BOOK I.

DEFINITIONS.

1. A point is that which has no part.
2. A line is breadthless length.
3. The extremities of a line are points.
4. A straight line is a line which lies evenly with the points on itself.
5. A surface is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A plane surface is a surface which lies evenly with the straight lines on itself.
8. A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called rectilinear.
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.
11. An obtuse angle is an angle greater than a right angle.
12. An acute angle is an angle less than a right angle.
13. A boundary is that which is an extremity of anything.
14. A figure is that which is contained by any boundary or boundaries.
15. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another;
16. And the point is called the centre of the circle.

17. A diameter of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

18. A semicircle is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.

19. Rectilineal figures are those which are contained by straight lines, trilateral figures being those contained by three, quadrilateral those contained by four, and multilateral those contained by more than four straight lines.

20. Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle that which has two of its sides alone equal, and a scalene triangle that which has its three sides unequal.

21. Further, of trilateral figures, a right-angled triangle is that which has a right angle, an obtuse-angled triangle that which has an obtuse angle, and an acute-angled triangle that which has its three angles acute.

22. Of quadrilateral figures, a square is that which is both equilateral and right-angled; an oblong that which is right-angled but not equilateral; a rhombus that which is equilateral but not right-angled; and a rhomboid that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called trapezia.

23. Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

POSTULATES.

Let the following be postulated:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance.
4. That all right angles are equal to one another.
POSTULATES.

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1. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS.
1. Things which are equal to the same thing are also equal to one another.
2. If equals are added to equals, the wholes are equal.
3. If equals are subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

DEFINITIONS.

A point is that which has no part.

A straight line is a line which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

An exactly parallel line of plane (uers) in the singular is found in Aristotle, Metaph. 1035 b 33, μέτοχος μείζονες καλος καὶ χαλέπιος, literally “There is a part even of the form;” Buxtorf translates as if the plural were used, “Thele glicht es,” and the meaning is simply “even the form is divisible into parts.” Accordingly it would be quite justifiable to translate in this case “A point is that which is indivisible into parts.”

Marianus Capella (5th c. A.D.) alone or almost alone translated differently, “Punctum est cuius pars nihil est,” “a point is that which of a part is nothing.” Notwithstanding that Max Simon (Gewich und die sechs platonischen Bücher, 1903) has adopted this translation (on grounds which I shall presently mention), I cannot think that it gives any sense. If a part of a point is nothing, Euclid might as well have said that a point is itself “nothing,” which of course he does not do.

Pre-Euclidean definitions.

It would appear that this was not the definition given in earlier textbooks; for Aristotle (Post. l. 6, 141 b 20), in speaking of “the definition” of point, line, and surface, says that they all define the prior by means of the posterior; a point as an extremity of a line, a line of a surface, and a surface of a sol

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Rectilinear figures are those which are contained by straight lines, trilateral figures being those contained by three, quadrilateral those contained by four, and multilateral those contained by more than four straight lines.

Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle that which has two of its sides alone equal, and a scalene triangle that which has its three sides unequal.

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BOOK I

Join $OP$, and draw another radius $OQ$ such that the angle $POQ$ is equal to the angle $MOP$.

The intersection, $M'$, of $OQ$ with the circle satisfies the required condition.

For $MM'$ meets $OP$ at right angles in $S$.

Therefore, in the right-angled triangle $MSP$, $MS$ is not greater than $MP$ (it is less, unless $MP$ coincides with $MS$, when it is equal).

Therefore $MS$ is not greater than $\frac{a}{2}$, so that $MM'$ is not greater than $a$.

BOOK I. PROPOSITIONS.

**Proposition 1.**

On a given finite straight line to construct an equilateral triangle.

Let $AB$ be the given finite straight line.

Thus it is required to construct an equilateral triangle on the straight line $AB$.

With centre $A$ and distance $AB$ let the circle $BCD$ be described; \[\text{[Post. 3]}\]

again, with centre $B$ and distance $BA$ let the circle $ACE$ be described; \[\text{[Post. 3]}\]

and from the point $C$, in which the circles cut one another, to the points $A$, $B$ let the straight lines $CA$, $CB$ be joined. \[\text{[Post. 1]}\]

Now, since the point $A$ is the centre of the circle $CDB$, $AC$ is equal to $AB$. \[\text{[Def. 15]}\]

Again, since the point $B$ is the centre of the circle $CAE$, $BC$ is equal to $BA$. \[\text{[Def. 15]}\]

But $CA$ was also proved equal to $AB$;

therefore each of the straight lines $CA$, $CB$ is equal to $AB$.

And things which are equal to the same thing are also equal to one another; \[\text{[C. N. 1]}\]

therefore $CA$ is also equal to $CB$.

Therefore the three straight lines $CA$, $AB$, $BC$ are equal to one another.