GDP is a measure of output, not welfare. 
Or, HOS meets the SNA

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THE ISSUE TODAY

How should exogenous changes in the **terms of trade** (the export price relative to the import price) be treated? Do they change GDP? Or both welfare and GDP? Or just welfare?

To answer this, I use two textbook models of a small open economy. I ask

- How would an economic theorist answer these question?
- Would a national income accountant, with access to all the necessary data and applying SNA 2008, agree?
The two models (both HOS)

**Model 1**
The country produces both goods which are for final consumption. Good 1 is exported, good 2 imported.

(any textbook of international trade)

**Model 2**
The country specialises on good 1. Good 1 is for final consumption and export, good 2 is an imported intermediate input (e.g. energy) into good 1.

(Bruno and Sachs 1985; Barsky and Kilian 2002; Blinder and Rudd 2008; Kehoe and Ruhl 2008)
Anticipating the conclusions

• In both models, the theorist would agree that GDP is constant (at least under perfect competition) but welfare (consumption) increases.

• The national income accountant, applying SNA 2008, would agree.

• Agreement depends on use of Divisia indices.
Why use Divisia indices?

- Divisia indices have excellent theoretical properties:
  - Price index times quantity index = value index
  - They are consistent in aggregation
- These two properties not shared by any discrete index number in common use.
- BUT Divisia indices cannot be calculated exactly and in practice discrete approximations (e.g. chained Fisher or chained Törnqvist) must be used.
Hecksher-Ohlin-Samuelson (HOS) model of a trading economy

- Small open economy. Terms of trade are exogenous.
- 2 goods. The country exports good 1.
- Technology and factor endowments (land and labour) are fixed. *No saving or investment.*
- Constant returns to scale and perfect competition, so prices equal marginal costs and factors are paid the value of their marginal products.
Model 1

- TT': terms of trade line
- PP: production possibility curve
- U₀U₀': indifference curve
- Production is at point P₀
- Consumption is at point C₀
- Exports: CD (= OD-OC)
- Imports: AB (= OB-OA)
Model 1, after improvement in terms of trade

The economy moves *around* the production possibility frontier. Welfare rises.
Divisia index of real GDP (Y) and of real GDI (C)

The Divisia index of aggregate output $Y$ (real GDP) is defined as:

$$ \hat{Y} = s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 $$

But along the production possibility frontier

$$ s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 = 0 $$

Therefore under the assumptions of Model 1

$$ \hat{Y} = 0 $$

But real GDI: $ \hat{C} = s_{M}^{GDP} (\hat{P}_1 - \hat{P}_2) > 0 $
Effect of a *large* change in the terms of trade

The Divisia index of aggregate output gives the instantaneous growth rate at a point in time. The total proportionate change over a finite interval $(0, T)$ is given by integration:

$$\log \left( \frac{Y(T)}{Y(0)} \right) = \int_0^T \hat{Y}(t) dt = \int_0^T \left[ s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 \right] dt$$

$$= 0 \text{ in this case.}$$

(This is the continuous analogue of a discrete chain index.)
Model 2

The country specialises on good 1. Good 1 is for final consumption and export, good 2 is an imported intermediate input (e.g. energy) into good 1.

(Bruno and Sachs 1985; Barsky and Kilian 2002; Blinder and Rudd 2008; Kehoe and Ruhl 2008)
Kehoe and Ruhl (2008):
under *perfect competition*, a *small* change in the terms of trade has *no* effect on GDP, provided GDP is measured by a *chained Fisher* index. But consumption rises.

My result:
(1) Under *perfect competition*, even a *large* improvement in the terms of trade has *no* effect on GDP, provided GDP is measured by a *Divisia* index. But consumption *rises*.
(2) Under *imperfect competition*, an *improvement* in the terms of trade *raises* GDP (and also consumption).
National accounts in Model 2

Supply Use relationships:
\[ P_1Y_1 = P_1C_1 + P_1X_1 \]

Nominal GDP:
\[
\begin{align*}
GDP(E) &:= P_E E \equiv P_1C_1 + P_1X_1 - P_2M_2 \\
GDP(O) &:= P_Y Y \equiv P_Y Y_1 - P_2M_2
\end{align*}
\]

Easy to see that:
\[
GDP(E) = GDP(O)
\]
\[
\hat{P}_E = \hat{P}_Y, \quad \hat{E} = \hat{Y}
\]

Divisia index for real GDP:
\[
\hat{Y} = (1 + s_M^{GDP})\hat{Y}_1 - s_M^{GDP} \hat{M}_2
\]
Model 2: theory

Gross output production function for good 1:

\[ Y_1 = Y_1(R, L, M_2, \tau) \]

Land \((R)\), labour \((L)\) and technology \((\tau)\) assumed fixed. So fall in relative price of imported energy \((M)\) leads to increase in volume of imports which raises output of good 1:

\[ \dot{Y}_1 = \frac{\partial Y_1}{\partial M_2} \dot{M}_2 \]

or

\[ \dot{Y}_1 = \left( \frac{s_M^{GDP}}{1 + s_M^{GDP}} \right) \dot{M}_2 > 0 \quad s_M^{GDP} : \text{import share in GDP} \]

assuming inputs are paid the value of their marginal products, i.e. *perfect competition.*
Back to the national accounts ...

Empirically, the national income accountant in model 2 would then find:

\[
\hat{Y} = (1 + s_M^{GDP}) \hat{Y}_1 - s_M^{GDP} \hat{M}_2 = 0
\]

i.e. no change in GDP. Also, no change in (trend of) GDP deflator \((P_Y)\)

But welfare (real consumption of good 1) rises:

\[
\hat{C}_1 = -s_M^{GDP} (\hat{X}_1 - \hat{M}_2)
\]

\[
= s_M^{GDP} (\hat{P}_1 - \hat{P}_2) > 0
\]

NB: the trade balance is always zero but the volume of imports rises faster than that of exports when the terms of trade improve. Also, empirically, growth of consumption equals growth of real GDI.
Large versus small changes

These results are for small changes in the terms of trade. What about large changes taking place over a finite time interval \((0,T)\)?

In model 2:
\[
\log \left( \frac{Y(T)}{Y(0)} \right) = \int_{0}^{T} \hat{Y}(t) dt = 0
\]
\[
\log \left( \frac{C(T)}{C(0)} \right) = \int_{0}^{T} \hat{C}(t) dt > 0
\]
Model 2 under imperfect competition

Now input prices are set equal to MR x the marginal product, so

\[ \hat{Y} = S_M^{GDP} \left( \frac{P_1}{MR_1} - 1 \right) \hat{M}_2 > 0 \]

i.e. GDP increases (and, as in the perfect competition case, consumption rises too)
Summary

• In both models, GDP is **constant** under perfect competition but welfare (real consumption, also equal here to real GDI) **rises** when the terms of trade improve.
Coming down to earth a bit ...

• Data are discrete, not continuous. So Divisia indices can’t be calculated.
Two theoretical approaches to national income accounting

1. *Discrete approach* (now orthodox amongst economists though not national income accountants). Assume economic behaviour is described *exactly* by a flexible functional form, a quadratic mean of order $s$ (Diewert 1976). Then changes in quantities and prices are measured *exactly* by superlative index numbers, e.g. Fisher ($s=2$) or Törnqvist ($s=0$).

2. *Continuous approach* (advocated here). Assuming only optimizing behaviour by agents, changes in quantities and prices are measured by Divisia index numbers. These can be approximated in practice by chained indices such as chained Fisher or chained Törnqvist. Or maybe better approximations can be developed in future?
Conclusions

• GDP is a measure of output, not welfare.
• But welfare can be measured within the framework of the SNA, e.g. as a first step by real GDI.
• Divisia indices provide a sound conceptual basis for measuring prices and volumes in the national accounts.
THE END
ADDITIONAL SLIDES
François Divisia (1889-1964)