

Consumption and Portfolio Decisions When Expected Returns Are Time Varying

Erratum

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Our article “Consumption and Portfolio Decisions When Expected Returns Are Time Varying” (*Quarterly Journal of Economics* 114:433–495, May 1999) contained an error in the estimation of the time-series model that was the basis for the calibration exercise in the paper. This erratum describes in detail the estimation error and presents new calibration results based on a correct estimation of the model. These results strengthen, rather than weaken, the conclusions of the paper: When the model is calibrated to US stock market data it implies that intertemporal hedging motives greatly increase the average demand for stocks by investors whose risk-aversion coefficients exceed one. The optimal portfolio policy also involves timing the stock market. Failure to time or to hedge can cause large welfare losses relative to the optimal policy.

I The problem

The article “Consumption and Portfolio Decisions When Expected Returns Are Time Varying” that we published in the *Quarterly Journal of Economics* in May 1999 contained an error in the estimation of the time-series model that was the basis for the calibration exercise in the paper. The error is in the timing of the data used to estimate the model of Table I. In constructing quarterly stock returns from monthly data, returns were incorrectly lagged one month. Specifically, we regressed quarterly log stock returns for quarters December-February, March-May, June-August, etc., in excess of T-bill rates for quarters January-March, April-June, July-September, etc., onto log dividend price ratios computed as of the end of December, March, June, etc. The correct regression should have involved quarterly log stock returns for quarters January-March, May-June, July-September, etc.¹ This invalidates both the parameter estimates and the implied consumption and portfolio rules. Conditional on the reported parameter estimates, however, the reported consumption and portfolio rules are correct.

Table I in this Erratum shows the correct estimation results. The subtle timing error in the QJE paper has the effect of reducing the predictability of stock returns, and more important, it changes the correlation of stock return innovations with D/P innovations from -0.96 (the true value) to -0.73 (the value used in the QJE paper).

We discovered the problem in the course of working on a new paper, “A Multivariate Model of Strategic Asset Allocation,” available from our web pages. This new paper builds on the methodology of the QJE paper but estimates a richer econometric model of bond and stock returns.

II The impact on the empirical results of the paper

Tables II through VII and Figures I through III are corrected versions of the tables and figures with the same numbers in the QJE paper. Table II shows that, as one would expect,

¹Or, conversely, regressing quarterly log excess stock returns for quarters December-February, March-May, etc., onto dividend-price ratios as of the end of November, February, etc.

the greater predictability of stock returns in the correct estimates increases the magnitude of intertemporal hedging demands for stocks. Total stock demand is almost invariant to risk aversion in the range 1–5, and the mean allocation to stocks for highly risk averse investors is larger than reported in the QJE paper. This increase is caused by a larger intertemporal hedging demand. For example, for investors with $\gamma = 40$, intertemporal hedging demand represents 93% (versus 65% in the QJE paper) of the total portfolio demand for stocks.

Conservative investors respond to the increase in the predictability of stock returns not only by holding more stocks on average, but also by increasing the sensitivity of their stock demands to changes in expected returns. Table II shows that the slope of the optimal portfolio policy for values of $\gamma \geq 10$ is at least 250% larger than the slopes reported in the QJE paper. Of course, this implies an increase in the volatility of stock allocations, as shown in new Figure III.

Figure I shows that the impact of the elasticity of intertemporal substitution of consumption on portfolio choice through the loglinearization parameter is more noticeable for low coefficients for relative risk aversion than it was in the QJE paper. However, new panels I.g and I.h, which plot the optimal portfolio policy for $\gamma = 10$ and 20, show that this impact diminishes as we consider larger coefficients of relative risk aversion.

The pattern of optimal consumption-wealth ratios across different values of γ and ψ does not differ from those reported in the QJE paper. Of course, the magnitudes are different. The most important difference regarding optimal consumption is shown in new Table V: Both the optimal consumption-wealth ratio and the consumption growth rate of conservative investors are more volatile in the new calibration. This is the result of the increase in the sensitivity of the optimal portfolio policy to shocks to the state variable.

Table VII shows the impact of the new parameter estimates on the utility costs of sub-optimal portfolio choice. Utility costs are larger in general. That is, both investors who time the market but do not hedge, and investors who hedge but do not time the market, have larger utility costs than in the lower-predictability environment in the QJE paper. Utility costs can be infinitely large for investors who are not allowed to time the market and are forced to hold the same large mean portfolio allocations as investors who hedge and time the market.

III The accuracy of the solution

Our analytical solutions are exact only in the limit where time is continuous, and for parameter values that imply a constant consumption-wealth ratio ($\psi = 1$ or constant expected returns). For other parameter values our solutions are only approximate. One way to assess their accuracy is to compare them with solutions obtained using standard numerical methods. In Campbell, Cocco, Gomes, Maenhout, and Viceira [1998], we have solved numerically for optimal policy functions using the incorrect parameter estimates reported in the QJE paper. We show that numerical solutions are very similar to the approximate analytical solutions. We are currently working on a new numerical solution that uses the empirically correct model. We will make these results available as soon as we have them ready.

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Campbell: <http://www.economics.harvard.edu/faculty/jcampbell/campbell.html>

Viceira: <http://www.people.hbs.edu/lviceira/>

TABLE I

Estimates of the Stochastic Process
for Returns (1947.1 - 1995.4)

(A) Restricted VAR(1):

$$\begin{pmatrix} r_{1,t+1} - r_f \\ d_{t+1} - p_{t+1} \end{pmatrix} = \begin{pmatrix} 0.243 \\ (0.069) \\ -0.146 \\ (0.073) \end{pmatrix} + \begin{pmatrix} 0.069 \\ (0.021) \\ 0.957 \\ (0.022) \end{pmatrix} (d_t - p_t) + \begin{pmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \end{pmatrix}$$

$$\Omega = \begin{pmatrix} 5.816E-3 & -5.850E-3 \\ (0.594E-3) & (0.610E-3) \\ -5.850E-3 & 6.397E-3 \\ (0.610E-3) & (0.653E-3) \end{pmatrix} \quad R^2 = \begin{pmatrix} 0.052 \\ 0.910 \end{pmatrix}$$

(B) Derived model:

$$\begin{aligned} r_{1,t+1} - r_f &= x_t + u_{t+1} \\ x_{t+1} &= 1.005E-2 + \underset{(0.005)}{\quad} + \underset{(0.022)}{0.957} (x_t - \mu) + \eta_{t+1} \end{aligned}$$

$$\begin{pmatrix} \sigma_u^2 & \sigma_{u,\eta} \\ \sigma_{u,\eta} & \sigma_\eta^2 \end{pmatrix} = \begin{pmatrix} 5.816E-3 & -0.402E-3 \\ (0.594E-3) & (0.13E-3) \\ -0.402E-3 & 0.034E-3 \\ (0.130E-3) & (0.019E-3) \end{pmatrix}$$

$$r_f = .082E-2 \quad \sigma_x^2/\sigma_u^2 = 6.175E-2 \quad \text{corr}(\eta, u) = -0.960$$

TABLE II
Optimal Portfolio Policy

| R.R.A. | E.I.S. | | | | | | | |
|---|--------|--------|--------|--------|--------|--------|--------|--------|
| (A) Exponentiated intercept: $a_0^* \times 100$ | | | | | | | | |
| | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| 0.75 | -50.09 | -36.24 | -27.86 | -24.72 | -20.97 | -19.12 | -18.54 | -18.27 |
| 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.50 | 52.19 | 40.71 | 32.21 | 28.90 | 24.81 | 22.81 | 22.20 | 21.91 |
| 2.00 | 77.50 | 62.48 | 50.33 | 45.45 | 39.43 | 36.42 | 35.49 | 35.04 |
| 4.00 | 105.86 | 91.61 | 78.21 | 72.19 | 64.61 | 60.60 | 59.37 | 58.78 |
| 10.0 | 91.78 | 87.08 | 81.80 | 79.38 | 75.53 | 73.28 | 72.55 | 72.19 |
| 20.0 | 64.87 | 65.24 | 65.60 | 65.78 | 66.05 | 66.21 | 66.26 | 66.29 |
| 40.0 | 39.79 | 41.86 | 44.09 | 45.26 | 47.21 | 48.44 | 48.87 | 49.08 |
| (B) Slope: a_1 | | | | | | | | |
| | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| 0.75 | 185.09 | 189.61 | 193.19 | 194.77 | 196.90 | 198.07 | 198.45 | 198.63 |
| 1.00 | 171.96 | 171.96 | 171.96 | 171.96 | 171.96 | 171.96 | 171.96 | 171.96 |
| 1.50 | 154.32 | 150.27 | 146.75 | 145.21 | 143.16 | 142.07 | 141.73 | 141.56 |
| 2.00 | 142.07 | 136.56 | 131.54 | 129.34 | 126.43 | 124.89 | 124.40 | 124.15 |
| 4.00 | 112.59 | 106.91 | 101.29 | 98.67 | 95.24 | 93.37 | 92.79 | 92.50 |
| 10.0 | 73.14 | 71.09 | 68.77 | 67.71 | 66.00 | 65.00 | 64.67 | 64.51 |
| 20.0 | 46.91 | 47.07 | 47.24 | 47.32 | 47.44 | 47.51 | 47.54 | 47.55 |
| 40.0 | 27.39 | 28.35 | 29.39 | 29.93 | 30.85 | 31.42 | 31.62 | 31.72 |

TABLE III
Mean Optimal Percentage Allocation to Stocks and
Percentage Mean Hedging Demand Over Mean Total Demand

| R.R.A. | E.I.S. | | | | | | | |
|--|--------|--------|--------|--------|--------|--------|--------|--------|
| (A) Mean optimal percentage allocation to stocks: | | | | | | | | |
| $\alpha_t = [a_0^* + a_1(\mu + \sigma_u^2/2)] \times 100$ | | | | | | | | |
| | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| 0.75 | 189.66 | 209.36 | 222.37 | 227.57 | 234.06 | 237.44 | 238.50 | 239.02 |
| 1.00 | 222.74 | 222.74 | 222.74 | 222.74 | 222.74 | 222.74 | 222.74 | 222.74 |
| 1.50 | 252.08 | 235.35 | 222.29 | 216.99 | 210.24 | 206.84 | 205.78 | 205.27 |
| 2.00 | 261.52 | 239.36 | 220.72 | 212.99 | 203.19 | 198.18 | 196.62 | 195.85 |
| 4.00 | 251.69 | 230.09 | 209.41 | 199.99 | 187.97 | 181.55 | 179.56 | 178.60 |
| 10.0 | 186.51 | 179.16 | 170.88 | 167.08 | 161.02 | 157.47 | 156.32 | 155.75 |
| 20.0 | 125.63 | 126.21 | 126.79 | 127.07 | 127.50 | 127.75 | 127.84 | 127.88 |
| 40.0 | 75.26 | 78.58 | 82.15 | 84.03 | 87.17 | 89.14 | 89.83 | 90.17 |
| (B) Fraction due to hedging demand (percentage): | | | | | | | | |
| $[\alpha_{t, \text{hedging}}(\mu; \gamma, \psi) / \alpha_t(\mu; \gamma, \psi)] \times 100$ | | | | | | | | |
| | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| 0.75 | -56.59 | -41.85 | -33.55 | -30.50 | -26.88 | -25.08 | -24.52 | -24.25 |
| 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.50 | 41.09 | 36.91 | 33.20 | 31.57 | 29.37 | 28.21 | 27.84 | 27.66 |
| 2.00 | 57.41 | 53.47 | 49.54 | 47.71 | 45.19 | 43.80 | 43.36 | 43.14 |
| 4.00 | 77.88 | 75.80 | 73.41 | 72.16 | 70.38 | 69.33 | 68.99 | 68.82 |
| 10.0 | 88.06 | 87.57 | 86.97 | 86.67 | 86.17 | 85.86 | 85.75 | 85.70 |
| 20.0 | 91.14 | 91.18 | 91.22 | 91.24 | 91.27 | 91.28 | 91.29 | 91.29 |
| 40.0 | 92.60 | 92.91 | 93.22 | 93.37 | 93.61 | 93.75 | 93.80 | 93.82 |

TABLE IV

Optimal Consumption-Wealth Ratio and
Long-Term Expected Log Return on Wealth

| R.R.A. | E.I.S. | | | | | | | |
|---|--------|------|-------|------|------|------|------|------|
| (A) Consumption-Wealth ratios: | | | | | | | | |
| $C_t/W_t = \exp\{E[c_t - w_t]\} \times 100$ | | | | | | | | |
| | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| 0.75 | 0.43 | 1.53 | 2.61 | 3.14 | 3.96 | 4.46 | 4.62 | 4.71 |
| 1.00 | 0.52 | 1.53 | 2.54 | 3.03 | 3.77 | 4.21 | 4.36 | 4.43 |
| 1.50 | 0.65 | 1.53 | 2.43 | 2.87 | 3.51 | 3.88 | 4.00 | 4.06 |
| 2.00 | 0.75 | 1.53 | 2.35 | 2.74 | 3.32 | 3.64 | 3.75 | 3.81 |
| 4.00 | 0.99 | 1.53 | 2.12 | 2.41 | 2.83 | 3.06 | 3.14 | 3.18 |
| 10.0 | 1.32 | 1.53 | 1.77 | 1.89 | 2.07 | 2.18 | 2.22 | 2.24 |
| 20.0 | 1.56 | 1.53 | 1.51 | 1.50 | 1.48 | 1.47 | 1.46 | 1.46 |
| 40.0 | 1.74 | 1.53 | 1.32 | 1.20 | 1.01 | 0.89 | 0.85 | 0.83 |
| (B) Long-Term expected log return on wealth: | | | | | | | | |
| $E[r_{p, t+1}] \times 100$ | | | | | | | | |
| | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| 0.75 | 4.56 | 4.57 | 4.56 | 4.56 | 4.54 | 4.53 | 4.53 | 4.53 |
| 1.00 | 4.61 | 4.61 | 4.61 | 4.61 | 4.61 | 4.61 | 4.61 | 4.61 |
| 1.50 | 4.55 | 4.56 | 4.55 | 4.54 | 4.52 | 4.51 | 4.51 | 4.51 |
| 2.00 | 4.47 | 4.47 | 4.44 | 4.42 | 4.38 | 4.36 | 4.36 | 4.35 |
| 4.00 | 4.22 | 4.17 | 4.09 | 4.04 | 3.96 | 3.92 | 3.90 | 3.90 |
| 10.0 | 3.55 | 3.49 | 3.42 | 3.39 | 3.33 | 3.29 | 3.28 | 3.28 |
| 20.0 | 2.70 | 2.71 | 2.72 | 2.72 | 2.73 | 2.73 | 2.73 | 2.73 |
| 40.0 | 1.80 | 1.85 | 1.91 | 1.95 | 2.00 | 2.03 | 2.04 | 2.05 |

TABLE V

Volatility of Consumption Growth and
Volatility of the Log Consumption-Wealth Ratio

| R.R.A. | E.I.S. | | | | | | | |
|---|--------|-------|-------|-------|-------|-------|-------|-------|
| (A) Volatility of consumption growth: | | | | | | | | |
| $\sigma(\Delta c_{t+1} - E_t[\Delta c_{t+1}]) \times 100$ | | | | | | | | |
| | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| 0.75 | 40.26 | 31.71 | 25.46 | 22.96 | 19.83 | 18.28 | 17.81 | 17.59 |
| 1.00 | 40.00 | 30.10 | 22.98 | 20.18 | 16.74 | 15.08 | 14.58 | 14.35 |
| 1.50 | 39.12 | 28.17 | 20.19 | 17.08 | 13.36 | 11.63 | 11.13 | 10.90 |
| 2.00 | 37.96 | 26.88 | 18.58 | 15.35 | 11.51 | 9.79 | 9.31 | 9.10 |
| 4.00 | 33.22 | 23.38 | 15.43 | 12.21 | 8.41 | 6.84 | 6.46 | 6.30 |
| 10.0 | 23.29 | 17.09 | 11.48 | 9.01 | 5.92 | 4.72 | 4.50 | 4.43 |
| 20.0 | 15.40 | 11.79 | 8.25 | 6.57 | 4.37 | 3.54 | 3.41 | 3.39 |
| 40.0 | 9.14 | 7.26 | 5.27 | 4.26 | 2.90 | 2.39 | 2.33 | 2.32 |
| (B) Volatility of the consumption-wealth ratio: | | | | | | | | |
| $\sigma(c_{t+1} - w_{t+1} - E_t[c_{t+1} - w_{t+1}]) \times 100$ | | | | | | | | |
| | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| 0.75 | 10.84 | 0.00 | 8.08 | 11.42 | 15.80 | 18.11 | 18.84 | 19.19 |
| 1.00 | 10.62 | 0.00 | 7.90 | 11.14 | 15.36 | 17.57 | 18.26 | 18.60 |
| 1.50 | 10.18 | 0.00 | 7.62 | 10.73 | 14.73 | 16.84 | 17.49 | 17.82 |
| 2.00 | 9.75 | 0.00 | 7.40 | 10.41 | 14.30 | 16.34 | 16.98 | 17.29 |
| 4.00 | 8.34 | 0.00 | 6.69 | 9.48 | 13.15 | 15.09 | 15.70 | 16.00 |
| 10.0 | 5.75 | 0.00 | 5.23 | 7.66 | 11.04 | 12.93 | 13.54 | 13.84 |
| 20.0 | 3.78 | 0.00 | 3.81 | 5.74 | 8.63 | 10.38 | 10.97 | 11.26 |
| 40.0 | 2.23 | 0.00 | 2.45 | 3.76 | 5.86 | 7.19 | 7.65 | 7.88 |

TABLE VI
Optimal Consumption Rules Implied
by Restricted Portfolio Rules

| | <i>Portfolio Rule</i> | <i>Optimal Consumption Rule Given Portfolio Rule</i> |
|------------|--|--|
| Hedging | Timing $\alpha_t = a_0^* + a_1(x_t + \sigma_u^2/2)$ | $c_t - w_t = b_0^* + b_1^*(x_t + \sigma_u^2/2) + b_2(x_t + \sigma_u^2/2)$ |
| | No-Timing $\alpha_t = a_0^* + a_1(\mu + \sigma_u^2/2)$ | $c_t - w_t = b_0^{h,nt} + b_1^{h,nt}(x_t + \sigma_u^2/2)$ |
| No-Hedging | Timing $\alpha_t = \frac{x_t + \sigma_u^2/2}{\gamma\sigma_u^2}$ | $c_t - w_t = b_0^{nh,t} + b_1^{nh,t}(x_t + \sigma_u^2/2) + b_2^{nh,t}(x_t + \sigma_u^2/2)$ |
| | No-Timing $\alpha_t = \frac{\mu + \sigma_u^2/2}{\gamma\sigma_u^2}$ | $c_t - w_t = b_0^{nh,nt} + b_1^{nh,nt}(x_t + \sigma_u^2/2)$ |

TABLE VII

Percentage Mean Value Function When the Optimal Portfolio Rule is Unrestricted, and Percentage Loss in the Value Function Under Alternative, Restricted Portfolio Rules

| R.R.A. | | Timing | | | | | | | | No-Timing | | | | | | | |
|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----------|--------|--------|--------|--------|--------|---------|--------|
| | | E.I.S. | | | | | | | | E.I.S. | | | | | | | |
| | | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| Hedging | 0.75 | 363.13 | 31.71 | 13.75 | 10.66 | 8.06 | 7.08 | 6.82 | 6.69 | -99.49 | -93.83 | -85.25 | -80.72 | -74.14 | -70.37 | -69.14 | -68.53 |
| | 1.00 | 170.61 | 25.87 | 12.10 | 9.53 | 7.31 | 6.46 | 6.23 | 6.12 | -98.85 | -92.42 | -83.78 | -79.38 | -73.12 | -69.59 | -68.46 | -67.90 |
| | 1.50 | 67.36 | 18.86 | 9.98 | 8.09 | 6.37 | 5.69 | 5.51 | 5.42 | -97.18 | -90.11 | -81.69 | -77.61 | -71.94 | -68.85 | -67.87 | -67.39 |
| | 2.00 | 37.22 | 14.72 | 8.61 | 7.14 | 5.76 | 5.20 | 5.04 | 4.97 | -95.30 | -88.26 | -80.37 | -76.64 | -71.60 | -68.92 | -68.08 | -67.68 |
| | 4.00 | 10.70 | 7.51 | 5.67 | 5.06 | 4.39 | 4.08 | 4.00 | 3.95 | -87.53 | -83.08 | -79.12 | -77.84 | -78.04 | -82.04 | -100.00 | - |
| | 10.0 | 3.02 | 2.92 | 2.81 | 2.76 | 2.67 | 2.63 | 2.61 | 2.60 | -68.68 | -73.97 | -91.25 | - | - | - | - | - |
| | 20.0 | 1.60 | 1.60 | 1.60 | 1.60 | 1.59 | 1.59 | 1.59 | 1.59 | -50.13 | -63.42 | - | - | - | - | - | - |
| | 40.0 | 1.08 | 1.05 | 1.00 | 0.97 | 0.92 | 0.87 | 0.86 | 0.85 | -32.50 | -48.87 | - | - | - | - | - | - |
| No-Hedging | | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 | 1/.75 | 1.00 | 1/1.5 | 1/2 | 1/4 | 1/10 | 1/20 | 1/40 |
| | 0.75 | -70.64 | -26.31 | -12.21 | -8.72 | -5.58 | -4.37 | -4.04 | -3.89 | -99.37 | -92.72 | -83.13 | -78.16 | -71.01 | -66.93 | -65.61 | -64.96 |
| | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -98.85 | -92.42 | -83.78 | -79.38 | -73.12 | -69.59 | -68.46 | -67.90 |
| | 1.50 | -66.48 | -37.21 | -20.96 | -15.94 | -10.86 | -8.77 | -8.19 | -7.92 | -97.79 | -92.12 | -85.17 | -81.71 | -76.83 | -74.14 | -73.28 | -72.86 |
| | 2.00 | -81.12 | -60.42 | -42.62 | -35.38 | -26.82 | -22.83 | -21.64 | -21.08 | -96.63 | -91.60 | -85.86 | -83.11 | -79.39 | -77.43 | -76.82 | -76.52 |
| | 4.00 | -83.07 | -75.59 | -67.94 | -64.00 | -58.47 | -55.32 | -54.31 | -53.82 | -91.26 | -88.15 | -85.35 | -84.28 | -83.42 | -83.44 | -83.57 | -83.65 |
| | 10.0 | -67.81 | -68.12 | -69.14 | -69.80 | -71.25 | -72.51 | -73.01 | -73.28 | -74.37 | -75.77 | -77.91 | -79.47 | -82.83 | -85.81 | -87.08 | -87.77 |
| | 20.0 | -49.48 | -53.30 | -58.60 | -61.83 | -67.98 | -72.82 | -74.71 | -75.73 | -54.69 | -59.19 | -65.13 | -68.94 | -76.44 | -82.67 | -85.24 | -86.65 |
| 40.0 | -31.54 | -36.06 | -42.64 | -47.08 | -56.55 | -65.25 | -69.10 | -71.26 | -35.09 | -49.19 | -47.43 | -52.46 | -63.38 | -73.73 | -78.38 | -81.04 | |

Notes to the Tables

Table 1: Table 1 reports ML estimates of the stochastic process driving expected and unexpected returns in the model. These estimates are based on quarterly returns, dividends and prices from CRSP for the period 1947:1 - 1995:4. Stock market data is for the CRSP value-weighted market portfolio inclusive of the NYSE, AMEX and NASDAQ markets, and the short-term nominal interest rate is the 3-month Treasury bill yield from the Riskfree File on the CRSP Bond tape. Panel A reports ML point estimates and standard errors (in parentheses) of a restricted VAR(1) model (see equation [24] in text) for excess log returns and the log dividend-price ratio. Panel B reports estimates for the parameters defining the stochastic structure of the model. These estimates and their standard errors (in parentheses) are derived from the estimates in Panel A. Standard errors are obtained using the delta method.

Table 2: Panel A reports the optimal percentage allocation per quarter to stocks when the expected gross excess return is zero for different levels of relative risk aversion and elasticities of intertemporal substitution. Panel B reports the quarterly change - in percentage points - in the optimal allocation to stocks when the expected log excess return increases by one percent per quarter. These numbers are all based on the parameter estimates for the return process reported in Table 2 (Sample period 1947:1-1995:4). **The values in the main diagonal correspond to the power utility case.**

Table 3: Panel A reports the mean optimal percentage allocation per quarter to stocks, for different levels of relative risk aversion and elasticities of intertemporal substitution. Panel B reports the percentage mean hedging demand over mean total demand, i.e., the fraction of the mean allocation due to hedging demand. Mean hedging demand is calculated as $\alpha_{t,hedging}(\mu; \gamma, \psi) = \alpha_t(\mu; \gamma, \psi) - \alpha_t(\mu; 1, \psi)/\gamma$. These numbers are all based on the parameter estimates for the return process reported in Table 2 (Sample period 1947:1-1995:4). **The values in the main diagonal correspond to the power utility case.**

Table 4: Panel A reports percentage exponentiated mean optimal log consumption-wealth ratios per quarter, i.e., 100 times the exponential of $E[c_t - w_t] = b_0^* + b_1^*(\mu + \sigma_u^2/2) + b_2(\sigma_x^2 + \mu^2 + \mu\sigma_u^2 + \sigma_u^4/4)$, for different levels of relative risk aversion and elasticities of intertemporal substitution. Panel B reports the percentage unconditional mean of the quarterly log return on wealth. These numbers are all based on the parameter estimates for the return process reported in Table 2 (Sample period 1947:1-1995:4). **The values in the main diagonal correspond to the power utility case.**

Table 5: Panel A reports percentage unconditional standard deviation of quarterly log consumption innovations for different levels of relative risk aversion and elasticities of intertemporal substitution, while Panel B reports the percentage unconditional standard deviation of innovations in the quarterly log

consumption-wealth ratio. These numbers are all based on the parameter estimates for the return process reported in Table 2 (Sample period 1947:1-1995:4). **The values in the main diagonal correspond to the power utility case.**

Table 6: The second column in Table 6 describes the consumption rule followed by an investor who adjusts consumption optimally given the portfolio rule described in the first column of the table. The first row describes the optimal consumption rule implied by the unconstrained optimal portfolio rule. This rule is state-dependent and includes a hedging component. Therefore, the first row of the table describes the solution to the intertemporal optimization problem we solve in section 3. The second row describes the optimal consumption rule followed by an investor who follows a suboptimal portfolio rule consisting in allocating to stocks each period a fixed fraction of her savings that equals the average allocation to stocks implied by the optimal portfolio rule. Therefore, this investor ignores timing in her portfolio decisions, though she allows for (imperfect) hedging. The third row of the table describes the optimal consumption rule followed by an investor who follows a myopic portfolio rule. This suboptimal portfolio rule ignores hedging, but it is time-dependent. Finally, the fourth row of the table describes the optimal portfolio rule followed by an investor that ignores both hedging and timing and invests in stocks each period a fixed fraction of her savings that equals the average myopic allocation to stocks.

Table 7: The panel on the upper, left corner of the table reports the unconditional mean of the value function under the optimal consumption and portfolio rules. The panel on the upper, right corner reports the percentage loss in the value function when the portfolio rule is fixed at the mean value of the optimal portfolio rule and consumption adjusts optimally. The panel on the lower, left corner reports the percentage loss in the value function when the portfolio rule is myopic (see equation [21] in text) and consumption adjusts optimally. The panel on the lower, right corner reports the percentage loss in the value function when the portfolio rule is fixed at the mean value of the myopic rule and consumption adjusts optimally. ”-” indicates that there is an infinite loss in value. These numbers are based on the parameter values for the return process presented in Table 1. These values are estimates for the period 1947:1-1995:4. **The values in the main diagonal correspond to the power utility case.**

Figure 1.c: $\gamma = 0.75$

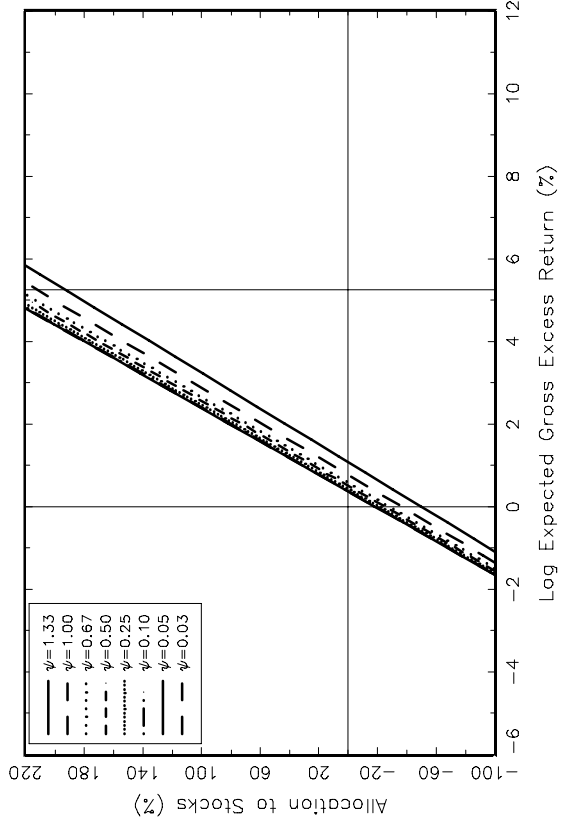


Figure 1.d: $\gamma = 4.00$

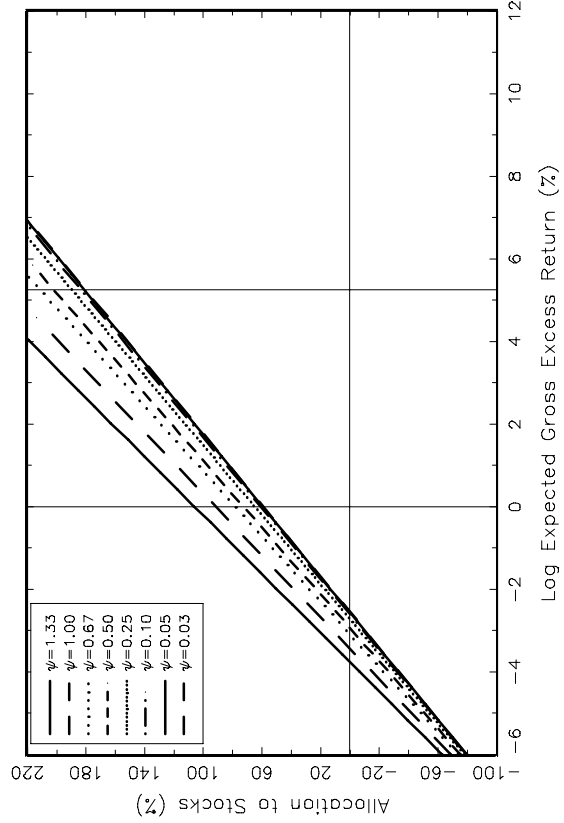


Figure 1.a: $\psi = 1.33$

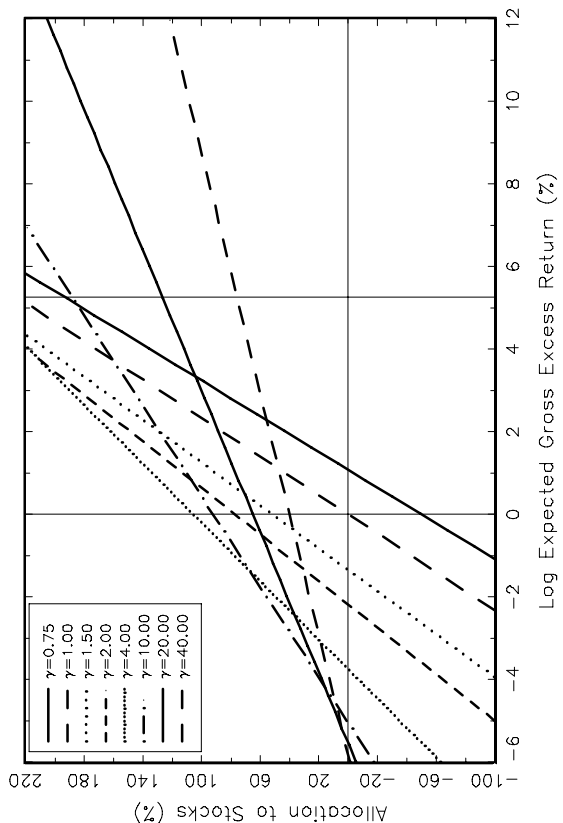


Figure 1.b: $\psi = 0.25$

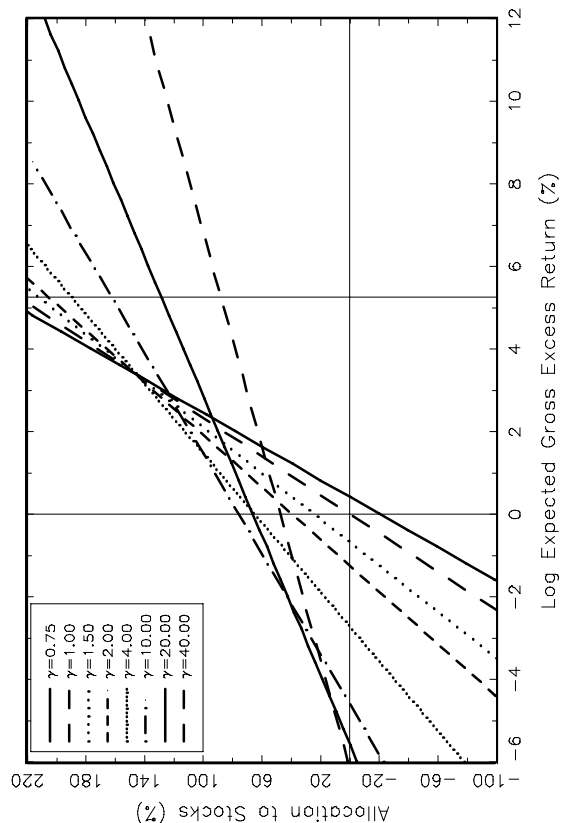


Figure l.g: $\gamma = 10.00$

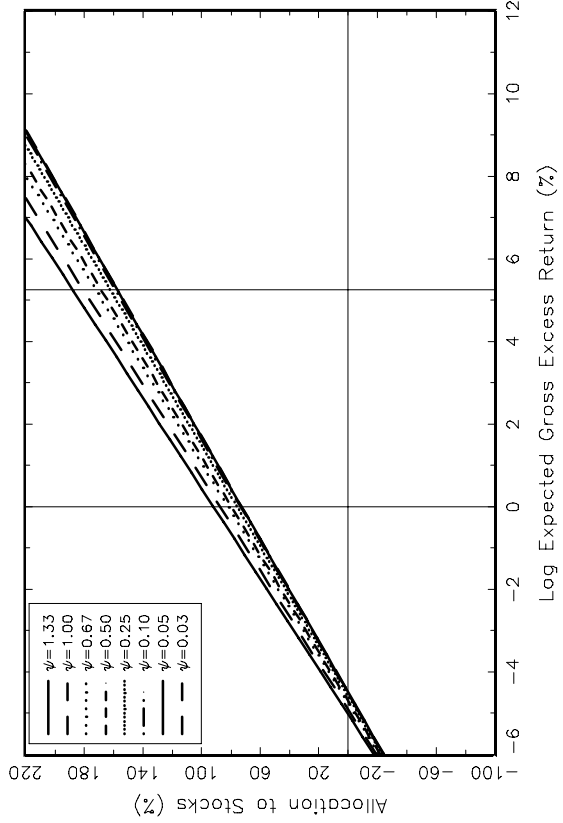


Figure l.h: $\gamma = 20.00$

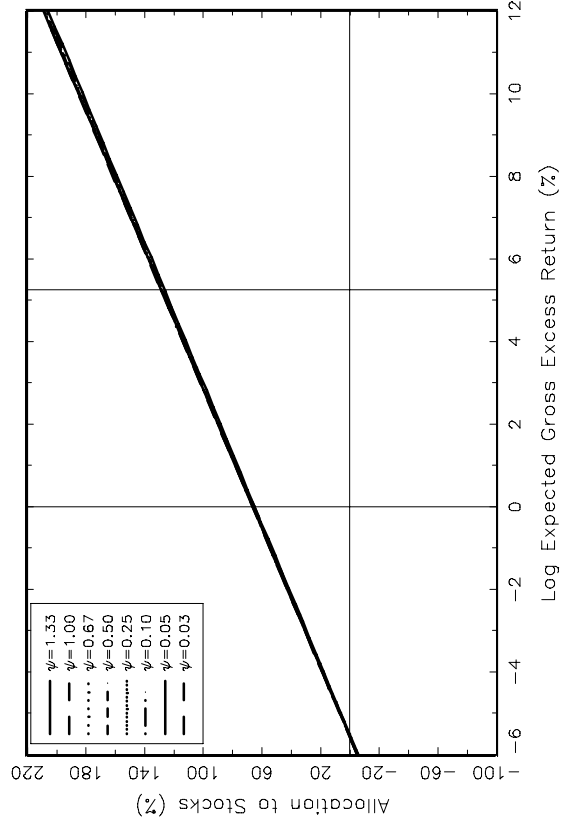


Figure l.e: $\psi = 0.10$

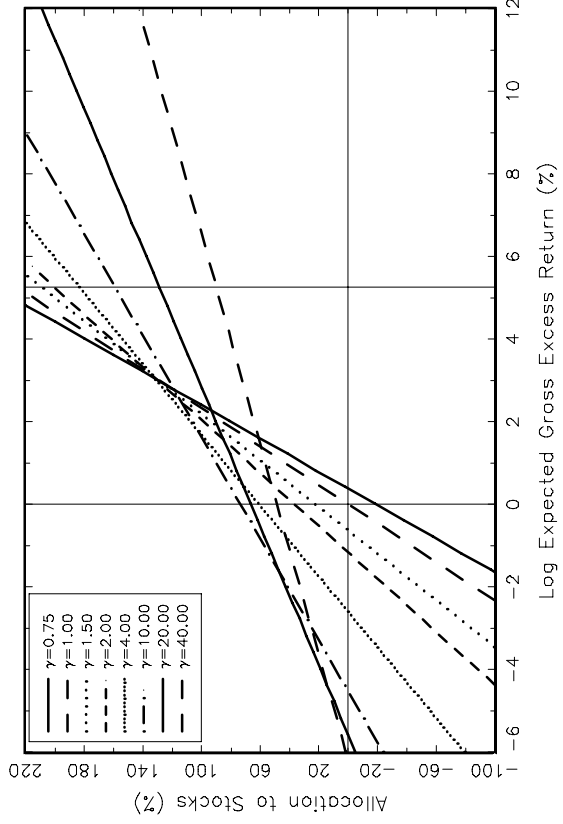


Figure l.f: $\psi = 0.05$

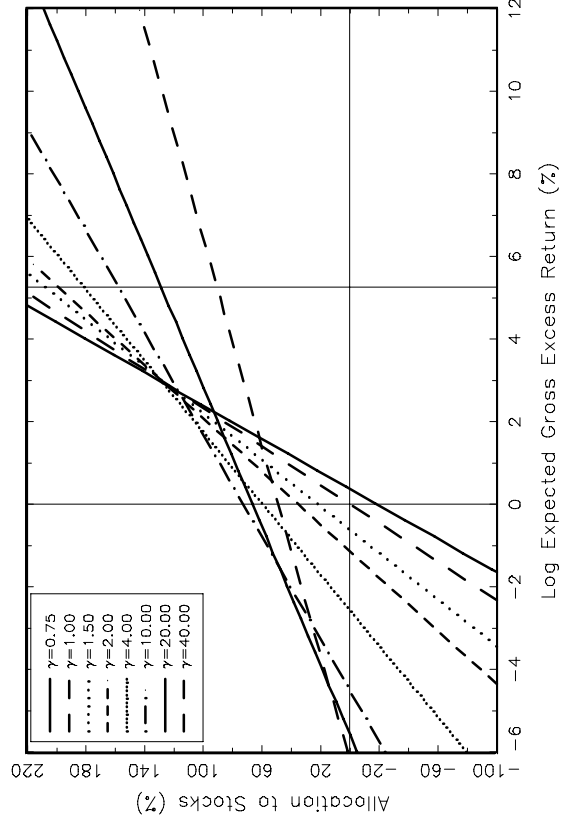


Figure II.a: $\psi = 1.33$

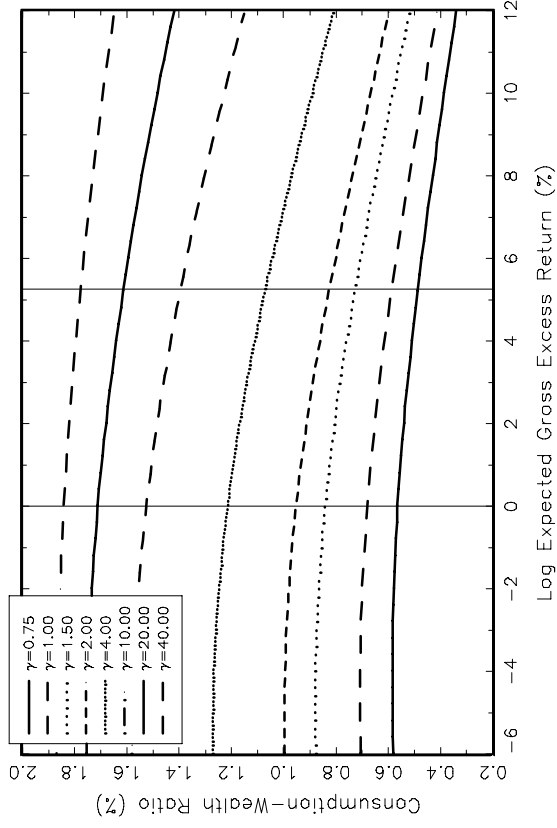


Figure II.c: $\gamma = 0.75$

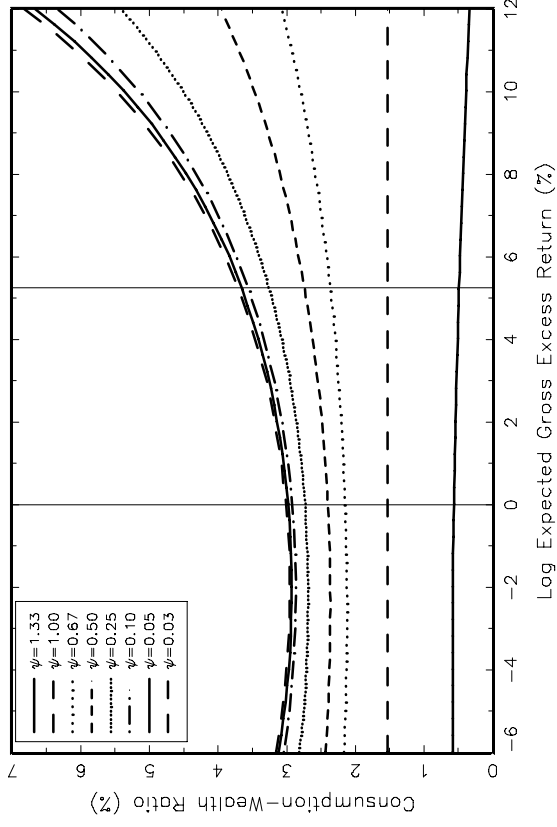


Figure II.b: $\psi = 0.25$

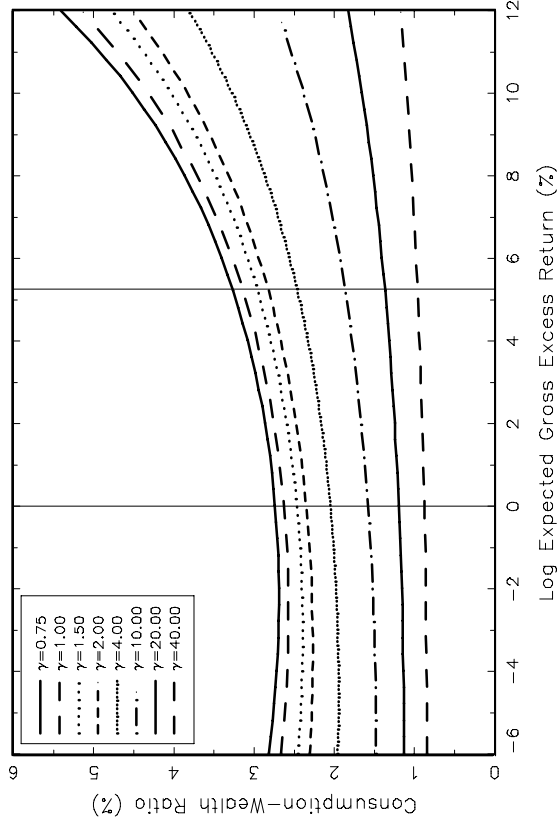


Figure II.d: $\gamma = 4.00$

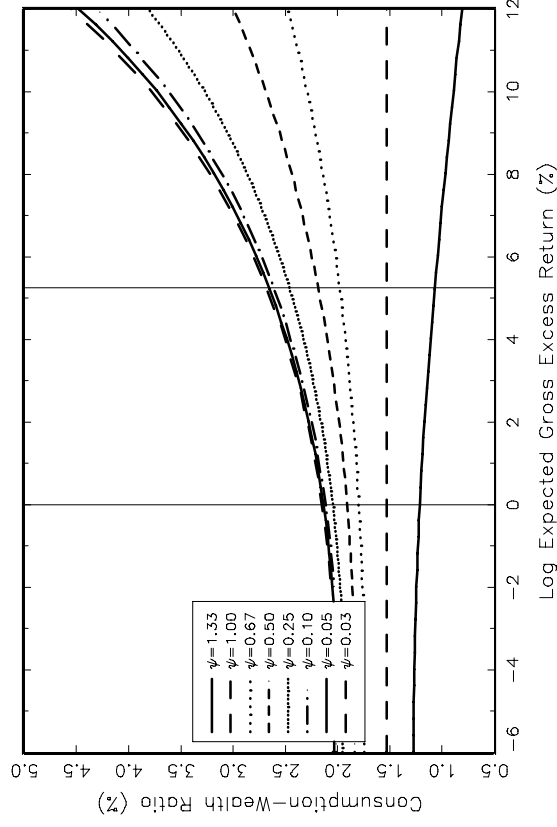


Figure II.g: $\gamma = 10.00$

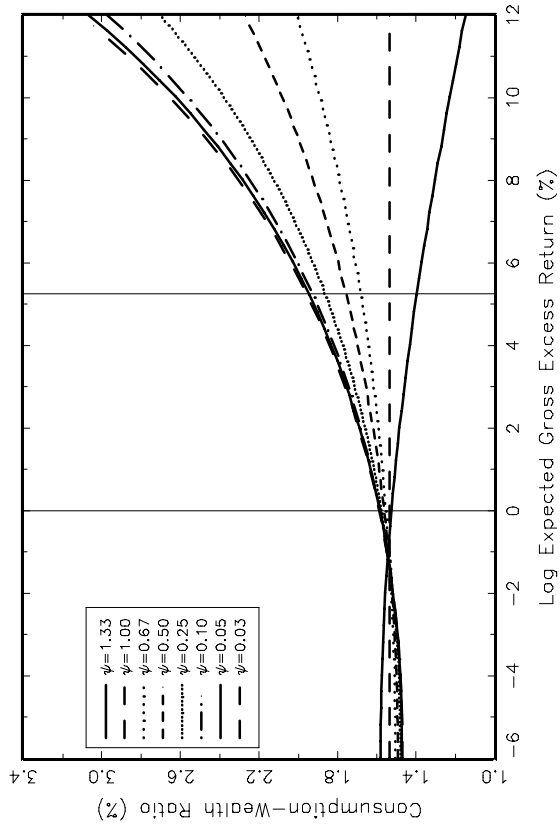


Figure II.h: $\gamma = 20.00$

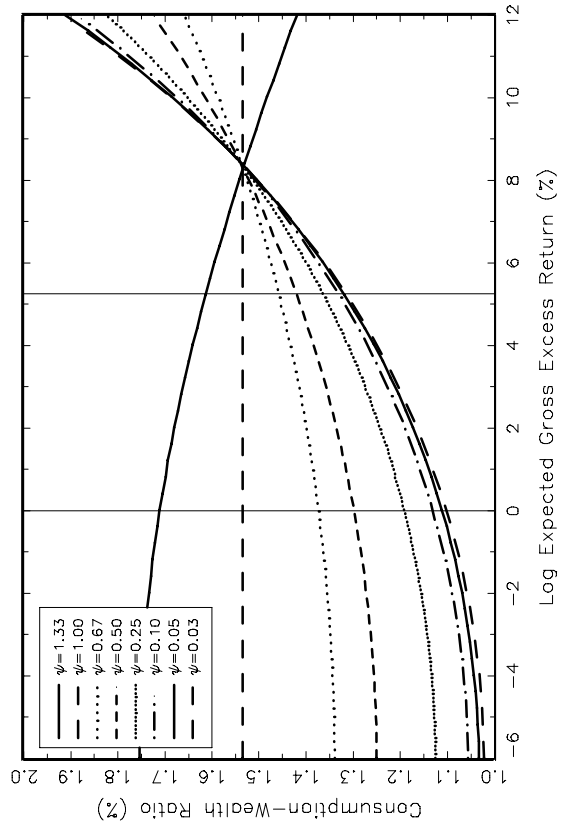


Figure II.e: $\psi = 0.10$

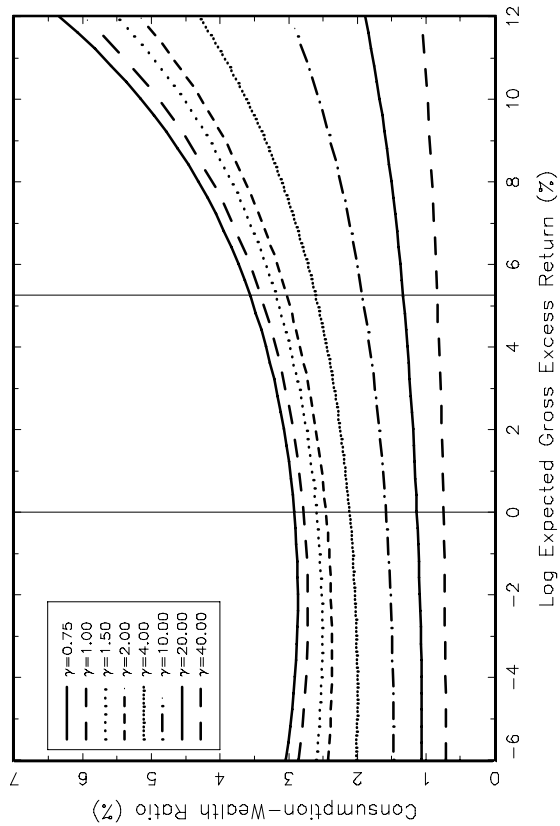


Figure II.f: $\psi = 0.05$

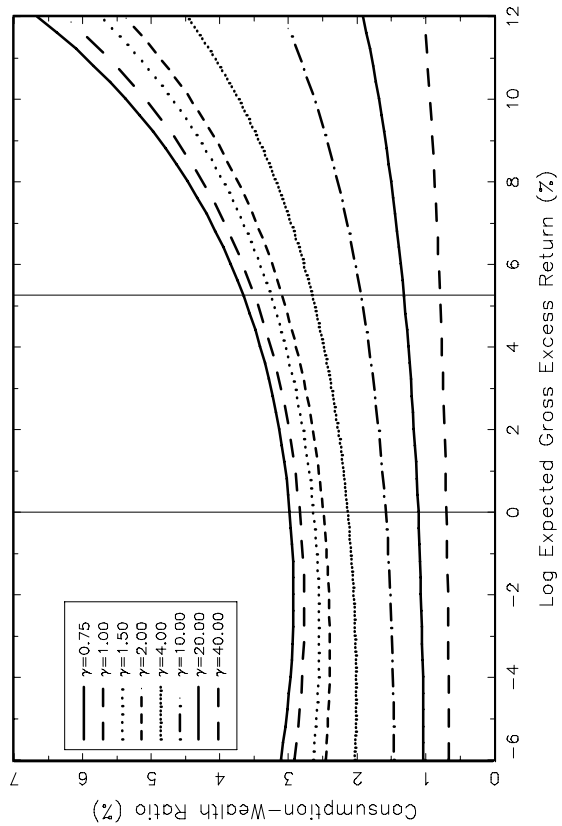


Figure III: $\psi = 0.25$ and $\gamma = 4.00, 20.00$

