

THURSDAY SEMINAR 2018-19

The goal of the seminar is to understand Ralph Cohen's proof of the immersion conjecture. The original reference is [Coh85], and two expository accounts are [CT86, Lan81]. After an introductory lecture, the seminar will cover some of the following topics, not necessarily in the order given.

- (1) Background on the Steenrod algebra and Stiefel–Whitney classes. Wu classes. The relationship between Poincaré duality and the Steenrod algebra. Atiyah duality.
 - Tentative length: 1 lecture
 - Tentative speakers: ???
 - References: [Wu50], [MS74], [Mil01], [Ada69, Section 3]
- (2) Stiefel–Whitney classes of manifolds. Massey's theorem and the ideal I_n .
 - Tentative length: 1 lecture
 - Tentative speakers: Bena
 - References: [Mas60], [BP64], [CT86, I.1]
- (3) Immersion theory. Reduction of the theorem to a lifting problem.
 - Tentative length: 2 lectures
 - Tentative speakers: Emily, Sander
 - References: [CT86, I], [Wal16, Chapter 6], [Hir59], [HP64], [Las70], [Gro86], [EM02]
- (4) The immersion conjecture up to bordism.
 - Tentative length: 1 lecture
 - Tentative speakers: ???
 - [CT86, I.2] [Tho54]
- (5) Configuration space models and stable splittings. Braid groups and their cohomology. Braid Thom spectra as Brown–Gitler spectra. Deduction of Mahowald's theorem.
 - Tentative length: ≥ 2 lectures
 - Tentative speakers: Ben and maybe someone else
 - References: [Boe87, Fuk70, Coh79] [CT86, II.2]
- (6) Uniqueness and versal property of Brown–Gitler spectra. Dieudonné modules. Deduction of the immersion conjecture up to Thom spectra.
 - Tentative length: 1-2 lectures
 - Tentative speakers: ???
 - References: [GLM92, BG73, BP77] [CT86, II]
- (7) De-Thomification obstruction theory. Construction and versal property of BO/I_n .
 - Tentative length: ≥ 2 lectures
 - Tentative speakers: ???
 - References: [BP79], [CT86, III.1], [Bro69, Section 6]
- (8) Stable splittings of classifying spaces of classical groups.
 - Tentative length: 1 lecture
 - Tentative speakers: ???
 - References: [Sna79]

- (9) The lift of BO/I_n .
- Tentative length: ???
 - Tentative speakers: ???
 - References: [Coh85] [CT86, III.2]

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