Bargaining with Endogenous Learning

Jetlir Duraj

Harvard University
Motivation

In some bargaining situations initial private information about the value of trade plays a smaller role, in contrast to information learned over time.

The buyer or the seller may learn privately about value of the good with the aim of influencing bargaining positions.

Learning takes time, is endogenous and is typically costly.

Learning creates option value that is strategic in nature:  
- when receiving good news can choose to withhold new information to trade at a bargain,  
- when receiving bad news can disclose to get a lower price or can walk away entirely.
Case study: Sula Vineyards

(taken from Zeisberger et al. 2017 case studies book on private equity and venture capital (VC))

Deepak Shahdadpuri: VC

Rajeev Samant: Wine producer (WP)

VC previous investment experience in tech sector
WP own background: engineering (Stanford and Oracle alumnus)
VC acted as an informal advisor for a long time to WP;
feelings of mutual trust and respect (symmetric information at the start)
Case study: Sula Vineyards

Wine industry in India in 2004: nascent, consumption low due to cultural and regulatory obstacles.

Signs it could take off due to changing consumer demographics and a tendency to looser regulations.

⇒ Challenge: Valuation!

What are reasonable growth projections without prior industry experience in India?

How will market structure and competition react once demand increases?
Case study: Sula Vineyards

Approach of VC team of analysts: use wine industry expansion experiences in China and New Zealand to develop a model of projections for India’s expansion

Parallels: changing tides of consumer attitudes and government policies

Based on analysis VC negotiated investment of a 35% stake for $3.5m

Sula Vineyards dominates the Indian wine market today, which has grown steadily since 2010 by an annual compounded growth rate of over 14%

---

1 https://www.indianwineacademy.com/item_4_418.aspx
Other examples of endogenous learning in bargaining

- Author/publisher negotiations at the ‘book stage’

- Buying patent rights to a new invention

- Government sale of a natural resource: e.g. buyer performs tests on value of resource before final acquisition decision

- Real estate developer performs title exam and hires market analysts to estimate value of a house in a year’s time
Bargaining literature


Standard literature assumes there is initial private information for free!

Focus is Coase-conjecture dynamics – as frequency of interaction speeds up:
- delay till agreement vanishes
- bargaining outcome becomes efficient
- informed party receives maximized surplus

Some recent bargaining literature
This paper

starts out with the *simplest set up*: 
- exclusive bargaining between a buyer and a seller 
- independent values  
- parties start off with symmetric information 

and *adds active learning* with the aim of *influencing* directly the *bargaining positions*:

- buyer can learn privately and endogenously over time about value, can disclose verifiably what she learns and/or can walk away at any time; learning takes time and is costly

‘Small’ give-away of results: outcomes not as extreme as in the standard model...
Research questions

Results give answers to the following

**Question 1:** What does the possibility to learn imply when compared to

- learning new information for Buyer is impossible
- Buyer knows value at the start (initial private information)

**Question 2:** When can we assume information is exogenous without loss of generality?
Research questions

**Question 3:** What is the optimal information acquisition given strategic incentives, when information acquisition is costly?

**Question 4:** How can we quantify the potential efficiency loss and delay in agreement when information is costly and endogenously acquired?

What about comparative statics?
Negative values and walking away

Here focus on case where trade is always efficient ex-ante for brevity.

Situation with similar strategic behavior:
- Buyer valuation can be negative ⇒ trade not always ex-ante efficient
- Buyer can walk away from bargaining at any time

Analysis analogous because: independent values and the buyer learns with the aim of actively influencing bargaining positions.

Behavior is similar, payoffs and efficiency interpretations change.
Baseline model

**Players:** One *buyer* and one *seller*. Seller has no value for the good, makes offers at the end of every period, **neither** party knows (buyer’s) value $\theta$ of good ex-ante $\theta \in \{\bar{\nu}, \nu\}$, $\bar{\nu} > \nu \geq 0$ (seller-offer game, (weak) gap case)

**Common prior** of high value $\pi_0 \in (0, 1)$. Let $\hat{\nu} = \pi_0 \bar{\nu} + (1 - \pi_0)\nu$

**Time:** discrete $t = \Delta, 2\Delta, 3\Delta, \ldots$ ($\Delta > 0$), infinite horizon

**common discount factor** $\delta = e^{-r\Delta} \in (0, 1)$
Baseline model

**Information:**

**Seller:** never receives any information, unless buyer discloses

**Buyer:** every period with probability $\mu = 1 - e^{-\lambda \Delta}, \lambda > 0$ opportunity to learn about $\theta$ (arrival is private information).

Decides privately on experiment when opportunity to learn arrives.

Observes privately outcome of experiment.

Major simplifying assumption: buyer can exploit only one opportunity to learn

**Information acquisition:** Experiments: $E_a : \{v, \bar{v}\} \rightarrow \Delta(\{H, L\})$ with

$$E_a(\bar{v})(H) = E_a(v)(L) = a \in \left[\frac{1}{2}, 1\right]$$

$a$ is accuracy, chosen whenever opportunity to learn arrives

$\lambda$ is the learning intensity
Bargaining with Endogenous Learning

Accuracy vs. intensity

Two dimensions of the learning effort:

- **exploration/search** for information sources (intensity), e.g. *researching new ideas for an experiment/searching for new data sources/looking for the ‘right’ experts, etc*

and

- **exploitation**: exploiting the information source (accuracy), e.g. *how large should scale of experiment be/how large a dataset to acquire/how many external experts to hire, etc*

Here mostly focus on second dimension, **endogenous intensity** as an extension
Model

Disclosure decision: In every period, after opportunity to learn and learning, buyer can reveal updated valuation verifiably, or not disclose at all.

In particular: Buyer can delay disclosure.

Timing within a period $t \geq 1$:

- Opportunity to learn arrives or not
- If opportunity arrives, \textbf{buyer} decides how much to learn
- If \textbf{buyer} learns, decides whether to disclose new valuation
- Seller quotes a price
- Buyer accepts or rejects price
Two distinct types of exploitation costs – results similar in both cases

**Deterministic variable costs of accuracy:** Informativeness of experiment $E_a$ is $I(a) = \frac{a}{1-a}$ and costs according to $c : [1, \infty) \to \mathbb{R}_+$ with the properties

- $c(1) = 0$ and $c'(1) = 0$
- $c$ is strictly convex and increasing
- $\lim_{I \to \infty} c'(I) = +\infty$. 
Stochastic fixed costs of accuracy: Conditional on an opportunity to learn having arrived and independent of everything else, a fixed cost \( c \in (0, \infty) \) is drawn, distributed according to a distribution \( F \).

If buyer pays current cost, can pick accuracy \( a \) at no additional cost, otherwise waits for future opportunities and draws.

\( F \) satisfies the following

- \( F \) is continuous and has finite first moment
- (Possibility of arbitrarily low costs) \( F \) puts positive probability on every neighborhood of zero
Histories and equilibrium concept

Informally:

Private information of buyer is: arrival of opportunity to learn, accuracy chosen and learning outcome

Public information: seller’s belief over the current valuation of the buyer

We call these current valuations throughout: the buyer’s type

It is a sufficient statistic of the buyer’s private information

In all model versions: We look for perfect Bayesian equilibria (PBE)
**Notation**

$\Delta$ is the time-length of the commitment

$\frac{1}{\Delta}$ frequency of interaction for buyer and seller

Many of the results hold with the quantifier *for all $\Delta$ small enough*, i.e. when players can commit to their actions only for a small period of time

- *near the HFL* means *for all $\Delta$ small enough* (HFL = high-frequency limit)

- for limit statements: *in the HFL* means the limit is *w.r.t. $\Delta \to 0$*
Reservation prices

Fix a PBE, a buyer type $w$ and a private history $h$ which ends just before the seller has the possibility to quote a price.

The reservation price of type $w$ after $h$ denoted $r(w, h)$ is the highest price type $w$ is willing to pay after $h$ given continuation play; $r(w, h) < w$ whenever trade in the future results in positive surplus with positive probability to the buyer.
Some properties of all PBEs with and without costs

Lemma

In any PBE in both the costless and costly information cases it holds:

i. every seller-offer after a history $h$ is at least as large, as the lowest type seller thinks is possible with positive probability after $h$,

ii. after a public history $h$, the seller puts positive probability only on reservation prices $r(w, h)$, for $w$ he thinks is possible with positive probability after $h$,

iii. after every private history the buyer with the highest reservation price accepts an offer equal to that reservation price with positive probability,

iv. after any disclosure event the seller quotes a price equal to the current buyer valuation with probability one.
Disclosure decision and strategic option value

Lemma

In all PBEs in both the costless and costly information cases it holds:

i. the buyer with good news never discloses, whenever PBE has strictly positive buyer payoff,

ii. the buyer with bad news is indifferent between disclosing or not disclosing at all bad news,

iii. there are no strict incentives to delay disclosure of bad news.

Strategic option value of buyer: buyer with good news pools with buyer who has not learned yet to get good at a bargain; comes from discretionary nature of disclosure

If learning is public there is no strategic option value
Equilibrium refinements: Disclosure

Equilibria without disclosure following bad news don’t survive small ‘overhead costs’ of bargaining, or small inventory costs for the good

Focus in the following on PBEs in which the lowest type discloses immediately and the bargaining ends after that (disclosure equilibria)
Prelude on seller updating: positive and negative selection

Given refinement on disclosure:

- No disclosure is evidence of higher types $\Rightarrow$ belief of seller shifts up $\Rightarrow$ positive selection

- Rejection of a price is evidence of lower types $\Rightarrow$ belief of seller shifts down $\Rightarrow$ negative selection

Which of the two forces wins in equilibrium?
Equilibrium refinements: Stationarity

Definition

Say that an equilibrium is \textit{stationary} if

i. Buyer’s on-path actions depend only on her current type and seller’s current belief over buyer types

ii. Seller’s on-path actions depend only on his belief distribution over buyer types

iii. If off-path play leads to a seller-belief that happens with positive probability on-path, ensuing play of seller follows his on-path strategies

A stationary equilibrium is called \textit{strongly stationary}, if as long as bargaining goes on, seller’s belief on-path is concentrated on type $\hat{v}$. 
Costless learning

Whenever information is free and useful to buyer, it is as if it is exogenous

Proposition

Every PBE with costless learning satisfies one of the following:

i. buyer payoff is zero,

ii. buyer payoff is strictly positive and she learns conclusively, whenever the opportunity to learn arrives

Proposition gives a micro-foundation in costless case for assumption of: exogenous, one-shot and conclusive learning opportunity
Costs of learning and buyer payoff

Theorem
Fix \(\pi_0, \underline{v}, \bar{v}\), and \(r\). It holds,

i. If learning is costless, there are stationary (though not strongly stationary) PBE with zero buyer payoff whenever \(\lambda \Delta\) is large enough,

ii. If learning is costless, all PBEs have strictly positive buyer payoff whenever \(\lambda \Delta\) is small enough,

iii. If learning is costly, every PBE has a positive buyer payoff for any \(\Delta\).

Details on the stationary equilibria in i.
Intuition if learning is costless

i. When $\lambda \Delta$ is large seller knows that with high probability buyer learns very soon after bargaining starts. Non-disclosure is a strong indicator of good news. Buyer with good news cannot successfully pool with buyer who has not learned yet

$\implies$ positive selection can win

ii. When $\lambda \Delta$ is small the probability that buyer has learned before any fixed period $K$ is small. Thus, seller cannot ask for very high prices at the beginning of the game. This results in informational rents

$\implies$ positive selection never wins near the HFL
Intuition if learning is costly

iii. to get zero buyer payoff the seller needs to quote at the end of every period on-path the reservation price of highest type that is possible with positive probability (high-price equilibrium)

Formal definition of high-price equilibrium

In a high-price equilibrium, option value from learning disappears $\implies$ buyer would never learn due to costs $\implies$ quoting $\hat{v}$ optimal on-path $\implies$ buyer wants to learn to create option value

$\implies$ positive selection never wins if learning is costly
Can negative selection win?

Not near the HFL: the seller never screens first type $\bar{v}$ and then type $\hat{v}$ in disclosure equilibria

Intuition: seller updates twice within a period as long as bargaining continues
- upon non-disclosure
- and
- after a price rejection (traditional channel)

First round ‘weakens’ effects of second round, especially near the HFL and near the beginning of game (probability of learning is small)

An implication of learning + disclosure + lack of commitment by seller!

No negative selection near the HFL even in the presence of costs
Stationary equilibria

- Next look for (strongly) stationary equilibria in which buyer with bad news discloses immediately.

Mixed pricing: mixing with probability $p$ in $[0, 1]$ between a price $p_H$ which makes buyer with good news indifferent between buying now or continuing and a price $p_L$ which makes type $\hat{v}$ indifferent between buying now or continuing.

Buyer with good news accepts $p_H$ immediately, type $\hat{v}$, who has option value, accepts $p_L$ with positive probability $q(\Delta)$.

Pure pricing: seller charges $p_L$ with probability one as long as no disclosure type $\hat{v}$ accepts $p_L$ with positive probability $q(\Delta)$.

$\implies$ stationary belief of type $\hat{v}$ is 1 as long as bargaining goes on.
$\implies$ strong stationarity.
Stationary equilibria in costless case

Reservation price relations

\[ \bar{v} - p_H = \delta(p(\bar{v} - p_H) + (1 - p)(\bar{v} - p_L)), \quad \text{type } \bar{v} \]

\[ \hat{v} - p_L = \frac{\mu \pi_0(\bar{v} - p_H) + \delta(1 - \mu)(1 - p)(\hat{v} - p_L)}{1 - \delta p(1 - \mu)}, \quad \text{type } \hat{v} \]

Can be solved for \( p_H(p), p_L(p) \).
Seller pricing in stationary equilibria

Let $V_\Delta(q, p)$ be stationary seller-value. Define $U(0) = \frac{\mu \pi_0}{1 - \mu + \mu \pi_0}$. For mixed pricing need:

$$U(0) p_H(p) + (1 - U(0)) \delta V_\Delta(q, p)$$

$$= p_L(p)(U(0) + (1 - U(0))q) + (1 - U(0))(1 - q) \delta V_\Delta(q, p)$$

seller – indifference

For pure pricing need: $\leq (p_L$-optimality)

Seller pricing in equilibrium is the same for costly and costless learning
Stationary equilibria in costless case

Definition

Say that a sequence of strongly stationary equilibria indexed by period-length $\Delta > 0$ with $\Delta \to 0$ converges in the HFL, if as $\Delta \to 0$

i. the average price quoted by Seller upon non-disclosure $\hat{p}(\Delta) = p(\Delta) \cdot p_H(\Delta) + (1 - p(\Delta)) \cdot p_L(\Delta)$ converges,

ii. the sequence $q(\Delta)$ of acceptance probabilities of Buyer type $\hat{\nu}$ for price $p_L(\Delta)$ satisfies

$$\frac{q(\Delta)}{\Delta} \to \nu,$$

for some $\nu \in [0, \infty]$.

Say that a HFL corresponds to some $\nu$ if there exists a sequence of strongly stationary equilibria such that $\frac{q(\Delta)}{\Delta}$ converges to $\nu$ as $\Delta \to 0$. 
Theorem

1) [Existence near the HFL] Strongly stationary equilibria with both pure and mixed pricing exist near the HFL.

2) [Uniqueness near the HFL] Near the HFL, every strongly stationary equilibrium with mixed pricing is unique up to the mixing probability \( p \in (0, 1) \) of Seller.

Near the HFL, every strongly stationary equilibrium with pure pricing is unique up to the acceptance probability \( q \) of Buyer of type \( \hat{v} \).
Theorem (continued)

3) [Delay in the HFL] There exists HFL of strongly stationary equilibria corresponding to any $\kappa \in [0, \infty]$.

- In any HFL of strongly stationary equilibria with mixed pricing $\kappa$ is 0 and positive expected delay is $\frac{1}{\lambda}$.
- In any HFL of strongly stationary equilibria with pure pricing $\kappa$ is in $(0, \infty]$ and expected delay is given by

$$\begin{cases} \frac{1}{\lambda + \kappa}, & \text{if } \kappa \in (0, \infty), \\ 0, & \text{if } \kappa = \infty. \end{cases}$$
Theorem (continued)

4) **[Pricing in the HFL]** In any HFL of strongly stationary equilibria prices converge to

\[
\psi = \frac{r\hat{v} + \lambda(1 - \pi_0)v}{r + \lambda(1 - \pi_0)}.
\]

5) **[Payoff and efficiency properties in the HFL]** Buyer and Seller payoffs in any converging sequence of strongly stationary equilibria with \( \frac{q(\Delta)}{\Delta} \rightarrow \kappa \) are unique. Buyer and Seller payoffs lie in \((v, \hat{v}]\) for all \( \kappa \in [0, \infty] \).

The efficiency loss in the HFL of equilibrium sequences with \( \kappa \in [0, \infty) \) is given by

\[
\frac{r}{r + \lambda + \kappa} \hat{v}.
\]

There is no efficiency loss in the HFL of sequences with \( \kappa = \infty \).
Intuition

- As frequency of interaction speeds up, trade happens at non-extreme prices and payoffs of buyer and seller are non-extreme.

- buyer has strategic option associated with learning:
  \[\Rightarrow\] receives non-trivial surplus

- but learning takes time and is costly and seller knows this
  \[\Rightarrow\] seller receives non-trivial surplus as well

- non-extreme payoffs \[\Rightarrow\] non-extreme trading prices
Intuition

- Whenever there is a positive gap between values of buyer and seller, learning is inefficient, but occurs in all stationary equilibria

  - if buyer does not exercise strategic option value she receives zero payoff
  - for large frequency of interaction inefficiency may be small or maximal

- Maximal inefficient delay is achieved in equilibria with mixed seller-pricing: buyer waits till she gets the information to end the game with either of the prices
Intuition

- In the HFL the seller asks for a flat price $\psi$, unless he sees evidence that $\theta = v$ and subsequently revise price down to $v$;

Delay due to learning leads to inefficiency which is larger the smaller $\lambda$ and $\kappa$

Delay may occur despite vanishing price spread in the HFL;

Delay is not due to seller screening, but because buyer waits to realize strategic option value from learning
Stationary equilibria with costly exploitation

Case of deterministic costs: Recall deterministic accuracy costs

Bayesian updates:

\[ \bar{w}(a) = \frac{a\pi_0\bar{v} + (1 - a)(1 - \pi_0)v}{a\pi_0 + (1 - a)(1 - \pi_0)} , \quad w(a) = \frac{(1 - a)\pi_0\bar{v} + a(1 - \pi_0)v}{(1 - a)\pi_0 + a(1 - \pi_0)} , \]

for the valuations after getting good/bad news with experiment of accuracy \( a \).

\[ v < w(a) < \hat{v} < \bar{w}(a) < \bar{v} \] whenever \( a > \frac{1}{2} \).
Denote $V_A(a, \hat{p}) = (\pi_0 a + (1 - \pi_0)(1 - a))(\bar{w} - \hat{p})$ the strategic option value when buyer uses experiment of accuracy $a$ and faces average price $\hat{p} = pp_H + (1 - p)p_L$

Reservation price relations

\[
\bar{w}(a) - p_H = \delta(\bar{w}(a) - \hat{p}), \quad \text{type } \bar{w}
\]

\[
\hat{v} - p_L = \frac{\delta \mu}{1 - \delta + \delta \mu} (V_A(a, \hat{p}) - c(l(a))), \quad \text{type } \hat{v}
\]
Stationary equilibria with deterministic variable costs

Optimal learning

\[(OL - Intensive) \quad a \in \arg \max \{ V_A(\bar{a}, \hat{p}) - c(I(\bar{a})) \}, \]

and

\[(OL - Extensive) \quad V_A(a, \hat{p}) - c(I(a)) \geq \hat{v} - p_L. \]

Easy to see: OL-Extensive is implied by reservation price relations for buyer types, OL-Intensive and assumptions on costs.
Stationary equilibria with stochastic fixed costs

Now the rate of learning is endogenous and different from \( \lambda \)!

Suppose an opportunity to learn arrives and let \( V_A = \pi_0 (\bar{v} - \hat{p}) \) the stationary continuation payoff after learning and \( V_N = \hat{v} - p_L \) the stationary continuation payoff after not learning

Buyer pays costs \( c \) iff \( c \leq V_A - V_N \)

\[ \implies \text{the stationary probability of learning is} \]
\[ \mu(\hat{p}) = \mu_0 F (\pi_0 (\bar{v} - \hat{p}) - (\hat{v} - p_L(\hat{p}))) . \]

\[ \mu_0 = 1 - e^{-\lambda \Delta} \text{ is the intensity of arrival of opportunities to learn} \]
Stationary PBEs with stochastic fixed costs

\( \bar{v} \)-indifference and seller-indifference as before whereas \( \hat{v} \)-indifference becomes

\[
\hat{v} - p_L = \frac{\delta \mu}{1 - \delta(1 - \mu)} (\pi_0(\bar{v} - \hat{p}) - E_F[c|c \leq V_A - V_N])
\]

Right hand side is strategic option value from learning times effective discount rate
Existence and uniqueness near HFL

Proposition

Pick any \( \pi_0, v, \bar{v} \) and \( \lambda, r \). In both cases of accuracy costs the following holds.

1) [Existence near the HFL] Strongly stationary equilibria with pure pricing always exist near the HFL.

There exists an open neighborhood \( \mathcal{N} \) of \( \hat{v} \) such that strongly stationary equilibria with mixed pricing and average price upon non-disclosure \( \hat{p} \in \mathcal{N} \) exist near the HFL, whenever the following condition is satisfied

\[
(P) \quad r > \lambda \text{ if } \pi_0 \leq \frac{1}{2} \quad \text{or} \quad r > \sqrt{2} \lambda \text{ if } \pi_0 > \frac{1}{2}
\]
Existence and uniqueness near HFL

Proposition (continued)

2) [Uniqueness near the HFL] For any fixed average price $\hat{p} \in \mathcal{N}$ the quantities $p_L(\hat{p}, \Delta), p_H(\hat{p}, \Delta), p(\hat{p}, \Delta)$ are uniquely determined in every strongly stationary equilibrium.

In the case of deterministic variable costs, $a(\hat{p})$ is unique.

In the case of mixed pricing the acceptance probability $q(\Delta, \hat{p})$ is unique.
The conditions $r > \lambda$ if $\pi_0 \leq \frac{1}{2}$ or $r > \sqrt{2}\lambda$ if $\pi_0 > \frac{1}{2}$ can be relaxed, though not by much.

They say that the rate of arrival of information opportunities is not too high when compared to the impatience parameter $r$. 
Analysis in the HFL

Focus in exposition mostly on deterministic variable costs for brevity

Theorem

1) [Delay in the HFL] In any HFL of strongly stationary equilibria expected delay is given by

\[
\begin{cases}
\frac{1}{\lambda + \kappa}, & \text{if } \kappa \in [0, \infty) \\
0, & \text{if } \kappa = \infty.
\end{cases}
\]

2) [Pricing in the HFL] In both cases of accuracy costs the price spread in the HFL of a sequence of strongly stationary equilibria with mixed pricing is bounded away from zero, and the low price is charged with vanishingly small probability.
3) \textit{[Payoff and efficiency properties in the HFL]} Buyer and Seller payoffs in any converging sequence of strongly stationary equilibria with $\frac{q(\Delta)}{\Delta} \to \kappa$ are unique. Buyer and Seller \textit{payoffs lie in} $(v, \hat{v})$ for all $\kappa \in [0, \infty]$. 

The efficiency loss in the HFL of equilibrium sequences with $\kappa \in [0, \infty)$ is given by 

\[
\frac{r}{r + \lambda + \kappa} \hat{v} + \frac{\lambda}{r + \lambda + \kappa} c(I(a(\hat{p}))).
\]
Stationary equilibria with costly learning

Differences to costless learning:

- for mixed pricing, a price-spread is necessary even in the limit to subsidize information costs incurred with positive probability

For $\Delta$ positive but small, seller promises to charge a low price occasionally to incentivize learning

But probability to learn in a single period vanishes as $\Delta \to 0$

$\implies$ the seller promises the low price within a period less and less often as $\Delta \to 0$

$\implies$ delay is again due to buyer waiting to realize the strategic option value
Differences to costless learning (continued):

- With costs some of the ex-ante surplus is wasted due to costs of learning

Inefficiency due to costly exploitation increases in $\lambda$ and falls with $r, \kappa$

Inefficiency overall falls with $\lambda$, because option value from learning is always greater than costs of exploitation incurred in equilibrium
Analysis in the HFL with stochastic fixed costs

Intuitions parallel the case of deterministic variable costs

Expected delay in real time is now equilibrium-dependent and equal to

\[
\begin{align*}
\frac{1}{\lambda \tilde{\mu}(\hat{\rho}) + \kappa}, & \quad \text{if } \kappa \in [0, \infty), \\
0, & \quad \text{if } \kappa = \infty.
\end{align*}
\]

Reason: rate of learning and of arrival of opportunity to learn diverge
Extension: endogenous intensity

At the beginning of every period the buyer picks $\mu$ for the cost $C(\Delta, \mu)$ where

- the function $C : (0, \infty) \times [0, 1) \to \mathbb{R}_+$ is differentiable

- it satisfies

$$\lim_{\Delta \to 0} \frac{C(\Delta, \lambda \Delta)}{\Delta} = f(\lambda), \quad \lim_{\Delta \to 0} \frac{\partial}{\partial \mu} C(\Delta, \Delta \lambda) = f'(\lambda)$$

uniformly on $\lambda > 0$,

and

$f : [0, \infty) \to \mathbb{R}_+$ is differentiable, strictly increasing and convex with $f(0) = f'(0) = 0$ and $\lim_{\lambda \to \infty} f'(\lambda) = +\infty$

Example: $C(\Delta, \mu) = \Delta \cdot f(\mu)$ for $f$ as above
Stationary equilibria with endogenous exploration

Focus on deterministic variable costs for brevity

FOC for the choice of intensity in the HFL is

\[ f'(\lambda) \frac{r + \lambda}{r} = V_A(a(\hat{p}), \hat{p}) - c(I(a(\hat{p}))) \]

\[ V_A(a(\hat{p}), \hat{p}) - c(I(a(\hat{p}))) \] is the net-benefit from learning

The efficiency loss in the HFL is given by

\[ \frac{r}{r + \lambda(\hat{p}) + \kappa} \hat{v} + \frac{\lambda(\hat{p})}{r + \lambda(\hat{p}) + \kappa} c(I(a(\hat{p}))) + \frac{f(\lambda(\hat{p}))}{r + \lambda(\hat{p}) + \kappa}. \]

Additional efficiency loss due to endogenous search for information sources
Comparative statics in the HFL

Compare two stationary equilibria in the HFL with the same $\hat{\rho}$:

- $\hat{\rho}$ is a sufficient statistic for the construction
- $\hat{\rho}$ is potentially empirically observable
Comparative statics in the HFL

Results in words:

- $a$ is, roughly speaking, reverse-U-shaped in the prior $\pi_0$: when buyer is more uncertain ex-ante about her gains from trade, she picks higher accuracy

- $\lambda$ is increasing in $r$ and $\pi_0$:

a more impatient or more ex-ante optimistic buyer searches faster for information sources

- in the stochastic costs model $\lambda$ falls if $F$ FOSD-increases:

less incentives to search for information sources if exploiting them costs more

Proposition for comparative statics
Extension: pre-learning negotiations

Add ex-ante stage: seller first approaches buyer before she can start to learn.

Diagram:

- Seller
- Buyer
- p
- (p, \hat{v} - p)
- Accept
- Reject

Play bargaining game from t=1 on.
Extension: pre-learning negotiations

Proposition

All perfect Bayesian equilibria of the game with pre-learning negotiations are efficient. In particular, all perfect Bayesian equilibria feature agreement at time $t = 0$. 
Extension: pre-learning negotiations

seller offers a price equal to \((\hat{v} - \text{an equilibrium payoff of buyer})\), thus pricing the opportunity to learn

\[ \Rightarrow \text{inefficiency in the game only because buyer can start learning before approaching seller} \]
Conclusions

- in contrast to case with pure initial private information and to case where learning is impossible:

both buyer and seller ensure non-negligible surplus from the negotiations, as interaction speeds up

- as interaction speeds up, trade necessarily happens at non-extreme prices
Conclusions

- if interaction is not fast enough, costs are necessary for positive buyer payoff; costs are always sufficient for positive buyer payoff

- delay is indeterminate in stationary equilibria: ranges from near zero to rate of arrival of information source $\rightarrow$ approximate efficiency is feasible as interactions speed up

- can characterize optimal information acquisition $\rightarrow$ comparative static results feasible
Appendix: Additional results and details
Some recent bargaining literature

Recent literature on delay:

- Due to arrival of outside options: Fuchs, Skrzypacz 2010; Hwang 2015; Hwang, Li 2017 (both public and private, also studies disclosure decision); Lomys 2018

Differences to this paper

- Bargaining and public, exogenous news: Daley, Green 2019 (seller has initial private information, interdependence of values necessary but not sufficient for delay)

- Bargaining in which both parties learn exogenously and privately: Esö, Wallace 2019 (considers disclosure of hard evidence as here, learning is costless, values are interdependent, no delay can occur)
Some recent bargaining literature

Literature on endogenous evolution of valuations in bargaining situations:

- Ravid 2019: rationally inattentive buyer on past prices and good value, no info disclosure decision, seller has private initial info

- due to buyer endogenously acquiring information about value: this project
Other related literature

- Strategic information transmission without commitment in static settings has a long tradition: started by Milgrom ’81; Crawford, Sobel ’82, and many others...
  still active: e.g. Lipnowski, Ravid 2019

- Information acquisition: few papers with dynamic costly information acquisition
In contrast to outside options literature, here:

1. There is no informational asymmetry at the start of the game. It appears at a random date due to learning at a random time during the bargaining process.

2. The valuations change in a MPS way, the mechanism how this turns into a FOSD-shift is an equilibrium property, rather than ad-hoc.
Formalism for buyer histories

Let $\mathcal{H}_{S,t}$ histories of length $t$ of seller.

\begin{align*}
\mathcal{H}_{B,t}^L &= \mathcal{H}_{B,t-1} \oplus \{o, \neg o\}, \\
\mathcal{H}_{B,t}^D &= \mathcal{H}_{B,t-1} \oplus \{a \in [0, 1], \text{signal} = H, L : (o, a, \text{signal}, d), (o, \neg d)\}, \\
\text{and finally} \\
\mathcal{H}_{B,t} &= \mathcal{H}_{B,t-1} \oplus \{p \in \mathbb{R}_+ : (\neg o, p, \text{reject}), (\neg o, p, \text{accept})\} \\
&\quad \cup \mathcal{H}_{B,t}^D \oplus \{p \in \mathbb{R}_+ : ((p, \text{accept}), (p, \text{reject}))\}.
\end{align*}
An agenda on endogenous learning and bargaining

Relevant questions:
- is there initial private information?
- which party is learning endogenously?
- is the learning private or public?
- can information acquired be disclosed? Is disclosure verifiable? Can it be garbled?
- how does endogenous learning affect trade efficiency, how does it interact with competitive pressure?
- Is there scope for limited commitment and institutional design?
Definition

Call a PBE a high-price equilibrium if after every on-path history in which it is the seller’s turn to move, he quotes with probability one the reservation price of the highest type that is possible with positive probability after that history.

It is easy to show that a PBE has zero buyer payoff if and only if it is a high-price equilibrium.
Belief dynamics of the seller in stationary high-price equilibria

Suppose seller starts period $t \geq 2$ with belief $\gamma$ that $\bar{v}$ has appeared. Then upon non-disclosure she updates to

$$U(\gamma) = \frac{\gamma + (1 - \gamma) \mu \pi_0}{1 - (1 - \gamma) \mu (1 - \pi_0)}.$$  

Suppose upon non-disclosure seller asks for $p_H$ s.t., given continuation play, $\theta = \bar{v}$ mixes between accept and reject when faces $p_H$.

$\theta = \hat{v}$ has strict incentives to reject $p_H$

Then if $\theta = \bar{v}$ accepts $p_H$ with probability $q$, upon rejection of $p_H$ seller updates to

$$B(U(\gamma), q) = \frac{U(\gamma)(1 - q)}{U(\gamma)(1 - q) + 1 - U(\gamma)}.$$
Stationary beliefs in high-price equilibria (from $t = 2$ on)

Under equilibrium properties above:

Upon non-disclosure belief of seller goes up, upon rejection of higher price belief of seller goes down.

Condition for stationary beliefs with these strategies is

$$B(U(\gamma), q(\gamma)) = \gamma.$$ 

Can be solved for $q(\gamma)$ for every $\gamma \in (0, 1]$. 
Stationary high-price equilibrium with zero buyer surplus

Seller asks for $p_t = \bar{v}$ on-path when no disclosure.
Buyer accepts $p_t = \bar{v}$ if $\bar{v}$ with probability $q_1 \in (0, 1)$ in $t = 1$ and $q(\gamma)$ in $t \geq 2$, where

$$U(0)q_1 \cdot \bar{v} + (1 - U(0)q_1) \delta W(\gamma) \geq \hat{v},$$

seller-optimality at $(t = 1)$ and

$$U(\gamma)q(\gamma) \cdot \bar{v} + (1 - U(\gamma)q(\gamma)) \delta W(\gamma) \geq \hat{v},$$

seller-optimality at $(t \geq 2)$ with

$$\gamma = B(U(0), q_1), \quad W(\gamma) = \frac{\mu}{1 - \delta + \delta \mu}(\gamma \bar{v} + (1 - \gamma)\hat{v}).$$

Can show: optimality at $t = 1$ implies optimality at $t \geq 2$. 
PBE with zero buyer surplus

Proposition

Let \( U(0) = \frac{\mu \pi_0}{1 - \mu + \mu \pi_0} \) and \( W = \frac{\mu}{1 - \delta + \delta \mu} (U(0)\bar{v} + (1 - U(0))\hat{v}) \). Then whenever the parameters satisfy

\[
(C - \text{high}) \quad U(0)\bar{v} + (1 - U(0))\delta W > \hat{v},
\]

there exists a stationary disclosure PBE where the buyer’s payoff is zero and the seller only asks for the highest value \( \bar{v} \) whenever game continues and there is no disclosure of learning outcomes.

For fixed other parameters of the game, \((C - \text{high})\) is always satisfied when \( \mu \) is near enough to one.
Sequential screening of valuations

Say that a PBE has **sequential screening of valuations (SSV)** if and only if

1. as long as there is no disclosure, the seller quotes a strictly decreasing sequence of deterministic prices \( \{r_l, l \leq K\} \) \((K \leq \infty)\), starting with the reservation price of type \( \bar{v} \)

2. the sequence of beliefs of the seller \( \gamma_l(\bar{v}) \) at the beginning of every period \( l \geq 2 \) is strictly decreasing over time in the FOSD-sense

One can show that whenever \( v > 0 \), just the logic of the seller’s pricing delivers no SSV near the HFL

Case \( v = 0 \) is not trivial. I impose stationarity and require an equilibrium refinement to treat that case
A refinement: ‘Divinity in bargaining’

After an off-path history resulting from the rejection of a price, if the pool of buyer types contains only one type who was indifferent between accepting and rejecting and all other types the seller deemed feasible had a strict incentive to accept the rejected price, then put **probability one on the type who was indifferent** between accepting and rejecting.
No SSV near HFL

Proposition

For all $\Delta$ small enough, there are no disclosure PBE which satisfy the following properties.

1. $v > 0$
2. SSV

or

1. $v = 0$
2. Stationarity,
3. SSV
4. ‘Divinity in bargaining’.
Proof idea for no SSV

In the case of HFL: (Case $\nu > 0$) There needs to be a first period where the screening of the highest type begins. Fix $T$ such a period. Then, the first price quoted on-path that screens out intermediate type $\hat{v}$, is higher the higher the probability of $\bar{v}$ upon non-disclosure at $T$. But this probability goes to zero as $\Delta \to 0$. So reservation price of $\hat{v}$ needs to be arbitrarily small as $\Delta \to 0$. This can’t happen under the assumptions in the Proposition.

Additional assumptions for case $\nu = 0$ allow a uniform estimate of the last price quoted in any purported SSV PBE. Estimate shows that that price is bounded away from zero near HFL and this gives a contradiction.
Comparative statics in the HFL

Proposition

1. Suppose there are two strongly stationary equilibria in the HFL with the same average price $\hat{p}$ and all parameters the same, except for $r_1 > r_2$. Then the equilibrium intensity is higher for $r_1$ than $r_2$. Equilibrium accuracy in the model with deterministic costs is the same in both cases.

2. Suppose there are two strongly stationary equilibria with the same average price $\hat{p}$ and all parameters the same, except for $\pi_0^1 > \pi_0^2$. Equilibrium accuracy in the model with deterministic costs is higher for $\pi_0^1$ if $\frac{\bar{v} + v}{2} > \hat{p}$, whereas it is higher for $\pi_0^2$ if $\frac{\bar{v} + v}{2} < \hat{p}$. Equilibrium intensity is always higher for $\pi_0^1$.

3. Suppose in the case of stochastic costs there are two strongly stationary equilibria with the same average price $\hat{p}$ and all parameters the same, except for $F_1 >_{FOSD} F_2$. Then $\lambda_1$ is lower than $\lambda_2$. 

Back