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Supplemental information

Dynamically preferred state with strong

electronic fluctuations from electrochemical

synthesis of sodium manganate

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Figure S2. *In-situ* synchrotron XRD and Rietveld refinement for Na_{1/2}NiO₂. (bottom) Enlarged region with the experimentally identified superstructure peaks labelled by vertical blue arrows. The broad background is subtracted in the bottom plot.



Figure S3. *In-situ* synchrotron XRD and Rietveld refinement for Na_{1/3}NiO₂. (bottom) Enlarged region with the experimentally identified superstructure peaks labelled by vertical blue arrows. The broad background is subtracted in the bottom plot.



Figure S4. *In-situ* synchrotron XRD and Rietveld refinement for Na_{2/3}CoO₂. Refined with the 2% restriction of relative internal coordinate change starting from the DFT structure. (bottom) Enlarged region with the experimentally identified superstructure peaks labelled by vertical blue arrows. The broad background is subtracted in the bottom plot.



Figure S5. *In-situ* synchrotron XRD and Rietveld refinement for Na_{1/2}CoO₂. (bottom) Enlarged region with the experimentally identified superstructure peaks labelled by vertical blue arrows. The broad background is subtracted in the bottom plot.



Figure S6. *In-situ* synchrotron XRD and Rietveld refinement for Na_{1/3}CoO₂. Refined with the 2% restriction of relative internal coordinate change starting from the DFT structure. (bottom) Enlarged region with the experimentally identified superstructure peaks labelled by vertical blue arrows. The broad background is subtracted in the bottom plot. This ordering shows weak superstructure peak signals in the *in-situ* XRD pattern.



Figure S7. *In-situ* synchrotron XRD measurement of Na_xNiO_2 with an electrochemical charge and discharge rate of C/50. Each XRD scan corresponds to 1% of Na composition change. The O3 and P3 ordered single-phase regions are marked by darker red and darker blue in the phase evolution bar. Lighter red and lighter blue regions in the bar correspond to two-phase regions. The XRD scans corresponding to the ordered single-phase regions are also highlighted in red in the right figure.



Figure S8. *In-situ* synchrotron XRD measurement of Na_xCoO₂ at C/50 rate. Each scan corresponds to 1% of Na composition change. The O3 and P3 ordered single-phase regions are marked by darker red and darker blue. Lighter red and lighter blue regions correspond to two-phase regions. The XRD scans corresponding to the ordered single-phase regions are also highlighted in red.



Figure S9. Structure evolution of Na_xNiO_2 , Na_xCoO_2 , Na_xMnO_2 solved from *in situ* SXRD. Definition of the P3 and O3 type TMO₂ stacking (3D model on the leftmost) and the stacking of Na (2D projection) in the major ordering structures presented in Fig. 1 of the Main Text. The 2D projections are along hexagonal-*c* crystallographic direction.



Figure S10. Measured and calculated phase diagrams of Na_x**NiO**₂. All the DFT predicted major ground state orderings at discrete x values for Na_xNiO₂ agree with experimental SXRD refinements (Fig. S1 to S3). The phase boundaries given by the solid shaded areas (top part) are determined by *in situ* SXRD (Fig. S7) and the shaded areas with skewed lines (bottom part) are those determined by the data points from DFT computations. The numbers on the top x-axis of the figure are the boundary compositions obtained from *in situ* SXRD (Fig. S7).



(experiment)			0 -Na _{1/2} WinO ₂ (Di T)			0 -Na _{1/2} NITO ₂ (DI T)																	
			_		V	~		-		v	~		~		V	x	У	z		V	x	У	z
	X	У	z		•	×	У	<u> </u>		v	*	У	2	Mn1	3.80	0	0.016	0.5	Mn1	3.90	0	0.016	0.5
Mn1	0	0	0.5	Mn1	3.80	0	0	0.5	Mn1	3.18	0	0	0.5	Mo2	2.90	0.5	0.270	0.5	Mp2	2.00	0.5	0.270	0.5
Mn2	0.5	0 739	0.5	Mn2	3.12	0.5	0.749	0.5	Mn2	3.47	0.5	0.744	0.5	IVIT12	3.60	0.5	0.270	0.5	WINZ	3.90	0.5	0.270	0.5
	0.0	0.100	0.0											Mn3	3.12	0	0.519	0.5	Mn3	3.17	0	0.518	0.5
Mn3	0	0.5	0.5	Mn3	3.80	0	0.5	0.5	Mn3	3.94	0	0.5	0.5	Mn4	3 12	0.5	0 767	0.5	Mn4	3 17	0.5	0 767	0.5
01	0.195	0.730	0.677	01		0.186	0.750	0.696	01		0.227	0.733	0.687	IVIII-	0.12	0.0	0.101	0.0	IVIII-4	0.17	0.0	0.101	0.0
	0.704		0.005			0.740		0.075			0.704	0.5	0.007	01		0.226	0.754	0.184	01		0.227	0.733	0.68
02	0.724	0.5	0.665	02		0.718	0.5	0.675	02		0.731	0.5	0.687	02		0.726	0.531	0.684	02		0.731	0.531	0.68
03	0.699	0	0.642	O3		0.715	0	0.666	O3		0.716	0	0.669			0.707	0	0.000			0.740	0	0.00
N.d.	0.5	0.5	0	N A		0.5	0.5	0	AL-A		0.5	0.5	0	03		0.727	0	0.683	03		0.716	0	0.66
Nati	0.5	0.5	0	Nai		0.5	0.5	0	Nati		0.5	0.5	0	Na1		0.5	0.471	0	Na1		0.503	0.471	0
Na2	0	0	0	Na2		0	0	0	Na2		0	0	0	NI-O		0	0.040	0	NI-0		0	0.044	-
														Na2		0	0.813	0	Na2		0	0.814	0

d

,	c				Na _{1/2} M					
	Parameter	EXP	DFT (δ^+)	DFT (δ^-)	DFT (v^-)	DFT (v_0^-)	Difference $(\delta^+, \%)$	Difference (δ ⁻ , %)	Difference (v ⁻ , %)	Difference (v_0^-, %)
	a (Å)	4.96	5.27	5.04	5.00	5.01	6.3	1.6	0.8	1.0
	b (Å)	5.95	5.76	5.90	6.07	6.07	-3.2	-0.8	2.0	2.0
	c (Å)	5.81	5.79	5.80	5.78	5.78	-0.3	-0.2	-0.5	-0.5
	β (°)	104.49	101.38	103.69	105.03	104.95	3.0	-0.8	0.5	0.4

-							
Parameter	δ^+	δ-		v_0^-	Refine internal coordinates (EXP)		
a (Å)	4.962	4.963	4.963	4.963	4.961		
b (Å)	5.951	5.951	5.951	5.951	5.948		
c (Å)	5.811	5.810	5.809	5.809	5.808		
β (°)	104.507	104.508	104.516	104.516	104.485		
R _{wp}	5.089	4.849	4.941	4.920	3.51		

	Na _{1/3} I	MnO ₂		Na _{5/8} MnO ₂					
Parameter	EXP	EXP DFT		Parameter	EXP	DFT	Difference(%)		
a (Å)	5.06	5.20	2.8	a (Å)	11.53	11.59	0.5		
b (Å)	8.45	8.68	2.7	b (Å)	5.64	5.78	2.5		
c (Å)	5.91	5.92	0.2	c (Å)	17.50	17.65	0.9		
β (°)	107.85	108.68	0.8	β (°)	144.88	144.45	-0.3		

Na_{1/2}MnO₂

Figure S11. Lattice parameters, bond lengths and internal coordinates of δ^+ , δ^- , v^- and $v_0^$ structures. (a) δ^+ , δ^- , v^- and v_0^- structures of Na_{1/2}MnO₂ from DFT relaxations at experimental lattice parameters. The numbers show the length (in Å) of each Mn-O bond, as well as the Mn-Mn distances along b direction. The corresponding connection lines between atoms are marked with the same color as the numbers. (b) The internal coordinates of Na_{1/2}MnO₂ refined from insitu SXRD experiment, and the four x = 1/2 states from DFT relaxations. The valences of the Mn ions are also labeled in the "V" column for the DFT cases. (c) The lattice parameters obtained from *in-situ* SXRD refinement (EXP) and DFT relaxed energy minimum states (DFT) for x = 1/3, 5/8, and the four candidate states for x = 1/2. The difference (%) between calculated and experimental ones are also presented. (d) Refined lattice parameters and goodness of fit (Rwp) by fixing internal coordinates from δ^- , δ^- , v^- and v_0^- structures at x = 1/2, compared with the result from fully refined internal coordinates (EXP).



Figure S12. Structure models of Na_{5/8}**MnO**₂ (a) and Na_{1/3}**MnO**₂ (b). Figures from left to right are 3D structure, pure Mn³⁺ plane and other alternating planes mixed with Mn³⁺/Mn⁴⁺ for Na_{5/8}MnO₂ and pure Mn⁴⁺ for Na_{1/3}MnO₂, respectively. Blue plane in each 3D model marks the pure Mn³⁺ atomic plane. Note that in both *in-situ* SXRD refinements and DFT simulations, Na_{5/8}MnO₂ and Na_{1/3}MnO₂ show the strongly elongated JT distortion of Mn³⁺O₆, same as δ^+ -Na_{1/2}MnO₂ in DFT.



Figure S13. Original and perturbed band structures for Na_xMnO₂. (a) x = 1. (b) x = 5/8. (c-g) x = 1/2 (δ^+ , δ^- , v^- , v_0^- states). (h) x = 1/3. The O_{JT} perturbation is generated as the elongation of a pair of O_{JT} ions by 0.1 Å along the JT long-axis directions of the elongation mode. The perturbations to the Mn^{3.5+} dimer either shorten the dimer bond (strengthening dimerization) or elongate it (removing the dimer). Blue and green markers represent the band occupation of the Mn *d* and the O *p* orbitals in the JT active MnO₆ octahedron of the pure charge stripe, respectively. The color near *E*_f blends blue and green before perturbation, indicating a pd hybridization. At x = 1/2 (δ^+), 1/3 and 5/8, they are the d_{z^2} and the p_z orbitals of the two O_{JT} oxygens that are hybridized to the Mn³⁺ ion, while at x = 1/2 (δ^-), they are the $d_{x^2-y^2}$ and p_x or p_y orbitals of the two represent the band occupation of the other four oxygens that hybridize to the Mn^{3.5+} ion. At x=1/2, the orange and red markers represent the band occupation of the Mn³⁺ $d_{x^2-y^2}$ and the p_x or p_y orbitals of the four oxygens that hybridize to the Mn^{3.5+} ion. At x=1/2, the orange and red markers represent the band occupation of the Mn³⁺ $d_{x^2-y^2}$ and the p_x or p_y orbitals of the four oxygens that hybridize to the Mn^{3.5+} ion. At x=1/2, the orange and red markers represent the band occupation of the Mn³⁺ $d_{x^2-y^2}$ and the p_x or p_y orbitals of the four oxygens that hybridize to the Mn³⁺ $d_{x^2-y^2}$ and the p_x or p_y orbitals of the four oxygens that hybridize with it, respectively, in the Mn³⁺ $d_{x^2-y^2}$ and the p_x or p_y orbitals of the four oxygens that hybridize

and the $O_{JT} p_z$ orbitals, respectively, also in the mixed valence stripe. For v^- and v_0^- states at x = 1/2, we also use orange for Mn³⁺ ion and red for oxygen ion, indicating their negative JT distortion for $d_{x^2-y^2}$ and p_x / p_y orbitals. Note that for the δ^- state at x = 1/2 the JT distortion is different from the other three states at x = 1/2 and the two states at x = 1/3 and 5/8, due to its abnormal JT distortion with the weak compression on the unique Mn^{3.5+}O₆. However, for comparison, the perturbation in all three cases are generated on the same oxygen ions, i.e., the O_{JT} defined in x=1/3 and x=5/8, and along the JT long axes of the elongation mode. The O_{JT} perturbations are applied to one of the Mn³⁺O₆ for x = 1, 1/2 (δ^+ , v^- , v_0^-), 1/3; to one of the Mn³⁺O₆ in the pure Mn³⁺ stripe at x = 5/8; to one of the Mn^{3.5+}O₆ at x = 1/2 (δ^-). The 3D diagrams show the corresponding Brillouin zone definition. Na_{5/8}MnO₂ shows the most obvious de-hybridization by perturbation, while Na_{1/3}MnO₂ shows an energy suppression to the perturbed d_{z^2} and p_z orbitals.



Figure S14. Original and perturbed DOS for Na_xMnO₂. (a) x = 1. (b) x = 5/8. (c-g) x = 1/2 (δ^+ , δ^- , v^- , v_0^- states). (h) x = 1/3. The O_{JT} perturbation is generated as the elongation of a pair of O_{JT} ions by 0.1 Å along the JT long-axis directions of the elongation mode. The perturbations to the Mn^{3.5+} dimer either shorten the dimer bond (strengthening dimerization) or elongate it (removing the dimer) in (e). The DOS of the p_z orbital of an elongated O_{JT} and the d_{z^2} orbital of the Mn³⁺ in the octahedral center are plotted for x = 1/3. For x = 5/8, Mn^{3+, (2)} is the Mn³⁺ in the pure Mn³⁺ stripe (Fig. S12) whose JT oxygens are elongated. Mn^{3+, (1)} is the Mn³⁺ in the mixed Mn^{3+/4+} plane/stripe. For the δ^- state at x = 1/2, the $d_{x^2-y^2}$ orbitals of the Mn³⁺ in the Mn³⁺ stripe, as well as p_x or p_y orbitals of the four non-O_{JT} oxygens that hybridize with them are plotted. For the δ^+ state at x = 1/2, the d_{z^2} orbital for Mn³⁺ in the Mn³⁺ stripe, as well as

 p_z orbitals of the two O_{JT} oxygens are plotted. For the v^- and v_0^- states at x = 1/2, we also use orange for Mn³⁺ ion and red for oxygen ion, indicating their negative JT distortions with $d_{x^2-y^2}$ and p_x / p_y orbitals. Note that for the δ^- state at x = 1/2 the JT distortion is different from the other three states at x = 1/2 and the two states at x = 1/3 and 5/8 due to its abnormal JT distortion with the weak compression on the unique Mn^{3.5+}O₆. However, for comparison, the perturbation in all three cases are generated on the same oxygen ions, i.e., the O_{JT} defined in x=1/3 and x=5/8, and along the elongated JT direction. The O_{JT} perturbations are applied to one of the Mn³⁺O₆ for x = 1, 1/2 (δ^+ , v^- , v_0^-), 1/3; to one of the Mn³⁺O₆ in the pure Mn³⁺ stripe at x = 5/8; to one of the Mn^{3.5+}O₆ at x = 1/2 (δ^-).



Figure S15. Charge density difference (CCD) resulted from the perturbation of ions for δ^- **-Na**_{1/2}**MnO**₂. (a) Perturbation of a pair of O_{JT} ions along the JT long-axis. (b) Perturbation of a Mn^{3.5+} ion along the *b* direction. The result corresponds to Fig. 2(e) in the Main Text. More reddish and more blueish regions in CCD indicate larger amounts of electron increase and decrease, respectively. Blue arrows indicate charge flux flow directions, and black empty arrows indicate the ion perturbation direction. The section in (a) goes through the oxygen plane defined by the 3 oxygen ions in the CCD (inset), which clearly shows the charge fluxes from the perturbed O_{JT} ions to the oxygens of the neighboring Mn^{3.5+}O₆ stripe. Multiple such CCD sections along different planes are needed in order to illustrate the entire flow pathways shown in (a, left). The section in (b) goes through the plane at the center between the Mn layer and O layer, and a plane including Mn and four oxygens as shown in the insets. It shows the zig-zag fluxes along the *b* direction, as well as across the Mn^{3.5+} stipes. For clarity, only relevant ions are shown in the CCD plots. Note that here we illustrate some visual evidence of the charge fluxes. The actual charge flux direction and strength are determined by quantitative calculations using the entire 3D density.



Figure S16. Comparison of *in-situ* SXRD (λ = 0.4134 Å) collected at 11-BM of APS at ANL and *ex-situ* SXRD (λ = 0.1844 Å) collected at 28-ID-2 of NSLS II at BNL for Na_{1/2}MnO₂. The plot for *ex-situ* SXRD is scaled to 0.4134 Å as well. Both SXRD patterns are after the background subtraction.



Figure S17. Zoom-in plots around the frequencies of the green and red arrows in Fig. 3(a). They represent for (left) the Na phonon modes, and (right) the oxygen phonon modes, from comparison with DFT simulations in Fig. S18 and S19.



Figure S18. DFT calculated phonon spectra of Na_{1/3}MnO₂ and δ^- -Na_{1/2}MnO₂.



Figure S19. Comparison between the calculated phonon spectra of δ^- -Na_{1/2}MnO₂ (blue) and the measured Raman spectra (green) for Na_{1/2}MnO₂.



Figure S20. Phonon coupling map for Na_xMnO₂ states at x = 1, x = 5/8, x = 1/3 and δ^+ , δ^- , v^- , and v_0^- states at x=1/2. Figure 3d of the Main Text is generated using these results. (a) comparison among all seven states with a high cutoff of 0.04 eV. (b) Comparison between x=1 and δ^- states with a high cutoff of 0.2 eV.



Figure S21. Neutron diffraction spectra measured at different temperatures from 2.5K to 300K for Na_{1/2}MnO₂. The red arrows indicate magnetic peaks marked with (hkl) labels.



Figure S22. Heatmap of the evolution of neutron diffraction spectra with decreasing temperature in the 2θ angle ranges. (left) Measurement around the 10-1 Bragg magnetic peak. (right) Measurement around the 10-3 (right) Bragg magnetic peak. The plots correspond to Fig. 4b and 4c in the Main Text. The more reddish and blueish regions indicate stronger and weaker neutron signals, respectively. The inset shows the integrated intensity (square of the order parameter) versus temperature, with a mean field fit (solid curve) to estimate the Néel ordering temperature of 47(1) K.



Figure S23. DFT prediction of ground state magnetic ordering and calculated magnetic coupling constants $J_{ij}^{(n)}$ in δ^- -Na_{1/2}MnO₂. (a) ground state magnetic ordering. (b) calculated magnetic coupling constants. The subscript 3 indicates it is Mn³⁺ or Mn^{3.5+} that participates in the coupling, and 4 indicates it is Mn⁴⁺, while the superscript indicates the inequivalent interactions. Positive J means ferromagnetic coupling and negative means antiferromagnetic coupling. The ground state magnetic ordering in (a) from DFT prediction is consistent with neutron refinement.



Figure S24. Neutron diffraction and neutron PDF refinements of δ^+ and δ^- states at x = 1/2 for Na_xMnO₂. (top) Neutron diffraction refinements. (bottom) Neutron PDF refinements.



Figure S25. Inelastic neutron scattering of the magnon spectra at the magnetic zone center of Na_{1/2}MnO₂. (a)T = 4.8 K. (b) T = 20 K. (c) T = 30 K. (d) T = 60 K. Δ denotes the gap value. The solid curves are the fits to a step function at the gap energy plus intrinsic lifetime broadening and a delta function at zero energy for the elastic scattering, convoluted with the instrumental resolution. At base temperature a sharp gap of 2.30 meV is observed. With increasing temperature, the gap softens, the magnetic scattering broadens, and by 60 K the gap has closed, and only quasi-elastic scattering is found. The inset in (d) shows the temperature dependence of the gap and the solid-dashed blue curve is fitted using $(T_N - T)^{1/3}$ power law assuming the gap is zero at T_N.



Figure S26. Typical phonon modes and their effect on magnetic coupling strength $J_{33}^{(1)}$. (a) Illustration of phonon modes # 22 and 25 in Fig. 3c. Vectors are scaled movements of atoms. Purple and red vectors highlight the special Mn^{3.5+} dimer component and the elongation O_{JT} component. Note: 1 meV = 8.065 cm⁻¹. (b) Magnetic coupling strength $J_{33}^{(1)}$ of the δ^- state between nearest Mn^{3.5+} ions along *b* vs. applying phonon mode amplitudes of Mn^{3.5+} dimer mode (# 22, blue dots), O_{JT} mode (# 25, orange dots) and both modes together (black dots) to the δ^- structure, respectively. The two modes are applied together based on the calculated anharmonic coupling phase. The blue and orange triangles are the phonon induced transient magnetic moment difference between the two dimer Mn^{3.5+} ions in the Mn^{3.5+} *b* stripe.



Figure S27. The average oscillation amplitude of each ion in δ^- -Na_{1/2}MnO₂ along x, y, and z directions versus temperature based on the DFT calculated phonon spectra.



Figure S28. Neutron diffraction refinements. (top) v^- and v_0^- states at x = 1/2 Na_xMnO₂. (bottom) two-phase diffraction refinements of $\delta^- + v^-$ and $\delta^- + v_0^-$.



Figure S29. One possible Na_{1/2}MnO₂ large supercell that combines blocks of δ^- and mixed v^-/v_0^- regions to better refine SXRD. Mn valence ordering pattern in a larger supercell (green rectangle) with δ^- type and mixed v^-/v_0^- type Na ordering regions alternating along *a* direction and with the cell size doubled along *b* direction. The orange rectangle shows the primitive cell size corresponding to the four DFT low-energy states in Na_{1/2}MnO₂ (Fig. 1b).



Figure 30. XRD simulation and refinement based on an enlarged supercell with blocks of δ^- type and ν^-/ν_0^- type Na orderings alternating along *a* direction and with the cell size doubled along *b* direction. (a) Simulated XRD vs experiment. (b) Rietveld refinement. The thermal displacement parameter B_{eq} in the refinement is fitted within the [-0.5, 0.5] range.