The sub-optimality of optimal household tax schedules.*

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Abstract

In this paper, we derive optimal tax formulae for a Nash-bargaining collective household model and show how tax schedules play a dual role: defining households' objective function and their budget sets. By targeting threat points, filing options can improve upon a single tax schedule. We calibrate our model to the U.S. economy and evaluate the consequence of eliminating the option to file individually; we find welfare impacts one third as large as the ones from a substantial reform that eliminates joint filing. Finally, we apply mechanism design to our collective household framework, characterize the set of implementable allocations using the revelation principle, generalize our optimal taxation results to arbitrary tax schedules and show conditions on allocations that guarantee implementation with a fall-back schedule as a filing option to a principal schedule. ***Keywords:*** *Optimal Taxation; Collective Households; Nash bargaining; Taxation Principle.* ***JEL Codes:*** *D13; H21; H31.*

ALMOST everything we know about optimal distributive policy has implicitly assumed single-person households or households that behave like a single person, the so-called ***unitary*** approach. Unfortunately, not only is this assumption rejected by mounting empirical evidence—see the references in Browning and Chiappori (1998)—but it also precludes the discussion of important policy questions like intra-household distribution and household formation. Our goal is to bring optimal

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tax theory up to date with recent advances in Family Economics. We extend some of the most central results from optimal tax theory to a ***collective*** household model with endogenous marriage.

In our model, spouses reach efficient decisions following a Nash-bargaining protocol, as in Manser and Brown (1980) and McElroy and Horney (1981). An essential input to the Nash bargaining is the determination of threat points; we assume ***internal*** within-marriage threat points given by ***disagreement*** utilities obtained if spouses act non-cooperatively, as in Lundberg and Pollak (1993). These within-marriage channels can explain the rationale behind policies that explicitly target at women.[1] In the marriage stage, people anticipate their married utilities and observe non-material payoffs for each match. The marriage market then defines a matching problem with non-transferable utility. Because of our marriage market assumption, we can break policy analysis into two stages: the impact on utilities after marriage and the new utilities' impact on the marriage market. This separation simplifies policy analysis with endogenous marriage.[2]

Our first contribution is to derive an optimal tax formula for ***income-splitting*** joint tax schedules which considers that taxes may affect threat points. We use a tax perturbation approach that has become standard for optimal tax characterization following Piketty (1997), Dahlby (1998), and Saez (2001). When taxes *do not* affect threat points, our optimal formula highlights the ***dissonance*** between actual power distribution within the couple and society's preferred power distribution. When taxes *do* affect threat points, our optimal formula includes novel non-local terms because disagreement earnings can differ from observed earnings. In contrast, standard formulae only allow tax perturbations to affect two (local) channels: the marginal retention rate and the virtual income.[3]

Our second contribution is to show that the ***taxation principle*** fails when threat points are endogenous to policy. In particular, the joint tax schedule does not affect threat points if spouses can file taxes separately, and the optimal tax formula does not include the non-local terms.[4] Moreover, if these non-local terms are positive, then a ***fall-back*** schedule used as a filing option strictly improves a tax system without filing options. There are two important consequences for the optimal taxation literature: First, the set of tax instruments affects whether we should rely upon unitary or collective models; more precisely, it depends if this set of instruments can or cannot keep threat points fixed. Second, the idea that elasticities used in tax

[1] E.g., *Bolsa Família* in Brazil and *Oportunidades* in Mexico.

[2] For a discussion and alternatives, see Browning et al. (2014) and Pollak (2019).

[3] Similar considerations appear when the government taxes behavioral agents, see Farhi and Gabaix (2020).

[4] Tax systems in the U.S., Germany, and Ireland offer the option to file tax separately, for example.

formulae should be optimally chosen becomes all the more important; by targeting threat points, the planner can affect these elasticities and change optimal tax prescriptions.[5]

Our third contribution is to evaluate the welfare impact of removing filing options from the U.S. tax system. With parametric utilities and tax schedules, we compare removing filing options with mandatory individual taxation. For most families, their tax liabilities are lower with joint taxation, and a unitary model would predict removing filing options to have little welfare impact. In contrast, the welfare impact of removing the option to file separately, both ex-ante and ex-post the marriage market, is about a third as large as the impact of mandatory individual taxation in our model. We also explore how gender-based taxation is affected by exogenous or endogenous threat points. Exogenous threat points imply the counter-intuitive result that lowering women's marginal tax rate decreases their utility in marriage, whereas endogenous threat points imply the opposite.[6]

Our final contribution is an extension for collective households of Mirrlees's mechanism design framework. We show that the ***revelation principle*** survives if threat points are solely a function of household transactions in disagreement. We use the revelation principle to show three facts: *First*, the taxation principle's failure for income-splitting schedules generalizes to an arbitrary schedule. *Second*, if auxiliary instruments can hold threat points fixed, we recover a *conditional* taxation principle. A tax schedule is as powerful as a direct mechanism with the same threat points; a significant implication is that any improvement over a tax schedule is through instruments that better target threat points. *Third*, filing options expand implementation for tax systems but *do not* implement all incentive-compatible allocations. We characterize which allocations a tax system with filing options can implement.

We know of no other use of mechanism design with a collective model and view the framework developed in this paper as a relevant methodological contribution. It is also worth mentioning that our framework may also apply to other cooperative relationships characterized by the efficient allocation of resources, e.g., micro-credit programs, partnerships, joint ventures, and collusive firms.[7]

Section 1 describes the economy, the household behavior in agreement and disagreement, and the marriage market. Section 2 derives optimal tax formulae for exogenous and endogenous threat points and show the role of filing options to tar-

[5]For a similar consideration in different contexts, see Kopczuk and Slemrod (2002) and Moore and Slemrod (2020).

[6]This counter-intuitive result holds for more general unitary models—see Alves et al. (2018)—which explains the findings in Cremer et al. (2016).

[7]We thank an anonymous referee for suggesting this observation.

get threat points. In Section 3, the quantitative relevance of Section 2 is assessed. Section 4 develops a general collective framework and applies a mechanism design approach to extend the previous findings; we relegate proofs in Section 4 to Appendix A.5.

Literature Review

The literature on optimal taxation of multi-person households started with Boskin and Sheshinski (1983). They considered linear parametric taxes applied to a unitary household and found optimal taxes lower for wives than for husbands because women have larger labor supply elasticities. As it used to be typical, Boskin and Sheshinski assumed that the household's choices have normative meaning. Apps and Rees (1988) were the first to point out that the household welfare function, which rationalizes the household's choices, may have no normative meaning because the society ultimately cares about the spouses' utilities. They called this distinction dissonance.

One branch from the literature has kept the assumption of unitary households and has extended characterization to optimal non-linear taxes—e.g., Kleven et al. (2009), Immervoll et al. (2011), Cremer et al. (2016), and Alves et al. (2018). They employed different assumptions to overcome the challenge of multi-dimensional types in a multi-person household: Kleven et al. (2009) assumed that primary earners decide at the intensive margin and that secondary earners decide at the extensive margin. Immervoll et al. (2011) only considered extensive margin decisions. Cremer et al. (2016) used a stylized model with finite types. Alves et al. (2018) considered only income-splitting taxation and identical iso-elastic preferences.

Another branch has abandoned the unitary framework and has considered the policies' impact on the spouses' power balance. To the best of our knowledge, Alesina et al. (2011), Bastani (2013) and Gayle and Shephard (2019) have been the only attempts to characterize optimal taxes for collective households—as defined by Browning et al. (2006). Alesina et al. (2011) and Bastani (2013) used a Nash-bargaining household model with single's utilities as threat points. Both assumed quasi-linear utilities and linear tax schedules, and they abstracted from marriage considerations. Furthermore, Alesina et al. (2011) did not consider heterogeneity in productivity, while Bastani (2013) assumed spouses in a couple are equally productive.[8] In contrast, Gayle and Shephard (2019) assumed a spouse's power within a couple to be the "price" that clears the marriage market. In their model, spouses can credibly promise to forgo power after-marriage to get a better match.

[8] Bastani (2013), in his quantitative analysis, relaxed the quasi-linearity assumption and considered internal threat points.

An essential feature of our approach is internal threat points: the utilities attained if spouses act non-cooperatively. The family economics literature has emphasized the assumption of internal threat points—e.g., Lundberg and Pollak (1993) and Chen and Wooley (2001)—partly because many sources have suggested its relevance: First, some programs designed to target within-marriage power balance—e.g., *Bolsa Família* in Brazil and *Oportunidades* in Mexico, see Chiappori and Mazzocco (2017). Second, a growing empirical literature—e.g. Mazzocco (2007); Armand et al. (2020). Third, both theoretical and experimental findings in bargaining games—e.g., Bergstrom (1996), Binmore (1985) and Binmore et al. (1989).

With internal threat point, we are in a bargaining-in-marriage approach (Pollak, 2019), which provides a very tractable model for policy evaluation. We leverage this tractability to derive optimal tax formulae comparable to those in Diamond (1998), Saez (2001), and Jacquet et al. (2013), thus going beyond numerical results—e.g., Gayle and Shephard (2019)—and linear taxes—e.g., Alesina et al. (2011); Bastani (2013).

As far as we know, our work is the first to apply a mechanism design approach to a collective model. This contribution allows us to discuss how auxiliary non-rate policies impact allocations, something about which the literature was silent.[9]

1 Environment

The economy is inhabited by a continuum of people of two genders, $i = f, m$. They derive utility from two goods: consumption, c, and leisure, l. We denote their (continuous, increasing, and strictly quasi-concave) utility function by $u_i(c, l)$. The utility function may depend on gender, but it is otherwise identical across people; however, people can have different labor market productivity θ_i, which belongs to a finite set Θ.[10] A ***person*** is defined by his/her productivity θ_i and gender i. Two people of different genders can form a ***couple***, and we represent couples by an ordered pair of productivities, $\boldsymbol{\theta} = (\theta_f, \theta_m)$.[11]

A person's life has two stages. The first stage is the ***marriage*** market, which defines distributions of couples, μ, and singles of each gender, μ_f^{s} and μ_m^{s}, as a match-

[9]We use the term non-rate policy—as in Moore and Slemrod (2020)—to represent any tax system aspect beyond the tax schedule's shape. In particular, filing options are non-rate instruments. Their effect depends on who is entitled to choose which tax schedule applies.

[10]Except for the existence of a stable marriage-market matching in Subsection 1.2 and the quantitative analysis, Section 3, this assumption is for simplicity only.

[11]Except for the existence of a stable marriage-market matching in Subsection 1.2, this assumption is for simplicity only. We could define couples as an *unordered* pair of productivities and ignore gender. However, since one motivation for our analysis is gender-based policies, the simplifying assumption of spouses with different genders streamlines the presentation.

ing equilibrium. We detail this marriage process in Subsection 1.2. The second stage in a person's life is the ***transaction*** market, in which a person decides how much to earn, z, and how much consumption good, y, to purchase. We denote a person's transaction by $x := (y, -z)$, which belongs to a (closed, convex) set X.

Every person has a time endowment of 1. For singles, a transaction $x = (y, -z)$ affords a consumption/leisure bundle, (c, l), such that $(c, l) \leq (y, 1 - z/\theta_i)$. For couples, however, transactions $\boldsymbol{x} = (x_f, x_m)$ do not map directly to consumption/leisure bundles because spouses can reallocate consumption between themselves. That is, the consumption/leisure bundle for a couple satisfies

$$c_f + c_m \leq \alpha(y_f + y_m) \quad \text{and} \quad (l_f, l_m) \leq \left(1 - \frac{z_f}{\theta_f}, 1 - \frac{z_m}{\theta_m}\right),$$

where α represents consumption gains of scale for couples in agreement. The set of possible transactions for couples is denoted by $\mathbf{B} \subset X^2$. In Section 2, we show how a tax system defines $\mathbf{B}$ as a budget set. Finally, since people in a couple have conflicting preferences, we assume that spouses bargain over transactions and consumption division. We detail this bargaining process in Subsection 1.1.

We consider two sources of ***informational asymmetry***. First, productivity is private information. A single person is the only one to know his/her productivity, and spouses in a couple are the only ones to know each other productivity. Second, except for the spouses, no one knows how consumption is allocated within the couple. In contrast, transactions can be externally observed and, importantly, taxed.

The ***feasible allocations*** in the economy are represented by a transformation function G, which maps feasible allocations to (weakly) negative numbers and not-feasible ones to (strictly) positive numbers.

1.1 Couples: Conflict and Cooperation

To understand how a couple chooses transactions and consumption, we model the couple's decision as a Nash bargaining. We first discuss how a non-cooperative game between the spouses define the internal threat points. Then we formally specify the household welfare function used to choose transactions and within-couple consumption.

Choices in disagreement. Following Lundberg and Pollak (1993), we use as threat points the utilities from a non-cooperative game that arises as an agreement eludes spouses. Each spouse independently chooses his/her transactions, $\bar{x}_i = (\bar{y}_i, \bar{z}_i)$, to maximize utility. The disagreement transactions are the equilibrium of this game. In the following sections, we provide examples of disagreement games, but what

is relevant for now is that the equilibrium disagreement transactions, $\bar{\boldsymbol{x}} = (\bar{x}_f, \bar{x}_m)$, imply a pair of disagreement utilities $\bar{\boldsymbol{u}}(\boldsymbol{\theta}) = (\bar{u}_f(\boldsymbol{\theta}), \bar{u}_m(\boldsymbol{\theta}))$, which will be used as threat points for the cooperative Nash bargaining.

Choices in agreement. Given the disagreement utilities $\bar{\boldsymbol{u}}(\boldsymbol{\theta})$ as threat points, a couple $\boldsymbol{\theta}$ chooses transactions in its budget set, $\mathbf{B}$, and consumptions to maximize

$$\max_{\boldsymbol{x}, c_f, c_m} \left[u_f\left(c_f, 1 - \frac{z_f}{\theta_f}\right) - \bar{u}_f(\boldsymbol{\theta}) \right] \left[u_m\left(c_m, 1 - \frac{z_m}{\theta_m}\right) - \bar{u}_m(\boldsymbol{\theta}) \right]$$

subject to $c_f + c_m \leq \alpha(y_f + y_m)$ and $\boldsymbol{x} = (y_f, z_f; y_m, z_m) \in \mathbf{B}$.[12]

The Nash bargaining solution is always efficient, consistent with the assumptions of cooperation and symmetric information within a couple.[13] For a couple $\boldsymbol{\theta}$, the equilibrium transactions and utilities are denoted by $\boldsymbol{x}(\boldsymbol{\theta})$ and $\boldsymbol{u}(\boldsymbol{\theta})$.

1.2 Marriage Markets: Who Marries Whom?

To understand how policy affects the marriage-market distributions $(\mu, \mu_f^{\mathrm{s}}, \mu_m^{\mathrm{s}})$, we assume that people anticipate their utilities from marrying or staying single. However, ***material utilities*** $\boldsymbol{u}(\boldsymbol{\theta})$ are not the only relevant payoff at the marriage stage: we also consider an ***emotional payoff*** $\boldsymbol{\xi}(\boldsymbol{\theta})$. Furthermore, we introduce a stochastic ***courtship shock*** $\boldsymbol{\epsilon}(\theta_i)$ to allow people with the same gender and productivity to prefer different matches. More precisely, a person, say a woman with productivity θ_f, ranks her match with a man θ_m by

$$v_f(\theta_f, \theta_m, \boldsymbol{\epsilon}) := u_f(\theta_f, \theta_m) + \xi_f(\theta_f, \theta_m) + \epsilon(\theta_m).$$

By comparing her ranking of different men and her utility as single, she defines an ordering $\succeq$ of matches. For this section, we denote a θ_i-person and his/her ordering by $\vartheta_i := (\theta_i, \succeq)$. Because ϵ is random, even people with the same productivities may have different orderings. We denote the distribution of person-orderings for gender i as $\pi_i(\vartheta_i)$.

[12] It is well known that this functional form follows from a series of axioms in Nash (1950) if the set of possible utilities is convex. The budget set $\mathbf{B}$ may generate non-convex utility sets, so we formally use the axioms from Zambrano (2016). He replaces Nash's *symmetry* axiom by a *preference for symmetry* axiom to handle non-convex utility possibility sets.

[13] Under special conditions, efficiency can also arise in non-cooperative household models, see Becker (1991) and Grossbard-Shechtman (1993). More generally, showing that Nash's (1950) axiomatic solution can arise in a non-cooperative game defines the so-called "Nash Program." It is within this context that the theoretical case for internal threat points is made (Binmore, 1985; Binmore et al., 1989; Bergstrom, 1996).

We assume that people cannot commit to after-marriage transfers; therefore, the marriage market solves a marriage problem with non–transferable utility (Gale and Shapley, 1962). The only difference from the traditional setting—Roth and Sotomayor (1990)—is that we have a continuum of people so a stable matching ν is defined by a (probability) coupling between π_f and π_m such that

$$\sum_{\vartheta_m} \nu(\vartheta_f, \vartheta_m) \leq \pi_f(\vartheta_f) \text{ for all } \vartheta_f, \quad \text{and} \quad \sum_{\vartheta_f} \nu(\vartheta_f, \vartheta_m) \leq \pi_m(\vartheta_m) \text{ for all } \vartheta_m;$$

and such that all matches are stable: no married person would rather be single, and no two people would prefer to end their marriage (or singleness) to marry each other.[14]

The mass of single women ϑ_f is given by $\pi_f(\vartheta_f) - \sum_{\vartheta_m} \nu(\vartheta_f, \vartheta_m)$, and the mass of single men ϑ_m is given by $\pi_m(\vartheta_m) - \sum_{\vartheta_f} \nu(\vartheta_f, \vartheta_m)$. Given a matching ν for person-orderings, we ignore the ordering information and get distributions of couples, μ, and singles, μ_f^{s} and μ_m^{s}.

2 Optimal Taxation

Now that we have described our economic environment, we can derive optimal tax formulae for households when taxes affect threat points, and when they do not. We first describe, through some simple examples, how taxes affect couples' decisions. Then, we formally derive optimal tax formulae with exogenous and endogenous threat points. Finally, we describe how filing options can hold threat points fixed and change optimal tax implications.

This section restricts the government's policy instruments to a tax schedule for singles, $T^{\text{s}} : \mathbb{R}_+ \mapsto \mathbb{R}$, and a joint (income-splitting) tax schedule for couples, $T : \mathbb{R}_+ \mapsto \mathbb{R}$.[15] Many tax authorities use this type of joint taxation—e.g., in the U.S., Germany, and Ireland—which makes its study interesting on its own. Moreover, with income-splitting tax schedules, we can compare our optimal tax formulae with traditional single-agent formulae. In Subsection 2.1, this comparison highlights the multi-person nature of the household and the effect of endogenous threat points. We refer to a tax system with only one tax schedule for couples as a ***one-schedule***

[14] We borrowed this definition from Bodoh-Creed (2016).

[15] An income-splitting tax schedule maps earnings, z_f and z_m, into a joint after-tax income, $y = y_f + y_m$, by $y = z_f + z_m - T(z_f + z_m)$. In Section 4, we relax this assumption and allow for an arbitrary joint tax schedule.

tax system. The tax schedule T defines a budget set

$$\mathbf{B}_T := \left\{ (y_f, z_f; y_m, z_m) \middle| y_f + y_m \leq [z_f + z_m - T(z_f + z_m)] \right\}.$$

With income-splitting tax schedules, we only need to know the sum of earnings, $z = z_f + z_m$, and the sum of after-tax incomes, $y = y_f + y_m$, to check if transactions belong to the budget set. For this reason, we sometimes abuse notation and say $(y, z) \in \mathbf{B}_T$ to represent $(y_f, z_f; y_m, z_m) \in \mathbf{B}_T$ for $z = z_f + z_m$ and $y = y_f + y_m$. We denote the set of all utility pairs attainable in agreement for a couple $\boldsymbol{\theta}$ by $\mathbf{U}^{\mathrm{a}}(\boldsymbol{\theta}; T)$. All the examples in this section are such that disagreement utilities, $\bar{\boldsymbol{u}} = (\bar{u}_f, \bar{u}_m)$, lie in the interior of $\mathbf{U}^{\mathrm{a}}(\boldsymbol{\theta}; T)$.

2.1 Tax Equilibrium

To derive optimal taxes, we first need to understand how taxes affect a couple's choices. There are two important channels: taxes define a budget set, and taxes can affect threat points through the disagreement game. To provide some intuition, we start with exogenous threat points; then, we consider how choices change when taxes affect threat points.

Exogenous threat points. Given a couple $\boldsymbol{\theta}$, assume that policy does not affect threat points and fix them at $\bar{\boldsymbol{u}} = (\bar{u}_f, \bar{u}_m)$. Because an income-splitting tax schedule only depends on the sum of the earnings, $z = z_f + z_m$, it is useful to define a household welfare function $U(y, z; \boldsymbol{\theta}, \bar{\boldsymbol{u}})$ over bundles (y, z) by

$$\max_{c_f, z_f \geq 0} \left[u_f\left(c_f, 1 - \frac{z_f}{\theta_f}\right) - \bar{u}_f \right] \left[u_m\left(\alpha y - c_f, 1 - \frac{z - z_f}{\theta_m}\right) - \bar{u}_m \right]. \tag{1}$$

We denote the individual utilities by $\boldsymbol{u}(y, z; \boldsymbol{\theta}, \bar{\boldsymbol{u}}) = (u_f(y, z; \boldsymbol{\theta}, \bar{\boldsymbol{u}}), u_m(y, z; \boldsymbol{\theta}, \bar{\boldsymbol{u}}))$. With this household welfare function, the couple behaves as a single agent and chooses transactions $(y, z) \in \mathbf{B}_T$ to maximize $U(\cdot; \boldsymbol{\theta}, \bar{\boldsymbol{u}})$, that is, it solves

$$\max_{z \geq 0} U(z - T(z), z; \boldsymbol{\theta}, \bar{\boldsymbol{u}}). \tag{2}$$

As for single agents, we can define the standard set of elasticities of taxable income (ETI): the uncompensated elasticity, $\varepsilon(z; \boldsymbol{\theta})$; the income elasticity, $\eta(z; \boldsymbol{\theta})$; and the compensated elasticity, $\varepsilon^{\mathrm{c}}(z; \boldsymbol{\theta}) = \varepsilon(z; \boldsymbol{\theta}) + \eta(z; \boldsymbol{\theta})$.[16]

[16] We use the definition from Jacquet et al. (2013) and Scheuer and Werning (2017), which takes into account non-linearities of the tax schedule. See Appendix A.3.

Let us consider a simple example with the tax schedule $T(z) = z - \lambda z^{1-\rho}$. This functional form has been used as far back as Musgrave (1959) and Feldstein (1969) to study the U.S. income tax schedule's progressivity. It has constant progressivity[17] and has been shown to provide a good approximation for the U.S. effective tax schedule when suitably calibrated, e.g., Bénabou (2000, 2002); Heathcote et al. (2017). Because of its recent revival by the work of Heathcote, Storesletten, and Violante (2017), we call this specification an HSV tax schedule.

Example 2.1. *Given threat points $\bar{\boldsymbol{u}}$, assume spouses have a* log-log *utility, $u_i(c, l) = \log(c) + A\log(l)$. With this utility and an HSV tax schedule, the couple solves*[18]

$$U(y, z; \boldsymbol{\theta}, \bar{\boldsymbol{u}}) = \max_{\beta} \left((1+A)\log(\beta) + \log(\alpha y) + A\log(\theta_f + \theta_m - z) - \bar{u}_f\right)$$
$$\left((1+A)\log(1-\beta) + \log(\alpha y) + A\log(\theta_f + \theta_m - z) - \bar{u}_m\right),$$

where β denotes the wife's share of consumption, $\beta := \frac{c_f}{\alpha y}$. Then, the solution to $\max_z U(z - T(z), z; \boldsymbol{\theta}, \bar{\boldsymbol{u}})$ is given by

$$z = z_f + z_m = \frac{(1-\rho)\Big(\theta_f + \theta_m\Big)}{A + 1 - \rho}.$$

It is also possible to write the ETIs for this problem.

Notice that the particular combination of income-splitting taxation and log-log utilities leads to household earnings, z, that do not depend on $\bar{\boldsymbol{u}}$. This does not hold in general.

Endogenous threat points. Now we need to understand how spouses decide in disagreement when they face the same income-splitting schedule. Because transactions do not map directly to consumption, we need to define a disagreement partitioning rule. In general, these ***partitions***, $p_f(\cdot) = 1 - p_m(\cdot)$, could depend on the full set of disagreement transactions, $\bar{\boldsymbol{x}} = (\bar{y}_f, \bar{z}_f; \bar{y}_m, \bar{z}_m)$. However, we focus on partitions that only depend on earnings, $\bar{z}_f$ and $\bar{z}_m$, because an income-splitting schedule only defines y, not the individual after-tax incomes.

In disagreement and given her husband's earnings, $\bar{z}_m$, a wife θ_f chooses $\bar{z}_f$ to solve

$$\max_{\bar{z}_f} u_f\left(\bar{c}_f, 1 - \frac{\bar{z}_f}{\theta_f}\right) \quad \text{s.t.} \quad \bar{c}_f = p_f(\bar{z}_f, \bar{z}_m)\left(\bar{z}_f + \bar{z}_m - T(\bar{z}_f + \bar{z}_m)\right).$$

[17] For all z, $z[1 - T'(z)]/(z - T(z)) = 1 - \rho$.

[18] Ignoring the non-negativity constraint on earnings, z_f and z_m, we take it into account in our quantitative exercise (Section 3).

The husband solves a similar problem, and disagreement transactions are the equilibrium of this game.

Once we consider that threat points react to taxes, couples may not behave as a single agent. In particular, the elasticities derived in the previous section are conditional on threat points and only consider local substitution and income effects.

To see that taxes above or below the observed earnings, z, can impact a couples' decision we provide two simple examples. The following example illustrates how the disagreement earnings $\bar{z} = \bar{z}_f + \bar{z}_m$ can be below the observed earnings. It uses a partition that does not depend on spouses' choices.

Example 2.2. *Let preferences and tax schedules be as in Example 2.1. In disagreement each spouse chooses his/her effort, z_i, taking z_{-i} as given under the agreed consumption rule, $c_i = p_i y$, where p_i is independent of choices. Under this partitioning rule, the Nash equilibrium for the disagreement game under the HSV schedule is*

$$\bar{z}_i = \frac{(1-\rho)\theta_i + A(\theta_i - \theta_{-i})}{2A + 1 - \rho},$$

for $i, -i = f, m$.

In this case,

$$\bar{z} = \bar{z}_f + \bar{z}_m = \frac{(1-\rho)\big(\theta_f + \theta_m\big)}{2A + 1 - \rho} < \frac{(1-\rho)\big(\theta_f + \theta_m\big)}{A + 1 - \rho} = z$$

Changes in the marginal tax rate at $\bar{z}$ will affect the couple's decision in agreement beyond the usual income effect channel. It is not always the case that $\bar{z} < z$. The next example considers a more natural partitioning rule, in which the labor supply affects the partition.

Example 2.3. *Assume that preferences and taxes are as in Example 2.2. Let consumption be shared in disagreement according to the partition,*

$$c_i = \frac{\bar{z}_i^{1-\rho}}{\bar{z}_i^{1-\rho} + \bar{z}_{-i}^{1-\rho}} y.$$

To get a simple solution, we focus on the symmetric case, $\theta := \theta_f = \theta_m$,

$$\bar{z} = \bar{z}_f + \bar{z}_m = \frac{(1-\rho)\theta}{A/2 + 1 - \rho} > \frac{(1-\rho)2\theta}{A + 1 - \rho} = z.$$

A subtle lesson from these examples is that, since disagreement income choices may differ from the observed choices, tax perturbation evaluation with "local" statistics may lead to errors.

2.2 Optimal Tax Formulae

After seeing how policy can change threat points, we now derive optimal tax formulae for both the case where the policy does not, and when it does affect threat points. We focus only on the optimal tax schedule for married couples since the problem for singles is standard. We also ignore the endogeneity of couples formation and derive optimal formulae for a given marriage distribution. In Appendix A.1, we show how to adapt our tax formulae to consider an endogenous marriage market.

Exogenous threat points. If we fix threat points, we face an (almost) standard optimal taxation program because couples behave as single agents with preferences U, as defined in (1). One issue is the potential dissonance between how the household welfare function U ranks bundles (y, z), and how the society ranks these bundles. That is, society does not need to attach a normative value to U. We retain the notation from the previous section for the ETIs, $\varepsilon(z;\boldsymbol{\theta})$, $\eta(z;\boldsymbol{\theta})$, and $\varepsilon^{\mathrm{c}}(z;\boldsymbol{\theta})$, with the understanding that they are defined conditional on threat points, $\bar{\boldsymbol{u}} = \bar{\boldsymbol{u}}(\boldsymbol{\theta})$.

For concreteness, assume that the planner solves a weighted utilitarian objective where $w_i(\boldsymbol{\theta})$, $i = f, m$ is the weight attached to a gender i spouse in a couple $\boldsymbol{\theta}$. Leaving the dependency on the disagreement utility $\bar{\boldsymbol{u}}$ implicit we can define

$$W(z, y; \boldsymbol{\theta}) = w_f(\boldsymbol{\theta}) u_f\left(z, y; \boldsymbol{\theta}\right) + w_m(\boldsymbol{\theta}) u_m\left(z, y; \boldsymbol{\theta}\right). \tag{3}$$

Whereas U is the household welfare function, W reflects society's valuation of different bundles consumed by the household.

The welfare impact of a small increase in z around the optimum is

$$\begin{aligned} \mathrm{d}W &= \left[\frac{\partial W(z, y; \boldsymbol{\theta})}{\partial y}\left[1 - T'(z)\right] + \frac{\partial W(z, y; \boldsymbol{\theta})}{\partial z}\right] dz \\ &= -\frac{\partial W(z, y; \boldsymbol{\theta})}{\partial y}\left[\frac{\partial U(z, y; \boldsymbol{\theta})/\partial z}{\partial U(z, y; \boldsymbol{\theta})/\partial y} - \frac{\partial W(z, y; \boldsymbol{\theta})/\partial z}{\partial W(z, y; \boldsymbol{\theta})/\partial y}\right] \mathrm{d}z, \end{aligned} \tag{4}$$

which is only zero if the marginal rates of substitution coincide for U and W at the equilibrium transactions. If $\frac{\partial U(z,y;\boldsymbol{\theta})/\partial z}{\partial U(z,y,\boldsymbol{\theta})/\partial y} \neq 0$, we define the ***dissonance*** term, at $y = z - T(z)$, by

$$\Delta_y(z; \boldsymbol{\theta}) := 1 - \frac{\partial W(z, y; \boldsymbol{\theta})/\partial z}{\partial W(z, y; \boldsymbol{\theta})/\partial y}\left(\frac{\partial U(z, y; \boldsymbol{\theta})/\partial z}{\partial U(z, y; \boldsymbol{\theta})/\partial y}\right)^{-1}, \tag{5}$$

which captures the difference in the social and private value of disposable income,

y, relative to earnings, z.[19] If we define $W_y(z;\boldsymbol{\theta}) := \partial W(z, z - T(z);\boldsymbol{\theta})/\partial y$, we have

$$\mathrm{d}W = [1 - T'(z)]W_y(z;\boldsymbol{\theta})\Delta_y(z;\boldsymbol{\theta})\mathrm{d}z. \tag{6}$$

Another issue is that different couples may have the same earnings, and we must ***aggregate*** across them to evaluate tax policies. To do that, start with the marriage distribution, $\mu(\boldsymbol{\theta})$, which induces a joint distribution on z and $\boldsymbol{\theta}$ by

$$\phi(z(\boldsymbol{\theta}), \boldsymbol{\theta}) = \mu(\boldsymbol{\theta})$$

and $\phi(z, \boldsymbol{\theta}) = 0$ if $z \neq z(\boldsymbol{\theta})$. From which we define the marginal, $\phi(z)$, and conditional $\phi(\boldsymbol{\theta}; z)$ distributions. For any variable v, we also define the following averages,

$$\tilde{v}(z) := \int v(z;\boldsymbol{\theta})\phi(\boldsymbol{\theta};z)\mathrm{d}\boldsymbol{\theta} \quad \text{and} \quad \hat{v}(z) := \int v(z;\boldsymbol{\theta})\frac{W_y(z;\boldsymbol{\theta})\phi(\boldsymbol{\theta};z)}{\widetilde{W}_y(z)}\mathrm{d}\boldsymbol{\theta}. \tag{7}$$

With these definitions, the revenue effect of a $\mathrm{d}\tau$ perturbation of the tax system at the interval $[z', z' + \mathrm{d}z']$ is given by[20]

$$\frac{\mathrm{dRevenue}}{\mathrm{d}\tau\mathrm{d}z'} = \int_{z'}^{\infty}\left(\frac{T'(z)}{1 - T'(z)}\tilde{\eta}(z) + 1\right)\phi(z)\,\mathrm{d}z - \frac{T'(z')}{1 - T'(z')}\tilde{\varepsilon^{\mathrm{c}}}(z')z'\phi(z').$$

The welfare effect, which can be computed from equation (6), is given by

$$\begin{aligned}\frac{\mathrm{dWelfare}}{\mathrm{d}\tau\mathrm{d}z'} = \int_{z'}^{\infty}\widetilde{W}_y(z)\left(\hat{\Delta}_y(z)\hat{\eta}(z) + \widehat{\mathrm{Cov}}[\Delta_y, \eta](z) - 1\right)\phi(z)\,\mathrm{d}z - \\ - \widetilde{W}_y(z')\left(\hat{\Delta}_y(z)\hat{\varepsilon^{\mathrm{c}}}(z) + \widehat{\mathrm{Cov}}[\Delta_y, \varepsilon^{\mathrm{c}}](z)\right)z'\phi(z'),\end{aligned}$$

where $\widehat{\mathrm{Cov}}$ is the covariance under the distribution $W_y(z;\boldsymbol{\theta})\phi(\boldsymbol{\theta};z)/\widetilde{W}_y(z)$. At the optimum, these effects cancel out, and we get Proposition 1.

Proposition 1. *If threat points are invariant to tax perturbations, then the optimal labor*

[19] This concept is similar to the one in the behavioral public finance literature when "experience" and "decision" utilities differ, as shown by Farhi and Gabaix (2020).

[20] We follow a procedure that parallels the one in Alves et al. (2018). It differs in two ways: First, they hold spouses' Pareto weights fixed, we hold threat points fixed. Both assumptions lead to "rational" household behavior. However, they have distinct consequences for the curvature of households indifference curves, hence their ETIs. Second, we go beyond the utilitarian case, and allow the planner to assign any weight to different productivity types.

income tax formula is implicitly defined by

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\tilde{\varepsilon}^{\mathrm{c}}(z)} \frac{1-\Phi(z)}{z\phi(z)} \mathbb{E}\Bigg[\frac{T'(Z)}{1-T'(Z)} \tilde{\eta}(Z) + 1 - \widetilde{W}_y(Z) + \\ + \widetilde{W}_y(Z) \left(\hat{\Delta}_y(Z)\hat{\eta}(Z) + \widehat{\mathrm{Cov}}[\Delta_y, \eta](Z) \right) \Big| Z \geq z \Bigg] \\ - \frac{\widetilde{W}_y(z)}{\tilde{\varepsilon}^{\mathrm{c}}(z)} \left(\hat{\Delta}_y(z)\hat{\varepsilon}^{\mathrm{c}}(z) + \widehat{\mathrm{Cov}}[\Delta_y, \varepsilon^{\mathrm{c}}](z) \right) \quad (8)$$

The terms involving Δ_y capture the consequences of dissonance. They arise whenever the planner and the couple disagree about the marginal cost and benefit from increasing effort. It is one reason why this tax formula differs from the one found in Saez (2001)—or in most of the literature that followed. The second is the aggregation of different couples with the same earnings, which appears in the covariances and averages.[21] However, the aggregation consequences disappear if each z is associated with a different couple $\boldsymbol{\theta}$.

Corollary 1. *If for all $z \in \mathbb{R}_+$, there is at most one $\boldsymbol{\theta}$ with $z(\boldsymbol{\theta}) = z$, then (8) simplifies to*[22]

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\varepsilon^{\mathrm{c}}(z)} \frac{1-\Phi(z)}{z\phi(z)} \mathbb{E}\left[\frac{T'(Z)}{1-T'(Z)} \eta(Z) + 1 + W_y(Z) \left(\Delta_y(Z)\eta(Z) - 1 \right) \Big| Z \geq z \right] \\ - W_y(z)\Delta_y(z)$$

Example 2.4 below consider a particular case in which dissonance terms vanish and the optimal tax formula, if threat points are held fixed, coincides with unitary optimal taxes.

Example 2.4. *Let preferences be as in Example 2.1, with exogenous disagreement utilities. For this class of preferences and any welfare function $W(u_f, u_m)$ we have*

$$\frac{\partial W/\partial z}{\partial W/\partial y} = \frac{-Ay}{\theta_f + \theta_m - z} = \frac{\partial U/\partial z}{\partial U/\partial y},$$

and we have no dissonance, $\Delta_y = 0$. The optimal tax formula simplifies to a standard

[21] See Gerritsen (2016), for similar aggregation issues in the context of behavioral public finance.

[22] The same is true if the marginal social value of income, $W_y(z; \boldsymbol{\theta})$, and the relevant elasticities are the same for all couples earning z.

formula,

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\varepsilon^{\mathrm{c}}(z)} \frac{1-\Phi(z)}{z\phi(z)} \mathbb{E}\left[\frac{T'(Z)}{1-T'(Z)} \eta(Z) + 1 - \widetilde{W}_y(Z) \middle| Z \geq z \right].$$

The dissonance term disappears not because the planner and the couple agree on their welfare evaluation. Instead, it is the combination of log-log preferences and income-splitting taxation. The log-log utility implies that the the cost of producing z and the benefit of having y only differ between the planner and the couple because of the usual fiscal externality.

Endogenous threat points. We derive optimal tax formula (8) under the assumption that policy does not affect $\bar{\boldsymbol{u}}$. However, if threat points depend on disagreement choices, the tax schedule ought to play an essential role in defining these threat points. In this section, we allow the tax perturbation to affect threat points.

Changes in threat points have two effects in a household. They affect the total labor earnings, z, and change how resources are distributed within the couple. The second channel is relevant for welfare but has no impact on revenues. For this reason, we first derive the ***revenue*** effect. Consider a couple of type $\boldsymbol{\theta}$. If there is a marginal tax increase at z' the impact on $z = z(\boldsymbol{\theta})$ through threat points is given by

$$z\varepsilon^{\mathrm{c}}(z;\boldsymbol{\theta}) \underbrace{\sum_{i=f,m} \frac{-\partial \log\left(-\frac{\partial U/\partial z}{\partial U/\partial y}(y,z;\boldsymbol{\theta},\bar{\boldsymbol{u}})\right)}{\partial \bar{u}_i} \frac{\partial \bar{u}_i(\boldsymbol{\theta})}{\partial \tau}\bigg|_{z'}}_{=:\Gamma_U(z;z',\boldsymbol{\theta})}. \tag{9}$$

with $y = z - T(z)$, and $\bar{\boldsymbol{u}} = \bar{\boldsymbol{u}}(\boldsymbol{\theta})$. The term Γ_U represents how changes in threat points affect the couple's value of after-tax income relative to labor earnings. If we integrate (9) over all z and add to the revenue effect from the previous section, we get the total revenue effect of a marginal tax increase at z',

$$\begin{aligned}\frac{\mathrm{dRevenue}}{\mathrm{d}\tau \mathrm{d}z'} = &-\frac{T'(z')}{1-T'(z')}\tilde{\varepsilon^{\mathrm{c}}}(z')z'\phi(z') + \int_{z'}^{\infty} \left(\frac{T'(z)}{1-T'(z)}\tilde{\eta}(z) + 1 \right) \phi(z)\,\mathrm{d}z \\ &+ \int_0^{\infty} T'(z)z \left(\tilde{\varepsilon^{\mathrm{c}}}(z)\tilde{\Gamma}_U(z;z') + \widetilde{\mathrm{Cov}}[\varepsilon^{\mathrm{c}}, \Gamma_U(\cdot;z')](z) \right) \phi(z)\,\mathrm{d}z.\end{aligned}$$

For the ***welfare*** effect, the impact via changes in z is only relevant if there is dissonance, Δ_y. For this reason, we define $\Gamma_W(z;z',\boldsymbol{\theta}) := \Delta_y(z;\boldsymbol{\theta})\Gamma_U(z;z',\boldsymbol{\theta})$. We

also need to consider the impact on welfare from redistribution between spouses,

$$\underbrace{\left[w_f \frac{\partial u_f}{\partial c} - w_m \frac{\partial u_m}{\partial c}\right]}_{=:\Delta_f(\boldsymbol{\theta})} \underbrace{\sum_{i=f,m} \left\{ \frac{\partial c_f}{\partial \bar{u}_i} - \alpha[1 - T'(z)] \frac{\partial \check{\mathfrak{z}}_f}{\partial \bar{u}_i} \right\} \frac{\partial \bar{u}_i}{\partial \tau}\bigg|_{z'}}_{=:\Gamma_f(\boldsymbol{\theta};z')},$$

where the functions should be evaluated at their arguments in equilibrium for a $\boldsymbol{\theta}$ couple. The first term, Δ_f, is a different dimension of "dissonance" and captures the difference in social and private value of wives' consumption relative to their husbands' consumption. The second term, Γ_f, measures how much the wives' consumption is affected by changes in threat points. When we consider these two new channels in equation (6), we get that the welfare effect of a marginal change in the tax rate at z' is given by

$$\begin{aligned}
\frac{\mathrm{dWelfare}}{\mathrm{d}\tau \mathrm{d}z'} = &- \widetilde{W}_y(z') \left(\hat{\Delta}_y(z) \hat{\varepsilon^{\mathrm{c}}}(z) + \widehat{\mathrm{Cov}}[\Delta_y, \varepsilon^{\mathrm{c}}](z) \right) z' \phi(z') \\
&+ \int_{z'}^{\infty} \widetilde{W}_y(z) \left(\hat{\Delta}_y(z) \hat{\eta}(z) + \widehat{\mathrm{Cov}}[\Delta_y, \eta](z) - 1 \right) \phi(z) \, \mathrm{d}z \\
&+ \int_0^{\infty} z \widetilde{W}_y(z) \left(\tilde{\varepsilon^{\mathrm{c}}}(z) \tilde{\Gamma}_W(z; z') + \widetilde{\mathrm{Cov}}[\varepsilon^{\mathrm{c}}, \Gamma_W(\cdot; z')](z) \right) \phi(z) \, \mathrm{d}z \\
&+ \int_0^{\infty} \left(\tilde{\Delta}_f(z) \tilde{\Gamma}_f(z; z') + \widetilde{\mathrm{Cov}}[\Delta_f, \Gamma_f(\cdot; z')](z) \right) \phi(z) \, \mathrm{d}z
\end{aligned}$$

The revenue and welfare effects cancel out at the optimum, and we get the next proposition.

Proposition 2. *If threat points are affected by the tax schedule in place then the optimal tax formula from Proposition 1 must be modified, and is given by*

$$\begin{aligned}
\frac{T'(z)}{1 - T'(z)} = &\textit{ RHS of } (8) \\
&+ \frac{1}{\tilde{\varepsilon^{\mathrm{c}}}(z) z \phi(z)} \mathbb{E} \Big[T'(Z) Z \left(\tilde{\varepsilon^{\mathrm{c}}}(Z) \tilde{\Gamma}_U(Z; z) + \widetilde{\mathrm{Cov}}[\varepsilon^{\mathrm{c}}, \Gamma_U(\cdot; z)](Z) \right) \\
&\quad + Z \widetilde{W}_y(Z) \left(\tilde{\varepsilon^{\mathrm{c}}}(Z) \tilde{\Gamma}_W(Z; z) + \widetilde{\mathrm{Cov}}[\varepsilon^{\mathrm{c}}, \Gamma_W(\cdot; z)](Z) \right) \\
&\quad + \tilde{\Delta}_f(Z) \tilde{\Gamma}_f(Z; z) + \widetilde{\mathrm{Cov}}[\Delta_f, \Gamma_f(\cdot; z)](Z) \Big]. \quad (10)
\end{aligned}$$

The terms with Γ_U and Γ_W capture the impact of changes in earnings, z, through threat points. The first for revenue, the second for welfare. The terms with Δ_f capture the impact on welfare of redistribution within the couple. This last term implies that redistribution within couples may affect welfare even if earnings do

not change with threat point. We illustrate this possibility in the example below.

Example 2.5. *Let preferences be as in Example 2.1. As in example 2.4 the dissonance term vanishes. On top of that, we also have that $\Gamma_U = 0$. This implies that the only new term for the optimal taxation formula comes from reallocation within couples*

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\varepsilon^{\mathrm{c}}(z)}\frac{1-\Phi(z)}{z\phi(z)}\mathbb{E}\left[\frac{T'(Z)}{1-T'(Z)}\eta(Z) + 1 - \widetilde{W}_y(Z)\Big| Z \geq z\right] + \frac{1}{\varepsilon^{\mathrm{c}}(z)z\phi(z)}\mathbb{E}\left[\tilde{\Delta}_f(Z)\tilde{\Gamma}_f(Z;z) + \widetilde{\mathrm{Cov}}[\Delta_f, \Gamma_f(\cdot;z)](Z)\right].$$

If we also have exogenous partitioning rules, as in Example 2.2, then $\frac{\partial \bar{u}_f}{\partial \tau}\big|_{z'} = \frac{\partial \bar{u}_m}{\partial \tau}\big|_{z'}$ for any tax perturbation. It follows that

$$\Gamma_f(\boldsymbol{\theta}; z') = \frac{\alpha(2\beta-1)(1+A)[z(\boldsymbol{\theta}) - T(z(\boldsymbol{\theta}))]}{u_f(\boldsymbol{\theta}) - \bar{u}_f(\boldsymbol{\theta}) + u_m(\boldsymbol{\theta}) - \bar{u}_m(\boldsymbol{\theta}) + 2(A+1)}\left.\frac{\partial \bar{u}_f}{\partial \tau}\right|_{z'},$$

and

$$\left.\frac{\partial \bar{u}_i}{\partial \tau}\right|_{z'} = \left(\frac{1+A}{\bar{z} - T(\bar{z})} - \frac{AT''(\bar{z})}{(1-T'(\bar{z}))^2}\right)\begin{cases} \bar{\eta}(\bar{z}) & \textit{if} \quad \bar{z} > z' \\ \bar{\varepsilon}^{\mathrm{c}}(\bar{z}) & \textit{if} \quad \bar{z} = z' \\ 0 & \textit{if} \quad \bar{z} < z' \end{cases}.$$

We have derived two different formulae for optimal taxes. Expression (8) applies if the government can hold threat points fixed as it perturbs the tax schedule. Propositions 1 and 2 and Example 2.2, in contrast, highlight the impact of tax schedules on threat points.

A legitimate question is whether it is possible to hold threat points fixed or whether the planner must always consider its endogeneity. For the latter possibility, the crucial feature is that the same schedule defines the budget set for couples in agreement and disagreement. We now argue that this need not always be the case. A feature present in the U.S. tax system may allow for threat points to be held fixed: filing options.

2.3 Filing Options

In many countries, like the U.S. or Germany, spouses may choose whether to file jointly, roughly using an income-splitting rule like the one we have been considering thus far, or separately.[23] Under a progressive schedule, the household budget set

[23] In the U.S., household taxes are based on total household income, $z_f + z_m$, but a spouse may opt to file individually using a *fall-back schedule* that considers only his/her income. When one spouse opts to file individually, this obliges the other to do the same.

associated with individual filing is a strict subset of the budget set associated with joint filing. Nevertheless, this does not mean that filing options are irrelevant.

We refer to the schedule that can be chosen unilaterally by either spouse as a ***fall-back*** schedule and to the schedule used by spouses in agreement as the ***principal*** schedule. A system comprised of a principal schedule and a fall-back schedule used as a filing option is called ***two-schedule tax system***.

Example 2.6. *Assume that the planner allows spouses to file individually, and if any of the two spouses opt to do it, the other must too.*

To take progressivity into account and keep the filing option neutral under agreement, we defined the fall-back tax schedule as $\overline{T}(z) = z - \bar{\lambda} z^{1-\rho}$ *for* $\bar{\lambda} = 2^{-\rho}\lambda$.[24] *Consider the special case,* $\theta_f = \theta_m = \theta$*, and the disagreement partitioning rule is exogenous. In this case, the disagreement earnings are, with and without filing options,*

$$\bar{z}_i' = \frac{\theta(1-\rho)}{A+1-\rho} \quad \textit{and} \quad \bar{z}_i = \frac{\theta(1-\rho)}{2A+1-\rho},$$

respectively.

In Example 2.6, no couple in agreement would strictly prefer to file individually, whereas at least one spouse prefers to file individually for any couple in disagreement. If the optimal tax system has these features, then the principal schedule is characterized by the optimal formula with exogenous threat points, (8), instead of (10). Of course, just saying that different formulae arise with and without filing options says little about welfare. There is an immediate sense in which one cannot do worse with filing options.

Proposition 3. *The set of allocations that can be implemented with filing options contains all allocations implemented without filing options.*

Proof. Take an allocation that is implemented by a tax schedule $T(\cdot)$ without filing options. Now offer this schedule as the filing option schedule. The same threat points result, and new allocations may be induced by a different choice, $\tilde{T}(\cdot)$ for the principal schedule. □

Although the set inclusion above need not be strict, it will, in general. By comparing optimal tax formulae (8) and (10), we obtain a sufficient condition for an optimal one-schedule tax system to be strictly dominated for a given welfare metric.

[24] In words, each spouse tax liability is half of the taxes for a couple that earns in total twice her/his earnings. This roughly corresponds to the U.S. tax schedule when filing separately.

Proposition 4. *Suppose that for an optimal tax schedule there are earnings z such that the new terms displayed in proposition 2 are strictly positive, that is*

$$\mathbb{E}\Big[T'(Z)Z\left(\tilde{\varepsilon^{\mathrm{c}}}(Z)\tilde{\Gamma}_U(Z;z)+\widetilde{\mathrm{Cov}}[\varepsilon^{\mathrm{c}},\Gamma_U(\cdot;z)](Z)\right)$$
$$+Z\widetilde{W_y}(Z)\left(\tilde{\varepsilon^{\mathrm{c}}}(Z)\tilde{\Gamma}_W(Z;z)+\widetilde{\mathrm{Cov}}[\varepsilon^{\mathrm{c}},\Gamma_W(\cdot;z)](Z)\right)$$
$$+\tilde{\Delta}_f(Z)\tilde{\Gamma}_f(Z;z)+\widetilde{\mathrm{Cov}}[\Delta_f,\Gamma_f(\cdot;z)](Z)\Big]>0,$$

then having a fall-back schedule can improve strictly over the optimal one-schedule tax system.

Proof. Starting with the optimal tax schedule T, add a fall-back schedule $\overline{T}$ equal to T except at a neighborhood of z where we increase taxes marginally. Because $\overline{T}$ has higher taxes than T, it does not affect equilibrium decisions *conditional on threat points*. $\overline{T}$ only effect is through threat points and is given by the new terms displayed above. Because these terms add up to something positive, society is strictly better with this two-schedule tax system. □

3 A Calibrated Model

In this section, we assess the quantitative relevance of internal threat points. We start with a two-schedule tax system calibrated to the U.S. economy and evaluate the effect of removing filing options on equilibrium decisions. We find effects about one-third as large as imposing mandatory individual filing. Next, we consider the effect of reducing taxes on women relative to men. Besides being an interesting exercise of gender-based taxation, it supports our assumption of endogenous threat points. Since, with exogenous threat points, we get the counter-intuitive result that lower taxes hurt wives.

3.1 Model

We use a log-log utility, $u(c,l)=\log(c)+A\log(l)$, as in our previous examples. For tax schedules, we use the HSV specification, as introduced in Example 2.1. Singles have an after-tax income of $y_i=\frac{\lambda}{2}(2z_i)^{1-\rho}$; spouses filing ***individually*** face the singles' schedule, and we denote this tax-schedule budget set by $\mathrm{B_I}$; whereas spouses filing ***jointly*** have an after-tax income of $y_f+y_m=\lambda(z_f+z_m)^{1-\rho}$, and we denote the budget set by $\mathrm{B_J}$.

In ***disagreement***, each spouse chooses transactions $\bar{x}_i = (\bar{y}_i, -\bar{z}_i)$ given his/her spouse's transactions, $\bar{x}_{-i} = (\bar{y}_{-i}, -\bar{z}_{-i})$, to solve

$$\max \quad \log(\bar{c}_i) + A \log\left(1 - \frac{\bar{z}_i}{\theta_i}\right) \quad \text{s.t.} \quad \bar{c}_i = \frac{\bar{z}_i^{1-\rho}}{\bar{z}_i^{1-\rho} + \bar{z}_{-i}^{1-\rho}}(\bar{y}_f + \bar{y}_m),\ (\bar{x}_f, \bar{x}_m) \in \mathbf{B}.$$

If spouses do not have the option to file individually, we use $\mathbf{B} = \mathbf{B}_{\mathrm{J}}$. In contrast, if they have the option, we use $\mathbf{B} = \mathbf{B}_{\mathrm{I}}$ because at least one spouse prefers to file individually in disagreement.[25] Both with filing options or without it, it is simple to check that the disagreement game has a unique pure-strategy Nash equilibrium. The consumption partition, $\bar{z}_i^{1-\rho}/(\bar{z}_i^{1-\rho} + \bar{z}_{-i}^{1-\rho})$, is the same used in Example 2.3. This partitioning rule allows spouses' earnings to affect how much of the total (after-tax) income they get. It is such that a spouse filing individually has a disagreement utility equal to the utility of a single person with the same productivity.

In ***agreement***, the couple solves

$$\begin{aligned} \max \quad & \left[\log(c_f) + A \log\left(1 - \frac{z_f}{\theta_f}\right) - \bar{u}_f\right]\left[\log(c_m) + A \log\left(1 - \frac{z_m}{\theta_m}\right) - \bar{u}_m\right] \\ \text{s.t.} \quad & c_f + c_m \leq \alpha(y_f + y_m) \quad \text{and} \quad \boldsymbol{x} = (y_f, z_f; y_m, z_m) \in \mathbf{B}_{\mathrm{J}} \end{aligned}$$

given the disagreement utilities, $\bar{u}_f$ and $\bar{u}_m$. The ***total utility*** of a woman with productivity θ_f that marries a man with productivity θ_m is given by

$$u_f(\theta_f, \theta_m) + \xi_f(\theta_f, \theta_m).$$

As defined in section 1.2, u_f is her material utility and ξ_f her emotional payoff. The total utility for a man is defined similarly. Furthermore, we assume that the courtship shock, ϵ, follows an extreme value type 1 distribution, which generates a smooth distribution of orderings for each person. The matches are then based on a Gale-Shapley algorithm – see Appendix A.4 for details.

This model would be under-identified without further assumptions since we have $2|\Theta|^2$ values of ξ_i to calibrate but only observe $|\Theta|^2$ shares of couples (θ_f, θ_m). To identify the model, we assume the emotional payoff to be equally split, that is $\xi_f(\theta_f, \theta_m) = \xi_m(\theta_f, \theta_m)$.

Table 1: Parameters

A	α	ρ	λ
$2(1-\rho)$	1.2	0.237	2.1

3.2 Parameters

We divide the population of men and women into five levels of productivity and calibrate their distribution from the implied labor supply decision using the 2018 Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS) for people with ages between 16 and 64. We calibrate $\boldsymbol{\xi}$ based on the total utility that would be consistent with the observed marriage market. The other parameters are presented in Table 1. The importance of leisure relative to consumption, A, is set to match the fraction of working hours; the gains of scale when in agreement, α, are set to 20%; and the tax schedule parameters, ρ and λ, are taken from Heathcote et al. (2017) and to get a government expenditure of 20% of GDP, respectively.

3.3 Quantitative Findings

Baseline. As a baseline, we allow for filing options and use a gender-neutral tax schedule. These assumptions imply that the material utility only depends on each spouse's productivity but not on gender, that is, for any θ and θ', $u_f(\theta, \theta') = u_m(\theta', \theta)$. On the other hand, the emotional payoff is equally split within a couple, that is, we assume $\xi_f(\theta, \theta') = \xi_m(\theta, \theta')$. Table 2 shows the calibrated utilities and emotional payoffs and the ex-ante (expected) total utility by productivity level. We see that the material utility of high-productivity people is not much affected by whom they marry, while low-productivity ones benefit by marrying high-productivity spouses. Based on that, we would expect many marriages between low-productivity men and high-productivity women; however, we do not see that on the data, which the model explains by having a negative or small emotional payoff when the wife has a higher productivity than the husband.

Removing the option to file individually. Table 3 displays the effect of removing filing options under the assumption that the emotional payoffs, ξ, and the tax schedule parameters, ρ and λ, are unchanged.

[25]Spouses with lower productivity than their partners face a higher marginal tax rate with the joint tax schedule than with the individual one.

We see that the option to file individually protects the spouse with lower productivity. Removing it does not impact the utility of high-productivity spouses much, but substantially reduces the utility of low-productivity ones. It impacts the marriage market as well, increasing the number of couples with asymmetric productivities. In the baseline, low-productivity people want to marry high-productivity ones, but high-productivity people do not want to marry them back. Removing filing options increases the utility for productive people married to unproductive ones, which increases asymmetric marriages.

Tax couples as singles. To assess our findings' quantitative relevance, we compare the welfare impact of removing filing options with considerably larger reform that replaces joint filing with individual filing, that is taxing everyone as if singles. As in the previous exercise, we keep the emotional payoffs, ξ, and tax schedule parameters, ρ and λ, unchanged.

Table 4 displays changes in utility and marriage market outcomes. The removal of filing options has an effect comparable to this major reform, even though filing options have only an indirect impact through disagreement utilities. These last two exercises suggest that taxation jointness—how much we should consider the earnings together or separately in a couple—can have a large impact through disagreement. This dimension must be taken into account to discuss optimal jointness.

Gender-based taxation. Essential to all the quantitative findings is how much policy affects the disagreement game. The next tax reform provides some compelling reasons why it must have a non-trivial impact. We consider the effect of a small increase on married men's taxes and show that it generates counter-factual consequences if we ignore threat points' endogeneity. More precisely, a couple filing jointly has after-tax income given by

$$\lambda\big((1+\delta_f)z_f + (1-\delta_m)z_m\big)^{1-\rho}$$

with $\delta_m = 0.01$ for all couples, while δ_f varies by couple to keep their total tax liability unchanged. The same δs apply to their earnings if filing individually.

Table 5 shows the counter-intuitive result that women are worse off by being taxed more lightly when threat points are exogenous. The explanation is that this policy intervention decreases the labor cost for women, makes them work more than before, and thus decreases their utility. In contrast, for endogenous threat points, this intervention positively impacts the women's disagreement utility, and the overall effect is that women are better off with lower taxes.

A consequence of the exercise is that gender-based taxation results— e.g., Boskin

and Sheshinski (1983); Alesina et al. (2011); Bastani (2013); Gayle and Shephard (2019)—depend on assumptions about threat points and the available instruments.

Table 2: Baseline utilities.
Productivity, θ, increases down and right

material utility, u_f: women

	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	1.68	1.95	2.22	2.42	2.55
	2.69	2.72	2.78	2.86	2.93
θ_f	3.12	3.12	3.14	3.18	3.22
	3.40	3.38	3.39	3.41	3.44
$\Downarrow$	3.61	3.58	3.59	3.60	3.61

material utility, u_m: men

	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	1.68	2.69	3.12	3.40	3.61
	1.95	2.72	3.12	3.38	3.58
θ_f	2.22	2.78	3.14	3.39	3.59
	2.42	2.86	3.18	3.41	3.60
$\Downarrow$	2.55	2.93	3.22	3.44	3.61

emotional payoff, ξ: women

	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.97	1.39	1.73	1.85	2.11
	0.01	0.97	1.49	1.52	1.44
θ_f	-0.97	0.01	1.05	1.29	1.26
	-0.89	-0.31	0.16	0.81	0.89
$\Downarrow$	-0.78	-0.46	0.17	0.52	1.45

emotional payoff, ξ: men

	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.97	1.39	1.73	1.85	2.11
	0.01	0.97	1.49	1.52	1.44
θ_f	-0.97	0.01	1.05	1.29	1.26
	-0.89	-0.31	0.16	0.81	0.89
$\Downarrow$	-0.78	-0.46	0.17	0.52	1.45

ex-ante total utility: women

	$\Rightarrow$		θ_f		$\Rightarrow$
	3.41	3.63	3.79	3.74	4.21

ex-ante total utility: men

	$\Rightarrow$		θ_m		$\Rightarrow$
	2.09	3.53	4.34	4.74	5.14

singles' utility: women

	$\Rightarrow$		θ_f		$\Rightarrow$
	1.49	2.54	2.96	3.23	3.43

singles' utility: men

	$\Rightarrow$		θ_m		$\Rightarrow$
	1.49	2.54	2.96	3.23	3.43

Table 3: Remove Filling Options — % change from Baseline.
Productivity, θ, increases down and right

material utility, u_f: women

%	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.00	-1.83	-4.57	-5.59	-6.24
	0.43	0.00	-0.08	-0.35	-0.76
θ_f	0.69	0.04	0.00	-0.02	-0.09
	0.68	0.14	0.01	0.00	-0.01
$\Downarrow$	0.69	0.24	0.05	0.01	0.00

material utility, u_m: men

%	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.00	0.43	0.69	0.68	0.69
	-1.83	0.00	0.04	0.14	0.24
θ_f	-4.57	-0.08	0.00	0.01	0.05
	-5.59	-0.35	-0.02	0.00	0.01
$\Downarrow$	-6.24	-0.76	-0.09	-0.01	0.00

ex-ante total utility: women

%	$\Rightarrow$		θ_f		$\Rightarrow$
	-1.54	-0.30	-0.21	-0.14	-0.20

ex-ante total utility: men

%	$\Rightarrow$		θ_f		$\Rightarrow$
	-1.92	0.08	0.27	0.27	0.27

couples' mass, μ

%	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	-0.69	0.59	1.42	1.45	1.45
	-2.25	-1.15	-0.62	-0.41	-0.23
θ_f	2.75	-0.73	-0.75	-0.84	-0.89
	3.27	1.81	-0.81	-0.88	-1.05
$\Downarrow$	4.16	2.86	-1.04	-0.90	-1.06

single women's mass, μ_f^s

%	$\Rightarrow$		θ_f		$\Rightarrow$
	-4.21	2.89	2.62	1.06	1.12

single men's mass, μ_m^s

%	$\Rightarrow$		θ_f		$\Rightarrow$
	0.65	-0.58	-0.75	-0.88	-1.06

Table 4: Tax Couples as Singles — % change from Baseline.
Productivity, θ, increases down and right

material utility, u_f: women

%	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.00	-6.69	-12.89	-16.37	-17.68
	-0.84	0.00	-0.78	-1.94	-3.21
θ_f	-1.29	-0.06	0.00	-0.02	-0.09
	-1.63	-0.28	0.02	0.00	-0.01
$\Downarrow$	-1.91	-0.47	-0.08	0.03	0.00

material utility, u_m: men

%	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.00	-0.84	-1.29	-1.63	-1.91
	-6.69	0.00	-0.06	-0.28	-0.47
θ_f	-12.89	-0.78	0.00	0.02	-0.08
	-16.37	-1.94	-0.02	0.00	0.03
$\Downarrow$	-17.68	-3.21	-0.09	-0.01	0.00

ex-ante total utility: women

%	$\Rightarrow$		θ_f		$\Rightarrow$
	-7.28	-0.41	0.34	0.30	0.44

ex-ante total utility: men

%	$\Rightarrow$		θ_f		$\Rightarrow$
	-3.49	-0.48	-0.49	-0.62	-0.79

couples' mass, μ

%	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	5.44	-0.48	-2.29	-3.44	-4.10
	-5.41	1.38	1.52	1.11	1.05
θ_f	-9.63	-0.18	1.71	2.15	2.46
	-5.90	-4.12	0.69	2.08	2.88
$\Downarrow$	-7.04	-3.00	-0.91	1.45	2.76

single women's mass, μ_f^s

%	$\Rightarrow$		θ_f		$\Rightarrow$
	5.94	-1.63	-3.56	-3.06	-3.59

single men's mass, μ_m^s

%	$\Rightarrow$		θ_f		$\Rightarrow$
	2.91	1.80	1.71	2.08	2.76

Table 5: Gender-Based Taxation — Marginal increase to men's tax from Baseline.
Productivity, θ, increases down and right

material utility, $\partial u_f / \partial \delta_f$: women
exogenous threat point

∂	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	-0.11	-0.83	-4.37	-7.70	-7.24
	-0.07	-0.11	-0.15	-0.21	-0.31
θ_f	-0.07	-0.09	-0.11	-0.12	-0.13
$\Downarrow$	0.00	-0.09	-0.10	-0.11	-0.11
	0.00	-0.09	-0.10	-0.11	-0.11

material utility, $\partial u_m / \partial \delta_f$: men
exogenous threat point

∂	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.10	0.53	1.66	1.60	0.85
	0.07	0.09	0.19	0.35	0.61
θ_f	0.05	0.05	0.09	0.15	0.24
$\Downarrow$	0.00	0.02	0.05	0.09	0.14
	0.00	0.00	0.03	0.06	0.09

endogenous threat point

∂	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.59	3.10	6.71	8.64	11.74
	0.07	0.59	1.21	2.02	3.16
θ_f	-0.02	0.27	0.59	0.95	1.38
$\Downarrow$	0.05	0.14	0.35	0.58	0.84
	0.04	0.07	0.23	0.40	0.58

endogenous threat point

∂	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	-0.61	-0.84	-1.27	-2.14	-2.98
	-0.36	-0.61	-0.68	-0.73	-0.79
θ_f	-0.21	-0.53	-0.61	-0.65	-0.68
$\Downarrow$	-0.25	-0.47	-0.57	-0.61	-0.64
	-0.24	-0.42	-0.53	-0.58	-0.61

couples' mass elasticity, $\partial \log \mu / \partial \delta_f$
exogenous threat point

∂	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	-0.32	0.17	1.02	0.95	0.37
	-0.30	-0.39	-0.45	-0.30	0.14
θ_f	1.90	-0.31	-0.55	-0.50	-0.24
$\Downarrow$	0.71	1.03	-0.59	-0.56	-0.34
	1.01	1.33	-0.62	-0.59	-0.38

couples' mass elasticity, $\partial \log \mu / \partial \delta_f$
endogenous threat point

∂	$\Rightarrow$		θ_m		$\Rightarrow$
$\Downarrow$	0.25	-0.07	-0.46	-1.02	-1.50
	0.22	0.31	0.14	0.39	0.68
θ_f	-0.62	0.24	0.20	0.46	0.78
$\Downarrow$	-1.18	-1.22	0.25	0.50	0.83
	-1.90	-2.13	0.29	0.54	0.86

4 Constrained Efficiency and Tax Systems

In this section, we expand our previous analysis in two dimensions. First, we consider arbitrary joint tax schedules instead of only income-splitting ones. Second, we allow for arbitrary auxiliary policy instruments, instead of only filing options.

To proceed with these generalizations, we shift from a tax perturbation to a mechanism design approach. Toward this end, our household decision model must be specified to allow for well-defined choices under a general mechanism. With a expanded definition of a type, we show that the revelation principle holds; and we use this central mechanism-design result to understand the limits of tax schedules and filing options. As for income-splitting schedules, the taxation principle also fails for an arbitrary tax schedule. Even though filing options do not implement all incentive-feasible allocations, they do expand the set of implementable allocations, and we characterize this set.

In our context, a ***mechanism***, $\mathcal{M}$, is defined by a choice sets for each spouse, S_i, $i = f, m$; and a function mapping choices into transactions, $\boldsymbol{x} : S_f \times S_m \mapsto X \times X$.[26] The choice sets and the $\boldsymbol{x}$ function define the set of attainable transactions under $\mathcal{M}$, which is denoted by $\mathbf{X}_{\mathcal{M}} := \boldsymbol{x}\,(S_f \times S_m)$. Throughout this section, a bold lower-case letter refers to a pair, and each element of the pair is denoted by a non-bold version with a subscript, e.g., $\boldsymbol{x} = (x_f, x_m)$.

4.1 The Household Decision under a General Mechanism

To understand how a couple behaves under a mechanism, we need to define disagreement and agreement choices. For a couple $\boldsymbol{\theta}$ with transactions $\boldsymbol{x}$, we denote the ***agreement utility set*** by $\mathbf{U}^{\mathrm{a}}(\boldsymbol{\theta}; \boldsymbol{x})$. Then, we summarize how resources are allocated in disagreement by a function that maps transactions into a pair of ***disagreement utilities***, $\bar{\boldsymbol{u}}(\cdot; \boldsymbol{\theta})\colon X^2 \mapsto \mathbb{R}^2$. Our results require only two general assumptions: disagreement utilities solely depend on the disagreement transactions, not on the specific mechanism, $\mathcal{M}$, and couples in agreement can get the disagreement utilities with the same transactions, $\bar{\boldsymbol{u}}(\boldsymbol{x}; \boldsymbol{\theta}) \in \mathbf{U}^{\mathrm{a}}(\boldsymbol{\theta}; \boldsymbol{x})$.[27]

Disagreement game equilibrium. Under the mechanism $\mathcal{M} = \{S_f, S_m, \boldsymbol{x}\}$, a wife chooses s_f to maximize her disagreement utility given her husband's choice, s_m.

[26] We do not explicitly consider singles in this section; however, we can accommodate them if we add a dummy productivity type and define someone "married" to a this new productivity as a single.

[27] The second assumption is only to avoid the (pathological) situation in which the agreement has no solution because no transaction gives more utility than the threat points.

Her choice defines a reaction function, $\bar{\jmath}_f \colon S_m \times \Theta^2 \mapsto S_f$, by

$$\bar{\jmath}_f(s_m; \boldsymbol{\theta}) := \operatorname*{argmax}_{s \in S_f} \bar{u}_f\left(\boldsymbol{x}(s, s_m); \boldsymbol{\theta}\right).$$

We analogously define $\bar{\jmath}_m$. The equilibrium choices in disagreement, $\bar{\boldsymbol{s}}^*$, are a fixed point of these reaction functions; $\bar{s}_f^* = \bar{\jmath}_f(\bar{s}_m^*; \boldsymbol{\theta})$ and $\bar{s}_m^* = \bar{\jmath}_m(\bar{s}_f^*; \boldsymbol{\theta})$. Moreover, the pair $\bar{\boldsymbol{s}}^*$ implies disagreement utilities under $\mathcal{M}$; $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{\theta}) := \bar{\boldsymbol{u}}\left(\boldsymbol{x}(\bar{\boldsymbol{s}}^*); \boldsymbol{\theta}\right)$.

We focus on pure strategy for simplicity, since we could define the disagreement utilities as expected utilities. Multiplicity is a more delicate issue. To handle it, we need spouses to agree on how an equilibrium is selected and the mechanism not to affect this selection.[28]

Agreement equilibrium. Given disagreement utilities $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{\theta})$, we define a household welfare function by

$$U_{\mathcal{M}}(u_f, u_m; \boldsymbol{\theta}) := \begin{cases} \left[u_f - \bar{u}_{f,\mathcal{M}}(\boldsymbol{\theta})\right]\left[u_m - \bar{u}_{m,\mathcal{M}}(\boldsymbol{\theta})\right] & \text{if} \quad u_i \geq \bar{u}_{i,\mathcal{M}}(\boldsymbol{\theta}), i = f, m \\ 0 & \text{otherwise.} \end{cases}$$

From a household welfare function aggregating individual utilities, we can define a household welfare function over bundles, $U_{\mathcal{M}}(\boldsymbol{x}; \boldsymbol{\theta}) := \max_{\boldsymbol{u} \in \mathbf{U}^{\mathrm{a}}(\boldsymbol{\theta}; \boldsymbol{x})} U_{\mathcal{M}}(u_f, u_m; \boldsymbol{\theta})$. The optimal choices are then given by $\boldsymbol{s}_{\mathcal{M}}(\boldsymbol{\theta}) := \operatorname{argmax} U_{\mathcal{M}}(\boldsymbol{x}(s_f, s_m); \boldsymbol{\theta})$. The equilibrium transactions are $\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}) := \boldsymbol{x}(\boldsymbol{s}_{\mathcal{M}}(\boldsymbol{\theta}))$, and the equilibrium utilities are $\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{\theta})$.

The ***allocation implemented*** by $\mathcal{M}$ is denoted by $\mathbf{A}_{\mathcal{M}}$ and is given by transactions $\boldsymbol{x}_{\mathcal{M}}(\cdot)$, utilities $\boldsymbol{u}_{\mathcal{M}}(\cdot)$, and couples distribution $\mu_{\mathcal{M}}(\cdot)$. It is essential to include the equilibrium utilities because they summarize how the mechanism impacts the household welfare function through threat points. Recalling that the function G is the economy's technology, allocation $\mathbf{A}_{\mathcal{M}}$ is ***feasible*** if and only if

$$G\left(\sum_{\boldsymbol{\theta}} \sum_{i=f,m} x_{i,\mathcal{M}}(\boldsymbol{\theta}) \mu_{\mathcal{M}}(\boldsymbol{\theta})\right) \leq 0.$$

4.2 Implementable Allocations

With a definition of the allocation implemented by a general mechanism, we can ask what the set of all implementable allocations is. To answer this question, we show that the revelation principle—Myerson (1979); Dasgupta et al. (1979); Harris and Townsend (1981)—remains valid if a spouse's type includes his/her productivity,

[28] We thank an anonymous referee for this remark.

his/her spouse's productivity, and whether the couple is in agreement or not.

A ***type*** is given by $(\theta_f, \theta_m, t) \in \Theta^2 \times \{a, d\}$, in which t represents agreement, a, or disagreement, d. A ***direct mechanism***, $\mathcal{D} = (S_f, S_m, \boldsymbol{x})$, has types as choice sets, that is $S_f = S_m = \Theta^2 \times \{a, d\}$.

Proposition 5 (revelation principle)**.** *Let $\mathbf{A}_{\mathcal{M}} = \{\boldsymbol{x}_{\mathcal{M}}, \boldsymbol{u}_{\mathcal{M}}, \mu_{\mathcal{M}}\}$ be the allocation implemented by $\mathcal{M}$. Then there exists a direct mechanism, $\mathcal{D}$, such that spouses truthfully report their types, and $\mathbf{A}_{\mathcal{D}} = \mathbf{A}_{\mathcal{M}}$.*

Proof. See Appendix A.5. □

Even though spouses in a couple share the same type, the planner cannot use this information to implement all the feasible allocations—as in Crémer and McLean (1988). Incentive compatibility matters for the implementable allocations because spouses act cooperatively in agreement.

Incentive-feasible allocations. An allocation $\mathbf{A}_{\mathcal{D}} = \{\boldsymbol{x}_{\mathcal{D}}, \boldsymbol{u}_{\mathcal{D}}, \mu_{\mathcal{D}}\}$, with $\boldsymbol{x}_{\mathcal{D}}(\boldsymbol{\theta}) = \boldsymbol{x}(\boldsymbol{\theta}, a, \boldsymbol{\theta}, a)$ for all $\boldsymbol{\theta}$, is incentive feasible if:

i) For all $\boldsymbol{\theta}$,

$$(\boldsymbol{\theta}, d) \in \bar{\mathcal{I}}_f(\boldsymbol{\theta}, d; \boldsymbol{\theta}) = \underset{\boldsymbol{\theta}',\, t'}{\operatorname{argmax}}\, \bar{u}_f\left(\boldsymbol{x}(\boldsymbol{\theta}', t', \boldsymbol{\theta}, d); \boldsymbol{\theta}\right),$$

and, similarly, $(\boldsymbol{\theta}, d) \in \bar{\mathcal{I}}_m(\boldsymbol{\theta}, d; \boldsymbol{\theta})$.

ii) For every couple $\boldsymbol{\theta}$ and $(\boldsymbol{\theta}', t', \boldsymbol{\theta}'', t'') \in (\Theta \times \{a, d\})^2$,

$$U_{\mathcal{D}}(\boldsymbol{x}(\boldsymbol{\theta}, a, \boldsymbol{\theta}, a); \boldsymbol{\theta}) \geq U_{\mathcal{D}}(\boldsymbol{x}(\boldsymbol{\theta}', t', \boldsymbol{\theta}'', t''); \boldsymbol{\theta}). \tag{11}$$

iii) $\mu_{\mathcal{D}}$ is a stable matching of the marriage problem.

iv) The allocation is feasible: $G\left(\sum_{\boldsymbol{\theta}} \sum_{i=f,m} x_i(\boldsymbol{\theta}, a, \boldsymbol{\theta}, a)\mu_{\mathcal{D}}(\boldsymbol{\theta})\right) \leq 0$.

Constraint (i) guarantees that truth-telling is best for spouses in disagreement. Given the threat points defined in the disagreement game, constraint (ii) guarantees that truth-telling is best for a couple in agreement. In (iii), more than one distribution μ could work as a stable matching, and we assume a given Gale-Shapley algorithm selects one; constraint (iii) is the only one directly affected by the emotional payoff, $\boldsymbol{\xi}$, and the courtship shock, ϵ. In (iv), feasibility is only required under agreement because the disagreement allocation does not happen in equilibrium.

4.3 Allocations implemented by a Tax Systems

To map a ***one-schedule*** tax system, T, to a mechanism, define the choice set as transactions, $S_i := X$. A transaction is feasible if the tax payments, $(z_f - y_f, z_m - y_m)$ are feasible given the labor earnings, (z_f, z_m); if a transaction is feasible we denote it by $(z_f - y_f, z_m - y_m) \in T(z_f, z_m)$, and we denote the set of feasible transaction for the tax schedule T by the budget set

$$\mathbf{B}_T := \Big\{(x_f, x_m) \in X^2 \; ; (z_f - y_f, z_m - y_m) \in T(z_f, z_m)\Big\}.$$

Our notation accommodates both tax schedules that specify a particular division of tax liabilities and tax schedules that only specify a couple's total tax liability—e.g., an income-splitting tax schedule.

If we add a second schedule $\overline{T}$ as filing option to T, we get a ***two-schedule*** tax system. Then, we can map the tax system to a mechanism with a choice set $S_i := X \times \{a, d\}$. If both spouses report a, then they use T; but if one of them reports d, then both must use $\overline{T}$.

The next proposition characterizes the allocations that can be implemented by a tax system. We denote the mechanism induced by a tax system as $\mathcal{T}$.

Proposition 6 (Tax-Implementable Allocations)**.** *With differentiable utility functions, an allocation $\mathbf{A}_{\mathcal{M}}$ can be implemented by a tax system $\mathcal{T}$ if and only if the equilibrium transactions are the same, $\boldsymbol{x}_{\mathcal{T}}(\cdot) = \boldsymbol{x}_{\mathcal{M}}(\cdot)$, and, for every couple $\boldsymbol{\theta}$, their disagreement utilities under $\mathcal{T}$ satisfy*

$$\bar{\boldsymbol{u}}_{\mathcal{T}}(\boldsymbol{\theta}) = p\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{\theta}) + (1-p)\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{\theta})$$

for some $p \leq 1$.

Proof. See Appendix A.5. □

Proposition 6 clarifies that, even if a tax system induces the same equilibrium transaction as a mechanism $\mathcal{M}$, it must *also* induce threat points that lead to the same distribution of utilities between spouses. If threat points were exogenous, then the tax system could focus on implementing the same equilibrium transactions. In that case, even a one-schedule tax system would be sufficient by the taxation principle.

With endogenous threat points, a one-schedule tax system may not suffice, and a two-schedule tax system can be used to target threat points more precisely. The planner has two instruments for two objectives: T to induce choices conditional on threat points, and $\overline{T}$ to induce better threat points. The proposition below shows that if no spouses have the same earnings in disagreement, a two-schedule tax system is

sufficient to implement any (incentive-feasible) allocation.[29]

Proposition 7. *Given an mechanism $\mathcal{M}$, suppose that the disagreement labor earnings, $\bar{z}_{f,\mathcal{M}}(\cdot)$ and $\bar{z}_{m,\mathcal{M}}(\cdot)$, are both one-to-one, that is we can recover $\boldsymbol{\theta}$ from each spouse separately, then there is a two-schedule tax system that implements $\mathbf{A}_{\mathcal{M}}$.*

Proof. See Appendix A.5 □

However, even a two-schedule tax system may not implement all incentive-feasible allocations for a subtle reason. If spouses from different couples have the same earnings, then a tax schedule is limited in ways that the direct mechanism is not. For example, if the wives from two couples have low productivity and decide not to work, then a tax schedule cannot costlessly separate their husbands. A different instrument—e.g., a means-tested program—can be useful in that case, since only a wife of a not-too-productive husband would enroll in the program.

4.4 Implementable Threat Points

A two-schedule tax system may fail to implement an incentive-feasible allocation because it is restricted in ways that a direct mechanism is not. A natural question is what restricts the implementable threat points. The only restriction is that no couple in agreement envies a disagreement transaction. The planner is unrestricted otherwise because spouses in disagreement behave non-cooperatively.

Given a mechanism $\mathcal{M}$, any feasible transaction $\mathbf{X}_{\mathcal{M}}$ can be used as transactions in disagreement without affecting the decisions in agreement because couples already had access to these transactions in agreement.

Proposition 8. *Given a mechanism $\mathcal{M}$ and a couple $\boldsymbol{\theta}$, define the set of disagreement utilities attainable with feasible transactions as $\mathbf{U}^{\mathrm{d}}_{\mathcal{M}}(\boldsymbol{\theta}) := \{\bar{\boldsymbol{u}}(\boldsymbol{x};\boldsymbol{\theta}) \mid \boldsymbol{x} \in \mathbf{X}_{\mathcal{M}}\}$. Then, for any $\bar{\boldsymbol{u}} \in \mathbf{U}^{\mathrm{d}}_{\mathcal{M}}(\boldsymbol{\theta})$, there is a another mechanism $\mathcal{N}$ such that $\boldsymbol{x}_{\mathcal{N}}(\boldsymbol{\theta}') = \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}')$ and $\boldsymbol{u}_{\mathcal{N}}(\boldsymbol{\theta}') = \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{\theta}')$ for all $\boldsymbol{\theta}' \neq \boldsymbol{\theta}$; and $\bar{\boldsymbol{u}}_{\mathcal{N}}(\boldsymbol{\theta}) = \bar{\boldsymbol{u}}$.*

Proof. See Appendix A.5 □

The non-cooperative behavior of spouses in disagreement allows for a mechanism to affect threat points at a zero cost. The only restriction is that no couple in agreement would instead announce to be in disagreement, and Proposition 8 provides a sufficient condition for this to be the case. In contrast, a necessary condition is that disagreement transactions cannot be strictly larger than any agreement transactions.

[29] These conditions are only sufficient. A planner could implement an allocation in which spouses from different couples have the same earnings if their partners did not envy each other.

Proposition 9. *For a given $\bar{\boldsymbol{u}}$, assume that for all $\boldsymbol{x}$ for which $\bar{\boldsymbol{u}} = \bar{\boldsymbol{u}}(\boldsymbol{x}, \boldsymbol{\theta})$ there is $\boldsymbol{\theta}'$ such that $\boldsymbol{x}_m(\boldsymbol{\theta}') < \boldsymbol{x}$. Then, $\bar{\boldsymbol{u}}$ cannot be chosen without changing the equilibrium allocation.*

Proof. See Appendix A.5 □

Propositions 8 and 9 define a set of implementable threat points for a couple $\boldsymbol{\theta}$ without changing the equilibrium transactions for other couples. In Appendix A.2, we offer a heuristic account of how a planner could use this latitude to improve upon an initial allocation.

4.5 Generalizing Types and Preferences

For simplicity, our analysis has restricted heterogeneity to be uni-dimensional and preferences to be defined over a consumption/leisure bundle, but our results do not depend on these assumptions. Indeed, if we let productivities and transactions be a subset of a general Euclidean space, then all the definitions would go through; furthermore, these definitions already allow for transactions to map to an arbitrary set of goods within a household.

This extension can incorporate relevant policy aspects of households: e.g., household production, with separate productivity for home duties; public goods; and other restrictions on utility transferability within a household.

5 Conclusion

We have extended optimal taxation to a Nash-bargaining collective household with threat points endogenous to policy; endogenous threat points impact optimal tax formulae and the set of relevant policy instruments. When the planner only have access to one tax schedule, the optimal tax formulae depend on non-standard statistics that are not solely a function of marginal retention rates and virtual income. These non-local effects connect collective household taxation to the taxation of behavioral agents as in Farhi and Gabaix (2020). However, exploring the structure of a collective household, we went further and showed that filing options—found in the U.S. tax system—can eliminate these non-local effects. This new result implies a failure of the taxation principle; auxiliary policy instruments can expand the set of implementable allocations.

To understand how relevant this failure can be, we calibrated our model to the U.S. economy and considered a counterfactual without filing options. We found effects comparable to replacing the whole U.S. joint-filing system with mandatory individual filing.

We have also provided an extension of Mirrlees's framework to a collective household and showed these results to be more general. The taxation principle fails even for a general tax schedule. Moreover, we characterize the allocations implemented with a tax schedule and auxiliary instruments, e.g., filing options.

Our model relies on internal threat points and delivers a tractable framework both from an empirical and analytical perspective. If we depart from internal threat points and allow for pre-marital contracts to directly affect the household decision, we lose some of the tractability. However, unless bargaining in marriage is entirely irrelevant, the taxation principle is bound to fail, and our results will still matter for optimal redistributive policies.

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A Appendix

A.1 Tax Formula with Endogenous Marriage Market

The tax formulae in Propositions 1 and 2 assumed a fixed distribution of households. However, if a policy affects utility, then it changes the value of each marriage market potential outcome; whether to marry and whom to marry is a choice that depends on expected utility from marriage. A formula encompassing the full impact of perturbations requires taking these marriage market effects into account.

Because the marriage equilibrium does not directly impact the family decision for a given type, we can decompose the effect of tax perturbations in changes in decisions conditional on the marriage distribution and changes in the distribution of couples ignoring mechanical and behavioral effects.

The effect on revenues from a tax increase at z' through changes in the distribution of couples is given by

$$\underbrace{\int_{\boldsymbol{\theta}} T(z(\boldsymbol{\theta}))\left.\frac{\partial \mu(\boldsymbol{\theta})}{\partial \tau}\right|_{z'} \mathrm{d}\boldsymbol{\theta}}_{\text{(I)}} + \sum_{i=f,m} \int_{\theta_i} T_i^{\mathrm{s}}(z_i^{\mathrm{s}}(\theta_i))\left.\frac{\partial \mu_i(\theta_i)}{\partial \tau}\right|_{z'} \mathrm{d}\theta_i. \tag{12}$$

Similarly, for welfare we have,

$$\underbrace{\int_{\boldsymbol{\theta}} W(\boldsymbol{\theta})\left.\frac{\partial \mu(\boldsymbol{\theta})}{\partial \tau}\right|_{z'} \mathrm{d}\boldsymbol{\theta}}_{\text{(II)}} + \sum_{i=f,m} \int_{\theta_i} W_i^{\mathrm{s}}(\theta_i)\left.\frac{\partial \mu_i(\theta_i)}{\partial \tau}\right|_{z'} \mathrm{d}\theta_i. \tag{13}$$

The first term in (12), can be rewritten as

$$
\begin{aligned}
(\mathrm{I}) = \int_z T(z) \int \frac{1}{\mu(\boldsymbol{\theta})} \left.\frac{\partial \mu(\boldsymbol{\theta})}{\partial \tau}\right|_{z'} \mu(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}\, \mathrm{d}z \\
= \int_z T(z) \int \left.\frac{\partial \log \mu(\boldsymbol{\theta})}{\partial \tau}\right|_{z'} \phi(\boldsymbol{\theta}; z)\, \mathrm{d}\boldsymbol{\theta} \phi(z)\, \mathrm{d}z \\
= \int_z T(z) \left.\frac{\partial \widetilde{\log \mu(z)}}{\partial \tau}\right|_{z'} \phi(z)\, \mathrm{d}z.
\end{aligned}
$$

Recall that ~ denotes an average over all households with the same earnings as defined in (7).

For welfare, because $W(\boldsymbol{\theta})$ can differ even for couples with the same earnings, a covariance term appears in the formula, and the first term in (13) becomes

$$
(\mathrm{II}) = \int_z \left\{ \tilde{W}(z) \left.\frac{\partial \widetilde{\log \mu(z)}}{\partial \tau}\right|_{z'} + \widetilde{\mathrm{Cov}} \left[W, \left.\frac{\partial \log \mu(\cdot)}{\partial \tau}\right|_{z'} \right](z) \right\} \phi(z)\, \mathrm{d}z.
$$

With these modifications, add (12) and (13) to get the term that correspond to the effect of changes in the marriage market distribution,

$$
\begin{aligned}
\mathbb{E} \left[\left(\widetilde{W}(z) + T(z) \right) \left.\frac{\partial \widetilde{\log(\mu)}}{\partial \tau}\right|_{z'} (z) + \widetilde{\mathrm{Cov}} \left[W, \left.\frac{\partial \log(\mu)}{\partial \tau}\right|_{z'} \right](z) \right] \\
+ \sum_{i=f,m} \int_\theta \left(W_i^{\mathrm{s}}(\theta) + T_i^{\mathrm{s}}(\theta) \right) \left.\frac{\partial \mu_i^{\mathrm{s}}}{\partial \tau}\right|_{z'} (\theta)\, \mathrm{d}\theta. \quad (14)
\end{aligned}
$$

The optimal tax formula taking into account the endogenous marriage market is given by adding (14) to the effects identified in Section 2 with an exogenous distribution of types—Propositions 1 and 2.

The impact of changes in taxes on the distribution of types can be hard to obtain in closed-form but can be simulated given a particular marriage market algorithm. An example is given in Section 3 when we consider gender-based taxation.

A.2 Better Allocations

From Subsection 4.3, we know the planner enjoys much latitude to manipulate threat points. Here we discuss two ways in which the planner can use this latitude produce better allocations.

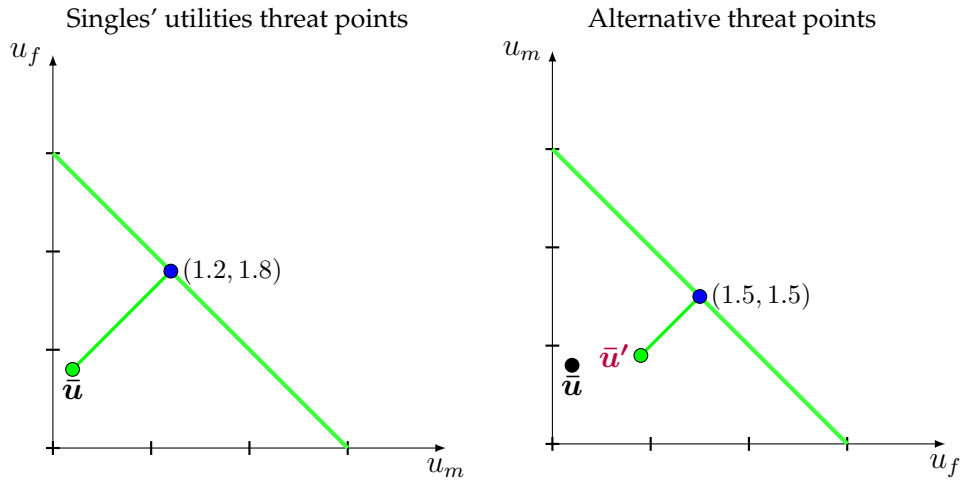

Figure 1: **Aligning Objectives** The panel in the left displays the set of individually rational utility pairs, when the threat point — green (light) dot — is induced by the tax schedule in place. The blue (dark) dot denotes the optimal choice for the couple. If the planner is able to induce a different threat point — green dot in the right panel — a new choice distribution of utilities is induced for the same transactions.

Threat points and dissonance Assume that utility is transferable between spouses. This example's convenient feature is that households choose transactions to maximize their surplus regardless of how it will be shared, as formalized in Lemma 1.

Under this assumption, the relevant utility possibility set for a given couple is displayed in Figure 1. The pair of utilities that arise from the Nash bargaining is given by a 45 degrees line from $(\bar{u}_f, \bar{u}_m)$ to the frontier of the utility possibility set. For concreteness, assume that these threat points arise from a disagreement game played under the tax schedule in place.

Assume that the planner's objective is to maximize the sum of the log of people's utilities. In this case, the household and the planner's objectives are aligned if and only if $\bar{u}_f = \bar{u}_m$, as this would preserve couples' transactions while increasing the value of the planner's objective.[30]

So if $\bar{\boldsymbol{u}}'$ in Figure 1 belongs to $\mathbf{U}_T^{\mathrm{d}}(\boldsymbol{\theta})$, defined in Proposition 8, the planner will manipulate the tax schedule to implement this disagreement utility.

Threat points and the ETI. In Figure 2, the convex set bounded by the green (light) curve is the utility possibility set for a couple $\boldsymbol{\theta}$ who has chosen $\boldsymbol{x} = (y_f, -z_f, y_m, -z_m)$. The set bounded by the blue (dark) curve is the utility possibility set had the couple

[30] da Costa and Diniz (2016) explore the consequences of filing options for various piecewise linear tax schedules with transferable utilities. Although they take a different approach regarding the disagreement game, the rationale for their results is the same—see the discussion of their approach in Chiappori and Mazzocco (2017).

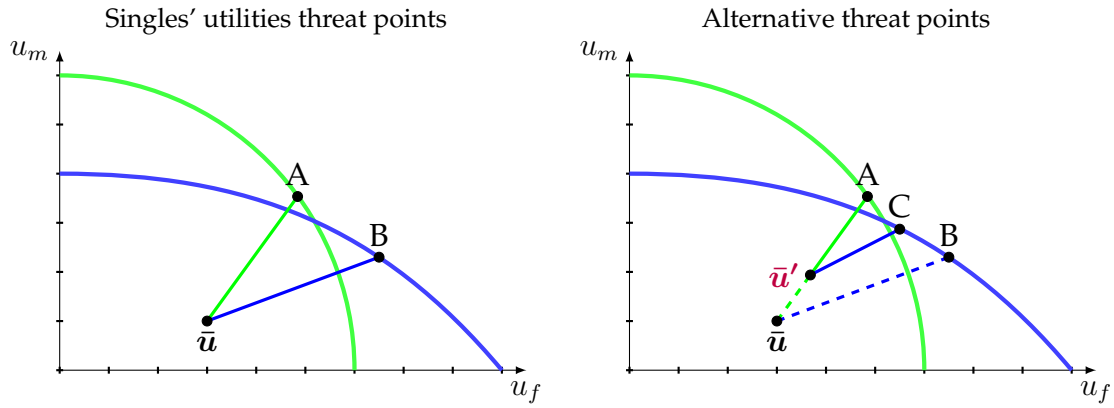

Figure 2: **Breaking Indifference:** The green (light) curve is the frontier of the household's utility possibility for a true report. The blue (dark) curve is the utility frontier for the same couple for the case in which a false report $\boldsymbol{\theta}' \neq \boldsymbol{\theta}$ is made. Point B denotes the utility pair the couple would attain in case of a lie. If the threat points changes from $\bar{\boldsymbol{u}} = (\bar{u}_f, \bar{u}_m)$ to $\bar{\boldsymbol{u}}' = (\bar{u}'_f, \bar{u}'_m)$ equilibrium choices are not changed, yet the deviation utility pair changes from B to C.

chosen $\boldsymbol{x}'$, instead. Point A in the left panel denotes the optimal choice of utilities for the household given $\boldsymbol{x}$. In contrast, point B denotes the chosen utility pair were the couple to choose transactions $\boldsymbol{x}'$, instead.

The green (light) line connecting $(\bar{u}_f, \bar{u}_m)$ to point A defines the set of all potential threat points (closer to A than $\bar{\boldsymbol{u}}$) that have A as the optimal choice for this couple, given $\boldsymbol{x}$. The analogous locus for the couple if $\boldsymbol{x}'$ was chosen is the blue (dark) line connecting $\bar{\boldsymbol{u}}$ to B.

Assume that, when the threat point is $\bar{\boldsymbol{u}}$, points A and B produce the same value for the Nash product. That is, the couple is just indifferent between transactions $\boldsymbol{x}$ and $\boldsymbol{x}'$.[31] The couple's choice, $\boldsymbol{x}$, is, by assumption, the one preferred by the planners' metric. Because the household is just indifferent, the planner cannot increase taxes for transaction $\boldsymbol{x}$, as this would lead the couple to prefer $\boldsymbol{x}'$ strictly.

Now, assume that the planner can replace $\bar{\boldsymbol{u}}$ by $\bar{\boldsymbol{u}}'$ in Figure 2.[32] Point A remains the optimal choice for the couple if it chooses $\boldsymbol{x}$ while point B is no longer optimal for the couple if it chooses $\boldsymbol{x}'$. A is now strictly better than C or any other utility pair attainable with transactions $\boldsymbol{x}'$; hence, $\boldsymbol{x}$ strictly preferred to $\boldsymbol{x}'$, and some room is created for a larger tax reform. We formalize this argument in Lemma 2.

Essentially, by moving along the curve, we are reducing the flexibility that a couple has to substitute the utility of one spouse for the other—Figure 3. We are in practice changing the relevant elasticities. This reform is useful whenever the

[31]Recall that axiom 'Preference for symmetry', as defined by Zambrano (2016), substitutes for 'symmetry' to deal with the non-convexity in the utility possibility sets.

[32]In Example 2.6, this type of change is attained with the introduction of filing options if spouses have the same productivity.

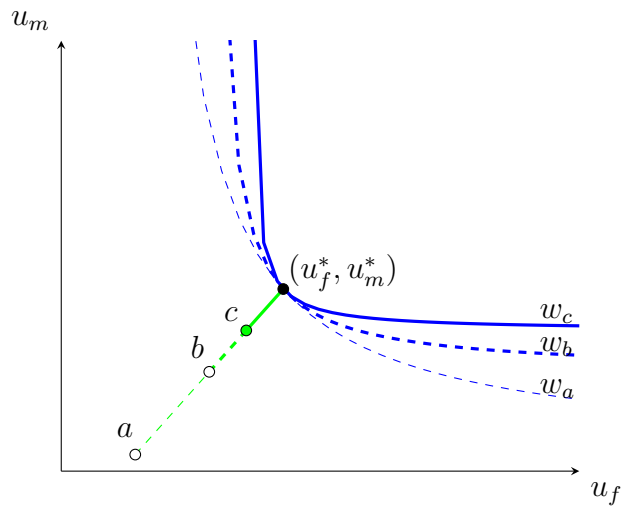

Figure 3: **Changing Elasticities.** The figure shows how the set of utility pairs which are preferred to (u_f^*, u_m^*) shrinks as we move along the curve connecting point a to point (u_f^*, u_m^*).

original allocation is distorted due to incentive provision, which we formalize in Lemma 3.

A.3 Elasticities

Given a tax schedule T and a type $\boldsymbol{\theta}$ couple, we have, as defined in (2), $z(\boldsymbol{\theta}) := \operatorname{argmax}_z U(z, z - T(z) \mid \boldsymbol{\theta}, \bar{\boldsymbol{u}}(\boldsymbol{\theta}))$. Consider the local perturbation

$$z(1-\tau, I; \boldsymbol{\theta}) := \operatorname*{argmax}_z U(z, (1-\tau)z - T(z) + I \mid \boldsymbol{\theta}, \bar{\boldsymbol{u}}(\boldsymbol{\theta})).$$

We have

$$\frac{\partial z}{\partial(1-\tau)} = \frac{U_{zy}z + U_y - U_{yy}\frac{U_z}{U_y}z}{U_y T'' - U_{zz} + 2U_{zy}\frac{U_z}{U_y} - U_{yy}\left(\frac{U_z}{U_y}\right)^2}$$

and

$$\frac{\partial z}{\partial I} = \frac{U_{zy} - U_{yy}\frac{U_z}{U_y}}{U_y T'' - U_{zz} + 2U_{zy}\frac{U_z}{U_y} - U_{yy}\left(\frac{U_z}{U_y}\right)^2},$$

where the functions are evaluated at $z(1, 0; \boldsymbol{\theta})$, that is at $\tau = I = 0$.

The relevant elasticities are then given by,

$$\varepsilon(z \mid \boldsymbol{\theta}) := \frac{\partial z}{\partial(1-\tau)} \frac{1 - T'(z)}{z},$$

$$\eta(z \mid \boldsymbol{\theta}) := -\frac{\partial z}{\partial I}(1 - T'(z)),$$

and

$$\varepsilon^{c}(z \mid \boldsymbol{\theta}) := \varepsilon(z \mid \boldsymbol{\theta}) + \eta(z \mid \boldsymbol{\theta})$$

A.4 Calibrated Model Marriage Market

In section 3, we calibrate the productivity using the fact that for symmetric couples with productivity θ we have[33]

$$\theta = \frac{1}{2}\frac{A+1-\rho}{1-\rho} z(\theta, \theta).$$

This implies a distribution of types in the population.

Given this distribution, a guess for the emotional payoff, $\boldsymbol{\xi}$, the equilibrium utilities for each possible couple, and our assumption that courtship shocks follow an extreme value type 1 distribution, we can compute the implied distribution for extended types $\vartheta = (\theta, \succeq)$.

Next, we split the population into two identical sets. In one, we apply a Gale-Shapley algorithm in which women propose first, while in the other, the men start the courtship. This algorithm is gender-neutral, in the sense that the equilibrium would be the same if we swapped the gender of all participants (except for the gender).

This procedure does not necessarily generate a stable match. It will be stable within each of the two sets, but not necessarily across, unless the pairs of extended types with positive matching mass, $\nu(\vartheta_f, \vartheta_m) > 0$, is the same in both sets.[34] This is the case in our numerical calibration for the baseline and all exercises.

Using this algorithm, we calibrate the emotional payoff $\boldsymbol{\xi}$ to match the observed shares of couples in the population.

A.5 Proofs

Proof of Proposition 5. Given a mechanism, $\mathcal{M} = \{S_i, \boldsymbol{x}\}$, and its allocation, $\mathbf{A}_{\mathcal{M}} = \{\boldsymbol{x}_{\mathcal{M}}, \boldsymbol{u}_{\mathcal{M}}, \mu_{\mathcal{M}}\}$, we divide the proof in two steps. First, we show that if threat points are fixed at $\bar{\boldsymbol{u}}_{\mathcal{M}}$, then we can find a direct mechanism that truthfully implements the equilibrium allocation in agreement. Then we show that this direct mechanism implements the threat points truthfully.

[33] We get the same productivities if we use the singles to calibrate the model.

[34] By contradiction, if there was a woman and a man from different sets that would prefer to dissolve their respective marriages and marry each other, then, because the matches with positive probability are the same in both sets and preferences depend only on the extended type, there would be a woman and a man from the same set that would prefer to dissolve their respective marriages. Contradiction with a stable match within each set.

Define a direct mechanism $\mathcal{D} = \{S_i', \boldsymbol{x}'\}$ with $S_i' = \Theta^2 \times \{a, d\}$ and

$$\boldsymbol{x}'(\boldsymbol{\theta}, t, \boldsymbol{\theta}', t') = \begin{cases} \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}) & \text{if } \boldsymbol{\theta} = \boldsymbol{\theta}' \text{ and } t = t' = a \\ \bar{\boldsymbol{x}}_{\mathcal{M}}(\boldsymbol{\theta}) & \text{if } \boldsymbol{\theta} = \boldsymbol{\theta}' \text{ and } t = t' = d \\ \mathbf{0} & \text{otherwise} \end{cases}.$$

Direct truthful mechanism given $\bar{\boldsymbol{u}}_{\mathcal{M}}$. For any $\boldsymbol{\theta}$, and report $(\boldsymbol{\theta}', t', \boldsymbol{\theta}'', t'')$, if threat points are given by $\bar{\boldsymbol{u}}_{\mathcal{M}}$, then we have

$$\begin{aligned} U(\boldsymbol{x}'(\boldsymbol{\theta}, a, \boldsymbol{\theta}, a); \boldsymbol{\theta}) &= U(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}); \boldsymbol{\theta}) \\ &\geq \max\{U(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}'); \boldsymbol{\theta}), U(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}''); \boldsymbol{\theta}), U(\bar{\boldsymbol{x}}_{\mathcal{M}}(\boldsymbol{\theta}'); \boldsymbol{\theta}), U(\bar{\boldsymbol{x}}_{\mathcal{M}}(\boldsymbol{\theta}''); \boldsymbol{\theta}), U(\mathbf{0}; \boldsymbol{\theta})\} \\ &\geq U(\boldsymbol{x}'(\boldsymbol{\theta}', t', \boldsymbol{\theta}'', t''); \boldsymbol{\theta}), \end{aligned}$$

which implies that telling the truth is an equilibrium of the direct mechanism $\mathcal{D}$ and, by construction, this equilibrium implements the same allocation as $\mathcal{M}$.

Direct truthful mechanism implements $\bar{\boldsymbol{u}}_{\mathcal{M}}$. Consider a couple in disagreement with productivities $\boldsymbol{\theta}$, to show that $\bar{\boldsymbol{u}}_{\mathcal{M}}$ is an equilibrium assume that one of the spouses, say the wife, announces $(\boldsymbol{\theta}, d)$. Any different announcement from the husband will result in a zero allocation, so it is optimal to announce $(\boldsymbol{\theta}, d)$ as well.

Alternative direct truthful mechanism. There are other direct truthful mechanisms that implement the same allocation without extreme punishment for lies. For instance, a direct mechanism with an allocation function given by

$$\boldsymbol{x}''(\boldsymbol{\theta}, t, \boldsymbol{\theta}', t') = \begin{cases} \boldsymbol{x}\big(s_{f,\mathcal{M}}(\boldsymbol{\theta}), s_{m,\mathcal{M}}(\boldsymbol{\theta}')\big) & \text{if } t = a, t' = a \\ \boldsymbol{x}\left(\bar{s}^*_{f,\mathcal{M}}(\boldsymbol{\theta}), s_{m,\mathcal{M}}(\boldsymbol{\theta}')\right) & \text{if } t = d, t' = a \\ \boldsymbol{x}\left(s_{f,\mathcal{M}}(\boldsymbol{\theta}), \bar{s}^*_{m,\mathcal{M}}(\boldsymbol{\theta}')\right) & \text{if } t = a, t' = d \\ \boldsymbol{x}\left(\bar{s}^*_{f,\mathcal{M}}(\boldsymbol{\theta}), \bar{s}^*_{m,\mathcal{M}}(\boldsymbol{\theta}')\right) & \text{if } t = d, t' = d \end{cases}.$$

also has truth-telling as an equilibrium by essentially the same argument as for $\mathcal{D}$.

□

Proof of Proposition 6. Given a couple $\boldsymbol{\theta}$ and a mechanism $\mathcal{M}$, suppose a tax system $\mathcal{T}$ implements the same transactions as $\mathcal{M}$ and induces disagreement utilities in the line defined by $\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{\theta})$ and $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{\theta})$. Then, by Lemma 2, the utility division for $\mathcal{T}$ and $\mathcal{M}$ are the same, that is $\boldsymbol{u}_{\mathcal{T}}(\boldsymbol{\theta}) = \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{\theta})$.

In the other direction, if transactions differ, then by definition the tax system $\mathcal{T}$ and the mechanism $\mathcal{M}$ have different allocations. So assume they have the same transactions, $\boldsymbol{x}_{\mathcal{T}}(\cdot) = \boldsymbol{x}_{\mathcal{M}}(\cdot)$. Given a couple $\boldsymbol{\theta}$, the utility frontier of $\mathbf{U}^{\mathrm{a}}(\boldsymbol{\theta}; \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}))$ is smooth if we have differentiable utility functions; therefore, by Lemma 2, the only way $\mathcal{T}$ can implement the same utility split as $\mathcal{M}$ is if disagreement utilities lie in the line defined by $\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{\theta})$ and $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{\theta})$. □

Proof of Proposition 7. First, conditional on threat points, the standard taxation principle guarantees that we can find a budget set B_T that implements $\boldsymbol{x}_{\mathcal{M}}(\cdot)$.

Second, define a fall-back schedule $\overline{T}$ so that

$$B_{\overline{T}} = \left\{ (\bar{x}_f, \bar{x}_m) \in X^2 \text{ ; there exists } \boldsymbol{\theta} \text{ such that } (\bar{x}_f, \bar{x}_m) = (\bar{x}_{f,\mathcal{M}}(\boldsymbol{\theta}), \bar{x}_{m,\mathcal{M}}(\boldsymbol{\theta})) \right\}$$

Because $\bar{z}_{i,\mathcal{M}}(\cdot)$ are one-to-one, if a spouse from a couple $\boldsymbol{\theta}$ uses the fall-back schedule $B_{\overline{T}}$ and chooses earnings $\bar{z}_{i,\mathcal{M}}(\boldsymbol{\theta})$, then the only available transaction for the couple is $(\bar{x}_{f,\mathcal{M}}(\boldsymbol{\theta}), \bar{x}_{m,\mathcal{M}}(\boldsymbol{\theta}))$. Therefore $B_{\overline{T}}$ induces the same disagreement utilities as $\mathcal{M}$. Furthermore, because all transaction in $B_{\overline{T}}$ were feasible in $\mathcal{M}$, the fall-back schedule doesn't affect the equilibrium allocation. □

Proof of Proposition 8. Given $\bar{\boldsymbol{u}} \in \mathbf{U}^{\mathrm{d}}_{\mathcal{M}}(\boldsymbol{\theta})$, let $\bar{\boldsymbol{x}}$ be such that $\bar{\boldsymbol{u}} = \bar{\boldsymbol{u}}(\bar{\boldsymbol{x}}; \boldsymbol{\theta})$. The proof of proposition 5 shows that we can construct a direct mechanism $\mathcal{D} = \{\Theta^2 \times \{a, d\}, \boldsymbol{x}\}$ such that $\bar{s}^*_{f,\mathcal{D}}(\boldsymbol{\theta}) = \bar{s}^*_{m,\mathcal{D}}(\boldsymbol{\theta}) = (\boldsymbol{\theta}, d)$ as long as the allocation function $\boldsymbol{x}$ punishes harshly lies. In particular, we can implement $\bar{\boldsymbol{x}}$ in disagreement for couple $\boldsymbol{\theta}$ without affecting the disagreement choices of any other couple.

For a couple $\boldsymbol{\theta}' \neq \boldsymbol{\theta}$, if their disagreement utility are the same, and if transaction $\bar{\boldsymbol{x}}$ was feasible at the original mechanism $\mathcal{M}$, that is, if $\bar{\boldsymbol{x}} \in \mathbf{X}_{\mathcal{M}}$, then their optimal allocation is unchanged and we get $\boldsymbol{x}_{\mathcal{D}}(\boldsymbol{\theta}') = \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}')$. □

Proof of Proposition 9. For any $\boldsymbol{\theta}'$, $U_{\mathcal{M}}(\boldsymbol{x}; \boldsymbol{\theta}')$ is strictly increasing in $\boldsymbol{x}$.[35] Therefore, couple $\boldsymbol{\theta}'$ would change their equilibrium allocation if $\boldsymbol{x} > \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta}')$ was feasible. □

A.6 Lemmata

Lemma 1. *Assume that utilities representing people's preferences are of the form $u_i(c, l) = c + \hbar(l)$ for $\hbar(\cdot)$ strictly increasing and concave. Then, optimal transactions are independent of threat points.*

[35] With respect to the component-wise partial order on X^2.

Proof. For transactions $\boldsymbol{x}$, define $h_i = \hbar(1 - z_i/\theta_i)$, $i = f, m$. Given these transaction the household solves

$$\max_{c_f} \left(c_f + h_f - \bar{u}_f\right)\left(\alpha y - c_f + h_m - \bar{u}_m\right),$$

for $y = y_f + y_m$.

At the optimum, for the maximization problems above, we have

$$c_f + h_f - \bar{u}_f = \alpha y - c_f + h_m - \bar{u}_m,$$

and the value of the program, for a given $\boldsymbol{x}$ is, therefore,

$$\frac{1}{4}\Big[\alpha y + (h_f - \bar{u}_f) + (h_m - \bar{u}_m)\Big]^2.$$

It follows that transaction $\boldsymbol{x}$ is preferred to $\boldsymbol{x}'$ if and only if

$$\Big[\alpha y + (h_f - \bar{u}_f) + (h_m - \bar{u}_m)\Big]^2 \geq \Big[\alpha y' + (h'_f - \bar{u}_f) + (h'_m - \bar{u}_m)\Big]^2,$$

which is equivalent to $\alpha y + h_f + h_m \geq \alpha y' + h'_f + h'_m$ □

Lemma 2. *Given a mechanism $\mathcal{M}$, assume*

$$U_m(\boldsymbol{x}_m(\boldsymbol{\theta}); \boldsymbol{\theta}) = U_m(\boldsymbol{x}'; \boldsymbol{\theta})$$

for some $\boldsymbol{x}'$ such that

$$\boldsymbol{u}_m(\boldsymbol{\theta}) \notin \underset{\boldsymbol{u} \in \mathbf{U}^{\mathrm{a}}(\boldsymbol{\theta}; \boldsymbol{x}')}{\operatorname{argmax}} U_m(\boldsymbol{u}; \boldsymbol{\theta}).$$

That is, $\boldsymbol{x}'$ gives the same Nash product but generates a different split in utilities.

Assume that there is another mechanism $\mathcal{N}$ that satisfies

$$\bar{\boldsymbol{u}}_n(\boldsymbol{\theta}) = p\boldsymbol{u}_m(\boldsymbol{\theta}) + (1 - p)\bar{\boldsymbol{u}}_m(\boldsymbol{\theta})$$

for $p \in (0, 1)$. That is, $\mathcal{N}$ moves the disagreement utility of $\mathcal{M}$ closer to its agreement utility.

Then,

$$U_n(\boldsymbol{x}_m(\boldsymbol{\theta}); \boldsymbol{\theta}) > U_n(\boldsymbol{x}'; \boldsymbol{\theta}).$$

Under the disagreement utilities induced by $\mathcal{N}$, the original transaction is strictly preferred to $\boldsymbol{x}'$.

Proof. Without loss, let the spouses utilities at the original threat point be $(0, 0)$. Let (u_f, u_m) be the corresponding agreement solution. We know, in this case, that $u_f u_m \geq u'_f u'_m$ for any feasible utility pair (u'_f, u'_m).

Using that the function $f(x) = (x-p)\left(\frac{1}{x} - p\right)$ is uniquely maximized at $x = 1$ it follows that if $u_f \neq u'_f$, then

$$\left(\frac{u'_f}{u_f} - p\right)\left(\frac{u'_m}{u_m} - p\right) \leq \left(\frac{u'_f}{u_f} - p\right)\left(\frac{u_f}{u'_f} - p\right) < \left(1 - p^2\right)$$

if $u'_f > pu_f$.

Multiply both sides by $u_f u_m$ to get

$$(u'_f - pu_f)(u'_m - pu_m) < (1-p)^2 u_f u_m = (u_f - pu_f)(u_m - pu_m)$$

The same proof shows that if the utility possibility set is smooth and convex, then we implement the same split in utilities if and only if $p \leq 1$.

A simpler way to see this is to notice that we can also assume, without loss of generality, that $u := u_f = u_m$ by rescaling the utilities. By symmetry, if the UPS is smooth and convex, the Nash solution must be (u, u) if and only if the threat points are also symmetric, that is, something of the form (pu, pu) with $p \leq 1$. □

Lemma 3. *For a given mechanism $\mathcal{M}$, assume that there is another mechanism $\mathcal{N}$ such that it induces the same equilibrium allocation as $\mathcal{M}$, for a subset $\mathbf{\Theta}_> \subset \Theta^2$ it increases disagreement utilities as in Lemma 2, and, for the rest, it does not affect the disagreement utilities.*

Define the set $\mathbf{\Theta}_< \subset \Theta^2$ of households such that for $\mathcal{M}$ their allocation is either envied, in the sense of a binding incentive-compatibility constraint, by no one or only envied by households in the set $\mathbf{\Theta}_>$. Then, one must be true: No transaction for households in $\mathbf{\Theta}_<$ is distorted, or; the allocation $\boldsymbol{x}_{\mathcal{M}}(\cdot)$ is constrained inefficient.

Proof. By Lemma 2, households in $\mathbf{\Theta}_<$ are envied by no one for the mechanism $\mathcal{N}$. Therefore, either their allocations is not distorted or it can be improved while respecting incentive compatibility, which implies that $\boldsymbol{x}_{\mathcal{N}}(\cdot) = \boldsymbol{x}_{\mathcal{M}}(\cdot)$ is constrained inefficient. □