IGEM, an Inter-temporal General Equilibrium Model of the U.S. Economy with Emphasis on Growth, Energy and the Environment

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Table of Contents

1. The Inter-temporal General Equilibrium Model (IGEM)

1.1. Overview of the Model

The Intertemporal General Equilibrium Model (IGEM) is a dynamic model of the U.S. economy which describes growth due to capital accumulation, technical change and population change. It is a multi-sector model that tracks changes in the composition of industry output, as well as changes in input mix used by each industry, including energy use. It also depicts changes in consumption patterns due to demographic changes, price and income effects.

The main driver of economic growth in this model is capital accumulation and technological change. Capital accumulation arises from savings of a household that is modeled as an economic actor with "perfect foresight." Aggregate household consumption and savings are chosen to maximize a utility function that is a discounted sum of the stream of future consumption. Within each period, the consumption – or demand – side of the model is driven by a detailed model of household demand that includes demographic characteristics.

The production – or supply – side of the model characterizes the industrial structure in detail. 35 industries are identified, of which 21 are manufacturing and 5 are energy related, these are listed in Table 1. Each industry produces output using capital, labor, energy and non-energy intermediate inputs using constant returns to scale technology. The production technology used changes over time due to both exogenously specified changes and endogenous changes from price effects. Coal, refined oil and gas are separately identified energy inputs. The output from domestic industries is supplemented by imports from the rest of the world to form the total supply of each commodity.

There are four main sectors of the economy in IGEM: business, household, government and the rest of the world. The flow of goods and factors among these sectors is illustrated in Figure 1. The boxes on the right side of the diagram represent the five groups on the demand side for commodities -- consumption, investment, government, exports and intermediate purchases. The business sector is represented by the boxes on the left; labor, capital and intermediate inputs flow into the producer box, and domestic commodities flow out. All markets for goods and factors are assumed to be competitive. Prices of commodities adjust to equate the supply from domestic and foreign producers to the demand in each period, as represented at the bottom of Figure 1.

This model is implemented econometrically, by which is meant that the parameters governing the behavior of producers and consumers are statistically estimated over a time series dataset that is constructed specifically for this purpose. This is in contrast to many other multi-sector models that are calibrated to the economy of one particular year. These data are based on a system of national accounts developed by Jorgenson (1980) that integrates the capital accounts with the National Income Accounts. These capital accounts include an equation linking the price of investment goods to the stream of future rental flows, a link that is essential to modeling the dynamics of growth.

This model is an extension and revision of the one used in Jorgenson and Wilcoxen (1993), and Ho and Jorgenson (1994) to analyze environment and trade policies[1]. The following sections describe the main features of the model.

1.2. The production and supply of commodities.

Energy consumption per person, like most goods, depends on the price of energy and the level of income. These, in turn, ultimately depend on technology, and to some extent, on world supplies. General progress in technology means a rising level of real incomes, progress in particular energy technologies means lower energy prices or lower energy requirements. A careful specification of producer behavior and technical change is thus essential for analyses of future energy trends and responses to energy and environmental policies. The response of firms to changes in prices determines to an important degree the ability of the producers to substitute other inputs for energy. In the long run, productivity growth allows both higher personal consumption and pollution reduction. The exact specification and parameterization of the production and technical change are therefore very important and described in detail in this section.

The business sector of the model is subdivided into 35 industries as listed in Table 1. There are two additional sectors that are not private businesses, but also hire labor and

[1] Jorgenson and Wilcoxen (1993) is reprinted as Chapter 1 of Jorgenson (1998), and Ho and Jorgenson (1994) is Chapter 8.

capital, these are the government and household sectors. There are 21 manufacturing industries, 4 mining industries, and 1 transportation industry. Five of the industries are labeled as energy producers, Coal Mining (industry 3), Oil and Gas Extraction (4), Petroleum Refining (16), Electric Utilities (30) and Gas Utilities (31). Seven are classified as intensive energy using industries, these are industries with value share of energy inputs in total output exceeding 4% in 1995.

The output of the business sector also is subdivided into 35 commodities; each commodity is the primary product of one of the industries. Many industries produce secondary products as well, for example, Petroleum Refining produces commodities that are the primary output of the Chemicals industry. Joint production of this kind is allowed for in the model.

The technology of each industry is represented by means of an econometric model of producer behavior. As noted in the Introduction the parameters of these production functions are estimated over a database constructed for this purpose, based on a system of national accounts developed by Jorgenson (1980). This database includes a time series of inter-industry transactions tables covering the period 1958-2000.

These input-output (IO) tables consist of a *use* matrix and a *make* matrix. The use matrix gives the inputs used by each industry -- intermediate commodities, non-competing imports, capital and labor. It also gives the commodity use by each category of final demand -- consumption, investment, government, exports and imports. The use matrix is illustrated in Figure 2. The make matrix gives the amount of each commodity produced by each industry and is illustrated in Figure 3.

The IO tables include the value of capital and labor input. The system of accounts includes a division of this value into price and quantity. The quantity of capital input is constructed by aggregating over a detailed set of capital assets, ranging from computers to office buildings. Similarly the quantity of labor input is constructed by aggregating over demographic groups, ranging from young workers with high school education to old workers with masters degrees. (A detailed description of the methods to calculate capital and labor input, and the data sources, is given in the Jorgenson, Ho and Stiroh (2003)).

The approach of calculating inputs by aggregating over detailed categories and econometrically estimating production function parameters over a time series dataset stands in contrast to most other multi-sector models, static or dynamic. A simple sum of capital stocks will have ignored the rapidly rising ratio of computers to structures, a phenomenon that is captured by IGEM's index of capital input. Similarly, a simple sum of labor hours ignores the rising ratio of college educated to non-college workers, which raises IGEM's quantity index of labor input substantially. The common method of calibrating the use of intermediate inputs to one year's IO matrix, instead of using an entire time series, ignores the changing pattern of input use. A parallel assumption that is typically made is that input-output material coefficients are fixed, i.e., there is no substitution between steel and plastic, for example.[2] IGEM's approach does not impose such assumptions as it embodies estimates of the elasticities of substitution among productive inputs using time series data.

1.2.1. Notation

The general system of notation within IGEM employs Roman letters for economic variables and Greek letters for estimated model parameters. The *t* subscript denotes time, *i* indexes commodities and *j* indexes industries.

Q_j	quantity of output of industry *j*
$P_{Q,j}$	price of output to producer in industry *j*
$P_{QT,j}$	price of output to purchasers from industry *j*
$X_{i,j}$	quantity of commodity input *i* into industry *j*
P_i^X	price of commodity *i* to buyers
K_j	quantity of capital input into *j*
L_j	quantity of labor input into *j*
E_j	index of energy intermediate input into *j*
M_j	index of total nonenergy intermediate input into *j*
$P_{E,j}$	price of energy intermediate input into *j*
$P_{M,j}$	price of total nonenergy intermediate input into *j*
$P_{K,j}$	price of total capital input to industry *j*
$P_{L,j}$	price of total labor input to industry *j*
v	value shares
QC_i	quantity of domestically produced commodity *i*
$P_{C,i}$	price of domestically produced commodity *i*

[2] Some models specify a Cobb-Douglas form for material inputs instead of this "Leontief" style fixed coefficients. This means that the elasticity of substitution is assumed to be one. In contrast, the approach in IGEM estimates the elasticities of substitution, allowing them to be different among inputs and industries.

$M_{j,i}$ MAKE matrix; value of commodity *i* made by industry *j*

1.2.2. Top tier production function with technical change

The production function may be expressed abstractly as producing output from capital, labor, *m* intermediate inputs, non-competing imports (X_N) and technology (t), and thus for industry j:

(1) $$Q_j = f(K_j, L_j, X_{1,j}, X_{2,j}, ...X_{m,j}, X_{Nj}, t), \qquad j=1,2,...35$$

This is too general to be tractable and, so, it is assumed that inputs are chosen based on a multi-stage allocation. At the first stage, the value of each industry output is allocated to four input groups -- capital, labor, energy and non-energy materials:

(2) $$\begin{aligned} Q_j &= f(K_j, L_j, E_j, M_j, t); \\ E_j &= E(X_{3j}, X_{4j} X_{16j} X_{30j} X_{31j}) \\ M_j &= M(X_{1j}, ...X_{35j}, X_{Nj}) \end{aligned}$$

The second stage allocates the energy and non-energy materials groups to the individual intermediate commodities. The components of the energy group are Coal, Oil and Gas Extraction, Petroleum Refining, Electric Utilities, and Gas Utilities. The materials group includes all the other 30 commodities listed in Table 1 as well as non-competing imports (X_{Nj}). This last item are imports that are regarded as having no close domestic substitutes and include goods such as coffee and foreign port services.

Production is assumed to occur under constant returns to scale and the value of industry output is equal to the sum of the values of all inputs:

(3) $$\begin{aligned} P_{Qjt} Q_{jt} &= P_{Kjt} K_{jt} + P_{Ljt} L_{jt} + P_{Ejt} E_{jt} + P_{Mjt} M_{jt} \\ P_{Ejt} E_{jt} &= p^X_{3t} X_{3jt} + p^X_{7t} X_{7jt} + .. + p^X_{11t} X_{11jt} \\ P_{Mjt} M_{jt} &= p^X_{1t} X_{1jt} + p^X_{2t} X_{2jt} + .. + p^X_{Nt} X_{Njt} \end{aligned}$$

It is more convenient to work with the dual cost function instead of the direct quantity function in equation (2)[3]. The cost function expresses the unit output price as a function of all the input prices and technology, $P_{Qj} = p(P_{Kj}, P_{Lj}, P_{Ej}, P_{Mj}, t)$. The form of the cost function is chosen as the translog form:

[3] The dual function is equivalent to the primal function; all the information expressed in one is recoverable from the other.

(4) $$\ln P_{Qt} = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln P_{it} + \tfrac{1}{2} \sum_{i,k} \beta_{ik} \ln P_{it} \ln P_{kt} + \sum_{i=1}^{n} \ln P_{it} f_{it} + f_{pt}$$

i,k={K,L,E,M}

where the industry *j* subscript is dropped for simplicity, and α_0, α_i and β_{ik} are parameters that are estimated separately for each industry. The f_{it}'s are state variables representing biases in technical change and f_{pt} is the state variable for the level of neutral technology. f_{pt} is referred to as the price technology term. These *f*'s are unknown functions of time and are estimated using the Kalman filter (see Jorgenson et al. (2004) and Jorgenson and Hui Jin (2005)).

The above formulation has a more flexible form for technology than that in the previous version of IGEM. In Jorgenson (1998) the cost function was written in a parametric form:

(5) $$\ln P_{Qt} = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_{it} + \tfrac{1}{2} \sum_{i,k} \beta_{ik} \ln p_{it} \ln p_{kt} + \sum_{i=1}^{n} \beta_{it} \ln p_{it} g(t) + \alpha_t g(t) + \tfrac{1}{2} \beta_{tt} g(t)^2$$

where the g(t) function was an index of the level of technology and was assumed to have a logistic form. This new version of IGEM does not impose an explicit parametric form on g(t).

The substitution terms are the same in equations (4) and (5) and are described in detail in Jorgenson (1986). The reason for choosing the translog form is that it is rich enough to allow for substitution among all inputs and for biases in technical change while yielding a simple linear input demand equation. Differentiating equation (4) with respect to the log of input prices yields input share equations. For example, the demand for capital is derived from the capital share equation:

(6) $$v_{Kt} = \frac{P_K K}{P_Q Q} = \alpha_K + \sum_{k} \beta_{Kk} \ln P_{kt} + f_{Kt}$$

In more compact vector notation the cost function and share equation may be written as:

(4') $$\ln P_{Qt} = \alpha_0 + \boldsymbol{\alpha}' \ln \mathbf{p}_t + \tfrac{1}{2} \ln \mathbf{p}_t' \mathbf{B} \ln \mathbf{p}_t + \ln \mathbf{p}_t' \mathbf{f}_t + f_{pt} + \varepsilon_t^p$$

(6') $$\mathbf{v}_t = \boldsymbol{\alpha} + \mathbf{B} \ln \mathbf{p}_t + \mathbf{f}_t + \boldsymbol{\varepsilon}_t^v$$

where $\mathbf{p} = (P_K, P_L, P_E, P_M)'$, $\mathbf{v} = (v_K, v_L, v_E, v_M)'$, $\mathbf{f}_t = (f_{Kt}, f_{Lt}, f_{Et}, f_{Mt})'$ and $\mathbf{B} = [\beta_{ik}]$. The ε_t^p and $\boldsymbol{\varepsilon}_t^v$ terms are stochastic variables with mean zero that are added for the econometric estimation.

The α_i's may be thought of as the average value share of input *i* in output value. When β_{ik}'s are zero, the cost function reduces to the Cobb-Douglas form, and the primal output function becomes the familiar $Q_t = A_t K_t^{\alpha_K} L_t^{\alpha_L} E_t^{\alpha_E} M_t^{\alpha_M}$.

The β_{ik}'s are the share elasticities and represent the degree of substitutability among the K,L,E,M inputs. They capture the prices responsiveness of demands for inputs, e.g. how a higher price for energy leads to more demand for capital input. Constant returns to scale in production where the value shares sum to unity and homogeneity restrictions on the cost function (i.e., doubling of all input prices doubles the output price) imply that:

(7) $\alpha_K + \alpha_L + \alpha_E + \alpha_M = 1$

$\sum_i \beta_{ik} = 0$ for each k

That the cost function be symmetric implies that:

(8) $\beta_{ik} = \beta_{ki}$

In order to guarantee a well defined interior solution for the model there is also the requirement that the cost function to be "locally concave". This condition implies that:

(9) $\mathbf{B} + \mathbf{v}_t \mathbf{v}_t' - \mathbf{V}_t$,

must be non-positive definite at each *t* in the sample period, where $\mathbf{V}_t$ is a diagonal matrix with the value shares along the diagonal.

Turning now to the state variables for technology, if, for example, f_{Kt} is trending upwards then we say that technical change is "capital-using". Alternatively, if f_{Kt} is trending downwards then technical change is "capital-saving". When technical change is input-i using that means that higher relative prices for input-i will slow down technical progress. IGEM's cost function with both β_{ik} and f_{it} allows the separation of price induced changes in input ratios from those that result from changes in technology.

Productivity growth translates into a fall in output price given input prices. The productivity change between *t-1* and *t* is given by:

$$(10) \qquad \Delta T_t = -\sum_{i=1}^{n} \ln P_{it} (f_{it} - f_{i,t-1}) - (f_{pt} - f_{p,t-1})$$

The price technology term, f_{pt}, is non-stationary but the difference, $\Delta f_{pt} = f_{pt} - f_{pt-1}$, is found to be stationary. The state variables f_{it} are stationary. To implement the cost function (4) we express these technology state variables as a vector auto-regression (VAR). Let $\mathbf{F} = (f_k , f_l , f_e , f_m , \Delta f_p)'$ denote the entire vector of stationary state variables. The transition equations are assumed to be governed by:

$$(11) \qquad \mathbf{F}_t = \mathbf{\Phi F}_{t-1} + u_t$$

where u_t is a random variable with mean zero and $\mathbf{\Phi}$ is a matrix of estimated coefficients of the first-order VAR.

The goal in choosing the above state space representation of technology is to allow in IGEM both a flexible representation of complex behavior in the sample period and a feasible but controlled representation of technical change for the projection period. Specifically, IGEM is a model with infinitely lived households in consumption and, so, requires simulation to a steady-state (i.e., zero-growth) solution in all model inputs and outputs. In turn, this requires that trends in factor biases and neutral technical change, projected from observed history, transition to constants in steady state. This transition is presumed to begin after 25 to 30 years and is completed within another 25 to 30 years, reflecting a conservative approach toward a distant and very uncertain future.

1.2.3. Lower tiers production function for intermediate inputs

In the subsequent stages of production, the energy and material aggregates are allocated to the *m* individual commodities. To repeat equation (2) for the second stage:

$$(12) \qquad E_j = E(X_{3j,} X_{4j} , X_{16j} , X_{30j} , X_{31j}); \qquad M_j = M(X_{1j} ,...X_{35j} , X_{Nj})$$

where the components of the non-energy materials (M) aggregate are every other commodity in Table 1 except for the five energy commodities. Also included in M is non-competing imports which is a “commodity” not produced by any domestic industry. It is denoted as X_N.

The demand for these detailed commodities by each industry *j* also is derived from a translog cost function. These sub-tier cost functions have a simpler form than equation (4) in that they do not have the technology terms. This is due to the requirements of consistent aggregation. To illustrate these sub-tier functions, the cost function for the energy aggregate is written as:

(13) $$\ln P_{Et} = \alpha_0 + \sum_{i=\{3,4,16,30,31\}} \alpha_i \ln P_{it}^X + \tfrac{1}{2}\sum_{i,k} \beta_{ik} \ln P_{it}^X \ln P_{kt}^X$$

while the share equation for the first component of aggregate energy (coal mining) is:

(14) $$v_3 = \frac{P_3^X X_3}{P_E E} = \alpha_3 + \sum_{k=3,4,16,30,31} \beta_{3k} \ln P_k^X \ ,$$

Again, the β_{ik} 's are share elasticities representing the degree of substitution among the 5 types of energy, and the α 's are the average input value shares.

The long list of items in the materials aggregate, M(.), requires that it too be arranged in a multi-stage manner. The entire tier structure for producer behavior in each industry is given in Table 2. The M(.) aggregate consist for 5 sub-aggregates – Construction, Agricultural materials, Metallic materials, Nonmetallic materials and Services materials. These sub-aggregates, in turn, are functions of other groups and so on until all the *m* commodities are accounted for. Each node in the tier structure employs a cost equation as written by a generalized equation (13).

1.2.4. Relation between commodities and industries, and output taxes.

One of the taxes that are explicitly identified in the model is production (or sales) taxes. This introduces a wedge between the seller and buyer prices. Denoting the buyer price of industry output by $P_{QT,j}$:

(15) $$P_{QT,j} = (1 + tt_j)P_{Q,j}$$

Each industry makes a primary commodity and many make secondary products that are the primary output of some other industry. In the *make* matrix the M_{ji} element represents the value of the *i*th commodity produced by industry *j*. Thus, the *i*th column of the *make* matrix indicates which industries contribute to the *i*th commodity, while the *j*th row shows which commodities are made by industry *j*.

The value of industry *j* output is $P_{QT,j}Q_j$; let the price, quantity and value of commodity *i* be denoted by, P_{Ci}, QC_i, VQC_i respectively, all from the buyer's point of view. For column *i* of the make matrix, let the shares contributed by the various industries to that commodity in the base year be denoted:

$$(16) \qquad m_{ji} = \frac{M_{ji,t=T}}{VQC_{i,t=T}}; \qquad \sum_j m_{ji} = 1$$

For row *j*, let the shares of the output of industry *j* be allocated to the various commodities be denoted:

$$(17) \qquad m_{ji}^{row} = \frac{M_{ji,t=T}}{P_{QT,j}Q_{j,t=T}}; \qquad \sum_i m_{ji}^{row} = 1$$

These shares are assumed fixed for all periods after the base year. Equivalently, the production function of commodities is a simple Cobb-Douglas aggregate of the output from the various industries where the component weights are these base year shares. Thus, the price of domestic commodity *i* is given as:

$$(18) \qquad P_{Ci} = P_{Q1}^{m_{1i}} ... P_{Qm}^{m_{mi}} \qquad \text{for i=1,2,....m}$$

The values and quantities then are given by:

$$(19) \qquad VQC_{it} = \sum_j m_{ji}^{row} P_{Qjt} Q_{Yjt} \qquad \text{for i=1,2,....m}$$

$$(20) \qquad QC_i = \frac{VQC_i}{P_{Ci}}$$

1.3. Household model

To capture differences among households, the household sector is subdivided into demographic groups including region of residence. Each household is treated as a consuming unit, i.e. it is the unit maximizing some utility function over all commodities in IGEM, including leisure.

As currently specified, demographic differences in IGEM are limited to the allocation of commodity consumption. These differences do not enter the allocation of time between work and leisure nor do they enter the allocation of income between consumption and saving. IGEM's household model thus has three stages. At the first stage, lifetime income is allocated to consumption and saving in each period. This consumption consists of commodities and leisure and is referred to as "full consumption". In the second stage, full consumption is allocated to total goods and services and leisure. In the third stage, total goods and services are allocated to IGEM's various energy and non-energy commodities. This third stage is actually a series of stages and is where the detailed demographic information appears.

1.3.1. Notation

F_t	quantity of full consumption
C_t	quantity of aggregate goods consumption
L_t^{leis}	quantity of aggregate leisure
LS_t	quantity of aggregate labor supply
$\bar{L}_t$	quantity of aggregate time endowment
KS_t	quantity of aggregate capital stock at end of period t
n_t	growth rate of population index
P_t^F, P_t^C, P_t^{leis}	price of F_t, C_t and L_t^{leis}
P_t^L	price of labor to employer, economy average
P_t^K	price of capital (rental rate), economy average
r_t	rate of return between t-1 and t
Y_t^{hh}	household disposable income
S_t^{hh}	household savings

1.3.2. Household model 1st stage (intertemporal optimization)

At this level, the aggregate household maximizes an additively separable intertemporal utility function:

$$(21) \qquad U = \sum_{t=0}^{\infty} N_0 \prod_{s=1}^{t} \left(\frac{1+n_s}{1+\rho} \right) \ln F_t$$

subject to a lifetime budget constraint:

$$(22) \qquad W_0^F \geq \sum_{t=0}^{\infty} \left(\prod_{s=1}^{t} \frac{1}{1+r_s} \right) P_t^F F_t$$

where F_t is the per capita full consumption in period t, ρ is the econometrically estimated rate of time preference, N_0 is the initial population, and n_s is the population growth rate in period s. W_0^F is the aggregate household's "full wealth" at time 0, P_t^F is the price of full consumption and r_s is the rate of return between $s-1$ and s (i.e., the spot market interest rate). The term "full wealth" refers to the present value of future earnings from the supply of tangible assets and labor, plus transfers from the government and imputations for the value of leisure. Tangible assets include domestic physical capital, government bonds and net foreign assets.

Equations (21) and (22) are common to Cass-Koopmans type growth models occurring in standard macroeconomics textbooks (e.g. Barro and Sala-i-Martin 1995). Intertemporal optimality is expressed in a so-called Euler equation and requires that:

(23) $$\frac{F_t}{F_{t-1}} = \frac{(1+n_t)(1+r_t)}{1+\rho}\frac{P_{t-1}^F}{P_t^F}$$

The Euler equation is forward-looking, so that the current level of full consumption incorporates expectations about all future prices and discount rates. The solution in IGEM includes this forward-looking relationship in every period. The future prices and discount rates determined by the model enter full consumption for earlier periods through the assumption of perfect foresight (or rational expectations if there was uncertainty in the model). The value of full consumption in any period is the key element in deriving household saving in that period.

1.3.3. Household model 2nd stage (goods and leisure)

Once each period's full consumption is determined, it is subsequently divided into aggregate personal consumption expenditures (commodities) and leisure time. The determination of leisure is also the determination of labor supply. Full consumption at time t is viewed as a utility function of aggregate commodities (C_t) and leisure (L_t^{leis}) at t:

(24) $$F_t = F(C_t, L_t^{leis})$$

and the value of full consumption is the sum of the values of goods consumption and leisure:

(25) $$P_t^F F_t = P_t^C C_t + P_t^{leis} L_t^{leis}$$

For this stage of the household model, it is assumed that the utility function is homothetic, i.e. the income elasticities for goods and leisure are one. The producer model used the cost dual instead of the direct production function. Here again, it more convenient to use the indirect utility function, $V_t^F = V(P_t^C, P_t^{leis}, Y_t^{hh})$, to derive the demand for aggregate consumption and leisure. The translog form of the indirect utility function under the assumption of homotheticity results in the following constant returns to scale "cost function" for the price of full consumption[4]:

(26) $$\ln P^F = \alpha_c \ln P^C + \alpha_{leis} \ln P^{leis} + \tfrac{1}{2} \sum_{i,j=\{C.leis\}} \beta_{ij} \ln P^i \ln P^j$$

The demand for goods consumption and leisure is derived in a manner identical to that for input demands in the producer model (equation 6):

(27) $$\frac{P^C C}{P^F F} = \alpha_C + \beta_{cc} \ln P^C + \beta_{cl} \ln P^{leis}$$

Given the demand for leisure, the quantity of labor supply, *LS*, is the exogenous time endowment minus leisure:

(28) $$LS_t = \overline{L}_t - L_t^{leis}$$

The time endowment $\overline{L}_t$ is aggregated from population data by detailed demographic groups and using wage rates as weights.

In equation (22) for the lifetime budget constraint, W_0^F represents the present value of the stream of household full income, that is, tangible income plus the imputation for leisure value. Household tangible income is the sum of after-tax labor income, capital income, interest income from government bonds (B^G), interest income from net foreign assets (B^*), and transfers from the government (G^{hh}):

(29) $$Y_t^{hh} = (1-tl)P_t^L LS_t + (1-tk)P_t^K KS_{t-1} + i(B_{t-1}^G) + i(B_{t-1}^*) + G_t^{hh} - twP_{t-1}^{KS} KS_{t-1}$$

where *KS* is the stock of capital owned by households, P_t^K is the rental price of aggregate capital, and *tl* and *tk* are tax rates on labor and capital income respectively. *tw* is the wealth (estate) tax put on the value of capital stock whose price is P^{KS}.

The difference between the price of leisure and the wage paid by employers is the labor tax:

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[4] This indirect utility function for full consumption is first used in Jorgenson and Yun (1986)..

(30) $P^{leis} = (1 - tl)P^L$

The capital price (P^K) and tax (tk) has a similar interpretation[5].

Private household saving then is simply tangible income less consumption, non-tax payments to the government and transfers to rest-of-the-world:

(31) $S_t^{hh} = Y_t^{hh} - P_t^C C_t - R_t^N - H_t^{row}$

1.3.4. Household model 3rd stage

Once the total value of spending on commodities is determined in the second stage, it then is allocated to all IGEM's available. The allocation of aggregate consumption is done according to the household demand model in Jorgenson and Slesnick (1987). Households are divided into various demographic types by income (expenditure) class, age, sex and race of head, family size, and type and region of residence, and the demands for goods and services are indexed by household types. Total personal consumption is the aggregate over all the household types.

In the producer model, the 35 intermediate inputs entered via a tier structure with the top tier written as *Q=f(K,L,E,M)*. The household commodity model is similarly a function of 35 items, and the top tier is a function of five commodity groups – energy, food, non-durables, capital services, and services. Let the prices of these groups be denoted P_{EN}, P_{FD}, P_{ND}, P_K, P_{SV}, and the value of total expenditures by household k be M_k:

(32) $M_k = P_{EN} C_{EN}^k + P_{FD} C_{FD}^k + P_{ND} C_{ND}^k + P_K C_K^k + P_{SV} C_{SV}^k$

The indirect utility function for household k, $V(P_{EN}, ..P_{SV}, M_k; A_k)$, is written in translog form as:

(33) $$\ln V_k = \sum_i \alpha_i \ln \frac{P_i}{M_k} + \tfrac{1}{2} \sum_{i,j} \beta_{ij} \ln \frac{P_i}{M_k} \ln \frac{P_j}{M_k} + \sum_i \beta_{Ai} A_k \ln \frac{P_i}{M_k}$$

$$i, j = \{EN, FD, ND, K, SV\}$$

where A_k is a vector of demographic dummies and α_i, β_{ij} and β_{Ai} are parameters that are estimated from historical data.

[5] Further features about the actual tax system is left out of this description to avoid too much unimportant detail. These include the property tax, estate tax and non-tax payments. These are, however, included in the accounts of the economy and in the actual code of the model.

In order to derive an aggregate demand function, restrictions are imposed on the parameters as explained in Jorgenson and Slesnick (1987). With these restrictions the share demand equations are derived as:

(34) $$w^k = \frac{\alpha + \mathbf{B}\ln p - \mathbf{B\iota}\ln M_k + \beta_A A_k}{-1 + \mathbf{\iota B}\ln p}$$

where w^k is the vector of shares, $(P_{EN}C_{EN}^k / M_k, ..., P_{SV}C_{SV}^k / M_k)$, and p is the vector of the 5 prices. $\mathbf{B} = [\beta_{ij}]$ and $\mathbf{\iota}$ is a vector of 1's. The aggregate demand for these 5 commodity groups is the sum over all households:

(35) $$w = \frac{\sum_k w^k M_k}{\sum_k M_k} = \frac{1}{-1 + \mathbf{\iota B}\ln p}[\alpha + \mathbf{B}\ln p - \mathbf{B\iota}\sum_k \frac{M_k \ln M_k}{M} + \beta_A \sum_k \frac{M_k A_k}{M}]$$

The total economy spending by all households is the value of consumption from the second stage, eqs (25) and (27):

(36) $$M_t = \sum_k M_{kt} = P_t^C C_t$$

and the aggregate share vector is:

(37) $$w = (P_{EN}C_{EN} / M, ..., P_{SV}C_{SV} / M)'$$

The demands for the five commodity groups, $C_{EN}, C_{FD}, C_{ND}, C_K, C_{SV}$, are allocated to the individual commodities identified in the model. These groups are based on the definitions in the Consumer Expenditure Surveys and reconciled with the categories in the Personal Consumption Expenditures (PCE) in the National Accounts. These detailed categories in the National PCE for 35 items are given in Table 3 and, below, their prices and quantities are denoted by P_i^N and C_i^N. The tier structure allocating the five consumption groups to these detailed C_i^N is organized like that for the production function and is given in Table 4. There is a total of 16 sub-tier functions and they are written in a manner identical to the example in equation (13) and (14) for the production energy sub-tier, that is, the price of energy for the household is a function of the prices of gasoline, fuel oil, coal, electricity, and gas:

(38) $$P_{EN} = f(P_6^N, P_7^N, P_8^N, P_{18}^N, P_{19}^N)$$

Using these sub-tier cost functions yields the aggregate demands for all 35 NIPA-PCE items $\{C_i^N\}$.

The commodity outputs from the producer models are classified by input-output categories. The official benchmark IO tables from the Bureau of Economic Analysis come with bridge tables that link the NIPA-PCE categories to the IO classification. For example, they show how "nondurable toys and sport supplies" (item 13 in Table 3) is made up of deliveries from Chemicals, Miscellaneous Manufacturing, Trade, Transportation, etc. Using this bridge table, denoted $\mathbf{H}=[H_{ij}]$, gives consumer demands in terms of their IO classification:

(39) $$P_i^X C_i = \sum_j H_{ij} P_j^N C_j^N$$

1.4. Investment and the cost of capital

The primary factors of production in this model are capital and labor. Capital here includes structures, producer's equipment, land, inventories, and consumer durables. This differs from the official investment in the National Income and Product Accounts (NIPA) which records consumer durables as part of Personal Consumption Expenditures[6]. Capital here is assumed to be the capital owned by the private sector. Government owned capital is accounted for separately.

Capital is mobile and moves costlessly from one industry to another within any period. Investment goods are converted costlessly into capital stock; there are no installation or adjustment costs. These assumptions mean that producer optimization reduces to minimizing the cost of production in period *t* (equation 4) without the necessity of considering future prices. Also, with an aggregate household owning all the capital with perfect foresight, the saving decision is the investment decision[7]. However, it is important to present the savings-investment decision in a manner that clarifies the economy's cost of capital, a key determinant of overall growth.

The owner of the stock of capital may be thought of as choosing the path of investment by maximizing the stream of capital income subject to a capital accumulation

[6] Land is in the "fixed, non-reproducible" asset category, and is not part of Investment in GDP (land is transferred, not produced). The rental from land is, of course, included in the income side of GDP.

constraint. Let KS_t denote the aggregate capital stock at the end of period t, which is to be distinguished from the flow of capital services K_{jt} in the industry production function equation (2). Let P_t^K denote the rental price of a unit of this stock, the model maximizes the discounted rental income net of purchases of aggregate new investment:

(40) Max $\sum_{t=0}^{\infty} \frac{(1-tk)P_t^K KS_{t-1} - P_t^I I_t^a}{\prod_{s=0}^{t}(1+r_s)}$

(41) s.t. $KS_t = (1-\delta)KS_{t-1} + I_t^a$

The after tax capital income term, $(1-tk)P_t^K KS_{t-1}$, is the same as that in the household income equation (29), and the discount rate r_s is the same as that in the Euler equation (23). I_t^a is the quantity of aggregate investment and P_t^I is its price. (Certain tax details in the model, such as depreciation allowances, are not represented above so as to focus on the model's main points.)

Aggregate investment is actually a basket of commodities ranging from computers to structures. This basket changed substantially in the sample period. An index of the quality of aggregate investment, ψ_t^I, is employed to keep track of the changing composition. Accordingly, Eq (41) is actually written as:

(42) $KS_t = (1-\delta)KS_{t-1} + \psi_t^I I_t^a$.

This refinement is ignored below to keep the description simple but is used in the actual model.

The solution of the maximization problem gives the Euler equation:

(43) $(1+r_t)P_{t-1}^I = (1-tk)P_t^K + (1-\delta)P_t^I$

There is a simple interpretation of this equation. If P_{t-1}^I dollars were put in a bank in period *t-1* it would earn a gross return of $(1+r_t)P_{t-1}^I$ at t. On the other hand, if P_{t-1}^I dollars were used to buy one unit of capital goods it would collect rent for one period, and the depreciated capital would be worth $(1-\delta)P_t^I$ in period t prices.

[7] Other types of growth models with adjustment costs of investment would have a distinct investment function, i.e. distinct from the household savings function.

The assumption that there are no installation costs means that new investment goods are linearly substitutable for old capital; that is, the price of capital stock is equal to the price of aggregate investment:

(44) $P_t^{KS} = P_t^I$

Equations (40-44) say that, in equilibrium, the price of one unit of capital stock (P^{KS}) is the present value of the discounted stream of rental payments, or capital service flows (P^K)[8]. In the perfect foresight equilibrium path of the solution, the capital rental prices, interest rates and stock prices for each period are such that equation (43) holds. This incorporates the forward-looking dynamics of asset pricing into the model of intertemporal equilibrium. There is also the backward-looking asset accumulation equation (41).

With equation (44) the Euler equation (43) can be rewritten as the well-known cost of capital equation (Jorgenson 1963):

(45) $P_t^K = \dfrac{1}{(1-tk)}[(r_t - \pi_t) + \delta(1+\pi_t)]P_{t-1}^{KS}$

where $\pi_t = (P_t^{KS} - P_{t-1}^{KS}) / P_{t-1}^{KS}$ is the asset inflation rate. This rental price of aggregate capital is the endogenous price that equates the demand for capital by the 35 industries and households with the supply given by KS_{t-1}. When property taxes (taxes based on the value of assets) are included the cost of capital equation becomes:

(45') $P_t^K = [\dfrac{1}{(1-tk)}((r_t - \pi_t) + \delta(1+\pi_t)) + tp]P_{t-1}^{KS}$

The quantity of total investment demanded by the household/investor is I_t^a when the price is $P_t^{KS} = P_t^I$. This aggregate demand for producer durables, consumer durables and inventories is allocated as demand for the *m* individual commodities − Construction of new structures, Machinery, Electric Machinery, Instruments, etc. − by means of a simple production function:

(46) $I^a = I(I_1, I_2, ..., I_m)$

[8] In a model with uncertainty, this would be stated as, "the present value of the expected stream discounted at risk adjusted rates...".

The m types of commodity inputs are the same set as the commodities demanded by the household and producers. In the same way that demands for intermediate inputs are derived from a nested tier of translog price functions in equations (12-14) and Table 2, investment commodity demands are derived from a nested structure of investment price functions[9]. (The details are in Appendix E of Ho (1989)). The price of aggregate investment is thus a function of the prices of commodities:

(47) $P^I = P(P_1^X, P_2^X, ..., P_m^X)$

The value of total investment is thus:

(48) $P^I I^a = \sum_{i=1}^{m} P_i^X I_i$

The value, $P_i^X I_i$, is the ith row of the Investment column in the *use* table (part of the total final demand F in Figure 2).

In summary, capital formation is the outcome of intertemporal optimization. Decisions today are based on expectations of future prices and rates of return, including the world prices of energy. Policies, announced today, that change future rules affect today's decisions.

1.5. Government

The government plays several important roles in IGEM. Government spending affects welfare directly (e.g. through transfer payments) and, indirectly, through public capital that improves private sector productivity. Taxes introduce wedges between buyers and sellers and distort the allocation of resources. IGEM does not incorporate a sophisticated model for public goods and taxation (e.g. median voter models) but instead treats the government sectors in a relatively simple fashion. They are not regarded as optimizing agents. Tax rates and the overall budget deficit are set exogenously as specified by current law and "officially" projected trends conditioned by it. Expenditures on individual commodities are set as simple share functions.

[9] In the household sub-model in section 1.3 the demand for individual commodities is specified with a rich consumption function including demographics and estimated with Consumer Expenditure Survey data. There is no corresponding theory of investment commodity demand.

Following the National Income and Product Accounts, general government purchases are distinguished from government enterprises. The latter are treated as part of the business sector; it is industry number 35. This section considers only the purchases of finished goods and services by federal, state and local governments. The accounting system developed in Jorgenson (1980) regards the social insurance system as internal to the household sector; social security taxes are regarded as private savings and the insurance trust funds regarded like private assets.

1.5.1. General Government Revenues and Expenditures

The taxes that are explicitly recognized are sales taxes, import tariffs, the capital income tax, labor income taxes, the property tax, and the wealth (estate) tax. Sales taxes tt_j were defined in equation (15), the labor tax *tl* was used in equation (29) and (30), and the capital income tax *tk* was used in equation (29) and (45). The property tax appears in the cost of capital equation (45), while the wealth tax is in equation (29). Tariffs, *tr*, are described later in equation (55). There is also an item called non-tax receipts that includes various fees charged by the government (denoted R^N) appearing in equation (31). The final revenue item is the profit or surplus from government enterprises (R^{ent}).

These tax formulations are an abstraction of the complex actual system that includes depreciation allowances, tax credits, "alternative minimum tax", etc. The tax rates are developed from historical data in a manner that replicates actual revenues; they are close to, but not identical with statutory rates[10]. For labor income, there is also the distinction between marginal and average tax rates. For example, in the definition of the price of leisure (equation 30) the labor tax rate is the marginal rate.

Government expenditures fall into 4 major categories – goods and services from the private sector, transfers to households and foreigners, interest payment on debt to households and foreigners, and subsidies. The first three are denoted by V^{GG}, $G^{hh} + G^{row}$ and $i(B^G_{t-1}) + i(B^{G*}_{t-1})$. Subsidies are regarded as negative sales taxes and included in the calculation of tt_j in equation (15). Transfers and interest payments are set exogenously, scaled to preliminary projections of the economy and population and aligned with the

[10] For example, the tax paid on labor income is part of personal income taxes and follow the complex federal and state government income tax rules.

"official" forecasts from the Congressional Budget Office (CBO 2003, 2004). The total spending on commodities (including labor and capital) is V^{GG}, and this is allocated to the individual commodities using shares from the base year:

(49) $P_{it}^{X} G_{it} = \alpha_i^G V_t^{GG} \qquad P_{Gt}^{L} L_{Gt} = \alpha_L^G V_t^{GG} \qquad P_{Gt}^{K} K_{Gt} = \alpha_K^G V_t^{GG}$

The value, $P_i^X G_i$, is the *i*th row of the Government column in the *use* table in Figure 2.

1.5.2. Government Deficits and Debts

The total revenue of the government is thus:

(50)
$$Rev = \sum_j tt_j P_{Y,j} Y_j + \sum_i tr_i P_{M,i} M_i + tkP^K KS_{t-1} + tlP^L LS + tpP^{KS} KS_{t-1} + twP^{KS} KS_{t-1} + R^N + R^{ent}$$

Total government expenditures are:

(51) $Exp = V^{GG} + G^{hh} + G^{row} + i(B_{t-1}^G) + i(B_{t-1}^{G*})$

In the National Income and Product Accounts (NIPA) there is a distinction made between current expenditures and investment spending, and between current receipts and capital transfers. This results in a "current deficit" that is distinct from "net borrowing requirement". No such distinction is made in IGEM. Here, the public deficit of the government is total outlays less total revenues, a concept similar to the official "net borrowing requirement." Denoting the deficit by ΔG:

(52) $\Delta G = Exp - Rev$

The difference between IGEM's accounting of the deficit and NIPA is the treatment of the social insurance surplus. The deficit in IGEM is, conceptually, the NIPA borrowing requirement plus the social insurance fund surplus.

These deficits add to the public debt. Total government debt is separated into that held by US residents and that held by foreigners but in IGEM only the net total debt, $B^G + B^{G*}$ matters. Notationally:

(53) $B_t^G + B_t^{G*} = B_{t-1}^G + B_{t-1}^{G*} + \Delta G_t + \delta^{BG}$

The official accounts of the stock of debt[11] unfortunately are not reconciled with the official deficits given in NIPA. There is, therefore, a discrepancy term, δ^{BG}, in the above

[11] These are given in the *Flow of Funds, Assets and Liabilities* published by the Federal Reserve Bank.

equation. The accounting in equation (53) is in book value terms; there is also an exogenous capital gains term that is omitted.

To summarize, tax rates are set exogenously (and are not necessarily constant in the forecast period), and as is the overall deficit of federal, state and local government. The model generates economic activity and, hence, tax revenues are endogenous. Government transfers and net interest are set exogenously and, so, the remaining item in equation (51), total general government spending on goods (V^{GG}) is determined residually.

1.6. Rest of the world (exports, imports and total supply)

IGEM is a national, one-country model, which is to say that the supply of goods from the rest of the world (ROW) is not modeled explicitly for each commodity. Similarly the demand for U.S. exports is not driven by endogenous world growth rates as is done in multi-country models. IGEM follows the treatment that is standard in one-country models, that is, imports and domestic output are regarded as imperfect substitutes where the elasticities of substitution are not infinite. This is often referred to as the "Armington" assumption and is reasonable at IGEM's level of aggregation[12]. It is also assumed that U.S. demand is not sufficient to change world relative prices.

The total supply of commodity *i* is an aggregate of the domestic and imported varieties:

(54) $$XS_{it} = XS(QC_i, M_i, t)$$

The domestic commodity supply is given in equation (20), while M_i denotes the quantity of competitive imports[13]. This is to be distinguished from non-competing imports described in Section 1.2. The price of competitive imports is the world price multiplied by an "exchange rate" (*e*), plus tariffs (*tr*):

(55) $$P_{M,it} = (1 + tr_{it}) e_t P^*_{M,it}$$

[12] That is, while one may regard the imports of steel of a particular type as perfectly substitutable, the output of the entire Primary Metals sector is a basket of many commodities and would have an estimated elasticity that is quite small.

[13] The notation M_j denoted above the inputs of non-energy materials into the industry production function. The distinction here from M_i as commodity imports should be clear from the context.

e_t is technically the world relative price and its role will be made clear after the discussion of the current account balance below.

The supply function is similar to the production model given in equations (1)-(6). The demand for domestic and imported varieties is derived from a translog price function for the total supply price:

(56) $$\ln P_{it}^X = \alpha_{ct} \ln P_{C,it} + \alpha_{mt} \ln P_{M,it} + \tfrac{1}{2} \sum_{j,k \in \{C,M\}} \beta_{jk} \ln P_{j,it} \ln P_{k,it}$$

(57) $$\frac{P_{M,it} M_{it}}{P_{it}^X XS_{it}} = \alpha_{mt} + \beta_{MM} \ln \frac{P_{M,it}}{P_{C,it}}$$

(58) $$P_{it}^X XS_{it} = P_{C,it} QC_{it} + P_{M,it} M_{it}$$

It should be noted that there now is a closed loop in the flow of commodities. Producers purchase intermediate inputs at price P_i^X and sell output at price $P_{QT,j}$. Prices of intermediates, P_{it}^X, are the prices given in equation (56), that is, the prices of total supply. It is assumed that all buyers buy the same bundle of domestic and imported varieties for each type *i*.

Imports into the U.S. have been rising rapidly during the sample period, not just in absolute terms but as a share of domestic output. As explained in Ho and Jorgenson (1994), this is modeled by indexing the parameter α_m in equations (56 and 57) by time, allowing the share to rise exogenously over time. The β_{MM} coefficient is the share elasticity and, for most goods, is a fairly elastic parameter.

As noted in Section 1.2, one of the inputs into the industry production functions is non-competing imports. These are goods that do not have close U.S. substitutes, e.g. coffee. The demand for these are derived in the nested structure of the production function, the value of such imports by industry *j* is $P_{NC,j} X_{Nj}$.

The demand for U.S. exports should depend on world prices and world incomes. Since these are not modeled endogenously, IGEM begins with an exogenous projection of world incomes and demands (X_{it0}^x). It is assumed that the world price of commodity *i* relative to commodity *k*, ($P_{C,i}^* / P_{C,k}^*$), is not affected by U.S. market outcomes. With these projections, the export demand for commodity *i* is written as a function of domestic prices and the effective world price $eP^*_{C,i}$. Normalizing units such that the world price is 1 yields:

(59) $$X^x_{it} = X^x_{it0}\left(\frac{P_{C,it}}{e_t}\right)^{\eta_i}$$

The estimates of the export elasticity coefficient are also reported in Ho and Jorgenson (1994).

The current account balance is exports minus both types of imports, plus exogenous net interest payments and transfers:

(60) $$CA_t = \sum_i P_{C,i}X^x_i - \sum_i e_t P^*_{Mi}M_i - \sum_j eP^*_{NC,j}X_{Nj} + i(B^*_{t-1}) - i(B^{G*}_{t-1}) - H^{row}_t - G^{row}_t$$

This current account surplus adds to the stock of net U.S. foreign assets, which is equivalent to net private claims on ROW minus net government debt to the ROW:

(61) $$B^*_t - B^{G*}_t = B^*_{t-1} - B^{G*}_{t-1} + CA_t$$

The closure of the foreign sector is treated in various ways in different models. One may either set the current account exogenously and let the world relative price, e_t, move to align exports and imports with it, or set e_t and let the *CA* balance be endogenous. IGEM adopts the former method; that is, the price of imports and exports move with the endogenous e_t so that equation (60) is satisfied.

1.7. Market balances

In IGEM with constant returns to scale and factor mobility, equilibrium prices clear all markets at zero profits each period.

In the commodity markets, the demands in the economy consist of intermediate demands by producers, household consumption, investor demand, government demand and exports. The supply is given by equation (54). In equilibrium we have, for each *i*,:

(62) $$P^X_i XS_i = \sum P^X_i X_{ij} + P^X_i(C_i + I_i + G_i) + P_{C,i}X^x_i$$

In the capital market, the demand for capital input from all industries and households equals the supply:

(63) $$P^K_t KS_{t-1} = \sum_j P_{K,jt}K_{jt}$$

Since capital is mobile across sectors, there is only one price for capital rental that is needed to clear the market. However, in the data, widely different rates of return are observed. To reconcile this, the industry rental price is assumed to be a fixed multiple of the economy's endogenous rental price:

(64) $P_{K,jt} = \psi_j^K P_t^K$

Similarly, in the labor market, the assumption of mobile labor requires the industry specific labor price to be a constant times the economy's market clearing price:

(65) $P_t^L LS_t = \sum_j P_{L,jt} L_{jt}$; $P_{L,jt} = \psi_j^L P_t^L$

The government deficit (equation 52) is satisfied by endogenous spending on goods and services, V^{GG}, and the current account surplus (equation 60) is satisfied by endogenous changes in the world relative price, e_t. The final item is the saving and investment relation:

(66) $S_t^{hh} = P_t^I I_t^a + \Delta G_t + CA_t$

Household saving is first allocated to the two exogenous items – lending to the government to finance the public deficit and lending to the rest of the world. The remainder is allocated to investment in domestic capital. As explained in earlier, in IGEM there are no separate saving and investment decisions; equation (66) holds as a consequence of household intertemporal optimization[14].

IGEM is homogenous in prices. Doubling all prices leaves the equilibrium unchanged. Therefore, any price may be chosen for the purposes of normalization. In IGEM, the after-tax price of labor received by households as the selected as the numeraire and is exogenous to model simulations. In addition, any one of IGEM's equations is implied by Walras Law, that is, if *n-1* equations hold, the n^{th} also will hold. In the current implementation of the model, the labor market equation (equation 65) is dropped and is checked at solution to see that it indeed holds.

1.8. Data underlying the model

[14] In other models where investment is derived separately, e.g. due to sector specific reasons, an endogenous interest rate will clear this S=I equation.

The important data issue relating to the production component of the IGEM model is to identify the price and quantity data that correspond to the concepts laid out in the official input-output tables and that are consistent with the demand components of the model.

The dollar values from the input-output tables are obviously the ones to use to characterize the nominal output of the industries ($P_{QT,jt}Q_{jt}$). IGEM's principal data source is the time series of IO tables put together by the Bureau of Labor Statistics (BLS), Office of Occupational Statistics and Employment Projections. These are constructed from the benchmark tables published every 5 years by the Bureau of Economic Analysis (BEA). This dataset gives the value of output and intermediate inputs of all sectors for 1983-2000. These are combined with an earlier BLS series for 1977-96, and an even earlier version of an internal IO dataset (Jorgenson 1998), giving a sample period of 1958-2000. The BLS dataset also comes with industry prices for the entire 1958-2000 period that are based on their Producer Price Indices (PPI). These are used as $P_{QT,jt}$.

The details of the construction of industry output and K,L,E,M inputs are given in Jorgenson, Ho and Stiroh (2003). The industry capital stock and capital input are derived from the BEA's Capital Stock Study which includes information on investment by 60 asset categories. The industry labor input are derived from detailed demographic and wage data in the annual Current Population Survey and decennial Census.

The data for the final demand for commodities are also made consistent with the benchmark Input-Output tables in the BLS time series. The consumption data for the third stage is taken from the NIPA Personal Consumption Expenditures as described in Jorgenson and Slesnick (1987). This is related to the IO commodity classification using a bridge table like that given in *Benchmark Input-Output Accounts for the U.S. Economy 1992*, Table D[15]. The data for aggregate labor supply and full consumption is described in Jorgenson and Yun (2001) and are derived from population time series cross classified by gender, age and education. The BLS IO series also provide the investment, government exports and imports by the IO commodities. The investment data from the BEA Capital Stock Study may be reconciled with the IO classification via the official IO bridge table (op. cit. *Benchmark* Table E). The government purchases are derived from the annual NIPA government expenditures by broad categories (e.g. *Survey of Current Business* August 2002, p 61, Table 3.7). The export and import data are taken from the detailed Census trade data and reconciled with the official NIPA goods and services trade accounts (*Survey of Current Business* August 2002, p 68, Table 4.3)

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[15] *Survey of Current Business*, November 1997, page 50.

2. Projections of exogenous variables

IGEM simulates the future growth and structure of the U.S. economy over the intermediate term of 25 to 30 years, after which growth is gradually slowed so as to achieve a necessary model closure by means of a zero-growth steady state. The time path of model outcomes is conditional on projections of key exogenous variables that ultimately stabilize to yield the steady state results. Among the most important of these variables are the total population, the time endowment of the working-aged population, the overall government deficit, the current account deficit, labor and capital quality, world prices and government tax policies. Many of these are developed from published sources, "official" and otherwise. The remaining variables are projected from the historical data that underlie the model and its estimation.

The key variable is population growth and demographic change. Population projections are taken from the U.S. Bureau of the Census by sex and individual year of age.[16] During the sample period the population is allocated to educational attainment categories using data from the Current Population Survey in a way parallel to the calculations of labor input described in Jorgenson, Ho and Stiroh (2003). Each adult is given 14 hours a day of time endowment to be used for work and leisure. This quantity of hours for each sex-age-education category then is weighted by labor compensation rates and aggregated to form the national time endowment. The index used is the translog index and the methodology is described in Ho (1989, Appendix C).

Projections beyond the sample period use the Census Bureau forecasts by sex and age. It is assumed that the educational attainment of those aged 35 or younger will be the same as the last year of the sample period; that is, a person who becomes 22 years old in 2020 will have the same chance of having a BA degree as a person in 2000. Those aged 55 and over carry their education attainment with them as they age; that is, the educational distribution of 70 year olds in 2010 is the same as that of 60 year olds in 2000. Those between 35 and 55 have a complex adjustment that is a mixture of these two

[16] Data may be taken from the Bureau of the Census website, data pre1980 in http://eire.census.gov/ popest/archives/pre1980/popclockest.txt, data for 1980-90 in *U.S. Population Estimates by Age, Sex, Race, and Hispanic Origin: 1980 to 1999*, and data 1990+ in

assumptions to allow a smooth improvement of educational attainment that is consistent with the observed profile in 2000.

The results of these calculations, shown in Figure 4, are that population is expected to grow at just under 1.0% per year through 2025, reaching in excess of 460 million by 2060. In addition, the slow improvement of educational attainment means that the time endowment grows only at a modestly faster rate of 1.1% through 2025 and matches population growth thereafter.

The non-price-induced (i.e., the autonomous or exogenous) component of total factor productivity (TFP) growth for each sector is projected using the Kalman filter in equation (11) above, curtailed to achieve steady state by 2050. To illustrate this procedure, Figure 5 plots results for selected industries while Figure 6 provides a historical perspective for the projections for all industries. A negative f_{pt} reduces output prices below costs while a positive f_{pt} raises them above costs (see equation (4)). More importantly, a falling f_{pt} means that the relative price of output is falling more rapidly, i.e. there is positive TFP growth from a quantity perspective. As an example, in Electric Utilities, the sample period, 1958-2000, shows the f_{pt} term first falling, then rising and then falling again. Beyond 2000, IGEM's baseline projections portray, to varying degrees, steadily improving productivity in 30 of IGEM's 35 sectors. Leading the list in projected TFP growth is the well known IT producer, Electrical Machinery. There are, to be sure, several key sectors with negative projected productivity growth including the large Construction and Services industries.

Projecting the factor biases of equations (4) and (6) is accomplished in a manner that is identical to projecting autonomous TFP. Figures 7 and 8 show the results for Electric Utilities and Electrical Machinery, respectively. Beyond 2000, Electric Utilities are projected to be energy-saving. Initially, they are projected to be capital- and labor-using and materials-saving but this reverses toward the end of the current decade. The high technology Electrical Machinery industry is projected to continue to be capital-using and labor-, energy- and materials-saving.

eire.census.gov/popest/data/national/tables/intercensal/intercensal.php. These population data are revised to match the latest censuses (e.g. 1981 data is revised to be consistent with the 1990 Census).

Two other important assumptions that determine the shape of the economy are the government and trade deficits. To achieve a steady-state condition, the levels of government and rest-of-world indebtedness must stabilize to some invariant level in the future. This requires that the government budget and current account deficits trend ultimately to zero balances. The current base case assumptions are plotted in Figure 9. The government deficit follows the forecasts of the Congressional Budget Office (CBO 2003) for the next 10 years and then is set to track to a zero balance by 2038. The current account deficit is presumed to shrink steadily so that it reaches a zero balance by 2030. These simplifying assumptions allow the simulation a smooth transition path to steady state which permits easier computation along the way. These deficits are determinants of long run growth to the extent of their influence on base case capital formation but are substantially less important than the demographic and productivity drivers.

3. Emissions projections and abatement opportunities in IGEM

3.1. Introduction

The Inter-temporal General Equilibrium Model (IGEM) is equipped with a number of array-based "externality" variables that are conceptually and empirically defined to suit the needs of a particular analysis. For example, in one configuration, there are four variables aiding in the assessment of the benefits and costs of climate change and climate change mitigation policies. These are:

1. Carbon emissions arising from fossil fuel use in millions of metric tons, carbon dioxide equivalent (MMTCO$_2$E);
2. Fossil fuel use in physical units, quadrillion Btu;
3. An IGEM construct of total GHG emissions less those arising from agriculture and from the residential and commercial sectors;
4. A composite of total GHG in MMTCO$_2$E covering all gases arising from all sources.

"Externalities" in IGEM are considered as joint outputs or products of the economic activities represented within its structure. These may be process related in that they arise from the creation and manufacture of a particular good or service or they may be product related in that they arise from the economy's use of a particular good or service. In either case, the annual level of each composite externality is jointly determined by the production and consumption activities that give rise to it and, in turn, these activities are associated with the processes and products of domestic industries and with corresponding U.S. imports.

3.2. Emissions projections

The development of IGEM's externality coefficients for energy and the environment is derived from detailed historical data appearing in EPA's *2004 Inventory of U.S. Greenhouse Gas Emissions and Sinks, 1990-2002* and EIA's *Monthly Energy Review*. These series are sorted and aggregated (see Table 5) to create the energy and emissions totals corresponding to the four externality variables defined above. The totals then are expressed relative to the underlying sector-specific economic outputs that give rise to them. It is worth noting that none of the externality coefficients is trendless over the period 1990-2000 which further highlights the difficulties in projecting them (see Figure 10).

In developing baseline projections, there are four inter-related issues. These are:

1. What weight should be attached to each emission factor when dealing with such aggregated sectors?
2. How should emissions coefficients change over time to reflect compositional changes within a sector?
3. To what extent should historical or anticipated mitigation be stripped from or preserved in coefficient trends?
4. To what degree are externality outcomes to be calibrated either to historical data or to "official" projections?

Ideally, and data permitting, analyses should be conducted for each gas and each economic activity; that is, trend first and then aggregate. This solves the problems of weighting and compositional changes and gets the baseline "right." Invariably, however, time and data are unaffordable luxuries. More often than not, aggregation occurs prior to trending. The biases that this introduces in baseline emissions paths can be overcome, however, through development and use of alternative base cases that are directionally appropriate to these biases.

Decisions on trends in mitigation are conditional on the objectives and circumstances of the particular analysis to which the model is being applied. Changes in emissions intensities are both market and policy driven. The extent to which policy driven mitigation is to be left in or stripped from the emissions coefficients depends on whether the particular policy is part of the current assessment. If it is independent then the effects of mitigation should remain; however, if the analysis is retrospective in nature and a portion of the observed mitigation is policy dependent then it should be parsed from the emissions coefficients. The process of isolating the market and policy causes of changes in emissions intensities is obviously much easier the more disaggregated are the data used in their construction.

Calibration is also a matter that depends on the particular analysis; it is generally more important in comparative assessments than it is in those in which a model analysis stands alone. Matching or tracking emissions levels, be they historical or projected, requires either calibrating the variables that drive emissions (and) or adjusting the joint production of emissions per unit of economic activity.

In the current base case, the details of energy use (coal, oil, gas and electricity) in IGEM are consistent with historical data and, generally, with the projections from EIA's 2005 Annual Energy Outlook (AEO). Emissions are calibrated to match the 2000 levels represented in EPA's 2004 emissions inventory. The emissions coefficients for fossil fuels (coal, oil and gas) are held temporally fixed while a common trend, dampening to achieve steady state, is adopted for those coefficients attached to all other economic activities (e.g., agriculture, chemicals, metal manufacturing, electricity transmission and distribution, etc.). For the future, in developing baseline emissions paths, each of the underlying relationships between emissions outcomes and their driving forces merits more independent analysis and evaluative scrutiny. With its diversity of detail, IGEM then could reflect more fully the payoffs from bottom-up investigations of emissions sources.

3.3. Endogenizing exogenous abatement opportunities

Were the emissions intensities of output unresponsive to market or policy driven changes and were all market and technological possibilities fully represented within a model's structure, there would be no need for additional work. Marginal abatement cost schedules derived from model simulations would accurately characterize the economic costs associated with all of the substitutions and all of the market and technological changes that follow from the implementation of a particular mitigation strategy. But emissions intensities are not unresponsive to market circumstances or policy initiatives, and a given model may not fully represent all of the market and technical opportunities that may serve future mitigation. To the extent that abatement possibilities, above and beyond those implicit in a given model, ***and*** their associated costs can be identified, there naturally emerges the question of integration. IGEM employs the following process in endogenizing these external abatement opportunities.

1. For each GHG and each economic activity, those mitigation possibilities are identified that are likely to be adequately represented in IGEM's response to a given policy initiative. These are considered to be internal to IGEM as are the economic costs associated with their implementation. All other possibilities are external to IGEM and require external abatement cost schedules. Currently, all foreseeable abatement opportunities related to carbon emissions are viewed as internal; that is, marginal abatement cost schedules derived from IGEM simulations accurately portray all the economic costs of their intermediate-term mitigation. External to IGEM are those abatement opportunities related to residential and commercial mitigation strategies, non-CO_2 greenhouse gases, international greenhouse gas permit trading, and domestic sequestration (see Table 6 and Figure 11).
2. IGEM is simulated to ascertain its response to the particular mitigation policy. This generates an initial marginal abatement cost (MAC) schedule that serves as the starting point of an iterative process. Typically, this step involves imposing an emissions constraint and observing its corresponding path of permit prices or introducing a path of permit prices and observing its corresponding abatement.
3. The marginal abatement cost schedule from step two (or step six below) is summed horizontally with those cost schedules external to IGEM to create an aggregate marginal abatement cost schedule.
4. The targeted or required level of abatement then is "read" from this schedule and the allocation of abatement to each of the external and internal categories is determined. Because some abatement is being provided from sources external to IGEM, the constraint in IGEM is relaxed or, equivalently, permit prices are reduced.
5. Having determined the abatement benefits from external sources, it is also necessary to calculate and introduce their economic costs. These are determined by integrating the areas underneath the external MAC schedules in accordance

with their allocated amounts of abatement and introducing these costs directly into IGEM. International permit trading is treated as a factor payment (e.g., rent on a tangible asset or income on a financial asset) and is presumed to substitute for a portion of the current account deficit that arises from trade and, accordingly, affects the overall saving-investment balance. The costs associated with domestic sequestration are assumed to be borne entirely by IGEM's agriculture, forestry and fisheries sector. All other costs are allocated to emissions generating activities in proportion to their contributions to baseline GHG emissions. In addition, all costs save those associated with international permit trading are introduced as factor-neutral, or unbiased, changes in input-to-output relationships.

6. IGEM then is re-simulated with less "internal" abatement (or lower permit prices) arising from more "external" abatement purchased with the now endogenized, additional input requirements implicit in the external abatement cost schedules. This yields a new schedule of IGEM marginal abatement costs.
7. Steps three through six are repeated until IGEM's (internal) marginal abatement cost schedule no longer changes from one iteration to another; experience has shown this to be anywhere from one to six iterations of the aforementioned.

The procedure outlined above, though different mechanically, is identical in spirit and outcome to that implemented in the Emissions Prediction and Policy Analysis (EPPA) Model of MIT's Joint Program on the Science and Policy of Climate Change (Hyman, et al., 2002). The iterative process adopted here sacrifices the computational efficiency of the MIT approach to gain fuller use of the informational content portrayed in the external MAC schedules, most specifically, the areas of "no regrets," their precise curvatures and the points at which they become inelastic. Beyond these differences, both approaches succeed in offering quite reasonable ways to endogenize those market and technological abatement opportunities (and their costs) that are identified as lying outside the boundaries of the possibilities inherent in a model's responses.

4. A base case simulation

IGEM's baseline path for the economy evolves through four phases. The near term, e.g., 2000-2010, represents a continuation of recent trends and conditions. The intermediate term, e.g., 2010-2025, reflects the onset of trends to eliminate the nation's budget and trade deficits. The long term, e.g., 2025-2060, involves a systematic transition of all variables to their final-stage, zero-growth, steady-state levels. Factor biases and autonomous productivity trends stabilize. Budget and trade deficits vanish. Tax rates and foreign commodity prices become invariant. Throughout the first three of these phases, there is a gradual slowing in the rates of population and labor force expansion and in the external forces governing productivity and factor substitution. In the case of the latter, there are still the interactions of these with IGEM's emerging patterns of relative prices and so the forces of price-induced technical change are still at work. Beyond 2060, the remaining two of IGEM's driving variables, population and the labor force, stabilize and the economy ceases to grow. This steady-state condition of zero growth is not a prediction; rather, it is an assumption of necessity for the model's solution.

The trends above in their historical context are evident in the data on aggregate spending and inputs to production shown in Table 7. Growth in real GDP and personal consumption is initially in the 2.5 to 3.5% range but averages less than 1.0% over the interval from 2025 to 2060. Growth in capital input, arising from gross investment net of depreciation (capital consumption), and the availability of labor follow similar patterns of declining growth over time. A substantial decline in labor growth is more immediate as population growth slows and households demand more leisure. Subsequently, a slowdown in capital growth occurs as budget and trade deficits are narrowed and households maintain their spending behavior. Finally, aggregate productivity shows a modest decline to one-half of one percent, 2000-2025, but averages only 0.1% per annum, 2025-2060. The productivity trend reflects the combined influences of price-induced technical change and the autonomous productivity projections described in the Section 2.

Sectoral trends on total factor productivity (TFP) are displayed in Figures 12 (simulated) and 13 (historical). For the near and intermediate terms, TFP growth rates appear as extensions of their long-run histories. Within the context of an overall slowing, productivity growth in U.S. agriculture and high technology manufacturing continues to lead and flourish while that in services, publishing, motor vehicles and construction continues to lag and languish.

Growth in the total output of the U.S. economy, including all intermediate goods and services as well as value added and all final spending (GDP), averages around 2.0% over the period 2000-2025. The projected composition, portrayed in Figures 14 and 15, again evolves as an extension of historic market behavior (Figure 16). High technology manufacturing and the financial sector continue to enjoy relatively more rapid growth while the mining, metals, energy and agricultural sectors continue to grow less rapidly. Domestic motor vehicle manufacturing and construction are also among the relatively slower growing industries.

Of particular relevance to this analysis are the emerging patterns of energy use and greenhouse gas emissions. Figure 15 provides evidence of the changing mix of energy inputs. All of the energy sectors experience slower than average rates of growth over the period 2000-2025. Domestic oil and gas extraction and coal production are the slowest growing, natural gas and electric utility outputs are the fastest growing and growth in petroleum refinery output lies in between. As shown in Table 8, aggregate fossil fuel use tracks the overall economy but at a slower rate. The carbon emissions from fossil fuel use grow initially at an even slower rate reflecting the changing relative mix of energy inputs toward oil and gas and away from coal. Beyond 2010, this change in relative importance has largely occurred and the carbon emissions associated with fossil fuel use grow in line with the corresponding physical quantity.

As discussed in Section 4, the (physical) energy and emissions coefficients for fossil fuels (coal, oil and gas) are constant over time while a common and declining trend is adopted for the emissions coefficients attached to all other economic activities (e.g., agriculture, chemicals, metal manufacturing, electricity transmission and distribution, etc.). Thus, in these latter cases, there are degrees of “autonomous” change reflected in the base case emissions projections. This is evidenced in the projections of greenhouse gases presented in Table 8. Greenhouse gas emissions grow more slowly than fossil fuel use and the emissions from same because of the structural changes in the mix of economic activities and because of the representation of observed behavior in the form of “autonomous” efficiency improvements.

Projected energy- and emissions-efficiency improvements continue well into the future but at rates that are somewhat slower than historically observed (Table 9). The annual reduction in the energy-intensity of real GDP averages 1.0%, 2000-2010, with emissions efficiency improvements averaging 1.2% for the carbon from fossil fuel use and 1.4% for total greenhouse gases. The annual rates of energy- and emissions-efficiency improvement diminish as the economy heads toward steady state, averaging 0.2%, 2025-2060. It should be noted that these diminishing rates of efficiency improvement also are consistent with the broader trends of recent history (Table 9).

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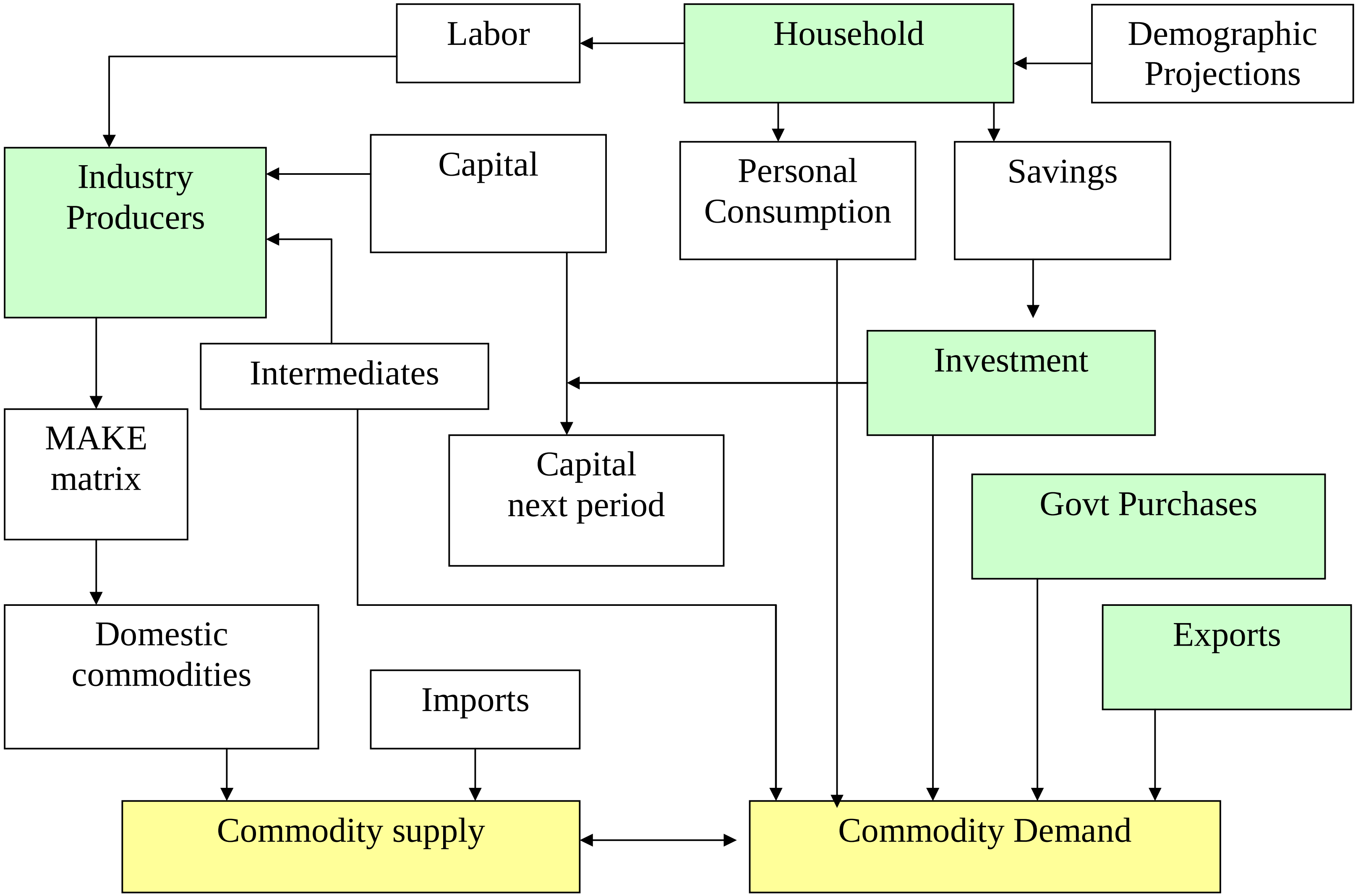
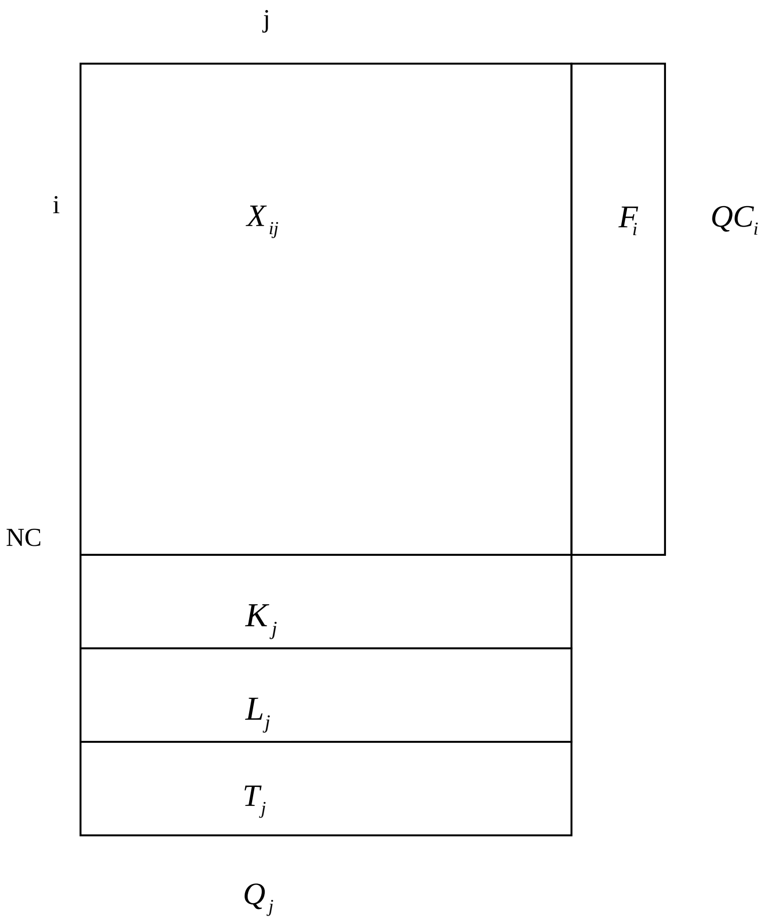
Figure 1: Flow of goods and factors in IGEM





Q_j: industry j output
QC_i: quantity of domestic commodity i
K: capital input
L: labor input
T: sales tax
NC: noncompeting imports
X_{ij}: quantity of intermediate input i into j
F_i: final demand for commodity i (C+I+G+X-M)
M_{ji}: quantity of commodity i made by industry j

Figure 2: Input-output USE table.

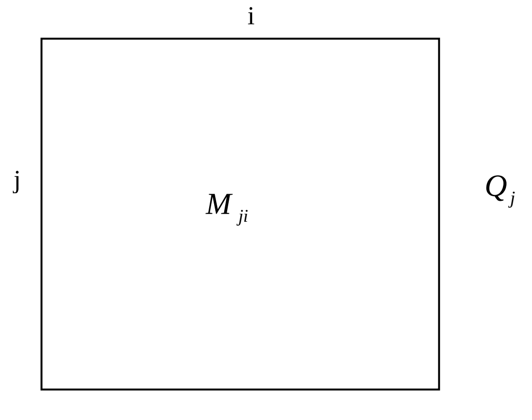


Figure 3: Input-output MAKE table.

Table 1: IGEM's Industry and Commodity Detail

Sector	Description
1	Agriculture, forestry, fisheries
2	Metal mining
3	Coal mining
4	Crude oil and gas extraction
5	Non-metallic mineral mining
6	Construction
7	Food and kindred products
8	Tobacco manufactures
9	Textile mill products
10	Apparel and other textile products
11	Lumber and wood products
12	Furniture and fixtures
13	Paper and allied products
14	Printing and publishing
15	Chemicals and allied products
16	Petroleum refining
17	Rubber and plastic products
18	Leather and leather products
19	Stone, clay and glass products
20	Primary metals
21	Fabricated metal products
22	Non-electrical machinery
23	Electrical machinery
24	Motor vehicles
25	Other transportation equipment
26	Instruments
27	Miscellaneous manufacturing
28	Transportation and warehousing
29	Communications
30	Electric utilities (services)
31	Gas utilities (services)
32	Wholesale and retail trade
33	Finance, insurance and real estate
34	Personal and business services
35	Government enterprises

Table 2. Tier structure of industry production function.

	Symbol	Name	Components
1	Q	Gross output	capital, labor, energy, materials Q=f(K,L,E,M)
2	E	Energy	coal mining, petroleum & gas mining, petroleum refining, electric utilties, gas utilities E=f(X3,X4,X16,X30,X31)
3	M	Materials (nonenergy)	Construction, Agriculture Mat, Metallic Mat, Nonmetallic Mat, Services Mat M=f(X6,MA,MM,MN,MS)
4	MA	Agriculture materials	Agriculture, Food manuf, Tobacco, Textile-apparel, Wood-paper MA=f(X1,X7,X8,TA,WP)
5	MM	Metallic Materials	Fab-other metals, Machinery mat, Equipment MM=f(FM,MC,EQ)
6	MN	Nonmetallic Materials	Nonmetal mining, Chemicals, Rubber, Stone, Misc manuf MN=f(X5,X15,X17,X19,X27)
7	MS	Services Materials	Transportation, Trade, FIRE, Services, OS MS=f(X28,X32,X33,X34,OS)
8	TA	Textile-apparel	Textiles, Apparel, Leather TA=f(X9,X10,X18)
9	WP	Wood-paper	Lumber-wood, Furniture, Paper, Printing WP=f(X11,X12,X13,X14)
10	OS	Other services	Communications, Govt. enterprises, NC imports OS=f(X29,X35,X_N)
11	FM	Fab-other Metals	Metal mining, Primary metals, Fabricated metals FM=f(X2,X20,X21)
12	MC	Machinery materials	Ind. Machinery, Electric Machinery MC=f(X22,X23)
13	EQ	Equipment	Motor vehicles, Other transp equip, Instruments EQ=f(X24,X25,X26)

Table 3. Commodities classified by NIPA Personal Consumption Expenditures

	IGEM classes	NIPA PCE classes
1	Food	Food purchased for off-premise consumption
2	Meals	Purchased Meals and Beverages
3	Meals-Employees	Food furnished employees incl. farms
4	Shoes	Shoes
5	Clothing	Clothing and accesories except shoes; Clothing military
6	Gasoline	Gasoline and Oil
7	Coal	Fuel Oil and Coal
8	Fuel	Fuel Oil and Coal
9	Tobacco	Tobacco products
10	Cleaning	Cleaning and misc. household supplies and paper
11	Furnishings	Semi-durable house furnishings
12	Drugs	Drug preparations and sundries
13	Toys	Nondurable toys and sport supplies
14	Stationery	Stationery and writing supplies
15	Imports	Expenditures abroad by US residents
16	Reading	Magazines, newspapers; Flowers and potted plants
17	Rental	Tenant-occupied nonfarm; Farm dwellings; Housing-other
18	Electricity	Electricity
19	Gas	Gas
20	Water	Water and sanitary services
21	Communications	Telephone and Telegraph
22	Labor	Domestic service
23	Other household	Household Operation- Other
24	Own transportation	User-operated transportation services
25	Transportation	Purchased local transportation; Intercity transportion
26	Medical Services	Physicians; Dentists; Other professional; Hospitals & homes
27	Health Insurance	Health Insurance
28	Personal services	Cleaning, storage, repair; Cothing-Other; Barbershops etc.
29	Financial services	Brokerage; Bank service; Services without payment; Expense of life insurance and pension plans
30	Other services	Legal services, Funeral & burial, Personal business-other
31	Recreation	Repair; Admissions to spectator amusements; Clubs; Commercial participant amusements; pari-mutuel; Recreation-other
32	Education Inst.	Education and research; Religious and welfare activities
33	Foreign Travel	Foreign Travel by US residents
34	Owner maintenance	Imputations for maintenance of owner occupied housing
35	Durables	Imputed rental value from all durable classes: Jewelry and watches; Furniture; ..... Video and audio goods

Note: NIPA-PCE classes are those given in the National Accounts in the annual *Survey of Current Business* , e.g. (SCB August 2001, Table 2.4).

Table 4. Tier structure of consumption function.

	Symbol	Name	Components
1	V	Consumption	**Energy, Food, Nondurables, Capital, Services group**
			V=f(EN,FD,ND,K,SV)
2	EN	Energy	Gasoline, **Fuel Coal**, Electricity, Gas
			EN=f(C6,FC,C18,C19)
3	FD	Food	Food, Meals, Meals-employees, Tobacco
			FD=f(C1,C2,C3,C9)
4	ND	Nondurables	**Clothing-shoe**, **Household Nondurables**, Drugs, **Nondurable misc**
			ND=f(CS,HHN,C12,NDM)
5	K	Capital services	Capital service flow from household capital
			K=f(C35)
6	SV	Services	**Housing-tenant, Household services, Transportation, Medical**
			Services-misc
			SV=f(HS, HHS, TR, MD, SVM)
7	FC	Fuel Coal	Coal, Fuel Oil
			FC=f(C7,C8)
8	CS	Cothing Shoe	Clothing, Shoes
			CS=f(C4,C5)
9	HHN	Household	Cleaning, Furnishings
		Nondurables	HHN=f(C10,C11)
10	NDM	Nondurables	Toys, Stationery, Imports, Reading
		miscellaneous	NDM=f(C13,C14,C15,C16)
11	HS	Housing tenant	Rental, Owner maintainence
		Services	HS=f(C17,C34)
12	HHS	Household	Water, Communications, Labor, Other household
		services	HHS=f(C20,C21,C22,C23)
13	TR	Transportation	Own transportation, transportation
			TR=f(C24,C25)
14	MD	Medical	Medical services, Health Insurance
			MD=f(C26,C27)
15	SVM	Services	Personal services, **Business services, Recreation**, Education inst.
		miscellaneous	SVM=f(C28,BS,RR,C32)
16	BS	Business	Financial services, Other services
		Services	BS=f(C29,C30)
17	RR	Recreation	Recreation, Foreign Travel
			RR=f(C31,C33)

Table 5. Greenhouse Gas Emissions - By Gas, Activity and Sector

	IGEM Sector	MMTCO2E 1990	MMTCO2E 2000	MMTCE 1990	MMTCE 2000
CO2					
Coal					
Residential	3	2.4	1.1	0.7	0.3
Commercial	3	12.1	8.6	3.3	2.3
Industrial	3	150.3	133.8	41.0	36.5
Electricity Generation	3	1515.9	1890.5	413.4	515.6
U.S. Territories	3	0.6	0.9	0.2	0.2
Natural Gas					
Residential	31	238.8	270.3	65.1	73.7
Commercial	31	142.6	174.3	38.9	47.5
Industrial	31	421.6	473.8	115.0	129.2
Transportation	31	35.9	35.5	9.8	9.7
Electricity Generation	31	176.0	280.7	48.0	76.6
U.S. Territories	31	-	0.7	-	0.2
Petroleum					
Residential	16	98.3	107.8	26.8	29.4
Commercial	16	69.5	54.2	19.0	14.8
Industrial	16	394.7	392.1	107.6	106.9
Transportation	16	1422.3	1714.2	387.9	467.5
Electricity Generation	16	100.1	90.4	27.3	24.7
U.S. Territories	16	33.1	44.4	9.0	12.1
Ammonia Production and Urea Application	15	19.3	19.6	5.3	5.3
Soda Ash Manufacture and Consumption	15	4.1	4.2	1.1	1.1
Titanium Dioxide Production	15	1.3	1.9	0.4	0.5
Phosphoric Acid Production	15	1.5	1.4	0.4	0.4
Carbon Dioxide Consumption	15	0.9	1.0	0.2	0.3
Cement Manufacture	19	33.3	41.2	9.1	11.2
Lime Manufacture	19	11.2	13.3	3.1	3.6
Limestone and Dolomite Use	19	5.5	6.0	1.5	1.6
Iron and Steel Production	20	85.4	65.7	23.3	17.9
Aluminum Production	20	6.3	5.7	1.7	1.6
Ferroalloys	20	2.0	1.7	0.5	0.5
Geothermal*	30	0.4	0.4	0.1	0.1
Natural Gas Flaring	31	5.8	5.8	1.6	1.6
Waste Combustion	34	10.9	18.0	3.0	4.9

CH4					
Enteric Fermentation	1	117.9	115.7	32.2	31.6
Manure Management	1	31.0	38.0	8.5	10.4
Stationary Sources - Wood residential	1	8.2	7.7	2.2	2.1
Rice Cultivation	1	7.1	7.5	1.9	2.0
Agricultural Residue Burning	1	0.7	0.8	0.2	0.2
Coal Mining	3	81.9	56.2	22.3	15.3
Abandoned Coal Mines	3	3.4	4.4	0.9	1.2
Petrochemical Production	15	1.2	1.7	0.3	0.5
Petroleum Systems	16	28.9	23.5	7.9	6.4
Mobile Sources	16	5.0	4.4	1.4	1.2
Iron and Steel Production	20	1.3	1.2	0.4	0.3
Natural Gas Systems	31	122.0	125.7	33.3	34.3
Landfills	34	210.0	199.3	57.3	54.4
Wastewater Treatment	34	24.1	28.4	6.6	7.7
N2O					
Agricultural Soil Management	1	262.8	289.7	71.7	79.0
Manure Management	1	16.2	17.7	4.4	4.8
Field Burning of Agricultural Residues	1	0.4	0.5	0.1	0.1
Stationary Sources - Coal	3	4.4	5.2	1.2	1.4
Nitric Acid	15	17.8	19.6	4.9	5.3
Adipic Acid	15	15.2	6.0	4.1	1.6
N2O Product Usage	15	4.3	4.8	1.2	1.3
Mobile Sources	16	50.7	57.4	13.8	15.7
Stationary Sources - Petroleum	16	5.5	6.1	1.5	1.7
Stationary Sources - Natural Gas	31	2.7	3.1	0.7	0.9
Human Sewage	34	12.8	15.3	3.5	4.2
Waste Combustion	34	0.4	0.4	0.1	0.1
HFCs PFCs and SF6					
Substitution of Ozone Depleting Substances	15	0.3	75.1	0.1	20.5
HCFC-22 Production	15	35.0	29.8	9.5	8.1
Magnesium Production and Processing	15	5.4	3.2	1.5	0.9
Aluminum Production	20	18.1	8.9	4.9	2.4
Semiconductor Manufacture	23	2.9	6.3	0.8	1.7
Electrical Transmission and Distribution	30	29.2	15.9	8.0	4.3
Total GHG		6128.9	7038.7	1671.5	1919.6
Non-covered GHG		1008.0	1093.9	274.9	298.3
Residential and Commercial		563.7	616.3	153.7	168.1
Agricultural		444.3	477.6	121.2	130.3
Covered GHG		5120.9	5944.8	1396.6	1621.3
Covered as Percentage of Total GHG		83.6%	84.5%	83.6%	84.5%

MMTCO2E - Millions of metric tons, carbon dioxide equivalent
MMTCE - Millions of metric tons, carbon equivalent

Table 6. Marginal Abatement Cost Schedules

Cost in 2000 dollars per metric ton, carbon dioxide equivalent

Abatement in millions of metric tons, carbon dioxide equivalent (MMTCO2E)

	Covered and unlimited		Non-covered and limited				
	Internal to IGEM	External to IGEM					
Cost	IGEM	Non-CO2 GHG	Non-covered HH & Small Bus	International Trading	Stavins-Richard Sequestration	Total Limited Offsets	Total
-$5.45	0.0	2.7	0.0	8.1	0.0	8.1	10.8
-$2.73	0.0	5.6	0.0	31.4	0.0	31.4	37.0
$0.00	0.0	77.8	0.0	95.5	0.0	95.5	173.3
$2.73	302.5	123.9	23.3	667.3	0.0	690.6	1117.0
$4.33	484.7	143.1	35.7	992.3	0.0	1028.0	1655.8
$5.45	595.8	156.4	43.7	1219.1	194.3	1457.1	2209.4
$8.18	854.3	181.3	60.8	1666.8	572.0	2299.7	3335.3
$10.91	1100.4	207.9	78.9	2023.9	876.3	2979.1	4287.3
$13.64	1340.5	263.7	96.1	2349.0	1136.7	3581.8	5186.0
$27.27	2349.6	302.6	173.2	3330.9	2134.0	5638.1	8290.3
$40.91	3283.5	324.7	242.1	3922.0	2882.0	7046.0	10654.2
$54.55	4084.7	346.4	301.1	3947.8	3509.0	7758.0	12189.1

Table 7. Characteristics of Base Case Growth - The Economy

Average Annual Growth Rates in Percent

	Historical		Simulated	
	1960-2000	2000-2010	2010-2025	2025-2060
Demand Side				
GDP	3.42%	2.65%	1.60%	0.77%
Household Spending	3.60%	3.52%	1.53%	0.69%
Supply Side				
Value Added	3.39%	2.47%	1.38%	0.71%
Capital Input	4.03%	3.72%	1.23%	0.40%
Labor Input	1.87%	0.85%	0.68%	0.68%
Productivity	0.62%	0.50%	0.48%	0.14%

Table 8. Characteristics of Base Case Growth - Energy and Emissions
Average Annual Growth Rates in Percent

	Historical 1990-2000	2000-2010	Projected 2010-2025	2025-2060
Fossil Fuel Use	1.60%	1.65%	1.14%	0.56%
GHG Excl. Agriculture, Residential & Commercial	1.51%	1.24%	1.01%	0.55%
GHG - Total	1.40%	1.23%	0.96%	0.54%
Carbon from Fossil Fuel Use	1.66%	1.50%	1.08%	0.55%

Table 9. Characteristics of Base Case Growth - Energy and Emissions Intensities

Average Annual Growth Rates in Percent

	Historical 1990-2000	2000-2010	Projected 2010-2025	2025-2060
Fossil Fuel Use	-1.62%	-1.00%	-0.46%	-0.21%
GHG Excl. Agriculture, Residential & Commercial	-1.71%	-1.41%	-0.59%	-0.22%
GHG - Total	-1.82%	-1.43%	-0.64%	-0.24%
Carbon from Fossil Fuel Use	-1.56%	-1.15%	-0.52%	-0.22%

Trends in energy and emissions per unit real GDP

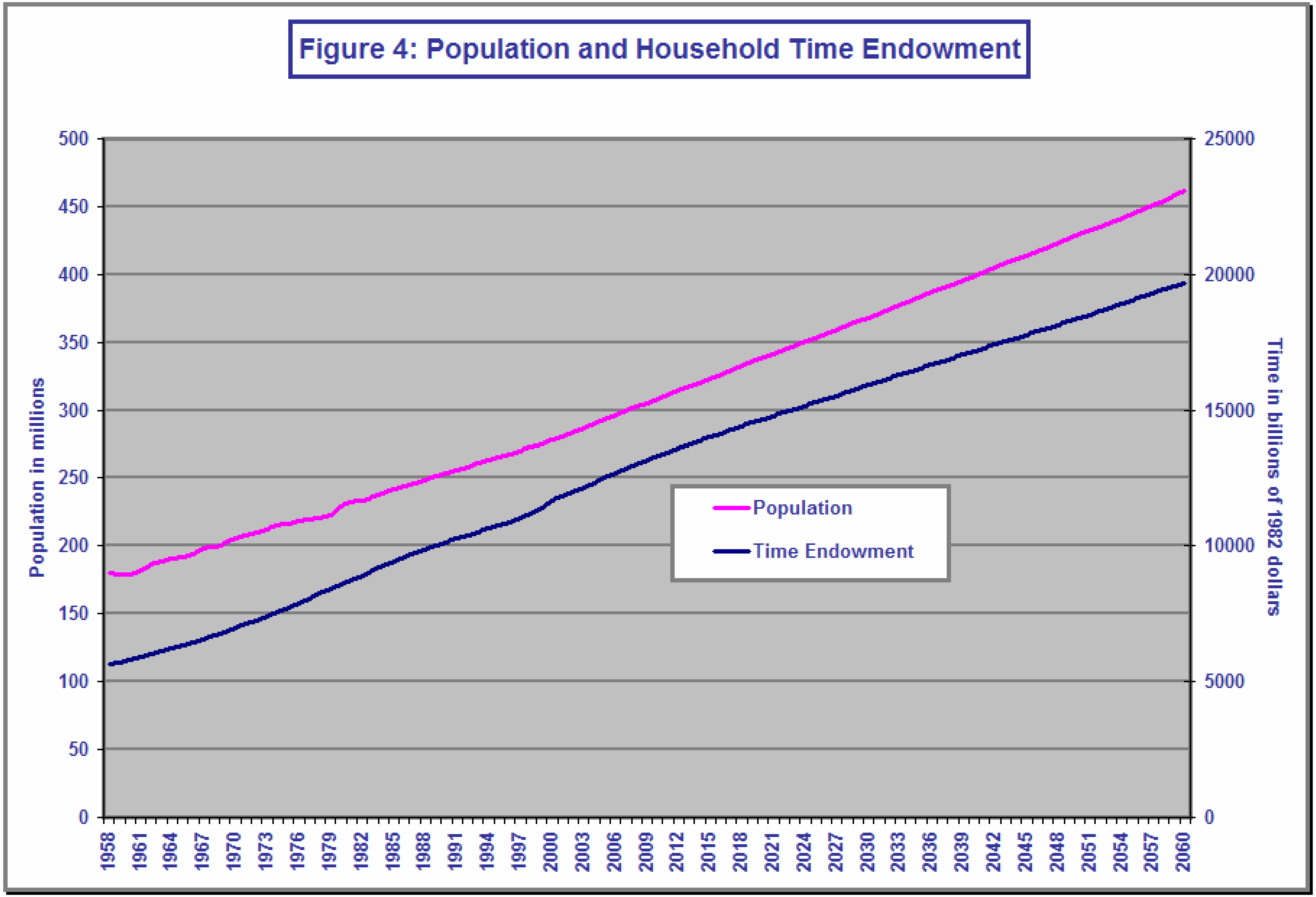


Figure 4: Population and Household Time Endowment
Population in millions
Time in billions of 1982 dollars
Population
Time Endowment
500
450
400
350
300
250
200
150
100
50
0
25000
20000
15000
10000
5000
0
1958
1961
1964
1967
1970
1973
1976
1979
1982
1985
1988
1991
1994
1997
2000
2003
2006
2009
2012
2015
2018
2021
2024
2027
2030
2033
2036
2039
2042
2045
2048
2051
2054
2057
2060

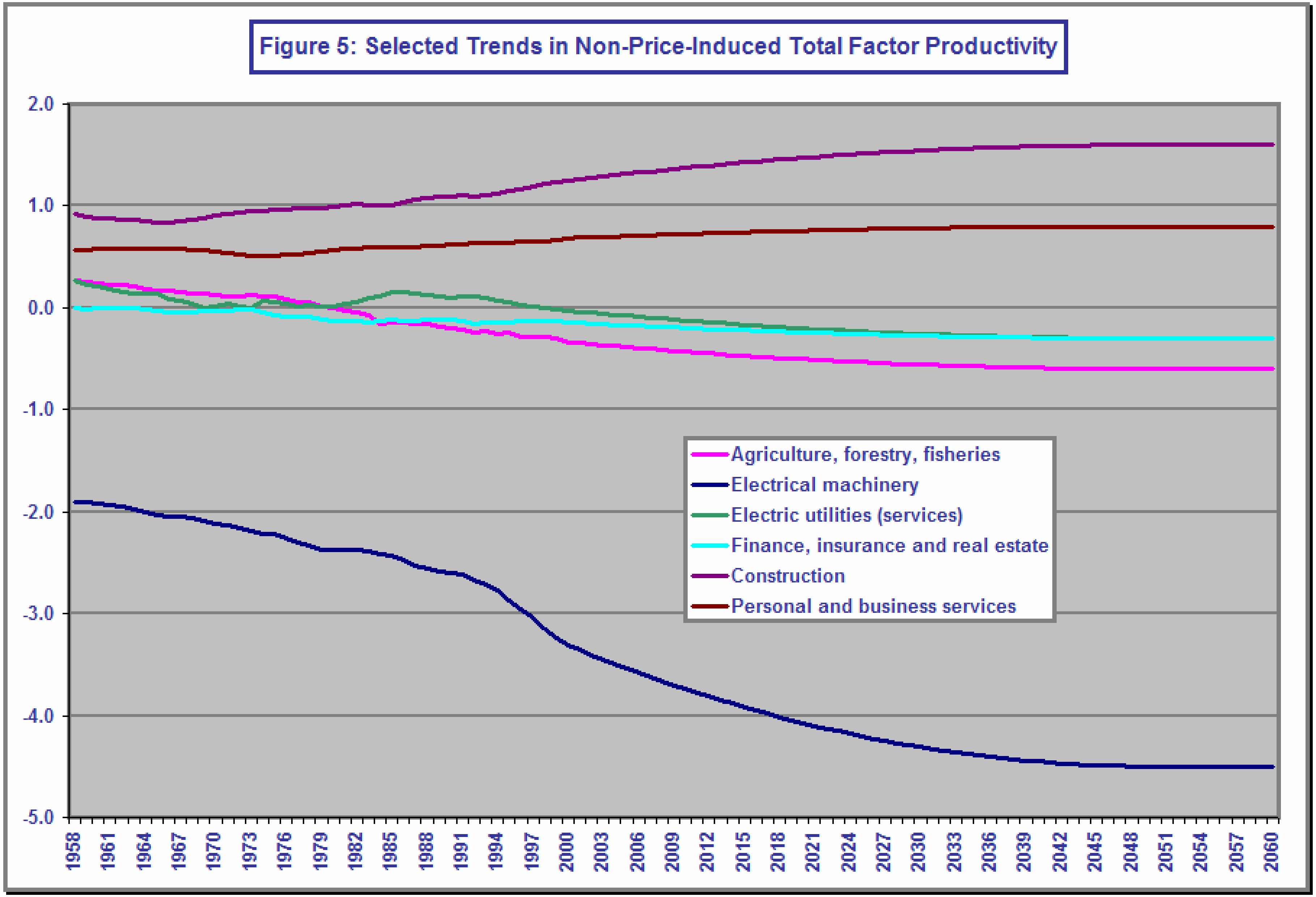


Figure 5: Selected Trends in Non-Price-Induced Total Factor Productivity
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
1958
1961
1964
1967
1970
1973
1976
1979
1982
1985
1988
1991
1994
1997
2000
2003
2006
2009
2012
2015
2018
2021
2024
2027
2030
2033
2036
2039
2042
2045
2048
2051
2054
2057
2060
Agriculture, forestry, fisheries
Electrical machinery
Electric utilities (services)
Finance, insurance and real estate
Construction
Personal and business services

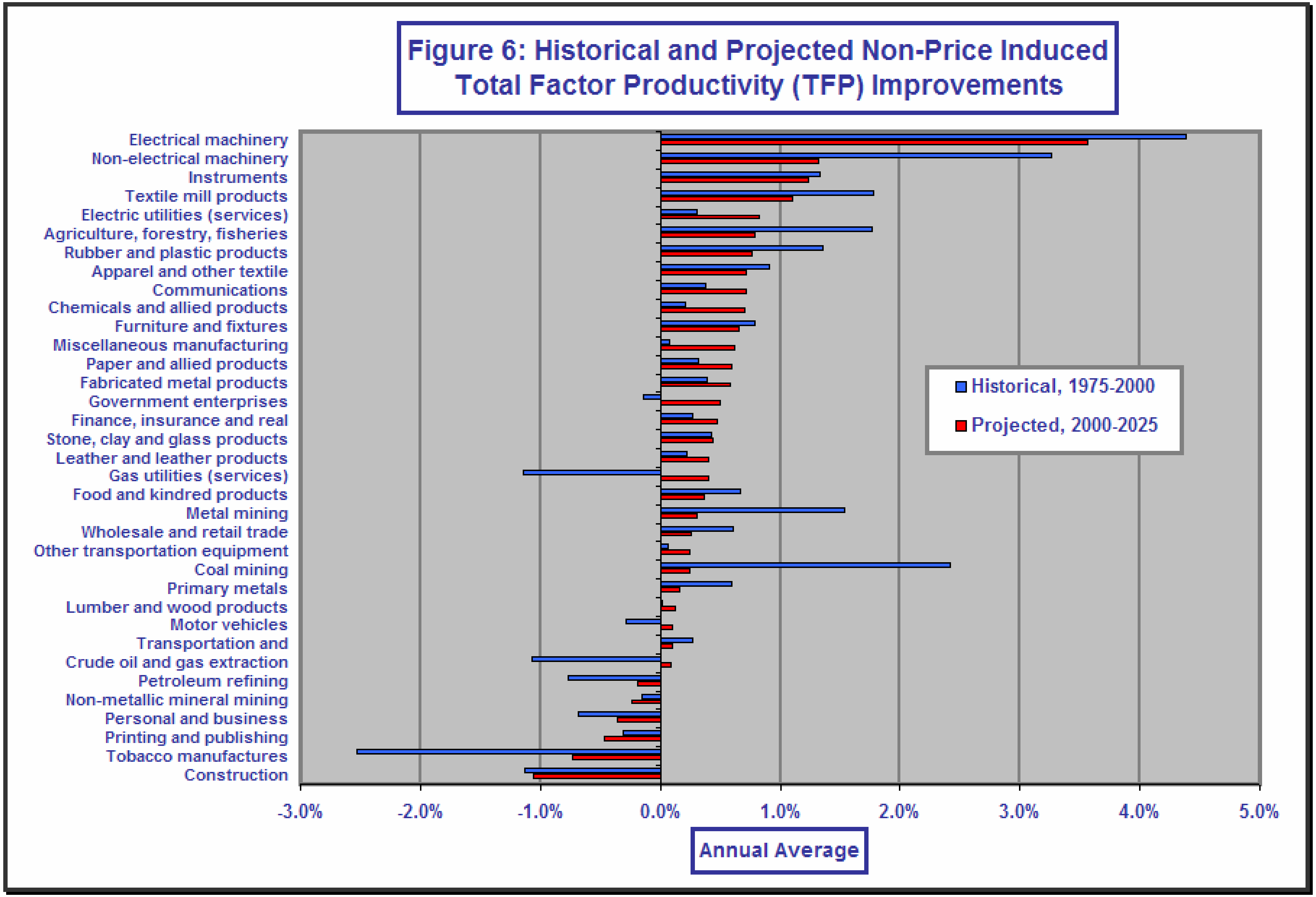


Figure 6: Historical and Projected Non-Price Induced Total Factor Productivity (TFP) Improvements
Electrical machinery
Non-electrical machinery
Instruments
Textile mill products
Electric utilities (services)
Agriculture, forestry, fisheries
Rubber and plastic products
Apparel and other textile
Communications
Chemicals and allied products
Furniture and fixtures
Miscellaneous manufacturing
Paper and allied products
Fabricated metal products
Government enterprises
Finance, insurance and real
Stone, clay and glass products
Leather and leather products
Gas utilities (services)
Food and kindred products
Metal mining
Wholesale and retail trade
Other transportation equipment
Coal mining
Primary metals
Lumber and wood products
Motor vehicles
Transportation and
Crude oil and gas extraction
Petroleum refining
Non-metallic mineral mining
Personal and business
Printing and publishing
Tobacco manufactures
Construction
Historical, 1975-2000
Projected, 2000-2025
-3.0%
-2.0%
-1.0%
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
Annual Average

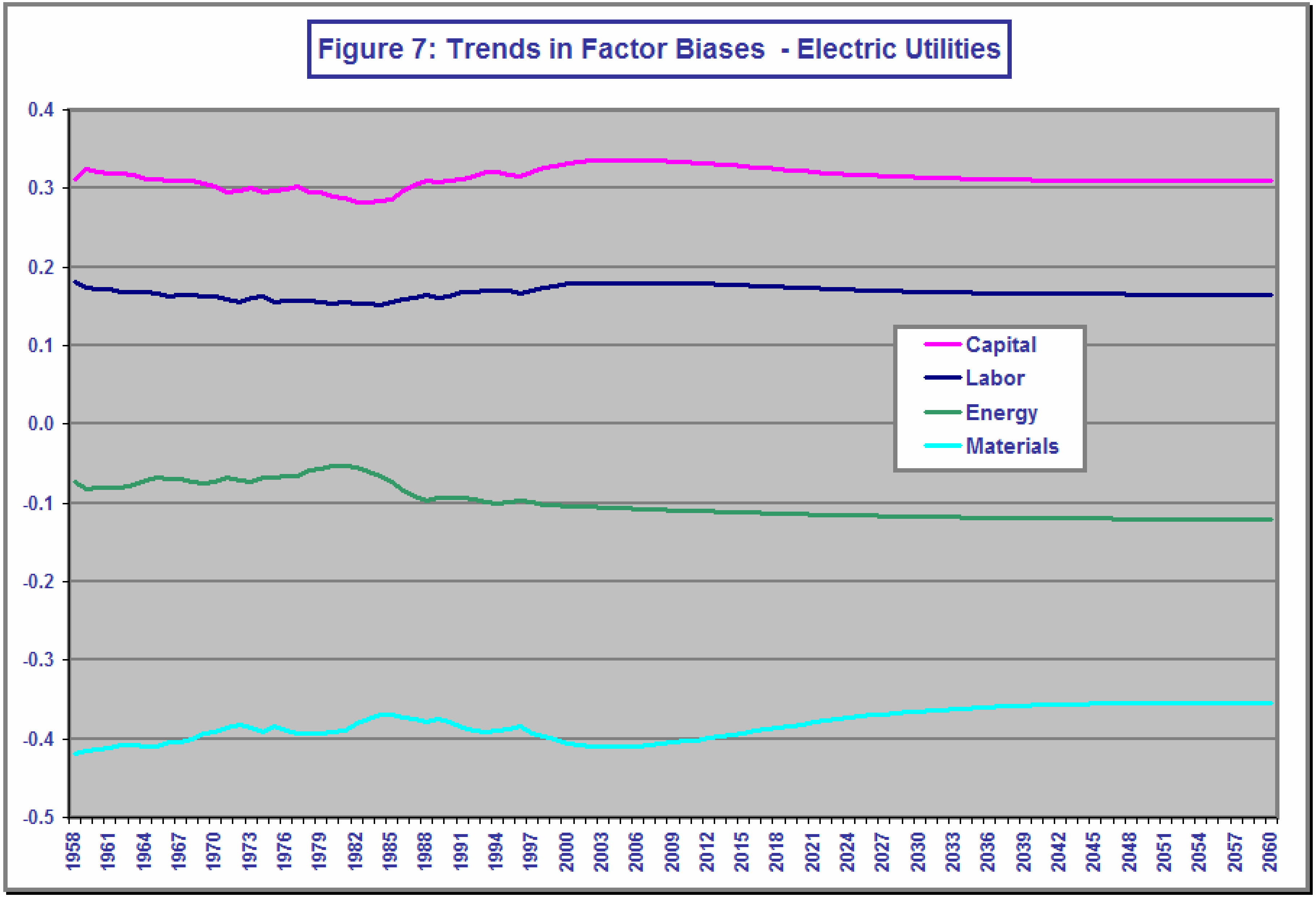


Figure 7: Trends in Factor Biases - Electric Utilities
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
Capital
Labor
Energy
Materials
1958
1961
1964
1967
1970
1973
1976
1979
1982
1985
1988
1991
1994
1997
2000
2003
2006
2009
2012
2015
2018
2021
2024
2027
2030
2033
2036
2039
2042
2045
2048
2051
2054
2057
2060

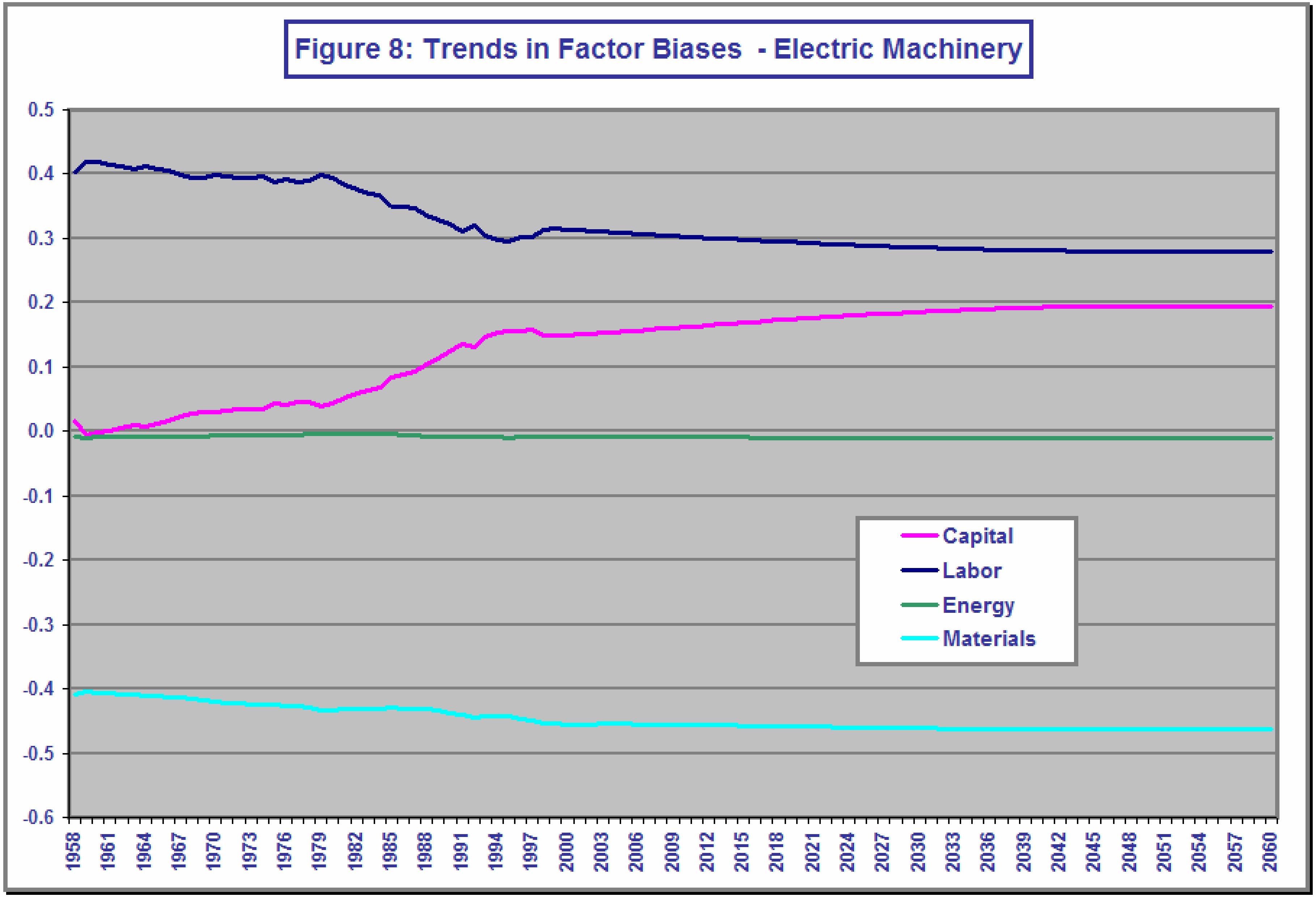
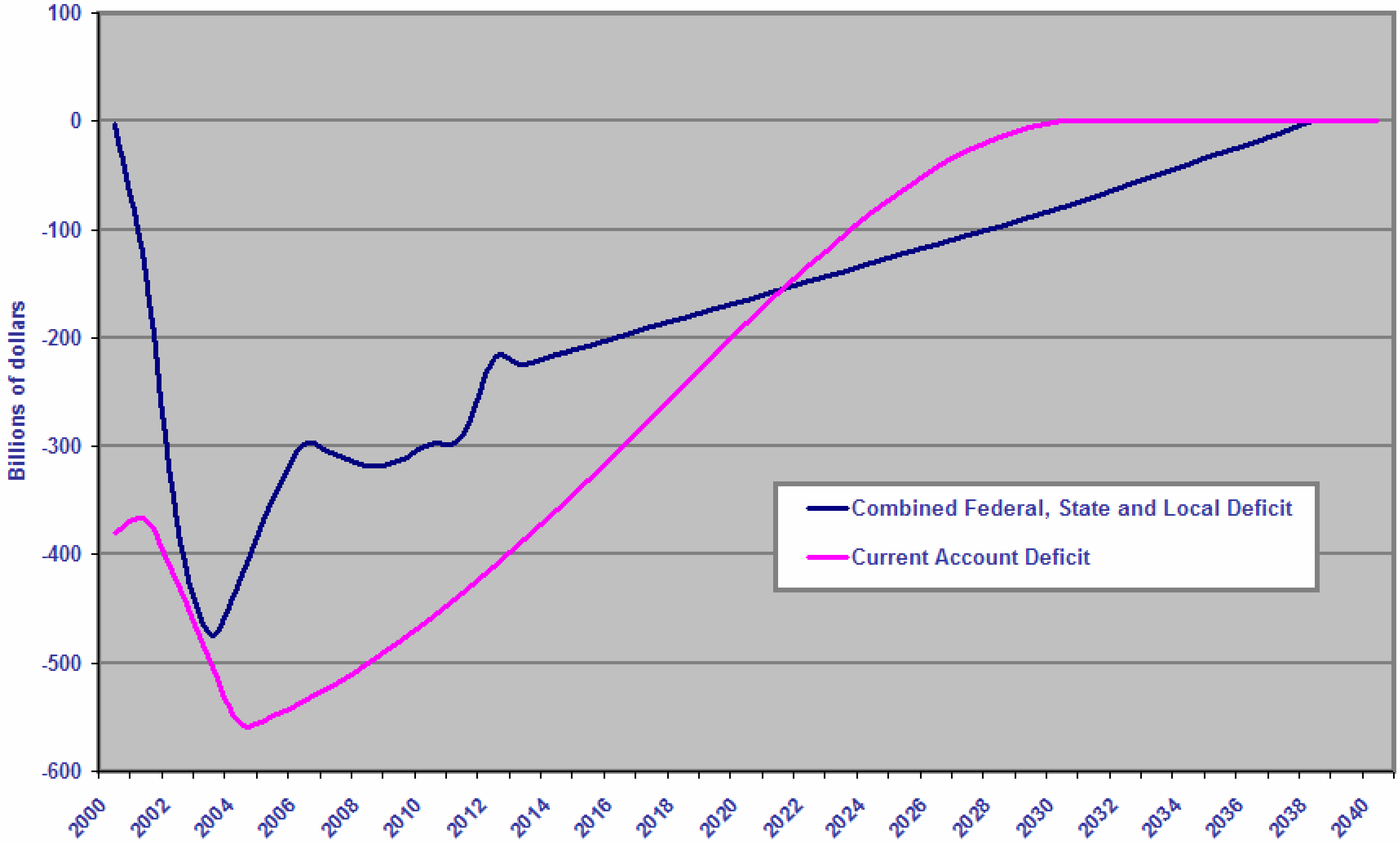


Figure 8: Trends in Factor Biases - Electric Machinery
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
Capital
Labor
Energy
Materials
1958 1961 1964 1967 1970 1973 1976 1979 1982 1985 1988 1991 1994 1997 2000 2003 2006 2009 2012 2015 2018 2021 2024 2027 2030 2033 2036 2039 2042 2045 2048 2051 2054 2057 2060

Figure 9: Government and Current Account Deficits



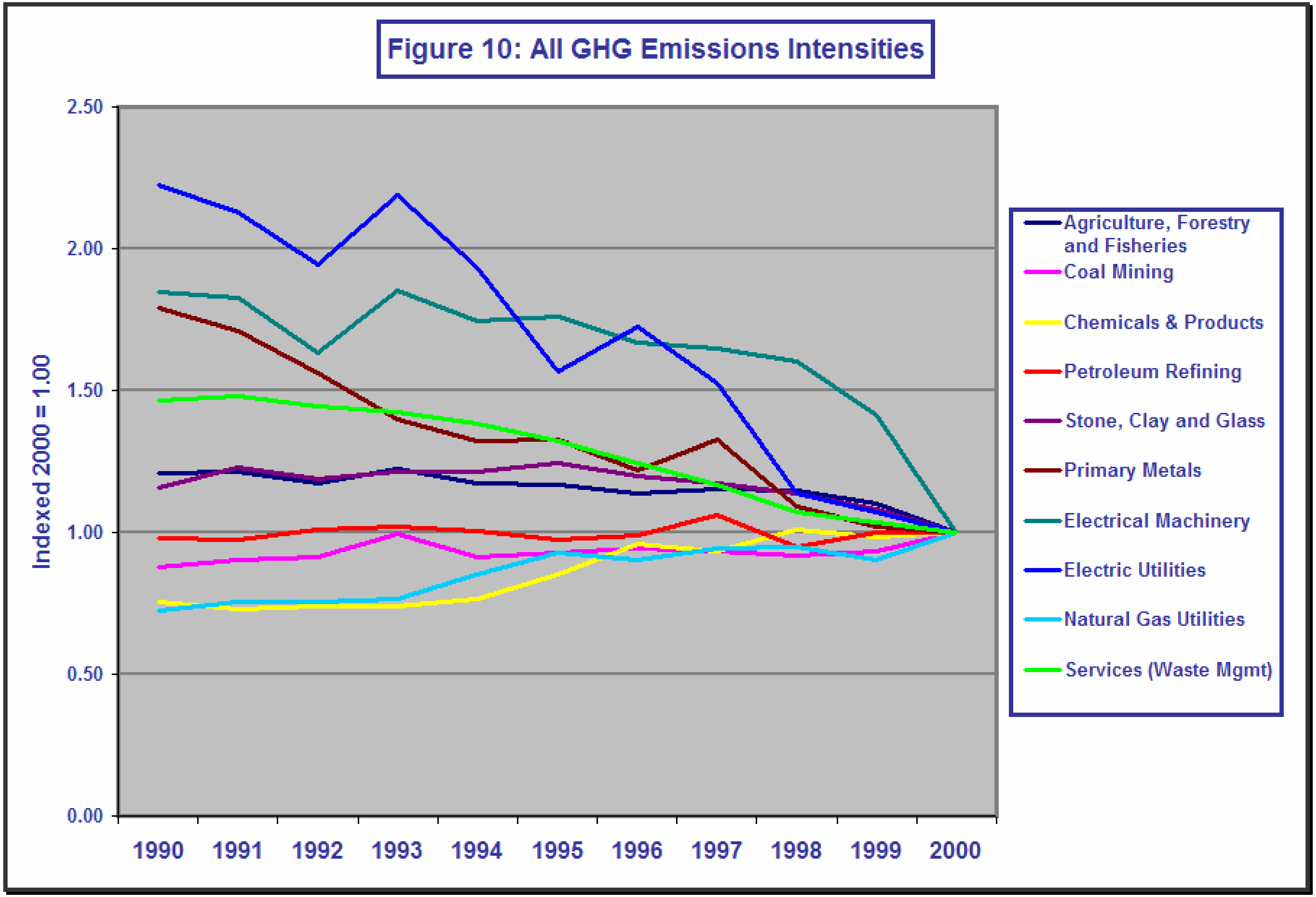


Figure 10: All GHG Emissions Intensities
Indexed 2000 = 1.00
0.00
0.50
1.00
1.50
2.00
2.50
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Agriculture, Forestry and Fisheries
Coal Mining
Chemicals & Products
Petroleum Refining
Stone, Clay and Glass
Primary Metals
Electrical Machinery
Electric Utilities
Natural Gas Utilities
Services (Waste Mgmt)

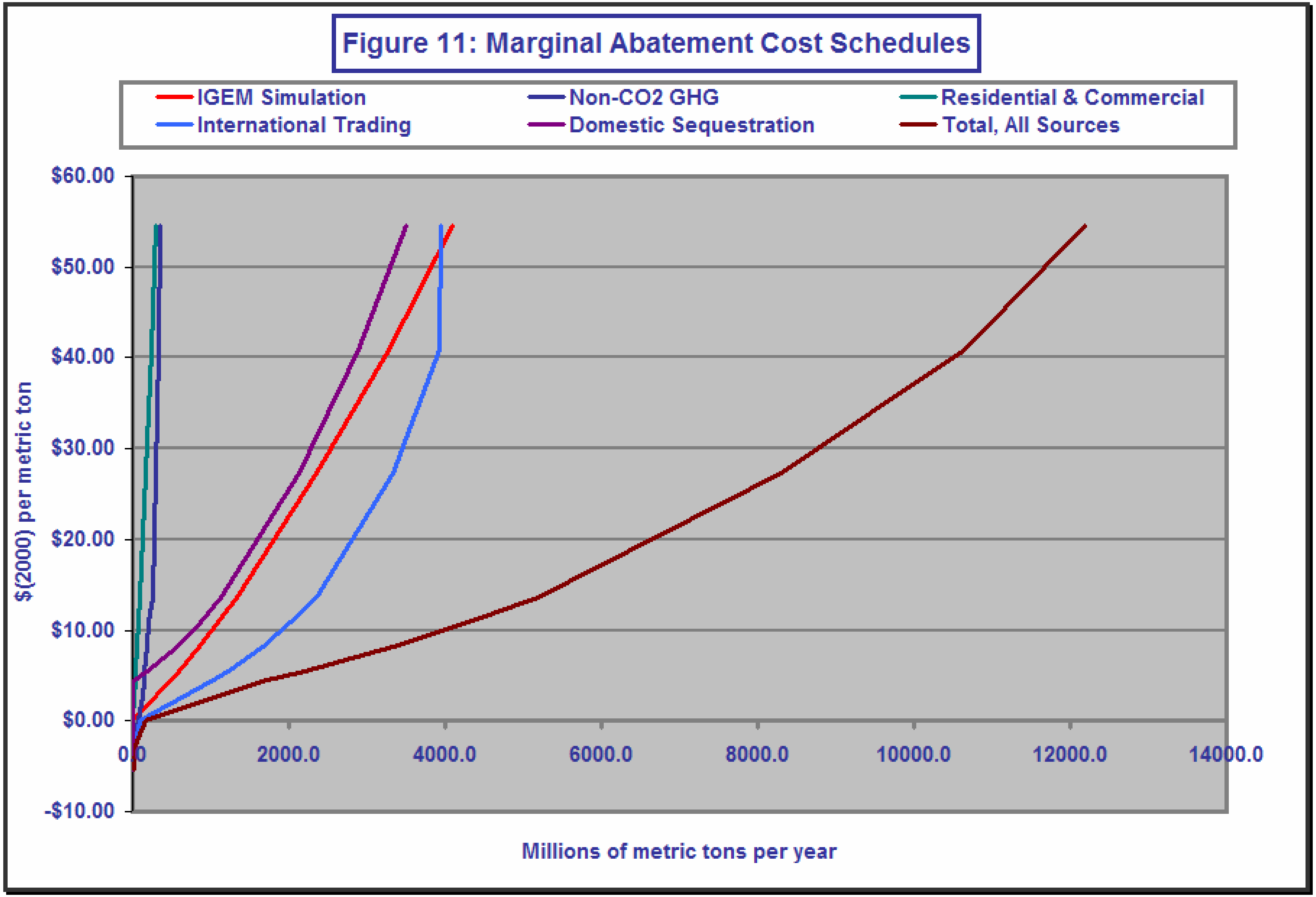


Figure 11: Marginal Abatement Cost Schedules
IGEM Simulation
Non-CO2 GHG
Residential & Commercial
International Trading
Domestic Sequestration
Total, All Sources
$60.00
$50.00
$40.00
$30.00
$20.00
$10.00
$0.00
-$10.00
$(2000) per metric ton
0.0
2000.0
4000.0
6000.0
8000.0
10000.0
12000.0
14000.0
Millions of metric tons per year

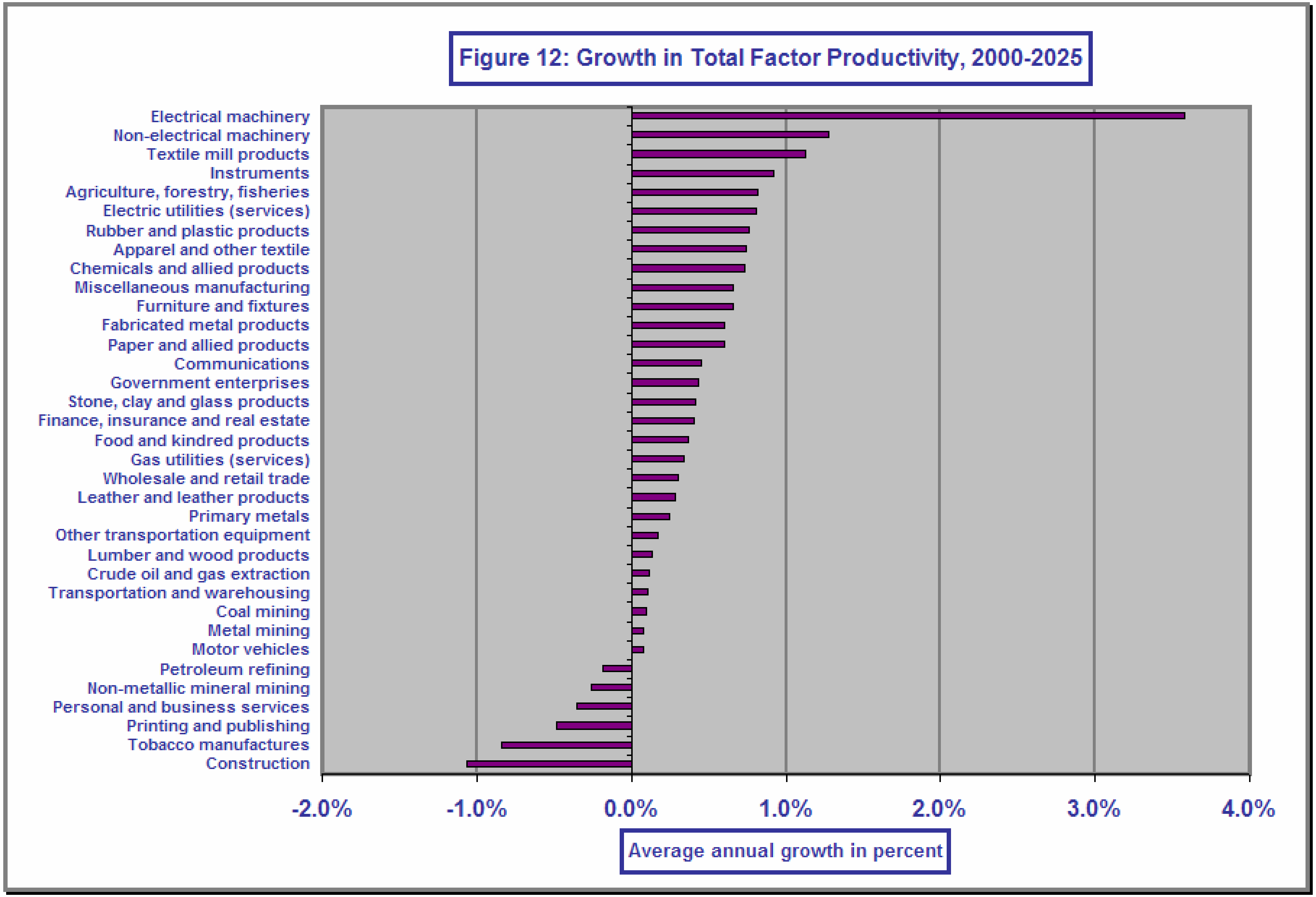


Figure 12: Growth in Total Factor Productivity, 2000-2025
Electrical machinery
Non-electrical machinery
Textile mill products
Instruments
Agriculture, forestry, fisheries
Electric utilities (services)
Rubber and plastic products
Apparel and other textile
Chemicals and allied products
Miscellaneous manufacturing
Furniture and fixtures
Fabricated metal products
Paper and allied products
Communications
Government enterprises
Stone, clay and glass products
Finance, insurance and real estate
Food and kindred products
Gas utilities (services)
Wholesale and retail trade
Leather and leather products
Primary metals
Other transportation equipment
Lumber and wood products
Crude oil and gas extraction
Transportation and warehousing
Coal mining
Metal mining
Motor vehicles
Petroleum refining
Non-metallic mineral mining
Personal and business services
Printing and publishing
Tobacco manufactures
Construction
-2.0%
-1.0%
0.0%
1.0%
2.0%
3.0%
4.0%
Average annual growth in percent

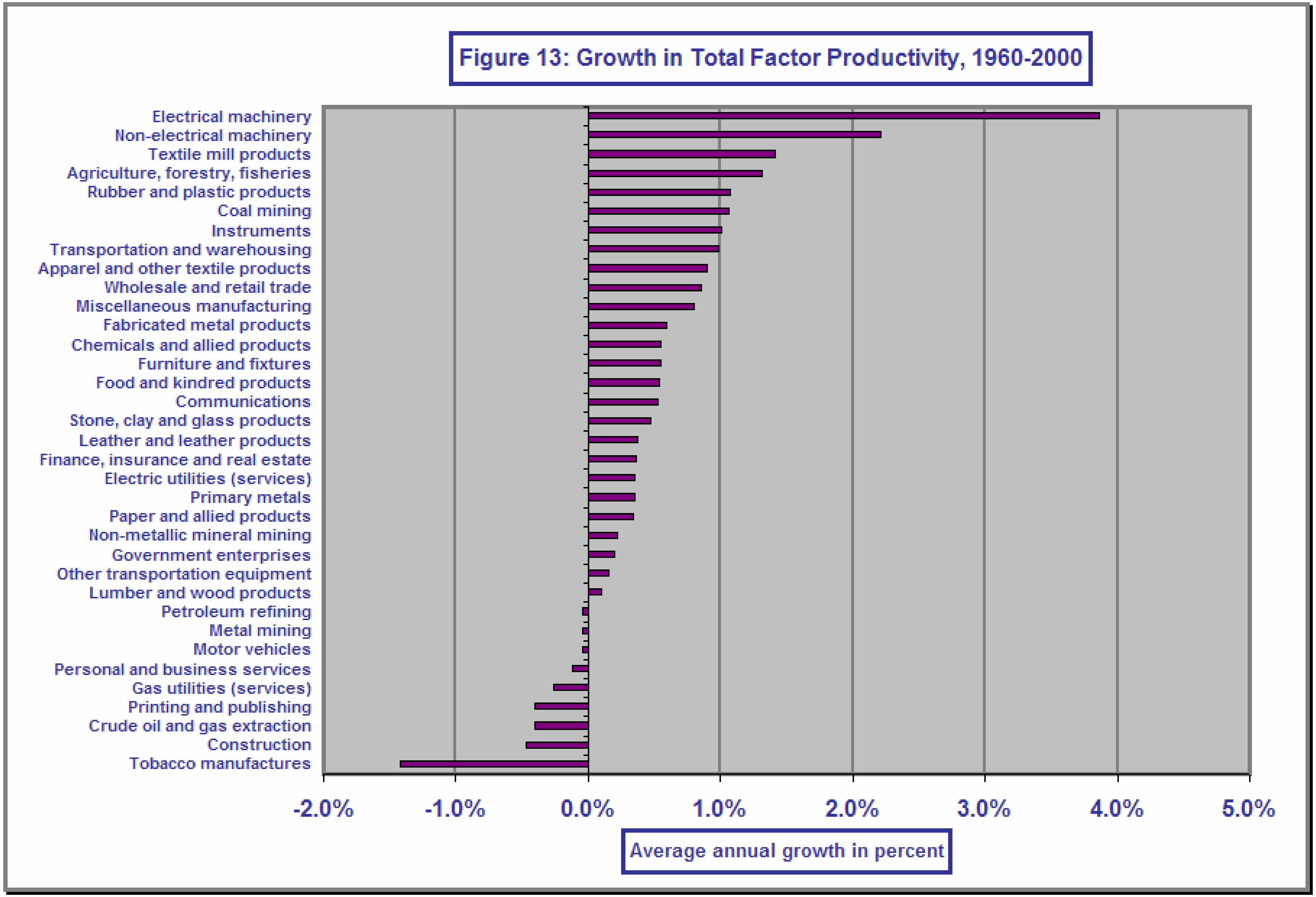


Figure 13: Growth in Total Factor Productivity, 1960-2000
Electrical machinery
Non-electrical machinery
Textile mill products
Agriculture, forestry, fisheries
Rubber and plastic products
Coal mining
Instruments
Transportation and warehousing
Apparel and other textile products
Wholesale and retail trade
Miscellaneous manufacturing
Fabricated metal products
Chemicals and allied products
Furniture and fixtures
Food and kindred products
Communications
Stone, clay and glass products
Leather and leather products
Finance, insurance and real estate
Electric utilities (services)
Primary metals
Paper and allied products
Non-metallic mineral mining
Government enterprises
Other transportation equipment
Lumber and wood products
Petroleum refining
Metal mining
Motor vehicles
Personal and business services
Gas utilities (services)
Printing and publishing
Crude oil and gas extraction
Construction
Tobacco manufactures
-2.0%
-1.0%
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
Average annual growth in percent

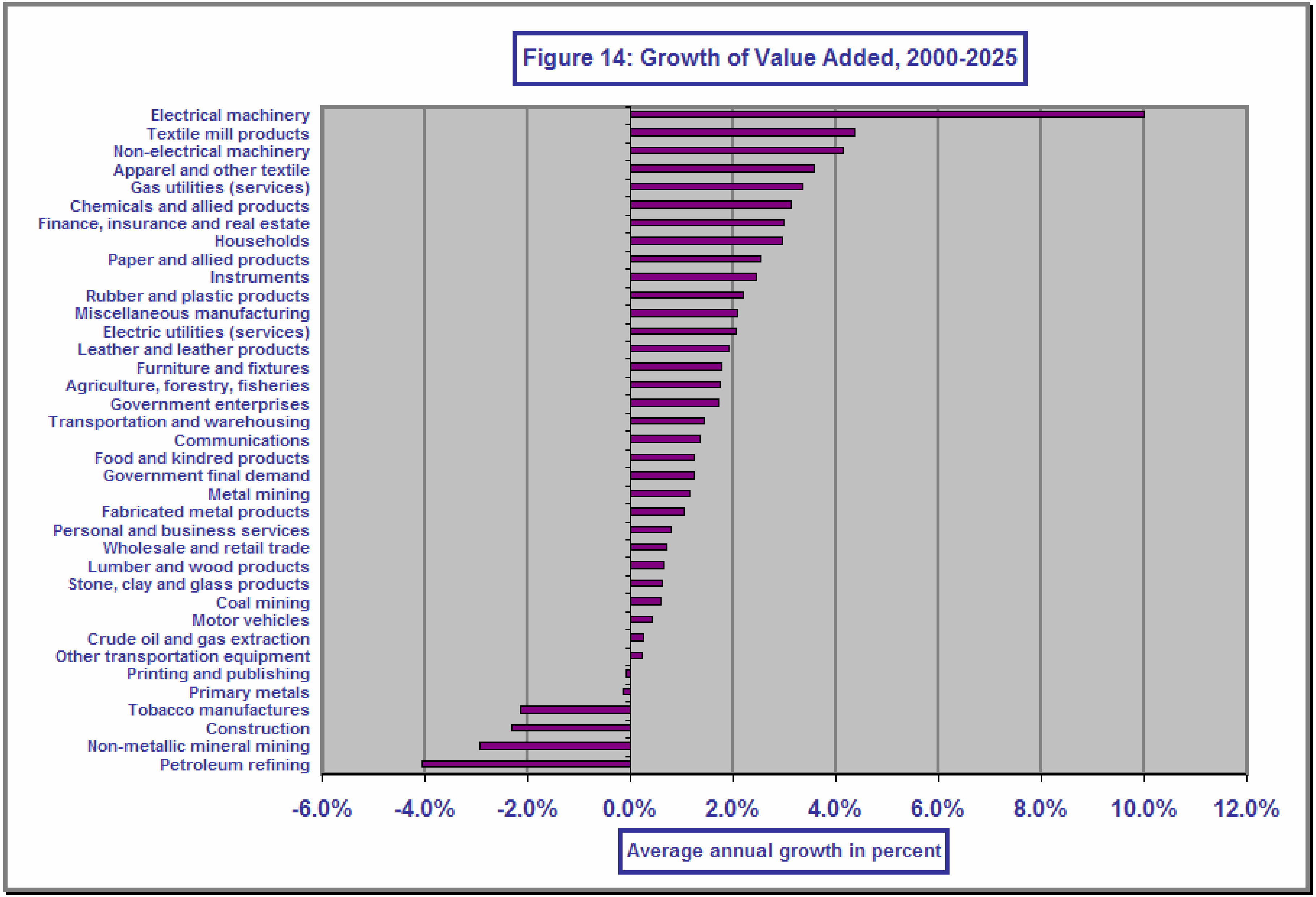


Figure 14: Growth of Value Added, 2000-2025
Electrical machinery
Textile mill products
Non-electrical machinery
Apparel and other textile
Gas utilities (services)
Chemicals and allied products
Finance, insurance and real estate
Households
Paper and allied products
Instruments
Rubber and plastic products
Miscellaneous manufacturing
Electric utilities (services)
Leather and leather products
Furniture and fixtures
Agriculture, forestry, fisheries
Government enterprises
Transportation and warehousing
Communications
Food and kindred products
Government final demand
Metal mining
Fabricated metal products
Personal and business services
Wholesale and retail trade
Lumber and wood products
Stone, clay and glass products
Coal mining
Motor vehicles
Crude oil and gas extraction
Other transportation equipment
Printing and publishing
Primary metals
Tobacco manufactures
Construction
Non-metallic mineral mining
Petroleum refining
-6.0%
-4.0%
-2.0%
0.0%
2.0%
4.0%
6.0%
8.0%
10.0%
12.0%
Average annual growth in percent

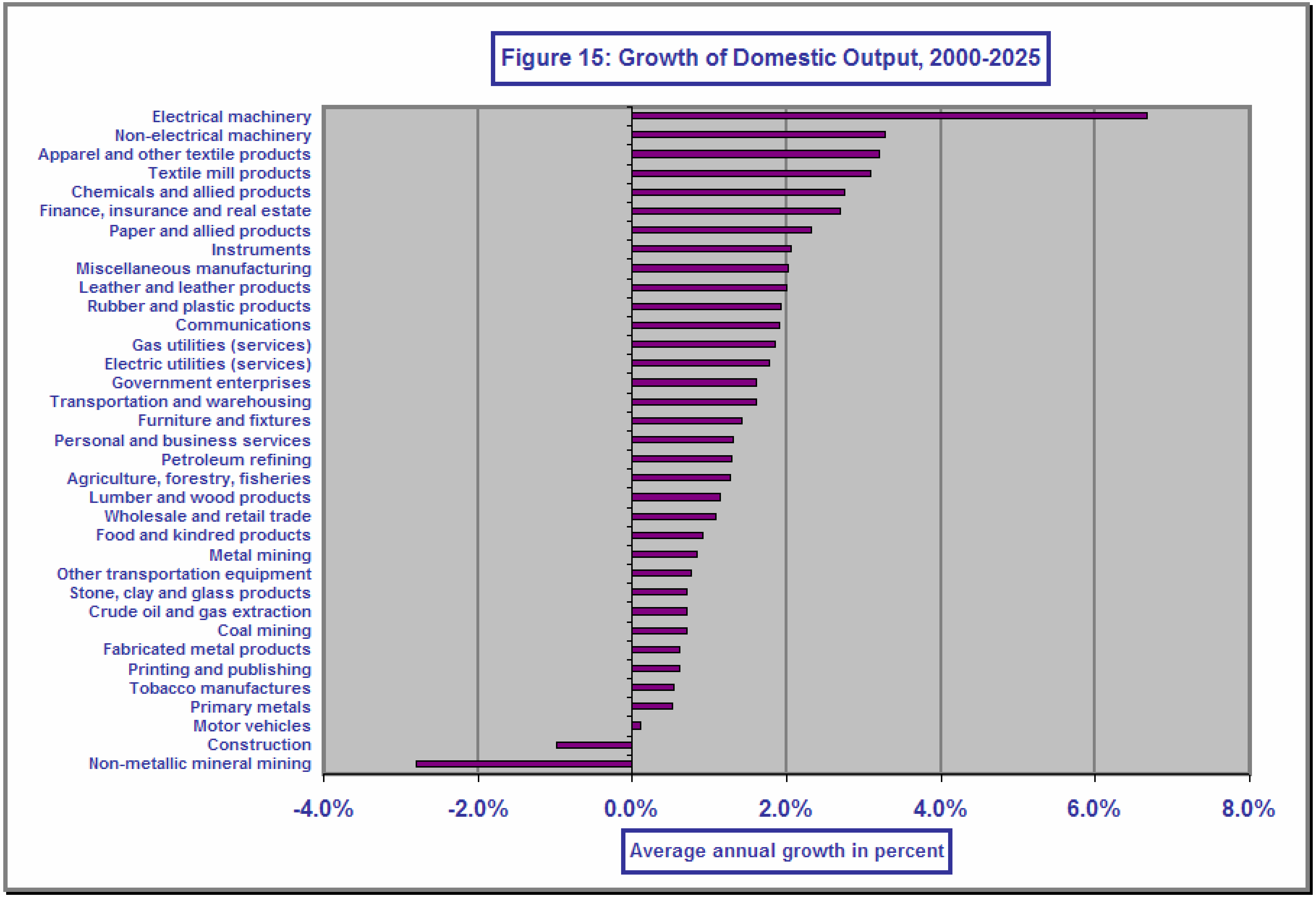


Figure 15: Growth of Domestic Output, 2000-2025
Electrical machinery
Non-electrical machinery
Apparel and other textile products
Textile mill products
Chemicals and allied products
Finance, insurance and real estate
Paper and allied products
Instruments
Miscellaneous manufacturing
Leather and leather products
Rubber and plastic products
Communications
Gas utilities (services)
Electric utilities (services)
Government enterprises
Transportation and warehousing
Furniture and fixtures
Personal and business services
Petroleum refining
Agriculture, forestry, fisheries
Lumber and wood products
Wholesale and retail trade
Food and kindred products
Metal mining
Other transportation equipment
Stone, clay and glass products
Crude oil and gas extraction
Coal mining
Fabricated metal products
Printing and publishing
Tobacco manufactures
Primary metals
Motor vehicles
Construction
Non-metallic mineral mining
-4.0%
-2.0%
0.0%
2.0%
4.0%
6.0%
8.0%
Average annual growth in percent

Figure 16: Growth of Domestic Output, 1960-2000