & & & & & & & \\
\(\sigma_{\xi_{f,p}}\) & 0.00 & 10.0 & 0.635 & 0.021 & & & & & & & & \\
\(\sigma_{\xi_{y}}\) & 0.00 & 10.0 & 0.001 & 0.000 & & & & & & & & \\
\(\sigma_{j_d}\) & 0.00 & 0.01 & 0.000 & 0.000 & & & & & & & & \\
\(\sigma_{j_f}\) & 0.00 & 0.01 & 0.000 & 0.000 & & & & & & & & \\
\(\sigma_{j_f}\) & 0.00 & 0.01 & 0.000 & 0.000 & & & & & & & & \\
\(\sigma_{\xi_{d,c}}\) & -20.0 & 20.0 & -0.070 & 0.143 & & & & & & & & \\
\(\sigma_{\xi_{f,c}}\) & -20.0 & 20.0 & -0.688 & 0.000 & & & & & & & & \\
\(\sigma_{\xi_{f,l}}\) & -20.0 & 20.0 & -1.758 & 0.044 & & & & & & & & \\
\(\sigma_{\xi_{f,m}}\) & -20.0 & 20.0 & 0.794 & 0.000 & & & & & & & & \\
\hline
\end{tabular}
\caption{Estimation results for the model without SVS. LB and UB stands for Lower Bound and Upper Bound respectively.}
\end{table}
Table 4: Estimation results for the model 1SVS. \( LB \) and \( UB \) stands for Lower Bound and Upper Bound respectively, \( p_{reg_a=2|reg_{a-1}=1} \) for the probability of switching to regime \( a \) at period \( t \) if we are in regime \( b \) at period \( t-1 \), and \( \sigma(\xi_{y|reg_{a}=a}) \) for the standard deviation of the corresponding shock if we are in regime \( a \).
<table>
<thead>
<tr>
<th>Priors</th>
<th>Posterior</th>
<th>Priors</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB</td>
<td>UB</td>
<td>Mean</td>
<td>Std.</td>
</tr>
<tr>
<td>$p_{reg=2</td>
<td>reg=-1} = 1$</td>
<td>0.04 0.35</td>
<td>0.042 0.001</td>
</tr>
<tr>
<td>$p_{reg=1</td>
<td>reg=-1} = 2$</td>
<td>0.04 0.35</td>
<td>0.131 0.017</td>
</tr>
<tr>
<td>$\pi_{d,m}$</td>
<td>-0.01 0.00</td>
<td>-0.006 0.000</td>
<td>$\nu_d$</td>
</tr>
<tr>
<td>$\pi_{f,m}$</td>
<td>-20.0 20.0</td>
<td>-6.807 0.054</td>
<td>$\nu_f$</td>
</tr>
<tr>
<td>$\pi_{d,r}$</td>
<td>1.00 20.0</td>
<td>3.253 0.027</td>
<td>$h_{d,c}$</td>
</tr>
<tr>
<td>$\pi_{f,r}$</td>
<td>-0.01 0.00</td>
<td>-0.008 0.000</td>
<td>$h_{f,c}$</td>
</tr>
<tr>
<td>$\pi_{f,u}$</td>
<td>-20.0 20.0</td>
<td>-7.564 0.068</td>
<td>$\rho_{dx}$</td>
</tr>
<tr>
<td>$\pi_{f,v}$</td>
<td>1.00 20.0</td>
<td>11.44 0.100</td>
<td>$\rho_{dx}$</td>
</tr>
<tr>
<td>$\pi_{f,w}$</td>
<td>0.00 0.01</td>
<td>0.001 0.000</td>
<td>$\rho_{dx}$</td>
</tr>
<tr>
<td>$\eta_{d,u}$</td>
<td>1.00 1.00</td>
<td>0.979 0.001</td>
<td>$\rho_{f,v}$</td>
</tr>
<tr>
<td>$\eta_{d,j}$</td>
<td>0.969 0.002</td>
<td>0.969 0.002</td>
<td>$\rho_{f,v}$</td>
</tr>
<tr>
<td>$\eta_{d,m}$</td>
<td>-1.00 1.00</td>
<td>0.883 0.010</td>
<td>$\rho_{f,v}$</td>
</tr>
<tr>
<td>$\eta_{d,r}$</td>
<td>-1.00 1.00</td>
<td>0.184 0.006</td>
<td>$\rho_{d,dr}$</td>
</tr>
<tr>
<td>$\eta_{d,p}$</td>
<td>-0.954 0.003</td>
<td>0.954 0.003</td>
<td>$\rho_{d,sp}$</td>
</tr>
<tr>
<td>$\eta_{f,u}$</td>
<td>-1.00 1.00</td>
<td>0.447 0.027</td>
<td>$\rho_{d,dr}$</td>
</tr>
<tr>
<td>$\eta_{f,l}$</td>
<td>-1.00 1.00</td>
<td>0.907 0.006</td>
<td>$\rho_{d,dr}$</td>
</tr>
<tr>
<td>$\eta_{f,m}$</td>
<td>-0.954 0.004</td>
<td>0.954 0.004</td>
<td>$\rho_{d,dr}$</td>
</tr>
<tr>
<td>$\eta_{f,r}$</td>
<td>-0.957 0.004</td>
<td>0.957 0.004</td>
<td>$\rho_{d,dr}$</td>
</tr>
<tr>
<td>$\eta_{f,y}$</td>
<td>-0.951 0.005</td>
<td>0.951 0.005</td>
<td>$\rho_{f,v}$</td>
</tr>
<tr>
<td>$\eta_{y}$</td>
<td>-1.00 1.00</td>
<td>0.958 0.001</td>
<td>$\rho_{d,dr}$</td>
</tr>
<tr>
<td>$1/\sigma_{d,c}$</td>
<td>-20.0 20.0</td>
<td>0.734 0.001</td>
<td>$\pi_{d}(j)$</td>
</tr>
<tr>
<td>$1/\sigma_{d,t}$</td>
<td>-20.0 20.0</td>
<td>0.102 0.079</td>
<td>$\pi_{f}(j)$</td>
</tr>
<tr>
<td>$1/\sigma_{d,m}$</td>
<td>-20.0 20.0</td>
<td>1.774 0.038</td>
<td>$\pi_{f}(j)$</td>
</tr>
<tr>
<td>$A_{d}$</td>
<td>-20.0 20.0</td>
<td>-0.015 0.017</td>
<td>$\tau_{d,tr}$</td>
</tr>
<tr>
<td>$1/\sigma_{f,c}$</td>
<td>-20.0 20.0</td>
<td>-0.687 0.002</td>
<td>$\sigma(\xi_{d,a})$</td>
</tr>
<tr>
<td>$1/\sigma_{f,t}$</td>
<td>-20.0 20.0</td>
<td>-1.650 0.043</td>
<td>$\sigma(\xi_{d,j})$</td>
</tr>
<tr>
<td>$1/\sigma_{f,m}$</td>
<td>-20.0 20.0</td>
<td>0.793 0.000</td>
<td>$\sigma(\xi_{d,m})$</td>
</tr>
<tr>
<td>$A_{f}$</td>
<td>-20.0 20.0</td>
<td>8.600 4.547</td>
<td>$\sigma(\xi_{d,p})$</td>
</tr>
<tr>
<td>$\varphi_{d,dr}$</td>
<td>0.00 1000</td>
<td>0.182 0.004</td>
<td>$\sigma(\xi_{f,a})$</td>
</tr>
<tr>
<td>$\varphi_{d,sp}$</td>
<td>0.00 1000</td>
<td>0.082 0.000</td>
<td>$\sigma(\xi_{f,j})$</td>
</tr>
<tr>
<td>$\varphi_{d,dr}$</td>
<td>0.00 1000</td>
<td>17.33 1.478</td>
<td>$\sigma(\xi_{f,m})$</td>
</tr>
<tr>
<td>$\varphi_{f,dr}$</td>
<td>0.00 1000</td>
<td>0.216 0.007</td>
<td>$\sigma(\xi_{f,p})$</td>
</tr>
<tr>
<td>$\varphi_{f,dr}$</td>
<td>0.00 1000</td>
<td>0.002 0.000</td>
<td>$\sigma(\xi_{f,p})$</td>
</tr>
<tr>
<td>$\varphi_{f,dr}$</td>
<td>0.00 1000</td>
<td>0.010 0.000</td>
<td>$\sigma(\xi_{f,p})$</td>
</tr>
<tr>
<td>$\varphi_{f,dr}$</td>
<td>0.00 1000</td>
<td>0.000 0.000</td>
<td>$\sigma(\xi_{f,p})$</td>
</tr>
<tr>
<td>$\varphi_{f,dr}$</td>
<td>0.00 1000</td>
<td>0.000 0.000</td>
<td>$\sigma(\xi_{f,p})$</td>
</tr>
<tr>
<td>$s_{d}$</td>
<td>0.00 1.00</td>
<td>0.618 0.010</td>
<td>$\sigma(\xi_{d,dr})$</td>
</tr>
<tr>
<td>$s_{f}$</td>
<td>0.00 1.00</td>
<td>0.063 0.002</td>
<td>$\sigma(\xi_{f,dr})$</td>
</tr>
</tbody>
</table>

Table 5: Estimation results for the model 3SVS. LB and UB stands for Lower Bound and Upper Bound respectively, $p_{reg=a|reg=-1} = b$ for the probability of switching to regime $a$ at period $t$ if we are in regime $b$ at period $t-1$, and $\sigma(\xi_{y|reg=a})$ for the standard deviation of the corresponding shock if we are in regime $a$.  

44
D Variance decompositions

Tables 6 to 8 present short- and long-run variance decomposition of variables with respect to the shocks for each model.

The variance decomposition for nonlinear models requires additional comments. Variance decomposition coefficients, \( \forall i \in \{d, f\} \) and \( \forall j \in \{u, m, l, p, r, y\} \), are computed with respect to the following function:

\[
VD_t (x, \xi_{i,j}) = 1 - \frac{E [x_t^2]_{\xi_{i,j} \sim N (0, 0)} - (E [x_t]_{\xi_{i,j} \sim N (0, 0)}^2)}{E [x_t^2]_{\xi_{i,j} \sim N (0, \sigma (\xi_{i,j}))} - (E [x_t]_{\xi_{i,j} \sim N (0, \sigma (\xi_{i,j}))}^2)},
\]

where \( x_t \) is the value of the variable of interest (for which variance decomposition is computed) in period \( t \), \( \xi_{i,j} \) is the shock of interest, \( \sigma (\cdot) \) is the standard error operator, \( E [\cdot] \) is the expectation operator, and \( N \) is the normal Law. This formula expresses the proportion by which the variable of interest variance is lower if the shock of interest were equal to zero for all periods.

The sum of \( VD_t (x, \xi) \) for each shock gives 1 in the case of the linear model. However, this does not hold for nonlinear models. The sum is close to 1 for most variables, but there are some exceptions.

All the models present variance decomposition in line with the literature. Domestic price markup-shock (\( \xi_{d,p} \)) plays a predominant role in domestic prices (\( p_{d,t} \)) in both the short run and long run. However, domestic preference shock (\( \xi_{d,u} \)) plays a substantial role in domestic wages (\( w_{d,t} \)) in the short run and an important role in domestic consumption (\( c_{d,t} \)) in the long run—which should be greater if we do not consider domestic firms’ production, \( y_{d,t} (j) \).

Foreign shocks play a role in the dynamics of several variables, especially foreign preference shock (\( \xi_{f,u} \)). This shock drives the dynamics of several foreign as well as domestic variables in the long run. This shows how the EA (domestic) is still dependent on the US economy (foreign) and US households’ preferences.

Moreover, we observe that in the long term, financial markets are almost entirely dependent on the US economy (foreign) shocks and technology progress shocks for all versions of the model. Foreign shocks play an important role in domestic long-term interest rates, but not for corresponding bonds, in short-term horizons. Thus, the model reproduces domination by the US in financial markets.

Interestingly, foreign money demand shocks (\( \xi_{f,m} \)) play an important role in the dynamics of several domestic variables in the long run (rather than in the short run), such as the exchange rate, domestic long-run interest rates, and most domestic bond quantities, including those bought by central banks. This finding shows how important US money demand shocks are for the EA’s economic dynamics, a result that is in line with the closed-economy and linear DSGE literature (Benchimol and Fourçans, 2017).
In addition, foreign worked hours’ shocks have a substantial impact on domestic economic dynamics, particularly for bonds’ positions and the exchange rate.

The explanatory power of technology progress shock ($\xi_y$) is relatively small for the short-term horizon. This shock explains 4–6% of domestic output growth and 14–17% of foreign output growth, depending on the model version. This is substantially smaller than the 25.4% for Europe (Lombardo and McAdam, 2012). The long-term explanatory power is 35–43% for domestic output growth and 59–62% for foreign output growth, depending on the model version, which is larger than that found by Lombardo and McAdam (2012) (27.3% for a 20-quarter horizon). A similar picture is related to inflation’s explanatory power. The short-term values are 5.7–9.3% for domestic inflation and 13.7–22.5% for foreign inflation, while the long-term values are 19.2–24.7% and 41.8–45.1%, respectively. This differs from 26.9% and 27.7%, respectively, in the models of the EA (Lombardo and McAdam, 2012). Small open-economy models produce similar long-term values (16–21%) for Canada, Spain, and Sweden (Guerrón-Quintana, 2013). However, the long-term explanatory power of technology progress shock for inflation differs significantly for different countries: from 13% for Australia to 54% for Belgium (Guerrón-Quintana, 2013).

Variance decomposition of foreign inflation is slightly unusual. The markup shocks are usually key for inflation: 64.9% for EA inflation (Guerrón-Quintana, 2013) and more than 80% for US inflation (Smets and Wouters, 2007). However, our model explains 44.9–50.6% of short-term domestic inflation with domestic markup shock ($\xi_{d,p}$) and 0.4–1.9% of short-term foreign inflation with foreign markup shock ($\xi_{f,p}$).

There are not many differences between models in terms of variance decomposition of variables with respect to structural shocks, except when considering domestic worked hours and domestic money demand shocks. These shocks impact domestic bonds’ positions of households and central banks in different manners. This shows that including SVSs in domestic and foreign monetary policies diminishes the role of domestic money demand while increasing the role of worked hours on domestic bonds’ positions.
<table>
<thead>
<tr>
<th></th>
<th>Short run variance decompositions</th>
<th></th>
<th>Long run variance decompositions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\xi_{d,u}$ $\xi_{d,l}$ $\xi_{d,m}$ $\xi_{d,p}$</td>
<td>$\xi_{d,u}$ $\xi_{d,l}$ $\xi_{d,m}$ $\xi_{d,p}$</td>
<td>$\xi_{d,u}$ $\xi_{d,l}$ $\xi_{d,m}$ $\xi_{d,p}$</td>
</tr>
<tr>
<td>$b_{d,d,sr,t}$</td>
<td>0.0 55.2 42.5 0.1</td>
<td>$b_{d,d,lr,t}$</td>
<td>0.0 55.3 42.4 0.1</td>
</tr>
<tr>
<td>$b_{d,f,lr,t}$</td>
<td>0.2 0.1 0.0 0.2</td>
<td>$b_{d,f,lr,t}$</td>
<td>0.1 0.1 0.0 0.2</td>
</tr>
<tr>
<td>$b_{d,g,t}$</td>
<td>1.1 0.0 0.0 5.4</td>
<td>$b_{d,g,t}$</td>
<td>0.5 0.0 0.0 5.0</td>
</tr>
<tr>
<td>$b_{d,lr,t}$</td>
<td>1.1 0.0 0.0 5.4</td>
<td>$b_{d,lr,t}$</td>
<td>0.5 0.0 0.0 5.0</td>
</tr>
<tr>
<td>$c_{d,t}$</td>
<td>19.2 2.5 10.9 2.4</td>
<td>$c_{d,t}$</td>
<td>19.2 2.5 10.9 2.4</td>
</tr>
<tr>
<td>$c_{f,t}$</td>
<td>0.2 0.0 0.0 0.2</td>
<td>$c_{f,t}$</td>
<td>0.2 0.0 0.0 0.2</td>
</tr>
<tr>
<td>$c_{p,t}$</td>
<td>6.0 1.7 0.0 2.0</td>
<td>$c_{p,t}$</td>
<td>6.0 1.7 0.0 2.0</td>
</tr>
<tr>
<td>$m_{d,t}$</td>
<td>3.2 54.4 37.5 1.5</td>
<td>$m_{d,t}$</td>
<td>3.2 54.4 37.5 1.5</td>
</tr>
<tr>
<td>$m_{f,t}$</td>
<td>0.3 0.0 0.0 0.5</td>
<td>$m_{f,t}$</td>
<td>0.3 0.0 0.0 0.5</td>
</tr>
<tr>
<td>$p_{d,t}$</td>
<td>34.8 4.2 25.8 50.6</td>
<td>$p_{d,t}$</td>
<td>34.8 4.2 25.8 50.6</td>
</tr>
<tr>
<td>$p_{f,t}$</td>
<td>0.6 0.0 0.0 0.2</td>
<td>$p_{f,t}$</td>
<td>0.6 0.0 0.0 0.2</td>
</tr>
<tr>
<td>$r_{d,t}$</td>
<td>0.6 0.1 98.6 0.6</td>
<td>$r_{d,t}$</td>
<td>0.6 0.1 98.6 0.6</td>
</tr>
<tr>
<td>$r_{f,t}$</td>
<td>0.6 0.0 0.0 0.5</td>
<td>$r_{f,t}$</td>
<td>0.6 0.0 0.0 0.5</td>
</tr>
<tr>
<td>$w_{d,t}$</td>
<td>64.4 2.8 0.1 14.2</td>
<td>$w_{d,t}$</td>
<td>64.4 2.8 0.1 14.2</td>
</tr>
<tr>
<td>$w_{f,t}$</td>
<td>0.0 0.0 0.0 0.1</td>
<td>$w_{f,t}$</td>
<td>0.0 0.0 0.0 0.1</td>
</tr>
<tr>
<td>$y_{d,t}(j)$</td>
<td>2.8 6.6 0.1 28.3</td>
<td>$y_{d,t}(j)$</td>
<td>2.8 6.6 0.1 28.3</td>
</tr>
<tr>
<td>$y_{f,t}(j)$</td>
<td>0.5 0.2 0.0 0.0</td>
<td>$y_{f,t}(j)$</td>
<td>0.5 0.2 0.0 0.0</td>
</tr>
</tbody>
</table>

Table 6: Short and long run variance decompositions for the model without SVS
<table>
<thead>
<tr>
<th></th>
<th>( \xi_{du} )</th>
<th>( \xi_{dj} )</th>
<th>( \xi_{dm} )</th>
<th>( \xi_{dp} )</th>
<th>( \xi_{fu} )</th>
<th>( \xi_{fm} )</th>
<th>( \xi_{fr} )</th>
<th>( \xi_{fp} )</th>
<th>( \xi_{y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_{d,t} )</td>
<td>0.0</td>
<td>50.2</td>
<td>49.4</td>
<td>0.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>( b_{f,t} )</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.4</td>
<td>43.5</td>
<td>16.9</td>
<td>30.5</td>
<td>4.2</td>
</tr>
<tr>
<td>( b_{d,t} )</td>
<td>0.0</td>
<td>50.1</td>
<td>49.3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( b_{d,t} )</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>4.4</td>
<td>0.7</td>
<td>59.9</td>
<td>20.1</td>
<td>14.4</td>
<td>0.7</td>
</tr>
<tr>
<td>( b_{d,t} )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.4</td>
<td>0.0</td>
<td>68.6</td>
<td>16.1</td>
<td>0.6</td>
<td>5.5</td>
</tr>
<tr>
<td>( b_{f,t} )</td>
<td>0.6</td>
<td>0.0</td>
<td>1.2</td>
<td>0.0</td>
<td>63.1</td>
<td>16.8</td>
<td>1.8</td>
<td>8.9</td>
<td>0.6</td>
</tr>
<tr>
<td>( c_{d,t} )</td>
<td>7.0</td>
<td>17.4</td>
<td>0.6</td>
<td>74.2</td>
<td>1.9</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>( c_{f,t} )</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>49.8</td>
<td>0.5</td>
<td>18.1</td>
<td>22.2</td>
</tr>
<tr>
<td>( e_{t} )</td>
<td>3.9</td>
<td>1.4</td>
<td>0.0</td>
<td>2.8</td>
<td>1.6</td>
<td>46.3</td>
<td>19.6</td>
<td>22.2</td>
<td>4.9</td>
</tr>
<tr>
<td>( m_{d,t} )</td>
<td>2.8</td>
<td>50.1</td>
<td>43.5</td>
<td>0.9</td>
<td>3.4</td>
<td>0.4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>( m_{f,t} )</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.8</td>
<td>0.1</td>
<td>51.8</td>
<td>22.0</td>
<td>6.6</td>
<td>0.5</td>
</tr>
<tr>
<td>( p_{d,t} )</td>
<td>36.0</td>
<td>3.7</td>
<td>0.0</td>
<td>22.3</td>
<td>44.9</td>
<td>1.8</td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( p_{f,t} )</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.8</td>
<td>0.1</td>
<td>39.2</td>
<td>0.8</td>
<td>36.2</td>
<td>4.6</td>
</tr>
<tr>
<td>( r_{d,t} )</td>
<td>0.5</td>
<td>98.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( r_{f,t} )</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.9</td>
<td>0.1</td>
<td>64.6</td>
<td>0.4</td>
<td>0.2</td>
<td>8.8</td>
</tr>
<tr>
<td>( r_{d,t} )</td>
<td>1.4</td>
<td>21.7</td>
<td>11.9</td>
<td>3.3</td>
<td>0.6</td>
<td>44.9</td>
<td>9.3</td>
<td>6.7</td>
<td>0.1</td>
</tr>
<tr>
<td>( r_{f,t} )</td>
<td>1.4</td>
<td>0.0</td>
<td>0.0</td>
<td>1.3</td>
<td>0.8</td>
<td>14.3</td>
<td>3.6</td>
<td>10.8</td>
<td>0.6</td>
</tr>
<tr>
<td>( w_{d,t} )</td>
<td>66.7</td>
<td>1.9</td>
<td>0.1</td>
<td>11.0</td>
<td>0.6</td>
<td>1.7</td>
<td>0.4</td>
<td>2.7</td>
<td>0.0</td>
</tr>
<tr>
<td>( w_{f,t} )</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
<td>64.5</td>
<td>0.2</td>
<td>10.8</td>
<td>16.6</td>
</tr>
<tr>
<td>( g_{d,t} )</td>
<td>3.4</td>
<td>4.7</td>
<td>0.0</td>
<td>29.1</td>
<td>2.3</td>
<td>7.1</td>
<td>3.2</td>
<td>8.9</td>
<td>0.2</td>
</tr>
<tr>
<td>( g_{f,t} )</td>
<td>0.3</td>
<td>0.3</td>
<td>0.0</td>
<td>0.6</td>
<td>1.8</td>
<td>33.3</td>
<td>2.0</td>
<td>15.1</td>
<td>15.3</td>
</tr>
</tbody>
</table>

Table 7: Short and long run variance decompositions for the 1SVD model
### Short run variance decompositions

<table>
<thead>
<tr>
<th></th>
<th>$\xi_{d,t}$</th>
<th>$\xi_{d,l}$</th>
<th>$\xi_{d,m}$</th>
<th>$\xi_{d,p}$</th>
<th>$\xi_{d,r}$</th>
<th>$\xi_{f,u}$</th>
<th>$\xi_{f,m}$</th>
<th>$\xi_{f,r}$</th>
<th>$\xi_{f,p}$</th>
<th>$\xi_{u}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{d,t}$</td>
<td>0.0</td>
<td>76.4</td>
<td>23.2</td>
<td>0.4</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$b_{d,l}$</td>
<td>0.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.3</td>
<td>0.8</td>
<td>51.0</td>
<td>18.1</td>
<td>19.3</td>
<td>4.1</td>
<td>2.3</td>
</tr>
<tr>
<td>$b_{d,m}$</td>
<td>0.0</td>
<td>76.5</td>
<td>23.0</td>
<td>0.4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>$b_{d,p}$</td>
<td>0.0</td>
<td>4.6</td>
<td>58.8</td>
<td>19.4</td>
<td>10.9</td>
<td>0.6</td>
<td>0.9</td>
<td>0.9</td>
<td>1.9</td>
<td>0.1</td>
</tr>
<tr>
<td>$b_{d,r}$</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>68.7</td>
<td>16.3</td>
<td>0.4</td>
<td>5.2</td>
<td>2.1</td>
</tr>
<tr>
<td>$b_{f,t}$</td>
<td>1.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.0</td>
<td>63.6</td>
<td>17.7</td>
<td>1.4</td>
<td>8.6</td>
<td>2.0</td>
</tr>
<tr>
<td>$b_{f,l}$</td>
<td>8.4</td>
<td>16.0</td>
<td>0.2</td>
<td>73.5</td>
<td>7.3</td>
<td>0.6</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>$b_{f,m}$</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.3</td>
<td>0.6</td>
<td>49.2</td>
<td>0.8</td>
<td>9.9</td>
<td>26.6</td>
<td>2.8</td>
</tr>
<tr>
<td>$b_{f,p}$</td>
<td>6.0</td>
<td>1.5</td>
<td>0.0</td>
<td>2.7</td>
<td>2.9</td>
<td>44.7</td>
<td>22.6</td>
<td>16.7</td>
<td>4.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### Long run variance decompositions

<table>
<thead>
<tr>
<th></th>
<th>$\xi_{d,t}$</th>
<th>$\xi_{d,l}$</th>
<th>$\xi_{d,m}$</th>
<th>$\xi_{d,p}$</th>
<th>$\xi_{d,r}$</th>
<th>$\xi_{f,u}$</th>
<th>$\xi_{f,m}$</th>
<th>$\xi_{f,r}$</th>
<th>$\xi_{f,p}$</th>
<th>$\xi_{u}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{d,t}$</td>
<td>18.4</td>
<td>2.5</td>
<td>13.2</td>
<td>2.9</td>
<td>14.9</td>
<td>25.2</td>
<td>13.0</td>
<td>9.7</td>
<td>3.3</td>
<td>7.8</td>
</tr>
<tr>
<td>$b_{d,l}$</td>
<td>0.0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.6</td>
<td>0.7</td>
<td>31.3</td>
<td>22.2</td>
<td>39.7</td>
<td>2.7</td>
<td>13.2</td>
</tr>
<tr>
<td>$b_{d,m}$</td>
<td>16.2</td>
<td>2.2</td>
<td>11.9</td>
<td>2.8</td>
<td>13.7</td>
<td>26.6</td>
<td>14.2</td>
<td>10.6</td>
<td>3.7</td>
<td>9.6</td>
</tr>
<tr>
<td>$b_{d,p}$</td>
<td>5.4</td>
<td>0.0</td>
<td>0.0</td>
<td>3.6</td>
<td>3.7</td>
<td>24.0</td>
<td>21.9</td>
<td>39.9</td>
<td>1.9</td>
<td>10.7</td>
</tr>
<tr>
<td>$b_{d,r}$</td>
<td>0.5</td>
<td>0.0</td>
<td>0.1</td>
<td>3.6</td>
<td>1.6</td>
<td>25.8</td>
<td>22.0</td>
<td>41.4</td>
<td>2.1</td>
<td>15.5</td>
</tr>
<tr>
<td>$b_{f,t}$</td>
<td>12.1</td>
<td>0.0</td>
<td>0.2</td>
<td>1.1</td>
<td>0.6</td>
<td>31.6</td>
<td>13.2</td>
<td>33.5</td>
<td>2.1</td>
<td>29.4</td>
</tr>
<tr>
<td>$b_{f,l}$</td>
<td>3.0</td>
<td>0.0</td>
<td>0.0</td>
<td>2.7</td>
<td>2.6</td>
<td>37.9</td>
<td>23.4</td>
<td>33.5</td>
<td>4.1</td>
<td>8.7</td>
</tr>
<tr>
<td>$b_{f,m}$</td>
<td>13.5</td>
<td>4.6</td>
<td>15.9</td>
<td>10.8</td>
<td>12.0</td>
<td>21.4</td>
<td>9.9</td>
<td>8.4</td>
<td>2.7</td>
<td>4.9</td>
</tr>
<tr>
<td>$b_{f,p}$</td>
<td>4.1</td>
<td>0.0</td>
<td>0.1</td>
<td>2.0</td>
<td>0.4</td>
<td>44.9</td>
<td>6.0</td>
<td>13.2</td>
<td>2.6</td>
<td>5.0</td>
</tr>
<tr>
<td>$b_{f,r}$</td>
<td>2.7</td>
<td>0.0</td>
<td>0.0</td>
<td>2.7</td>
<td>0.6</td>
<td>37.9</td>
<td>23.4</td>
<td>33.5</td>
<td>4.1</td>
<td>8.7</td>
</tr>
</tbody>
</table>

Table 8: Short and long run variance decompositions for the 3SVD model