# Switching Volatility in a Nonlinear Open Economy: Online Appendix*

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**Abstract**

This online appendix describes the derivations, variables, and parameters used in our paper.

*Keywords:* DSGE, Volatility Shocks, Markov Switching, Open Economy, Financial Crisis, Nonlinearities.

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# 1 Derivations

Our generic model is a symmetric two-country model in which domestic ($d$) and foreign ($f$) households maximize their country-specific utility subject to their country-specific budget constraints, firms maximize their respective benefits, and central banks follow their respective *ad-hoc* Taylor-type rules and budget constraints.

## 1.1 Domestic households

The following equations are expressed in terms of stationary variables. The relation between the initial and stationary variables is logarithmic after normalization by price and/or the level of exogenous technological progress.[1]

The optimization problem of domestic households, corresponding to Eq. 1 to Eq. 4 in the paper, expressed in terms of the stationary variables is

$$E_t\left[\sum_{t=0}^{\infty}\varepsilon^u_{d,t-1}\left(\begin{array}{l}\frac{\left(e^{c_{d,t}}-e^{h_{d,C,t}}\right)^{1-1/\sigma_{d,C}}}{1-1/\sigma_{d,C}}-e^{\phi^L_{d,t}}\frac{\left(e^{l_{d,t}}\right)^{1+1/\sigma_{d,L}}}{1+1/\sigma_{d,L}}+e^{\phi^M_{d,t}}\frac{\left(e^{m_{d,t}}\right)^{1-1/\sigma_{d,M}}}{1-1/\sigma_{d,M}}\\ -\frac{\varphi_{d,s,d}}{2}\left(\frac{b_{d,s,d,t}h_{d,C}}{e^{h_{d,C,t}}}-\mu_{d,s,d}\right)^2-\frac{\varphi_{d,l,d}}{2}\left(\frac{b_{d,l,d,t}h_{d,C}}{e^{h_{d,C,t}}}-\mu_{d,L,d}\right)^2\\ -\frac{\varphi_{d,s,f}}{2}\left(\frac{b_{d,s,f,t}h_{d,C}}{e^{h_{d,C,t}}}-\mu_{d,s,f}\right)^2-\frac{\varphi_{d,l,f}}{2}\left(\frac{b_{d,l,f,t}h_{d,C}}{e^{h_{d,C,t}}}-\mu_{d,l,f}\right)^2\end{array}\right)\right]\rightarrow\max_{B,C,L,M}, \tag{1}$$

$$\begin{array}{l}e^{c_{d,t}}+e^{m_{d,t}}+b_{d,s,d,t}e^{-r_{d,s,t}}+b_{d,l,d,t}e^{-r_{d,l,t}}+b_{d,s,f,t}e^{f_t-r_{f,s,t}}+b_{d,l,f,t}e^{f_t-r_{f,l,t}}=\\ +b_{d,s,d,t-1}e^{-p_{d,t}-\phi^y_t}+b_{d,l,d,t-1}e^{-p_{d,t}-\phi^y_t}\left(\left(1-s_d\right)e^{-r_{d,l,t}}+s_d\right)+e^{w_{d,t}+l_{d,t}}+\\ b_{d,s,f,t-1}e^{f_t-p_{f,t}-\phi^y_t}+b_{d,l,f,t-1}e^{f_t-p_{f,t}-\phi^y_t}\left(\left(1-s_f\right)e^{-r_{f,l,t}}+s_f\right)+e^{m_{d,t}-p_{d,t}-\phi^y_t}+d_{d,t}\end{array}. \tag{2}$$

The utility function consists of consumption, with part of consumption behavior attributed to habits in preferences, labor, and money in the utility. A friction similar to the portfolio adjustment costs *à la* Schmitt-Grohé and Uribe (2003) or price rigidity *à la* Rotemberg (1982a) is added into the utility function for each type of bond position.[2] This is required when two agents with different intertemporal preferences trade the same security, especially bonds.[3]

We use the following external consumption habits in the quadratic costs because the bond position should be expressed in terms of household consumption:[4]

$$e^{h_{d,C,t}}=h_{d,C}e^{c_{d,t-1}}. \tag{3}$$

Eq. 3 describes how consumption habits are formed and the parameter $h_{d,C}$ contributes to the rigidity.

The market consists of domestic and foreign one-period bonds and long-term bonds. Long-term bonds pay a share ($s_d$, $s_f$) of their nominal in each period. Thus, long-term bonds with a nominal of 1 euro produce $s_d$ euros in the first period, $s_d\left(1-s_d\right)$ in the second, $s_d\left(1-s_d\right)^2$ in the third, and so on.

[1]For instance, $c_{D,t}=\ln\left(C_{D,t}/Z_t\right)$ or $m_{D,t}=\ln\left(M_{D,t}/(Z_tP_{D,t})\right)$. The relation between the exchange rate and stationary exchange rate (real exchange rate) is $f_t=\ln\left(e_tP_{F,t}/P_{D,t}\right)$. Because the bond position could be negative, we do not take logs. See Section 2 for more details about these transformations.

[2]This is equivalent to a credit borrowing constraint.

[3]Otherwise, agents could find an optimal solution in having a minus infimum position that is unrealistic. Alternatively, restricting the negative values could modulate the rigidity. However, while simple quadratic costs produce a smoothed restriction, this would be more complicated.

[4]The use of consumption habits allows us simplify the optimization with respect to consumption.

The first-order conditions of domestic households are

$$-e^{\phi^M_{d,t}}\left(e^{m_{d,t}}\right)^{-1/\sigma_{d,M}}e^{m_{d,t}}-\lambda_{d,t}e^{m_{d,t}}+\lambda_{d,t+1}e^{\phi^u_{d,t}+m_{d,t}-p_{d,t+1}-\phi^y_{t+1}}=0, \tag{4}$$

$$-e^{\phi^L_{d,t}}\left(e^{l_{d,t}}\right)^{1/\sigma_{d,L}}e^{l_{d,t}}+\lambda_{d,t}e^{l_{d,t}+w_{d,t}}=0, \tag{5}$$

$$\left(e^{c_{d,t}}-h_{d,C}e^{c_{d,t-1}}\right)^{-1/\sigma_{d,C}}e^{c_{d,t}}-\lambda_{d,t}e^{c_{d,t}}=0, \tag{6}$$

$$-\varphi_{d,s,d}\left(b_{d,s,d,t}e^{-c_{d,t-1}}-\mu_{d,s,d}\right)e^{-c_{d,t-1}}-\lambda_{d,t}e^{-r_{d,s,t}}+\lambda_{d,t+1}e^{\phi^u_{d,t}-p_{d,t+1}-\phi^y_{t+1}}=0, \tag{7}$$

$$\begin{array}{l}-\varphi_{d,l,d}\left(b_{d,l,d,t}e^{-c_{d,t-1}}-\mu_{d,l,d}\right)e^{-c_{d,t-1}}-\lambda_{d,t}e^{-r_{d,L,t}}+\\+\lambda_{d,t+1}e^{\phi^u_{d,t}-p_{d,t+1}-\phi^y_{t+1}}\left(\left(1-s_d\right)e^{-r_{d,L,t+1}}+s_d\right)=0\end{array}, \tag{8}$$

$$-\varphi_{d,s,f}\left(b_{d,s,f,t}e^{-c_{d,t-1}}-\mu_{d,s,f}\right)e^{-c_{d,t-1}}-\lambda_{d,t}e^{f_t-r_{f,t}}+\lambda_{d,t+1}e^{\phi^u_{d,t}+f_{t+1}-p_{f,t+1}-\phi^y_{t+1}}=0, \tag{9}$$

$$\begin{array}{l}-\varphi_{d,l,f}\left(b_{d,l,f,t}e^{-c_{d,t-1}}-\mu_{d,l,f}\right)e^{-c_{d,t-1}}-\lambda_{d,t}e^{f_t-r_{f,l,t}}+\\+\lambda_{d,t+1}e^{\phi^u_{d,t}+f_{t+1}-p_{f,t+1}-\phi^y_{t+1}}\left(\left(1-s_f\right)e^{-r_{f,l,t+1}}+s_f\right)=0\end{array}. \tag{10}$$

These first-order conditions (Eq. 4 to Eq. 10), which are easier to compute in stationary variables, can be rewritten in nonstationary variables (Eq. 11 to Eq. 17), which are more readable, as follows:

$$-\varepsilon^M_{d,t}\left(\frac{M_{d,t}}{Z_tP_{d,t}}\right)^{-1/\sigma_{d,M}}\frac{M_{d,t}}{Z_tP_{d,t}}-\lambda_{d,t}\frac{M_{d,t}}{Z_tP_{d,t}}+\frac{\lambda_{d,t+1}Z_tP_{d,t}}{Z_{t+1}P_{d,t+1}}\frac{\varepsilon^u_{d,t}}{\varepsilon^u_{d,t-1}}\frac{M_{d,t}}{Z_tP_{d,t}}=0, \tag{11}$$

$$-\varepsilon^L_{d,t}\left(L_{d,t}\right)^{1/\sigma_{d,L}}L_{d,t}+\lambda_{d,t}L_{d,t}W_{d,t}=0, \tag{12}$$

$$\left(\frac{C_{d,t}}{Z_t}-h_{d,C}\frac{C_{d,t-1}}{Z_{t-1}}\right)^{-1/\sigma_{d,C}}\frac{C_{d,t}}{Z_t}-\lambda_{d,t}\frac{C_{d,t}}{Z_t}=0, \tag{13}$$

$$-\varphi_{d,s,d}\left(\frac{B_{d,s,d,t}Z_{t-1}}{P_{d,t}C_{d,t-1}Z_t}-\mu_{d,s,d}\right)\frac{Z_t}{C_{d,t}}-\lambda_{d,t}\frac{1}{R_{d,s,t}}+\frac{\lambda_{d,t+1}Z_tP_{d,t}}{Z_{t+1}P_{d,t+1}}\frac{\varepsilon^u_{d,t}}{\varepsilon^u_{d,t-1}}=0, \tag{14}$$

$$\begin{array}{l}-\varphi_{d,l,d}\left(\frac{B_{d,l,d,t}Z_{t-1}}{P_{d,t}C_{d,t-1}Z_t}-\mu_{d,l,d}\right)\frac{Z_{t-1}}{C_{d,t-1}}-\lambda_{d,t}\frac{1}{R_{d,l,t}}+\\+\frac{\lambda_{d,t+1}Z_tP_{d,t}}{Z_{t+1}P_{d,t+1}}\frac{\varepsilon^u_{d,t}}{\varepsilon^u_{d,t-1}}\left(\left(1-s_d\right)\frac{1}{R_{d,l,t+1}}+s_d\right)=0\end{array}, \tag{15}$$

$$-\varphi_{d,s,f}\left(\frac{B_{d,s,f,t}}{P_{f,t}Z_t}\frac{Z_{t-1}}{C_{d,t-1}}-\mu_{d,s,f}\right)\frac{Z_{t-1}}{C_{d,t-1}}-\lambda_{d,t}\frac{e_{d,t}P_{f,t}}{P_{d,t}R_{f,s,t}}+\frac{\lambda_{d,t+1}Z_tP_{f,t}}{Z_{t+1}P_{f,t+1}}\frac{e_{d,t+1}P_{f,t+1}}{P_{d,t+1}}\frac{\varepsilon^u_{d,t}}{\varepsilon^u_{d,t-1}}=0, \tag{16}$$

$$\begin{array}{l}-\varphi_{d,l,f}\left(\frac{B_{d,l,f,t}}{P_{f,t}Z_t}\frac{Z_{t-1}}{C_{d,t-1}}-\mu_{d,l,f}\right)\frac{Z_{t-1}}{C_{d,t-1}}-\lambda_{d,t}\frac{e_{d,t}P_{f,t}}{P_{d,t}R_{f,l,t}}+\\+\frac{\lambda_{d,t+1}Z_tP_{f,t}}{Z_{t+1}P_{f,t+1}}\frac{e_{d,t+1}P_{f,t+1}}{P_{d,t+1}}\frac{\varepsilon^u_{d,t}}{\varepsilon^u_{d,t-1}}\lambda_{d,t+1}\left(\left(1-s_f\right)\frac{1}{R_{f,l,t+1}}+s_f\right)=0\end{array}. \tag{17}$$

## 1.2 Foreign households

The foreign household's problem, corresponding to Eq. 1 to Eq. 4 in the paper, is symmetric:

$$E_t\left[\sum_{t=0}^{\infty}\varepsilon^u_{f,s-1}\left(\begin{array}{l}\frac{\left(e^{c_{f,t}}-e^{h_{f,C,t}}\right)^{1-1/\sigma_{f,C}}}{1-1/\sigma_{f,C}}-e^{\phi^L_{f,t}}\frac{\left(e^{l_{f,t}}\right)^{1+1/\sigma_{f,L}}}{1+1/\sigma_{f,L}}+e^{\phi^M_{f,t}}\frac{\left(e^{m_{f,t}}\right)^{1-1/\sigma_{f,M}}}{1-1/\sigma_{f,M}}\\-\frac{\varphi_{f,s,f}}{2}\left(\frac{b_{f,s,f,t}h_{f,C}}{e^{h_{f,C,t}}}-\mu_{f,s,f}\right)^2-\frac{\varphi_{f,l,f}}{2}\left(\frac{b_{f,l,f,t}h_{f,C}}{e^{h_{f,C,t}}}-\mu_{f,l,f}\right)^2\\-\frac{\varphi_{f,s,d}}{2}\left(\frac{b_{f,s,d,t}h_{f,C}}{e^{h_{f,C,t}}}-\mu_{f,s,d}\right)^2-\frac{\varphi_{f,l,d}}{2}\left(\frac{b_{f,l,d,t}h_{f,C}}{e^{h_{f,C,t}}}-\mu_{f,l,d}\right)^2\end{array}\right)\right]\rightarrow\max_{B,C,L,M}, \tag{18}$$

$$\begin{array}{l} e^{c_{f,t}} + e^{m_{f,t}} + b_{f,s,f,t}e^{-r_{f,s,t}} + b_{f,l,f,t}e^{-r_{f,l,t}} + b_{f,s,d,t}e^{-f_t - r_{d,s,t}} + b_{f,l,d,t}e^{-f_t - r_{d,l,t}} = \\ +b_{f,s,f,t-1}e^{-p_{f,t} - \phi_t^y} + b_{f,l,f,t-1}e^{-p_{f,t} - \phi_t^y}\left(\left(1 - s_f\right)e^{-r_{f,l,t}} + s_f\right) + e^{m_{f,t} - p_{f,t} - \phi_t^y} + \\ b_{f,s,d,t-1}e^{-f_t - p_{d,t} - \phi_t^y} + b_{f,l,d,t-1}e^{-f_t - p_{d,t} - \phi_t^y}\left(\left(1 - s_d\right)e^{-r_{d,l,t}} + s_d\right) + e^{w_{f,t} + l_{f,t}} + d_{f,t} \end{array} . \quad (19)$$

The main difference between these two optimization problems, Eq. 1 to Eq. 2 and 18 to Eq. 19, resides in the use of the exchange rate variable.

### 1.3 Domestic firms

The firm's problem with a linear production function and price adjustment costs as in Rotemberg (1982a,b) corresponds to Eq. 5 to Eq. 8 in the paper and can be rewritten as follows.

The firm maximizes the following expression:

$$\frac{1}{P_{d,t}Z_t}E_t\left[\sum_{s=0}^{\infty}\left(\prod_{k=0}^{s-1}R_{d,s,t+k}\right)^{-1}\left(D_{d,t+s} - \varphi_{d,P}\left(\frac{P_{F,d,t}\left(j\right)}{P_{F,d,t-1}\left(j\right)e^{\overline{p_d}v+(1-v)p_{d,t-1}}} - 1\right)^2 P_{d,t+s}Y_{d,t+s}\right)\right] \quad (20)$$

with respect to $D$, $P$, $Y$, and $L$, and the following constraints

$$D_{d,t} + W_tL_t\left(j\right) = P_{F,d,t}\left(j\right)Y_{F,d,t}\left(j\right), \quad (21)$$

$$Y_{F,d,t}\left(j\right) = \left(\frac{P_{F,d,t}\left(j\right)}{P_{d,t}}\right)^{-\varepsilon_{d,t}^{\theta}}\omega_d Y_{d,t} + \left(\frac{P_{F,d,t}\left(j\right)}{F_tP_{f,t}}\right)^{-\varepsilon_{f,t}^{\theta}}(1-\omega_f)Y_{f,t}, \quad (22)$$

$$Y_{F,d,t}\left(j\right) = A_dZ_tL_{d,t}\left(j\right). \quad (23)$$

This problem (Eq. 20 to Eq. 23) can be rewritten in terms of stationary variables (Eq. 24 to Eq. 27):

$$E_t\left[\begin{array}{c} \sum_{s=0}^{\infty} e^{-\sum_{j=1}^{s}\left(r_{d,s,t+j-1} - \phi_{t+j}^y - p_{d,t+j}\right)} \\ \left(d_{d,t+s} - \varphi_{d,P}\left(e^{p_{F,d,t}(j) - p_{F,d,t-1}(j) + p_{d,t} - (\overline{p_d}v + (1-v)p_{d,t-1})} - 1\right)^2 e^{y_{d,t+s}}\right) \end{array}\right], \quad (24)$$

$$d_{d,t} + e^{w_t + l_t(j)} = e^{p_{F,d,t}(j) + y_{F,d,t}(j)}, \quad (25)$$

$$e^{y_{F,d,t}(j)} = e^{-\phi_{d,t}^{\theta}p_{F,d,t}(j) + y_{d,t}}\omega_d + (1-\omega_F)e^{-\phi_{f,t}^{\theta}(p_{F,d,t}(j) - f_t) + y_{f,t}}, \quad (26)$$

$$e^{y_{F,d,t}(j)} = e^{a_d + l_{d,t}(j)}. \quad (27)$$

The production function (Eq. 23) includes a unit root technology process $Z_t$ and a country-specific total factor productivity parameter. Firms sell their products in domestic and foreign markets at the same price. The demand function (Eq. 26) results from the usual CES aggregation.

## 1.4 Domestic firms

After simplification, firms' first-order condition is

$$
\begin{aligned}
& e^{p_{F,d,t}(j)} \left( e^{-\phi^{\theta}_{d,t} p_{F,d,t}(j) + y_{d,t}} \omega_d + (1-\omega_f) e^{-\phi^{\theta}_{f,t}(p_{F,d,t}(j) - f_t) + y_{f,t}} \right) + \\
& + \left( e^{p_{F,d,t}(j)} - e^{w_{d,t} - a_d} \right) \left( -\phi^{\theta}_{d,t} e^{-\phi^{\theta}_{d,t} p_{F,d,t}(j) + y_{d,t}} \omega_d - \phi^{\theta}_{f,t} (1-\omega_f) e^{-\phi^{\theta}_{f,t}(p_{F,d,t}(j) - f_t) + y_{f,t}} \right) - \\
& - \varphi_{d,P} \left( e^{p_{F,d,t}(j) - p_{F,d,t-1}(j) + p_{d,t} - (\overline{p_d} v + (1-v) p_{d,t-1})} - 1 \right) e^{y_{d,t} + p_{F,d,t}(j)} + \\
& + \varphi_{d,P} \left( e^{p_{F,d,t+1}(j) - p_{F,d,t}(j) + p_{d,t+1} - (\overline{p_d} v + (1-v) p_{d,t})} - 1 \right) e^{y_{d,t+1} + p_{F,d,t}(j) - \left( r_{d,s,t} - \phi^{y}_{t+1} - p_{d,t+1} \right)} = 0
\end{aligned}, \tag{28}
$$

which can be rewritten in terms of nonstationary variables as

$$
\begin{aligned}
& \frac{P_{F,d,t}(j)}{P_{d,t}} \left( \left( \frac{P_{F,d,t}(j)}{P_{d,t}} \right)^{-\varepsilon^{\theta}_{d,t}} Y_{d,t} \omega_d + (1-\omega_f) \left( \frac{P_{F,d,t}(j) P_{d,t}}{e_{d,t} P_{d,t} P_{f,t}} \right)^{-\varepsilon^{\theta}_{f,t}(p_{F,d,t}(j) - f_t)} Y_{f,t} \right) + \\
& + \left( \frac{P_{F,d,t}(j)}{P_{d,t}} - \frac{w_{d,t}}{A_d} \right) \left( -\varepsilon^{\theta}_{d,t} \left( \frac{P_{F,d,t}(j)}{P_{d,t}} \right)^{-\varepsilon^{\theta}_{d,t}} Y_{d,t} \omega_d - \varepsilon^{\theta}_{f,t} (1-\omega_f) \left( \frac{P_{F,d,t}(j) P_{d,t}}{e_{d,t} P_{d,t} P_{f,t}} \right)^{-\varepsilon^{\theta}_{f,t}\left(p_{F,d,t}(j) - f_t\right)} Y_{f,t} \right) - \\
& - \varphi_{d,P} \left( \frac{P_{F,d,t}(j)}{P_{F,d,t-1}(j)} e^{\cdot - \overline{p_d} v} \left( \frac{P_{d,t-1}}{P_{d,t-2}} \right)^{\cdot - (1-v)} - 1 \right) Y_{d,t} \frac{P_{F,d,t}(j)}{P_{d,t}} + \\
& + \varphi_{d,P} \left( \frac{P_{F,d,t+1}(j)}{P_{F,d,t}(j)} e^{\cdot - \overline{p_d} v} \left( \frac{P_{d,t}}{P_{d,t-1}} \right)^{\cdot - (1-v)} - 1 \right) \frac{Y_{d,t+1} Z_{t+1} P_{d,t+1}}{R_{d,s,t} Z_t P_{d,t}} \frac{P_{F,d,t}(j)}{P_{d,t}} = 0
\end{aligned}. \tag{29}
$$

## 1.5 Foreign firms

The foreign firm's problem consists of maximizing

$$
\frac{1}{P_{f,t} Z_t} E_t \left[ \sum_{s=0}^{\infty} \left( \prod_{k=0}^{s-1} R_{f,s,t+k} \right)^{-1} \left( D_{f,t+s} - \varphi_{f,P} \left( \frac{P_{F,f,t}(j)}{P_{F,f,t-1}(j) \, e^{\overline{p_f} v + (1-v) p_{f,t-1}}} - 1 \right)^2 P_{f,t+s} Y_{f,t+s} \right) \right] \tag{30}
$$

with respect to $D$, $P$, $Y$, and $L$, and the following constraints:

$$
D_{f,t+s} + W_{f,t+s} L_{f,t+s}(j) = P_{F,f,t}(j) \, Y_{F,f,t+s}(j), \tag{31}
$$

$$
Y_{F,f,t}(j) = \left( \frac{P_{F,f,t}(j)}{P_{f,t}} \right)^{-z_{f,\theta,t}} \omega_f Y_{f,t} + \left( \frac{F_t P_{F,f,t}(j)}{P_t} \right)^{-z_{d,\theta,t}} (1-\omega_d) Y_{d,t}, \tag{32}
$$

$$
Y_{F,f,t}(j) = Z_t A_f L_{f,t}(j). \tag{33}
$$

The foreign firm's problem (Eq. 30 to Eq. 33) can be rewritten in terms of variables, similar to domestic firms, as follows:

$$
E_t \left[ \sum_{s=0}^{\infty} e^{-\sum_{j=1}^{s} \left( r_{f,s,t+j-1} - \phi^{y}_{t+j} - p_{f,t+j} \right)} \left( d_{f,t+s} - \varphi_{f,P} \left( e^{p_{F,f,t}(j) - P_{F,f,t-1}(j) + p_{f,t} - (\overline{p_f} v + (1-v) p_{f,t-1})} - 1 \right)^2 e^{y_{f,t+s}} \right) \right], \tag{34}
$$

$$
d_{f,t+s} + e^{w_{f,t+s} + l_{f,t+s}(j)} = e^{p_{F,f,t}(j) + y_{F,f,t+s}(j)}, \tag{35}
$$

$$
e^{y_{F,f,t}(j)} = e^{-\phi^{\theta}_{f,t} p_{F,f,t}(j) + y_{f,t}} \omega_f + (1-\omega_d) e^{-\phi^{\theta}_{d,t}(p_{F,f,t}(j) + f_t) + y_{d,t}}, \tag{36}
$$

$$
e^{y_{F,f,t}(j)} = e^{a_f + l_{f,t}(j)}. \tag{37}
$$

## 1.6 Governments

Central banks follow a Taylor-type rule, and their budget constraints (Eq. 10 and Eq. 11 in the paper) can be rewritten as stationary variables:

$$r_{d,s,t} = \gamma_{d,R}\left(r_{d,s,t-1}\right) + \left(1-\gamma_{d,R}\right)\left(\gamma_{d,P}\left(p_{d,t}-\overline{p}\right) + \gamma_{d,Y}\left(y_{d,t}-\overline{y_d}\right) + \gamma_{d,f}\left(f_t - \overline{f}\right)\right) + \phi^R_{d,t}, \tag{38}$$

$$b_{d,G,t}e^{-r_{d,s,t}} = b_{d,G,t-1}e^{-p_{d,t}-\phi^y_t} + \left(e^{m_{d,t}} - e^{m_{d,t-1}-p_{d,t}-\phi^y_t}\right), \tag{39}$$

$$r_{f,s,t} = \gamma_{f,R}\left(r_{f,s,t-1}\right) + \left(1-\gamma_{f,R}\right)\left(\gamma_{f,P}\left(p_{f,t}-\overline{p}\right) + \gamma_{f,Y}\left(y_{f,t}-\overline{y_f}\right) + \gamma_{f,f}\left(f_t - \overline{f}\right)\right) + \phi^R_{f,t}, \tag{40}$$

$$b_{f,G,t}e^{-r_{f,s,t}} = b_{f,G,t-1}e^{-p_{f,t}-\phi^y_t} + e^{m_{f,t}} - e^{m_{f,t-1}-p_{f,t}-\phi^y_t}. \tag{41}$$

## 1.7 Equilibrium

Domestic demand, corresponding to Eq. 12 in the paper, consists of consumption only:

$$y_{d,t} = c_{d,t}, \tag{42}$$

$$y_{f,t} = c_{f,t}. \tag{43}$$

The aggregate price level, corresponding to Eq. 9 in the paper, consists of the usual CES aggregation:

$$1 = e^{p_{F,d,t}(1-\phi^\theta_{d,t})}\left(\omega_d\right) + e^{(p_{F,f,t}+f_t)(1-\phi^\theta_{d,t})}\left(1-\omega_d\right), \tag{44}$$

$$1 = e^{p_{F,f,t}(1-\phi^\theta_{f,t})}\left(\omega_f\right) + e^{(p_{F,d,t}-f_t)(1-\phi^\theta_{f,t})}\left(1-\omega_f\right). \tag{45}$$

Each bond should be bought by someone (Eq. 13 and Eq. 14 in the paper), which can be rewritten as stationary variables as follows:

$$b_{d,s,d,t} + b_{f,s,d,t} + b_{d,G,t} = 0, \tag{46}$$

$$b_{d,s,f,t} + b_{f,s,f,t} + b_{f,G,t} = 0, \tag{47}$$

$$b_{d,l,d,t} + b_{f,l,d,t} = 0, \tag{48}$$

$$b_{d,l,f,t} + b_{f,l,f,t} = 0. \tag{49}$$

## 2 Summary of the variables

| Variable | Description | Stationary variable |
|---|---|---|
| $B_{d,s,d,t}$ | Domestic bonds bought by domestic households | $b_{d,s,d,t} = B_{d,s,d,t}/P_{d,t}Z_t$ |
| $B_{d,s,f,t}$ | Foreign bonds bought by domestic households | $b_{d,s,f,t} = B_{d,s,f,t}/P_{f,t}Z_t$ |
| $B_{d,G,t}$ | Domestic bonds bought by the domestic central bank | $b_{d,G,t} = B_{d,G,t}/P_{d,t}Z_t$ |
| $B_{f,s,d,t}$ | Domestic bonds bought by foreign households | $b_{f,s,d,t} = B_{f,s,d,t}/P_{d,t}Z_t$ |
| $B_{f,s,f,t}$ | Foreign bonds bought by foreign households | $b_{f,s,f,t} = B_{f,s,f,t}/P_{f,t}Z_t$ |
| $B_{f,G,t}$ | Domestic bonds bought by the foreign central bank | $b_{f,G,t} = B_{f,G,t}/P_{f,t}Z_t$ |
| $B_{d,l,d,t}$ | Domestic long-term bonds bought by domestic households | $b_{d,l,d,t} = B_{d,l,d,t}/P_{d,t}Z_t$ |
| $B_{f,l,d,t}$ | Domestic long-term bonds bought by foreign households | $b_{f,l,d,t} = B_{f,l,d,t}/P_{d,t}Z_t$ |
| $B_{d,l,f,t}$ | Foreign long-term bonds bought by domestic households | $b_{d,l,f,t} = B_{d,l,f,t}/P_{f,t}Z_t$ |
| $B_{f,l,f,t}$ | Foreign long-term bonds bought by foreign households | $b_{f,l,f,t} = B_{f,l,f,t}/P_{f,t}Z_t$ |
| $C_{d,t}$ | Consumption of domestic households | $c_{d,t} = \ln(C_{d,t}/Z_t)$ |
| $C_{f,t}$ | Consumption of foreign households | $c_{f,t} = \ln(C_{f,t}/Z_t)$ |
| $D_{d,t}$ | Dividends of domestic firms | $d_{d,t} = D_{d,t}/P_{d,t}Z_t$ |
| $D_{f,t}$ | Dividends of foreign firms | $d_{f,t} = D_{f,t}/P_{f,t}Z_t$ |
| $e_{D,t}$ | Exchange rate (domestic currency per unit of the foreign currency) | $f_t = \ln(e_{d,t}P_{f,t}/P_{D,t})$ |
| $L_{d,t}$ | Domestic labor | $l_{d,t} = \ln(L_{d,t})$ |
| $L_{f,t}$ | Foreign labor | $l_{f,t} = \ln(L_{f,t})$ |
| $M_{d,t}$ | Money stock held by domestic households | $m_{d,t} = \ln(M_{d,t}/P_{d,t}Z_t)$ |
| $M_{d,t}$ | Money stock held by domestic households | $m_{d,t} = \ln(M_{d,t}/P_{d,t}Z_t)$ |
| $P_{d,t}$ | Aggregate price level in the domestic market | $p_{d,t} = \ln(P_{d,t}/P_{d,t-1})$ |
| $P_{f,t}$ | Aggregate price level in the foreign market | $p_{f,t} = \ln(P_{f,t}/P_{f,t-1})$ |
| $P_{F,d,t}(j)$ | Price of goods of domestic firm $j$ | $p_{F,d,t}(j) = \ln(P_{F,d,t}(j)/P_{d,t})$ |
| $P_{F,f,t}(j)$ | Price of goods of foreign firm $j$ | $p_{F,f,t}(j) = \ln(P_{F,f,t}(j)/P_{f,t})$ |
| $R_{d,s,t}$ | Domestic interest rate | $r_{d,s,t} = \ln(R_{d,s,t})$ |
| $R_{f,s,t}$ | Foreign interest rate | $r_{f,s,t} = \ln(R_{f,s,t})$ |
| $R_{d,l,t}$ | Domestic long-term interest rate | $r_{d,l,t} = \ln(R_{d,l,t})$ |
| $R_{f,l,t}$ | Foreign long-term interest rate | $r_{f,l,t} = \ln(R_{f,l,t})$ |
| $W_{d,t}$ | Domestic wage | $w_{d,t} = \ln(W_{d,t}/P_{d,t}Z_t)$ |
| $W_{f,t}$ | Foreign wage | $w_{f,t} = \ln(W_{f,t}/P_{f,t}Z_t)$ |
| $Y_{d,t}$ | Domestic demand | $y_{d,t} = \ln(Y_{d,t}/Z_t)$ |
| $Y_{f,t}$ | Foreign demand | $y_{f,t} = \ln(Y_{f,t}/Z_t)$ |
| $Y_{F,d,t}$ | Domestic GDP (production of domestic firms) | $y_{F,d,t} = \ln(Y_{F,d,t}/Z_t)$ |
| $Y_{F,f,t}$ | Foreign GDP (production of foreign firms) | $y_{F,f,t} = \ln(Y_{F,f,t}/Z_t)$ |
| $\lambda_{d,t}$ | Domestic household Lagrange multiplier | $\lambda_{d,t} = \lambda_{d,t}$ |
| $\lambda_{f,t}$ | Foreign household Lagrange multiplier | $\lambda_{f,t} = \lambda_{f,t}$ |

Table 1: Description of the variables and corresponding stationary expressions.

| | | |
|---|---|---|
| $\varepsilon^u_{d,t}$ | Domestic households' intertemporal preference shock | $\phi^u_{d,t} = \ln\left(\varepsilon^u_{d,t}/\varepsilon^u_{d,t-1}\right)$ |
| $\varepsilon^u_{f,t}$ | Foreign households' intertemporal preference shock | $\phi^u_{f,t} = \ln\left(\varepsilon^u_{f,t}/\varepsilon^u_{f,t-1}\right)$ |
| $\varepsilon^L_{d,t}$ | Domestic households' labor supply shock | $\phi^L_{d,t} = \ln\left(\varepsilon^L_{d,t}\right)$ |
| $\varepsilon^L_{f,t}$ | Foreign households' labor supply shock | $\phi^L_{f,t} = \ln\left(\varepsilon^L_{f,t}\right)$ |
| $\varepsilon^M_{d,t}$ | Domestic households' liquidity preference shock | $\phi^M_{d,t} = \ln\left(\varepsilon^M_{d,t}\right)$ |
| $\varepsilon^M_{f,t}$ | Foreign households' liquidity preference shock | $\phi^M_{f,t} = \ln\left(\varepsilon^M_{f,t}\right)$ |
| $\varepsilon^R_{d,t}$ | Domestic monetary policy shock | $\phi^R_{d,t} = \varepsilon^R_{d,t}$ |
| $\varepsilon^R_{f,t}$ | Foreign monetary policy shock | $\phi^R_{f,t} = \varepsilon^R_{f,t}$ |
| $\varepsilon^\theta_{d,t}$ | Domestic demand elasticity shock | $\phi^\theta_{d,t} = \varepsilon^\theta_{d,t}$ |
| $\varepsilon^\theta_{f,t}$ | Foreign demand elasticity shock | $\phi^\theta_{f,t} = \varepsilon^\theta_{f,t}$ |
| $\varepsilon_t$ | Technological progress shock | $\phi^y_t = \ln\left(\varepsilon_t/\varepsilon_{t-1}\right)$ |

Table 2: Description of the shock variables and corresponding stationary expressions.

# 3 Summary of the parameters

1. Parameters ($\overline{\eta}_{i,j}$; $\eta_{i,j}$; std of $\xi^j_{i,t}$) for each exogenous process $\phi^j_{i,t}$.

2. Monetary policy rule's parameters: $\gamma_{d,R}$; $\gamma_{d,P}$; $\gamma_{d,Y}$; $\gamma_{d,f}$; $\gamma_{f,R}$; $\gamma_{f,P}$; $\gamma_{f,Y}$; $\gamma_{f,f}$.

3. Bonds' rigidity's parameters: $\varphi_{d,s,d}$; $\varphi_{d,s,f}$; $\varphi_{d,l,d}$; $\varphi_{d,s,f}$; $\varphi_{f,s,d}$; $\varphi_{f,s,f}$; $\varphi_{f,l,d}$; $\varphi_{f,l,f}$; $\mu_{d,s,d}$; $\mu_{d,s,f}$; $\mu_{d,l,d}$; $\mu_{d,l,f}$; $\mu_{f,s,d}$; $\mu_{f,s,f}$; $\mu_{f,l,d}$; $\mu_{f,s,f}$;

4. Price rigidity's parameters: $\mu_{d,P}$; $\mu_{f,P}$.

5. Habits' parameters: $h_{d,C}$; $h_{f,C}$.

6. Long-term bonds' parameters: $s_d$; $s_f$.

7. Indexation parameters: $v_d$; $v_f$.

8. Demand structure preferences' parameters: $\omega_D$; $\omega_f$.

# 4 Distance correlations

Table 3 to Table 5 present the distance correlations (Székely et al., 2007) of the models' variables. Distance correlation allows us to analyze linear and nonlinear dependency between variables, whereas Pearson correlation (standard coefficient of the correlation) is sensitive only to the linear relationships between two variables. Because we use nonlinear models, we must assess both the linear and the nonlinear relationships between the variables.

The domestic bonds bought by the central bank ($b_{d,g,t}$) are highly correlated with the short-run domestic bonds bought by domestic households in the local currency ($b_{d,d,sr,t}$). In addition, these bonds are highly correlated with domestic consumption ($c_{d,t}$). The quantity of bonds bought by households denotes the ability of households to intervene in markets, denoted by their increasing ability to consume. By buying domestic bonds, the central bank intervention plays a substantial role in this increasing ability of households.

Comparing Table 3 with Table 4, consider that an SVS to technological progress substantially decreases the distance correlation between foreign wages ($w_{f,t}$) and the foreign bonds bought by the central bank ($b_{f,g,t}$) as well as the distance correlation between foreign wages and foreign money stock ($m_{f,t}$).

Table 3 to Table 5 show that the distance correlation between the foreign bonds bought by the central bank ($b_{f,g,t}$) and the short-run domestic bonds bought by domestic households in the foreign currency ($b_{d,f,sr,t}$) increases in the 3SVS model compared with in the other models.

The model without SVSs displays more comovement than the other models.

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| | $b_{d,d,sr,t}$ | $b_{d,f,sr,t}$ | $b_{d,g,t}$ | $b_{d,d,lr,t}$ | $b_{d,f,lr,t}$ | $b_{f,g,t}$ | $c_{d,t}$ | $c_{f,t}$ | $e_t$ | $m_{d,t}$ | $m_{f,t}$ | $p_{d,t}$ | $p_{f,t}$ | $r_{d,t}$ | $r_{f,t}$ | $r_{d,lr,t}$ | $r_{f,lr,t}$ | $w_{d,t}$ | $w_{f,t}$ | $y_{d,t}(j)$ | $y_{f,t}(j)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $b_{d,d,sr,t}$ | 1,00 | 0,60 | 0,97 | 0,47 | 0,67 | 0,50 | 0,90 | 0,39 | 0,40 | 0,52 | 0,46 | 0,52 | 0,23 | 0,94 | 0,80 | 0,71 | 0,74 | 0,45 | 0,53 | 0,89 | 0,35 |
| $b_{d,f,sr,t}$ | 0,60 | 1,00 | 0,61 | 0,19 | 0,81 | 0,79 | 0,51 | 0,50 | 0,58 | 0,81 | 0,76 | 0,36 | 0,26 | 0,57 | 0,55 | 0,50 | 0,74 | 0,36 | 0,45 | 0,52 | 0,53 |
| $b_{d,g,t}$ | 0,97 | 0,61 | 1,00 | 0,38 | 0,75 | 0,52 | 0,85 | 0,45 | 0,40 | 0,54 | 0,52 | 0,49 | 0,23 | 0,95 | 0,80 | 0,68 | 0,79 | 0,42 | 0,51 | 0,84 | 0,41 |
| $b_{d,d,lr,t}$ | 0,47 | 0,19 | 0,38 | 1,00 | 0,30 | 0,23 | 0,35 | 0,30 | 0,16 | 0,19 | 0,27 | 0,26 | 0,15 | 0,30 | 0,20 | 0,88 | 0,26 | 0,65 | 0,85 | 0,27 | 0,26 |
| $b_{d,f,lr,t}$ | 0,67 | 0,81 | 0,75 | 0,30 | 1,00 | 0,90 | 0,56 | 0,47 | 0,29 | 0,57 | 0,92 | 0,43 | 0,24 | 0,63 | 0,58 | 0,52 | 0,78 | 0,44 | 0,58 | 0,56 | 0,44 |
| $b_{f,g,t}$ | 0,50 | 0,79 | 0,52 | 0,23 | 0,90 | 1,00 | 0,28 | 0,38 | 0,17 | 0,42 | 0,97 | 0,33 | 0,17 | 0,37 | 0,34 | 0,38 | 0,55 | 0,44 | 0,52 | 0,27 | 0,35 |
| $c_{d,t}$ | 0,90 | 0,51 | 0,85 | 0,35 | 0,56 | 0,28 | 1,00 | 0,33 | 0,37 | 0,52 | 0,22 | 0,56 | 0,30 | 0,88 | 0,75 | 0,60 | 0,70 | 0,27 | 0,33 | 0,98 | 0,33 |
| $c_{f,t}$ | 0,39 | 0,50 | 0,45 | 0,30 | 0,47 | 0,38 | 0,33 | 1,00 | 0,26 | 0,62 | 0,32 | 0,36 | 0,39 | 0,44 | 0,48 | 0,55 | 0,57 | 0,44 | 0,32 | 0,30 | 0,97 |
| $e_t$ | 0,40 | 0,58 | 0,40 | 0,16 | 0,29 | 0,17 | 0,37 | 0,26 | 1,00 | 0,57 | 0,21 | 0,22 | 0,16 | 0,36 | 0,27 | 0,36 | 0,38 | 0,20 | 0,31 | 0,41 | 0,26 |
| $m_{d,t}$ | 0,52 | 0,81 | 0,54 | 0,19 | 0,57 | 0,42 | 0,52 | 0,62 | 0,57 | 1,00 | 0,35 | 0,33 | 0,25 | 0,56 | 0,59 | 0,52 | 0,71 | 0,37 | 0,26 | 0,54 | 0,64 |
| $m_{f,t}$ | 0,46 | 0,76 | 0,52 | 0,27 | 0,92 | 0,97 | 0,22 | 0,32 | 0,21 | 0,35 | 1,00 | 0,34 | 0,18 | 0,30 | 0,21 | 0,34 | 0,54 | 0,43 | 0,56 | 0,24 | 0,30 |
| $p_{d,t}$ | 0,52 | 0,36 | 0,49 | 0,26 | 0,43 | 0,33 | 0,56 | 0,36 | 0,22 | 0,33 | 0,34 | 1,00 | 0,28 | 0,50 | 0,39 | 0,36 | 0,39 | 0,39 | 0,32 | 0,60 | 0,32 |
| $p_{f,t}$ | 0,23 | 0,26 | 0,23 | 0,15 | 0,24 | 0,17 | 0,30 | 0,39 | 0,16 | 0,25 | 0,18 | 0,28 | 1,00 | 0,27 | 0,23 | 0,20 | 0,26 | 0,19 | 0,45 | 0,28 | 0,33 |
| $r_{d,t}$ | 0,94 | 0,57 | 0,95 | 0,30 | 0,63 | 0,37 | 0,88 | 0,44 | 0,36 | 0,56 | 0,30 | 0,50 | 0,27 | 1,00 | 0,85 | 0,69 | 0,78 | 0,35 | 0,35 | 0,88 | 0,40 |
| $r_{f,t}$ | 0,80 | 0,55 | 0,80 | 0,20 | 0,58 | 0,34 | 0,75 | 0,48 | 0,27 | 0,59 | 0,21 | 0,39 | 0,23 | 0,85 | 1,00 | 0,55 | 0,86 | 0,44 | 0,21 | 0,74 | 0,42 |
| $r_{d,lr,t}$ | 0,71 | 0,50 | 0,68 | 0,88 | 0,52 | 0,38 | 0,60 | 0,55 | 0,36 | 0,52 | 0,34 | 0,36 | 0,20 | 0,69 | 0,55 | 1,00 | 0,62 | 0,64 | 0,76 | 0,55 | 0,53 |
| $r_{f,lr,t}$ | 0,74 | 0,74 | 0,79 | 0,26 | 0,78 | 0,55 | 0,70 | 0,57 | 0,38 | 0,71 | 0,54 | 0,39 | 0,26 | 0,78 | 0,86 | 0,62 | 1,00 | 0,42 | 0,38 | 0,69 | 0,55 |
| $w_{d,t}$ | 0,45 | 0,36 | 0,42 | 0,65 | 0,44 | 0,44 | 0,27 | 0,44 | 0,20 | 0,37 | 0,43 | 0,39 | 0,19 | 0,35 | 0,44 | 0,64 | 0,42 | 1,00 | 0,66 | 0,24 | 0,40 |
| $w_{f,t}$ | 0,53 | 0,45 | 0,51 | 0,85 | 0,58 | 0,52 | 0,33 | 0,32 | 0,31 | 0,26 | 0,56 | 0,32 | 0,45 | 0,35 | 0,21 | 0,76 | 0,38 | 0,66 | 1,00 | 0,27 | 0,26 |
| $y_{d,t}(j)$ | 0,89 | 0,52 | 0,84 | 0,27 | 0,56 | 0,27 | 0,98 | 0,30 | 0,41 | 0,54 | 0,24 | 0,60 | 0,28 | 0,88 | 0,74 | 0,55 | 0,69 | 0,24 | 0,27 | 1,00 | 0,28 |
| $y_{f,t}(j)$ | 0,35 | 0,53 | 0,41 | 0,26 | 0,44 | 0,35 | 0,33 | 0,97 | 0,26 | 0,64 | 0,30 | 0,32 | 0,33 | 0,40 | 0,42 | 0,53 | 0,55 | 0,40 | 0,26 | 0,28 | 1,00 |

Table 3: Distance correlations for the model without an SVS

| | $b_{d,d,sr,t}$ | $b_{d,f,sr,t}$ | $b_{d,g,t}$ | $b_{d,d,lr,t}$ | $b_{d,f,lr,t}$ | $b_{f,g,t}$ | $c_{d,t}$ | $c_{f,t}$ | $e_t$ | $m_{d,t}$ | $m_{f,t}$ | $p_{d,t}$ | $p_{f,t}$ | $r_{d,t}$ | $r_{f,t}$ | $r_{d,lr,t}$ | $r_{f,lr,t}$ | $w_{d,t}$ | $w_{f,t}$ | $y_{d,t}(j)$ | $y_{f,t}(j)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $b_{d,d,sr,t}$ | 1,00 | 0,60 | 0,96 | 0,50 | 0,70 | 0,55 | 0,89 | 0,37 | 0,40 | 0,51 | 0,52 | 0,54 | 0,22 | 0,92 | 0,77 | 0,72 | 0,72 | 0,51 | 0,49 | 0,88 | 0,33 |
| $b_{d,f,sr,t}$ | 0,60 | 1,00 | 0,59 | 0,14 | 0,88 | 0,88 | 0,46 | 0,47 | 0,52 | 0,78 | 0,85 | 0,36 | 0,24 | 0,52 | 0,50 | 0,48 | 0,68 | 0,45 | 0,29 | 0,47 | 0,50 |
| $b_{d,g,t}$ | 0,96 | 0,59 | 1,00 | 0,39 | 0,74 | 0,54 | 0,84 | 0,42 | 0,41 | 0,53 | 0,55 | 0,50 | 0,23 | 0,94 | 0,78 | 0,69 | 0,79 | 0,42 | 0,49 | 0,83 | 0,39 |
| $b_{d,d,lr,t}$ | 0,50 | 0,14 | 0,39 | 1,00 | 0,25 | 0,19 | 0,36 | 0,28 | 0,17 | 0,18 | 0,24 | 0,27 | 0,17 | 0,30 | 0,20 | 0,85 | 0,27 | 0,62 | 0,87 | 0,29 | 0,24 |
| $b_{d,f,lr,t}$ | 0,70 | 0,88 | 0,74 | 0,25 | 1,00 | 0,95 | 0,55 | 0,45 | 0,26 | 0,57 | 0,95 | 0,42 | 0,23 | 0,61 | 0,58 | 0,52 | 0,77 | 0,48 | 0,44 | 0,55 | 0,42 |
| $b_{f,g,t}$ | 0,55 | 0,88 | 0,54 | 0,19 | 0,95 | 1,00 | 0,26 | 0,37 | 0,16 | 0,43 | 0,97 | 0,33 | 0,17 | 0,36 | 0,33 | 0,39 | 0,56 | 0,50 | 0,34 | 0,26 | 0,34 |
| $c_{d,t}$ | 0,89 | 0,46 | 0,84 | 0,36 | 0,55 | 0,26 | 1,00 | 0,35 | 0,36 | 0,53 | 0,22 | 0,56 | 0,31 | 0,88 | 0,76 | 0,60 | 0,69 | 0,27 | 0,31 | 0,98 | 0,34 |
| $c_{f,t}$ | 0,37 | 0,47 | 0,42 | 0,28 | 0,45 | 0,37 | 0,35 | 1,00 | 0,27 | 0,62 | 0,32 | 0,35 | 0,39 | 0,43 | 0,47 | 0,56 | 0,57 | 0,41 | 0,32 | 0,32 | 0,98 |
| $e_t$ | 0,40 | 0,52 | 0,41 | 0,17 | 0,26 | 0,16 | 0,36 | 0,27 | 1,00 | 0,56 | 0,20 | 0,22 | 0,17 | 0,36 | 0,27 | 0,37 | 0,38 | 0,23 | 0,31 | 0,40 | 0,28 |
| $m_{d,t}$ | 0,51 | 0,78 | 0,53 | 0,18 | 0,57 | 0,43 | 0,53 | 0,62 | 0,56 | 1,00 | 0,37 | 0,33 | 0,25 | 0,57 | 0,60 | 0,53 | 0,72 | 0,37 | 0,22 | 0,55 | 0,64 |
| $m_{f,t}$ | 0,52 | 0,85 | 0,55 | 0,24 | 0,95 | 0,97 | 0,22 | 0,32 | 0,20 | 0,37 | 1,00 | 0,33 | 0,19 | 0,30 | 0,20 | 0,36 | 0,56 | 0,48 | 0,42 | 0,24 | 0,30 |
| $p_{d,t}$ | 0,54 | 0,36 | 0,50 | 0,27 | 0,42 | 0,33 | 0,56 | 0,35 | 0,22 | 0,33 | 0,33 | 1,00 | 0,29 | 0,50 | 0,39 | 0,36 | 0,39 | 0,43 | 0,29 | 0,59 | 0,31 |
| $p_{f,t}$ | 0,22 | 0,24 | 0,23 | 0,17 | 0,23 | 0,17 | 0,31 | 0,39 | 0,17 | 0,25 | 0,19 | 0,29 | 1,00 | 0,27 | 0,24 | 0,20 | 0,25 | 0,19 | 0,49 | 0,30 | 0,33 |
| $r_{d,t}$ | 0,92 | 0,52 | 0,94 | 0,30 | 0,61 | 0,36 | 0,88 | 0,43 | 0,36 | 0,57 | 0,30 | 0,50 | 0,27 | 1,00 | 0,85 | 0,70 | 0,77 | 0,35 | 0,33 | 0,87 | 0,40 |
| $r_{f,t}$ | 0,77 | 0,50 | 0,78 | 0,20 | 0,58 | 0,33 | 0,76 | 0,47 | 0,27 | 0,60 | 0,20 | 0,39 | 0,24 | 0,85 | 1,00 | 0,56 | 0,85 | 0,45 | 0,19 | 0,75 | 0,41 |
| $r_{d,lr,t}$ | 0,72 | 0,48 | 0,69 | 0,85 | 0,52 | 0,39 | 0,60 | 0,56 | 0,37 | 0,53 | 0,36 | 0,36 | 0,20 | 0,70 | 0,56 | 1,00 | 0,65 | 0,61 | 0,75 | 0,54 | 0,54 |
| $r_{f,lr,t}$ | 0,72 | 0,68 | 0,79 | 0,27 | 0,77 | 0,56 | 0,69 | 0,57 | 0,38 | 0,72 | 0,56 | 0,39 | 0,25 | 0,77 | 0,85 | 0,65 | 1,00 | 0,43 | 0,39 | 0,67 | 0,54 |
| $w_{d,t}$ | 0,51 | 0,45 | 0,42 | 0,62 | 0,48 | 0,50 | 0,27 | 0,41 | 0,23 | 0,37 | 0,48 | 0,43 | 0,19 | 0,35 | 0,45 | 0,61 | 0,43 | 1,00 | 0,59 | 0,24 | 0,36 |
| $w_{f,t}$ | 0,49 | 0,29 | 0,49 | 0,87 | 0,44 | 0,34 | 0,31 | 0,32 | 0,31 | 0,22 | 0,42 | 0,29 | 0,49 | 0,33 | 0,19 | 0,75 | 0,39 | 0,59 | 1,00 | 0,25 | 0,26 |
| $y_{d,t}(j)$ | 0,88 | 0,47 | 0,83 | 0,29 | 0,55 | 0,26 | 0,98 | 0,32 | 0,40 | 0,55 | 0,24 | 0,59 | 0,30 | 0,87 | 0,75 | 0,54 | 0,67 | 0,24 | 0,25 | 1,00 | 0,29 |
| $y_{f,t}(j)$ | 0,33 | 0,50 | 0,39 | 0,24 | 0,42 | 0,34 | 0,34 | 0,98 | 0,28 | 0,64 | 0,30 | 0,31 | 0,33 | 0,40 | 0,41 | 0,54 | 0,54 | 0,36 | 0,26 | 0,29 | 1,00 |

Table 4: Distance correlations for the 1SVS model

| | $b_{d,d,sr,t}$ | $b_{d,f,sr,t}$ | $b_{d,g,t}$ | $b_{d,d,lr,t}$ | $b_{d,f,lr,t}$ | $b_{f,g,t}$ | $c_{d,t}$ | $c_{f,t}$ | $e_t$ | $m_{d,t}$ | $m_{f,t}$ | $p_{d,t}$ | $p_{f,t}$ | $r_{d,t}$ | $r_{f,t}$ | $r_{d,lr,t}$ | $r_{f,lr,t}$ | $w_{d,t}$ | $w_{f,t}$ | $y_{d,t}(j)$ | $y_{f,t}(j)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $b_{d,d,sr,t}$ | 1,00 | 0,59 | 0,96 | 0,55 | 0,69 | 0,53 | 0,90 | 0,36 | 0,40 | 0,51 | 0,50 | 0,54 | 0,23 | 0,93 | 0,78 | 0,74 | 0,73 | 0,51 | 0,56 | 0,89 | 0,30 |
| $b_{d,f,sr,t}$ | 0,59 | 1,00 | 0,60 | 0,16 | 0,85 | 0,87 | 0,49 | 0,45 | 0,53 | 0,80 | 0,83 | 0,36 | 0,25 | 0,53 | 0,50 | 0,48 | 0,68 | 0,41 | 0,37 | 0,51 | 0,47 |
| $b_{d,g,t}$ | 0,96 | 0,60 | 1,00 | 0,39 | 0,77 | 0,55 | 0,86 | 0,40 | 0,40 | 0,52 | 0,56 | 0,50 | 0,23 | 0,94 | 0,79 | 0,68 | 0,79 | 0,44 | 0,50 | 0,86 | 0,34 |
| $b_{d,d,lr,t}$ | 0,55 | 0,16 | 0,39 | 1,00 | 0,30 | 0,21 | 0,36 | 0,28 | 0,17 | 0,17 | 0,27 | 0,27 | 0,16 | 0,30 | 0,20 | 0,86 | 0,29 | 0,61 | 0,88 | 0,29 | 0,24 |
| $b_{d,f,lr,t}$ | 0,69 | 0,85 | 0,77 | 0,30 | 1,00 | 0,92 | 0,57 | 0,46 | 0,27 | 0,57 | 0,93 | 0,42 | 0,24 | 0,62 | 0,59 | 0,53 | 0,77 | 0,46 | 0,52 | 0,58 | 0,41 |
| $b_{f,g,t}$ | 0,53 | 0,87 | 0,55 | 0,21 | 0,92 | 1,00 | 0,28 | 0,37 | 0,16 | 0,43 | 0,97 | 0,33 | 0,17 | 0,37 | 0,34 | 0,39 | 0,53 | 0,46 | 0,42 | 0,28 | 0,33 |
| $c_{d,t}$ | 0,90 | 0,49 | 0,86 | 0,36 | 0,57 | 0,28 | 1,00 | 0,33 | 0,37 | 0,55 | 0,23 | 0,55 | 0,30 | 0,90 | 0,78 | 0,62 | 0,72 | 0,31 | 0,32 | 0,98 | 0,30 |
| $c_{f,t}$ | 0,36 | 0,45 | 0,40 | 0,28 | 0,46 | 0,37 | 0,33 | 1,00 | 0,26 | 0,58 | 0,33 | 0,37 | 0,42 | 0,41 | 0,45 | 0,53 | 0,54 | 0,40 | 0,33 | 0,31 | 0,97 |
| $e_t$ | 0,40 | 0,53 | 0,40 | 0,17 | 0,27 | 0,16 | 0,37 | 0,26 | 1,00 | 0,55 | 0,20 | 0,22 | 0,16 | 0,36 | 0,27 | 0,36 | 0,37 | 0,22 | 0,30 | 0,40 | 0,26 |
| $m_{d,t}$ | 0,51 | 0,80 | 0,52 | 0,17 | 0,57 | 0,43 | 0,55 | 0,58 | 0,55 | 1,00 | 0,36 | 0,33 | 0,25 | 0,56 | 0,60 | 0,52 | 0,71 | 0,36 | 0,23 | 0,57 | 0,59 |
| $m_{f,t}$ | 0,50 | 0,83 | 0,56 | 0,27 | 0,93 | 0,97 | 0,23 | 0,33 | 0,20 | 0,36 | 1,00 | 0,34 | 0,20 | 0,30 | 0,21 | 0,35 | 0,53 | 0,44 | 0,48 | 0,25 | 0,30 |
| $p_{d,t}$ | 0,54 | 0,36 | 0,50 | 0,27 | 0,42 | 0,33 | 0,55 | 0,37 | 0,22 | 0,33 | 0,34 | 1,00 | 0,29 | 0,50 | 0,39 | 0,36 | 0,38 | 0,39 | 0,30 | 0,59 | 0,33 |
| $p_{f,t}$ | 0,23 | 0,25 | 0,23 | 0,16 | 0,24 | 0,17 | 0,30 | 0,42 | 0,16 | 0,25 | 0,20 | 0,29 | 1,00 | 0,27 | 0,24 | 0,20 | 0,25 | 0,16 | 0,48 | 0,28 | 0,35 |
| $r_{d,t}$ | 0,93 | 0,53 | 0,94 | 0,30 | 0,62 | 0,37 | 0,90 | 0,41 | 0,36 | 0,56 | 0,30 | 0,50 | 0,27 | 1,00 | 0,85 | 0,70 | 0,77 | 0,38 | 0,34 | 0,91 | 0,35 |
| $r_{f,t}$ | 0,78 | 0,50 | 0,79 | 0,20 | 0,59 | 0,34 | 0,78 | 0,45 | 0,27 | 0,60 | 0,21 | 0,39 | 0,24 | 0,85 | 1,00 | 0,55 | 0,86 | 0,52 | 0,20 | 0,78 | 0,37 |
| $r_{d,lr,t}$ | 0,74 | 0,48 | 0,68 | 0,86 | 0,53 | 0,39 | 0,62 | 0,53 | 0,36 | 0,52 | 0,35 | 0,36 | 0,20 | 0,70 | 0,55 | 1,00 | 0,63 | 0,60 | 0,77 | 0,57 | 0,50 |
| $r_{f,lr,t}$ | 0,73 | 0,68 | 0,79 | 0,29 | 0,77 | 0,53 | 0,72 | 0,54 | 0,37 | 0,71 | 0,53 | 0,38 | 0,25 | 0,77 | 0,86 | 0,63 | 1,00 | 0,46 | 0,40 | 0,71 | 0,49 |
| $w_{d,t}$ | 0,51 | 0,41 | 0,44 | 0,61 | 0,46 | 0,46 | 0,31 | 0,40 | 0,22 | 0,36 | 0,44 | 0,39 | 0,16 | 0,38 | 0,52 | 0,60 | 0,46 | 1,00 | 0,61 | 0,28 | 0,35 |
| $w_{f,t}$ | 0,56 | 0,37 | 0,50 | 0,88 | 0,52 | 0,42 | 0,32 | 0,33 | 0,30 | 0,23 | 0,48 | 0,30 | 0,48 | 0,34 | 0,20 | 0,77 | 0,40 | 0,61 | 1,00 | 0,26 | 0,25 |
| $y_{d,t}(j)$ | 0,89 | 0,51 | 0,86 | 0,29 | 0,58 | 0,28 | 0,98 | 0,31 | 0,40 | 0,57 | 0,25 | 0,59 | 0,28 | 0,91 | 0,78 | 0,57 | 0,71 | 0,28 | 0,26 | 1,00 | 0,26 |
| $y_{f,t}(j)$ | 0,30 | 0,47 | 0,34 | 0,24 | 0,41 | 0,33 | 0,30 | 0,97 | 0,26 | 0,59 | 0,30 | 0,33 | 0,35 | 0,35 | 0,37 | 0,50 | 0,49 | 0,35 | 0,25 | 0,26 | 1,00 |

Table 5: Distance correlations for the 3SVS model