Asset Price Reactions to News at the Zero Lower Bound

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Abstract

This paper analyzes the reaction of interest rates and the stock market to macroeconomic news announcements (MNAs) at the zero lower bound (ZLB). I start by using a shadow rate term structure model to formulate three predictions for the sensitivity of interest rates to MNAs. First, “better”-than-expected macroeconomic data increases interest rates. Second, as the expected duration of the ZLB increases, whether because economic conditions are worse or because monetary policy changes, interest rates become less sensitive to macroeconomic data. Third, this attenuation in the sensitivity of interest rates is largest for intermediate-maturity rates. I verify these predictions by using a broad sample of MNAs and high-frequency intraday futures data on interest rates. Turning to stocks, I show that the stock market’s reaction to MNAs can be decomposed into an interest rate news term that is directly related to interest rates’ reaction to MNAs and a cash flow plus risk premium news term. Using the same sample of MNAs and high-frequency intraday futures data on the stock market, I empirically estimate the stock market’s sensitivity to macroeconomic data as well as that of the constituent news terms. Based on the interest rate news term alone, the expected duration of the ZLB should increase the sensitivity of stocks to macroeconomic news. The data furthermore suggests that the expected duration of the ZLB decreases the magnitude of the cash flow plus risk premium news term.

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1 Introduction

A fundamental question in economics is how asset prices impound news about macroeconomic fundamentals. When news arrives of higher-than-expected inflation or output and lower-than-expected unemployment, do asset prices appreciate or depreciate and by how much? I tackle this question by analyzing the sensitivity of interest rates and the stock market in the U.S. to a broad sample of macroeconomic news announcements (MNAs). By considering the time period from December 17th, 2008 to March 6th, 2014 during which the federal funds (FF) rate was at the zero lower bound (ZLB), I find that the extent to which the ZLB binds drives the reaction of interest rates and stocks to data surprises.

We can think of the extent to which the ZLB binds as the expected duration of the ZLB, which varies significantly from the end of 2008 to 2014 even though the nominal short rate is fixed at the lower bound throughout. Figure 1 shows two proxies for the expected duration of the ZLB. The right-hand graph plots the median anticipated number of quarters until the first rate hike based on primary dealer surveys by the Federal Reserve Bank of New York (FRBNY). We see large variations in the anticipated time until the first rate hike from a low of 4 quarters in mid-2011 to a high of 12 quarters in late 2012. The left-hand graph plots implied FF rates from FF futures contracts with 1 month to 24 months maturities. As the expected duration of the ZLB increases, we expect FF rates in the future to decrease, and it indeed appears that implied FF rates are negatively correlated with the surveyed time until the first rate hike. The expected duration of the ZLB can change either because economic conditions change or because the monetary policy reaction function that sets the nominal short rate changes. For example, the first vertical line in both graphs of Figure 1 corresponds to the August, 2011 Federal Open Market Committee (FOMC) meeting during which the Federal Reserve decided to implement “exceptionally low levels for the federal funds rate at least through mid-2013.” This decision was clearly a surprise as evidenced by the sharp drop in implied FF rates and the sharp increase in the surveyed time until the first rate hike. This increase in the expected duration of the ZLB could have been due to pessimism about the state of the economy or beliefs that monetary policy changed. This paper shows that regardless of whether the expected duration of the ZLB changes due to economic conditions or to monetary policy, asset price sensitivities to news change in a certain manner.

Focusing on interest rates first, I employ a shadow rate term structure model that incorporates the ZLB to make three predictions about the sensitivity of interest rates to MNAs. The first prediction is that “better”—than-expected or “positive” macroeconomic news (e.g., higher-than-expected inflation or output and lower-than-expected unemployment) increases interest rates at all maturities. The second prediction is that as the expected duration of the
ZLB increases, whether because economic conditions are worse or because monetary policy is less responsive to economic conditions, the reaction of interest rates to MNAs decreases at all maturities. The third and final prediction is that this attenuation in the sensitivity of interest rates to news is initially increasing in maturity and then decreasing in maturity. In other words, the attenuation in the sensitivity of interest rates is greatest for intermediate maturity rates. I verify each of these predictions in the data using a representative sample of 18 MNAs and high-frequency data on interest rate futures at various maturities. To do so, I measure the reactions of various interest rates in a tight ±5 minute window around MNAs and then compare time variation in these estimated reactions to the aforementioned proxies for the expected duration of the ZLB. As expected, “positive” MNAs increase interest rates, but greater expected duration of the ZLB (e.g., after the August, 2011 FOMC meeting) moderates this reaction. The extent of this moderation is moreover greatest for interest rates with maturities that are not too short and not too long.

For stocks, I initially measure the time-varying sensitivity of the stock market to data using the same sample of MNAs and high-frequency data on the S&P 500 futures contract. Based off the ideas of Campbell (1991) and Campbell and Ammer (1993), we know that when the stock market reacts to macroeconomic news, it must be due to a combination of a change in expected future dividends (cash flow news), a change in expected future interest rates (interest rate news), and a change in expected future excess returns (risk premium news). Using a simple Gordon growth model as in Boyd, Hu, and Jagannathan (2005), I decompose the reaction of the stock market to MNAs into an interest rate news term and a cash flow plus risk premium news term. The interest rate news term is exactly the sensitivity of interest rates to MNAs (adjusted by a factor). Based on this news term alone, I show that as the expected duration of the ZLB increases, we expect the sensitivity of stocks to macroeconomic data to increase as well. The empirical relationship between the expected duration of the ZLB and the sensitivity of stocks is less clear-cut because of time-variation in the offsetting cash flow and risk premium news terms. Intriguingly, the expected duration of the ZLB is negatively correlated with the magnitude of the cash flow and risk premium news terms.

The ideas presented above connect to several research areas in the literature. Many papers have looked at how the release of macroeconomic news affects asset prices; e.g., Andersen, Bollerslev, Diebold, and Vega (2003, 2007), Boyd, Hu, and Jagannathan (2005), and Faust, Rogers, Wang, and Wright (2007). These papers have asked nuanced questions about various aspects of asset price reactions to MNAs. How are the reactions different for stocks versus bonds versus foreign exchange? Do asset prices react differently to “better”-than-expected news compared to “worse”-than-expected news? And do asset reactions depend on whether
the economy is in an expansionary state or a contractionary state?

My research agenda explores asset price reactions to news from a different angle by considering the effect of the ZLB and monetary policy. Prior work taking this approach is necessarily limited due to the recentness of the ZLB. Swanson and Williams (2013, 2014), as the first papers to study how the ZLB influences interest rate sensitivities to MNAs, focus attention to this research area. They find that the sensitivity of interest rates to macroeconomic data during the ZLB was lower than in normal times, though longer maturity interest rates were still surprisingly responsive to data. The results presented in this paper differ from those of Swanson and Williams (2013, 2014) in a few ways. First, I focus on how variation throughout the ZLB, specifically the expected duration of the ZLB, drives asset sensitivities to MNAs as opposed to comparing sensitivities during the ZLB to that during normal times. Second, I utilize a shadow rate term structure model to analytically derive comparative statics on interest rate reactions to macroeconomic news and how these reactions should depend on the expected duration of the ZLB. As such, I am able to relate the findings to the large literature on term structure models and test precise predictions for how interest rates should react to MNAs. Third, I use high-frequency intraday data on asset prices to measure the effect of data surprises on assets instead of daily data. By focusing on a tight ±5 minute window around news announcements, I am able to more cleanly measure any asset reactions to data without worrying about events throughout the day that are unrelated to macroeconomic news. Finally, I analyze not only interest rates but also the stock market. I show through a decomposition how the reaction of interest rates to MNAs has direct implications for the reaction of the stock market to MNAs.

In related work, Raskin (2013) uses the reaction of interest rate options to macroeconomic data at the ZLB to assess how actions of the Federal Reserve altered perceptions of the monetary policy reaction function. Instead of focusing on specific Federal Reserve events such as the FOMC meeting in August, 2011 and testing how asset sensitivities change before and after these events, I choose to exploit the richer variation in asset sensitivities throughout the ZLB period.

The work of Swanson and Williams (2013, 2014) and Raskin (2013) suggest several reasons for why we might care about how the ZLB affects the sensitivity of assets to MNAs. One reason is that the sensitivity of interest rates to macroeconomic news measures the effectiveness of monetary policy. The existence of the ZLB prevents the Federal Reserve from using its traditional instrument of monetary policy, the federal funds rate, which is locked at the lower bound. A number of papers such as Eggertsson and Woodford (2003) argue that a central bank can still conduct monetary policy at the ZLB by influencing expectations of short rates in the future, which in turn can affect other asset prices and
the broader economy. The Federal Reserve has indeed employed this strategy of forward guidance extensively such as with the previously mentioned action taken at the August, 2011 FOMC meeting. If interest rates are reasonably sensitive to MNAs despite the ZLB, then monetary policy can still be effective because it is still possible for monetary policy to meaningfully move around expectations of short rates as well as longer-maturity rates. On the other hand, if interest rates are insensitive to MNAs at the ZLB, then the ZLB constrains the ability of monetary policy to influence interest rates.

The sensitivity of interest rates to MNAs is also informative about the effectiveness of fiscal policy. Work by Christiano, Eichenbaum, and Rebelo (2011) among others shows that the fiscal multiplier may be larger at the ZLB because interest rates do not “crowd out” fiscal spending as much. If interest rates are less sensitive to MNAs at the ZLB, then it may be true that a government spending shock does not increase interest rates much, so the fiscal multiplier is indeed larger. On the other hand, if interest rates are still reasonably sensitive to MNAs, the “crowding out” effect of rising interest rates in response to government spending may diminish the fiscal multiplier.

Finally, my results on the stock market’s reaction to macroeconomic news at the ZLB continue an avenue of inquiry on the relationship between monetary policy and the stock market. Papers that address this area include Rigobon and Sack (2003), Bernanke and Kuttner (2005), and Campbell, Pflueger, and Viceira (2014). Better understanding this relationship between stocks and monetary policy is important because monetary policy often has to work through financial assets such as the stock market in order to ultimately influence the macroeconomy.

The organization of the paper is as follows. Section 2 introduces the shadow rate term structure model and links the model to three predictions about the sensitivity of interest rates to macroeconomic news surprises. Section 3 first describes the data for MNAs and high-frequency asset prices before discussing the basic regression that tests the effect of MNAs on asset prices. Section 4 takes the model predictions about the sensitivity of interest rates to the data and presents supporting evidence. Section 5 analyzes the sensitivity of stocks to macroeconomic data and performs a decomposition of this sensitivity. Section 6 concludes.

2 Shadow Rate Term Structure Model

I start by building a basic term structure model and analytically exploring its implications for the sensitivity of interest rates to MNAs. The presence of the ZLB prevents the use of the canonical Gaussian affine term structure model to establish relationships among interest rates at various maturities. As such, I follow the literature in utilizing a shadow rate term
structure model that incorporates the ZLB. Introduced by Black (1995), the idea of the shadow rate has become particularly relevant since the 2008 financial crisis as the nominal short rate has remained stuck at the ZLB in the U.S. and other countries. Shadow rate term structure models have been approximately solved in both continuous time by Ichiue and Ueno (2013) and Krippner (2013) and in discrete time by Wu and Xia (2014).

The model I develop and analyze is a one-factor version of Wu and Xia (2014). I first set up the model and solve for the forward rate, which is the interest rate of interest. The focus of the analysis is not on the forward rate itself but rather on how the forward rate changes in response to news about macroeconomic fundamentals and the role of the ZLB therein. To clarify the role of the ZLB, I next show in the model that the expected duration of the ZLB increases when economic conditions deteriorate and/or when monetary policy is less responsive to the economy. Finally, I highlight three predictions of the model for the sensitivity of interest rates to MNAs.

2.1 Setup and Forward Rate

Let the nominal short rate $i_t$ equal the maximum of the shadow rate $s_t$ and some lower bound $\bar{i}$:

$$i_t = \max \{ s_t, \bar{i} \}$$

with $0 < \bar{i} \leq 0.25$ reflecting the reality that the Federal Reserve still pays interest on excess reserves. If the shadow rate is higher than the lower bound, the short rate equals the shadow rate; otherwise, Eq. (1) restricts the short rate from venturing below the lower bound.

Analogous to the eponymous Taylor rule discussed in Taylor (1993), the shadow rate is linear in a general state variable $x_t$:

$$s_t = \delta_0 + \delta_1 x_t.$$  \hspace{1cm} (2)

The magnitude of $\delta_1$ determines how strongly policy makers respond to the state variable in setting the short rate. For tractability purposes, I assume that $x_t$ is a one-dimensional proxy for economic conditions. In a Taylor rule, there are two state variables corresponding to the output gap and inflation relative to some benchmark, so we can also think of $x_t$ as reflecting some combination of the output gap and inflation. Low-dimensionality is justified by Bernanke and Boivin (2003) who show that a few factors (e.g., corresponding to principal components) can summarize the large number of data series in a data-rich environment.

The state variable $x_t$ follows an AR(1) process under the physical measure ($\mathbb{P}$):

$$x_{t+1} = \mu + \rho x_t + \sigma \epsilon_{t+1}$$
with $\epsilon_{t+1} \sim \mathcal{N}(0, 1)$. Assuming that the price of risk $\lambda_t$ is linear in the state variable (such that there is a constant risk premium term and a time-varying risk premium term)

$$\lambda_t = \lambda_0 + \lambda_1 x_t,$$

and the log stochastic discount factor is essentially affine

$$M_{t+1} = \exp \left( -i_t - \frac{1}{2} \lambda_t^2 - \lambda_t \epsilon_{t+1} \right),$$

the state variable $x_t$ also follows an AR(1) process under the risk neutral measure ($\mathbb{Q}$):

$$x_{t+1} = \mu^\mathbb{Q} + \rho^\mathbb{Q} x_t + \sigma^\mathbb{Q} \epsilon^\mathbb{Q}_{t+1}$$

(3)

with $\epsilon^\mathbb{Q}_{t+1} \sim \mathcal{N}(0, 1)$.

Given this setup, the forward rate $f_{n,t}$, defined as the interest rate on a 1-period investment to be made at $t + n$, is approximately

$$f_{n,t} \approx i + \sigma_n^\mathbb{Q} \left( \left( \frac{a_n + b_n x_t - i}{\sigma_n^\mathbb{Q}} \right) \Phi \left( \frac{a_n + b_n x_t - i}{\sigma_n^\mathbb{Q}} \right) + \phi \left( \frac{a_n + b_n x_t - i}{\sigma_n^\mathbb{Q}} \right) \right).$$

(4)

$\Phi$ and $\phi$ are the standard normal CDF and pdf, respectively. Appendix A sketches the derivation for the forward rate and provides expressions for $a_n$, $b_n$, and $\sigma_n^\mathbb{Q}$ in terms of model parameters.

2.2 Expected Duration of the ZLB

In addition to observing the forward rate, we can see how the model illustrates the extent to which the ZLB binds, loosely termed the expected duration of the ZLB. In particular, the probability the ZLB still binds at time $t + n$ is simply measured by the probability that the shadow rate at that time is still below the lower bound:

$$\Pr_{t}^\mathbb{Q} \left[ s_{t+n} < i \right] = \Phi \left( \frac{i - \bar{\sigma}_n - b_n x_t}{\sigma_n^\mathbb{Q}} \right).$$

(5)

Appendix A expresses $\bar{\sigma}_n$ in terms of model parameters and shows the conditional distribution of $s_{t+n}$, which directly implies the above probability. The expected duration of the ZLB in the model can change either because economic conditions change or because the responsiveness of monetary policy changes.

**Proposition 1** As economic conditions worsen ($x_t$ is more negative), the probability that
the ZLB is binding in the future increases: \( \partial/\partial x_t \left( Pr_t^Q \left[ s_{t+n} < \hat{i} \right] \right) < 0 \). Symmetrically, as monetary policy becomes less responsive to economic conditions (\( \delta_1 \) is smaller), the probability that the ZLB is binding in the future increases: \( \partial/\partial \delta_1 \left( Pr_t^Q \left[ s_{t+n} < \hat{i} \right] \right) < 0 \).

Appendix B works out the comparative statics in the above proposition, but the intuition is fairly straightforward. First, as economic conditions worsen at \( t \), the shadow rate is set lower in Eq. (2), which increases the probability that the shadow rate in the future remains below the lower bound. Second, consider a small Taylor rule coefficient in Eq. (2), which implies that the shadow rate is set close to \( \delta_0 \) and largely independent of economic conditions. Assuming that \( \hat{i} > \delta_0 \) such that it is still possible for the ZLB to bind in this scenario, an unresponsive policy rule means that the shadow rate in the future is also set close to \( \delta_0 \) and relatively less influenced by economic conditions then. This insight in turn explains the increased probability that the ZLB still binds.

2.3 Predictions for Forward Rate Sensitivity to MNAs

Finally, the model delivers three predictions on the sensitivity of interest rates to news about macroeconomic fundamentals and how this sensitivity interacts with the expected duration of the ZLB. This sensitivity is defined by \( \partial f_{n,t}/\partial x_t \), which quantifies how a change in our proxy for economic conditions affects the forward rate defined in Eq. (4). Though I focus on the forward rate, the predictions below hold more generally. For example, the predictions hold for the yield to maturity of a zero-coupon bond, which we can mechanically see by expressing the yield to maturity in terms of a series of forward rates. Proofs for each of the three propositions below are in Appendix B.

**Proposition 2** News that economic conditions are “better”-than-expected \( (x_t \) is more positive) increases forward rates at all maturities: \( \partial f_{n,t}/\partial x_t > 0 \).

Examples of “better”-than-expected macroeconomic news include higher-than-expected inflation or output and lower-than-expected unemployment. Such a surprise increases the shadow rate today via Eq. (2) and, through the persistent AR(1) process for \( x_t \), leads to a more positive state variable in the future that increases the shadow rate at that time as well. The resulting effect is higher interest rates.

**Proposition 3** As the ZLB is expected to bind for longer (probability that the shadow rate will be below the lower bound increases), whether because economic conditions are worse \( (x_t \)
is more negative) or because monetary policy is less responsive to economic conditions ($\delta_1$ is smaller), the sensitivity of forward rates to news about macroeconomic fundamentals decreases at all maturities: $\partial/\partial x_t (\partial f_{n,t}/\partial x_t) > 0$ and $\partial/\partial \delta_1 (\partial f_{n,t}/\partial x_t) > 0$.

We know from Proposition 1 that poorer economic conditions and a less responsive monetary policy reaction function both lead to an increase in the expected duration of the ZLB. Because the shadow rate is more likely to remain below the lower bound, Eq. (1) says that the nominal short rate is more likely to stay anchored at $i$. This anchoring of the nominal short rate prevents interest rates at all maturities from reacting as strongly to MNA surprises. Consider an edifying example in which the expected duration of the ZLB binds completely for a short time such that the nominal short rate is guaranteed to equal the lower bound within this time period. Then a small macroeconomic news surprise has no effect on short-dated interest rates, which are set as a constant. As I show in the data, this example corresponds to reality. Over the ZLB period, the Federal Reserve has credibly committed to setting the federal funds rate close to zero for at least a quarter or two. The near-perfect certainty of short maturity interest rates has resulted in essentially no sensitivity of these rates to data surprises.

**Proposition 4** The attenuation in the sensitivity of forward rates to news in Proposition 3 is greater for longer maturity forward rates ($n$ is larger) provided that the ZLB is sufficiently binding (probability that the shadow rate will be below the lower bound is sufficiently large): $\partial/\partial n (\partial/\partial x_t (\partial f_{n,t}/\partial x_t)) > 0$ and $\partial/\partial n (\partial/\partial \delta_1 (\partial f_{n,t}/\partial x_t)) > 0$. Otherwise, if the ZLB does not bind sufficiently, the attenuation in the sensitivity of rates to news is decreasing in maturity $n$.

In other words, if the expected duration of the ZLB is sufficiently long, shorter maturity interest rates are essentially set equal to the lower bound and are thus insensitive to macroeconomic news. As the expected duration of the ZLB increases, the sensitivity of short rates to data has little room to decrease. The sensitivity of longer maturity interest rates, however, has more room to fall, since long rates are less bound by the ZLB to begin with.

Conversely, if the expected duration of the ZLB is relatively short, neither short-dated nor long-dated interest rates are significantly constrained by the ZLB. As the expected duration of the ZLB increases, the sensitivity of both types of interest rates has room to decrease. Since short rates represent rates of interest over shorter time periods while long rates represent rates of interest over longer time periods, an increase in the expected duration of the ZLB has a more noticeable impact on the former than on the latter. For example, the
sensitivity of a 100-year bond to MNAs should not change much because the nominal short rate is expected to stay at zero for six months instead of three months. Thus, the magnitude of the sensitivity decrease is larger in short rates than in long rates.

3 Data and Basic Regressions

3.1 MNAs

Model predictions in hand, I turn to the data to assess how various assets react to macroeconomic news at the ZLB. Table 1 shows the full sample of MNAs considered in this paper from December, 2008 to March, 2014. For each data release, the table presents the name, units, number of observations, frequency, government agency or private-sector firm responsible, and timestamp. In all, I analyze 18 economic announcements, which cover the lion’s share of important MNAs. All of the announcements occur once a month with the exception of Initial Jobless Claims, which is a weekly data release. The data in my sample are released at either 8:30 AM, 9:15 AM, or 10:00 AM ET.

While the economic announcements are officially produced and released by various government agencies (e.g., the Bureau of Labor Statistics), and private-sector firms (e.g., the National Association of Realtors), I collect the data from Bloomberg. Specifically, for each event, I download both the actual data release and the expected data release, the latter of which comes from a survey of economists. The difference between the actual number and the expected number constitutes the news that impacts asset prices.

The metric of the news surprise that I use in this paper is the conventional standardized news variable employed by Balduzzi, Elton, and Green (2001) as well as many subsequent papers in the literature of asset price reactions to MNAs. For economic indicator $i$, the standardized news variable at time $t$ is

$$S_{i,t} = \frac{A_{i,t} - \mathbb{E}_{t-} [A_{i,t}]}{\hat{\sigma}_i}.$$  (6)

$A_{i,t}$ is the actual data, $\mathbb{E}_{t-} [A_{i,t}]$ is the expectation of the data from the Bloomberg survey, and $\hat{\sigma}_i$ is the sample standard deviation of $A_{i,t} - \mathbb{E}_{t-} [A_{i,t}]$. Consistent with the model, I define the variable such that a positive value corresponds to “better”-than-expected data; i.e., higher-than-expected inflation or output and lower-than-expected unemployment. For this reason, I multiply the Initial Jobless Claims and the Unemployment Rate data by -1 while keeping the sign of the other data unchanged, as shown by the sign column of Table 1. As the name suggests, the standardized news variable provides a single metric...
that is standardized across different types of news about macroeconomic fundamentals and allows for comparability. This is important because different types of news are released with different units.

The use of the standardized news variable in conjunction with data from Bloomberg is common in testing the effect of macroeconomic news surprises on asset prices. Many financial market participants use Bloomberg to get a sense of the consensus forecast for any given MNA and to see how the actual data compares to the forecasted data at the time of the release. The survey expectation is furthermore unbiased and unlikely to be stale, since economists can adjust their forecasts until the very last moment.

3.2 High-Frequency Asset Prices

I consider three different futures contracts for the asset prices in my analysis: Eurodollar Futures (ED), 10-Year U.S. Treasury Note Futures (TY), and E-mini S&P 500 Futures (ES). I obtain intraday tick data from Tick Data, a data vendor. The use of futures data is important because many MNAs are released early in the morning before equity markets officially open at 9:30 AM ET. Whereas some markets are less liquid at that early hour, futures markets are already active. Not surprisingly, financial market participants also tend to react to data surprises by trading in these futures directly.

I use ED and TY futures data to evaluate the impact of MNAs on interest rates. ED futures settle at maturity based on the spot 3-month LIBOR rate, so the ED future with a certain maturity corresponds roughly to the 3-month forward rate at that maturity. I construct series of generic ED contracts with different maturities in a straightforward manner. For ED\(_n\), the \(n\)-quarter(s) out ED future is simply the \(n\)th contract with \(n = 1, 2, \ldots, 16\). Each contract is rolled over a few days before maturity. The ED1 contract thus corresponds roughly to the 3-month forward rate that is 1 quarter out, while the ED16 contract is the 3-month forward rate that is 16 quarters out. By considering how the full range of ED1 to ED16 futures responds to MNA surprises, I am able to distinguish interest rate reactions at different maturities. Since the longest maturity I consider for ED futures is 16 quarters or approximately 4 years, I also analyze the front TY futures, which corresponds to a longer-dated interest rate.

Assessing the reaction of the stock market to macroeconomic news surprises is more straightforward: I directly use the front ES futures contract.
### 3.3 Basic Regressions for All Assets

The motivating regression in evaluating the impact of a MNA surprise on asset prices is simply to regress asset returns $R_t$ in a window around event $i$ on the $S_{i,t}$ standardized news variable defined in Eq. (6):

$$R_t = \alpha_i + \beta_i S_{i,t} + \epsilon_t. \quad (7)$$

$\beta_i$ measures the reaction of the asset to a news surprise and is the object of focus. The availability of high-frequency asset prices allows me to set the event window to $\pm 5$ minutes around the event, which is an interval comparable to other studies that have used intraday return data. It is thus likely that asset prices vary over the window only due to any surprise embedded in the news announcement.

Table 2 documents the $\hat{\beta}_i$ from estimating Eq. (7) for various asset-MNA combinations over the ZLB date range. For example, consider Housing Starts and the ED16 futures contract. A one unit positive standardized news surprise in Housing Starts increases the 3-month forward rate 4 years out approximately 2.15 bps, which is both statistically and economically significant. Examining Table 2, we clearly see that the reaction of ED1 to a news surprise is muted, as $\hat{\beta}_i$ is generally small and statistically insignificant. This finding is not surprising given that interest rates are almost certain to remain fixed near zero for the next several months due to the ZLB. Results for ED4, ED8, and ED16 futures contracts show that the reactions of longer-dated interest rates to a news surprise is both statistically and economically significant and in the predicted direction: a positive standardized news surprise increases interest rates. The column corresponding to the TY futures tells the same story: the TY futures data is left in terms of prices instead of yields, so a positive news surprise that increases yields decreases prices. The final column of Table 3 shows that a positive data surprise also has a positive, significant impact on the stock market across a broad range of economic indicators.

Finally, the last row of Table 3 displays the results of running Eq. (7) grouping together all 18 MNAs. A one unit positive standardized news surprise in any macroeconomic announcement leads to a 0.017 bps, 0.268 bps, 0.617 bps, and 0.831 bps increase in the forward rate implied from the ED1, ED4, ED8, and ED16 contracts, respectively. This same surprise leads the TY futures contract to decrease by 0.043% and the ES futures contract to increase by 0.097%.
4 Empirical Test of Model Predictions for Interest Rates

While Table 2 shows the responses of interest rates to macroeconomic news surprises, the table is insufficient to formally demonstrate the three predictions of the shadow rate term structure model. In particular, Table 2 does not make use of time variation in the sensitivity of interest rates to MNAs; rather, it presents one number to summarize the estimated sensitivity of a given interest rate to economic data over the entire ZLB period. I formally test the model predictions for interest rates by focusing on ED futures and using TY futures as a robustness check.

4.1 Methodology

For an ED future with maturity \( n \), I perform a daily 1-year rolling regression over the ZLB period analogous to Eq. (7):

\[
\Delta f_{n,t} = \alpha_n + \beta_n S_t + \epsilon_{n,t},
\]

(8)

with the regression grouping together all 18 MNAs identical to the last row of Table 2. The result of the rolling regression is a time series of \( \hat{\beta}_{n,t} \), the estimated sensitivity of the 3-month forward rate \( n \)-quarter(s) out over a 1-year window centered at \( t \). By grouping together all the MNAs together, Eq. (8) artificially imposes the constraint that \( f_{n,t} \) reacts in the same manner to surprises in different data releases. This constraint is clearly an oversimplification; e.g., it is well known that the Change in NFP (nonfarm payrolls) data release is the most closely watched economic indicator with the greatest impact on financial markets. The upshot of forcing interest rates to react the same to different macroeconomic series is the greater statistical power from estimating Eq. (8) over more observations. Furthermore, the time series of \( \hat{\beta}_{n,t} \) from estimating the regression over any specific data release without grouping has a similar shape to the time series of \( \hat{\beta}_{n,t} \) from estimating the regression over all 18 MNAs. Swanson and Williams (2014) suggest a nonlinear least squares regression to measure the time-varying sensitivity of interest rates in a way that does allow for differential reactions to different macroeconomic news. Their methodology imposes that the relative magnitudes of the sensitivity of interest rates to different data releases are constant over time. I choose to not employ their technique for two reasons. The first is that their method requires choosing a benchmark time period when interest rates react “normally” to macroeconomic data. The choice of this benchmark period can affect the sensitivity of interest rates to data outside of the benchmark period. More importantly, the time-varying sensitivity of interest rates from simply grouping together all the MNAs as I have done is similar in shape to that produced by the Swanson and Williams (2014) methodology.
Using the time series of $\hat{\beta}_{n,t}$, I regress $\hat{\beta}_{n,t}$ on a proxy for the expected duration of the ZLB:

$$\hat{\beta}_{n,t} = \gamma_n + \delta_n ZLB_t + \eta_{n,t}. \quad (9)$$

$\delta_n$ measures how the estimated sensitivity of $f_{n,t}$ to MNAs changes as the expected duration of the ZLB changes. The three model predictions then map neatly to the following testable implications.

Proposition 2 implies that $\hat{\beta}_{n,t} > 0$ for all $n$ and for all 1-year windows centered at $t$: positive news surprises increase interest rates. This result is well-established empirically, and we have already seen some evidence in Table 2. In the table, forward rates implied from ED contracts clearly jump higher in reaction to positive macroeconomic news. At the same time, prices fall (yields rise) of TY futures. The rolling regression methodology in this section confirms the prediction for all 1-year windows.

Proposition 3 implies that $\hat{\delta}_n < 0$ for all $n$ assuming that $ZLB_t$ is a positive proxy for the expected duration of the ZLB. As the ZLB is expected to bind for longer, which could be because economic conditions are worse or because monetary policy is less responsive to economic conditions, the sensitivity of interest rates to MNAs decreases. If $ZLB_t$ is a negative proxy for the expected duration of the ZLB, then the prediction mechanically switches to $\hat{\delta}_n > 0$.

Proposition 4, the last of the three predictions, implies that $|\hat{\delta}_n|$, the magnitude of $\hat{\delta}_n$, is hump-shaped in $n$ (first increasing, then decreasing). Consider four interest rates with the following maturities: $n_1 < n_2 \ll n_3 < n_4$. The attenuation in the sensitivity of interest rates to data is greater for the $n_2$-maturity rate than for the $n_1$-maturity rate because the sensitivity of the $n_1$-maturity rate is more constrained by the ZLB and has little room to decrease. In contrast, the sensitivity of the $n_2$-maturity rate has more room to fall as the ZLB becomes more binding. The attenuation in the sensitivity of interest rates to data is smaller for the $n_4$-maturity rate than for the $n_3$-maturity rate because neither rates are greatly constrained by the ZLB to begin with. Both the $n_3$- and $n_4$-maturity rates encompass a time period over which the ZLB is binding. As the expected duration of the ZLB increases incrementally, the ZLB is binding for more of the period encompassed by the $n_3$-maturity rate than for that encompassed by the $n_4$-maturity rate, which explains why the sensitivity of the former is attenuated more than that of the latter. For example, if the expected duration of the ZLB increased s.t. all rates out to $n_3$ flattened to the lower bound, the sensitivity of the $n_3$-maturity rate would drop to zero, while the sensitivity of the $n_4$-maturity rate would still be non-zero.
4.2 Survey Proxy for Expected Duration of the ZLB

Figure 2 illustrates the results of applying the abovementioned methodology. In particular, the figure shows the time-varying sensitivities of various interest rates to MNAs and how the expected duration of the ZLB affects these sensitivities. Focusing first on Panels A, B, C, and D, which correspond to the interest rates associated with the ED1, ED4, ED8, and ED16 futures contracts, respectively, the right-hand plots shows two time series. “fcsts,” plotted on the right y-axis, is the same variable shown in the right-hand plot of Figure 1: the median anticipated number of quarters until the first rate hike based on primary dealer surveys by the FRBNY. Since the FRBNY surveys primary dealers only once every month or every other month, I create a daily variable with a simple forward fill. The result is the “fcsts” variable that I use as the survey proxy for the expected duration of the ZLB; i.e., $ZLB_t$ in Eq. (9). The second time series plotted in each right-hand plot is the time-varying sensitivity of a given interest rate to MNAs $\hat{\beta}_{n,t}$ in Eq. (9). The FRBNY started publishing primary dealer surveys in January, 2011, so I necessarily restrict my analysis such that the earliest $t$ in Eq. (9) is 1/1/11. For each panel, the left-hand plot is simply a scatterplot of “beta” on “fcsts” that graphically demonstrates the relationship in Eq. (9). Each point in the scatterplot is color-coded based on the timestamp: red for 12/17/08 to 8/9/11 corresponding to the period of general forward guidance (“fwd”); green for 8/10/11 to 12/12/12 corresponding to the period of calendar guidance (“cdr”); and blue for 12/13/12 to 3/6/14 corresponding to the period of data thresholds (“dat”).

Verifying Proposition 2, we see in Panels A to D of Figure 2 that $\hat{\beta}_{n,t} > 0$: the dots in the left-hand scatterplots, or equivalently the time-series of “beta” in the right-hand plots, are generally positive. Positive MNA surprises increase interest rates at all maturities and at all points in time. That some of the $\hat{\beta}_{n,t}$ are essentially at zero is entirely plausible and corresponds with the second prediction that a binding ZLB reduces the sensitivity of interest rates to MNAs. In the few cases that $\hat{\beta}_{n,t}$ is negative (Panel A of Figure 2), the magnitudes are economically small and statistically indistinguishable from zero.

Verifying Proposition 3, we see in the scatterplots that there is a negative relationship between $\hat{\beta}_{n,t}$ and $ZLB_t$; i.e., $\hat{\delta}_n < 0$ in Eq. (9). Table 3 presents the $\hat{\delta}_n$ for $n = 1, \ldots, 16$: all the $\hat{\delta}_n < 0$ and are statistically significant. As primary dealers expect the ZLB to bind for longer and the first rate hike to be later, the sensitivity of interest rates at all maturities to MNAs becomes smaller. This link is particularly apparent in Panels B, C, and D for longer-maturity interest rates. In Panel A, on the other hand, the negative relationship is less clear for a simple reason: the ZLB binds to such an extent that the 1 quarter out 3-month forward rate is essentially fixed at the lower bound, and $\hat{\beta}_{1,t}$ is therefore approximately zero. Nonetheless, $\hat{\delta}_1$ is negative and significantly different from zero in Table 3. The economic
magnitudes of $\hat{\delta}_n$ are significant as well. For example, $\hat{\delta}_8 = -0.116$ in Table 3 implies that a one quarter increase in the expected duration of the ZLB lowers the sensitivity of the 8 quarters out 3-month forward rate 0.116 bps. As the scatterplot in Panel C shows, increasing the expected duration of the ZLB from 0 quarters to 10 quarters thus flattens the sensitivity of this forward rate from around 1.2 bps to a unit of standardized news to essentially nothing. Figure 2 also provides color on the effect of a monetary policy move such as the introduction of calendar guidance in August, 2011, a date marked by the first vertical line in the right-hand plots. By declaring that the federal funds rate would be held at exceptionally low levels until mid-2013, the Federal Reserve increased primary dealers’ expected duration of the ZLB approximately 4 quarters, from around 5 quarters to around 9 quarters. As predicted, the sensitivity of interest rates to MNAs subsequently plummeted.

Finally, verifying Proposition 3, we see in Table 3 that the magnitude of $\hat{\delta}_n$ is first increasing in $n$ and then decreasing. The attenuation in the sensitivity of interest rates to macroeconomic news is greatest for intermediate maturity rates, as the ZLB has already constrained the sensitivity of short-term rates yet is not particularly relevant for long-term rates.

Panel E of Figure 2 provides a sanity check of the results using the TY futures contract instead of ED futures. The main difference is that the left-hand-side variable $\Delta f_{n,t}$ in Eq. (8) is replaced by $R_t$, the % change in the price level of the TY contract in the ±5 minute window around a MNA. This switch reverses the signs of Propositions 2 and 3, as we now expect $\hat{\beta}_{n,t} < 0$ and $\hat{\delta}_n > 0$. This is indeed what Panel E demonstrates: positive MNA surprises decrease TY prices (increase TY yields), and the extent of this reaction declines as the expected duration of the ZLB increases.

4.3 Fed Funds Futures Proxy for Expected Duration of the ZLB

Figure 3 provides additional support for the model predictions using a different proxy for the ZLB: the implied FF rate from the FF24 futures contract, which is also one of the variables shown in the left-hand plot of Figure 1. The only difference between Figures 2 and 3 is that “FF24” replaces “fcsts” as a plotted variable in the right-hand plots and as the independent variable in the scatterplots. As the expected duration of the ZLB increases, the FF rate 24 months out should decrease. For example, after the introduction of calendar guidance in August, 2011, “FF24” decreases significantly consistent with the increase in the surveyed time until the first rate hike. Though not as direct a measure of the expected duration of the ZLB, the implied FF rate is never a stale proxy in the way that the survey proxy for the ZLB could be due to the infrequency of surveys. The implied FF rate is also available
before January, 2011, which allows the analysis to begin further back in time, though Figure 3 covers the same period as Figure 2 for the sake of comparability.

Since the $\hat{\beta}_{n,t}$ are the same in Figures 2 and 3, Panels A to D of Figure 3 support Proposition 2 in the same way: $\hat{\beta}_{n,t} > 0$ for most $(n,t)$ pairs and is otherwise indistinguishable from zero. The relationship between $\hat{\beta}_{n,t}$ and $ZLB_t$ in Figure 3 is positive in the scatterplots, which corroborates Proposition 3. The reason that Proposition 3 implies $\hat{\delta}_n > 0$ in Figure 3 is that the implied FF rate from the FF24 futures contract is a negative proxy for the expected duration of the ZLB: as the latter increases, the former decreases. Table 4 produces the quantitative estimates of $\hat{\delta}_n$ from performing the regression in Eq. (9). I drop the constant (restrict $\gamma_n = 0$) based on the economic intuition that if “FF24” were to equal zero, then the ZLB must be acutely binding and the 3-month forward rate $n$-quarter(s) out must be fixed at the lower bound. Consequently, these interest rates must be insensitive to MNAs. Though not explicitly stated, this logic is also at play in Figure 2. If primary dealers expect the first rate hike to be sufficiently far in the future, the predicted sensitivity of a given interest rate to macroeconomic news is zero, because the downward sloping regression line intersects the x-axis. As we see in Table 4, the $\hat{\delta}_n$ are greater than zero with statistically and economically significant magnitudes. A 100 bps drop in the implied FF rate decreases the reaction of the 16 quarters out 3-month forward rate to a unit of standardized news by 1.285 bps.

A seemingly incongruent feature of Table 4 is that the magnitude of $\hat{\delta}_n$ is monotonically increasing in $n$, which is at odds with the hump-shaped pattern implied by Proposition 4. That is, Table 4 suggests that the ZLB attenuates the sensitivity to MNAs of long-dated rates more than that of shorter-dated rates. This result is in fact supportive of Proposition 4. The reason is that by setting $\gamma_n = 0$ in Eq. (9), I anchor each scatterplot with a point at the origin that corresponds to the ZLB binding “forever” such that the sensitivity of interest rates to data is zero. As “FF24” decreases, the sensitivity of short-dated rates falls quickly to zero and remains there, which leads to a small magnitude of $\hat{\delta}_n$. As “FF24” decreases, the sensitivity of long-dated rates initially remains above zero. As “FF24” nears zero and the ZLB binds for an increasingly longer time, the sensitivity of these long-dated rates must eventually collapse to zero by construction, and the suddenness of this attenuation is reflected in the larger magnitude of $\hat{\delta}_n$. The pattern in $\hat{\delta}_n$ thus comes from using the linear specification in Eq. (9) to analyze the nonlinear relationship between “beta” and “FF24”. In results not shown, estimating an unconstrained Eq. (9) without restricting $\gamma_n = 0$ does lead to the magnitude of $\hat{\delta}_n$ having a hump-shaped pattern in $n$.

As in Figure 2, Panel E of Figure 3 provides a sanity check of the results using the TY futures contract instead of ED futures. We see that $\hat{\beta}_{n,t} < 0$ and $\hat{\delta}_n < 0$ as expected:
positive MNA surprises decrease TY prices (increase TY yields), and the extent of this reaction declines as the implied FF rate decreases.

5 Stock Market’s Sensitivity to MNAs

5.1 Methodology and Results

Having looked at the sensitivity of interest rates to MNAs and how the ZLB may have affected this sensitivity, I turn the analysis to another important asset: the stock market. I first use the empirical framework already presented for interest rates and let the data speak for itself. That is, I run a daily 1-year rolling regression over the ZLB period replacing the left-hand variable of Eq. (8) with $R_t$, the % change in the price level of the ES contract in the $\pm 5$ minute window around a MNA. Using the resulting time series of $\hat{\beta}_{s,t}$, I run Eq. (9) using the two proxies for the expected duration of the ZLB: the median surveyed time until the first rate hike and the implied FF rate from the FF24 futures contract. Analogous to Figures 2 and 3, Figure 4 shows the results of this methodology for stocks. Panel A presents findings using the survey proxy “fcsts” for $ZLB_t$, and Panel B uses the future FF rate “FF24” for $ZLB_t$. Focusing on the “beta” in the right-hand plots, we see that from early 2011 to mid-2012, stocks reacted on the order of 10 to 15 bps for every unit of standardized news. Starting in mid-2012, the sensitivity of stocks started dropping significantly, reaching a level of around 2 bps for every unit of standardized news in 2014. The scatterplots do not show a clear relationship between “beta” and the expected duration of the ZLB. Up until mid-2012, the sensitivity of stocks to MNAs seemed to increase as the ZLB became more binding. The subsequent sharp drop in the sensitivity of stocks to data does not have a clear explanation, however, for the expected duration of the ZLB stayed stable before moderately decreasing.

5.2 Decomposition: Interest Rate News vs. Cash Flow and Risk Premium News

To better understand the time-varying sensitivity $\hat{\beta}_{s,t}$ of stocks to economic data in Figure 4, I utilize a standard decomposition of stock returns. When stocks jump in response to economic data, we know based on the work of Campbell (1991) and Campbell and Ammer (1993) that the reaction could be due to any combination of a change in expected future dividends (cash flow news), a change in expected future interest rates (interest rate news), and a change in expected future excess returns (risk premium news). To see this decomposition algebraically,
I follow Boyd, Hu, and Jagannathan (2005) in using a simple Gordon growth model:

\[ P = \frac{D (1 + G)}{R + \Pi - G} \]  

with \( P \) the price level of the stock market, \( D \) the current dividends of constituent stocks, \( G \) a constant growth rate in the dividends, \( R \) a long-dated risk-free rate, and \( \Pi \) the equity risk premium. Defining \( S \) as the surprise in a given MNA, \( (dP/P) / dS \) is the percentage change in the stock market in response to a data surprise and maps to \( \hat{\beta}_{s,t} \). Using the Gordon growth model, we see that

\[
\frac{1}{P} \frac{dP}{dS} = \frac{1}{P} \frac{D \frac{dG}{dS} (R + \Pi - G)}{(R + \Pi - G)^2} - \left( \frac{dR}{dS} + \frac{d\Pi}{dS} - \frac{dG}{dS} \right) \frac{D}{P} (1 + G) \\
= - \frac{P}{D} \left( \frac{dR}{dS} + \frac{d\Pi}{dS} - \frac{dG}{dS} \right) \left( 1 + \frac{D}{P} \right) \\
\approx - \frac{P}{D} \left( \frac{dR}{dS} + \frac{d\Pi}{dS} - \frac{dG}{dS} \right)
\]

with the approximation in the last step holding for small \( G \) and small \( D/P \). As qualitatively stated previously, the reaction of stocks to macroeconomic news is due to a combination of cash flow news \( (dG/dS) \) term, interest rate news \( (dR/dS) \) term, and risk premium news \( (d\Pi/dS) \) term.

Focusing on the interest rate channel, the interest rate news term

\[ N_R \equiv - \left( \frac{P}{D} \right) \left( \frac{dR}{dS} \right) \]  

has two components: \( P/D \), the price-to-dividend ratio, and \( dR/dS \), the sensitivity of interest rates to MNAs. In general, the interest rate channel attenuates the sensitivity of stocks to economic data such that the absence of this channel would increase the magnitude of the stock market reaction to data. To see this, consider a positive MNA surprise. Based on Figure 4 and Table 2 in addition to the literature on the reaction of stocks to MNAs, we know that positive MNA surprises tend to increase the stock market, so the left-hand side of Eq. (11) is positive. We also know from Proposition 2 that positive MNA surprises increase interest rates, so \( dR/dS > 0 \). Thus the interest rate news term \( N_R < 0 \), which reduces the magnitude of the stock market reaction to positive MNA surprises. The intuition is simply that higher interest rates discount dividends more heavily, which reduces stock prices. The exact same reasoning works for negative MNA surprises: stocks tend to decrease \( ((dP/P) / dS < 0) \) as
do interest rates \((dR/dS < 0)\), so the interest rate news term \((N_R > 0)\) has an offsetting effect on the stock market due to the discounting of dividends at lower interest rates.

If the reaction of interest rates to MNAs offsets the reaction of stocks to MNAs, logic dictates that, all else equal, when the sensitivity of interest rates to MNAs is lower, the sensitivity of stocks to MNAs is higher. Since we have already seen from Proposition 3 that the sensitivity of interest rates to macroeconomic news decreases as the ZLB is expected to bind for longer, we might expect to see that longer expected duration of the ZLB increases the sensitivity of stocks to macroeconomic news. Turning to the data, the right-hand plots of Figure 4 suggest that this positive relationship between the expected duration of the ZLB and the sensitivity of stocks holds only from early 2011 to mid-2012 ("early period") but not from mid-2012 to the end of the sample ("late period"). In the early period, the surveyed time until the first rate hike rose and the implied FF rate 24 months out fell, so the sensitivity of interest rates to data decreased. At the same time, the sensitivity of stocks to data increased, as we would expect if the interest rate news term were dominant in Eq. (11). In the late period, the expected duration of the ZLB was stable before decreasing modestly, which resulted in a stable and then modestly increasing sensitivity of interest rates to data. The sensitivity of the stock market to data, however, decreased continuously and precipitously throughout this period.

We can actually calculate the magnitude of the interest rate news term \(N_R\) in the data. To proxy for \(P/D\) in Eq. (12), I use the price-to-dividend ratio of the S&P 500 shown as "dp" in the right-hand plot of Figure 5 (right y-axis). \(dR/dS\) in Eq. (12) is the number of basis points that \(R\) changes by due to a data surprise. Since \(R\) is the interest rate on a long-term risk-free bond, I measure \(dR/dS\) using the sensitivity of the forward rate from the ED16 contract; i.e., “beta” \(\hat{\beta}_{16,t}\) in Panel D of Figures 2 and 3. The right-hand plot of Figure 5 plots \(\hat{\beta}_{16,t}\) as “betas.ED16” (left y-axis). I use the ED16 contract because it is the longest-maturity ED futures contract in my data, and \(\hat{\beta}_{16,t}\) is conveniently already measured in basis points. Note that while the TY futures contract corresponds to a longer maturity interest rate than the ED16 contract, converting TY prices to yields is a complicating step.

We see a hump-shaped pattern in the interest rate news term, consistent with our intuition. The expected duration of the ZLB increased in the early period before decreasing in the late period, which resulted in the interest rate news term decreasing in magnitude (from 75 bps to 25 bps per unit of standardized news) and then increasing (back to 75 bps). It is clear that if only the interest rate news term were time-varying in Eq. (11), we would expect a positive relationship between the expected duration of the ZLB and the reaction of stocks to MNAs.
We cannot, however, ignore the reality that the change in the stock market in response to a data surprise also depends on cash flow and risk premium news. Time variation in the two former terms must offset time variation in interest rate news to produce the final time-varying sensitivity of stocks to data. I back out the cash flow and risk premium news terms by subtracting the interest rate news term from the overall reaction of stocks to data:

\[ N_{\text{CF+RP}} \equiv \frac{1}{P} \frac{dP}{dS} - N_R \approx -\frac{P}{D} \left( \frac{d\Pi}{dS} - \frac{dG}{dS} \right). \]

In the data, \( N_{\text{CF+RP}} \) is just the \( \hat{\beta}_{s,t} \) of stocks, plotted as “beta” in the right-hand graph of Figure 5, less the “only R” interest rate news term. The left-hand graph of Figure 5 plots the resulting cash flow and risk premium news terms as “PI\_G” on the right y-axis. As expected, these two news terms demonstrate time-varying sensitivity to MNAs; in particular, a hump-shaped pattern that is almost a reflection of the hump-shaped pattern in the interest rate news term. At the beginning of the early period, a standardized unit of news changed the combined expectations of future dividends and excess returns by 90 bps. This reaction decreased steadily to less than 50 bps by mid-2012 before increasing back to 75 bps toward the end of the sample.

The decomposition in Figure 5 yields a few interesting observations. First, the magnitudes of the interest rate news term and the combined cash flow and risk premium news terms are an order larger than the magnitude of \( \hat{\beta}_{s,t} \). Second, while \( N_{\text{CF+RP}} \) appears to be a reflection of \( N_R \), the symmetry is clearly not perfect. For example, at the beginning of 2011, cash flow and risk premium news reacted more strongly to data than interest rate news, so stocks on net increased by 10 to 15 bps to a positive unit of surprise. Toward the beginning of 2014, the magnitude of \( N_{\text{CF+RP}} \) and \( N_R \) terms were nearly identical, so stocks on net increased by only 2 bps to a positive unit of surprise. Finally, relating the expected duration of the ZLB to time-variation in the cash flow and risk premium news terms is intriguing. As the expected duration of the ZLB increased over the early period, the magnitude of \( N_{\text{CF+RP}} \) decreased. Then, as the expected duration of the ZLB stabilized before decreasing over the late period, the magnitude of \( N_{\text{CF+RP}} \) increased again. The result is a negative correlation between the expected duration of the ZLB and magnitude of the combined change in expected future dividends and expected future excess returns in response to macroeconomic news.

An obvious extension is to break \( N_{\text{CF+RP}} \) into its constituent cash flow news and risk premium news terms. This step would allow us to separately analyze time variation in how expectations of dividends react to macroeconomic news and how expectations of excess returns react to macroeconomic news. The challenge is finding reliable empirical measures of changes in expected dividends and expected excess returns over short time periods around
MNAs. Boyd, Hu, and Jagannathan (2005) attempt to decompose the reaction of stocks to unemployment news into each of the three news components by using monthly proxies for cash flow news and risk premium news. A primary concern with this strategy is that unemployment news only arrives once a month, so any monthly proxy necessarily captures information aside from unemployment releases. As I consider many MNAs throughout a month, it would be challenging to map the impact of a given MNA to monthly proxies for cash flow and risk premium news. Bernanke and Kuttner (2005) attempt a different decomposition of stocks to surprises in the FF rate. They use a monthly VAR based off the methodology of Campbell (1991) and Campbell and Ammer (1993). A monthly VAR for my purposes, however, would run into the same problems just mentioned.

6 Conclusion

This paper has shown how the reactions of interest rates and stocks to macroeconomic news have changed over the ZLB period. A key reason for these time-varying reactions is the expected duration of the ZLB. In a shadow rate term structure model, I analytically derive predictions for how interest rates should react to data surprises. First, “positive” data surprises increases interest rates. Second, as the expected duration of the ZLB increases, whether because economic conditions deteriorate or because monetary policy shifts, interest rates become less sensitive to data surprises. Third and finally, this attenuation in the sensitivity of interest rates is greater for medium-maturity rates. Each of these three predictions is statistically and economically significant in the data. Insofar as the extent to which interest rates move in response to MNAs contains information on the effectiveness of monetary policy and fiscal policy, my results provide policy insights.

The sensitivity of interest rates to news further affects the sensitivity of the stock market to news. If the former were the only factor influencing the latter, greater expected duration of the ZLB would increase the responsiveness of stocks to MNAs. This result does not appear to hold in the data because the reaction of the stock market to news also depends on changes in expected future dividends and changes in expected future excess returns. The expected duration of the ZLB does, however, correlate negatively with the magnitude of this cash flow plus risk premium news term. An interesting idea going forward is to separate cash flow news from risk premium news and see how the magnitude of each term varies over time.
References


Tables and Figures

Table 1: Full sample of MNAs from 12/17/08 to 3/6/14. Freq. refers to monthly (M) or weekly (W). Source uses the following acronyms: BEA, Bureau of Economic Analysis; BLS, Bureau of Labor Statistics; CB, Conference Board; Census, Census Bureau; ETA, Employment and Training Administration; Fed, Federal Reserve Board of Governors; and ISM, Institute for Supply Management.

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<th>Event Name</th>
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<th>Freq.</th>
<th>Source</th>
<th>Time (ET)</th>
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<td>M</td>
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<td>M</td>
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<td>M</td>
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<td>M</td>
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<td>M</td>
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<tr>
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<td>M</td>
<td>Census</td>
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</tr>
<tr>
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Table 2: Reactions to news during the ZLB. Regressions of asset returns on standardized news for each event $i$ and then grouping all events together (last row): $R_t = \alpha_i + \beta_i S_{i,t} + \epsilon_t$. The left-hand side variable $R_t$ is the asset return in a $\pm 5$ minute window around a MNA (basis points change in the forward rate associated with ED futures or % change in the price level associated with TY and ES futures). The right-hand side variable $S_{i,t}$ is defined as the actual data less the expected data based on a Bloomberg survey of economists, all normalized by the sample standard deviation of the actual less the expected difference: $S_{i,t} = (A_{i,t} - E_t - [A_{i,t}]) / \hat{\sigma}_t$. $t$-statistics (not shown) are based on heteroskedasticity-consistent standard errors. *** denotes significance at 1%, ** denotes significance at 5%, and * denotes significance at 10%.

<table>
<thead>
<tr>
<th>Event Name</th>
<th>ED1</th>
<th>ED4</th>
<th>ED8</th>
<th>ED16</th>
<th>TY</th>
<th>ES</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\beta}$ (bps)</td>
<td>$\hat{\beta}$ (bps)</td>
<td>$\hat{\beta}$ (bps)</td>
<td>$\hat{\beta}$ (bps)</td>
<td>$\hat{\beta}$ (%)</td>
<td>$\hat{\beta}$ (%)</td>
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<tr>
<td>Change in NFP</td>
<td>-0.340**</td>
<td>-1.58</td>
<td>0.110</td>
<td>2.511</td>
<td>-0.116</td>
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<tr>
<td>Consumer Confidence</td>
<td>0.059</td>
<td>0.486***</td>
<td>0.855***</td>
<td>0.868***</td>
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<td>0.196***</td>
</tr>
<tr>
<td>CPI</td>
<td>0.017</td>
<td>0.316*</td>
<td>0.491**</td>
<td>0.657**</td>
<td>-0.037***</td>
<td>-0.046</td>
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<tr>
<td>Durable Orders</td>
<td>-0.004</td>
<td>0.332**</td>
<td>0.834***</td>
<td>1.383***</td>
<td>-0.058***</td>
<td>0.123***</td>
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<tr>
<td>Existing Home Sales</td>
<td>0.027</td>
<td>0.264**</td>
<td>0.579***</td>
<td>0.704***</td>
<td>-0.036***</td>
<td>0.143***</td>
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<tr>
<td>Factory Orders</td>
<td>0.037</td>
<td>0.148</td>
<td>0.239</td>
<td>0.347*</td>
<td>-0.016</td>
<td>0.035*</td>
</tr>
<tr>
<td>Housing Starts</td>
<td>-0.066</td>
<td>0.393***</td>
<td>1.110***</td>
<td>2.153***</td>
<td>-0.080***</td>
<td>0.095***</td>
</tr>
<tr>
<td>Industrial Production</td>
<td>0.032</td>
<td>0.132</td>
<td>0.288**</td>
<td>0.323**</td>
<td>-0.017**</td>
<td>0.052***</td>
</tr>
<tr>
<td>Initial Jobless Claims</td>
<td>0.007</td>
<td>0.293***</td>
<td>0.563***</td>
<td>0.561**</td>
<td>-0.037***</td>
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</tr>
<tr>
<td>ISM Manufacturing</td>
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<tr>
<td>ISM Non-Manufacturing</td>
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<td>0.893***</td>
<td>1.628***</td>
<td>2.004***</td>
<td>-0.098***</td>
<td>0.162***</td>
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<tr>
<td>Leading Index</td>
<td>0.088*</td>
<td>0.148</td>
<td>0.305*</td>
<td>0.187</td>
<td>-0.014</td>
<td>0.074**</td>
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<tr>
<td>New Home Sales</td>
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<td>0.835***</td>
<td>1.410***</td>
<td>1.523***</td>
<td>-0.084***</td>
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<tr>
<td>Pending Home Sales</td>
<td>-0.015</td>
<td>0.327***</td>
<td>0.690***</td>
<td>0.822***</td>
<td>-0.037***</td>
<td>0.101***</td>
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<tr>
<td>Personal Income</td>
<td>0.034</td>
<td>0.090</td>
<td>0.054</td>
<td>0.118</td>
<td>-0.006</td>
<td>0.019**</td>
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<tr>
<td>PPI</td>
<td>0.052</td>
<td>0.552***</td>
<td>0.662**</td>
<td>0.694*</td>
<td>-0.039**</td>
<td>0.041</td>
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<td>Retail Sales</td>
<td>0.223***</td>
<td>0.992***</td>
<td>1.672***</td>
<td>1.659***</td>
<td>-0.096***</td>
<td>0.215***</td>
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<tr>
<td>Unemployment Rate</td>
<td>0.014</td>
<td>0.294</td>
<td>0.560</td>
<td>0.980</td>
<td>-0.027</td>
<td>0.037</td>
</tr>
<tr>
<td>All</td>
<td>0.017</td>
<td>0.268***</td>
<td>0.617***</td>
<td>0.831***</td>
<td>-0.043***</td>
<td>0.097***</td>
</tr>
</tbody>
</table>
Table 3: Attenuation in the sensitivity of ED futures due to the ZLB using the survey proxy. First-stage regression is a daily 1-year rolling regression over the ZLB period that aggregates all 18 MNAs: \( \Delta f_{n,t} = \alpha_n + \beta_n S_t + \epsilon_{n,t} \). \( \Delta f_{n,t} \) is the basis points change in the forward rate implied from the ED\( n \) contract in a ±5 minute window around a MNA. The right-hand side variable \( S_t \) aggregates \( S_{i,t} \) for MNA \( i \). \( S_{i,t} \) is defined as the actual data less the expected data based on a Bloomberg survey of economists, all normalized by the sample standard deviation of the actual less the expected difference: \( \frac{A_{i,t} - E_t - [A_{i,t}]}{\hat{\sigma}_i} \). Second-stage regression takes the estimated \( \hat{\beta}_{n,t} \) corresponding to the first-stage regression over a 1-year window centered at \( t \) and regresses \( \hat{\beta}_{n,t} \) on ZLB\( t \): \( \hat{\beta}_{n,t} = \gamma_n + \delta_n ZLB_t + \eta_{n,t} \). ZLB\( t \) is the median number of quarters until the first rate hike based on primary dealer surveys by the FRBNY. \( t \)-statistics (not shown) are based on heteroscedasticity-consistent standard errors. *** denotes significance at 1%, ** denotes significance at 5%, and * denotes significance at 10%.

<table>
<thead>
<tr>
<th>Rate</th>
<th>( \hat{\delta}_n )</th>
<th>Rate</th>
<th>( \hat{\delta}_n )</th>
<th>Rate</th>
<th>( \hat{\delta}_n )</th>
<th>Rate</th>
<th>( \hat{\delta}_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ED1</td>
<td>-0.007***</td>
<td>ED5</td>
<td>-0.079***</td>
<td>ED9</td>
<td>-0.124***</td>
<td>ED13</td>
<td>-0.093***</td>
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<tr>
<td>ED2</td>
<td>-0.003***</td>
<td>ED6</td>
<td>-0.107***</td>
<td>ED10</td>
<td>-0.122***</td>
<td>ED14</td>
<td>-0.094***</td>
</tr>
<tr>
<td>ED3</td>
<td>-0.027***</td>
<td>ED7</td>
<td>-0.112***</td>
<td>ED11</td>
<td>-0.117***</td>
<td>ED15</td>
<td>-0.078***</td>
</tr>
<tr>
<td>ED4</td>
<td>-0.050***</td>
<td>ED8</td>
<td>-0.116***</td>
<td>ED12</td>
<td>-0.091***</td>
<td>ED16</td>
<td>-0.082***</td>
</tr>
</tbody>
</table>
Table 4: Attenuation in the sensitivity of ED futures due to the ZLB using the FF24 proxy. First-stage regression is a daily 1-year rolling regression over the ZLB period that aggregates all 18 MNAs: $\Delta f_{n,t} = \alpha_n + \beta_n S_t + \epsilon_{n,t}$. $\Delta f_{n,t}$ is the basis points change in the forward rate implied from the ED$n$ contract in a ±5 minute window around a MNA. The right-hand side variable $S_t$ aggregates $S_{i,t}$ for MNA $i$. $S_{i,t}$ is defined as the actual data less the expected data based on a Bloomberg survey of economists, all normalized by the sample standard deviation of the actual less the expected difference: $S_{i,t} = (A_{i,t} - E_t - [A_{i,t}]) / \hat{\sigma}_i$.

Second-stage regression takes the estimated $\hat{\beta}_{n,t}$ corresponding to the first-stage regression over a 1-year window centered at $t$ and regresses $\hat{\beta}_{n,t}$ on ZLB$_t$: $\hat{\beta}_{n,t} = \gamma_n + \delta_n ZLB_t + \eta_{n,t}$ with the constraint that $\gamma_n = 0$. ZLB$_t$ is the implied FF rate (%) from the FF24 futures contract. $t$-statistics (not shown) are based on heteroscedasticity-consistent standard errors. *** denotes significance at 1%, ** denotes significance at 5%, and * denotes significance at 10%.

<table>
<thead>
<tr>
<th>Rate</th>
<th>$\hat{\delta}_n$</th>
<th>Rate</th>
<th>$\hat{\delta}_n$</th>
<th>Rate</th>
<th>$\hat{\delta}_n$</th>
<th>Rate</th>
<th>$\hat{\delta}_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ED1</td>
<td>0.019***</td>
<td>ED5</td>
<td>0.463***</td>
<td>ED9</td>
<td>0.988***</td>
<td>ED13</td>
<td>1.231***</td>
</tr>
<tr>
<td>ED2</td>
<td>0.056***</td>
<td>ED6</td>
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<td>ED10</td>
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<td>ED14</td>
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<tr>
<td>ED3</td>
<td>0.155***</td>
<td>ED7</td>
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<td>1.124***</td>
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<tr>
<td>ED4</td>
<td>0.304***</td>
<td>ED8</td>
<td>0.883***</td>
<td>ED12</td>
<td>1.185***</td>
<td>ED16</td>
<td>1.285***</td>
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</table>
Figure 1: Proxies for the expected duration of the ZLB. The left-hand plot is of the implied FF rate (\%) from FF futures with maturities 1 month to 24 months out. Higher-value lines correspond to longer-maturity FF futures. The right-hand plot is the median number of quarters until the first rate hike based on primary dealer surveys by the FRBNY. The two vertical lines correspond to the dates of the August, 2011 FOMC meeting (when calendar guidance was introduced) and the December, 2012 FOMC meeting (when data thresholds was introduced), respectively.
Figure 2: Time-varying sensitivities of interest rates to MNAs and the impact of the ZLB using the survey proxy. Focusing on Panel A, the right-hand plot shows two time series. “fcsts,” plotted on the right y-axis, is the median number of quarters until the first rate hike based on primary dealer surveys by the FRBNY. “beta,” plotted on the left y-axis, comes from a daily 1-year rolling regression that aggregates all 18 MNAs: \( \Delta f_{n,t} = \alpha_n + \beta_n S_t + \epsilon_{n,t} \). \( \Delta f_{n,t} \) is the basis points change in the forward rate implied from the ED\( n \) contract in a ±5 minute window around a MNA. The right-hand side variable \( S_t \) aggregates \( S_{i,t} \) for MNA \( i \). \( S_{i,t} \) is defined as the actual data less the expected data based on a Bloomberg survey of economists, all normalized by the sample standard deviation of the actual less the expected difference: 

\[
S_{i,t} = \frac{(A_{i,t} - E_t - [A_{i,t}])}{\hat{\sigma}_i}.
\]

“beta” is specifically the \( \hat{\beta}_{n,t} \) estimated parameter from running the aforementioned regression on a 1-year window centered at \( t \). The two vertical lines correspond to the dates of the August, 2011 FOMC meeting (when calendar guidance was introduced) and the December, 2012 FOMC meeting (when data thresholds was introduced), respectively. The left-hand plot is simply a scatterplot of “beta” on “fcsts.” Each point in the scatterplot is color-coded based on the timestamp: red for 12/17/08 to 8/9/11 corresponding to the period of general forward guidance (“fwd”); green for 8/10/11 to 12/12/12 corresponding to the period of calendar guidance (“cdr”); and blue for 12/13/12 to 3/6/14 corresponding to the period of data thresholds (“dat”). In a given period, lighter colors indicate earlier dates. For Panels A, B, C, and D, \( n = 1, 4, 8, \) and 16, respectively. Panel E replaces \( \Delta f_{n,t} \) with \( R_t \), the % change in the price level of the TY contract. Thus, \( \hat{\beta}_{n,t} \) is measured in basis points for Panels A, B, C, and D but in % for Panel E.

Panel A: ED1
Figure 2: Time-varying sensitivities of interest rates and the impact of the ZLB using the survey proxy (continued).

Panel B: ED4

Panel C: ED8
Figure 2: Time-varying sensitivities of interest rates and the impact of the ZLB using the survey proxy (continued).

Panel D: ED16

Panel E: TY
Figure 3: Time-varying sensitivities of interest rates to MNAs and the impact of the ZLB using the FF24 proxy. Focusing on Panel A, the right-hand plot shows two time series. “FF24,” plotted on the right y-axis, is the implied FF rate (%) from the FF24 futures contract. “beta,” plotted on the left y-axis, comes from a daily 1-year rolling regression that aggregates all 18 MNAs: \( \Delta f_{n,t} = \alpha_n + \beta_n S_t + \epsilon_{n,t} \). \( \Delta f_{n,t} \) is the basis points change in the forward rate implied from the ED\( n \) contract in a ±5 minute window around a MNA. The right-hand side variable \( S_t \) aggregates \( S_{i,t} \) for MNA \( i \). \( S_{i,t} \) is defined as the actual data less the expected data based on a Bloomberg survey of economists, all normalized by the sample standard deviation of the actual less the expected difference: \( S_{i,t} = (A_{i,t} - E_t - [A_{i,t}]) / \hat{\sigma}_i \). “beta” is specifically the \( \hat{\beta}_{n,t} \) estimated parameter from running the aforementioned regression on a 1-year window centered at \( t \). The two vertical lines correspond to the dates of the August, 2011 FOMC meeting (when calendar guidance was introduced) and the December, 2012 FOMC meeting (when data thresholds was introduced), respectively. The left-hand plot is simply a scatterplot of “beta” on “FF24.” Each point in the scatterplot is color-coded based on the timestamp: red for 12/17/08 to 8/9/11 corresponding to the period of general forward guidance (“fwd”); green for 8/10/11 to 12/12/12 corresponding to the period of calendar guidance (“cdr”); and blue for 12/13/12 to 3/6/14 corresponding to the period of data thresholds (“dat”). In a given period, lighter colors indicate earlier dates. For Panels A, B, C, and D, \( n = 1, 4, 8, \) and 16, respectively. Panel E replaces \( \Delta f_{n,t} \) with \( R_t \), the % change in the price level of the TY contract. Thus, \( \hat{\beta}_{n,t} \) is measured in basis points for Panels A, B, C, and D but in % for Panel E.

Panel A: ED1
Figure 3: Time-varying sensitivities of interest rates to MNAs and the impact of the ZLB using the FF24 proxy (continued).

Panel B: ED4

Panel C: ED8
Figure 3: Time-varying sensitivities of interest rates to MNAs and the impact of the ZLB using the FF24 proxy (continued).

Panel D: ED16

Panel E: TY
Figure 4: Time-varying sensitivities of stocks to MNAs and the impact of the ZLB. Focusing on Panel A, the right-hand plot shows two time series. “fcsts,” plotted on the right y-axis, is the median number of quarters until the first rate hike based on primary dealer surveys by the FRBNY. “beta,” plotted on the left y-axis, comes from a daily 1-year rolling regression that aggregates all 18 MNAs: \( R_t = \alpha_s + \beta_s S_t + \epsilon_{s,t} \). \( R_t \) is the % change in the price level of the ES contract in a ±5 minute window around a MNA. The right-hand side variable \( S_t \) aggregates \( S_{i,t} \) for MNA \( i \). \( S_{i,t} \) is defined as the actual data less the expected data based on a Bloomberg survey of economists, all normalized by the sample standard deviation of the actual less the expected difference: \( S_{i,t} = (A_{i,t} - \bar{E}_t - [A_{i,t}]) / \hat{\sigma}_i \). “beta” is specifically the \( \hat{\beta}_{s,t} \) estimated parameter from running the aforementioned regression on a 1-year window centered at \( t \). The two vertical lines correspond to the dates of the August, 2011 FOMC meeting (when calendar guidance was introduced) and the December, 2012 FOMC meeting (when data thresholds was introduced), respectively. The left-hand plot is simply a scatterplot of “beta” on “fcsts.” Each point in the scatterplot is color-coded based on the timestamp: red for 12/17/08 to 8/9/11 corresponding to the period of general forward guidance (“fwd”); green for 8/10/11 to 12/12/12 corresponding to the period of calendar guidance (“cdr”); and blue for 12/13/12 to 3/6/14 corresponding to the period of data thresholds (“dat”). In a given period, lighter colors indicate earlier dates. Panel B is identical to Panel A, except the former replaces “fcsts” with “FF24,” the implied FF rate (%) from the FF24 futures contract.

Panel A: Survey proxy for the ZLB
Figure 4: Time-varying sensitivities of stocks to MNAs and the impact of the ZLB (continued).

Panel B: FF24 proxy for the ZLB
**Figure 5: Decomposition of the stock market reaction to MNAs into interest rate news versus cash flow and risk premium news.** The right-hand plot shows two time series. “dp,” plotted on the right y-axis, is the dividend-to-price ratio of the S&P 500. “beta_ED16,” plotted on the left y-axis, is the sensitivity of the forward rate from the ED16 contract; i.e., “beta” $\hat{\beta}_{16,t}$ in Panel D of Figures 2 and 3. The left-hand plot shows three time series. “beta,” plotted on the left y-axis, is the sensitivity of the stock market from the ES contract; i.e., “beta” $\hat{\beta}_{s,t}$ in Figure 4. “only_R,” plotted on the right y-axis, is the portion of the stock market reaction to MNAs attributed to interest rate news: $N_R \equiv -(P/D) \left( \frac{dR}{dS} \right)$. I compute “only_R” by using “dp” to proxy for $P/D$ and $\hat{\beta}_{16,t}$ to proxy for $dR/dS$. “PLG,” also plotted on the right y-axis, is the portion of the stock market reaction to MNAs attributed to cash flow and risk premium news: $N_{CF+RP} \equiv -(P/D) \left( \frac{d\Pi}{dS} - \frac{dG}{dS} \right)$. I compute “PLG” simply by subtracting “only_R” from $\hat{\beta}_{s,t}$, as it is an accounting identity that a change in the stock market must come from either interest rate news or cash flow and risk premium news (or a combination). The two vertical lines correspond to the dates of the August, 2011 FOMC meeting (when calendar guidance was introduced) and the December, 2012 FOMC meeting (when data thresholds was introduced), respectively.
Appendix A

We know that the forward rate

\[ f_{n,t} = (n + 1) y_{n+1,t} - n y_{n,t} \]

with \( y_{n,t} \) the log of the yield to maturity of a zero-coupon bond of maturity \( n \) at time \( t \). We can write this statement as

\[
\begin{align*}
&f_{n,t} = \frac{1}{(n + 1)} \log \left( \exp \left( -i t + \mathbb{E}_t^Q \left[ \exp \left( -\sum_{j=1}^{n} i t_{j+1} \right) \right] \right) \right) + \log \left( \exp \left( -i t + \mathbb{E}_t^Q \left[ \exp \left( -\sum_{j=1}^{n-1} i t_{j+1} \right) \right] \right) \right) \\
&\approx \mathbb{E}_t^Q \left[ \sum_{j=1}^{n} i t_{j+1} \right] - \frac{1}{2} \mathbb{V}_t^Q \left[ \sum_{j=1}^{n} i t_{j+1} \right] - \mathbb{E}_t^Q \left[ \sum_{j=1}^{n-1} i t_{j+1} \right] + \frac{1}{2} \mathbb{V}_t^Q \left[ \sum_{j=1}^{n-1} i t_{j+1} \right]
\end{align*}
\]

(13)

\( \mathbb{E}_t^Q \left[ \cdot \right] \) is the conditional expectation and \( \mathbb{V}_t^Q \left[ \cdot \right] \) is the conditional variance. The first equality comes from the fact that \( -ny_{n,t} = p_{n,t} \), with \( p_{n,t} \) the log price that equals a discounted expectation of the terminal payoff. The approximation comes from the fact that \( \log \left( \mathbb{E} \left[ \exp (Z) \right] \right) \approx \mathbb{E} \left[ Z \right] + \frac{1}{2} \mathbb{V} \left[ Z \right] \) for any random variable \( Z \).

Substituting Eq. (1) into the first term in Eq. (13), we see that

\[
\begin{align*}
\mathbb{E}_t^Q \left[ i t_{n+1} \right] &= \mathbb{E}_t^Q \left[ \max \left\{ s_{t+n}, i \right\} \right] \\
&= \Pr_t^Q \left[ s_{t+n} < i \right] \times i + \Pr_t^Q \left[ s_{t+n} \geq i \right] \times \mathbb{E}_t^Q \left[ s_{t+n} \mid s_{t+n} \geq i \right].
\end{align*}
\]

(14)

Based on Eqs. (2) and (3), the shadow rate has a conditional normal distribution:

\[ s_{t+n} | I_t \sim \mathcal{N} \left( \bar{a}_n + b_n x_t, \sigma_n^Q \right) \]

(15)

with parameters defined as

\[
\begin{align*}
\bar{a}_n &\equiv \delta_0 + \delta_1 \mu^Q \frac{1 - (\rho^Q)^n}{1 - \rho^Q}, \\
b_n &\equiv \delta_1 (\rho^Q)^n, \quad \text{and} \\
\sigma_n^Q &\equiv \sqrt{\left( \delta_1 \sigma_e \right)^2 \frac{1 - (\rho^Q)^{2n}}{1 - (\rho^Q)^2}}.
\end{align*}
\]

By assumption that \( \delta_1 > 0 \) and \( \rho^Q > 0 \), we know that \( b_n > 0 \). The standard deviation \( \sigma_n^Q > 0 \) as well. Using the conditional normal distribution of the shadow rate in Eq. (15) and the formula for the first moment of the truncated normal distribution, Eq. (14) simplifies:
$$E_{n}^{Q} [i_{t+n}] = \frac{i}{\sigma_{n}^{Q}} \Phi \left( \frac{\overline{a}_{n} + b_{n}x_{t} - \frac{i}{\sigma_{n}^{Q}}}{} \right) + \phi \left( \frac{\overline{a}_{n} + b_{n}x_{t} - \frac{i}{\sigma_{n}^{Q}}}{} \right)$$

for \( g(x) = x\Phi(x) + \phi(x) \). In order to derive the second term in Eq. (13), Wu and Xia (2014) show and utilize the following approximations:

\[
\begin{align*}
V_{n}^{Q} [i_{t+n}] & \approx \Pr_{t}^{Q} [s_{t+n} \geq \frac{i}{\sigma_{n}^{Q}}] V_{t}^{Q} [s_{t+n}] \\
C_{n}^{Q} [i_{t+n-j}, i_{t+n}] & \approx \Pr_{t}^{Q} [s_{t+n-j} \geq \frac{i}{\sigma_{n}^{Q}}, s_{t+n} \geq \frac{i}{\sigma_{n}^{Q}}] C_{t}^{Q} [s_{t+n-j}, s_{t+n}] \\
& \approx \Pr_{t}^{Q} [s_{t+n} \geq \frac{i}{\sigma_{n}^{Q}}] C_{t}^{Q} [s_{t+n-j}, s_{t+n}] .
\end{align*}
\]

\( C_{n}^{Q} [\cdot] \) is the conditional covariance. The last approximation uses Bayes’ law and assumes that the shadow rate is persistent such that

\[
\Pr_{t}^{Q} [s_{t+n-j} \geq \frac{i}{\sigma_{n}^{Q}} | s_{t+n} \geq \frac{i}{\sigma_{n}^{Q}}] \approx 1.
\]

Making use of the approximations in Eqs. (17) and (18), the second term in Eq. (13) is

\[
\begin{align*}
\frac{1}{2} \left( V_{t}^{Q} \left[ \sum_{j=1}^{n} i_{t+j} \right] - V_{t}^{Q} \left[ \sum_{j=1}^{n-1} i_{t+j} \right] \right) & \approx \Pr_{t}^{Q} [s_{t+n} \geq \frac{i}{\sigma_{n}^{Q}}] \times \frac{1}{2} \left( V_{t}^{Q} \left[ \sum_{j=1}^{n} \sigma_{t+j} \right] - V_{t}^{Q} \left[ \sum_{j=1}^{n-1} \sigma_{t+j} \right] \right) \\
& = \Phi \left( \frac{\overline{a}_{n} + b_{n}x_{t} - \frac{i}{\sigma_{n}^{Q}}}{} \right) \times (\overline{a}_{n} - a_{n})
\end{align*}
\]

with parameter \( a_{n} \) defined as

\[
a_{n} \equiv \overline{a}_{n} - \frac{1}{2} (\delta \sigma_{t})^{2} \left( \frac{1 - (\rho^{Q})^{n}}{1 - \rho^{Q}} \right)^{2}.
\]

Simplifying Eq. (13) by substituting in expressions for the first term derived in Eq. (16) and second term derived in Eq. (19),

\[
\begin{align*}
f_{n,t} & \approx \frac{i}{\sigma_{n}^{Q}} \phi \left( \frac{\overline{a}_{n} + b_{n}x_{t} - \frac{i}{\sigma_{n}^{Q}}}{} \right) \times (\overline{a}_{n} - a_{n}) \\
& = \frac{i}{\sigma_{n}^{Q}} \phi \left( \frac{\overline{a}_{n} + b_{n}x_{t} - \frac{i}{\sigma_{n}^{Q}}}{} \right) + \sigma_{n}^{Q} \frac{\partial g \left( \frac{\overline{a}_{n} + b_{n}x_{t} - \frac{i}{\sigma_{n}^{Q}}}{} \right)}{\partial \overline{a}_{n}} \times (\overline{a}_{n} - a_{n}) \\
& \approx \frac{i}{\sigma_{n}^{Q}} \phi \left( \frac{\overline{a}_{n} + b_{n}x_{t} - \frac{i}{\sigma_{n}^{Q}}}{} \right) \times (\overline{a}_{n} - a_{n})
\end{align*}
\]

The last line comes from a first-order Taylor approximation. We thus obtain the forward rate \( f_{n,t} \) in Eq. (4).
Appendix B

I use the following definition in many of the proofs below:

\[ z = \frac{i - \bar{a}_n - b_n x_t}{\sigma_n^Q}. \]

B.1 Proof of Proposition 1

Eq. (5) gives the probability that the ZLB binds at some time \( t + n \). As \( x_t \) decreases, this probability increases:

\[ \frac{\partial}{\partial x_t} (\Pr_t^Q[\{s_{t+n} < \hat{i}\}] = \phi \left( \frac{i - \bar{a}_n - b_n x_t}{\sigma_n^Q} \right) \times \left( -\frac{b_n}{\sigma_n^Q} \right) < 0. \]

As \( \delta_1 \) decreases, the probability that the ZLB is still binding also increases under certain conditions:

\[ \frac{\partial}{\partial \delta_1} (\Pr_t^Q[\{s_{t+n} < \hat{i}\}] < 0. \]

To see this, expand this probability using the model parameters:

\[
\Pr_t^Q[\{s_{t+n} < \hat{i}\}] = \Phi \left( \frac{i - \bar{a}_n - b_n x_t}{\sigma_n^Q} \right)
= \Phi \left( \frac{\hat{i} - \delta_0 + \delta_1 \mu^{Q^{1-(\rho_Q)^n}} - \delta_1 (\rho_Q)^n x_t}{\delta_1 \sqrt{(\sigma_t)^2 1-(\rho_Q)^2n}} \right)
= \Phi \left( \frac{\hat{i} - \delta_0 + \mu^{Q^{1-(\rho_Q)^n}} - (\rho_Q)^n x_t}{\delta_1 \sqrt{(\sigma_t)^2 1-(\rho_Q)^2n} + \frac{\mu^{1-(\rho_Q)^n}}{1-(\rho_Q)^2n} - (\rho_Q)^n x_t}} \right).
\]

The second term in the CDF doesn’t depend on \( \delta_1 \). Assuming that \( \hat{i} - \delta_0 > 0 \), the first term becomes arbitrarily large as \( \delta_1 \) nears 0, which implies that the CDF becomes arbitrarily close to 1 in value.

B.2 Proof of Proposition 2

To show that \( \partial f_{n,t}/\partial x_t > 0 \), I take the comparative static

\[
\frac{\partial f_{n,t}}{\partial x_t} = \Phi \left( \frac{a_n + b_n x_t - i}{\sigma_n^Q} \right) \times b_n > 0.
\]
B.3 Proof of Proposition 3

To show that $\frac{\partial}{\partial x_t} (\frac{\partial f_{n,t}}{\partial x_t}) > 0$, I take the comparative static

$$\frac{\partial}{\partial x_t} (\frac{\partial f_{n,t}}{\partial x_t}) = \phi \left( \frac{a_n + b_n x_t - \tilde{\bar{}}}{\sigma_n^Q} \right) \times \frac{v_n^2}{\sigma_n^Q} > 0.$$ 

To show that $\frac{\partial}{\partial \delta_1} (\frac{\partial f_{n,t}}{\partial x_t}) > 0$, I first make the following claim.

**Claim 1** $\frac{\partial z}{\partial \delta_1} > 0$ for $\delta_1 < \tilde{\delta}$ for some $\tilde{\delta}$.

The comparative static

$$\frac{\partial}{\partial \delta_1} \left( \frac{\partial f_{n,t}}{\partial x_t} \right) = \phi(z) \times \frac{\partial z}{\partial \delta_1} b_n + \Phi(z) \times (\rho^Q)^n$$

is guaranteed to be positive when $\delta_1 < \tilde{\delta}$ because $\frac{\partial z}{\partial \delta_1} > 0$ and the other terms are all positive as well.

**B.3.1 Proof of Claim 1**

To sign $\frac{\partial z}{\partial \delta_1}$, consider only the numerator since the denominator is positive:

$$\frac{\partial}{\partial \delta_1} (a_n + b_n x_t - \tilde{\bar{}}) \times \sigma_n^Q - \frac{\partial}{\partial \delta_1} (\sigma_n^Q) \times (a_n + b_n x_t - \tilde{\bar{}}).$$

In the expressions below, the first line simplifies the above expression for the numerator and each successive arrow ignores positive terms that are factored out.

$$\frac{\partial}{\partial \delta_1} (a_n + b_n x_t - \tilde{\bar{}}) \times \sigma_n^Q - \frac{\partial}{\partial \delta_1} (\sigma_n^Q) \times (a_n + b_n x_t - \tilde{\bar{}})$$

$$\Rightarrow \mu^Q \frac{1 - (\rho^Q)^n}{1 - \rho^Q} - \delta_1 \sigma_\epsilon^2 \left( \frac{1 - (\rho^Q)^n}{1 - \rho^Q} \right)^2 + (\rho^Q)^n x_t$$

$$- \left( \frac{\delta_0}{\delta_1} + \mu^Q \frac{1 - (\rho^Q)^n}{1 - \rho^Q} - \frac{1}{2} \delta_1 \sigma_\epsilon^2 \left( \frac{1 - (\rho^Q)^n}{1 - \rho^Q} \right)^2 + (\rho^Q)^n x_t - \frac{i}{\delta_1} \right)$$

$$\Rightarrow \frac{i - \delta_0}{\delta_1} - \frac{1}{2} \delta_1 \sigma_\epsilon^2 \left( \frac{1 - (\rho^Q)^n}{1 - \rho^Q} \right)^2.$$ 

This last line must be positive in order for $\frac{\partial z}{\partial \delta_1} > 0$. This occurs when

$$i - \delta_0 > \frac{1}{2} (\delta_1)^2 \sigma_\epsilon^2 \left( \frac{1 - (\rho^Q)^n}{1 - \rho^Q} \right)^2 \quad \Rightarrow \quad \delta_1 < \sqrt{\frac{2}{\sigma_\epsilon^2} (i - \delta_0) \frac{1 - \rho^Q}{1 - (\rho^Q)^n}} \equiv \tilde{\delta}.$$
For $\delta_1$ that is not too large, we have $\partial z/\partial \delta_1 > 0$. This proof also relies on the assumption in Proposition 1 that $i - \delta_0 > 0$. Assuming that $\sigma_i$ is not too large, there should be a sufficiently large range of values for $\delta_1$ for which Claim 1 holds.

**B.4 Proof of Proposition 4**

The comparative static $\partial/\partial x_t (\partial/\partial x_t (\partial f_{n,t}/\partial x_t))$ is below:

$$
\frac{\partial}{\partial n} \left( \frac{\partial}{\partial x_t} \left( \frac{\partial f_{n,t}}{\partial x_t} \right) \right) = \frac{\partial}{\partial x_t} \left( \frac{\partial}{\partial n} \left( \frac{\partial f_{n,t}}{\partial x_t} \right) \right) \\
= \frac{\partial}{\partial x_t} \left( \Phi (z) \times \ln (\rho^Q) \times \delta_1 (\rho^Q)^n + \phi (z) \times \frac{\partial z}{\partial n} \delta_1 (\rho^Q)^n \right) \\
= \delta_1 (\rho^Q)^n \left( \phi (z) \times \ln (\rho^Q) \times \frac{\partial z}{\partial x_t} + \phi' (z) \times \frac{\partial z}{\partial x_t} \frac{\partial}{\partial n} + \phi (z) \times \frac{\partial}{\partial x_t} \frac{\partial z}{\partial n} \right) \left( \frac{\partial z}{\partial n} \right) \\
= \delta_1 (\rho^Q)^n \phi (z) \left( \ln (\rho^Q) \times \frac{b_n}{\sigma_n^Q} - z \times \frac{b_n}{\sigma_n^Q} \frac{\partial z}{\partial n} + \frac{\partial}{\partial x_t} \left( \frac{\partial z}{\partial n} \right) \right). \tag{20}
$$

To sign this expression, I make the following claim.

**Claim 2** $\partial z/\partial n > 0$ for $x_t < \bar{x}_1$ for some $\bar{x}_1$. Moreover, $\partial/\partial x_t (\partial z/\partial n) < 0$ and is independent of $x_t$.

Now consider the 3 terms inside the large parentheses in Eq. (20). The first term is negative because $\ln (\rho^Q) < 0$ and $b_n/\sigma_n^Q > 0$. The third term is also negative because $\partial/\partial x_t (\partial z/\partial n) < 0$ by Claim 2. Moreover, the first and third terms are independent of $x_t$ by Claim 2. Assume that $x_t < \bar{x}_1$ such that $\partial z/\partial n > 0$ by Claim 2. As $x_t$ decreases, $z$ becomes more negative, which implies that the second term is overall positive. For $x_t < \bar{x}_2$, the overall expression is positive, as the second term outweighs the first and third terms, which are independent of $x_t$. Define $\bar{x} \equiv \min \{\bar{x}_1, \bar{x}_2\}$. Then we have the final result that

$$
\frac{\partial}{\partial n} \left( \frac{\partial}{\partial x_t} \left( \frac{\partial f_{n,t}}{\partial x_t} \right) \right) > 0 \text{ for } x_t < \bar{x}. \tag{21}
$$

Evaluating the second part of Proposition 4, the comparative static $\partial/\partial x_t (\partial/\partial x_t (\partial f_{n,t}/\partial x_t))$ is

$$
\frac{\partial}{\partial n} \left( \frac{\partial}{\partial \delta_1} \left( \frac{\partial f_{n,t}}{\partial x_t} \right) \right) = \frac{\partial}{\partial n} \left( \phi (z) \times \frac{\partial z}{\partial \delta_1} b_n + \Phi (z) \times (\rho^Q)^n \right) \\
= \frac{\partial}{\partial n} \left( \phi (z) \times \frac{\partial z}{\partial \delta_1} b_n \right) + \frac{\partial}{\partial n} \left( \Phi (z) \times (\rho^Q)^n \right). \tag{22}
$$

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I make the following claims.

**Claim 3** \( \partial/\partial n (\Phi(z) \times (\rho^Q)^n) > 0 \) for \( x_t < \bar{x}_3 \), for some \( \bar{x}_3 \).

**Claim 4** \( \partial/\partial n (\phi(z) \times (\partial z/\partial \delta_1) b_n) > 0 \) for \( x_t < \bar{x}_4 \) for some \( \bar{x}_4 \) and for \( \delta_1 < \bar{\delta} \) for the same \( \bar{\delta} \) in Claim 1.

Assuming that \( \delta_1 < \bar{\delta} \) and \( x_t < \min\{x_3, x_4\} \equiv \bar{x}^* \), Claim 3 and Claim 4 imply that both terms in Eq. (22) are positive. Then we have the final result that

\[
\frac{\partial}{\partial n} \left( \frac{\partial}{\partial \delta_1} \left( \frac{\partial f_{n,t}}{\partial x_t} \right) \right) > 0 \text{ for } x_t < \bar{x}^* \text{ and } \delta_1 < \bar{\delta}.
\]  

(23)

Eqs. (21) and (23) show Proposition 4. When economic conditions are bad enough and the ZLB is sufficiently binding by Proposition 1, the attenuation in the sensitivity of forward rates to news in Proposition 3 is greater for longer maturity forward rates. On the other hand, when economic conditions are good enough and the ZLB does not bind severely by Proposition 1, the attenuation in the sensitivity of forward rates to news in Proposition 3 is smaller for longer maturity forward rates.

**B.4.1 Proof of Claim 2**

The comparative static \( \partial z/\partial n \) is

\[
\frac{\partial z}{\partial n} = \frac{\partial}{\partial n} \left( \frac{a_n + b_n x_t - \hat{i}}{\sigma_n^Q} \right) = \frac{\partial}{\partial n} \left( a_n + b_n x_t - \hat{i} \right) \times \sigma_n^Q - \frac{\partial}{\partial n} \left( \sigma_n^Q \right) \times (a_n + b_n x_t - \hat{i}) \left( \sigma_n^Q \right)^2.
\]

We first show that \( \partial/\partial x_t (\partial z/\partial n) < 0 \) and is independent of \( x_t \) by considering only the numerator of \( \partial z/\partial n \) and the terms involving \( x_t \) since the denominator is positive and independent of \( x_t \):

\[
\frac{\partial}{\partial n} \left( b_n x_t \right) \times \sigma_n^Q - \frac{\partial}{\partial n} \left( \sigma_n^Q \right) \times b_n x_t.
\]

In the expressions below, the first line expands the above expression for terms in the numerator related to \( x_t \) and each successive arrow ignores positive terms that are factored
\[
\ln \left( \rho^Q \right) \times \delta_1 \left( \rho^Q \right)^n x_t \sigma_n^Q = \frac{1}{2} \frac{1}{\sigma_n^Q} \left( \sigma_1 \sigma_e \right)^2 \frac{1}{1 - (\rho^Q)^2} \delta_1 \left( \rho^Q \right)^n x_t \\
\Rightarrow \ln \left( \rho^Q \right) \times \delta_1 \left( \rho^Q \right)^n x_t \left( \sigma_1 \sigma_e \right)^2 \frac{1}{1 - (\rho^Q)^2} + \left( \sigma_1 \sigma_e \right)^2 \frac{1}{1 - (\rho^Q)^2} \delta_1 \left( \rho^Q \right)^n x_t \\
\Rightarrow \ln \left( \rho^Q \right) \times x_t \left( 1 - (\rho^Q)^n \right) + \ln \left( \rho^Q \right) \times (\rho^Q)^n x_t \\
\Rightarrow \ln \left( \rho^Q \right) \times x_t.
\]

Since \( \rho^Q < 1 \) by assumption of stationarity, \( \ln \left( \rho^Q \right) < 0 \) and \( \partial / \partial x_t (\partial z / \partial n) < 0 \) and is moreover independent of \( x_t \). Since \( \partial z / \partial n \) is decreasing in \( x_t \), for \( x_t < \bar{x}_1 \) for some threshold \( \bar{x}_1 \), \( \partial z / \partial n > 0 \).

**B.4.2 Proof of Claim 3**

Expand the comparative static \( \partial / \partial n \left( \Phi \left( z \right) \times (\rho^Q)^n \right) \):

\[
\frac{\partial}{\partial n} \left( \Phi \left( z \right) \times (\rho^Q)^n \right) = \phi \left( z \right) \times \frac{\partial z}{\partial n} (\rho^Q)^n + \Phi \left( z \right) \times \ln \left( \rho^Q \right) \times (\rho^Q)^n \\
= (\rho^Q)^n \left( \phi \left( z \right) \times \frac{\partial z}{\partial n} + \Phi \left( z \right) \times \ln \left( \rho^Q \right) \right).
\]

Thus,

\[
\frac{\partial}{\partial n} \left( \Phi \left( z \right) \times (\rho^Q)^n \right) > 0 \iff \frac{\Phi \left( z \right)}{\phi \left( z \right)} < -\frac{1}{\ln \left( \rho^Q \right) \partial n}. \tag{24}
\]

We know from Claim 2 that \( \partial / \partial x_t (\partial z / \partial n) < 0 \) and is independent of \( x_t \), so the right-hand side of the inequality in Eq. (24) becomes arbitrarily positive as \( x_t \) becomes arbitrarily negative. Moreover, as \( x_t \) becomes arbitrarily negative, \( z \) becomes arbitrarily negative as well. Using the relationship that

\[
\lim_{z \to -\infty} \frac{\Phi \left( z \right)}{\phi \left( z \right)} = \lim_{-z \to \infty} \frac{1 - \Phi \left( -z \right)}{\phi \left( z \right)} \approx \frac{1}{-z},
\]

it’s clear that at some point, the left-hand side of the inequality in Eq. (24) becomes an arbitrarily small positive number. Thus the inequality in Eq. (24) holds for \( x_t < \bar{x}_3 \) for some \( \bar{x}_3 \), which proves Claim 3.
B.4.3 Proof of Claim 4

Expand the comparative static \( \partial / \partial n (\phi (z) \times (\partial z / \partial \delta_1) b_n) \):

\[
\frac{\partial}{\partial n} \left( \phi (z) \times \frac{\partial z}{\partial \delta_1} b_n \right) = \frac{\partial}{\partial n} \left( \phi (z) \times \delta_1 (\rho \jmath)^n \right) \frac{\partial z}{\partial \delta_1} + \left( \phi (z) \times \delta_1 (\rho \jmath)^n \right) \frac{\partial}{\partial n} \frac{\partial z}{\partial \delta_1} \\
= \left( \phi' (z) \times \frac{\partial z}{\partial n} \delta_1 (\rho \jmath)^n + \phi (z) \times \delta_1 \ln (\rho \jmath) \times (\rho \jmath)^n \right) \frac{\partial z}{\partial \delta_1} \\
+ \left( \phi (z) \times \delta_1 (\rho \jmath)^n \right) \frac{\partial}{\partial n} \frac{\partial z}{\partial \delta_1} \\
= \left( -z \phi (z) \times \frac{\partial z}{\partial n} \delta_1 (\rho \jmath)^n + \phi (z) \times \delta_1 \ln (\rho \jmath) \times (\rho \jmath)^n \right) \frac{\partial z}{\partial \delta_1} \\
+ \left( \phi (z) \times \delta_1 (\rho \jmath)^n \right) \frac{\partial}{\partial n} \frac{\partial z}{\partial \delta_1} \\
= \delta_1 \phi (z) \times (\rho \jmath)^n \left( -z \frac{\partial z}{\partial n} \frac{\partial z}{\partial \delta_1} + \ln (\rho \jmath) \frac{\partial z}{\partial \delta_1} + \frac{\partial}{\partial n} \frac{\partial z}{\partial \delta_1} \right). \tag{25}
\]

The proof for Claim 1 shows that \( \partial z / \partial \delta_1 \) is independent of \( x_t \), so the second and third terms of Eq. (25) are likewise independent of \( x_t \). Claim 1 also says that \( \delta_1 < \delta \) implies \( \partial z / \partial \delta_1 > 0 \). Then for \( x_t < x_4 \), we have \( z \) (increasing in \( x_t \)) sufficiently negative and \( \partial z / \partial n \) (decreasing in \( x_t \) based on Claim 2) sufficiently positive such that the first term in Eq. (25) is positive and of large enough magnitude that the overall expression \( \partial / \partial n (\phi (z) \times (\partial z / \partial \delta_1) b_n) > 0 \), which proves Claim 4.