Abstract—Soft tissue parameter estimation embodies a wide area of active research in academia with the goal of creating accurate finite element models for haptic simulations and needle insertion modeling. The rise of robotics in surgery has also opened up the possibility of robotically palpating tissue and using local mechanical properties of the tissue as biomarkers to localize metastases in vivo. Up until now, most tissue parameter estimation approaches utilize heuristic methodologies or quasi-static implementations which can fall victim to the time-dependent, viscoelastic properties that biological tissues exhibit. Approaches that consider tissue dynamics in parameter fitting typically employ primarily batch-based regressions which are not suitable for online implementation. In this work, we explore the possibility of using Kalman filter-based parameter estimation techniques to quantify viscoelastic tissue properties online in real time. In doing so, we develop a discretized model of the nonlinear tool/tissue dynamics and analyze the model for observability. The model is then linearized and three different Kalman filter methodologies (Extended, Unscented, and Adaptive Fading) are implemented to estimate inherent tissue properties based on a popular model of tissue dynamics (Kelvin-Voigt). Functionality is demonstrated in simulation, and an experimental platform is fabricated to test the filter properties in hardware. We use this platform to assess the efficacy of the three filters in a variety of different scenarios. Convergence is assessed by analyzing the trace of the error covariance. Ultimately, we have demonstrated that it is possible to repeatedly estimate the viscoelastic properties of biological tissue with sufficiently fast (<200 ms) convergence. This work could be a preliminary step in creating robust estimation algorithms that could potentially enable low-level robotic control decisions to be made based on the local mechanical footprint of the biological tissue.

Index Terms—Soft Tissue Modeling, Surgical Robotics, Parameter Estimation, Kalman Filtering

I. INTRODUCTION

The rise of robotics in minimally-invasive surgery has equipped surgeons with enhanced dexterity and control inside the anatomical workspace in ways that are unattainable or unintuitive with manual tools. Advanced robotic control systems are able to perform such feats as motion scaling, virtual fixturing, and tremor suppression to improve a surgeon’s performance in vivo. However, the lack of haptic feedback has been widely recognized as a barrier to innovation and widespread application, as surgeons are unable to physically feel the forces being imparted at the tool-tissue interface.

Although force sensing in minimally-invasive surgery is a widely-studied field, most efforts focus on force reflection and haptics applications, and few groups have explored the opportunity to use these sensing modalities to make low-level, automated decisions at the end-effector level. A potential application is to use on-board sensors and actuators to characterize the material properties of tissue in order to make informed control decisions based on the tissue’s local pathology in both a diagnostic and therapeutic scenarios.

Robotic tissue palpation has been identified as a promising means of localizing and diagnosing potentially metastatic tissue in vivo without having to acquire a biopsy and perform pathological tests. The basic approach is to probe or indent biological tissue at a known depth and record the axial force, fitting the results to a simple first-order spring model of the tissue. While many groups have focused on quasi-static (<0.1 Hz) palpation [1]–[3], the nonlinear and viscoelastic nature of biological tissue can lead to widely variable results if testing conditions are not replicated adequately. To get around this problem, applying dynamic (>10 Hz) mechanical waveforms to tissue could potentially generate more robust measurements that are less vulnerable to errors induced by viscoelasticity (relaxation/creep effects).

The idea of dynamic tissue characterization is not particularly new, and several groups have used piezoceramics [4], [5] and voice coil [6] elements to mechanically excite tissue at high frequencies. However, system identification approaches used in literature have heavily relied on batch-based regression processes (i.e. post-processing/fitting) or heuristics (i.e. observing the empirical Bode plot generated by a chirp signal and fitting a model to the observed data) with no real-time component. Okamura et. al. implemented a batch-based regression to determine stiffness and damping parameters involved in needle insertions [8], and Barbe et. al. performed a similar feat but with an on-line algorithm based on recursive least-squares [7]. In both cases, the authors were able to linearize the system for linear least squares by ignoring needle dynamics, which may not be a particularly robust assumption. In [7], the estimation process suffered from relatively slow convergence times (>1
seconds), and due to the mechanics of tissue puncture, it was not possible for the tissue stiffness estimate to converge as it was highly dependent on the interaction state.

To the author’s knowledge, no group has explored the possibility of applying Kalman filtering to a nonlinear model of tool/tissue interaction to enable real-time, online estimation of the dynamic parameters of an unknown tissue. In addition, most approaches in literature are not particularly concerned with estimate convergence time, as the intended applications were primarily for creating finite element or analytical models of tissue behavior. Developing robust estimation algorithms with fast (<200ms) convergence could lead to the development of ‘smart’ instruments and tissue cutting devices that can estimate the tissue parameters in real-time and make low-level control decisions based on the information (i.e. a ‘smart’ cutting device that will only cut a certain type of tissue). The goal of this work is to apply Kalman filter-based parametric identification techniques to characterize tissue in real-time based on a model of the tool/tissue interaction, as shown in Fig. 1. Local observability is affirmed via an analysis of the nonlinear system with respect to the Lie derivative. The system is linearized, and three types of Kalman filter (Extended, Unscented, and Adaptive Fading Extended) are compared both in simulation and hardware to determine unknown tissue parameters (stiffness, damping) based on deterministic inputs (displacement) and measured outputs (force). The efficacy of the estimation process is quantified in terms of convergence rate, repeatability, and fidelity to baseline measurements.

II. MODELING AND SIMULATION

In designing an optimal filter for parameter estimation, an accurate model must be developed that characterizes system behavior. In this section, we consider the material model used to parametrically characterize viscoelastic tissue behaviors, and develop a coupled system of equations for describing the interface between the tissue and the tool.

A. Material Model Selection

Several models exist for parametrically modeling viscoelastic behavior in biological tissue. These models vary in terms of accuracy with respect to constant-stress (creep) or constant-strain (relaxation) conditions, and complexity in terms of the number of parameters required to fully parameterize the model. Three popular models are listed in Table 1, along with the number of states and parameters required for characterization. Although the Standard Linear Solid and Generalized Maxwell models offer the most accurate results, they are typically difficult to calculate and require many parameters. Due to simplicity, we assume a Kelvin-Voigt model for the biological tissue characterized by an effective stiffness k and damping $\beta$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$N_{\text{states}}$</th>
<th>$N_{\text{params}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maxwell</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Kelvin-Voigt</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Std. Linear Solid</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Generalized Maxwell</td>
<td>$&gt;3$</td>
<td>$&gt;3$</td>
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TABLE I: Three popular viscoelastic models

B. Continuous and Discrete Dynamic Model

Optimal parameter estimation requires (1) a model of the real system (hidden), (2) observations of the system states (obfuscated), and (3) the current best-estimate of the system parameters. In this section, we will develop a nonlinear model of the tool/tissue dynamics (both continuous and discrete). As shown in Fig. 1, the tool is modeled as a mass-spring-damper system with mass $m$, spring constant $k_0$, and damping coefficient $\beta_0$. The input to the system is a known proximal displacement, and the interaction force between the tool and the tissue is measured by a force sensor. The exogenous input to the system is a known (measurable) displacement $u(t)$ and velocity $\dot{u}(t)$ which can vary as a function of time. The measured quantity is the reaction force between the tool and the tissue (given by $F(t)$). We can construct the continuous dynamics of this system as follows:

$$m\ddot{x}(t) + \beta_0(\dot{x}(t) - \dot{u}(t)) - kx(t) = \beta_0\dot{\alpha}(t)$$

$$F(t) = k_0(x(t) - u(t))$$

Note how we observe the state (i.e., the displacement of proof mass $m$) through the force measurement.

In order to implement the estimator in hardware, we need to convert this continuous model into discrete time. To model the dynamics of the tool-tissue interface, we discretize derivatives using first- and second-order finite difference approximations given a sample rate $\Delta t$. For parameter estimation, it is convenient to introduce an augmented state matrix $X = [x_1, x_2, k, \beta]^T$ which contains the tissue parameters to be estimated (here we assume parameters are constant). As such, we can construct a nonlinear system of equations describing the state and measurement evolution based on nonlinear functions $f_t(X_t, u_t)$ and $h_t(X_t, u_t)$.

$$X_{t+1} = f_t(X_t, u_t) + G_t \omega_t$$

$$Y_t = h_t(X_t, u_t) + v_t$$

where $\omega_t$ and $v_t$ are the process and measurement noise, respectively, and are characterized by the following:

$$E[\omega_t \omega_s] = \begin{cases} Q_t, & t = s \\ 0, & t \neq s \end{cases}$$

$$E[v_t v_s] = \begin{cases} R_t, & t = s \\ 0, & t \neq s \end{cases} \quad \forall t, s$$

Expanding $f_t(X_t, u_t)$ and $h_t(X_t, u_t)$:

$$x_{1,t+1} = x_{1,t} + \Delta t x_{2,t}$$
\[ x_{2,t+1} = x_{2,t} + \frac{\Delta t}{m} \left[ k_0 (u_t - x_{1,t}) + \beta_0 \left( \frac{u_t - u_{t-1}}{\Delta t} - x_{2,t} \right) \right] - k_t x_{1,t} - \beta_t x_{2,t} + w_t \]

(7)

\[ k_{t+1} = k_t \]

(8)

\[ \beta_{t+1} = \beta_t \]

(9)

Our measurement equation is as follows:

\[ y_t = k_0(x_{1,t} - u_t) + v_t \]

(10)

Estimating two parameters \( k_t, \beta_t \) in addition to the states \( x_{1,t}, x_{2,t} \) requires an input \( u(t) \) that is persistently exciting to the system, i.e., an input that satisfies the following condition:

\[ \det \Phi^T \Phi = \begin{bmatrix} R_u(0) & R_u(1) \\ R_u(1) & R_u(0) \end{bmatrix} \neq 0 \]

(11)

\[ R_u(\tau) = E[u(t)u(t-\tau)] \]

(12)

It can easily be shown that \( u(t) = A \sin(\omega t) \) satisfies the P.E. condition to \( O(2) \). Therefore, we can estimate the required parameters by applying a sinusoidal input which is known to be persistently exciting to the second order.

**C. Observability**

Any parameter estimation problem hinges on the system being observable. If certain states cannot be accessed through the measurement equation, then these states cannot be estimated. Whereas, in the case of a linear system, observability can be ascertained through the Cayley-Hamilton theorem, the observability of a nonlinear system is intimately related to the Lie derivative. If we can show that a matrix containing the Lie derivatives has rank \( n \), then the system can be said to be locally observable. The Lie derivative of a scalar function \( h : \mathbb{R}^n \to \mathbb{R} \) (the nonlinear measurement) with respect to vector field \( f : \mathbb{R}^n \to \mathbb{R}^n \) (the nonlinear state) is given by the following:

\[ L_f(h) = \frac{\partial h}{\partial x} f(x) \]

(13)

\[ G = \begin{bmatrix} L_0^1(h) \\ L_1^1(h) \\ L_2^1(h) \end{bmatrix} \]

(14)

where \( L_0^1(h) = h \) and higher-order derivatives are recursively related by \( L_f^n(h) = \frac{\partial}{\partial x} \left[ L_f^{n-1}(h) \right] f \). We want to show that the gradient of the Lie derivative vector \( G \in \mathbb{R}^{4 \times 1} \) has full rank:

\[ \frac{\partial G}{\partial X} = \begin{bmatrix} \frac{\partial L_0^0}{\partial x_1} & \frac{\partial L_0^0}{\partial x_2} & \frac{\partial L_0^0}{\partial k_1} & \frac{\partial L_0^0}{\partial k_2} \\ \frac{\partial L_1^0}{\partial x_1} & \frac{\partial L_1^0}{\partial x_2} & \frac{\partial L_1^0}{\partial k_1} & \frac{\partial L_1^0}{\partial k_2} \\ \frac{\partial L_2^0}{\partial x_1} & \frac{\partial L_2^0}{\partial x_2} & \frac{\partial L_2^0}{\partial k_1} & \frac{\partial L_2^0}{\partial k_2} \end{bmatrix} \]

(15)

By solving for the determinant of this matrix, we can derive conditions under which the system is (locally) observable:

\[ \det \left( \frac{\partial G}{\partial X} \right) = \frac{\Delta t^2}{m^3} \left[ x_{1,t}^2 (k_0 + k_t) + x_{2,t}^2 m + x_{1,t} x_{2,t} (\beta_0 + \beta_t) - x_{1,t} u_t \beta_0 - k_0 x_{1,t} u_t \right] \]

(16)

From this, we can conclude that the system only becomes unobservable when \( x_{1,t} = x_{2,t} = 0 \) (i.e., no motion in the system). As such, we can proceed with confidence knowing that our system is locally observable so long as it is being excited.

**D. Linearization**

We can see that the discretized model is nonlinear, as we cannot formulate a state transition matrix that doesn’t contain the states or parameters being estimated. Implementing linear filtering techniques requires the system to be linearized about the current state estimates \( \hat{X}_t \). Linearizing the system about \( \hat{X}_t \), we formulate the linearized state transition and output matrices \( F_t \) and \( H_t \), respectively.

\[ \frac{\partial f}{\partial X} \bigg| \hat{X}_t = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial k_1} & \frac{\partial f_1}{\partial k_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial k_1} & \frac{\partial f_2}{\partial k_2} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial k_1} & \frac{\partial f_3}{\partial k_2} \end{bmatrix} \hat{X}_t \]

(17)

\[ \frac{\partial h}{\partial X} \bigg| \hat{X}_t = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \frac{\partial h_1}{\partial k_1} & \frac{\partial h_1}{\partial k_2} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \frac{\partial h_2}{\partial k_1} & \frac{\partial h_2}{\partial k_2} \end{bmatrix} \hat{X}_t \]

(18)

**III. KALMAN FILTERING**

The goal of this section is to present the algorithms for three different flavors of Kalman filter that will be compared in simulation and in hardware.

**A. Extended Kalman Filter**

With the linearized system, we can implement an extended Kalman filter (EKF) to estimate both the states and the parameters involved in the linear system of equations. We
initialize with an a priori error covariance $P_0 = P_0 = \mathbb{R}^{4 \times 4}$ where $\dim(X) = 4$. We compute the Kalman gain $K_t \in \mathbb{R}^{4 \times 1}$ as follows:

$$K_t = P_{t|t-1}H_t^T(H_tP_{t|t-1}H_t^T + R_t)^{-1}$$

where $R_t \in \mathbb{R}^1$ is the measurement noise covariance. We use this to compute our apriori state estimate $\hat{X}_t$ based on the nonlinear system model $h(X_t, u_t)$.

$$\hat{X}_t = f(X_t, u_t)$$

We propagate the a priori estimate through the nonlinear output model $h(X_t, u_t)$:

$$\hat{X}_t = \hat{X}_{t-1} + K_t(y_t - h(X_t, u_t))$$

We then update our covariance and propagate it forward (using Joseph's form for $P_t$ for numerical stability, where $I$ is a 4 x 4 identity matrix):

$$P_t = (I - K_tH_t)P_{t|t-1}(I - K_tH_t)^T + K_tR_tK_t^T$$

$$P_{t+1|t} = F_tP_tF_t^T + G_tQ_tG_t^T$$

where $Q_t \in \mathbb{R}^{4 \times 4}$ is the process noise covariance and $G_t \in \mathbb{R}^{4 \times 4}$ is the process noise state matrix.

### B. Adaptive Forgetting

It is well known that when system information is only partially known or incorrect, the EKF may diverge or give biased estimates. This condition could potentially arise when the tissue properties abruptly change or go to zero (i.e. the analyzer dis-engages from the tissue). As properties change in time, the the EKF is not very reactive as past data and recent data are weighed equally. To mitigate this, an adaptive fading EKF (AFEKF) can be implemented which modifies the Extended algorithm by adding an exponential forgetting factor that is adapted in real time based on system covariance [9]. We introduce three new scalars: $M_t \in \mathbb{R}^1$, $C_t \in \mathbb{R}^1$, and $N_t \in \mathbb{R}^1$.

$$M_t = H_tF_tP_tF_t^T$$

$$C_t = H_tP_{t|t-1}H_t^T + R_t$$

$$N_t = C_t - H_tG_tQ_tG_t^TH_t^T - R_t$$

We use these to calculate the forgetting factor $\lambda_t$ which is updated in real-time based on the covariance.

$$\lambda_t = \max\{1, \alpha \text{trace}(N_tM_t^{-1})\}$$

Parameter $\alpha$ is a heuristic that allows us to further adjust the forgetfulness of the algorithm (where $0 < \alpha < 1.003$ has shown acceptable results). Finally, we modify the a priori state covariance as follows:

$$P_{t|t+1} = \lambda_tF_tP_tA_t^T + G_tQ_tG_t^T$$

A notable feature of this algorithm is that the covariance is typically slower to converge when compared to EKF as exponential forgetting forces the system to be 'less trusting' of old data.

### C. The Unscented Transform

It is known that the EKF can perform poorly in the face of highly nonlinear behavior which exhibit large deviations from the piece-wise linearized dynamics. Unscented Kalman Filters (UKF) attempt to overcome this by estimating the distribution of the state and propagating it through the state and measurement equations. The filter relies on the unscented transform. Given our state vector $X \in \mathbb{R}^{4 \times 1}$, we compute 9 sigma points ($2n + 1$ where $n = 4$) to characterize the distribution of $X$:

$$\hat{X}_{t-1}^0 = \hat{X}_{t-1}$$

$$\hat{X}_{t-1}^i = \hat{X}_{t-1} + \sqrt{n + \kappa} \sigma_i \mathbf{v}_i$$

$$\hat{X}_{t-1}^{i+n} = \hat{X}_{t-1} - \sqrt{n + \kappa} \sigma_i \mathbf{v}_i$$

where $\sigma_i$ and $\mathbf{v}_i$ is the $i$th eigenvalue and associated eigenvector in the prior state covariance matrix $P_x$. Each sigma point is weighed by:

$$W_0 = \frac{\kappa}{n + \kappa}$$

$$W_i = W_{i+n} = \frac{1}{2(n + \kappa)}$$

We propagate each sigma point through the deterministic part of the nonlinear state transition equation:

$$\hat{X}_{t|t-1}^{*i} = f(\hat{X}_{t-1}, u_{t-1}, t - 1)$$

and predict the mean of these propagated points:

$$\hat{X}_{t|t-1,s} = \sum_{i=0}^{2n} W_i \hat{X}_{t|t-1}^{*i}$$

The predicted covariance is:

$$P_{t|t-1,s} = \sum_{i=0}^{2n} W_i(\hat{X}_{t|t-1}^{*i} - \hat{X}_{t|t-1,s})(\hat{X}_{t|t-1}^{*i} - \hat{X}_{t|t-1,s})^T$$

$$+ Q_{t-1}$$

We use the eigenvectors and eigenvalues of covariance $P_{t|t-1,s}$ to generate a new set of sigma points, which are fed through the observation equation

$$\hat{y}_t = h(\hat{X}_{t|t-1}, t)$$

and predict the observation by:

$$\hat{y}_t = \sum_{i=0}^{2n} W_i \hat{y}_t$$
We compute the output estimation covariance using the linearized observation matrix:

\[ P_y = H_t P_{t-1} H_t^T + R_t \] (39)

If we set \( \kappa = 2 \), the sample mean \( \hat{y}_t \) and sample covariance \( P_y \) approach the true mean and variance to the second order. Finally, the Kalman gain can be computed:

\[ K_t = P_{t-1} H_t^T P_y^{-1} \] (40)

We use the Kalman gain to update the state estimate:

\[ \hat{X}_t = \hat{X}_{t|t-1} + K_t (y_t - \hat{y}_t) \] (41)

Lastly, we update the state estimate covariance with \( \hat{x}_t \) and the process repeats itself.

D. Simulation

To verify that the filters can successfully estimate both states \( x_1, x_2 \) and parameters \( k, \beta \) in the system, the nonlinear model in Equation (1) was implemented in Simulink using a continuous time, variable step solver (ode45). The three Kalman filters were added in parallel and implemented in discrete form with a sample rate \( \Delta t = 0.5ms \) (a 2 kHz sampling frequency). The Simulink block diagram is shown in Fig. 2. The actual tissue stiffness \( k_{act} \) and damping \( \beta_{act} \) were set initially (to assess filter convergence) and then doubled at time \( t = 2 \) (to assess filter robustness to model changes).

The estimated stiffness \( \hat{k}_t \) and damping \( \hat{\beta}_t \), are shown in Figs. 3 and 4, where the dotted lines correspond to the actual stiffness and damping values. We see that, in simulation, the filter estimates are able to converge to the correct parameter values \( k_{act} \) and \( \beta_{act} \) very quickly (<200ms). However, when the stiffness changes, only the AFEKF is able to estimate the new parameter values in a timely manner, as expected. We will confirm this result again in hardware.

IV. Experimental Setup

Simulations have shown that, assuming the model is correct, the three filter modalities can estimate the stiffness and damping of an unknown tissue analog, with varying degrees of robustness to model changes. To demonstrate this capability in hardware, a system was fabricated to test the Kalman filters on actual material analogs. A detailed description of the experimental setup is presented in the following subsections.

A. Hardware

The experimental setup, shown in Fig. 5 (left), consists of a high-bandwidth linear voice coil actuator (Moticont LVCM-025-038-01) which provides the input to the system. A linear encoder (Heidenhain AK LIDA 47 TTL) tracks the position of the moving coil which is constrained to move unidirectionally by linear bearings. A beam-type load cell (Omega LCL-005) is attached to the moving coil which provides the force measurement. The stiffness of the load cell provides the system stiffness constant \( k_0 \), and coupled with the proof mass, sets an upper-limit on the system bandwidth based on the resonant frequency \( (\omega_{3dB} = \frac{1}{2\pi} \sqrt{k_0 / m}) \). The entire system
is mounted on a Thor labs micrometer, angled perpendicularly to the mounting surface, to allow for fine positioning above the tissue surface.

The system was heuristically characterized to obtain the free-moving stiffness $k_0$ and damping $\beta_0$. An impulse was applied to the system, and a 2nd-order mass spring damper model was fitted to the results (shown in Fig. 6). From this analysis, $k_0 = 9700 \text{ N/m}$ and $\beta_0 = 0.8 \text{ N}\cdot\text{m/s}$. Given a mass of $0.04$ kg, the resonant frequency of the system is $78$ Hz. As such, for our experiments, we want to excite the tissue at significantly below this frequency, so that natural oscillations in the load cell do not contaminate our measurements.

**B. Electrical and Software**

A National Instruments USB-6002 DAQ provides the excitation for the voice coil, and also acquires data from the load cell and the encoder. The load cell, a thermally-compensated full-bridge type with precision-matched resistors, generates a differential voltage that is amplified by an instrumentation amplifier (AD627, Analog Devices) with a gain of 1000 (and resolution of about 1 mN) to generate a sensitivity of 0.5 V/N. A quadrature encoder IC (Avago HCTL-2032-SC), externally clocked by a 2 MHz square wave generated by a 555 timer in astable mode, generates a 32-bit output that can be interrogated by the DAQ at any time without depending on the DAQ’s internal clock for counting encoder pulses. In order to run the entire system off of a single-sided 0-10V power supply, a high-power push-pull amplifier creates a virtual ground at 5V, which biases the voice coil. Another push-pull amplifier generates the bi-directional required current to drive the voice coil based on the analog output signal from the DAQ. A 5V regulator provides TTL-level power for the encoder readhead and load cell signal conditioner. A schematic of this system is shown in Fig. 5 (right).

**V. EXPERIMENTAL VALIDATION**

Numerous experiments were performed to observe filter performance in response to a number of different variables.

Several tissue analogs of varying stiffness were molded out of silicone-based rubbers (EcoFlex 00-10, EcoFlex 00-50, VytaFlex 20, DragonSkin 20, all manufactured by Smooth-On, Easton, PA). Tests were performed under a number of conditions and data were collected at a rate of 2 kHz. Although the filters are recursive and can be implemented in real-time, for the purpose of demonstrating feasibility, the filters were applied in post-processing to generate state and parameter estimates.

**A. Baseline Functionality**

Preliminary tests were performed to demonstrate that the implemented filters demonstrated stability and decent convergence properties for unknown materials. A sample tissue analog was molded out of EcoFlex 00-10. The tissue was probed at a frequency of 10 Hz for two seconds. Given an input frequency of 10 Hz, a moving-average filter with a window of 200 samples was applied to the stiffness and damping estimates to smooth the output. Process noise and
measurement noise were empirically determined to be \( w_t = [0, 1 \times 10^{-8}, 0, 0]^T \) and \( v_t = 2.5 \times 10^{-5} \), respectively.

The results of the parameter estimation are shown in Fig. 7. Although it is difficult to verify the accuracy of the results, we can compare the estimated stiffness to that predicted by a first-order model of the contact mechanics, treating the tool as a cylinder with radius \( a \):

\[
k_{m} = \frac{2Ea}{(1-\nu^2)} = 440 \text{N/m}
\]  

(42)

where \( E \) is the 100% modulus of the material (specified by the manufacturer to be 55 MPa), \( a \) is the radius of the suction probe (3 mm), and \( \nu \) is the Poisson ratio (\( \nu = 0.5 \) for most rubbers). As we can see, the model prediction matches fairly well with the estimated stiffnesses, lending some validity to the estimates. However this model is quasi-static. We will explore the frequency characteristics of our estimates in the next section.

In Fig. 7(c), we have plotted the trace of the a priori covariance \( \hat{P}_{t\mid t-1} \) as a function of time. This provides us with a measure of the ‘uncertainty’ (i.e. variance of the error) in the filter estimates at any time \( t \). We observe how the covariance of the EKF and UKF decay almost identically, whereas the covariance of the AFEKF is a bit slower to decay due to exponential forgetting.

The estimated force \( \hat{F}_t = k_0(\hat{x}_{1,t} - u_t) \) is compared to the measured force signal in Fig. 8. All filters perform well in estimating the actual force, with more noise in the AFEKF estimate due to the forgetting factor.

B. Frequency Dependency

Viscoelastic materials combine properties of linear solids and Newtonian fluids, and as such, the material behavior is often dependent on the speed (or frequency) of loading. These complex behaviors are encapsulated in storage \( (E') \) and loss \( (E'') \) moduli, which represent elastic and viscous behavior, respectively, and combine to form a ‘complex modulus’ \( E^* = E' + iE'' = \sigma_0 e^{i\phi} \), where \( \sigma_0 \) is the applied stress, \( \epsilon_0 \) is the resulting strain, and \( \phi \) is the phase angle between the two. Typically, for viscoelastic materials, the storage modulus monotonically increases as a function of frequency, often by factors of 2 or more, asymptotically approaching some value \( E'^\infty \). The loss modulus typically peaks at some frequency \( f_{\text{peak}} \) and decays at higher frequencies, asymptotically approaching some value \( E''^\infty \).

We can draw a parallel between dynamic moduli and our estimated parameters \( (E' \rightarrow k_t \text{ and } E'' \rightarrow \beta_t) \), as they describe similar phenomena in the viscoelastic response.

To understand this behavior further and observe the implications of frequency dependency on our estimated parameters (specifically, the convergence properties), a frequency sweep was performed on EcoFlex 00-10 silicone rubber. Frequencies ranging from 1Hz to 30Hz were applied to the specimen, resulting in the behavior shown in Fig. 9. As expected, the estimated stiffness tends to increase as a function of frequency, leveling off at around 25 Hz. The damping also behaves as expected, with an initial decay leveling off at around 25 Hz. Estimates are relatively consistent across all filters with more error variance in the AFEKF as expected.

C. Stiffness Dependency

Four different viscoelastic materials with varying stiffnesses (EcoFlex 00-10, EcoFlex 00-50, VytalFlex 20, DragonSkin 20) were parametrically identified with the AFEKF to understand how filter robustness scales with stiffness. Similar to before, each specimen was probed at a frequency of 10 Hz for a duration of 2 seconds. The results are shown in Fig. 10. An interesting result is that the filter generally takes longer to converge with stiffer materials, and the steady-state error variance scales approximately proportionally to the increase...
in stiffness, as can be seen in Fig. 10 (c). In addition, we see from Fig. 10 (a) that the algorithm becomes less accurate at higher stiffnesses. Although it is not a 1:1 comparison, the dotted lines show the 100% moduli of each material as specified by the manufacturer, scaled and normalized by the estimated stiffness values. At higher stiffnesses, the estimated values tend to deviate from what we expect based on a simple linear model employing the specified material modulus. This could potentially be due to the tissue stiffness approaching the tool stiffness.

**D. Model Deviation**

As demonstrated in Section III B., EKF and UKF implementations are not particularly robust to model or parameter changes. The AFEKF attempts to overcome this limitation by introducing an exponential forgetting factor which weighs recent data more heavily than past data at the cost of more computational overhead and potential instability.

To experimentally understand the implications of exponential forgetting, a test was performed wherein the analyzer is positioned onto a stiff tissue analog, data is collected for 4 seconds, and then the analyzer is disengaged from the stiff tissue and positioned onto softer tissue, where data is collected for another 4 seconds. The results shown in Fig. 11 show that the AFEKF is very quick to respond to model changes, and we can even see the instance where the analyzer is completely disengaged from the tissue (where the stiffness and damping drop to zero). The EKF and UKF are very slow to converge, as expected, and provide very inaccurate measurements given the duration of contact. Observing the covariance, we see that the covariance traces for the EKF and UKF monotonically approach zero despite the fact that model changes. The adaptation of the AFEKF is evident in
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(a) Stiffness  (b) Damping  (c) Covariance Trace

Fig. 12: Characterizing unknown tissue stiffness profiles using real-time estimation. Black circles in the $x-y$ plane indicate the locations of the embedded nodules, and the radii of the circles indicate relative depth from the surface (ranging from 1mm, 3mm, and 5mm deep)

E. Tissue Mapping

A particularly important potential application of in vivo tissue characterization is the localization and marginalization of metastatic tissue. Tumor localization and marginalization is a necessary step in minimally-invasive tumor removal, and it is paramount to provide sufficient margins between healthy and diseased tissue to avoid seeding during resection. Diseased tissue can be several (2-3) factors stiffer than healthy tissue [10], and this stiffness footprint could potentially be exploited to locate tumors in vivo without requiring specimen extraction.

To explore the possibility of doing this in real-time, a tissue analog (EcoFlex 00-10, Smooth-On) was prepared with embedded nodules at different depths to simulate cancerous tissue, as shown in Fig. 13 (left). The sample was analyzed at discrete locations in a uniform 5x5 grid. One second of data was collected at each location, and the resulting stiffness, damping, and covariance trace profiles are shown in Fig. 12. The surface curve represents a 2D interpolation of the average of each filter estimate. The black circles on the $x-y$ plane represent the locations of the nodules, and the radius of each circle represents its relative depth below the surface. We can see that the estimators were able to detect the location of each nodule with relatively high fidelity, purely based on the stiffness characteristics of the tissue. We also observe once again that the estimator error variance is higher for stiffer tissues, as the trace($P_{t|t-1}$) map indicates greater uncertainty over the nodules.

F. Tests with Biological Tissue

Characterization tests were repeated on an actual biological tissue analog (porcine liver) to demonstrate that the filters can reliably and repeatably estimate the viscoelastic properties of real biological tissue. The experimental setup is shown in Fig. 13 (b). Fig. 14 shows the results of 5 subsequent characterization tests (10 Hz) performed at different locations on the porcine liver, where shaded error bars indicate the standard deviation amongst tests. We see that the results are fairly repeatable long-term once the transient effects of initial conditions have subsided. The conclusion from this experiment is that it is indeed possible to repeatably characterize unknown biological tissues using the filtering techniques described.

A frequency sweep test was also performed to see if the liver behaves as we would expect from a viscoelastic specimen. Again we see that the stiffness (akin to the storage modulus $E'$) monotonically increases as a function of frequency, where the asymptotic frequency (i.e. $f$ where $E' \rightarrow E'_\infty$) lies beyond tested frequencies. This agrees with our previous conception of viscoelastic behavior. If we examine the damping behavior, we observe the initial drop and leveling off as we would expect from loss modulus behavior; however at higher frequencies, the EKF and AFEKF estimates begins to blow up. This is likely not a physically meaningful result and could potentially be due to linearization error which the UKF was able to successfully overcome. This result speaks to the merits of using unscented filtering at higher frequencies to better capture the nonlinear behavior of the system. Once again, we see in Fig. 15 (c) that the covariance begins to increase after about 20 Hz, as experienced before during the baseline frequency characterization tests. As such, we conclude that biological soft tissue does indeed behave as we would expect from a viscoelasticity standpoint, and this behavior is adequately captured in the model presented herein at lower frequencies, above which, potentially unmodeled nonlinearities tend to cause the EKF and AFEKF to diverge.
VI. CONCLUSION

In this paper, we demonstrate the feasibility of using Kalman filtering techniques to estimate viscoelastic mechanical properties of biological tissue in real-time. A mathematical model of the tool and tissue interaction is derived, and the observability of the system was ascertained by considering the determinant of the gradient of the Lie derivative of the nonlinear output equation with respect to the nonlinear state equations. Three flavors of Kalman filter are derived and implemented in software to compare their performance. Simulated results proved that the approach was feasible, and an experimental system was developed to demonstrate functionality using actual materials and biological tissues. Numerous case studies are presented and analyzed with respect to speed of convergence and error covariance. Ultimately we found that, at lower input frequencies (1-15 Hz), the AFEKF provides superior performance and robustness to model changes. However, at higher (>15 Hz) frequencies, linearization errors cause the extended filtering modalities to become unstable, whereas the UKF is still able to provide reliable state and parameter estimates.

Future work could focus on modifying the UKF algorithm to incorporate adaptive forgetting, thereby maximizing robustness to both model changes and system linearity.

REFERENCES


