Not what this class is about!

→ EUCLID

- Fixing Euclid
- The parallel postulate
- Affine geometry
- Projective geometry
- Hyperbolic geometry
Theme: Algebra emerging from geometry
Meta-theme: Axioms vs. models
Today: Informal discussion of Euclid

Allowed moves:

1. Draw the line through two points
2. Draw the circle with a given center and radius

Q: What can we do?
Example (1.1) We can build an equilateral triangle.

Example (1.9) We can bisect an angle.
Example (5.2) We can move segments around

\[ |AF| = |DF| - |DA| = |DE| - |DB| = |BE| = |BC| \]
Ex (I.10) We can bisect a segment

Ex (I.11+12) We can draw perpendiculars to lines
BUILDING BLOCKS

- Incidence
- Betweenness
- Congruence
- Straightness
- Length

Mystery Numbers and algebra arise from incidence and straightness alone!
Thales' theorem: (Similar triangles) A line drawn parallel to the base of a triangle cuts the sides proportionally.

\[
\frac{|AQ|}{|QC|} = \frac{|AP|}{|PB|}
\]
Consequences of Thaless

\[ \text{(1)} \]

By Thaless, \[ \frac{|AC|}{a} = \frac{b}{1} \Rightarrow |AC| = ab \]
By Thales, $|\mu B| = b/a$

We can now construct any positive rational number, after choosing a reference length (recall that a rational number is one of the form $p/q$, where $p, q$ are integers).
Applying similar triangles (which follows from Thales) to $\triangle DOUB$ and $\triangle AOB$:

\[
\frac{1}{|WB|} = \frac{|AB|}{1} = \frac{1}{2} |WB|
\]

\[
\Rightarrow |WB|^2 = 2
\]

Claim $|WB|$ is not rational.

Proof: Suppose $|WB| = \frac{p}{q}$, where $p + q$ are positive integers with no common factor.
Then \( p^2 = 2q^2 \), so \( p^2 \) is even

\[ \Rightarrow p \text{ is even} \]

\[ \Rightarrow p^2 \text{ is divisible by 4} \]

\[ \Rightarrow q^2 = \frac{p^2}{2} \text{ is even} \]

\[ \Rightarrow q \text{ is even} \]

\[ \Rightarrow p + q \text{ have a common factor, contradicting our assumptions.} \]

\[ \square \]

Big question: What is the scope of ruler and compass constructions?