

Bridging Finite and Super Population Causal Inference

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Abstract

There are two general views in causal analysis of experimental data: the super population view that the units are an independent sample from some hypothetical infinite populations, and the finite population view that the potential outcomes of the experimental units are fixed and the randomness comes solely from the physical randomization of the treatment assignment. These two views differs conceptually and mathematically, resulting in different sampling variances of the usual difference-in-means estimator of the average causal effect. Practically, however, these two views result in identical variance estimators. By recalling a variance decomposition and exploiting a completeness-type argument, we establish a connection between these two views in completely randomized experiments. This alternative formulation could serve as a template for bridging finite and super population causal inference in other scenarios.

Keywords: Completeness; Finite population correction; Potential outcome; Simple random sample; Variance of individual causal effects

1. Introduction

Neyman (1923, 1935) defined causal effects in terms of potential outcomes, and proposed an inferential framework viewing all potential outcomes of a finite population as fixed and the treatment assignment as the only source of randomness. This finite population view allows for easy interpretation free of any hypothetical data generating process of the outcomes, and is used in a variety of contexts (e.g., Kempthorne 1952; Hinkelmann and Kempthorne 2008; Copas 1973; Rosenbaum

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2002; Imai 2008; Freedman 2008a,b; Rosenbaum 2010; Aronow and Middleton 2013; Aronow et al. 2014; Abadie et al. 2014; Miratrix et al. 2013; Ding 2016; Lin 2013; Middleton and Aronow 2015; Imbens and Rubin 2015; Chiba 2015; Rigdon and Hudgens 2015; Li and Ding 2016). This approach is considered desirable because, in particular, it does not assume the data is somehow a representative sample of some larger (usually infinite) population.

Alternative approaches, also using the potential outcomes framework, assume that the potential outcomes are independent and identical draws from a hypothetical infinite population. Mathematical derivations under this approach are generally simpler, but the approach itself can be criticized because of this typically untenable sampling assumption. Furthermore, this approach appears to ignore the treatment assignment mechanism.

That being said, it is well known that the final variance formulae from either approach tend to be quite similar. For example, while the variance of the difference-in-means estimator for a treatment-control experiment under an infinite population model is different from the one under Neyman (1923)'s finite population formulation, this difference is easily represented as a function of the variance of the individual causal effects. Furthermore, this difference term is unidentifiable and is often assumed away under a constant causal effect model (Neyman 1923, 1935; Hodges and Lehmann 1964; Rubin 1990; Reichardt and Gollob 1999; Freedman et al. 2007), or by appeals to the final estimators being “conservative.”

Strictly speaking, the infinite population variance estimate gives a conservative (overly large) estimate of the finite population variance for the difference in means. As deriving infinite population variance expressions, relative to finite population variance expressions, tends to be more mathematically straightforward, we might naturally wonder if we could use infinite population expressions as conservative forms of finite population expressions more generally. In this work we show that in fact we can assume an infinite population model as an assumption of convenience, and derive formula from this perspective. This shows that we can thus consider the resulting formula as focused on the treatment assignment mechanism and not on a hypothetical sampling mechanism, i.e., we show variance derivations under the infinite population framework can be used as conservative estimators in a finite context.

Mathematically, this result comes from a variance decomposition and a completeness-style argument that characterizes the connection and the difference between these two views. The vari-

ance decomposition we use has previously appeared in Imai (2008), Imbens and Rubin (2015), and Balzer et al. (2016). The completeness-style argument, which we believe is novel in this domain, then sharpens the variance decomposition by moving from an expression on an overall average relationship to one that holds for any specific sample. Our overall goal is simple: we wish to demonstrate that if one uses variance formula derived from assuming an infinite population sampling model, then the resulting inference one obtains will be correct with regards to the analogous samples-specific treatment effects (although it could be potentially conservative in that the standard errors may be overly large) regardless of the existence of any sampling mechanism.

2. Super Population, Finite Population, and Samples

Assume that random variables $(Y(1), Y(0))$ represent the pair of potential outcomes of an infinite super population, from which we take an independent and identically distributed (IID) finite population of size n :

$$\mathcal{S} = \{(Y_i(1), Y_i(0)) : i = 1, \dots, n\}.$$

We first discuss completely randomized experiments, and comment on other experiments in Section 4. For a completely randomized experiment, we randomly assign n_1 units to receive treatment, leaving the remaining $n_0 = n - n_1$ units to receive control. Let $Z = (Z_1, \dots, Z_n) \in \{0, 1\}^n$ be the treatment assignment vector, which takes a particular value $(z_1, \dots, z_n) \in \{0, 1\}^n$ with probability $n_1!n_0!/n!$, for any $(z_1, \dots, z_n) \in \{0, 1\}^n$ with $\sum_{i=1}^n z_i = n_1$. The observed outcome for unit i is then $Y_i^{\text{obs}} = Z_i Y_i(1) + (1 - Z_i) Y_i(0)$.

At the super population level, the average potential outcomes are $E\{Y(1)\}$ and $E\{Y(0)\}$, and the average causal effect is

$$\tau = E\{Y(1) - Y(0)\}.$$

The population variances of the potential outcomes and individual causal effect are

$$V_1 = \text{Var}\{Y(1)\}, \quad V_0 = \text{Var}\{Y(0)\}, \quad V_\tau = \text{Var}\{Y(1) - Y(0)\}.$$

At the finite population level, i.e., for a fixed sample \mathcal{S} , the average potential outcomes and

average causal effect are

$$\bar{Y}_1 = \frac{1}{n} \sum_{i=1}^n Y_i(1), \quad \bar{Y}_0 = \frac{1}{n} \sum_{i=1}^n Y_i(0), \quad \tau_{\mathcal{S}} = \bar{Y}_1 - \bar{Y}_0.$$

The corresponding finite population variances of the potential outcomes and individual causal effects are

$$S_1^2 = \frac{1}{n-1} \sum_{i=1}^n \{Y_i(1) - \bar{Y}_1\}^2, \quad S_0^2 = \frac{1}{n-1} \sum_{i=1}^n \{Y_i(0) - \bar{Y}_0\}^2, \quad S_\tau^2 = \frac{1}{n-1} \sum_{i=1}^n (\tau_i - \tau)^2,$$

where we use the divisor $n-1$ following the tradition of survey sampling (Cochran 1977). All these quantities are fully dependent on \mathcal{S} . In classical casual inference (Neyman 1923), the potential outcomes of these n experimental units, \mathcal{S} , are treated as fixed numbers. Equivalently, we can consider such causal inference to be conducted conditional on \mathcal{S} (e.g., Copas 1973; Rosenbaum 2002; Imbens and Rubin 2015; Rigdon and Hudgens 2015; Chiba 2015).

Regardless, we have two parameters—the population average treatment effect τ , and the sample average treatment effect $\tau_{\mathcal{S}}$. After collecting the data, we would want to draw inference about τ or $\tau_{\mathcal{S}}$.

Our primary statistics are the averages of the observed outcomes and the difference-in-means estimator:

$$\bar{Y}_1^{\text{obs}} = \frac{1}{n_1} \sum_{i=1}^n Z_i Y_i^{\text{obs}}, \quad \bar{Y}_0^{\text{obs}} = \frac{1}{n_0} \sum_{i=1}^n (1 - Z_i) Y_i^{\text{obs}}, \quad \hat{\tau} = \bar{Y}_1^{\text{obs}} - \bar{Y}_0^{\text{obs}}.$$

We also observe the sample variances of the outcomes under treatment and control using

$$s_1^2 = \frac{1}{n_1-1} \sum_{i=1}^n Z_i (Y_i^{\text{obs}} - \bar{Y}_1^{\text{obs}})^2, \quad s_0^2 = \frac{1}{n_0-1} \sum_{i=1}^n (1 - Z_i) (Y_i^{\text{obs}} - \bar{Y}_0^{\text{obs}})^2.$$

We do not have the sample analogue of S_τ^2 or V_τ because $Y_i(1)$ and $Y_i(0)$ are never jointly observed for any unit i in the sample.

We summarize the infinite population, finite population and sample quantities in Table 1.

Table 1: Means and variances at the super population, finite population and sample levels.

	means			variances		
	treatment	control	effect	treatment	control	effect
super population	$E\{Y(1)\}$	$E\{Y(0)\}$	$\tau = E\{Y(1) - Y(0)\}$	V_1	V_0	V_τ
finite population	\bar{Y}_1	\bar{Y}_0	$\tau_S = \bar{Y}_1 - \bar{Y}_0$	S_1^2	S_0^2	S_τ^2
sample	\bar{Y}_1^{obs}	\bar{Y}_0^{obs}	$\hat{\tau} = \bar{Y}_1^{\text{obs}} - \bar{Y}_0^{\text{obs}}$	s_1^2	s_0^2	—

3. Deriving Complete Randomization Results with an Independent Sampling Model

The three levels of quantities in Table 1 are connected via independent sampling and complete randomization. Neyman (1923), without reference to any infinite population and by using the assignment mechanism as the only source of randomness, represented the assignment mechanism via an urn model, and found

$$\text{Var}(\hat{\tau} \mid \mathcal{S}) = \frac{S_1^2}{n_1} + \frac{S_0^2}{n_0} - \frac{S_\tau^2}{n}. \quad (1)$$

He then observed that the final term was unidentifiable but nonnegative, and thus if we dropped it we would obtain an upper bound of the estimator’s uncertainty.

We next derive this result by assuming a hypothetical sampling mechanism from some assumed infinite super-population model of convenience. This alternative derivation of the above result, which can be extended to other assignment mechanisms, shows how we can interpret formulae based on super-population derivations as conservative formulae for finite-sample inference.

3.1. Sampling and randomization

To begin, note that IID sampling of \mathcal{S} from the super population implies three things: first, the finite population average causal effect τ_S satisfies $E(\tau_S) = \tau$, second

$$\text{Var}(\tau_S) = \text{Var} \left[\frac{1}{n} \sum_{i=1}^n \{Y_i(1) - Y_i(0)\} \right] = \frac{V_\tau}{n}, \quad (2)$$

and third, the sample variances are unbiased for the true variances:

$$E(S_1^2) = V_1, \quad E(S_0^2) = V_0, \quad E(S_\tau^2) = V_\tau. \quad (3)$$

Conditional on \mathcal{S} , randomization of the treatment Z is the only source of randomness. In a completely randomized experiment, the outcomes in the treatment group form a simple random sample of size n_1 from $\{Y_i(1) : i = 1, \dots, n\}$, and the outcomes in the control group form a simple random sample of size n_0 from $\{Y_i(0) : i = 1, \dots, n\}$. Therefore, classical survey sampling theory (Cochran 1977) for the sample mean and variance gives

$$E(\bar{Y}_1^{\text{obs}} | \mathcal{S}) = \bar{Y}_1, \quad E(\bar{Y}_0^{\text{obs}} | \mathcal{S}) = \bar{Y}_0, \quad E(\hat{\tau} | \mathcal{S}) = \tau_{\mathcal{S}}, \quad E(s_1^2 | \mathcal{S}) = S_1^2, \quad E(s_0^2 | \mathcal{S}) = S_0^2. \quad (4)$$

We do not use the notation $E_{\mathcal{S}}(\cdot)$, because, depending on the contexts, such notation could either indicate expectation conditional on \mathcal{S} or expectation averaged over \mathcal{S} . We therefore use conditional expectations and conditional variances explicitly.

If we do *not* condition on \mathcal{S} , then the independence induced by the assignment mechanism means the outcomes under treatment are IID samples of $Y(1)$ and the outcomes under control are IID samples of $Y(0)$, and furthermore these samples are independent of each other. This independence makes it straightforward to show that $\hat{\tau}$ is unbiased for τ with super population variance

$$\text{Var}(\hat{\tau}) = \text{Var}(\bar{Y}_1^{\text{obs}}) + \text{Var}(\bar{Y}_0^{\text{obs}}) = \frac{V_1}{n_1} + \frac{V_0}{n_0}. \quad (5)$$

This is the classic infinite population variance formula for the two sample difference-in-means statistic. We could use it to obtain standard errors by plugging in s_1^2 and s_0^2 for the two variances. This derivation, from (2)–(5), is motivated by the sampling assumption: the assignment mechanism makes the two samples independent, but it is the IID assumption and classic sampling theory that gives this result (along with asymptotical normality which allows for associated testing and confidence interval generation).

3.2. Connecting the finite and infinite population inference with a variance decomposition

We will now extend the above to indirectly derive the result on the variance of $\hat{\tau}$ in finite population inference without explicitly enumerating the potential outcomes. The variance decomposition formula implies

$$\text{Var}(\hat{\tau}) = E\{\text{Var}(\hat{\tau} | \mathcal{S})\} + \text{Var}\{E(\hat{\tau} | \mathcal{S})\} \quad (6)$$

$$\begin{aligned} &= E\{\text{Var}(\hat{\tau} | \mathcal{S})\} + \text{Var}(\tau_{\mathcal{S}}) \\ &= E\{\text{Var}(\hat{\tau} | \mathcal{S})\} + \frac{V_{\tau}}{n}, \end{aligned} \quad (7)$$

which further implies that the finite population variance of $\hat{\tau}$ satisfies (using (3))

$$E\{\text{Var}(\hat{\tau} | \mathcal{S})\} = \text{Var}(\hat{\tau}) - \frac{V_{\tau}}{n} = \frac{V_1}{n_1} + \frac{V_0}{n_0} - \frac{V_{\tau}}{n} = E\left\{\frac{S_1^2}{n_1} + \frac{S_0^2}{n_0} - \frac{S_{\tau}^2}{n}\right\}. \quad (8)$$

Compare to the classic variance expression (1), which is this without the expectation. Here we have that *on average* our classic variance expression holds. Now, because this is true for any infinite population, as it is purely a consequence of the IID sampling mechanism and complete randomization, we can close the gap between (1) and (8). Informally speaking, because (8) holds as an average over many hypothetical super populations, it should also hold for any finite population at hand, and indeed it does, as we next show using a “completeness” concept from statistics (Lehmann and Romano 2008).

3.3. A “Completeness” argument

First, define

$$f(\mathcal{S}) \equiv \frac{S_1^2}{n_1} + \frac{S_0^2}{n_0} - \frac{S_{\tau}^2}{n} - \text{Var}(\hat{\tau} | \mathcal{S}), \quad (9)$$

a function of a fixed finite sample \mathcal{S} , as the difference of the hypothesized finite sample variance formula and the actual finite sample variance. Formula (8) shows $E\{f(\mathcal{S})\} = 0$. Now we are going to show the stronger result $f(\mathcal{S}) = 0$.

For any given sample \mathcal{S} , we have fixed sample quantities: $U = (Y_1(1) - \bar{Y}_1, \dots, Y_n(1) - \bar{Y}_1)$, and $W = (Y_1(0) - \bar{Y}_0, \dots, Y_n(0) - \bar{Y}_0)$. Some algebra gives

$$\hat{\tau} - \tau_{\mathcal{S}} = \frac{1}{n_1} \sum_{i=1}^n Z_i U_i - \frac{1}{n_0} \sum_{i=1}^n (1 - Z_i) W_i. \quad (10)$$

According to (10), the finite population variance $\text{Var}(\hat{\tau} \mid \mathcal{S}) = E\{(\hat{\tau} - \tau_{\mathcal{S}})^2 \mid \mathcal{S}\}$ must be a quadratic form of (U, W) . Therefore, from (9), $f(\mathcal{S})$ must be a quadratic form of (U, W) , and therefore must have the following expression:

$$f(\mathcal{S}) = \sum_{i=1}^n \sum_{j=1}^n a_{ij} U_i U_j + \sum_{i=1}^n \sum_{j=1}^n b_{ij} W_i W_j + \sum_{i=1}^n \sum_{j=1}^n c_{ij} U_i W_j.$$

for some a_{ij} , b_{ij} and c_{ij} . $f(\mathcal{S})$ is the same regardless of the ordering of units, so by symmetry the constants a_{ij} , b_{ij} and c_{ij} must be the same regardless of i and j . This gives $a_{ii} = a_{11}$, $a_{ij} = a_{12}$, $b_{ii} = b_{11}$, $b_{ij} = b_{12}$, $c_{ii} = c_{11}$, $c_{ij} = c_{12}$, for all $i \neq j$. Thus

$$f(\mathcal{S}) = a_{11} \sum_{i=1}^n U_i^2 + a_{12} \sum_{i \neq j} U_i U_j + b_{11} \sum_{i=1}^n W_i^2 + b_{12} \sum_{i \neq j} W_i W_j + c_{11} \sum_{i=1}^n U_i W_i + c_{12} \sum_{i \neq j} U_i W_j.$$

Because $\sum_{i=1}^n U_i = 0$, we have

$$\sum_{i=1}^n U_i^2 + \sum_{i \neq j} U_i U_j = \left(\sum_{i=1}^n U_i \right)^2 = 0.$$

Similarly, as $\sum_{i=1}^n W_i = 0$, we have

$$\sum_{i=1}^n W_i^2 + \sum_{i \neq j} W_i W_j = 0, \quad \sum_{i=1}^n U_i W_i + \sum_{i \neq j} U_i W_j = \left(\sum_{i=1}^n U_i \right) \left(\sum_{i=1}^n W_i \right) = 0.$$

Use the above to replace our cross terms of $i \neq j$ with single index terms to write $f(\mathcal{S})$ as

$$f(\mathcal{S}) = a \sum_{i=1}^n U_i^2 + b \sum_{i=1}^n W_i^2 + c \sum_{i=1}^n U_i W_i,$$

where $a = a_{11} - a_{12}$, $b = b_{11} - b_{12}$, and $c = c_{11} - c_{12}$. Because $E\{f(\mathcal{S})\} = 0$, we have $aE(U_1^2) +$

$aE(W_1^2) + cE(U_1W_1) = 0$, where

$$E(U_1^2) = \frac{n-1}{n}\text{Var}\{Y(1)\}, \quad E(W_1^2) = \frac{n-1}{n}\text{Var}\{Y(0)\}, \quad E(U_1W_1)^2 = \frac{n-1}{n}\text{Cov}\{Y(1), Y(0)\}.$$

Thus,

$$a\text{Var}\{Y(1)\} + b\text{Var}\{Y(0)\} + c\text{Cov}\{Y(1), Y(0)\} = 0. \quad (11)$$

Because (11) holds for any populations regardless of its values of $\text{Var}\{Y(1)\}$, $\text{Var}\{Y(0)\}$ and $\text{Cov}\{Y(1), Y(0)\}$, it must be true that $a = b = c = 0$, which implies $f(\mathcal{S}) = 0$.

4. Discussion

Equation (8) relies on the assumption that the hypothetical infinite population exists, but Equation (1) does not. However, the completeness-style argument allowed us to make our sampling assumption only for convenience in order to prove (1) by, in effect, dropping the expectation on both sides of (8). Similar argument exists in the classical statistics literature; see Efron and Morris (1973) for the empirical Bayes view of Stein’s estimator. While the final result is, of course, not new, we offer it as it gives an alternative derivation that does not rely on asymptotics such as a growing superpopulation or a focus on the properties of the treatment assignment mechanism.

Using Freedman (2008a)’s results, Aronow et al. (2014) considered a super population with N units, with the finite population being a simple random sample of size n . Letting $N \rightarrow \infty$, we can obtain similar results. We go in the other direction: we use the variance decomposition (6) to derive the finite population variance from the super population one.

This decomposition approach also holds for other types of experiments. First, for a stratified experiment, each stratum is essentially a completely randomized experiment. Apply the result to each stratum, and then average over all strata to obtain results for a stratified experiment. Second, because a matched-pair experiment is a special case of a stratified experiment with two units within each stratum, we can derive the Neyman-type variance (cf. Imai 2008; Imbens and Rubin 2015) directly from that of a stratified experiment. Third, a cluster-randomized experiment is a completely randomized experiment on the clusters. If the causal parameters can be expressed as

cluster-level outcomes, then the result can be straightforwardly applied (cf. Aronow and Middleton 2013; Middleton and Aronow 2015). Fourth, for general experimental designs, the variance decomposition in (6) still holds, and therefore we can modify the derivation of the finite population variance according to different forms of (4) and (5).

In a completely randomized experiment, the finite population sampling variance of $\hat{\tau}$ in (1) depends on three terms: the first two can be unbiasedly estimated by s_1^2/n_1 and s_0^2/n_0 , but the third term S_τ^2/n is unidentifiable from the data. Assuming a constant causal effect model, $S_\tau^2 = 0$ and the variance estimators under both finite and super population inference coincide. However, Robins (1988), Aronow et al. (2014) and Ding and Dasgupta (2016) demonstrated that the treatment variation term S_τ^2 has a sharp lower bound that may be larger than 0, which allows for more precise variance estimators under the finite population view. This demonstrates that we can indeed make better inference conditional on the sample we have. On the other hand, in this work, we showed that assuming an infinite population, while not necessarily giving the tightest variance expressions, nonetheless gives valid (conservative) variance expressions from a finite-population perspective. We offer this approach as a possible method of proof that could ease derivations for more complex designs. More broadly, it is a step towards establishing that infinite population derivations for randomized experiments can be generally thought of as pertaining to their finite population analogs. Also see Samii and Aronow (2012) and Lin (2013), who provide alternative discussions of super population regression-based variance estimators under the finite population framework.

Our discussion is based on the frequentists' repeated sampling evaluations of the difference-in-means estimator for the average causal effect. In contrast, Fisher (1935) proposed the randomization test against the sharp null hypothesis that $Y_i(1) = Y_i(0)$ for all units, which is numerically the same as the permutation test for exchangeable units sampled from an infinite population (Lehmann and Romano 2008). This connection becomes more apparent when only the ranks of the outcomes are used to construct the test statistics, as discussed extensively by Lehmann (1975).

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References

- Abadie, A., Athey, S., Imbens, G. W., and Wooldridge, J. M. (2014). Finite population causal standard errors. Technical report, National Bureau of Economic Research.
- Aronow, P. M., Green, D. P., and Lee, D. K. K. (2014). Sharp bounds on the variance in randomized experiments. *The Annals of Statistics*, 42:850–871.
- Aronow, P. M. and Middleton, J. A. (2013). A class of unbiased estimators of the average treatment effect in randomized experiments. *Journal of Causal Inference*, 1:135–154.
- Balzer, L. B., Petersen, M. L., and Laan, M. J. (2016). Targeted estimation and inference for the sample average treatment effect in trials with and without pair-matching. *Statistics in Medicine*, 35:3717–3732.
- Chiba, Y. (2015). Exact tests for the weak causal null hypothesis on a binary outcome in randomized trials. *Journal of Biometrics and Biostatistics*, 6:doi:10.4172/2155–6180.1000244.
- Cochran, W. G. (1977). *Sampling Techniques*. John Wiley & Sons, 3rd edition.
- Copas, J. (1973). Randomization models for the matched and unmatched 2×2 tables. *Biometrika*, 60:467–476.
- Ding, P. (2016). A paradox from randomization-based causal inference (with discussion). *Statistical Science*, in press.
- Ding, P. and Dasgupta, T. (2016). A potential tale of two by two tables from completely randomized experiments. *Journal of American Statistical Association*, 111:157–168.
- Efron, B. and Morris, C. (1973). Stein’s estimation rule and its competitors—an empirical bayes approach. *Journal of the American Statistical Association*, 68:117–130.
- Fisher, R. A. (1935). *The Design of Experiments*. Edinburgh, London: Oliver and Boyd, 1st edition.
- Freedman, D., Pisani, R., and Purves, R. (2007). *Statistics*. W. W. Norton & Company, 4th edition.
- Freedman, D. A. (2008a). On regression adjustments in experiments with several treatments. *The Annals of Applied Statistics*, 2:176–196.

- Freedman, D. A. (2008b). Randomization does not justify logistic regression. *Statistical Science*, 23:237–249.
- Hinkelmann, K. and Kempthorne, O. (2008). *Design and Analysis of Experiments, Volume 1: Introduction to Experimental Design*. New Jersey: John Wiley & Sons, Inc., 2nd edition.
- Hodges, J. L. J. and Lehmann, E. L. (1964). *Basic Concepts of Probability and Statistics*. San Francisco: Holden-Day.
- Imai, K. (2008). Variance identification and efficiency analysis in randomized experiments under the matched-pair design. *Statistics in Medicine*, 27:4857–4873.
- Imbens, G. W. and Rubin, D. B. (2015). *Causal Inference for Statistics, Social and Biometrical Sciences: An Introduction*. Cambridge: Cambridge University Press.
- Kempthorne, O. (1952). *The Design and Analysis of Experiments*. New York: John Wiley and Sons.
- Lehmann, E. L. (1975). *Nonparametrics: Statistical Methods Based on Ranks*. San Francisco: Holden-Day, 1st edition.
- Lehmann, E. L. and Romano, J. P. (2008). *Testing Statistical Hypotheses*. New York : Wiley, 3rd edition.
- Li, X. and Ding, P. (2016). Exact confidence intervals for the average causal effect on a binary outcome. *Statistics in Medicine*, 35:957–960.
- Lin, W. (2013). Agnostic notes on regression adjustments to experimental data: Reexamining Freedman’s critique. *The Annals of Applied Statistics*, 7:295–318.
- Middleton, J. A. and Aronow, P. M. (2015). Unbiased estimation of the average treatment effect in cluster-randomized experiments. *Statistics, Politics and Policy*, 6:39–75.
- Miratrix, L. W., Sekhon, J. S., and Yu, B. (2013). Adjusting treatment effect estimates by post-stratification in randomized experiments. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 75:369–396.

- Neyman, J. (1923). On the application of probability theory to agricultural experiments. essay on principles (with discussion). section 9 (translated). reprinted ed. *Statistical Science*, 5:465–472.
- Neyman, J. (1935). Statistical problems in agricultural experimentation (with discussion). *Supplement to the Journal of the Royal Statistical Society*, 2:107–180.
- Reichardt, C. S. and Gollob, H. F. (1999). Justifying the use and increasing the power of a t test for a randomized experiment with a convenience sample. *Psychological Methods*, 4:117–128.
- Rigdon, J. and Hudgens, M. G. (2015). Randomization inference for treatment effects on a binary outcome. *Statistics in Medicine*, 34:924–935.
- Robins, J. M. (1988). Confidence intervals for causal parameters. *Statistics in Medicine*, 7:773–785.
- Rosenbaum, P. R. (2002). *Observational Studies*. New York: Springer, 2nd edition.
- Rosenbaum, P. R. (2010). *Design of Observational Studies*. New York: Springer.
- Rubin, D. B. (1990). Comment: Neyman (1923) and causal inference in experiments and observational studies. *Statistical Science*, 5:472–480.
- Samii, C. and Aronow, P. M. (2012). On equivalencies between design-based and regression-based variance estimators for randomized experiments. *Statistics and Probability Letters*, 82:365–370.