Money in the production function: A New Keynesian DSGE perspective

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Abstract

This article checks whether money is an omitted variable in the production process by proposing a microfounded New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model. In this framework, real money balances enter the production function, and money demanded by households is differentiated from that demanded by firms. By using a Bayesian analysis, our model weakens the hypothesis that money is a factor of production. However, the demand of money by firms appears to have a significant impact on the economy, even if this demand has a low weight in the production process.

Keywords: Money in the production function, DSGE, Bayesian estimation.

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1 Introduction

The theoretical motivation to empirically implement money in the production function originates from the monetary growth models of Levhari and Patinkin (1968), Friedman (1969), Johnson (1969), and Stein (1970), which include money directly in the production function. Firms hold money to facilitate production on the grounds that money enables them to economize on the use of other inputs and saves costs incurred by running short of cash (Fischer, 1974).

> *Real cash balances are at least in part a factor of production. To take a trivial example, a retailer can economize on his average cash balances by hiring an errand boy to go to the bank on the corner to get change for large bills tendered by customers. When it costs ten cents per dollar per year to hold an extra dollar of cash, there will be a greater incentive to hire the errand boy, that is, to substitute other productive resources for cash. This will mean both a reduction in the real flow of services from the given productive resources and a change in the structure of production, since different productive activities may differ in cash-intensity, just as they differ in labor- or land-intensity.*
>
> Milton Friedman (1969)

Considering real money balances to be a factor of production has numerous implications. Money would have a marginal physical productivity schedule like other inputs, firms' demands for real balances would be derived in the same way as other factor demand functions, changes in the stock of money would affect real output–contrary to the classical dichotomy which implies that money is neutral–and real balances might explain part of the growth rates of total factor productivity or the residual.

Sinai and Stokes (1972) present a very interesting test of the hypothesis that money enters the production function, suggesting that real balances could represent a missing variable that contributes to the attribution of the unexplained residual to technological changes. Ben-Zion and Ruttan (1975) conclude that as a factor of production, money seems to play an important role in explaining induced technological changes.

Short (1979) develops structural models based on Cobb and Douglas (1928) and generalized translog production functions, both of which provide a more complete theoretical framework to analyze the role of money in the production process. The empirical results obtained by estimating these two models indicate that the relationship between real cash balances and output, even after correcting for any simultaneity bias, is positive and statistically significant. The results suggest that it is theoretically appropriate to include a real cash balances variable as a factor input in a production function in order to capture the productivity gains derived from using money.

You (1981) finds that the unexplained portion of output variation virtually vanishes with the inclusion of real balances in the production function. In addition to labor and capital, real money balances turn out to be an important factor of production, especially for developing countries. The results of Khan and Ahmad (1985) are consistent with the hypothesis that real money balances are an important factor of production. Sephton (1988) shows that real balances are a valid factor of production within the confines of a constant elasticity of substitution (CES) production function. Hasan and Mahmud (1993) also support the hypothesis that money is an important factor in the production function and that there are potential supply side-effects of interest rate changes.

Recent developments in econometrics regarding cointegration and error-correction models provide a rich environment in which to reexamine the role of money in the production function. Moghaddam (2010) presents empirical evidence indicating that in a cointegrated space, different definitions of money serve as an input in the Cobb and Douglas (1928) production function.

At the same time, Clarida et al. (1999), Woodford (2003), and Galí (2008) develop New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models to explain the dynamics of the economy. However, none of the studies on New Keynesian DSGE models use money as an input in the production function.

This article departs from the existing theoretical and empirical literature by specifying a fully microfounded New Keynesian DSGE model in which money enters the production function. This feature generates a new inflation equation that includes money. Following Benchimol and Fourçans (2012), we introduce the new concept of flexible-price real money balances and, in order to close the model, a quantitative equation. We also analyze the dynamics of the economy by using Bayesian estimations and simulations to confirm or reject the potential influence of money in the dynamics of the Eurozone and to determine the weight of real money balances in the production process. By distinguishing between money used for productive and nonproductive purposes (Benhabib et al., 2001), this paper intends to solve the now old and controversial hypothesis about money in the production function proposed by Levhari and Patinkin (1968) and Sinai and Stokes (1972), and to more deeply analyze the role of these two components of the demand for money (demand from households and firms).

After describing the theoretical set-up in Section 2, we calibrate and estimate five models of the Euro area using Bayesian techniques in Section 3. Impulse response functions and variance decomposition are analyzed in Section 3.4, and we study the consequences of money in the production function hypothesis by comparing the monetary policy rules of models in Section 4. Section 5 concludes, and Section 6 presents additional results.

2 The model

The model consists of households that supply labor, purchase goods for consumption, and hold money and bonds, as well as firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their respective prices each period. Households and firms behave optimally: Households maximize their expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal interest rate. This model is inspired by Smets and Wouters (2003), Galí (2008), and Walsh (2017).

2.1 Households

We assume a representative, infinitely lived household, that seeks to maximize

$$E_t \left[\sum_{k=0}^{\infty} \beta^k U_{t+k} \right], \tag{1}$$

where U_t is the period utility function and $\beta < 1$ is the discount factor.

We assume the existence of a continuum of goods, represented by the interval $[0;1]$. The household decides how to allocate its consumption expenditures among different goods. This requires that the consumption index, C_t, be maximized for any given level of expenditure. $\forall t \in \mathbb{N}$ and, conditionally on such optimal behavior, the period budget constraint takes the form

$$P_t C_t + M_{n,t} + M_{p,t} + Q_t B_t \leq B_{t-1} + W_t N_t + M_{n,t-1} + M_{p,t-1}, \tag{2}$$

where P_t is an aggregate price index; $M_{n,t}$ and $M_{p,t}$ are nominal money held for nonproductive and productive purposes, respectively; B_t is the quantity of one-period, nominally risk-free discount bonds purchased in period t and maturing in period $t+1$ (each bond pays one unit of money at maturity and its price is Q_t, so that the short-term nominal rate i_t is approximately equal to $-\ln Q_t$); W_t is the nominal wage; and N_t is hours worked (or the measure of employed household members).

The above sequence of period budget constraints is supplemented with a solvency condition, such as $\forall t \lim_{n \longrightarrow \infty} E_t [B_n] \geq 0$.

Preferences are measured using a common time-separable utility function. Under the assumption of a household period utility given by

$$U_t = e^{\varepsilon_t^u} \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma e^{\varepsilon_t^n}}{1-\nu} \left(\frac{M_{n,t}}{P_t} \right)^{1-\nu} - \frac{\chi N_t^{1+\eta}}{1+\eta} \right), \tag{3}$$

consumption, labor supply, money demand, and bond holdings are chosen to maximize Eq. (1), subject to Eq. (2) and the solvency condition. This Money-in-the-Utility (MIU) function depends positively on the consumption of goods, C_t, positively on real money balances, $\frac{M_t}{P_t}$, and negatively on labor N_t. σ is the coefficient of the relative risk aversion of households or the inverse of the intertemporal elasticity of substitution, ν is the inverse of the elasticity of money holdings with respect to the interest rate, and η is the inverse of the elasticity of work effort with respect to the real wage (inverse of the Frisch elasticity of the labor supply).

The utility function also contains two structural shocks: ε_t^u is a general shock to preferences that affects the intertemporal substitution of households (preference shock) and ε_t^n is a shock to household money demand. γ and χ are positive scale parameters.

This setting leads to the following conditions[1], which, in addition to the budget constraint, must hold in equilibrium. The resulting log-linear version of the first-order condition corresponding to the demand for contingent bonds implies that

$$c_t = E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - \rho_c) - \sigma^{-1} E_t \left[\Delta \varepsilon_{t+1}^u \right], \tag{4}$$

where the lowercase letters denote the logarithm of the original aggregated variables, $\rho_c = -\ln(\beta)$, and Δ is the first-difference operator.

The demand for cash that follows from the household optimization problem is given by

$$\varepsilon_t^n + \sigma c_t - \nu m p_{n,t} - \rho_m = a_2 i_t, \tag{5}$$

where $mp_{n,t} = m_{n,t} - p_t$ are the log-linearized real money balances for nonproductive purposes, $\rho_m = -\ln(\gamma) + a_1$, and a_1 and a_2 are the resulting terms of the first-order Taylor approximation of $\ln(1 - Q_t) = a_1 + a_2 i_t$. More precisely,

[1] See Appendix A.

if we compute our first-order Taylor approximation around the steady-state interest rate, $\frac{1}{\beta}$, we obtain $a_1 = \ln\left(1 - \exp\left(-\frac{1}{\beta}\right)\right) - \frac{\frac{1}{\beta}}{e^{\frac{1}{\beta}}-1}$ and $a_2 = \frac{1}{e^{\frac{1}{\beta}}-1}$.

Real cash holdings has a positive relation with consumption, with an elasticity equal to σ/ν, and a negative relation with the nominal interest rate ($\frac{1}{\beta} > 1$, which implies that $a_2 > 0$). Below, we take the nominal interest rate as the central bank's policy instrument.

In the literature, due to the assumption that consumption and real money balances are additively separable in the utility function, the cash holdings of households do not enter any of the other structural equations: Accordingly, the equation above becomes a recursive function of the remainder of the system of equations. However, as in Sinai and Stokes (1972), Subrahmanyam (1980), and Khan and Ahmad (1985), because real money balances enter the aggregate supply, we will use this money demand equation (Eq. 5) to solve the equilibrium of our model. See, for instance, Ireland (2004), Andrés et al. (2009), and Benchimol and Fourçans (2012) for models in which money balances enter the aggregate demand equation without entering the production function.

The resulting log-linear version of the first-order condition corresponding to the optimal consumption-leisure arbitrage implies that

$$w_t - p_t = \sigma c_t + \eta n_t - \rho_n, \tag{6}$$

where $\rho_n = -\ln(\chi)$.

Finally, these equations represent the Euler condition for the optimal intratemporal allocation of consumption (Eq. 4), the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money for nonproductive use (Eq. 5), and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage (Eq. 6).

2.2 Firms

We assume a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the following Money-in-the-Production function[2]

$$Y_t(i) = e^{\varepsilon_t^a}\left(e^{\varepsilon_t^p}\frac{M_{p,t}}{P_t}\right)^{\alpha_m} N_t(i)^{1-\alpha_n} \tag{7}$$

[2] This approach is similar to Benchimol (2011) and Benchimol (2011).

where $\exp(\varepsilon_t^a)$ represents the level of technology, assumed to be common to all firms and to evolve exogenously over time, and ε_t^p is a shock to firm money demand.

All firms face an identical isoelastic demand schedule and take the aggregate price level, P_t, and aggregate consumption index, C_t, as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time t, only a fraction $1-\theta$ of firms, where $0<\theta<1$, can reset their prices optimally, whereas the remaining firms index their prices to lagged inflation.

2.3 Price dynamics

Let us assume a set of firms that do not reoptimize their posted price in period t. As in Galí (2008), using the definition of the aggregate price level and the fact that all firms that reset prices choose an identical price, P_t^*, leads to $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$. Dividing both sides by P_{t-1} and log-linearizing around $P_t^* = P_{t-1}$ yields

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right). \tag{8}$$

In this set-up, we do not assume that prices have inertial dynamics. Inflation results from the fact that firms reoptimize their price plans in any given period, choosing a price that differs from the economy's average price in the previous period.

2.4 Price setting

A firm that reoptimizes in period t chooses the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. We solve this problem to obtain a first-order Taylor expansion around the zero-inflation steady state of the firm's first-order condition, which leads to

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\right], \tag{9}$$

where $\widehat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady-state value, $mc = -\mu$, and $\mu = \ln\left(\varepsilon/\left(\varepsilon-1\right)\right)$ is the log of the desired gross markup.

2.5 Equilibrium

Market clearing in the goods market requires $Y_t(i) = C_t(i)$ for all $i \in [0,1]$ and all t. Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$; it follows that $Y_t = C_t$ must hold for all t. The above goods market clearing condition can be combined with the consumer's Euler equation to yield the equilibrium condition

$$y_t = E_t[y_{t+1}] - \sigma^{-1}(i_t - E_t[\pi_{t+1}] - \rho_c) - \sigma^{-1}E_t\left[\Delta\varepsilon^u_{t+1}\right]. \tag{10}$$

Market clearing in the labor market requires $N_t = \int_0^1 N_t(i)\,di$. Using Eq. 7 leads to

$$N_t = \int_0^1 \left(\frac{Y_t(i)}{e^{\varepsilon^a_t}\left(e^{\varepsilon^p_t}\frac{M_{p,t}}{P_t}\right)^{\alpha_m}}\right)^{\frac{1}{1-\alpha_n}} di \tag{11}$$

$$= \left(\frac{Y_t}{e^{\varepsilon^a_t}\left(e^{\varepsilon^p_t}\frac{M_{p,t}}{P_t}\right)^{\alpha_m}}\right)^{\frac{1}{1-\alpha_n}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha_n}} di, \tag{12}$$

where the second equality (Eq. 12) follows from the demand schedule and the goods market clearing condition. Taking logs leads to

$$(1-\alpha_n)\,n_t = y_t - \varepsilon^a_t - \alpha_m\varepsilon^p_t - \alpha_m mp_{p,t} + d_t, \tag{13}$$

where $mp_{p,t} = m_{p,t} - p_t$ are the log-linearized, real money balances for productive purposes and $d_t = (1-\alpha_n)\ln\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha_n}} di\right)$, where di is a measure of price (and therefore output) dispersion across firms[3].

Hence, the following approximate relation among aggregate output, employment, real money balances, and technology can be written as

$$y_t = \varepsilon^a_t + \alpha_m\varepsilon^p_t + (1-\alpha_n)\,n_t + \alpha_m mp_{p,t}. \tag{14}$$

An expression is derived for an individual firm's marginal cost in terms of the economy's average real marginal cost. Using the marginal product of labor,

$$mpn_t = \ln\left(\frac{\partial Y_t}{\partial N_t}\right) \tag{15}$$

$$= \ln\left(e^{\varepsilon^a_t}\left(e^{\varepsilon^p_t}\frac{M_{p,t}}{P_t}\right)^{\alpha_m}(1-\alpha_n)\,N_t^{-\alpha_n}\right) \tag{16}$$

$$= \varepsilon^a_t + \alpha_m\varepsilon^p_t + \alpha_m mp_{p,t} + \ln(1-\alpha_n) - \alpha_n n_t, \tag{17}$$

[3] In a neighborhood of the zero-inflation steady state, d_t is equal to zero up to a first-order approximation (Galí, 2008).

and the marginal product of real money balances,

$$\begin{aligned} mpmp_t &= \ln\left(\frac{\partial Y_t}{\partial \frac{M_t}{P_t}}\right) && (18)\\ &= \ln\left(e^{\varepsilon_t^a} e^{\varepsilon_t^p}\alpha_m \left(e^{\varepsilon_t^p}\frac{M_{p,t}}{P_t}\right)^{\alpha_m-1} {N_t}^{1-\alpha_n}\right) && (19)\\ &= \varepsilon_t^a + \alpha_m\varepsilon_t^p + \ln\left(\alpha_m\right) + \left(\alpha_m - 1\right) mp_{p,t} + \left(1-\alpha_n\right) n_t, && (20) \end{aligned}$$

we obtain an expression of the marginal cost,

$$\begin{aligned} mc_t &= \left(w_t - p_t\right) - mpn_t - mpmp_t && (21)\\ &= w_t - p_t - 2\left(\varepsilon_t^a + \alpha_m\varepsilon_t^p\right) - \left(2\alpha_m - 1\right) mp_{p,t} \\ &\quad - \left(1-2\alpha_n\right) n_t - \ln\left(\alpha_m\left(1-\alpha_n\right)\right). && (22) \end{aligned}$$

Using Eq. 14, we obtain an expression of n_t such that

$$n_t = \frac{1}{1-\alpha_n}\left(y_t - \varepsilon_t^a - \alpha_m\varepsilon_t^p - \alpha_m mp_{p,t}\right). \quad (23)$$

Plugging Eq. 23 into Eq. 22 leads to an expression of the marginal cost

$$\begin{aligned} mc_t &= \left(w_t - p_t\right) + \frac{2\alpha_n - 1}{1-\alpha_n} y_t + \frac{1-\alpha_n-\alpha_m}{1-\alpha_n} mp_{p,t} \\ &\quad - \ln\left(\alpha_m\left(1-\alpha_n\right)\right) - \frac{1}{1-\alpha_n}\varepsilon_t^a - \frac{\alpha_m}{1-\alpha_n}\varepsilon_t^p, && (24) \end{aligned}$$

where Eq. 24 defines the economy's average marginal product of labor, mpn_t, and the economy's average marginal product of real money balances, $mpmp_t$, in a way that is consistent with Eq. 14.

Using the fact that $mc_{t+k|t} = \left(w_{t+k} - p_{t+k}\right) - mpn_{t+k|t} - mpmp_{t+k|t}$, we obtain

$$\begin{aligned} mc_{t+k|t} &= \left(w_{t+k} - p_{t+k}\right) + \frac{2\alpha_n - 1}{1-\alpha_n} y_{t+k|t} + \frac{1-\alpha_m-\alpha_n}{1-\alpha_n} mp_{p,t+k} \\ &\quad - \frac{1}{1-\alpha_n}\varepsilon_{t+k}^a - \frac{\alpha_m}{1-\alpha_n}\varepsilon_{t+k}^p - \ln\left(\alpha_m\left(1-\alpha_n\right)\right) && (25)\\ &= mc_{t+k} + \frac{2\alpha_n - 1}{1-\alpha_n}\left(y_{t+k|t} - y_{t+k}\right) && (26)\\ &= mc_{t+k} - \varepsilon\frac{2\alpha_n - 1}{1-\alpha_n}\left(p_t^* - p_{t+k}\right), && (27) \end{aligned}$$

where Eq. 27 follows from the demand schedule, $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$, combined with the market-clearing condition ($y_t = c_t$).

Substituting Eq. 27 into Eq. 9 and rearranging terms yields

$$p_t^* - p_{t-1} = (1-\beta\theta)\,\Theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t\left[\widehat{mc}_{t+k}\right] + \sum_{k=0}^{\infty} (\beta\theta)^k E_t\left[\pi_{t+k}\right], \qquad (28)$$

where $\Theta = \frac{1-\alpha_n}{1-\alpha_n+\varepsilon(2\alpha_n-1)} \leq 1$.

Finally, combining Eq. 8 with Eq. 28 yields the inflation equation

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \lambda_{mc}\widehat{mc}_t, \qquad (29)$$

where $\widehat{mc}_t = mc_t - mc$ is the *real marginal cost gap* and $\lambda_{mc} = \Theta\frac{(1-\theta)(1-\beta\theta)}{\theta}$ is strictly decreasing in the index of price stickiness, θ, the measure of decreasing returns, α_n, and the demand elasticity, ε.

Next, a relation is derived between the economy's real marginal cost and a measure of aggregate economic activity. Note that independent of the nature of price setting, average real marginal cost can be expressed as

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t - mpmp_t & (30)\\ &= (\sigma y_t + \eta n_t - \rho_n) + \frac{2\alpha_n - 1}{1-\alpha_n} y_t + \frac{1-\alpha_m-\alpha_n}{1-\alpha_n} mp_{p,t} \\ &\quad - \frac{1}{1-\alpha_n}\varepsilon_t^a - \frac{\alpha_m}{1-\alpha_n}\varepsilon_t^p - \ln\left(\alpha_m\left(1-\alpha_n\right)\right) & (31)\\ &= \left(\frac{\eta+\alpha_n}{1-\alpha_n} - (1-\sigma)\right) y_t - \frac{1+\eta}{1-\alpha_n}\varepsilon_t^a - \alpha_m\frac{1+\eta}{1-\alpha_n}\varepsilon_t^p \\ &\quad + \left(1 - \alpha_m\frac{1+\eta}{1-\alpha_n}\right) mp_{p,t} - \rho_n - \ln\left(\alpha_m\left(1-\alpha_n\right)\right), & (32) \end{aligned}$$

where the derivation of Eqs. 31 and 32 makes use of the household's optimality condition (Eq. 6) and the (approximate) aggregate production relation (Eqs. 14 and 23).

Knowing that $\sigma > 0$, $0 \leq \alpha_n \leq 1$, and $\eta \geq 1$, it is obvious that $\sigma(1-\alpha_n)+\eta+2\alpha_n-1 > 0$. However, the sign of $(1-(1+\eta)\alpha_m - \alpha_n)$ coming from Eq. 32 appears undefined. In fact, it confirms some studies from Sinai and Stokes (1975, 1977, 1981, 1989) concluding that $1-\alpha_n > (1+\eta)\alpha_m > \alpha_m$. If this is the case, then $1-(1+\eta)\alpha_m-\alpha_n > 0$.

Furthermore, under flexible prices, the real marginal cost is constant and given by $mc = -\mu$. Defining the natural level of output, denoted by y_t^f, as the equilibrium level of output under flexible prices,

$$mc = \left(\frac{\eta+\alpha_n}{1-\alpha_n} - (1-\sigma)\right) y_t^f - \frac{1+\eta}{1-\alpha_n}\varepsilon_t^a - \alpha_m \frac{1+\eta}{1-\alpha_n}\varepsilon_t^p + \left(1-\alpha_m\frac{1+\eta}{1-\alpha_n}\right) mp_{p,t}^f - \rho_n - \ln\left(\alpha_m\left(1-\alpha_n\right)\right), \quad (33)$$

implies

$$y_t^f = \upsilon_a^y \varepsilon_t^a + \upsilon_p^y \varepsilon_t^p + \upsilon_m^y mp_{p,t}^f + \upsilon_c^y, \quad (34)$$

where

$$\begin{aligned} \upsilon_a^y &= \frac{1+\eta}{\eta+\alpha_n-(1-\sigma)(1-\alpha_n)} \\ \upsilon_p^y &= \frac{\alpha_m(1+\eta)}{\eta+\alpha_n-(1-\sigma)(1-\alpha_n)} \\ \upsilon_m^y &= \frac{\alpha_n+\alpha_m(1+\eta)-1}{\eta+\alpha_n-(1-\sigma)(1-\alpha_n)} \\ \upsilon_c^y &= \frac{(1-\alpha_n)\left(\ln\left(\alpha_m(1-\alpha_n)\right)+\rho_n-\mu\right)}{\eta+\alpha_n-(1-\sigma)(1-\alpha_n)}. \end{aligned}$$

Subtracting Eq. 33 from Eq. 32 yields

$$\widehat{mc}_t = \left(\frac{\eta+\alpha_n}{1-\alpha_n}-(1-\sigma)\right)\left(y_t-y_t^f\right) + \left(1-\alpha_m\frac{1+\eta}{1-\alpha_n}\right)\left(mp_{p,t}-mp_{p,t}^f\right), \quad (35)$$

where $y_t - y_t^f$ is the *output gap*, and $mp_{p,t} - mp_{p,t}^f$ is the *real money gap*, where money is used here only for production purposes. By combining Eqs. 29 and 35, we obtain our first equation relating inflation to its next-period forecast, output gap, and real money balances gap,

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \psi_x\left(y_t-y_t^f\right) + \psi_m\left(mp_{p,t}-mp_{p,t}^f\right) \quad (36)$$

where

$$\psi_x = \frac{\eta+\alpha_n-(1-\alpha_n)(1-\sigma)}{1-\alpha_n+\varepsilon(2\alpha_n-1)}(1-\theta)\left(\frac{1}{\theta}-\beta\right)$$

and

$$\psi_m = \frac{1-\alpha_n-\alpha_m(1+\eta)}{1-\alpha_n+\varepsilon(2\alpha_n-1)}(1-\theta)\left(\frac{1}{\theta}-\beta\right).$$

The second key equation describing the equilibrium of the New Keynesian model is obtained from Eq. 10:

$$y_t = E_t\left[y_{t+1}\right] - \sigma^{-1}\left(i_t - E_t\left[\pi_{t+1}\right]-\rho_c\right) - \sigma^{-1}E_t\left[\Delta\varepsilon_{t+1}^u\right]. \quad (37)$$

Henceforth, Eq. 37 is referred to as the dynamic IS equation.

The third key equation describes the behavior of real money balances. Rearranging Eq. 5 yields

$$mp_{n,t} = \frac{\sigma}{\nu} y_t - \frac{a_2}{\nu} i_t - \frac{\rho_m}{\nu} + \frac{1}{\nu} \varepsilon_t^n. \tag{38}$$

From Eq. 10, we obtain an expression for the *natural interest rate*,

$$i_t^f = \rho_c + \sigma E_t \left[\Delta y_{t+1}^f \right]. \tag{39}$$

Therefore, from Eqs. 39 and 38, we obtain an expression of the money demand of firms in the flexible-price economy such that

$$mp_{n,t}^f = \frac{\sigma}{\nu} y_t^f - \frac{a_2}{\nu} \sigma E_t \left[\Delta y_{t+1}^f \right] - \frac{\rho_m + \rho_c a_2}{\nu} + \frac{1}{\nu} \varepsilon_t^n \tag{40}$$

The last equation determines the interest rate through a smoothed Taylor-type rule,

$$i_t = (1 - \lambda_i) \left(\lambda_\pi (\pi_t - \pi^*) + \lambda_x \left(y_t - y_t^f \right) + M_{k,t} \right) + \lambda_i i_{t-1} + \varepsilon_t^i, \tag{41}$$

where λ_π and λ_x are policy coefficients reflecting the weight on the inflation and output gaps and the parameter $0 < \lambda_i < 1$ captures the degree of interest rate smoothing. ε_t^i is an exogenous *ad hoc* shock accounting for fluctuations in the nominal interest rate. π^* is an inflation target and $M_{k,t}$ is a money variable that is defined as follows: money does not enter the Taylor rule ($k = 1$), leading to a standard Taylor rule; money enters the Taylor rule by the way of one real money gap ($k = 2$–4); and money enters the Taylor rule by the way of two real money gaps ($k = 5$).

Table 1 describes $M_{k,t}$'s functional forms.

In the literature, money is generally introduced through a money growth variable (Ireland, 2003; Andrés et al., 2006, 2009; Canova and Menz, 2011; Barthélemy et al., 2011). However, Benchimol and Fourçans (2012) also introduce a *money-gap* variable and show that, at least in the Eurozone, it is empirically more significant than other money variable measures. Such a rule can also be derived from the optimization program of the central bank as a social planner (Woodford, 2003).

Finally, closing the model requires an additional equilibrium relation. For that purpose, we use the following quantitative equation:

$$P_t Y_t = e^{\zeta_t} M_t, \tag{42}$$

k	$M_{k,t}$
1	0
2	$\lambda_2 \left(mp_{p,t} - mp^f_{p,t} \right)$
3	$\lambda_3 \left(mp_{n,t} - mp^f_{n,t} \right)$
4	$\lambda_4 \left(mp_t - mp^f_t \right)$
5	$\lambda_5 \left(mp_{p,t} - mp^f_{p,t} \right) + \lambda_6 \left(mp_{n,t} - mp^f_{n,t} \right)$

Table 1: The money variable in the Taylor rule

where M_t represents the total nominal money stock and e^{ζ_t} is an exogenous time-varying velocity process defined in the next section. Taking logs, Eq. 42 leads to

$$y_t = mp_t + \zeta_t = mp_{n,t} + mp_{p,t} + \zeta_t \tag{43}$$

The corresponding flexible-price economy equation is similar (Eq. 46) to the previous relation.

2.6 DSGE model

Our DSGE model consists of eight equations and eight dependent variables: inflation, nominal interest rate, output, flexible-price output, real money balances held for production purpose, its flexible-price counterpart, real money balances held for nonproduction purpose, and its flexible-price counterpart.

Flexible-price economy

$$y^f_t = \upsilon^y_a \varepsilon^a_t + \upsilon^y_p \varepsilon^p_t + \upsilon^y_m mp^f_{p,t} + \upsilon^y_c \tag{44}$$

$$mp^f_{n,t} = \frac{\sigma}{\nu} y^f_t - \frac{a_2}{\nu} \sigma E_t \left[\Delta y^f_{t+1} \right] - \frac{\rho_m + \rho_c a_2}{\nu} + \frac{1}{\nu} \varepsilon^n_t \tag{45}$$

$$mp^f_{p,t} = y^f_t - mp^f_{n,t} - \zeta_t \tag{46}$$

Sticky-price economy

$$\pi_t = \beta E_t \left[\pi_{t+1} \right] + \psi_x \left(y_t - y^f_t \right) + \psi_m \left(mp_{p,t} - mp^f_{p,t} \right) \tag{47}$$

$$y_t = E_t \left[y_{t+1} \right] - \sigma^{-1} \left(i_t - E_t \left[\pi_{t+1} \right] - \rho_c \right) - \sigma^{-1} E_t \left[\Delta \varepsilon^u_{t+1} \right] \tag{48}$$

$$mp_{n,t} = \frac{\sigma}{\nu} y_t - \frac{a_2}{\nu} i_t - \frac{\rho_m}{\nu} + \frac{1}{\nu} \varepsilon^n_t \tag{49}$$

$$mp_{p,t} = y_t - mp_{n,t} - \zeta_t$$

$$i_t = (1-\lambda_i)\left(\lambda_\pi(\pi_t-\pi^*)+\lambda_x\left(y_t-y_t^f\right)+M_{k,t}\right)+\lambda_i i_{t-1}+\ \varepsilon_t^i \qquad (50)$$

As we have five historical variables, we have five microfounded shocks: technology shock (ε_t^a), shock to household money demand (ε_t^n), shock to firm money demand (ε_t^p), short-term interest rate or monetary policy shock (ε_t^i), and preference shock (ε_t^u).

Definition 1 *$\forall j \in \{a,n,p,i,u\}$, $\varepsilon_{j,t} = \rho_j \varepsilon_{j,t-1} + \xi_{j,t}$, where ρ_j is an autoregressive coefficient of the AR(1) processes and $\xi_{j,t}$ follows a normal i.i.d. process with a mean of zero and standard deviation of σ_j.*

Following the literature (Benk et al., 2008; Lothian, 2009), the velocity process, ζ_t, depends essentially on money shocks. Then, we choose the following specification for the time-varying velocity process.

Definition 2 *$\zeta_t = \zeta + \lambda_s(\lambda_{ms}\varepsilon_{n,t} + (1-\lambda_{ms})\varepsilon_{p,t})$, where ζ, λ_s and $0 < \lambda_{ms} < 1$ are parameters.*

3 Results

As in Smets and Wouters (2007) and An and Schorfheide (2007), we apply Bayesian techniques to estimate our DSGE models. We test five specifications of the Taylor rule (Table 1) under our assumption that money is part of the production function.

3.1 Eurozone data

In our model of the Eurozone, π_t is the inflation rate, measured as the yearly log-difference of the gross domestic product (GDP) deflator between one quarter and the same quarter of the previous year; y_t is output, measured as the logarithm of GDP; and i_t is the short-term (three-month) nominal interest rate. These data are extracted from the Euro-area Wide Model (AWM) database of Fagan et al. (2001). $mp_{n,t}$ and $mp_{p,t}$ are the real money demands of households and firms, respectively, and are measured as the logarithm of the Euro-area accounts series[4] divided by the GDP deflator. We detrend historical variables using a Hodrick-Prescott filter (with a standard coefficient for quarterly data of 1600).

[4]The money demand series of households and firms are referenced in the Euro-area accounts as S1M.A1.S.1.X.E.Z and S11.A1.S.1.X.E.Z, respectively (with the IEAQ.Q.I6.N.V.LE.F2B suffix). The sum of these two aggregates leads to M2.

y_t^f, the flexible-price output, $mp_{n,t}^f$, the flexible-price household real money balances, and $mp_{p,t}^f$, the flexible-price firm real money balances, are completely determined by structural shocks.

Notice that we deal with as many historical variables as shocks.

3.2 Calibration

We estimate all parameters except the discount factor (β), the inverse of the Frisch elasticity of the labor supply (η), the Calvo (1983) parameter (θ), and the elasticity of household demand for consumption goods (ε). β is set at 0.9926 so that the annual steady-state real interest rate is three percent and θ, η, and ε are set to 0.66, one, and six, respectively, as in Galí (2008) and Ravenna and Walsh (2006).

Following standard conventions, we calibrate beta distributions for parameters that fall between zero and one, inverted gamma distributions for parameters that need to be constrained as greater than zero, and normal distributions in other cases.

As our goal is to compare five versions of the model, we adopt the same priors in each version with the same calibration, depending of the Taylor rule specification. The calibration of σ is inspired by Rabanal and Rubio-Ramírez (2005) and Casares (2007). They choose risk aversion parameters of 2.5 and 1.5, respectively. In line with these values, we regard $\sigma = 2$ as corresponding to a standard risk aversion (Benchimol and Fourçans, 2012; Benchimol, 2014).

We calibrate our central parameter α_m, the share of money in the production process, with a prior mean of 0.25 and a large standard error (relative to its prior mean) of 0.2. Following Basu (1995), we assume that the share of working hours in the production process is around $\alpha_n = 0.5$.

As in Smets and Wouters (2003), the standard errors of the innovations are assumed to follow inverse gamma distributions and we choose beta distributions for shock persistence parameters; the backward component of the Taylor rules; output elasticities of labor, α_n; and real money balances, α_m, of the production function that should be less than one.

The scale parameters γ and χ are calibrated to 0.44 and one, respectively, as in Christiano et al. (2005), and the money velocity mean prior (ζ) is calibrated to 0.31 following Carrillo et al. (2007).

The smoothed Taylor rules (λ_i, λ_π, and λ_x) are calibrated following Gerlach-Kristen (2003), with priors analogous to those used by Smets and Wouters (2003) and Benchimol and Fourçans (2012). In order to take possible behaviors of the central bank into consideration, we assign a higher standard error to the Taylor rule coefficients. The *non-standard* parameters' mean priors of the augmented Taylor rules for $k = 2 - 5$ are calibrated to 0.5, with a large standard error (relative to its prior mean) of 0.2.

All the standard errors of shocks are assumed to be distributed according to inverted Gamma distributions, with prior means of 0.02. The latter distribution ensures that these parameters have a positive support. The autoregressive parameters are all assumed to follow beta distributions. All these distributions are centered around 0.75 and we take a common standard error of 0.1 for the shock persistence parameters, as in Smets and Wouters (2003).

The calibration of the parameters entering the time-varying component of velocity is quite new. The prior mean of λ_s is calibrated to one and, because this calibration exercise is new, we assume a large standard deviation (0.50) and a normal distribution. The prior mean of λ_{ms} is calibrated to 0.50 and theoretically constrained between zero and one. Thus, we assume a Beta distribution for λ_{ms}–which can be seen as a trade-off parameter between the two money demand shocks ($\varepsilon_{n,t}$ and $\varepsilon_{p,t}$). Its standard deviation is not assumed to be very large (0.1) with respect to its prior mean.

3.3 Estimations

The model is estimated using 52 observations of the Eurozone from 1999Q1 to 2012Q1 and the estimation of the implied posterior distribution of the parameters under the five configurations of the Taylor rule is conducted using the Metropolis-Hastings algorithm[5] (ten distinct chains of 300,000 draws each).

The real money balances parameter (α_m) of the *augmented* production function is estimated to be between 0.014 ($k = 4$) and 0.042 ($k = 1$). This result differs from that found by Sinai and Stokes (1972) for the same parameter (0.087).[6] The prior and posterior distributions are presented in Appendix B and estimates of the macro-parameters (aggregated structural parameters) are provided in Appendix E.

We use Bayesian techniques to estimate our model including money in the production function (see Table 2). We do not adopt the Short (1979) restriction involving constant returns to scale in the production function.[7]

The presence of a *money gap* in the Phillips curve (Eq. 47) supports different Taylor rule considerations. Here, we test our model under five Taylor rules

[5]See, for example, Smets and Wouters (2003), Smets and Wouters (2007), Adolfson et al. (2007), and Adolfson et al. (2008).

[6]Benchimol (2011) estimates α_m to be 0.064.

[7]This work has already been done in Benchimol (2011) and Benchimol (2011).

	Priors			Posteriors ($k=1$)				Posteriors ($k=2$)				Posteriors ($k=3$)				Posteriors ($k=4$)				Posteriors ($k=5$)			
	Law	Mean	Std.	Mean	Std.	5%	95%	Mean	Std.	5%	95%	Mean	Std.	5%	95%	Mean	Std.	5%	95%	Mean	Std.	5%	95%
α_n	Beta	0.50	0.20	0.5019	0.2774	0.1810	0.8347	0.4996	0.2773	0.1708	0.8295	0.5002	0.2774	0.1724	0.8317	0.5001	0.2773	0.1754	0.8310	0.5000	0.2773	0.1781	0.8305
α_m	Beta	0.25	0.20	0.0427	0.0006	0.0000	0.0720	0.0299	0.0003	0.0000	0.0728	0.0222	0.0006	0.0000	0.0471	0.0147	0.0002	0.0000	0.0330	0.0248	0.0003	0.0000	0.0505
ν	Normal	1.25	0.10	1.3567	0.0912	1.2044	1.5049	1.4009	0.0907	1.2542	1.5501	1.3270	0.0899	1.1795	1.4771	1.3571	0.0911	1.2049	1.5044	1.3715	0.0953	1.2206	1.5257
σ	Normal	2.00	0.10	1.8465	0.1004	1.6803	2.0096	1.8755	0.1030	1.7055	2.0427	1.8106	0.1083	1.6481	1.9803	1.8465	0.1004	1.6794	2.0086	1.8540	0.1028	1.6873	2.0241
γ	Normal	0.44	0.05	0.4399	0.0498	0.3578	0.5209	0.4100	0.0531	0.3237	0.4983	0.4585	0.0500	0.3770	0.5402	0.4388	0.0498	0.3560	0.5203	0.4288	0.0528	0.3435	0.5170
χ	Normal	1.00	0.10	0.9998	0.1000	0.8374	1.1695	0.9998	0.1000	0.8333	1.1625	1.0009	0.1000	0.8358	1.1661	0.9994	0.1000	0.8346	1.1635	1.0010	0.1000	0.8354	1.1612
λ_i	Beta	0.50	0.05	0.6005	0.0519	0.5181	0.6843	0.6121	0.0507	0.5294	0.6949	0.5875	0.0538	0.5022	0.6735	0.5988	0.0520	0.5160	0.6838	0.6028	0.0523	0.5194	0.6883
λ_π	Normal	3.50	0.20	3.4258	0.1981	3.1020	3.7526	3.3998	0.2009	3.0677	3.7269	3.4246	0.2026	3.0852	3.7416	3.4280	0.1988	3.1048	3.7617	3.4208	0.1994	3.0879	3.7408
λ_x	Normal	1.50	0.20	1.4118	0.2037	1.0724	1.7369	1.3933	0.2023	1.0582	1.7226	1.4305	0.2041	1.1101	1.7693	1.4197	0.2019	1.0952	1.7542	1.4129	0.2025	1.0782	1.7435
λ_k	Normal	0.50	0.20					0.6193	0.2134	0.2731	0.9737	0.4643	0.2725	0.0923	0.8320	0.4201	0.2019	0.0863	0.7450	0.3505	0.2323	-0.0116	0.7269
λ_6	Normal	0.50	0.20																	0.5621	0.2212	0.2065	0.9204
ζ	Normal	0.31	0.10	0.3082	0.0998	0.1425	0.4711	0.3076	0.0998	0.1421	0.4716	0.3110	0.0999	0.1484	0.4739	0.3084	0.0999	0.1432	0.4703	0.3070	0.0998	0.1442	0.4725
λ_s	Normal	1.00	0.50	2.0798	0.2858	1.6270	2.5183	2.0830	0.2694	1.6330	2.5265	2.0754	0.2974	1.6309	2.5201	2.0806	0.2885	1.6297	2.5169	2.0794	0.2652	1.6286	2.5167
λ_{ms}	Beta	0.50	0.10	0.1617	0.0420	0.0958	0.2299	0.1595	0.0398	0.0928	0.2244	0.1654	0.0428	0.0961	0.2330	0.1622	0.0418	0.0949	0.2286	0.1610	0.0411	0.0922	0.2254
ρ_a	Beta	0.75	0.10	0.9398	0.0217	0.9044	0.9753	0.9333	0.0234	0.8955	0.9720	0.9442	0.0202	0.9112	0.9779	0.9399	0.0209	0.9051	0.9757	0.9378	0.0216	0.9019	0.9750
ρ_u	Beta	0.75	0.10	0.9543	0.0267	0.9307	0.9775	0.9423	0.0151	0.9140	0.9722	0.9638	0.0279	0.9446	0.9842	0.9557	0.0142	0.9341	0.9785	0.9515	0.0165	0.9255	0.9777
ρ_i	Beta	0.15	0.01	0.1567	0.0104	0.1397	0.1736	0.1569	0.0104	0.1398	0.1739	0.1562	0.0104	0.1391	0.1729	0.1567	0.0104	0.1397	0.1736	0.1568	0.0104	0.1397	0.1738
ρ_p	Beta	0.75	0.10	0.7656	0.0832	0.6390	0.8979	0.7492	0.0902	0.6186	0.8851	0.7815	0.0763	0.6592	0.9077	0.7659	0.0791	0.6409	0.8956	0.7605	0.0851	0.6335	0.8949
ρ_n	Beta	0.75	0.10	0.8342	0.0520	0.7526	0.9191	0.8405	0.0510	0.7601	0.9235	0.8289	0.0528	0.7445	0.9156	0.8337	0.0508	0.7516	0.9179	0.8368	0.0509	0.7553	0.9222
σ_a	Invgamma	0.02	2.00	0.0071	0.0007	0.0058	0.0082	0.0068	0.0007	0.0056	0.0080	0.0073	0.0008	0.0061	0.0085	0.0071	0.0007	0.0058	0.0082	0.0070	0.0007	0.0058	0.0082
σ_u	Invgamma	0.02	2.00	0.1063	0.0454	0.0592	0.1520	0.0986	0.0200	0.0563	0.1417	0.1124	0.0585	0.0612	0.1635	0.1081	0.0279	0.0609	0.1566	0.1052	0.0280	0.0597	0.1521
σ_i	Invgamma	0.02	2.00	0.0290	0.0046	0.0211	0.0368	0.0258	0.0042	0.0182	0.0331	0.0341	0.0061	0.0240	0.0438	0.0314	0.0052	0.0226	0.0400	0.0300	0.0055	0.0203	0.0392
σ_p	Invgamma	0.02	2.00	0.0164	0.0027	0.0118	0.0209	0.0164	0.0026	0.0118	0.0209	0.0166	0.0028	0.0119	0.0210	0.0164	0.0026	0.0118	0.0208	0.0164	0.0025	0.0118	0.0209
σ_n	Invgamma	0.02	2.00	0.0169	0.0018	0.0139	0.0199	0.0174	0.0018	0.0143	0.0204	0.0166	0.0018	0.0135	0.0195	0.0169	0.0018	0.0139	0.0199	0.0171	0.0018	0.0140	0.0201
				Acceptation rate $\in [0.18; 0.19]$				Acceptation rate $\in [0.21; 0.22]$				Acceptation rate $\in [0.15; 0.16]$				Acceptation rate $\in [0.21; 0.22]$				Acceptation rate $\in [0.20; 0.21]$			
				Log data density: -437.29				Log data density: -435.91				Log data density: -438.51				Log data density: -438.74				Log data density: -438.14			

Table 2: Bayesian estimation of the model

(see Table 1), and the Bayesian estimation of the model with a *productive-money gap* ($k = 2$) yields the higher log marginal density (-435.91).

A robustness test regarding the numerical maximization of the posterior kernel is also conducted and indicates that the optimization procedure leads to a robust maximum for the posterior kernel. The convergence of the proposed distribution to the target distribution is satisfied. A diagnosis of the overall convergence for the Metropolis-Hastings sampling algorithm is provided in Appendix D and, following Ratto (2008), all estimations are stable.

3.4 Simulations

3.4.1 Impulse response functions

Appendix C presents the responses of key variables to structural shocks for each k.

In response to a preference shock, the inflation rate, output, output gap, firm real money balances, nominal interest rate, and real interest rate rise, whereas household real money holdings display a little undershooting process in the first few periods, then return to their steady-state value.

After a technology shock, the output gap, inflation rate, nominal interest rate, firm real money balances, and real interest rates decrease, whereas output and household real money balances rise.

In response to an interest rate shock, the inflation rate, output, and output gap fall. Interest rates and firm money demand rise. A positive monetary policy shock induces a fall in interest rates due to a sufficiently low degree of intertemporal substitution (i.e., the risk aversion parameter is sufficiently high), which generates a high-impact response of current relative to future consumption. This result has been noted in *inter alia*, Jeanne (1994), and Christiano et al. (1997).

Following a shock in the money demand of firms, interest rates, the output gap, and the real money holdings of firms decrease, whereas inflation and the real money holdings of households increase. These impulse response functions are similar to Smets and Wouters (2003) with regard to output, inflation, and interest rates. However, the responses following a shock in the money demand of households depends on the model specification (see Appendix C).

3.4.2 Variance decompositions

The analysis is conducted via unconditional and conditional variance decompositions (see Table 3) to compare the impact of shocks on variables across the models and over time.

For all models, most of the long-run variance in output comes from the technology shock (around 75%), about one-quarter of the output variance results from the interest rate shock (around 5-20%) and the remaining quarter occurs due to the other shocks. In the short run, most of the output variance comes from the monetary policy shock (around 63%), whereas around 28% is a result of the technology shock. The money demand of firms impacts output variance (and its flexible-price counterpart) due to the form of the production function (Eq. 7). Although we do not add a constant return-to-scale restriction to the production function, we know that such a restriction should also attribute a larger role to real money demand in explaining the variances of output and its flexible-price counterpart (Benchimol, 2011). However, in the short run, the share of flexible-price output variance explained by the shock in the money demand of firms is important (around 24%). This role decreases over longer horizons (to around 8%) and is in line with Moghaddam (2010). However, we must temper this result by the fact that we do not have a money supply shock in our framework, which is similar to the frameworks found in the literature (Benhabib et al., 2001; Ireland, 2004; Andrés et al., 2009; Benchimol and Fourçans, 2012; Benchimol, 2014).

A look at the conditional and unconditional inflation variance decompositions shows the overwhelming role of the interest rate shock, which explains more than 92% of inflation rate variance in the short run. This role decreases over time, whereas the role of the preference shock increases from around 6% in the short run to around 20% in the long run (except for $k = 2$). The other shocks play a minor role in inflation variance.

The variance of the nominal interest rate is dominated in the short run by the direct interest rate shock (monetary policy shock), whereas the preference shock does not play a significant role. The relative importance of each of these shocks changes over time. For longer horizons, there is an inversion over time–the preference shock explains almost 75% of the nominal interest rate variance, whereas the interest rate shock explains less than 21%.

Table 3 shows that the demands for real money are mainly explained by the money, technology, and interest rate shocks. In the short run, variance in the money demand of firms is essentially determined by its corresponding shock (around 68%) as well as that in the money demand of households (around 25%). However, the variance in household money demand is mainly driven by the interest rate shock (around 50%), its corresponding shock (around 25%), and the technology shock (around 15%). In the long run, variance in the money demand of firms is also driven by its corresponding shock (around 60%) and the households' money demand shock (around 30%) and the firms' money demand variance decomposition changes. The latter is mainly driven, in the long run, by the technology shock (around

	Quarter 1 ($k=1$)					Quarter 1 ($k=2$)					Quarter 1 ($k=3$)					Quarter 1 ($k=4$)					Quarter 1 ($k=5$)				
	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$
y_t	27.99	1.20	63.45	7.36	0.00	27.69	1.73	63.66	6.91	0.01	29.64	0.66	62.17	7.53	0.00	28.18	0.93	63.68	7.21	0.00	28.14	1.13	63.68	7.05	0.00
π_t	0.14	6.42	93.04	0.40	0.00	0.75	5.82	92.37	0.96	0.10	0.00	5.90	93.96	0.13	0.01	0.13	5.50	94.05	0.32	0.00	0.22	5.69	93.62	0.46	0.00
i_t	0.42	19.31	78.83	1.44	0.00	1.01	21.62	75.25	2.10	0.03	0.14	15.36	83.55	0.95	0.00	0.39	17.13	81.26	1.22	0.00	0.51	18.63	79.31	1.55	0.00
$mp_{p,t}$	0.56	0.15	4.26	69.92	25.12	0.51	0.13	3.48	70.31	25.57	0.64	0.15	5.09	68.96	25.17	0.62	0.15	4.74	69.42	25.08	0.62	0.15	4.55	68.98	25.71
$mp_{n,t}$	15.23	0.00	53.10	4.65	27.02	15.13	0.02	50.70	4.43	29.73	15.90	0.02	54.14	4.64	25.30	15.36	0.00	54.10	4.52	26.02	15.37	0.00	53.23	4.50	26.90
y_t^f	75.16	0.00	0.00	24.84	0.00	74.16	0.00	0.00	25.84	0.00	77.19	0.00	0.00	22.81	0.00	76.30	0.00	0.00	23.70	0.00	75.70	0.00	0.00	24.30	0.00
$mp_{p,t}^f$	1.44	0.00	0.00	72.05	26.51	1.41	0.00	0.00	71.29	27.30	1.58	0.00	0.00	72.14	26.28	1.48	0.00	0.00	71.84	26.67	1.50	0.00	0.00	71.22	27.28
$mp_{n,t}^f$	35.47	0.00	0.00	13.96	50.57	33.12	0.00	0.00	14.14	52.74	38.42	0.00	0.00	13.16	48.41	36.29	0.00	0.00	13.23	50.49	35.43	0.00	0.00	13.71	50.87
ζ_t	0.00	0.00	0.00	96.28	3.72	0.00	0.00	0.00	96.52	3.48	0.00	0.00	0.00	96.26	3.74	0.00	0.00	0.00	96.28	3.72	0.00	0.00	0.00	96.27	3.73

	Quarter 4 ($k=1$)					Quarter 4 ($k=2$)					Quarter 4 ($k=3$)					Quarter 4 ($k=4$)					Quarter 4 ($k=5$)				
	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$
y_t	52.21	0.70	37.41	9.68	0.00	52.09	1.01	38.12	8.77	0.01	54.11	0.37	35.40	10.11	0.00	52.49	0.54	37.27	9.70	0.00	52.67	0.66	37.59	9.08	0.00
π_t	0.23	10.82	88.50	0.45	0.00	1.41	8.28	89.03	1.08	0.19	0.01	11.57	88.25	0.14	0.02	0.21	9.64	89.77	0.37	0.00	0.41	9.43	89.64	0.52	0.01
i_t	1.07	50.30	46.28	2.36	0.00	2.43	52.70	41.79	3.03	0.05	0.39	44.01	53.79	1.80	0.01	1.05	46.75	50.04	2.16	0.00	1.31	49.10	47.10	2.49	0.00
$mp_{p,t}$	0.89	0.53	2.12	68.50	27.95	0.87	0.52	1.84	66.25	30.52	0.96	0.46	2.38	69.69	26.50	0.96	0.48	2.28	68.63	27.64	1.01	0.54	2.29	66.53	29.63
$mp_{n,t}$	27.66	0.96	30.40	6.04	34.95	26.83	0.93	28.40	5.37	38.47	29.09	0.86	30.95	6.35	32.75	27.93	0.90	30.83	6.02	34.33	27.79	0.93	30.24	5.68	35.36
y_t^f	82.34	0.00	0.00	17.66	0.00	82.46	0.00	0.00	17.54	0.00	83.07	0.00	0.00	16.93	0.00	82.73	0.00	0.00	17.27	0.00	83.14	0.00	0.00	16.86	0.00
$mp_{p,t}^f$	2.12	0.00	0.00	68.79	29.08	2.13	0.00	0.00	65.66	32.21	2.25	0.00	0.00	70.67	27.08	2.12	0.00	0.00	69.00	28.88	2.23	0.00	0.00	66.83	30.95
$mp_{n,t}^f$	43.13	0.00	0.00	11.02	45.86	39.91	0.00	0.00	10.40	49.70	46.50	0.00	0.00	10.99	42.52	43.51	0.00	0.00	10.65	45.84	42.72	0.00	0.00	10.44	46.84
ζ_t	0.00	0.00	0.00	95.75	4.25	0.00	0.00	0.00	95.58	4.42	0.00	0.00	0.00	96.07	3.93	0.00	0.00	0.00	95.82	4.18	0.00	0.00	0.00	95.53	4.47

	Quarter ∞ ($k=1$)					Quarter ∞ ($k=2$)					Quarter ∞ ($k=3$)					Quarter ∞ ($k=4$)					Quarter ∞ ($k=5$)				
	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$	$\xi_{a,t}$	$\xi_{u,t}$	$\xi_{i,t}$	$\xi_{p,t}$	$\xi_{n,t}$
y_t	75.42	0.36	18.51	5.70	0.00	73.71	0.55	20.40	5.33	0.01	78.20	0.18	15.87	5.75	0.00	75.88	0.28	18.07	5.77	0.00	75.82	0.34	18.59	5.25	0.00
π_t	0.41	21.60	77.57	0.41	0.00	2.73	13.08	82.92	1.03	0.23	0.05	28.85	70.95	0.12	0.02	0.40	20.68	78.57	0.35	0.00	0.79	18.83	79.89	0.48	0.01
i_t	1.47	77.71	19.58	1.23	0.00	3.33	75.69	19.31	1.64	0.03	0.55	77.30	21.21	0.94	0.00	1.48	76.08	21.27	1.17	0.00	1.82	76.83	20.06	1.28	0.00
$mp_{p,t}$	2.13	1.70	1.71	64.49	29.98	1.93	1.48	1.51	60.49	34.59	2.41	1.72	1.84	67.12	26.91	2.28	1.59	1.79	64.77	29.57	2.39	1.74	1.84	61.34	32.69
$mp_{n,t}$	47.69	2.78	17.88	4.25	27.41	43.79	2.44	17.41	3.75	32.61	51.89	2.75	17.08	4.46	23.82	48.24	2.63	17.79	4.28	27.06	47.42	2.69	17.65	3.89	28.36
y_t^f	92.00	0.00	0.00	8.00	0.00	91.63	0.00	0.00	8.37	0.00	92.65	0.00	0.00	7.35	0.00	92.14	0.00	0.00	7.86	0.00	92.51	0.00	0.00	7.49	0.00
$mp_{p,t}^f$	4.88	0.00	0.00	64.15	30.96	4.48	0.00	0.00	59.34	36.18	5.50	0.00	0.00	67.26	27.24	4.86	0.00	0.00	64.49	30.65	5.10	0.00	0.00	61.03	33.88
$mp_{n,t}^f$	62.67	0.00	0.00	6.49	30.84	56.28	0.00	0.00	6.30	37.42	68.11	0.00	0.00	6.27	25.62	62.94	0.00	0.00	6.30	30.76	61.63	0.00	0.00	6.01	32.36
ζ_t	0.00	0.00	0.00	95.18	4.82	0.00	0.00	0.00	94.56	5.44	0.00	0.00	0.00	95.86	4.14	0.00	0.00	0.00	95.28	4.72	0.00	0.00	0.00	94.68	5.32

Table 3: Variance decompositions

45%), its corresponding shock (around 23-32%), and the interest rate shock (around 17%).

It is also interesting to note that the same type of analysis applies to the flexible-price output variance decomposition. The technology shock, with a weight of around 91%, is the main explanatory factor of long-run variance in flexible-price output. In shorter time frames, flexible-price output variance is mainly explained by the shocks in technology (around 75%) and firm money demand. As previously explained, this result is attributable to the functional form of the production function.

Although flexible-price real money demand of firms is mainly impacted by money shocks (regardless of the time horizon), flexible-price real money demand of households is mainly driven by the technology shock in the long run (around 56%-68%) and its corresponding shock, whereas in the short run, it is mainly driven by its corresponding shock (household real money demand shock).

4 Interpretation

Do increases in the real money supply increase the productive capacity of the economy? Empirical and theoretical papers, ranging from Sinai and Stokes (1972) to Benchimol (2011), have attempted to answer this question by including real balances in an estimated aggregate production function. Estimates of the output elasticity of real money, using various definitions of money and various methods, range from about 0.02 to about 1.0 (Startz, 1984). Since the growth rate of real money balances is generally between plus or minus seven percent per annum, these elasticity estimates suggest that fluctuations in the real money supply can explain increases in aggregate supply on the order of statistical noise in our case. The output elasticity of real money balances is equal to 0.016 in our best model specification ($k = 2$). For all k, our results are at least ten times lower than those of Sinai and Stokes (1972, 1975, 1989) and Short (1979).

In our study, we do not assume constant returns to scale: our production function specification follows the hypothesis of Khan and Ahmad (1985)–decreasing return-to-scale hypothesis. This hypothesis gives *by-construction* no influence on money in the dynamics of the variables, despite its introduction into the production function[8]. However, because of the Cobb and

[8] Following Benchimol (2011), the decreasing returns to scale hypothesis is preferred over the constant returns to scale hypothesis. We do not follow the hypothesis of Short (1979), Startz (1984), Benzing (1989), and Chang (2002) of constant returns to scale for money in the production function.

Douglas (1928) assumption about the formulation of the production function, and even if the elasticity of the real money holdings of firms is attributed to statistical noise, the money demand of firms could impact at least output dynamics.

The simulations (see Table 3 and Appendix C) are close to those obtained in the literature and provide interesting results regarding the potential effect of money on output and flexible-price output under two different money demands (household and firm).

Interestingly, and even if money enters into the inflation equation, money shocks have almost no effect on the variance decomposition of inflation. This result is common in the literature on money in a non-separable utility function (Ireland, 2004; Andrés et al., 2009; Benchimol and Fourçans, 2012). Moreover, the estimated velocity means are in line with Carrillo et al. (2007) and do not change across models (see Appendix E).

Another interesting result is that the shock on firm money demand has an important influence on flexible-price output and in the Taylor rule. The estimated contribution of firms' money holdings in our money-augmented Taylor rule ($k = 2$) seems to be significant: this means that this shock could potentially impact monetary policy. This result does not mean that we *should* target firm money demand. It only tells us that this variable *could* be taken into account for policy analysis. Because firm money holding shocks impact macroeconomic variable dynamics in our framework, monetary policy should pay particular attention to the money demand of firms.

5 Conclusion

One of the most unsettled issues of the postwar economic literature involves the role of money as a factor of production. The notion of money as a factor of production has been debated both theoretically and empirically by a number of researchers in the past five decades. The question is whether money is an omitted variable in the production process.

In parallel, New Keynesian DSGE theory, combined with Bayesian analysis, has become increasingly popular in the academic literature and in policy analysis. The unique contribution of this paper is to build and test a micro-founded New Keynesian DSGE model that includes money in the production function. We depart from the existing theoretical and empirical literature by building a New Keynesian DSGE model *à la* Galí (2008) that includes money in the production function, and, as a consequence, in the inflation equation (Phillips curve). Closing the model leads to the new concept of flexible-price real money balances presented by Benchimol and Fourçans (2012).

Empirical support for money as an input along with labor has been mixed; thus, the issue appears to be unsettled. This paper, as in Benhabib et al. (2001), differentiates between money demanded by households and firms. This distinction between money that is used for productive and nonproductive purposes seems to be warranted. By testing our models with Bayesian techniques under different monetary policy rules, we show that even if the weight of real money balances in the production function is very low, the firm money shock has an important influence on flexible-price output and a significant impact on output. Part of this influence comes from the functional form of the production function (non-separability between money holdings and working hours).

With respect to our estimation of the weight of money in the production function, real money balances could be excluded from the production process. However, considering this hypothesis of money in the production function highlights the significant role of the firm money shock. Incorporating the real money balances variable as a factor input in a production function–in order to capture the productivity gains derived from using money–could lead to important monetary policy implications.

6 Appendix

A Optimization problem

Our Lagrangian is given by

$$L_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left(U_{t+k} - \lambda_{t+k} V_{t+k} \right) \right],$$

where

$$V_t = C_t + \frac{\Delta M_{n,t}}{P_t} + \frac{\Delta M_{p,t}}{P_t} + Q_t \frac{B_t}{P_t} - \frac{B_{t-1}}{P_t} - \frac{W_t}{P_t} N_t$$

and

$$U_t = e^{\varepsilon_t^u} \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma e^{\varepsilon_t^n}}{1-\nu} \left(\frac{M_{n,t}}{P_t} \right)^{1-\nu} - \frac{\chi N_t^{1+\eta}}{1+\eta} \right).$$

The first-order condition related to consumption expenditures is given by

$$\lambda_t = e^{\varepsilon_t^u} C_t^{-\sigma}, \tag{51}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint at time t.

The first-order condition corresponding to the demand for contingent bonds implies that

$$\lambda_t Q_t = \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right]. \tag{52}$$

The demand for cash held for nonproduction purposes that follows from the household optimization problem is given by

$$\gamma e^{\varepsilon_t^u} e^{\varepsilon_t^n} \left(\frac{M_{n,t}}{P_t} \right)^{-\nu} = \lambda_t - \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right], \tag{53}$$

which can be naturally interpreted as a demand for real balances. The latter is increasing in consumption and is inversely related to the nominal interest rate, as in conventional specifications.

Working hours following the household optimization problem are given by

$$\chi e^{\varepsilon_t^u} N_t^\eta = \lambda_t \frac{W_t}{P_t}. \tag{54}$$

From Eq. 51, we obtain

$$\lambda_t = e^{\varepsilon_t^u} C_t^{-\sigma} \Leftrightarrow U_{c,t} = e^{\varepsilon_t^u} C_t^{-\sigma}, \tag{55}$$

where $U_{c,t} = \left.\frac{\partial U_{k,t}}{\partial C_{t+k}}\right|_{k=0}$. Eq. 55 defines the marginal utility of consumption.

Hence, the optimal consumption/savings, real money balance, and labor supply decisions are described by the following conditions:

- Combining Eq. 51 with Eq. 52 yields

  $$Q_t = \beta E_t \left[\frac{e^{\varepsilon_{t+1}^u} C_{t+1}^{-\sigma}}{e^{\varepsilon_t^u} C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] \Leftrightarrow Q_t = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right], \tag{56}$$

  where $U_{c,t+1} = \left.\frac{\partial U_{k,t}}{\partial C_{t+k}}\right|_{k=1}$. Eq. 56 is the usual Euler equation for intertemporal consumption flows. It establishes that the ratio of marginal utility of future and current consumption is equal to the inverse of the real interest rate.

- Combining Eq. 51 and Eq. 53 yields

  $$\gamma \frac{e^{\varepsilon_t^n}}{C_t^{-\sigma}} \left(\frac{M_{n,t}}{P_t} \right)^{-\nu} = 1 - Q_t \Leftrightarrow \frac{U_{m,t}}{U_{c,t}} = 1 - Q_t, \tag{57}$$

  where $U_{m,t} = \left.\frac{\partial U_{k,t}}{\partial \left(M_{n,t+k}/P_{t+k} \right)}\right|_{k=0}$. Eq. 57 is the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money.

- Combining Eq. 51 and Eq. 54 yields

$$\chi \frac{N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \Leftrightarrow \frac{U_{n,t}}{U_{c,t}} = -\frac{W_t}{P_t}, \tag{58}$$

where $U_{n,t} = \left.\frac{\partial U_{k,t}}{\partial N_{t+k}}\right|_{k=0}$. Eq. 58 is the condition for the optimal consumption-leisure arbitrage, implying that the marginal rate of substitution between consumption and labor is equal to the real wage.

B Priors and posteriors

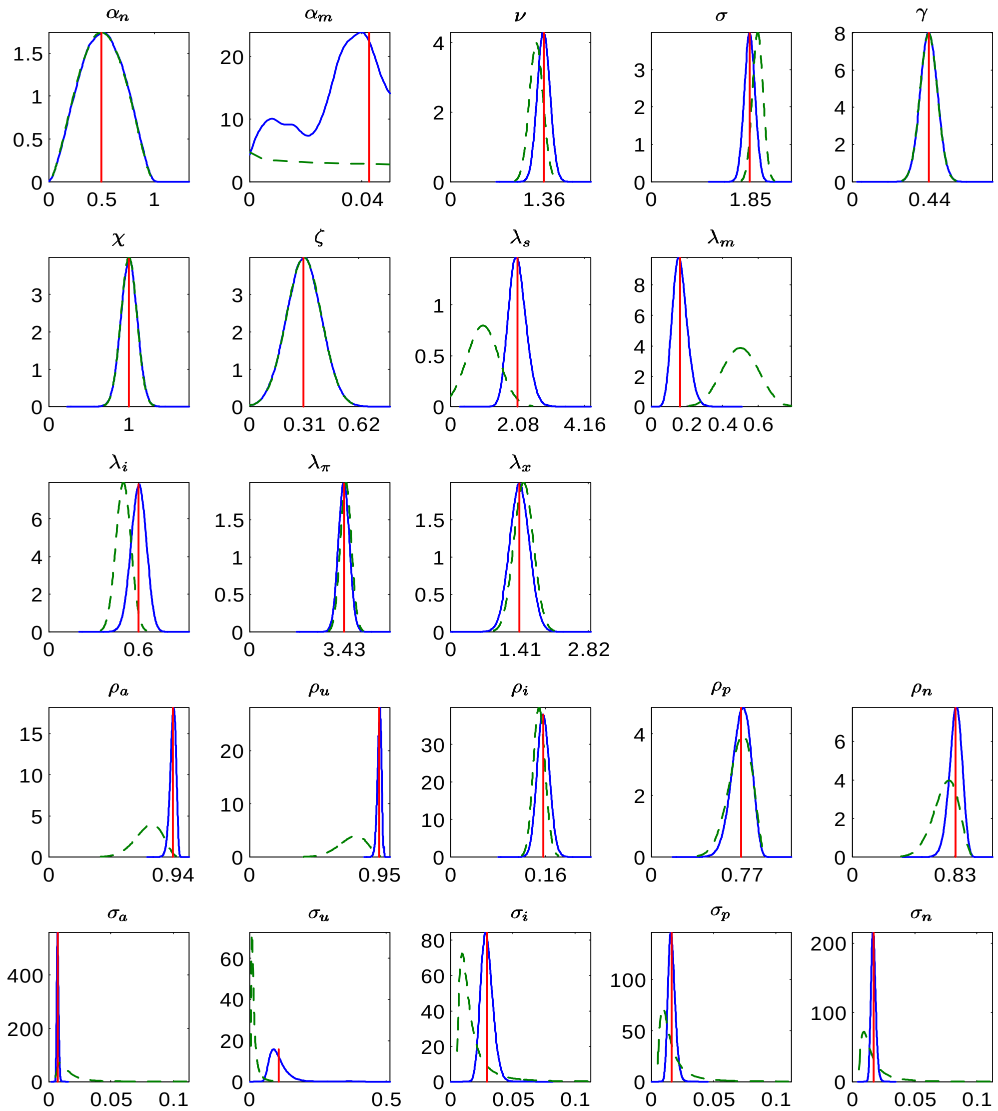

Figure 1: Priors and posteriors of the estimated parameters ($k = 1$).

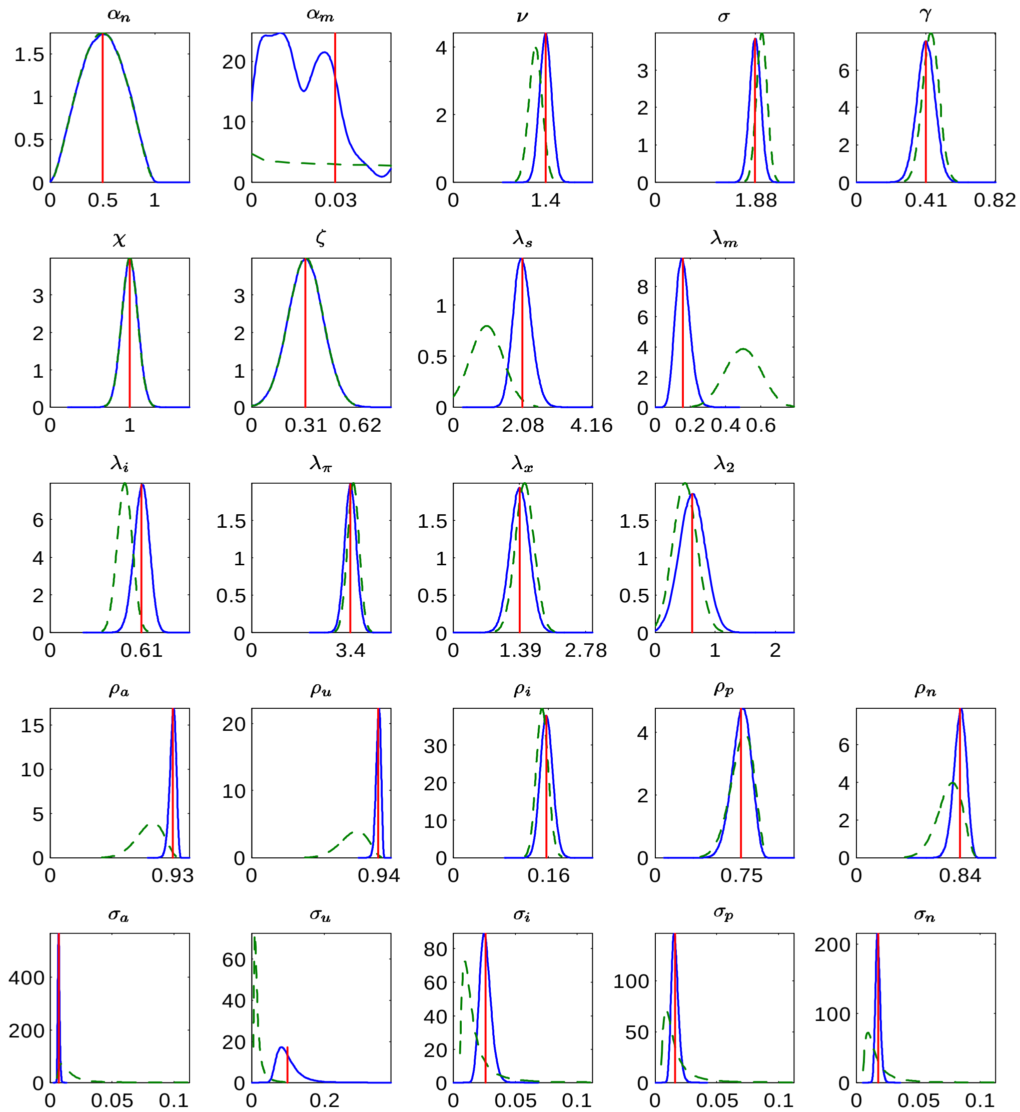

Figure 2: Priors and posteriors of the estimated parameters ($k = 2$).

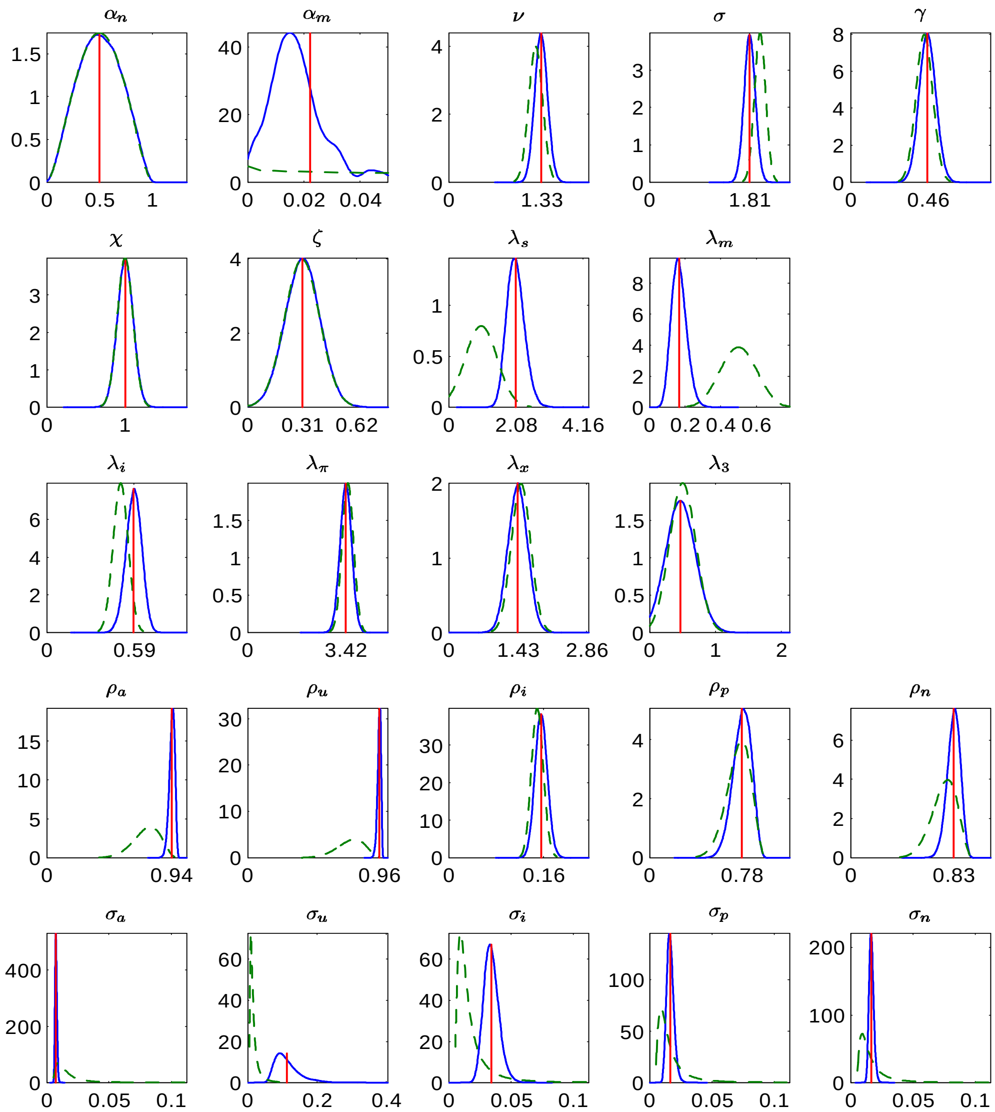

Figure 3: Priors and posteriors of the estimated parameters ($k = 3$).

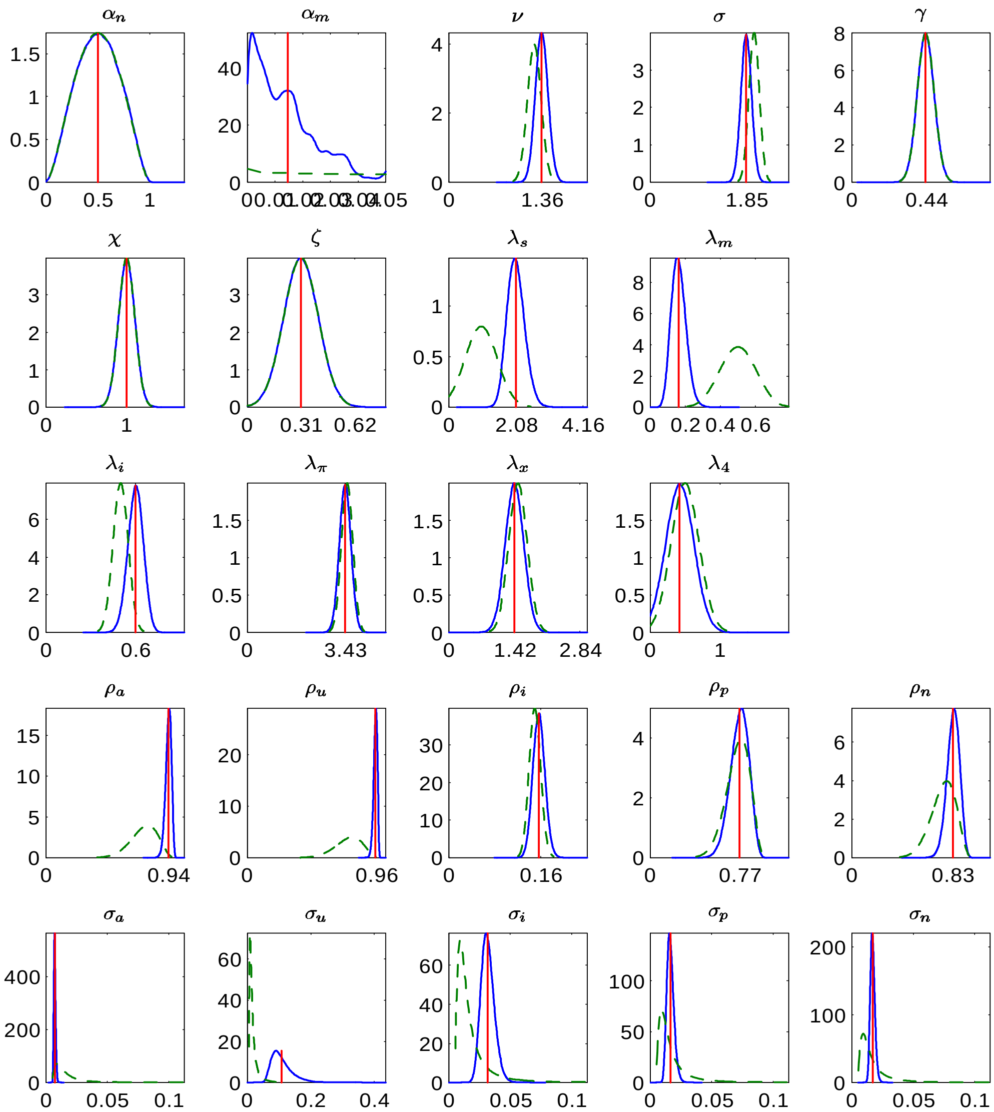

Figure 4: Priors and posteriors of the estimated parameters ($k = 4$).

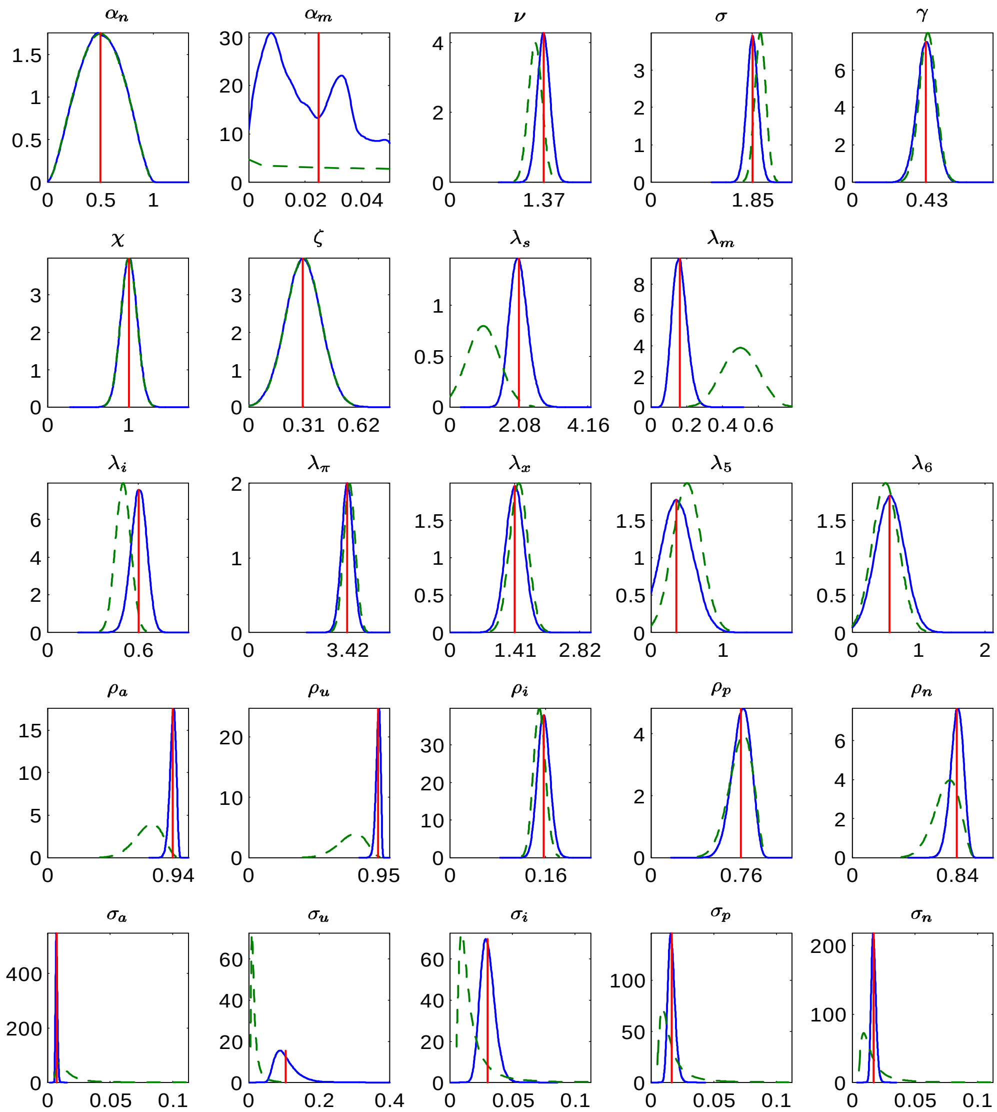

Figure 5: Priors and posteriors of the estimated parameters ($k = 5$).

C Impulse response functions

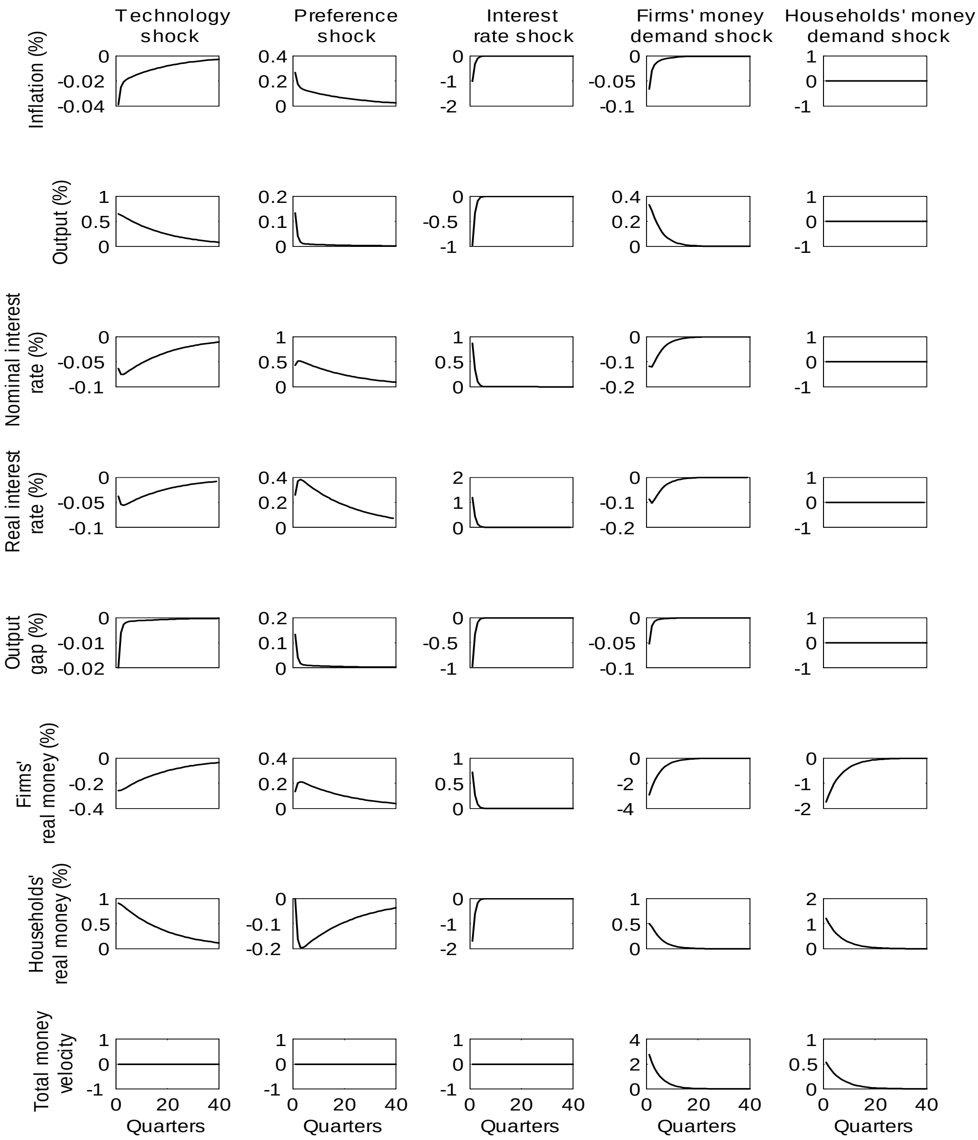

Figure 6: Impulse response function ($k = 1$)

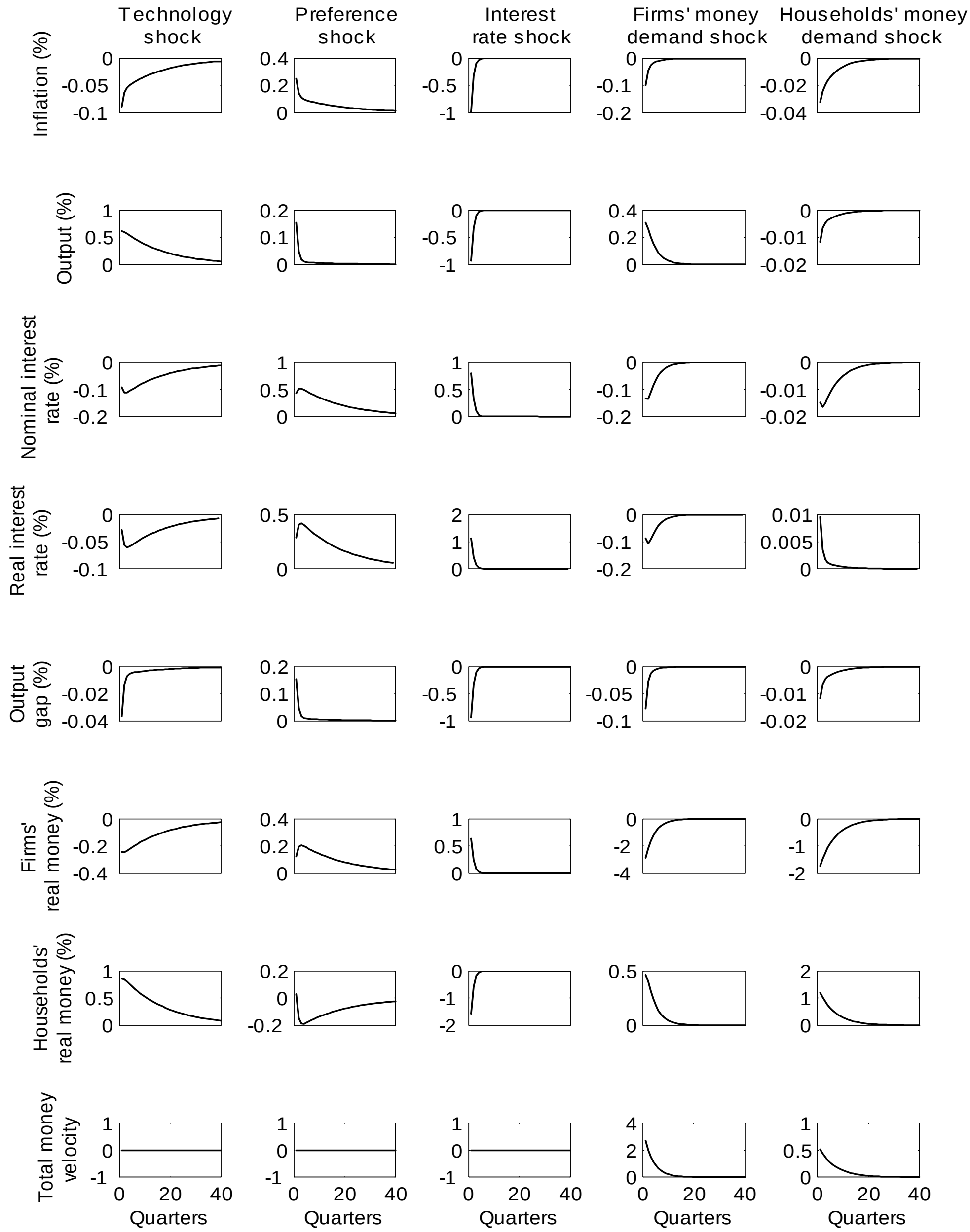

Figure 7: Impulse response function ($k = 2$)

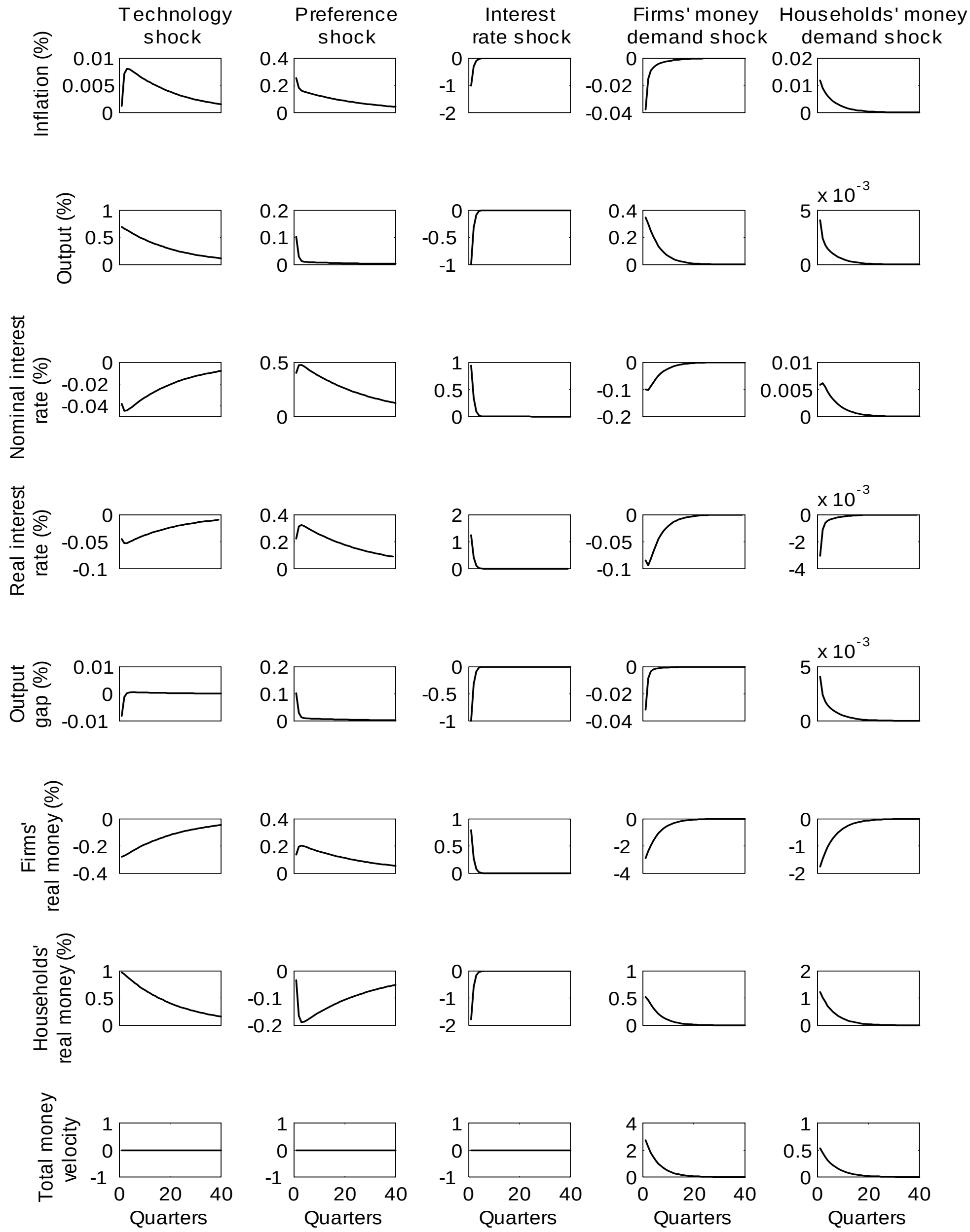

Figure 8: Impulse response function ($k = 3$)

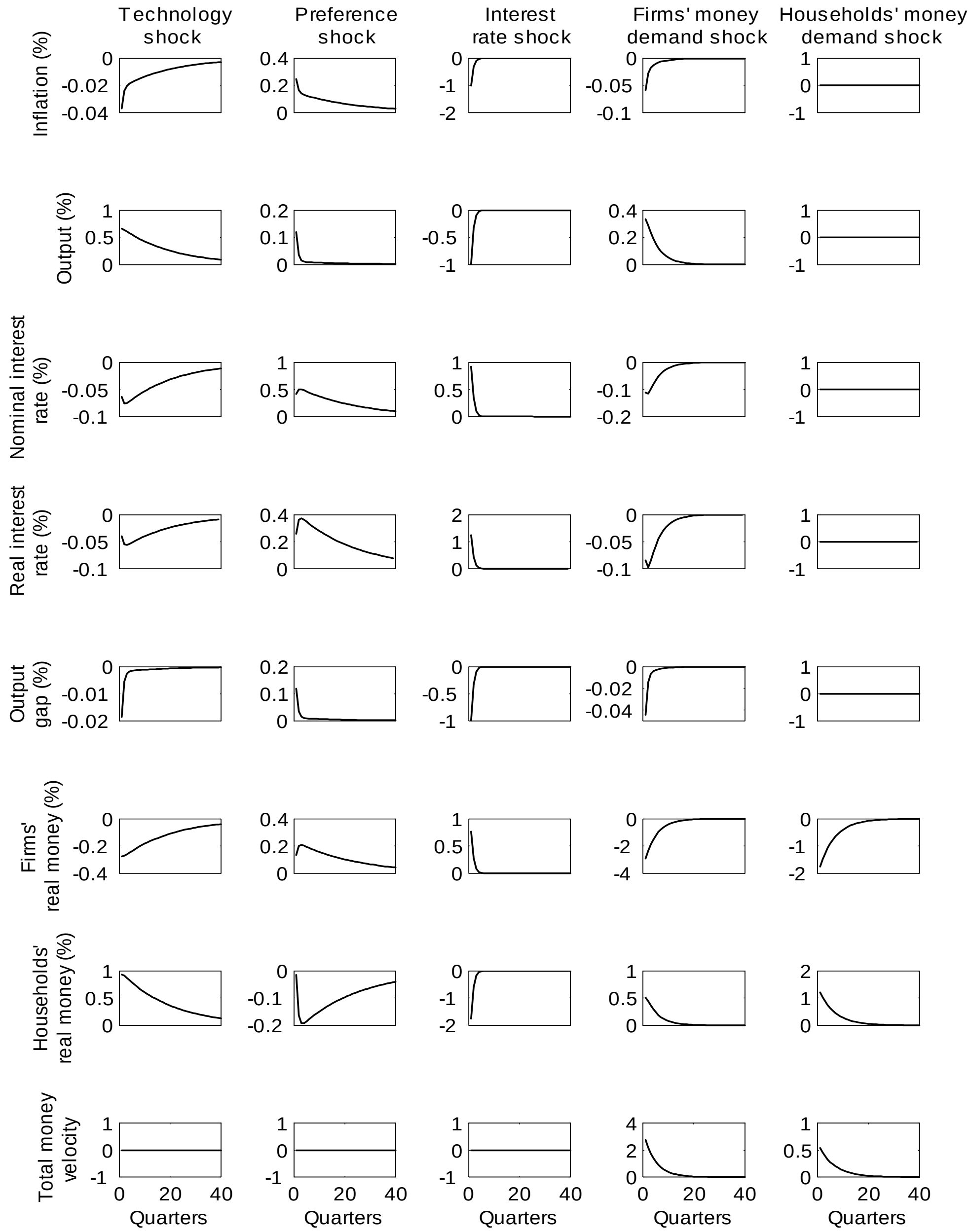

Figure 9: Impulse response function ($k = 4$)

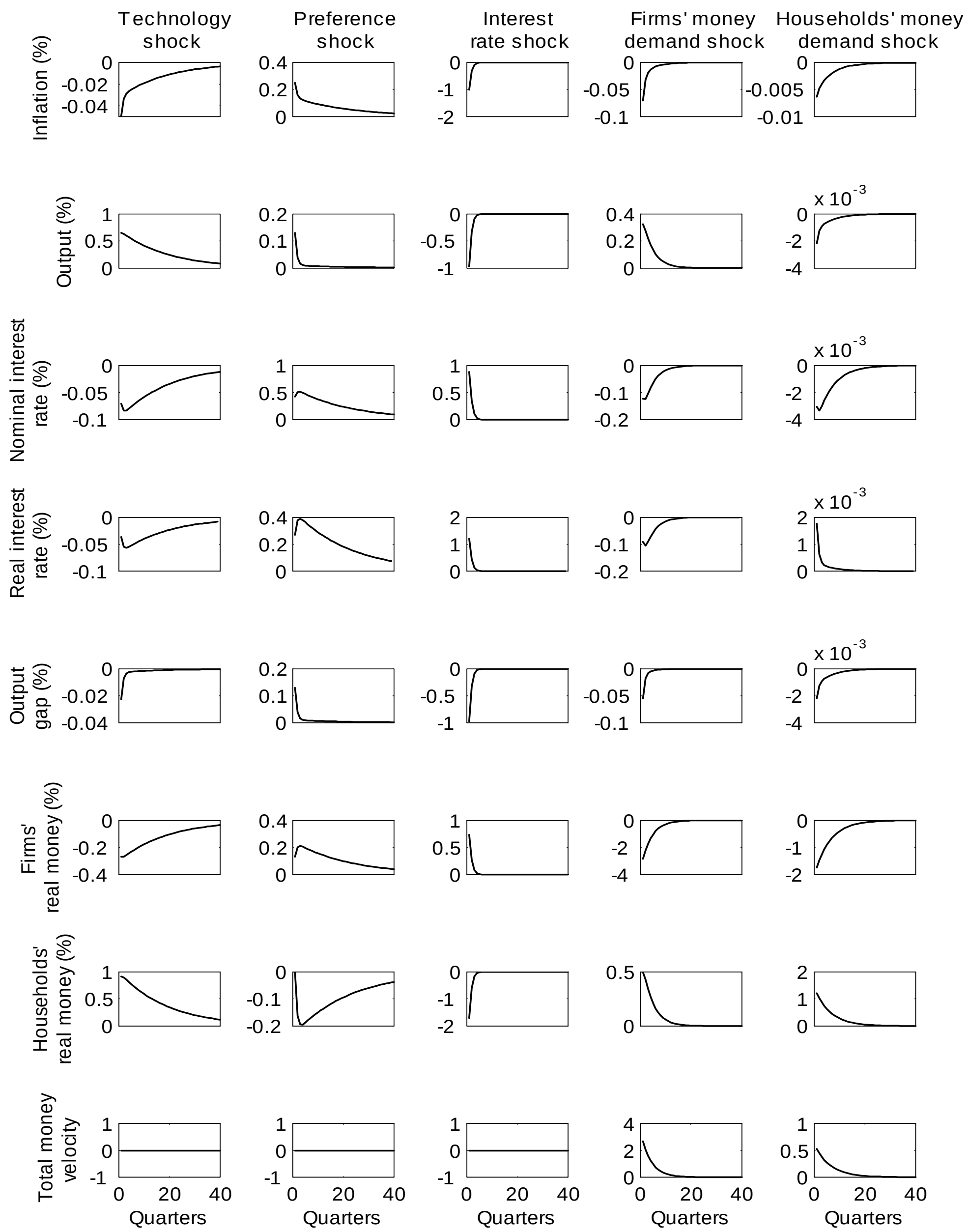

Figure 10: Impulse response function ($k = 5$)

D Robustness checks

Each graph represents specific convergence measures through two distinct lines that show the results within (red line) and between (blue line) chains (Geweke, 1999). Those measures are related to the analysis of the model parameter means (intervals), variances (m2), and third moments (m3). For each of the three measures, convergence requires both lines to become relatively horizontal and converge toward each other in both models[9].

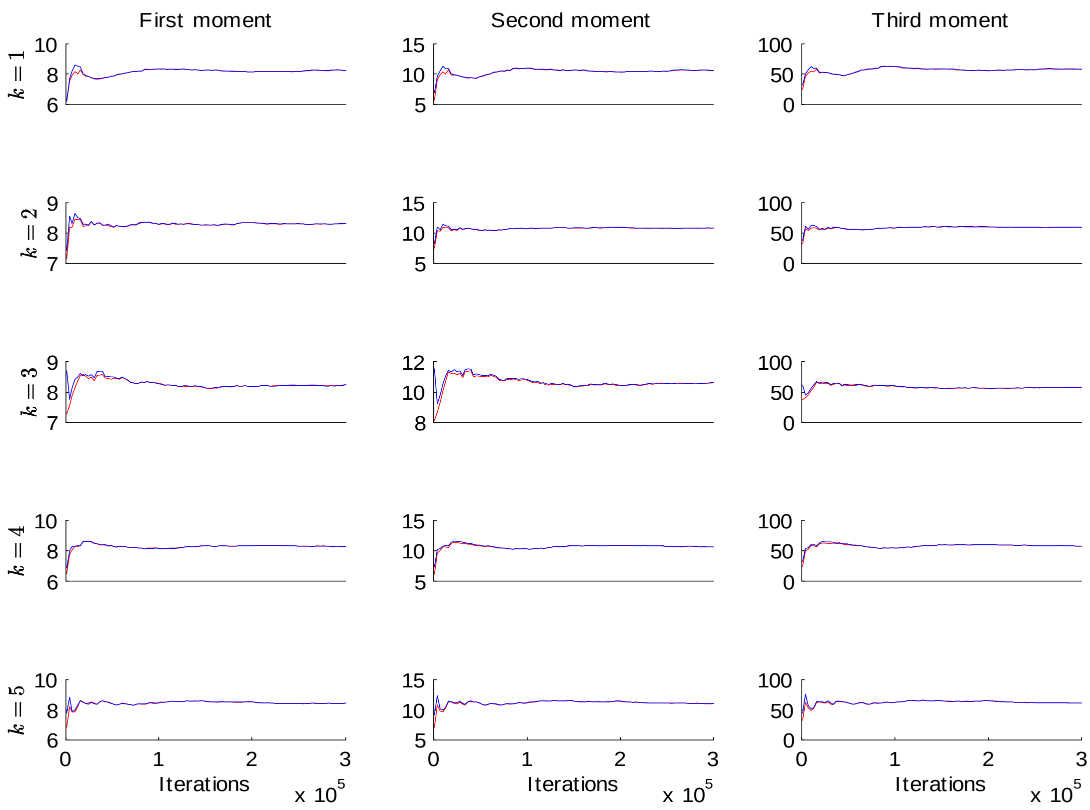

Figure 11: Metropolis-Hastings' convergence diagnostics

[9] Robustness analysis with respect to calibrated parameters is available upon request.

E Macro parameters

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
υ_a^y	1,0397	1,0321	1,0496	1,0398	1,0378
υ_p^y	0,0443	0,0308	0,0233	0,0152	0,0256
υ_m^y	-0,2145	-0,2273	-0,2390	-0,2446	-0,2337
υ_c^y	-1,0445	-1,1319	-1,2286	-1,3251	-1,1871
$\frac{\sigma}{\nu}$	1,3610	1,3388	1,3644	1,3606	1,3518
$\frac{\sigma}{\nu}a_2$	0,7828	0,7700	0,7847	0,7826	0,7775
$\frac{\rho_m+\rho_c a_2}{\nu}$	-0,1534	-0,0984	-0,1882	-0,1516	-0,1332
$\frac{1}{\nu}$	0,7370	0,7138	0,7535	0,7368	0,7291
ψ_x	0,6561	0,6943	0,6743	0,6819	0,6850
ψ_m	0,1407	0,1578	0,1611	0,1668	0,1601
$\frac{1}{\sigma}$	0,5415	0,5331	0,5523	0,5415	0,5393
$\frac{\rho_c}{\sigma}$	0,0040	0,0039	0,0041	0,0040	0,0040
$\frac{a_2}{\nu}$	0,4239	0,4105	0,4334	0,4238	0,4193
$\frac{\rho_m}{\nu}$	-0,1566	-0,1014	-0,1915	-0,1547	-0,1363
λ_i	0,6005	0,6121	0,5874	0,5987	0,6027
$\lambda_\pi(1-\lambda_i)$	1,3685	1,3187	1,4127	1,3753	1,3588
$\lambda_x(1-\lambda_i)$	0,5640	0,5404	0,5901	0,5695	0,5612
$\lambda_k(1-\lambda_i)$		0,2402	0,1915	0,1685	0,1392
$\lambda_6(1-\lambda_i)$					0,2232
$\exp(\zeta)$	1,3609	1,3602	1,3647	1,3612	1,3593
$\lambda_s\lambda_{ms}$	0,3363	0,3322	0,3433	0,3374	0,3347
$\lambda_s(1-\lambda_{ms})$	1,7434	1,7506	1,7321	1,7431	1,7446

Table 4: Macroparameter values

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