Optimal Defaults

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Default options have an enormous impact on household “choices.” Such effects are documented in the literature on 401(k) plans.\textsuperscript{1} Defaults affect 401(k) participation, savings rates, rollovers, and asset allocation. For example, when employees are automatically enrolled in their 401(k) plan, only a tiny fraction opt out, producing nearly 100% enrollment. But when employees are not automatically enrolled, less than half enroll on their own during their first year of employment.

Defaults matter for three key reasons modelled in this paper. First, acts of commission — e.g., opting out of a default — are costly. Second, these costs vary over time, generating an option value of waiting for a low cost period to opt out. Third, people are susceptible to procrastination. Even if they want to make a change, they have a tendency to delay that change longer than they should. These three effects imply that the choice of defaults can have significant welfare consequences.

If all employees share a common optimal savings rate, selecting an optimal default is trivial. But the calculation of an optimal default is not straightforward if employees have heterogeneous optimal savings rates. In such a world, it may sometimes be optimal to set defaults that are far away from the mean optimal savings rate. This effect arises for two reasons. First, a default that is far from a procrastinating employee’s optimal savings rate may make the employee better off than a default that is closer to the employee’s optimal savings rate. A “bad” default is more likely to motivate opting out than a good but imperfect default, potentially overcoming procrastination and improving the agent’s welfare. Second, our theory implies that optimal defaults are highly sensitive to the actual distribution of optimal savings rates. In particular, optimal defaults are often associated with the modal optimal savings rate, which is sometimes extreme (e.g., the plan’s minimum or maximum contribution rate). At the end of our paper, we illustrate these effects with calculations of optimal defaults for employees at four different companies.
1. A MODEL OF SAVINGS CHOICES

We adopt the model of Choi et al (2002b) to describe the 401(k) enrollment decisions of employees that have been newly hired at a firm. However, the model is general enough to describe any problem in which an actor decides when to move from a default state $s_D$ to an optimal state $s^*$. We assume that each employee at a firm has a fixed optimal savings rate $s^*$, with density function $f$ describing the distribution of these optimal savings rates for the employees in the firm. When new employees join the firm, they are automatically enrolled at a default savings rate of $s_D$. We assume that the firm uses a single default savings rate for all of its employees either because the firm does not observe an employee’s true type, $s^*$, or because of legal/practical costs of implementing employee-specific defaults.

Employees remain at the default $s_D$ unless they opt of the default by incurring a cost $c$. This opt-out cost is drawn each period and takes the value $c = 1$ with probability $\mu$ and value $c = 0$ with probability $1 - \mu$. When the agent opts out of the default, she sets her savings rate equal to her optimal savings rate $s^*$. Until the agent opts out of the default, the agent suffers a per-period flow loss of $L(s_D, s^*) \geq 0$, where the first argument is the current savings rate and the second argument is the optimal savings rate. After the agent has opted out of the default, the flow losses vanish.

Finally, we assume that agents are naive hyperbolic discounters, with discount function $1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots$. Such naive agents believe that their future selves will make choices that are consistent with their current preferences. We adopt such naive beliefs because they increase the force of procrastination, but our qualitative results would not change if we instead assumed that agents hold rational expectations. For simplicity and analytical tractability, we set $\delta = 1$ (no long-run discounting). We also adopt the standard “hyperbolic” assumption of $\beta < 1$.

We use the following timing convention. If the employee has not previously opted out of the default, the period begins with a loss of $L$. The employee then draws a current opt-out cost $c$ and
decides whether to delay action or to instead opt out now at cost $c$. If the employee delays, she will lose $L$ next period and also face an anticipated continuation value function $v(c')$, where $c'$ represents next period’s draw from the cost distribution. Hence, the employee chooses to opt of the default and pay $c$ if $c < \beta [L + Ev(c')]$. Assuming that ties are broken by immediate action, the employee’s strategy is thus

\[
\begin{align*}
\text{“Opt out only when } c = 0 \text{”} & \quad \text{if } 1 > \beta [L + Ev(c')] \\
\text{“Opt out at both } c = 0 \text{ and } c = 1 \text{”} & \quad \text{if } 1 \leq \beta [L + Ev(c')]
\end{align*}
\]

(1)

1.1. Naive expectations of the continuation value function $v(c)$. Since the employee is assumed to be a naive hyperbolic agent, the continuation value function is constructed under the (mistaken) belief that all future selves will exhibit no time discounting, since this is what today’s self wants those future selves to do. We now calculate the agent’s naive expectations of $v(c)$.

If the agent anticipates that she will always opt out in the future, then $Ev(c) = E(c) = \mu$. By contrast, if the agent anticipates that she will only opt out when $c = 0$, then $Ev(c) = \frac{\mu L}{1-\mu}$. Since, the naive agent anticipates that she will pick the optimal (patient) strategy in the future, she believes that

\[
Ev(c) = \begin{cases} 
\frac{\mu L}{1-\mu} & \text{if } L < 1 - \mu \\
\mu & \text{if } L \geq 1 - \mu
\end{cases}
\]

(2)

since $\frac{\mu L}{1-\mu} < \mu$ iff $L < 1 - \mu$.

1.2. Actual actions and welfare. Using (1) and (2), we can determine whether the agent will opt of the default when $c = 1$. It turns out that when $L < \frac{1}{\beta} - \mu$, the agent only opts out of the default when $c = 0$, but when the inequality is reversed, the agent opts out of the default at
both \( c = 0 \) and \( c = 1 \). It follows that the per-period probability of opting out of the default is

\[
p = \begin{cases} 
1 - \mu & \text{if } L < \frac{1}{\beta} - \mu \\
1 & \text{if } L \geq \frac{1}{\beta} - \mu 
\end{cases}
\]

Let \( w(c) \) represent the employee’s true (i.e., rational) expected total costs without short-run (\( \beta \)) discounting. Hence, \( \beta w(c) \) represents the employee’s interests at economic birth, which we assume occurred before she started working at the firm. Minimizing \( w(c) \) should be the goal of a social planner. If \( E(c|\text{opt out}) \) represents the true expected costs of opting out of a default, conditional on the agent choosing to opt out, then a recursive representation for \( Ew(c) \) is given by

\[
Ew(c) = pE(c|\text{opt out}) + (1 - p) [L + Ew(c)]
\]

\[
= \begin{cases} 
\frac{\mu L}{1 - \mu} & \text{if } L < \frac{1}{\beta} - \mu \\
\mu & \text{if } L \geq \frac{1}{\beta} - \mu 
\end{cases}
\]

To characterize the relationship between defaults and welfare, we examine the relationship between expected (dis)utility and \( L \). We stop suppressing \( L \) in our notation and consider \( W(L) = Euw(c)|L \), the expected losses for an agent with flow losses per period of \( L \).

In a standard model with exponential discounting (i.e., \( \beta = 1 \)), \( W(L) \) would be non-decreasing as flow costs \( L \) increase. But for hyperbolics (i.e., \( \beta < 1 \)), it will always be the case that \( W \) is non-monotonic in \( L \). To see this, note that \( W(L) = \mu \) when \( L = 1 - \mu \). This is the level of \( L \) at or above which an exponential agent should opt out of the default whatever the cost realization. But when \( c = 1 \), a hyperbolic agent will act only if \( L \geq \frac{1}{\beta} - \mu \), which is greater than \( 1 - \mu \). Hence, when \( 1 - \mu < L < \frac{1}{\beta} - \mu \), the hyperbolic agent is insufficiently motivated to act, and this motivational gap produces self-defeating procrastination. Finally, if \( L \) is above \( \frac{1}{\beta} - \mu \), the
procrastination effect vanishes and expected costs fall back to $\mu$, since the hyperbolic agent is now willing to act whatever the cost realization. Figure 1 plots the expected total cost function against the flow costs $L$, revealing the non-monotonicity that arises when $\beta < 1$. In a world with procrastination, moving the agent further from the optimum can make an agent better off, since it decreases the agent’s tendency to procrastinate.

1.3. The (benign) firm’s optimization problem. The firm is trying to pick an optimal default $s_D^*$ in the support of $f$ to minimize the social welfare function,

$$\int_{\underline{s}}^{\bar{s}} W(L(s_D, s^*))f(s^*)ds^*. \tag{3}$$

We adopt the cost function $L(s_D, s^*) = \kappa(s_D - s^*)^2$. We will minimize equation (3) numerically, using the actual distribution of optimal savings rates. However, for analytical tractibility, we also consider the case in which $f(s^*)$ is uniform over support $[\underline{s}, \bar{s}]$. Then, if $\beta < 1$,

$$s_D^* = \begin{cases} \frac{\bar{s} + \gamma}{2} & \text{if } \bar{s} - \underline{s} \text{ small} \\ \underline{s} + \sqrt{\kappa (1 - \mu)} & \text{or } \bar{s} - \sqrt{\kappa (1 - \mu)} & \text{if } \bar{s} - \underline{s} \text{ large} \end{cases}$$

Intuitively, when there is little variation in optimal savings rates, it is best to design a default that is in the middle of the range of optimal savings rates, since all employees will then be very close to their optimal savings rate and delays in opting out of the default won’t be very costly. By contrast, when there is a great deal of variation in optimal savings rates, it is better to design a default that is close to one of the two boundaries of the support. This “boundary” strategy reduces the proportion of employees that engage in costly procrastination, since the boundary strategy reduces the fraction of employees who fall in the “procrastination” interval $1 - \mu < L < \frac{1}{\beta} - \mu$.

Finally, note that if $\beta = 1$ and $f$ is uniform, then $s_D^* = \frac{\bar{s} + \gamma}{2}$ will always be an optimum because
the procrastination effect does not apply and there is no welfare gain from moving agents away from their optima.

It is also useful to emphasize another property of these models. This additional effect is easiest to understand if we assume that $f$ is a discrete density on the domain of feasible savings rates. As the cost of deviations rises ($\kappa \to \infty$), the optimal default converges to the mode of the distribution of $s^\ast$. This is because for large $\kappa$, all employees will immediately opt out of the default except those already at their optimum. Hence, the optimal social policy minimizes adjustment costs by setting the default equal to the most frequent value of $s^\ast$. We refer to this as the mode effect.

### 1.4. Calibration.

Our model has very few free parameters: the density of optimal savings rates, $f(s^\ast)$; the discount factor $\beta$; the scaling variable $\kappa$; and the probability of a high cost draw $\mu$. We further restrict this list by using individual employee data to pin down the density $f$ (see next section). We set $\beta = \frac{2}{3}$, reflecting a large body of experimental evidence and a growing body of field evidence. For example, Laibson, Andrea Repetto, and Jeremy Tobacman (2002) use the Method of Simulated Moments to estimate $\beta$ using household financial data. Their benchmark estimate is 0.61 with a standard error of 0.05.

Only $\kappa$ and $\mu$ remain to be calibrated. Before doing this, we need to pick units for the variables in our model. We assume that time units are periods of a pay cycle (about two weeks). We assume that utility units can be interpreted in terms of a money metric in which one unit of utility is equal in value to 1/10th of a pay cycle of income. So when the cost realization is high ($c = 1$), opting out of the default generates a time cost that is equal in value to 1/10 of the agent’s income during that pay cycle. We assume that such busyness is the norm, and set $\mu = 0.9$.

To set $\kappa$, we use the following thought experiment. What is the money-metric cost to a employee who is ten percentage points away from her optimal 401(k) savings rate? Let $x$ represent the loss
in units of 1/10th of one pay cycle of income. We will consider a range of values for \( x : 0.1, 1, 10 \). This translates into the following range of values for \( \kappa : 10, 100, 1000 \). We consider this wide range because we are agnostic about the appropriate calibration value and because we wish to explore the sensitivity of our results to the choice of \( \kappa \). However, if forced to choose, we would set \( \kappa = 100 \).

2. **Empirical analysis**

We compute the optimal default 401(k) savings rate for four companies, which we denote by their industry: Health, Office, Food and Finance. All are large employers with well-established 401(k) plans. There are two key differences in the 401(k) plan environments of these companies. First, Health and Office match employee contributions up to 6% of pay, while the other two companies have no match at all. Second, only Office and Food have a defined benefit pension plan in addition to a 401(k) plan. Other things equal, we would expect a lower desired 401(k) savings rate for employees in companies with a defined benefit pension.

The workforce demographics of our four companies also vary considerably. The median pay ranges from $25,000 per year in Food to $41,000 per year in Finance. Because Social Security replaces a higher fraction of income for low income employees, we would expect a higher desired savings rate for high income employees.

To estimate the distribution of optimal savings rates, we use two approaches. First, we report densities over 401(k) savings rates for employees with 3-5 years of tenure (density \( f_1 \)) and 5-7 years of tenure (density \( f_2 \)). We informally reason that such medium-tenure employees have been at a firm long enough to select their optimal savings rate, but not so long that tenure-driven selection effects dominate the data.

Second, we use a regression framework to control for demographic variables. We run an ordered logit regression in which the explanatory variable is the actual 401(k) contribution rate chosen by
each individual employee. We include non-participation, which implies a 0% contribution rate, as one of the categories. The control variables in the regressions are ln(pay), ln(age), ln(tenure), and a gender dummy. We then predict the distribution of contribution rates that would obtain if each employee had 30 years of tenure, holding other demographic characteristics constant (density $f_3$).

It turns out that these three methods for calculating the density of optimal savings rates yield very similar results. Within each of our four firms, $f_1$, $f_2$, and $f_3$ are very close, though the densities do vary meaningfully across firms. For example, $f_3$ implies that our four firms have respective mean optimal savings rates of 6.40%, 6.43%, 2.40%, and 8.82%.

With these densities in hand, we are now in a position to estimate the optimal savings rate by minimizing equation (3). We undertake this maximization for $3 \times 3 \times 4$ cases of interest: 3 different values for $\kappa$, 3 different ways of calculating the density $f$, and 4 different test companies. The results of these maximizations are reported in Table 1.

Table 1 documents five findings. First, the analysis reveals a high degree of heterogeneity in optimal defaults relative to the heterogeneity in mean optimal savings rates. The optimal defaults range from 0% to 15%, though as discussed above, the mean optimal savings rates only vary from 2.40% to 8.82%. Second, the optimal default calculation is extremely sensitive to distributional assumptions on $s^*$. Third, as $\kappa$ gets large, much of the variation in optimal defaults is driven by the mode effect. For $\kappa = 1000$, five out of twelve of the optimal defaults are equal to the modal optimal savings rate. Fourth, the optimal defaults vary in a sensible way with the underlying firm-specific attributes. Firms whose employees have a high motive to save turn out to have higher optimal defaults than firms whose employees have a low motive to save. For example, the employees at Food have a DB plan and a low average salary, and hence very low optimal defaults. By contrast, the employees at Finance have no DB plan, a high average salary, and a median optimal default of 14%. Finally, the optimal defaults tend to cluster in one of three regions: close to 0%, close to the
match threshold (6% for Health and Office), or close to the maximum contribution rate.

3. Extensions

This paper has presented a model of 401(k) enrollment that includes four key components: costs of opting out of a default, an option value of waiting to incur those costs, procrastination, and heterogeneity in optimal savings rates. One should also consider other psychological and economic factors when picking socially optimal defaults. First, some employees may interpret defaults as implicit advice, an issue that does not arise in the current model since each employee is assumed to know her true optimal savings rate. Second, defaults may be particularly sticky because of loss aversion. If the default is perceived to be a reference point, then deviations from that reference point may be psychologically aversive, since the resulting “gains” from the deviation (e.g., more saving) are weighted half as much as the resulting “losses” (e.g., less consumption). Third, choosing a long-run savings rate that is one percentage point too low is much more costly than choosing a long-run savings rate that is one percentage point too high (since retirement is short relative to working life and utility is concave), suggesting a desirable upward shading of optimal defaults. Fourth, optimal savings rates are not constant over time, but instead are more likely to trend up slowly with working age. Fifth, the firm may wish to pick an optimal default that weights some employees more heavily than others.

Finally, the model suggests one important generalization that we are now exploring (Choi et al 2002b). If procrastination effects are very strong, it will sometimes be optimal to pick a default that is so bad that all employees feel compelled to immediately opt out of the default. Such a setup is equivalent in practice to something that we call “active decision,” a regime that forces new employees to pick their own savings rate without the benefit of a fall-back default. In a world with significant procrastination, such active decision regimes are sometimes the best “defaults” of all.
4. References


Endnotes:

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2See Laibson (1997) for a discussion of hyperbolic discount functions and George A. Akerlof (1992) and Ted O’Donoghue and Matthew Rabin (1999) for a discussion of naïfs and procrastination. Note that the term “hyperbolic” is overly restrictive, since the important property of these preferences is simply that they are characterized by more discounting in the short-run than in the long-run.

3Details about the 401(k) plans, employee demographics, and optimal savings densities are available in an earlier expanded version of this paper.

Figure 1: Expected total losses as a function of flow cost per period

\[ W(L) = \text{expected total losses} \]

\[ \beta = \frac{2}{3} \]

\[ \beta = 1 \]

\[ \text{slope} = \frac{\mu}{1-\mu} \]

\[ L = 1 - \mu \]

\[ L = \frac{1}{\beta} - \mu \]
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Source: authors calculations. Densities are calculated with data from Hewitt Associates. See text for definitions of densities $f_1(s^*)$, $f_2(s^*)$, and $f_3(s^*)$. 

TABLE 1—Optimal Savings Rates, $\beta = 0.67$