Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds

John Y. Campbell, Adi Sunderam, and Luis M. Viceira[1]

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[1]Campbell: Department of Economics, Littauer Center, Harvard University, Cambridge MA 02138, USA, and NBER. Email john_campbell@harvard.edu. Sunderam: Harvard Business School, Boston MA 02163. Email asunderam@hbs.edu. Viceira: Harvard Business School, Boston MA 02163 and NBER. Email lviceira@hbs.edu. We acknowledge the extraordinarily able research assistance of Thummim Cho and Johnny Kang. We are grateful to anonymous referees and to Geert Bekaert, Andrea Buraschi, Jesus Fernandez-Villaverde, Wayne Ferson, Javier Gil-Bazo, Pablo Guerron, John Heaton, Ravi Jagannathan, Jon Lewellen, Monika Piazzesi, Pedro Santa-Clara, George Tauchen, Ivo Welch, and seminar participants at the 2009 Annual Meeting of the American Finance Association, Bank of England, European Group of Risk and Insurance Economists 2008 Meeting, Sixth Annual Empirical Asset Pricing Retreat at the University of Amsterdam Business School, Harvard Business School Finance Unit Research Retreat, Imperial College, Marshall School of Business, NBER Fall 2008 Asset Pricing Meeting, Norges Bank, Society for Economic Dynamics 2008 Meeting, Stockholm School of Economics, Tilburg University, Tuck Business School, and Universidad Carlos III in Madrid for helpful comments and suggestions. This material is based upon work supported by the National Science Foundation under Grant No. 0214061 to Campbell, and by Harvard Business School Research Funding.

Abstract

The covariance between US Treasury bond returns and stock returns has moved considerably over time. While it was slightly positive on average in the period 1953–2014, it was unusually high in the early 1980's and negative in the early 21st Century, particularly in the downturns of 2001 and 2007–09. This paper specifies and estimates a model in which the nominal term structure of interest rates is driven by four state variables: the real interest rate, temporary and permanent components of expected inflation, and the nominal-real covariance of inflation and the real interest rate with the real economy. The last of these state variables enables the model to fit the changing covariance of bond and stock returns. In the model, a high nominal-real covariance implies a high term premium and a concave yield curve. The decline in this covariance since the early 1980s has driven down our model-implied term premium on 10-year zero-coupon nominal Treasury bonds by about 2 percentage points.

1 Introduction

In recent years investors have come to regard US Treasury bonds as hedges, assets that perform well when other assets lose value, and more generally when bad macroeconomic news arrives. During both of the two most recent recessions, in 2001 and 2007–09, Treasury bonds performed well. In addition, since the turn of the century and particularly during these downturns, Treasury bond returns have been negatively correlated with stock returns at a daily frequency. In previous decades, however, Treasury bonds performed very differently; they were either uncorrelated or positively correlated with stock returns. The purpose of this paper is to highlight these changes in magnitude and switches in sign of the covariation between bonds and stocks, and to ask what they imply for bond risk premia and the shape of the term structure of interest rates.

To understand how a changing bond-stock covariance can affect the pricing of Treasury bonds, we specify and estimate a multifactor term structure model that incorporates traditional macroeconomic influences—real interest rates and expected inflation—along with a state variable driving the variance of real and nominal interest rates and their covariance with the macroeconomy. The model is set up so that all factors have an economic interpretation, and the covariance of bond returns with the macroeconomy can switch sign. To isolate the effect of the changing bond-stock covariance, the model assumes a constant price of risk, or equivalently a constant variance for the stochastic discount factor. We estimate the model using quarterly US time series, beginning in 1953 where possible and ending in 2014, for nominal and inflation-indexed bond yields, stock returns, realized and forecast inflation, and the realized second moments of bond and stock returns calculated from daily data within each quarter. The use of realized second moments, unusual in the term structure literature, forces our model to fit the historically observed changes in risks.

Our model shows that the risk premia of nominal Treasury bonds should have changed over the decades because of changes in the covariance between inflation and the real economy. The model predicts positive nominal bond risk premia in the early 1980s, when bonds covaried positively with stocks, and negative risk premia in the 2000s and particularly during the downturn of 2007–09, when bonds hedged equity risk. The maximum risk premium we estimate on a 10-year nominal zero-coupon Treasury bond is about 1.2% in 1980Q3, and the minimum is about -0.8% in 2008Q3. Thus the model generates a historical spread of about 2 percentage points in bond risk premia at this maturity.

A second implication of our model is that high bond risk premia are associated with a concave term structure of interest rates, specifically high interest rates at a maturity around 3 years relative to short- and long-term interest rates. In the model, a high bond-stock covariance implies a high volatility of bond returns. The high bond-stock covariance generates a high term premium and a steep yield curve at maturities of 1-3 years, while the high bond volatility lowers long-term yields through a Jensen's inequality or convexity effect. Thus, the concavity of the yield curve is correlated with the bond-stock covariance. In this fashion, our model explains the qualitative finding of Cochrane and Piazzesi (2005) that a tent-shaped linear combination of forward rates, with a peak at about 3 years, predicts excess bond returns at all maturities. However, the model also implies that many other factors affect the shape of the yield curve, so the predictability of bond returns from the Cochrane-Piazzesi factor is quite weak in the model. In other words, by incorporating only changes in the quantity of bond risk and ignoring time-variation in the price of bond risk, our model captures low-frequency variation in term premia associated with the changing bond-stock covariance, but misses some high-frequency variation in term premia.

To illustrate the basic observation that motivates this paper, Figure 1 plots the history of the realized covariance of 10-year nominal zero-coupon Treasury bonds with the CRSP value-weighted stock index, calculated using a rolling three-month window of daily data. For ease of interpretation, the figure also shows the history of the realized beta of Treasury bonds with stocks (the bond-stock covariance divided by the realized variance of stock returns), as this allows a simple back-of-the-envelope calculation of the term premium that would be implied by the simple Capital Asset Pricing Model (CAPM) given any value for the equity premium. The covariance (plotted on the left vertical scale) and beta (on the right vertical scale) move closely together, with the major divergences occurring during periods of low stock return volatility in the late 1960s and the mid-1990s.

Figure 1 displays a great deal of high-frequency variation in both series, much of which is attributable to noise in realized second moments. But it also shows substantial low-frequency movements. The beta of bonds with stocks was close to zero in the mid-1960's and mid-1970's, much higher with an average around 0.4 in the 1980's, spiked in the mid-1990's, and declined to negative average values in the 2000's. During the two downturns of 2001 and 2007–09, the average realized beta of Treasury bonds was about -0.2. Thus from peak to trough, the realized beta of

Treasury bonds has declined by about 0.6 and has changed its sign. According to the CAPM, this would imply that term premia on 10-year zero-coupon Treasuries should have declined by 60% of the equity premium.

Nominal bond returns respond both to expected inflation and to real interest rates. A natural question is whether the pattern shown in Figure 1 reflects a changing covariance of inflation with the stock market, or a changing covariance of real interest rates with the stock market. The data suggest that both factors play a role. To illustrate the importance of inflation, Figure 2 plots the covariance and beta of inflation shocks with stock returns, using a rolling three-year window of quarterly data and a first-order quarterly vector autoregression for inflation, stock returns, and the three-month Treasury bill yield to calculate inflation shocks. Because high inflation is associated with high bond yields and low bond returns, the figure shows the covariance and beta for realized deflation shocks (the negative of inflation shocks) which should move in the same manner as the bond return covariance and beta reported in Figure 1. Indeed, Figure 2 shows a similar history for the deflation covariance as for the nominal bond covariance.

Real interest rates also play a role in changing nominal bond risks. Since long-term Treasury inflation-protected securities (TIPS) were first issued in 1997, TIPS have had a predominantly negative beta with stocks as Campbell, Shiller, and Viceira (2009) emphasize and we illustrate below in Figure 4. Like the nominal bond beta, the TIPS beta was particularly negative in the downturns of 2001 and 2007–09. Thus not only the stock-market covariances of nominal bond returns, but also the covariances of two proximate drivers of those returns, inflation and real interest rates, change over time and occasionally switch sign. We design our term structure model to fit these facts.

The organization of the paper is as follows. Section 2 briefly reviews the related literature. Section 3 presents our model of the real and nominal term structures of interest rates. Section 4 describes our estimation method and presents parameter estimates and historical fitted values for the unobservable state variables of the model. Section 5 discusses the implications of the model for the shape of the yield curve and the movements of risk premia on nominal bonds. Section 6 concludes. An Appendix to this paper available online (Campbell, Sunderam, and Viceira 2016) presents details of the model solution and additional empirical results.

2 Literature Review

Despite the striking movements in the bond-stock covariance illustrated in Figure 1, this second moment has received relatively little attention in the enormous literature on the term structure of interest rates.[2] One reason for this neglect may be that until the last 15 years, the covariance was almost always positive and thus it was not apparent that it could switch sign. In the absence of a sign switch, a model of changing bond market volatility, with a constant correlation or even a constant covariance between bonds and stocks, might be adequate.

The early literature on the term structure of interest rates concentrated on testing the null hypothesis of constant bond risk premia, also known as the expectations hypothesis of the term structure (Shiller, Campbell, and Schoenholtz 1983, Fama and Bliss 1987, Stambaugh 1988, Campbell and Shiller 1991). Second-generation affine term structure models such as Cox, Ingersoll, and Ross (1985) modeled changes in bond market volatility and risk premia linked to the short-term interest rate. This approach encounters the difficulty that bond market volatility appears to move independently of the level of interest rates. In addition, the empirical link between bond market volatility and the expected excess bond return is weak, although some authors such as Campbell (1987) do estimate it to be positive.[3]

In the last ten years a large literature has specified and estimated essentially affine term structure models (Duffee 2002), in which a changing price of risk can affect bond market risk premia without any change in the quantity of risk, while risk premia are linear functions of bond yields (Dai and Singleton 2002, Sangvinatsos and Wachter 2005, Wachter 2006, Buraschi and Jiltsov 2007, Bekaert, Engstrom, and Xing 2009, Bekaert, Engstrom, and Grenadier 2010). Models such as those of Dai and Singleton (2002) and Sangvinatsos and Wachter (2005) achieve a good fit to the historical term structure, but this literature uses latent factors that are hard to interpret economically.

[2] Exceptions in the last decade include Guidolin and Timmermann (2006), Christiansen and Ranaldo (2007), David and Veronesi (2009), Baele, Bekaert, and Inghelbrecht (2010), and Viceira (2012).

[3] More recently, Piazzesi and Schneider (2006) and Rudebusch and Wu (2007) have built affine models of the nominal term structure in which a reduction of inflation uncertainty drives down the risk premia on nominal bonds towards the lower risk premia on inflation-indexed bonds. Similarly, Backus and Wright (2007) argue that declining uncertainty about inflation explains the low yields on nominal Treasury bonds in the mid-2000's.

Some papers have extended the essentially affine approach to model stock and bond prices jointly (d'Addona and Kind 2006, Bekaert, Engstrom, and Grenadier 2010). Eraker (2008) and Bansal and Shaliastovich (2013) price both stocks and bonds using the consumption-based long-run risks model of Bansal and Yaron (2004). However, while some of these papers allow the bond-stock covariance to move over time, none of them allow it to change sign.[4]

In this paper we want to model a time-varying covariance between state variables and the stochastic discount factor, which can switch sign, and we want out state variables to be interpretable macroeconomic variables. We take a direct approach and write a linear-quadratic term structure model like those of Beaglehole and Tenney (1991), Constantinides (1992), Ahn, Dittmar and Gallant (2002), Leippold and Wu (2003), and Realdon (2006). To solve our model, we use a general result on the expected value of the exponential of a non-central chi-squared distribution which we take from the Appendix to Campbell, Chan, and Viceira (2003). To estimate the model, we use a nonlinear filtering technique, the unscented Kalman filter, proposed by Julier and Uhlmann (1997) and reviewed by Wan and van der Merwe (2001).[5]

The filtering method we use for estimation can easily handle data series that are missing for part of our sample period. This allows us to include measurement equations for inflation-indexed bond yields and professional survey forecasts of inflation, during the period in which these variables are observed. In this way we can contribute to the literature on inflation-indexed bond yields and inflation forecasting (Gürkaynak, Sack, and Wright 2008, Campbell, Shiller, and Viceira 2009, Pflueger and Viceira 2011). Campbell, Shiller, and Viceira (2009) draws on an early unpublished version of this paper to present selected estimation results for our model, highlighting its implications for TIPS yields.

[4]Time-variation in the bond-stock covariance is also largely ignored by empirical papers such as Chen, Roll, and Ross (1986), Ferson and Harvey (1991), Fama and French (1993), and Koijen, Lustig, and Van Niewerburgh (2014) that specify reduced-form factor models, including both bond and equity factors, in order to price both bond and equity test assets. The same is true of a small empirical literature that decomposes nominal bond returns into shocks to real interest rates, inflation expectations, and risk premia, and estimates the covariances of these components with stock returns (Barsky 1989, Shiller and Beltratti 1992, Campbell and Ammer 1993).

[5]It is possible to embed linear-quadratic models within the affine class by augmenting the state vector, as pointed out by Duffie and Kan (1996) and Cheng and Scaillet (2007). In this spirit, Buraschi, Cieslak, and Trojani (2008) expand the state space of a nonlinear term structure model to obtain an affine model in which correlations can switch sign. We do not pursue this approach here as we do not need it to solve the model or to characterize the dynamics of our state variables.

3 A Quadratic Bond Pricing Model

We now present a term structure model that allows for time variation in the covariances between real interest rates, inflation, and the real economy. In the model, both real and nominal bond yields are linear-quadratic functions of the vector of state variables and, consistent with the empirical evidence, the conditional volatilities and covariances of excess returns on real and nominal assets are time varying. To highlight the effects of these changing conditional second moments on bond risk premia, we assume that the price of risk is constant, or equivalently that the stochastic discount factor is homoskedastic.

3.1 The SDF and the real term structure

We start by assuming that the log of the real stochastic discount factor (SDF), $m_{t+1} = \log(M_{t+1})$, follows the process:

$$-m_{t+1} = x_t + \frac{\sigma_m^2}{2} + \varepsilon_{m,t+1}. \tag{1}$$

As we have emphasized, the SDF innovation $\varepsilon_{m,t+1}$ in equation (1) is homoskedastic.[6] The drift x_t, however, follows an AR(1) process subject to both a heteroskedastic shock $\psi_t \varepsilon_{x,t+1}$ and a homoskedastic shock $\varepsilon_{X,t+1}$:

$$x_{t+1} = \mu_x (1 - \phi_x) + \phi_x x_t + \psi_t \varepsilon_{x,t+1} + \varepsilon_{X,t+1}. \tag{2}$$

The innovations $\varepsilon_{m,t+1}$, $\varepsilon_{x,t+1}$, and $\varepsilon_{X,t+1}$ are normally distributed, with zero means and constant variance-covariance matrix. We allow these shocks to be cross-correlated and adopt the notation σ_i^2 to describe the variance of shock ε_i, and σ_{ij} to describe the covariance between shock ε_i and shock ε_j. To reduce the complexity of the equations that follow, we assume that the shocks to x_t are orthogonal to each other; that is, $\sigma_{xX} = 0$.

[6] We have developed and estimated an extension of the model with a heteroskedastic SDF. Details of this more general model are available from the authors upon request. The more general specification captures the spirit of recent term structure models by Bekaert et al (2005), Buraschi and Jiltsov (2007), Wachter (2006) and others in which time-varying risk aversion drives time-varying bond risk premia.

The state variable x_t is the short-term log real interest rate. The price of a single-period zero-coupon real bond satisfies $P_{1,t} = \mathrm{E}_t \left[\exp \{ m_{t+1} \} \right]$,so that its yield $y_{1t} = -\log(P_{1,t})$ equals

$$y_{1t} = -\mathrm{E}_t \left[m_{t+1} \right] - \frac{1}{2} \mathrm{Var}_t \left(m_{t+1} \right) = x_t. \tag{3}$$

The model has an additional state variable, ψ_t, which governs time variation in the volatility of the real interest rate and its covariation with the SDF.[7] We assume that ψ_t follows a standard homoskedastic AR(1) process:

$$\psi_{t+1} = \mu_\psi \left(1 - \phi_\psi \right) + \phi_\psi \psi_t + \varepsilon_{\psi,t+1}. \tag{4}$$

Importantly, this process can change sign, so the covariance of the real interest rate with the SDF and the price of real interest rate risk can be either positive or negative. Because the model is observationally equivalent when both ψ_t and the shocks it multiplies switch sign, without loss of generality we normalize the model such that ψ_t has a positive mean.[8]

We allow for two shocks in the real interest rate because a single shock would imply a constant Sharpe ratio for real bonds. With only a heteroskedastic shock, the model would also imply that the conditional volatility of the real interest rate would be proportional to the covariance between the real interest rate and the real SDF; equivalently, the conditional correlation of the real rate and the SDF would be constant in absolute value with occasional sign switches. Our specification avoids these implausible implications while remaining reasonably parsimonious.

In this model, the log prices of real bonds are linear in x_t and quadratic in ψ_t:

$$p_{n,t} = A_n + B_{x,n} x_t + B_{\psi,n} \psi_t + C_{\psi,n} \psi_t^2, \tag{5}$$

[7] In an earlier version of this paper we assumed a homoskedastic process for the real interest rate, writing a model in which ψ_t only affects inflation and nominal interest rates. This generates a simpler affine real term structure of interest rates, but is inconsistent with time-variation in the covariance between TIPS returns and the real economy documented in this paper and by Campbell, Shiller, and Viceira (2009).

[8] Since ψ_t appears in the model as a multiplier for shocks whose variances are free parameters, the model also requires a normalization assumption to set the scale of ψ_t. We choose to normalize the variance of shocks to realized inflation to one, leaving the variance of shocks to ψ_t as a free parameter.

where the coefficients A_n, $B_{x,n}$, $B_{\psi,n}$, and $C_{\psi,n}$ solve a set of recursive equations given in the Appendix. These coefficients are functions of the maturity of the bond (n) and the coefficients that determine the stochastic processes for the state variables. From equation (3), $B_{x,1} = -1$ and the remaining coefficients are zero at $n = 1$.

The conditional risk premium on a two-period real bond is linear in ψ_t:

$$\mathrm{E}_t\left[r_{2,t+1} - r_{1,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left(r_{2,t+1} - r_{1,t+1}\right) = -(\sigma_{Xm} + \sigma_{xm}\psi_t). \tag{6}$$

To gain intuition about this risk premium, consider the simple case where $\sigma_{Xm} = 0$, $\sigma_{xm} < 0$, and $\psi_t > 0$. In this case the risk premium on a two-period real bond is positive, because with negative σ_{xm} and positive ψ_t, the real interest rate tends to rise in bad times and fall in good times. Since real bond returns move opposite the real interest rate, a two-period real bond is then a procyclical risky asset that commands a positive risk premium. However, if ψ_t changes sign and becomes negative, with the same fixed value of σ_{xm} the real interest rate tends to rise in good times and fall in bad times. This makes a two-period real bond a countercyclical asset that hedges against an economic downturn and commands a negative risk premium.

The conditional risk premium on an n-period bond, for $n > 2$, is no longer linear in ψ_t because of the quadratic term in equation (5). Empirically, however, the degree of nonlinearity is slight as we show in our empirical analysis below, and risk premia on real bonds of all maturities have the same sign as the risk premium on a two-period real bond.

3.2 Inflation and the nominal term structure

To price nominal bonds, we need a model for inflation. We assume that log inflation $\pi_t = \log(\Pi_t)$ follows a linear-quadratic conditionally heteroskedastic process:

$$\pi_{t+1} = \lambda_t + \xi_t + \frac{\sigma_\pi^2}{2}\psi_t^2 + \psi_t\varepsilon_{\pi,t+1}, \tag{7}$$

where ψ_t is given in (4) and expected log inflation is the sum of two components, a permanent component λ_t and a transitory component ξ_t.

The dynamics of these components are given by

$$\lambda_{t+1} = \lambda_t + \varepsilon_{\Lambda,t+1}, \tag{8}$$

and

$$\xi_{t+1} = \phi_\xi \xi_t + \psi_t \varepsilon_{\xi,t+1}. \tag{9}$$

The presence of an integrated component in expected inflation removes the need to include a nonzero mean in the stationary component of expected inflation.

We assume that the underlying shocks to realized inflation, the components of expected inflation, and conditional inflation volatility—$\varepsilon_{\pi,t+1}$, $\varepsilon_{\Lambda,t+1}$, $\varepsilon_{\xi,t+1}$, and $\varepsilon_{\psi,t+1}$—are again jointly normally distributed zero-mean shocks with a constant variance-covariance matrix. We allow these shocks to be cross-correlated with the shocks to m_{t+1} and x_{t+1}. We normalize the standard deviation of realized inflation shocks, σ_π, to one, but as already mentioned this is without loss of generality as ψ_t multiplies this shock and has a freely estimated standard deviation.

Our inclusion of two components of expected inflation gives our model the flexibility it needs to fit both persistent variation in long-term nominal interest rates and inflation, and transitory variation in short rates relative to long rates. The former requires persistent variation in expected inflation, while the latter requires transitory variation in some state variable. The persistence and volatility of the long-term inflation-indexed bond yield implies that the real interest rate is highly persistent, so under our assumption that a single AR(1) process drives the real interest rate, we need a transitory component of expected inflation to generate changes in the slope of the nominal yield curve.[9]

We use the same state variable ψ_t that drives changing volatility in the real term structure to drive changes in inflation volatility. This keeps our model parsimonious while capturing the inflation heteroskedasticity first modelled by Engle (1982) in a manner consistent with the common movements of nominal and inflation-indexed bond volatility documented by Campbell, Shiller, and Viceira (2009).[10]

[9] There are other specifications that could be used to fit these facts. We impose a unit root on the persistent component of expected inflation for convenience of model analysis and estimation, but a near-unit root would also be viable. Regime-switching models offer an alternative way to reconcile persistent fluctuations with stationary long-run behavior of interest rates (Garcia and Perron 1996, Gray 1996, Bansal and Zhou 2002, Ang, Bekaert, and Wei 2008). We could also allow the real interest rate to have both a persistent and transitory component, in which case expected inflation could be purely persistent. Our specification is consistent with Cogley, Primiceri, and Sargent (2010) and generalizes Stock and Watson (2007) to allow some persistence in the stationary component of inflation. Mishkin (1990) presents evidence that bond yield spreads forecast future changes in inflation, which is also consistent with our specification.

[10] Although not reported in the article, the correlation in their data between the volatility of

Equation (8) allows only a homoskedastic shock $\varepsilon_{\Lambda,t+1}$ to impact the permanent component of expected inflation, while equation (9) allows only a heteroskedastic shock $\varepsilon_{\xi,t+1}$ to impact the temporary component of expected inflation. We need at least one homoskedastic and one heteroskedastic shock to components of the inflation process for reasons similar to those that lead us to assume one shock of each type to the real interest rate process. We have experimented with allowing multiple shocks, both homoskedastic and heteroskedastic, to both components of inflation but have found that such a model is only weakly identified. For parsimony, therefore, we allow only a homoskedastic shock to long-term expected inflation and only heteroskedastic shocks to transitory expected and realized inflation.

The process for realized inflation, equation (7), is formally similar to the process for the log SDF (1) in that it includes a quadratic term. This term simplifies the process for the reciprocal of inflation by making the log of the conditional mean of $1/\Pi_{t+1}$ the negative of the sum of the two state variables λ_t and ξ_t. This in turn simplifies the pricing of short-term nominal bonds.

The real cash flow on a single-period nominal bond is simply $1/\Pi_{t+1}$. Thus the price of the bond is given by $P^{\$}_{1,t} = \mathrm{E}_t \left[\exp\left\{m_{t+1} - \pi_{t+1}\right\}\right]$,so the log short-term nominal rate $y^{\$}_{1,t+1} = -\log\left(P^{\$}_{1,t}\right)$ is

$$\begin{aligned} y^{\$}_{1,t+1} &= -\mathrm{E}_t\left[m_{t+1} - \pi_{t+1}\right] - \frac{1}{2}\mathrm{Var}_t\left(m_{t+1} - \pi_{t+1}\right) \\ &= x_t + \lambda_t + \xi_t - \sigma_{m\pi}\psi_t. \end{aligned} \tag{10}$$

The log nominal short rate is the sum of the log real interest rate, the two state variables that drive expected log inflation, and a term that accounts for the correlation between shocks to inflation and shocks to the stochastic discount factor. This term, $-\sigma_{m\pi}\psi_t$, is the expected excess return on a single-period nominal bond over a single-period real bond so it measures the inflation risk premium at the short end of the term structure.

The log price of a n-period zero-coupon nominal bond is a linear-quadratic function of the vector of state variables:

$$p^{\$}_{n,t} = A^{\$}_n + B^{\$}_{x,n}x_t + B^{\$}_{\lambda,n}\lambda_t + B^{\$}_{\xi,n}\xi_t + B^{\$}_{\psi,n}\psi_t + C^{\$}_{\psi,n}\psi_t^2, \tag{11}$$

where the coefficients $A^{\$}_n$, $B^{\$}_{i,n}$, and $C^{\$}_{i,n}$ solve a set of recursive equations given in

nominal US Treasury bond returns and the volatility of TIPS returns is slightly greater than 0.7.

the Appendix. From equation (10), $B^{\$}_{x,1} = B^{\$}_{\xi,1} = B^{\$}_{\lambda,1} = -1$, $C^{\$}_{z\psi,1} = \sigma_{m\pi}$, and the remaining coefficients are zero at $n = 1$.

Like risk premia in the real term structure, risk premia in the nominal term structure are increasing in ψ_t and are approximately (but not exactly) linear in ψ_t. When $\psi_t > 0$, realized inflation and transitory expected inflation are countercyclical, so nominal bonds are procyclical and investors demand a positive risk premium to hold them. When $\psi_t < 0$, these components of inflation are procyclical, so nominal bonds are countercyclical and become desirable hedges against business cycle risk.

3.3 Pricing equities

We want our model to fit the changing covariance of bonds and stocks, and so we must specify a process for the equity return within the model. One modelling strategy would be to specify a dividend process and solve for the stock return endogenously in the manner of Bekaert et al. (2005), d'Addona and Kind (2006), and Campbell, Pflueger, and Viceira (2015). However we adopt a simpler approach. Following Campbell and Viceira (2001), we model shocks to realized stock returns as a linear combination of shocks to the real interest rate and shocks to the log stochastic discount factor:

$$r_{e,t+1} - \mathrm{E}_t\, r_{e,t+1} = \beta_{ex}\varepsilon_{x,t+1} + \beta_{eX}\varepsilon_{X,t+1} + \beta_{em}\varepsilon_{m,t+1} + \varepsilon_{e,t+1}, \tag{12}$$

where $\varepsilon_{e,t+1}$ is an identically and independently distributed shock uncorrelated with all other shocks in the model. This shock captures movements in equity returns that are both unrelated to real interest rates and carry no risk premium because they are uncorrelated with the SDF.

Substituting (12) into the no-arbitrage condition $E_t\left[M_{t+1}R_{t+1}\right] = 1$, the Appendix shows that the equity risk premium is given by

$$\mathrm{E}_t\left[r_{e,t+1} - r_{1,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left(r_{e,t+1} - r_{1,t+1}\right) = \beta_{ex}\sigma_{xm} + \beta_{eX}\sigma_{Xm} + \beta_{em}\sigma^2_m. \tag{13}$$

The equity premium depends not only on the direct sensitivity of stock returns to the SDF, but also on the sensitivity of stock returns to the real interest rate and the covariance of the real interest rate with the SDF.

Equation (12) does not attempt to capture heteroskedasticity in stock returns. Although such heteroskedasticity is of first-order importance for understanding stock prices, we abstract from it here in order to maintain the parsimony of our term structure model. Moreover, as Figure 1 shows, the stock-bond covariance and the stock-bond beta move closely together, indicating that our assumption of homoskedastic stock returns is not overly restrictive for the purposes of studying the quantity of risk in nominal bonds.

The conditional covariance between the SDF and inflation also determines the covariance between the excess returns on real and nominal assets. Consider for example the conditional covariance between the real return on a one-period nominal bond and the real return on equities, both in excess of the return on a one-period real bond. This covariance is given by

$$\mathrm{Cov}_t\left(r_{e,t+1} - r_{1,t+1}, y^{\$}_{1,t+1} - \pi_{t+1} - r_{1,t+1}\right) = -\left(\beta_{ex}\sigma_{x\pi} + \beta_{em}\sigma_{m\pi}\right)\psi_t,$$

which moves over time and can change sign. This implies that we can identify the dynamics of the state variable ψ_t from the dynamics of the conditional covariance between equities and nominal bonds as well as real bonds.

4 Model Estimation

4.1 Data and estimation methodology

The term structure model presented in Section 3 generates bond yields which are linear-quadratic functions of a vector of latent state variables. We now use this model to study the postwar history of yields on US Treasury nominal and inflation-indexed bonds. Since our state variables are not observable, and the observable series have a nonlinear dependence on the latent state variables, we obtain maximum likelihood estimates of our model's parameters via a nonlinear Kalman filter. Specifically, we use the unscented Kalman filter estimation procedure of Julier and Uhlmann (1997).

The unscented Kalman filter is a nonlinear Kalman filter which works through deterministic sampling of points in the distribution of the innovations to the state variables, does not require the explicit computation of Jacobians and Hessians, and captures the conditional mean and variance-covariance matrix of the state variables accurately up to a second-order approximation for any type of nonlinearity, and up to a third-order approximation when innovations to the state variables are Gaussian.

Wan and van der Merwe (2001) describe in detail the properties of the filter and its practical implementation.

To implement the unscented Kalman filter, we specify a system of twelve measurement equations that relate observable variables to the vector of state variables. We sample the data at a quarterly frequency in order to minimize the impact of high-frequency noise in the measurement of some of our key variables—such as realized inflation—while keeping the frequency of observation reasonably high (Campbell and Viceira 2001, 2002). By not having to fit all the high-frequency monthly variation in the data, our estimation procedure can concentrate on uncovering the low-frequency movements in interest rates which our model is designed to capture.

Our first four measurement equations relate observable nominal bond yields to the vector of state variables, as in equation (11). We use yields on constant maturity 3-month, 1-year, 3-year, and 10-year zero-coupon nominal bonds sampled at a quarterly frequency for the period 1953Q1-2014Q4. These data are spliced together from two sources. From 1953Q1-1961Q1 we sample quarterly from the monthly dataset developed by McCulloch and Kwon (1993), and from 1961Q2-2014Q4 we sample quarterly from the daily dataset constructed by Gürkaynak, Sack, and Wright (GSW 2006, updated through 2014). We assume that bond yields are measured with errors, which are uncorrelated with each other and with the structural shocks of the model.

Our fifth measurement equation, (7), relates the observed inflation rate to expected inflation and inflation volatility, plus measurement error. We use the CPI as our observed price index in this measurement equation. We complement this measurement equation with another one that uses data on the median forecast of GDP deflator inflation one quarter ahead from the Survey of Professional Forecasters for the period 1968Q4-2014Q4. We relate this observed measure of expected inflation to the sum of equations (8) and (9) in our model plus measurement error. Before 1968Q4, we treat the survey forecast of inflation as missing, which can easily be handled by the Kalman filter estimation procedure.

The seventh measurement equation relates the observed yield on constant maturity Treasury inflation protected securities (TIPS) to the vector of state variables, via the pricing equation for real bonds (5). We obtain data on constant maturity zero-coupon 10-year TIPS dating back to 1999Q1 from GSW (2008). Before 1999, we treat the TIPS yield as missing, and as with nominal bond yields, we assume that real bond yields are measured with errors.

Figure 3 illustrates our real bond yield series. The decline in the TIPS yield since the year 2000, and the spike in the fall of 2008, are clearly visible in this figure. Campbell, Shiller, and Viceira (2009) document that this decline in the long-term real interest rate, and the subsequent sudden increase during the financial crisis, occurred in inflation-indexed bond markets around the world. In earlier data from the UK, long-term real interest rates were much higher on average during the 1980's and 1990's. Our model will explain such large and persistent variation in the TIPS yield primarily using persistent movements in the short-term real interest rate.

Our eighth measurement equation uses equity returns from the CRSP value-weighted index comprising the stocks traded in the NYSE, AMEX, and NASDAQ. This equation describes realized log equity returns $r_{e,t+1}$ using equations (12) and (13).

The last four measurement equations use the implications of our model for: (i) the conditional covariance between equity returns and real bond returns, (ii) the conditional covariance between equity returns and nominal bond returns, (iii) the conditional volatility of real bond returns, and (iv) the conditional volatility of nominal bond returns. The Appendix derives expressions for these time-varying conditional second moments, which are functions of ψ_t and therefore help us filter this state variable. Following Viceira (2012), we construct the analogous realized second moments using high-frequency data. We obtain daily stock returns from CRSP and calculate daily nominal bond returns from daily GSW nominal yields from 1961Q2 onwards, and daily real bond returns from daily GSW real yields from 1999Q1 onwards.[11] We then compute the variances and covariances realized over quarter t.

Realized variances and covariances in quarter t are expected variances and covariances at quarter $t-1$, plus shocks realized in quarter t. Unfortunately we cannot treat such shocks as pure measurement error because they may be contemporaneously correlated with innovations to the state variables of our model.[12] Accordingly we project realized variances and covariances onto information known at quarter $t-1$, and treat the fitted values as the conditional (expected) moments at quarter $t-1$ plus measurement error. For each realized variance and covariance, we use three pieces of information known at quarter $t-1$: the lagged value of the realized variance or

[11] We calculate daily returns on the n year bond from daily yields as $r_{n,t+1} = ny_{n,t} - (n-1/264)\, y_{n,t+1}$. We assume there are 264 trading days in the year, or 22 trading days per month. Prior to 1961:2, we calculate monthly returns from monthly McCulloch-Kwon nominal yields, and calculate variances and covariances using a rolling 12-month return window.

[12] We thank an anonymous referee for pointing out this issue.

covariance, the 3-month nominal Treasury yield, and the spread between the 10-year nominal yield and the 3-month nominal yield. Viceira (2012) shows these variables have strong predictive power for the realized second moments at quarter t. We also note that, because the realized second moments are persistent, the fitted values are quite similar to the realized second moments at quarter t.

The data used in these measurement equations are plotted in Figure 4. The two panels at the left show the projected covariances between daily stock and bond returns for nominal bonds (above) and inflation-indexed bonds (below, over a shorter sample), while the two panels at the right show the projected variances of daily nominal and real bond returns. The thick lines in each panel show a smoothed version of the raw data.

Figure 4 shows that both the stock-nominal bond covariance series and the nominal bond variance series increase in the early 1970's and, most dramatically, in the early 1980's. In the 1950's, and again in the 2000's, the stock-nominal bond covariance was negative, with downward spikes in the two recessions of the early 2000's and the late 2000's. The bottom part of the figure shows that the stock-real bond covariance series and the real bond variance series follow patterns similar to those of nominal bonds for the overlapping sample period.

Our model has a large number of shocks, and we have found that many of the covariances between these shocks are only very weakly identified, so that setting them to zero does not materially affect the empirical results. For parsimony we constrain some of these weakly identified covariances. The unconstrained parameters are the covariances of all shocks with the stochastic discount factor $(\sigma_{xm}, \sigma_{Xm}, \sigma_{\Lambda m}, \sigma_{\xi m}, \sigma_{\psi m}, \sigma_{\pi m})$, which are important because they determine the risk premia for different types of risk; and the covariance of the transitory component of expected inflation with realized inflation $(\sigma_{\xi\pi})$, which helps to deliver realistic dynamics for inflation and nominal interest rates. Other covariances, including all the covariances between shocks to the real interest rate and shocks to realized and expected inflation, are set to zero.

4.2 Parameter estimates

Table 1 Panel A presents quarterly parameter estimates over the period 1953Q1-2014Q4 and their asymptotic standard errors, calculated numerically using the outer product method. Panel A reports first means, then persistence coefficients, then shock volatilities, then loadings of stock returns on shocks, and finally those correlations between shocks that are estimated freely and not restricted to equal zero.[13] Table 1 Panel B converts the estimated shock volatilities into average conditional volatilities of the state variables.

The real interest rate is the most persistent state variable, with an autoregressive coefficient of 0.97 corresponding to a half life of about six years. This persistence reflects the observed variability and persistence of TIPS yields. The transitory component of expected inflation and the nominal-real covariance are less persistent processes in our model, with half-lives of about six and five quarters respectively. Of course the model also includes a permanent component of expected inflation. If we model expected inflation as a single stationary AR(1) process, as we did in the first version of this paper, we find expected inflation to be more persistent than the real interest rate. All persistence coefficients are precisely estimated, with very small standard errors.

The shock volatilities reported in panel A are fairly precisely estimated and vary considerably across shocks. Restating them in panel B as average one-quarter conditional volatilities of the annualized state variables for ease of interpretation, the estimated one-quarter conditional volatility of the homoskedastic shock to the annualized real interest rate is 32 basis points, and the average one-quarter conditional volatility of the heteroskedastic shock to the annualized real interest rate is 42 basis points. The conditional volatility of the homoskedastic permanent shock to annualized expected inflation is 27 basis points, the average conditional volatility of the heteroskedastic transitory shock to annualized expected inflation is 57 basis points, and the average conditional volatility of the heteroskedastic shock to annualized realized inflation is 209 basis points. All these numbers are plausible for quarterly innovation volatilities. Of course, the unconditional standard deviations of the real interest rate and the two components of expected inflation are much larger because of the high persistence of the processes; in fact, the unconditional standard deviation of the permanent component of expected inflation is undefined because this process has a unit root.

Returning to panel A, we estimate a large and statistically significant positive loading of stock returns on shocks to the negative of the log SDF (β_{em}). Naturally

[13] We report correlations instead of covariances to facilitate interpretation. We compute their standard errors from those of the primitive parameters of the model using the delta method. The Appendix reports covariances and their asymptotic standard errors.

this estimate implies a positive equity risk premium.[14] We also estimate a statistically significant positive loading β_{eX} of stock returns on homoskedastic shocks to the real interest rate, consistent with the tendency in recent data for real interest rates to decline during stock market downturns. The loading of stock returns on heteroskedastic shocks to the real interest rate is much smaller and statistically insignificant.

There is a statistically significant correlation of -0.19 between ξ and $-m$ shocks. Since ξ shocks are heteroskedastic, this negative correlation implies that the transitory component of expected inflation is countercyclical, generating a positive risk premium in the nominal term structure, when the state variable ψ_t is positive; but transitory expected inflation is procyclical, generating a negative risk premium, when ψ_t is negative. In addition, we estimate a marginally statistically significant negative correlation of almost -0.29 between x and $-m$ shocks. Since x shocks are heteroskedastic shocks to the real interest rate, this similarly implies a time-varying term premium on real bonds that is positive when ψ_t is positive.

The estimated correlations with $-m_t$ of other shocks to the term structure, the homoskedastic shocks X and Λ and realized inflation π, are all very small and statistically insignificant. This implies that bond risk premia are not just linear in ψ_t but almost proportional to it, and that short-term inflation risk is almost unpriced, so nominal three-month Treasury bills have an almost zero inflation risk premium.

Finally, we estimate a strongly positive and statistically significant correlation of 0.69 between ψ_t and $-m_t$ shocks. This implies that bonds tend to become better hedges (more negatively correlated with stocks) in bad times, consistent with recent experience during the global financial crisis although contrary to the pattern observed in the 1980s.[15]

[14] However, the equity premium in the model is substantially lower than in the data. Our estimates imply a maximum Sharpe ratio of 14%, while the Sharpe ratio for equities in our data is 40%. The estimation routine prefers to trade a counterfactually low maximum Sharpe ratio to improve the model's fit along other dimensions. In particular, raising the Sharpe ratio creates counterfactually high bond return volatilities.

[15] In earlier versions of this paper, with data ending in 2009, we estimated a slightly positive correlation between ψ_t and $-m_t$ shocks. Since bond risk premia rise with the quantity of risk, the short-sample finding was consistent with Ludvigson and Ng (2009), who report evidence that bond risk premia are countercyclically related to macroeconomic factors.

4.3 Fitted state variables

How does our model interpret the economic history of the last 60 years? That is, what time series does it estimate for the underlying state variables that drive bond and stock prices? Figure 5 shows our estimates of the four state variables in the model, with the real interest rate x_t in the top left panel. The model estimates a process for the real interest rate that is high on average, with a spike in the early 1980's, and becomes more volatile and declining in the second half of the sample. Higher-frequency movements in the real interest rate were often countercyclical in this period, as we see the real rate falling in the recessions of the early 1970's, early 1990's, early 2000's, and at the end of our sample period in 2007–09. The real interest rate also falls around the stock market crash of 1987. However there are important exceptions to this pattern, notably the very high real interest rate in the early 1980's, during Paul Volcker's campaign against inflation. Since the late 1990's the real interest rate generally tracks the TIPS yield, as shown in Figure 3. Thus the model attributes the history of long-dated TIPS yields mostly to changes in the short-term real rate x_t, with a supporting role for the state variable ψ_t.

The permanent component of expected inflation, in the top right panel of Figure 5, exhibits a familiar hump shape over the postwar period. It was low, even negative, in the mid-1950's, increased during the 1960's and 1970's, and reached a maximum value of about 10% in the first half of the 1980's. Since then, it has experienced a secular decline and remained close to 2% throughout the 2000's. This is consistent with the stability of professional survey forecasts of inflation, our sixth measurement variable, during this period.

The transitory component of expected inflation, in the bottom left panel, was particularly high in the late 1970's and early 1980's, indicating that investors expected inflation to decline gradually from a temporarily high level. The transitory component was predominantly negative from the mid 1980's to the mid 2000's, implying that our model attributes the generally high levels of yield spreads during this period at least partly to investor beliefs that inflation would increase in the future. By estimating a generally negative transitory component of expected inflation, the model is also able to explain simultaneously the low average nominal short-term interest rate and the high average real short-term interest rate in this period. During the last ten years, the transitory component of expected inflation has been more volatile and has reached positive values once again.

Finally, the bottom right panel of Figure 5 shows the time series of ψ_t. As we have noted, this variable is identified primarily through the covariance of stock returns and bond returns and the volatility of bond returns—both nominal and real. The state variable ψ_t exhibits low volatility and an average close to zero in the period leading up to the late 1970's, with briefly negative values in the late 1950's, and an upward spike in the early 1970's. It becomes much more volatile starting in the late 1970's through the end of our sample period. It rises to large positive values in the early 1980's and stays predominantly positive through the 1980's and 1990's. However, in the late 1990's it switches sign and turns predominantly negative, with particularly large downward spikes in the period immediately following the recession of 2001 and in the fall of 2008, at the height of the financial crisis of 2007–09. Thus ψ_t not only can switch sign, it has done so during the past twenty years. Overall, the in-sample average for ψ_t is positive, consistent with the positive unconditional mean estimated in Table 1.

The state variables we have estimated can be used to calculate fitted values for observed variables such as the nominal term structure, real term structure, realized inflation, analysts' median inflation forecast, and the realized second moments of bond and equity returns. Generally these fitted values track the observed data well, because our model is rich enough that it does not require measurement errors with high volatility to fit the data. As an illustration of this point, the left panels of Figure 6 show the realized variance of nominal bond returns (top panels) and covariance between bond and stock returns (bottom panels), along with their values fitted by the model. The right panels decompose the model-fitted values into the contributions of the state variable ψ_t and its square. These panels show that the bond-stock covariance is linear in ψ_t, while the behavior of the variance is dominated by the square ψ_t^2.

5 Term Structure Implications

5.1 Moments of bond yields and returns

Although our model fits the observed history of real and nominal bond yields, an important question is whether it must do so by inferring an unusual history of shocks, or whether the observed properties of interest rates emerge naturally from the properties of the model at the estimated parameter values. In order to assess this, Table 2 reports some important moments of bond yields and returns.

The table compares the sample moments in our historical data with moments calculated by simulating our model 10,000 times along a path that is 250 quarters (or 62 and a half years) long, and averaging time-series moments across simulations. Sample moments are shown in the first column and model-implied moments in the second column. The third column reports the fraction of simulations for which the simulated time-series moment is larger than the corresponding sample moment in the data. These numbers can be used as informal tests of the ability of the model to fit each sample moment. Although our model is estimated using maximum likelihood, these diagnostic statistics capture the spirit of the method of simulated moments (Duffie and Singleton 1993, Gallant and Tauchen 1996), which minimizes a quadratic form in the distance between simulated model-implied moments and sample moments.[16]

The first two rows of Table 2 report the sample and simulated means for nominal bond yield spreads, calculated using 3 and 10 year maturities, and the third and fourth rows look at the volatilities of these spreads. Our model provides a fairly good fit to yield spreads at the 3-year maturity, but it does somewhat understate the average 10-year spread and overstate the volatility of this spread.

The next four rows show how our model fits the means and standard deviations of realized excess returns on 3-year and 10-year nominal bonds. In order to calculate quarterly realized returns from constant-maturity bond yields, we interpolate yields between the constant maturities we observe, doing this in the same manner for our historical data and for simulated data from our models. The model slightly understates average realized excess returns, particularly at the 10-year maturity, but this may not be surprising in a sample period that ends with extremely low interest rates. It slightly overstates the volatility of realized excess returns at the 3-year maturity.

The next four rows of the table summarize our model description of TIPS yields. The model generates an average TIPS yield that is much higher than the observed average, and in fact none of our 10,000 simulations produce an average yield as low as the one observed in the data. We do not believe this is a serious problem, as our

[16] In Table 2 the short-term interest rate is a three-month rate and moments are computed using a three-month holding period. In the Appendix we report a table using a one-year short rate and holding period. This alternative table follows Cochrane and Piazzesi (2005), and shows us how our model fits lower frequency movements at the longer end of the yield curve. Results are comparable to those reported in Table 2.

estimates imply higher real interest rates earlier in our sample period, before TIPS were issued, than in the period since 1997 over which we measure the average TIPS yield. Thus the discrepancy may result in part from the short and unrepresentative period over which we measure the average TIPS yield in the data.

The model implies a small negative average real yield spread and a positive but tiny average realized excess return on TIPS. The difference between these two statistics reflects the effect of Jensen's Inequality; equivalently, it is the result of convexity in long-term bonds. The sign of the average risk premium on TIPS results from our negative estimate of ρ_{xm} in Table 1, which implies that the real interest rate is countercyclical on average.

5.2 Risk premia and the yield curve

In our model, all time variation in bond risk premia is driven by variation in bond risk, not by variation in the aggregate price of risk. It follows that long bond risk premia are almost exactly linear in the state variable ψ_t (with a very slight nonlinearity arising from the effect of ψ_t^2 on bond yields). Figure 7 illustrates this fact. The left panel plots the model's expected excess return on 3-year and 10-year nominal bonds over 3-month Treasury bills against ψ_t. The right panel of the figure shows the term structure of risk premia as ψ_t varies from its sample mean to its sample minimum and maximum. Risk premia spread out rapidly as maturity increases, and 10-year risk premia vary from -80 to 120 basis points.

The full history of our model's 10-year term premium is illustrated in Figure 8. The figure shows fairly stable risk premia close to zero during the 1950's and 1960's, a spike up to about 0.7% in the early 1970's, and a run up later in the 1970's to a peak of about 1.2% in 1980Q3. A long decline in risk premia later in the sample period was accentuated around the recession of the early 2000's and during the financial crisis of 2007–09, bringing the risk premium to its sample minimum of -0.8% in 2008Q3. This time series reflects the shape in the nominal-real covariance ψ_t illustrated in the bottom right panel of Figure 5.

Figure 8 also plots the model's implied Sharpe ratio on long-term bonds. This varies between 0.05 and -0.05, with its maximum value about one-third of the model's implied Sharpe ratio for equities (as noted above, the model underpredicts the realized Sharpe ratio on the US stock market during our sample period). It is noticeable that the Sharpe ratio does not spike as high during the early 1980's as the term premium does, because this was a period of unusually high bond market volatility. Instead, the Sharpe ratio remains elevated and fairly stable throughout the 1980s, declining only in the early 1990s. Similarly, the Sharpe ratio is more stable at a negative value during the global financial crisis and the subsequent economic downturn in the late 2000s.

An important question is how the shape of the yield curve responds to time-variation in bond risk premia. To isolate the effect of changing ψ_t, Figure 9 plots the log real and nominal yield curves generated by our model when ψ_t is at its in-sample mean, maximum, and minimum, while all other state variables are at their in-sample means. Thus the central line describes the yield curve—real or nominal—generated by our model when all state variables are evaluated at their in-sample mean. For simplicity we will refer to this curve as the "mean log yield curve."[17]

In both panels of Figure 9, increasing ψ_t from the sample mean to the sample maximum raises intermediate-term yields and lowers long-term yields, while decreasing ψ_t to the sample minimum lowers both intermediate-term and long-term yields. Thus ψ_t alters the concavity of both the real and nominal yield curves.

The impact of ψ_t on the concavity of the nominal yield curve results from two features of our model. First, nominal bond risk premia increase with maturity rapidly at intermediate maturities and slowly at longer maturities because intermediate maturities are exposed both to transitory and permanent shocks to expected inflation, and transitory shocks have greater systematic risk (a stronger correlation with the stochastic discount factor). When ψ_t is positive, this generates a steep yield curve at shorter maturities, and a flatter one at longer maturities. When ψ_t changes sign, however, the difference in risk prices pulls intermediate-term yields down more strongly than long-term yields.

Second, when ψ_t is far from zero bond returns are unusually volatile, and through Jensen's Inequality this lowers the bond yield that is needed to deliver any given expected simple return. This effect is stronger for long-term bonds; in the terminology of the fixed-income literature, these bonds have much greater "convexity" than short-

[17] Strictly speaking this is a misnomer for two reasons. First, the log real and nominal yield curves are non-linear functions of the vector of state variables. Second, the unconditional mean of the log nominal yield curve is not even defined, since one of the state variables follows a random walk. Thus at most we can compute a mean nominal yield curve conditional on initial values for the state variables.

or intermediate-term bonds. Therefore extreme values of ψ_t tend to lower long-term bond yields relative to intermediate-term yields.

Similar effects operate in the real term structure. Real bond risk premia are highly sensitive to ψ_t at intermediate maturities because real interest rate variation is transitory, and long-term real bonds have high convexity so their yields are driven down by high levels of bond volatility.

In the Appendix, we conduct similar analyses of term structure responses to our model's other state variables. Real interest rate shocks have highly persistent effects on both the real and nominal yield curve, while the permanent component of expected inflation shifts the nominal yield curve up and down, and the transitory component of expected inflation changes the slope of the nominal yield curve. These results can be related to Litterman and Scheinkman's (1991) "level", "slope", and "curvature" factors. In our model, the covariance of nominal and real variables ψ_t primarily drives the curvature factor while the other state variables primarily move the level and slope factors. Thus our model implies that the curvature factor should have predictive power for excess bond returns.

An empirical result of this sort has been reported by Cochrane and Piazzesi (CP, 2005). Using econometric methods originally developed by Hansen and Hodrick (1983), and implemented in the term structure context by Stambaugh (1988), CP show that a single linear combination of forward rates is a good predictor of excess bond returns at a wide range of maturities. CP work with a 1-year holding period and a 1-year short rate. They find that bond risk premia are high when intermediate-term interest rates are high relative to both shorter-term and longer-term rates; that is, they are high when the yield curve is strongly concave.

Our model interprets this phenomenon as the result of changes in the nominal-real covariance ψ_t. As ψ_t increases, the risk premium for transitory expected inflation rises. This strongly increases the intermediate-term yield, but it has a damped or even perverse effect on long-term yields because these yields respond primarily to the permanent component of expected inflation and the convexity of long bonds causes their yields to fall with volatility. Thus excess bond returns are predicted by the intermediate-term yield relative to the average of short- and long-term yields.

5.3 The predictability of bond returns

Despite this promising qualitative pattern, bond returns have very limited predictability in our model. Table 3 illustrates this point. In the first three rows we report the standard deviations of true expected 3-month excess returns within the model. The annualized standard deviation for the expected excess return on 3-year bonds is 10 basis points, and for the expected excess return on 10-year bonds it is 21 basis points.[18] This variation is an order of magnitude smaller than the annualized standard deviations of realized excess bond returns, implying that the true explanatory power of 3-month predictive regressions is very small in our model. There is similar variability of about 16 basis points in the true expected excess returns on TIPS.

The next three rows report the standard deviations of fitted values of Campbell-Shiller (1991, CS) predictability regressions of annualized nominal bond excess returns onto yield spreads of the same maturity at the beginning of the holding period. The standard deviations in the data are 102 basis points for 3-year bonds, and 270 basis points for 10-year bonds. These numbers are considerably larger than the true variability of expected excess returns in our model, implying that our model cannot match the behavior of these predictive regressions.

In artificial data generated by our model, predictive regressions deliver fitted values that are considerably more volatile than the true expected excess returns. The reason for this counterintuitive behavior is that there is important finite-sample bias in the CS regression coefficients of the sort described by Stambaugh (1999). In the case of regressions of excess bond returns on yield spreads, by contrast with the better known case of regressions of excess stock returns on dividend yields, the Stambaugh bias is negative (Bekaert, Hodrick, and Marshall 1997). In our model, where the true regression coefficient is positive but close to zero, the Stambaugh bias increases the standard deviation of fitted values by generating spurious negative coefficients. Nonetheless, the standard deviation of fitted values in the model is still much smaller than in the data, particularly for 10-year excess bond returns.

Another way to understand the difficulty here is to decompose the time-variation of the yield spread—the CS explanatory variable—into components due to transitory

[18] Yield interpolation for 3-month returns may exaggerate the evidence for predictability; however the same yield interpolation is used for simulated data from our models. We have used our simulations to examine the effect of interpolation. We find that interpolation does slightly increase measured bond return predictability, but the effect is modest.

expected inflation, the real interest rate, and the state variable ψ_t and its square. Figure 10 plots the actual and fitted yield spread in the left panel, and the components explained by each state variable in the right panel. Transitory shocks to expected inflation contribute most of the high-frequency variation, while the real interest rate contributes some lower-frequency variation, in particular explaining why the yield spread has been higher on average in the later part of the sample. The state variable ψ_t and its square have only very small effects on the spread. Even though ψ_t determines the risk premium, the variation in the risk premium is neither large nor persistent enough to be a dominant influence on the yield spread in our model. This also explains why the history of the expected excess bond return shown in Figure 8 does not resemble the history of the yield spread shown in Figure 10.

We obtain more promising results using a procedure that approximates the approach of Cochrane and Piazzesi (2005, CP). We regress excess bond returns on 1-, 3-, and 5-year forward rates at the beginning of the holding period, which allows the regression to predict bond returns from the overall shape, and specifically the concavity, of the yield curve rather than just its slope.[19] Figure 11 reports the average coefficients estimated in our model in the left hand panel, and the coefficients estimated in the data in the right hand panel. The two sets of coefficients both have the "tent" shape found by CP. Table 3 reports the standard deviations of fitted values, and shows that for CP regressions these standard deviations are comparable in the model and in the data, at least for predicting excess 3-year bond returns. Once again, however, this finding is largely driven by small-sample bias as the fitted values in the model have a much higher standard deviation than the true expected excess returns. Figure 12 presents a decomposition of the 3-year forward rate relative to the average of the 1-year rate and 5-year forward rate, comparable to Figure 10 for the yield spread. The figure shows that the temporary component of expected inflation is the most important driver of this CP "tent" variable, even though the real interest rate x_t, and ψ_t and its square, also have an influence.

Our results can be related to recent suggestions that unspanned factors may be important in bond pricing (Duffee 2011, Joslin, Priebsch, and Singleton 2014). An unspanned factor predicts both expected future interest rates and risk premia in such a way that it does not influence current bond yields. While our term structure model is not set up to include an unspanned factor, we do find that ψ_t has only subtle effects

[19] Cochrane and Piazzesi use five forward rates rather than three, and an annual rather than a quarterly holding period. They also impose proportionality restrictions across the regressions at different maturities, which we do not.

on the shape of the term structure, which is primarily driven by other factors in our model. Thus, the model implies that it will be hard to extract risk premia reliably from the shape of the yield curve; direct estimation of ψ_t from other macroeconomic data may help one to understand the behavior of bond risk premia.

We have explored an extension of our model that allows for time-variation in the aggregate price of risk, identifying this time-variation explicitly with the yield spread as in Wachter (2006) and others. This extension allows the model to explain more of the observed variation in bond risk premia, perhaps unsurprisingly given prior results in the literature. However, the low-frequency variation in the bond risk premium generated by changing bond risk remains present in that more complicated framework.

6 Conclusion

We have argued that term structure models must confront the fact that the covariances between nominal and real bond returns, on the one hand, and stock returns, on the other, have varied substantially over time and have changed sign. Analyses of asset allocation traditionally assume that broad asset classes have a stable structure of risk over time; our empirical results imply that for bonds at least, this assumption is seriously misleading.

We have added a changing covariance, which can change sign, to an otherwise standard term structure model with a constant price of risk and identifiable macroeconomic state variables. In our model real and nominal bond returns are driven by four state variables: the real interest rate, transitory and permanent components of expected inflation, and a state variable that governs the covariances of inflation and the real interest rate with the stochastic discount factor. The model implies that the risk premia of nominal bonds should have changed over the decades because of changes in the covariance between inflation and the real economy. The model predicts positive nominal bond risk premia in the 1980's, when bonds covaried strongly with stocks, and negative risk premia in the 21st Century and particularly during the downturns of 2001 and 2007–09, when bonds hedged equity risk. The model-implied decline in the risk premium on a 10-year zero-coupon bond is about 2% from peak to trough.

Our model is consistent with the qualitative finding of Cochrane and Piazzesi (2005) that a tent-shaped linear combination of forward rates, with a peak at about 3 years, predicts excess bond returns at all maturities. Since the model has a constant price of bond risk and explains risk premia only from time-variation in the quantity of bond risk, it does not replicate the high explanatory power of regressions that predict excess US Treasury bond returns from yield spreads and forward rates. However, the results do suggest that time-varying bond risk is important in understanding lower-frequency movements in bond risk premia.

Our results pose a new challenge to the asset pricing literature. A successful asset pricing model should jointly explain the time-variation in bond and stock risk premia along with the time-variation in the comovements of bond and stock returns. Our model is a first attempt to do this, but it does not reconcile the changing second moments of bond and stock returns with high-frequency variation in bond risk premia captured by the shape of the yield curve. We hope that future term structure research will address the challenge by extending the model presented here.

There are a number of ways in which this can be done. First and most obviously, one can allow for changes in risk aversion, or the volatility of the stochastic discount factor, following Duffee (2002), Dai and Singleton (2002), Bekaert, Engstrom, and Grenadier (2005), Wachter (2006), Buraschi and Jiltsov (2007), and Bekaert, Engstrom, and Xing (2009).

Second, one can model changing second moments in stock returns, possibly deriving those returns from primitive assumptions on the dividend process, as in the recent literature on affine models of stock and bond pricing (Bekaert, Engstrom, and Grenadier 2005, d'Addona and Kind 2006, Bekaert, Engstrom, and Xing 2009).

Third, one can allow both persistent and transitory variation in the nominal-real covariance, as we have done for expected inflation. This might allow our model to better fit both the secular trends and cyclical variation in the realized covariance between bonds and stocks.

Fourth, one can consider other theoretically motivated proxies for the stochastic discount factor. An obvious possibility is to look at realized or expected future consumption growth, as in recent papers on consumption-based bond pricing by Piazzesi and Schneider (2006), Eraker (2008), Hasseltoft (2009), Lettau and Wachter (2011), and Bansal and Shaliastovich (2013). A disadvantage of this approach is that consumption is not measured at high frequency, so one cannot use high-frequency data to track a changing covariance between bond returns and consumption growth. Alternatively, it may be fruitful to incorporate other variables that may influence the stochastic discount factor such as the supply of Treasury bonds (Greenwood and Vayanos 2014, Krishnamurthy and Vissing-Jorgensen 2012).

It will also be interesting to estimate our model using data from other countries, for example the UK, where inflation-indexed bonds have been actively traded since the mid-1980's. Evidence of bond return predictability is considerably weaker outside the US (Bekaert, Hodrick, and Marshall 2001, Campbell 2003) and may better fit the predictability generated by our model.

Finally, it is important to better understand the monetary and macroeconomic determinants of the bond-stock covariance. Within a new Keynesian paradigm, one possibility is that a positive covariance corresponds to an environment in which the Phillips Curve is unstable, perhaps because supply shocks are hitting the economy or the central bank is unable to anchor inflation expectations, while a negative covariance reflects a stable Phillips Curve. Campbell, Pflueger, and Viceira (2015) combine a small new Keynesian macroeconomic model with a consumption-based asset pricing model to explore this interpretation. They allow the price of risk to change at high frequencies, and find that changes in the price of risk amplify the effects of lower-frequency changes in the quantity of bond risk driven by changes in monetary policy and the variances of macroeconomic shocks.

The connection between the bond-stock covariance and the state of the macroeconomy should be of special interest to central banks. Many central banks use the breakeven inflation rate, the yield spread between nominal and inflation-indexed bonds, as an indicator of their credibility. The bond-stock covariance may be appealing as an additional source of macroeconomic information.

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Table 1: Parameter Estimates.

Description: Panel A of this table reports parameter estimates and standard errors for our model, estimated using maximum likelihood and an unscented Kalman filter. The first block reports the means of the real rate x_t and the state variable ψ_t which governs the time variation in both the volatility of inflation and the real rate and their covariance with the SDF, m_t; the second block reports persistence parameters for these two state variables and ξ_t, the transitory component of expected inflation; the third block reports the volatilities of shocks; the fourth block reports the loadings of equities on shocks; and the fifth block reports the correlations between the shocks. σ_π is not estimated but normalized to 1. Panel B reports average conditional volatilities of shocks to each state variable of interest in annualized percentage terms. We compute the average conditional volatilities of the heteroskedastic shock to the real interest rate, the components of expected inflation, and realized inflation as $(\mu_\psi^2 + \sigma_\psi^2)^{1/2}$ times the volatility of the underlying shocks.

Panel A: Parameter Estimates		
Parameter	Estimate	Std Err
μ_x x 10^3	11.756	0.859
μ_ψ x 10^3	3.577	1.132
ϕ_x	0.972	0.002
ϕ_ξ	0.885	0.007
ϕ_ψ	0.858	0.028
σ_m x 10^2	7.109	2.526
σ_x x 10^1	1.996	0.169
σ_X x 10^4	8.015	1.157
σ_Λ x 10^4	6.805	0.402
σ_ξ x 10^1	2.713	0.379
σ_π	1.000	
σ_ψ x 10^3	3.793	1.088
β_{em} x 10^1	0.836	0.044
β_{ex} x 10^2	0.164	5.616
β_{eX}	0.734	0.001
ρ_{xm}	-0.289	0.141
ρ_{Xm} x 10^3	0.127	0.082
$\rho_{\Lambda m}$ x 10^4	-0.695	0.441
$\rho_{\xi m}$	-0.193	0.074
$\rho_{\xi\pi}$	-0.163	0.612
$\rho_{\pi m}$ x 10^2	-0.342	11.147
$\rho_{\psi m}$	0.691	0.158

Panel B: Conditional Volatilities	
Parameter	Std Dev($\times$400)
σ_X	0.321
$(\mu_\psi^2 + \sigma_\psi^2)^{1/2}\sigma_x$	0.416
σ_Λ	0.272
$(\mu_\psi^2 + \sigma_\psi^2)^{1/2}\sigma_\xi$	0.566
$(\mu_\psi^2 + \sigma_\psi^2)^{1/2}\sigma_\pi$	2.085

Table 2: Sample and Implied Moments.

Description: This table reports sample and implied moments for yield spreads and excess returns. Yield spreads (YS) are calculated over the 3 month yield. Realized excess returns (RXR) are calculated over a 3 month holding period, in excess of the 3 month yield. Units are annualized percentage points. The model column reports means across 10,000 simulation replications. In each simulation, we use our estimated parameters values to simulate paths of the model state variables for 250 quarters. We then compute the relevant statistic in our simulated data. In the rightmost column, we report the fraction of simulation runs where the simulated value exceeds the value observed in the data. Data moments for the 10 year return require 117 month yields. We interpolate the 117 month yield linearly between the 5 year and the 10 year.† TIPS entries refer to the 10 year TIPS yield. We have this data 1999Q1-2014Q4.

Interpretation: The model somewhat underestimates mean yield spreads and excess returns, matches their standard deviations, and generates an average TIPS yield that is much higher than the observed average, implying that real interest rates were higher in the period before TIPS were issued.

Sample and Implied Moments			
Moment	Actual Data	Model	Above
3yr YS mean	0.62	0.45	0.27
10yr YS mean	1.27	0.63	0.17
3yr YS stdev	0.44	0.43	0.44
10yr YS stdev	0.70	0.86	0.81
3yr RXR mean	1.22	0.89	0.28
10yr RXR mean	2.73	1.36	0.17
3yr RXR stdev	4.21	4.71	0.81
10yr RXR stdev	11.11	10.87	0.38
10yr TIPS yield mean	1.89†	4.62	1.00
10yr TIPS YS mean		-0.10	
10yr TIPS RXR mean		0.10	
10yr TIPS RXR stdev		8.65	

Table 3: Predictive Regressions.

Description: This table reports sample and implied moments for bond return predictability. Realized excess returns (RXR) are calculated over a 3 month holding period, in excess of the 3 month yield. Expected excess returns (EXR) are model-implied expected excess returns over a 3 month holding period, in excess of the 3 month yield. Units are annualized percentage points. The model column reports means across 10,000 simulation replications. In each simulation, we use our estimated parameters values to simulate paths of the model state variables for 250 quarters. We then compute the relevant statistic in our simulated data. In the rightmost column, we report the fraction of simulation runs where the simulated value exceeds the value observed in the data. The $\sigma(\widehat{CP})$ row reports the standard deviation of the fitted values from a Cochrane-Piazzesi style regression of RXR on the estimated time-series of single CP tent factor. This tent factor is derived by regressing the average of the 2-,3-,4-, and 5- year RXRs on the 1-, 3-, and 5- year forward rates. The $\sigma(\widehat{CS})$ row reports the standard deviation of the fitted values from a Campbell-Shiller style regression of RXR on the same-maturity yield spread (YS) at the beginning of the holding period. Data moments for the 10 year return require 117 month yields. We interpolate the 117 month yield linearly between the 5 year and the 10 year.† TIPS entries refer to the 10 year TIPS yield. We have this data 1999Q1-2014Q4.

Interpretation: Changes in the quantity of bond risk explain some but not all the variation in expected bond excess returns, suggesting that other factors such as a time-varying price of risk are needed to fully explain time variation in bond risk premia.

Predictive Regressions

Moment	Actual Data	Model	Above
3yr EXR stdev		0.10	
10yr EXR stdev		0.21	
10yr TIPS EXR stdev		0.16	
3yr RXR $\sigma(\widehat{CS})$	1.02	0.29	0.01
10yr RXR $\sigma(\widehat{CS})$	2.70†	0.63	0.00
10yr TIPS RXR $\sigma(\widehat{CS})$		0.57	
3yr RXR $\sigma(\widehat{CP})$	0.71	0.63	0.35
10yr RXR $\sigma(\widehat{CP})$	1.74†	1.29	0.23

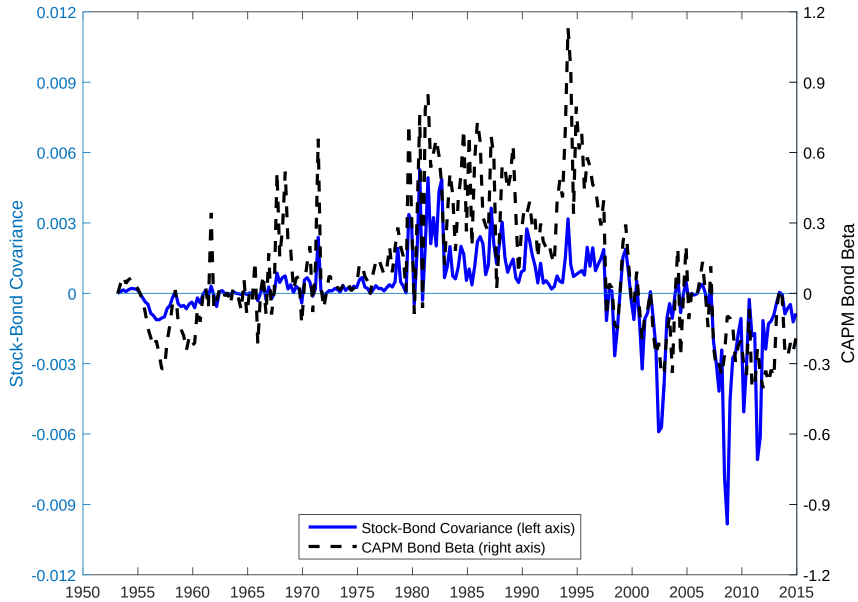

Figure 1: Time series of the stock-bond covariance and the CAPM beta of the 10-year nominal bond.

Description: The covariance and beta are estimated from a rolling window of 3 months of daily data.

Interpretation: The covariance and beta vary strongly and change sign over time. From 1950 to 1980, the average beta was about zero; from 1980 to 2000, it was about 0.4; and from 2000 to 2015, it was about -0.2.

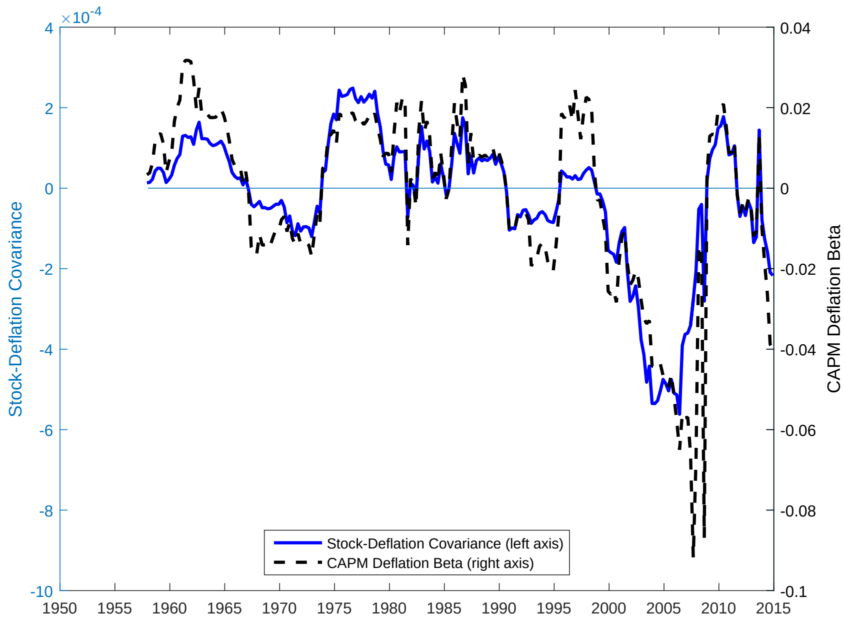

Figure 2: Time series of the covariance of stock returns with shocks to deflation (-1 times inflation) and the CAPM beta of deflation shocks.

Description: A first-order quarterly vector autoregression is estimated for inflation, stock returns, and the three-month Treasury bill yield to calculate inflation shocks. The stock return covariance and CAPM beta of deflation shocks (-1 times inflation shocks) are estimated from a rolling window of 20 quarters.

Interpretation: These time series are similar to the stock-bond covariance and CAPM bond beta, suggesting that the behavior of inflation plays an important role in driving the changing stock-bond covariance.

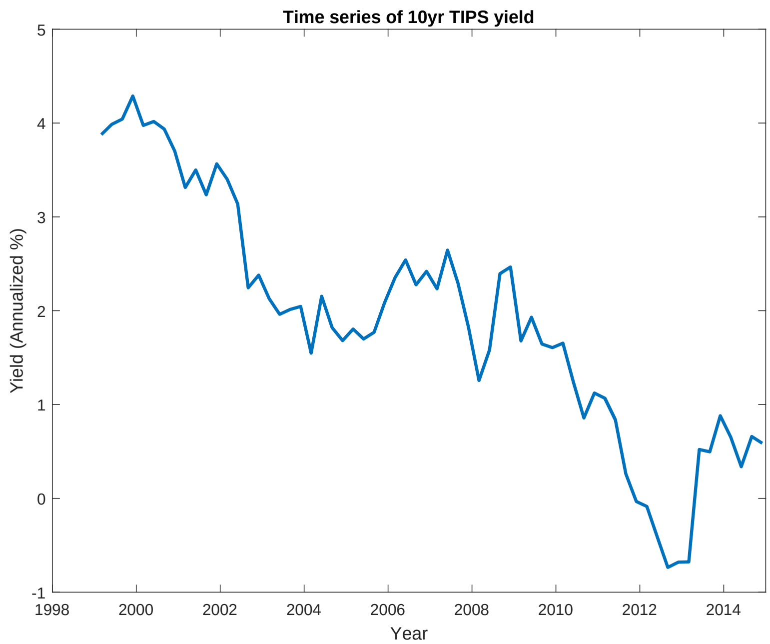

Figure 3: Time series of US 10-year inflation-indexed yields.

Interpretation: Inflation-indexed yields have fallen dramatically since 2000.

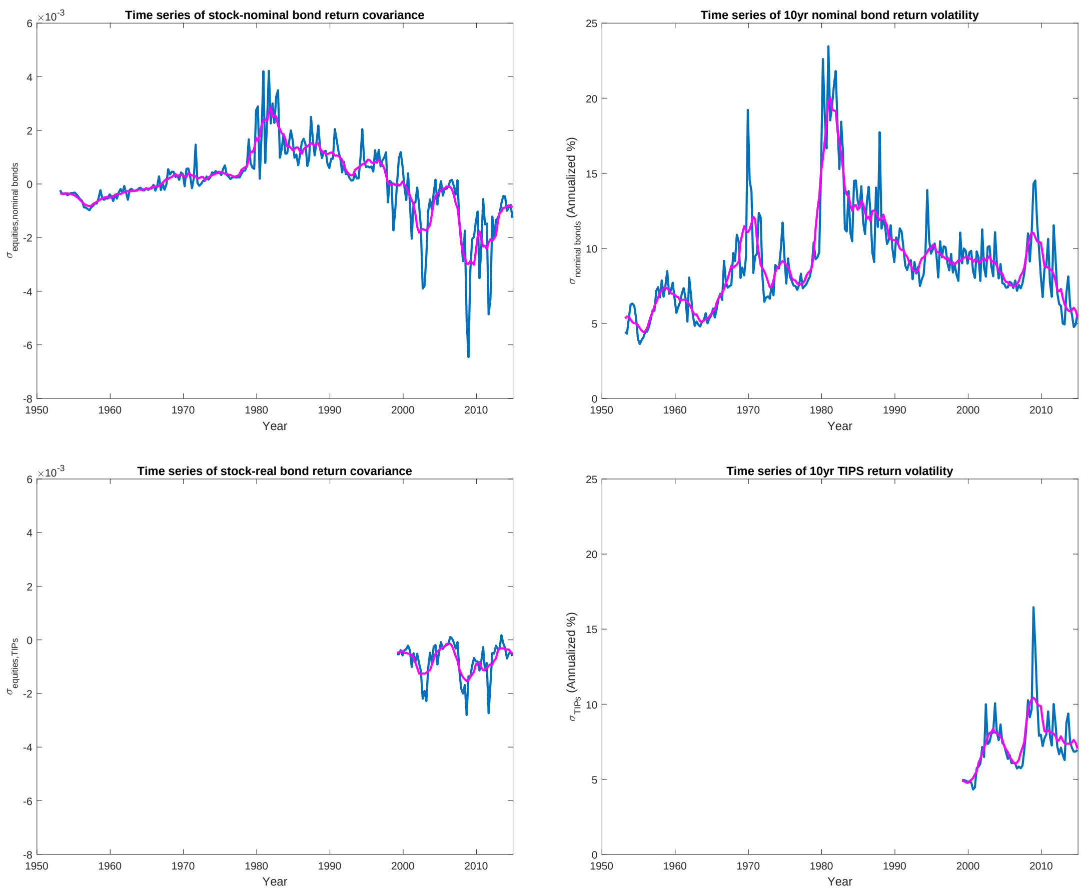

Figure 4: Time series of second moments.

Description: The figure on the top left shows the fitted value from a regression of the realized covariance between stock and 10-year nominal bond returns on lagged values of itself, the nominal short rate, and the yield spread. The figure on the top right shows the fitted value from a regression of the realized variance of nominal bond returns on lagged values of itself, the nominal short rate, and the yield spread. The bottom two figures are repetitions of the top two figures using real bond returns. The smoothed line in each figure is a 2-year equal-weighted moving average.

Interpretation: There is strong time-variation at both high and low frequencies in all of the second moments. Bond volatility increased in 1970, in the early 1980s, and during the 2008-2009 financial crisis.

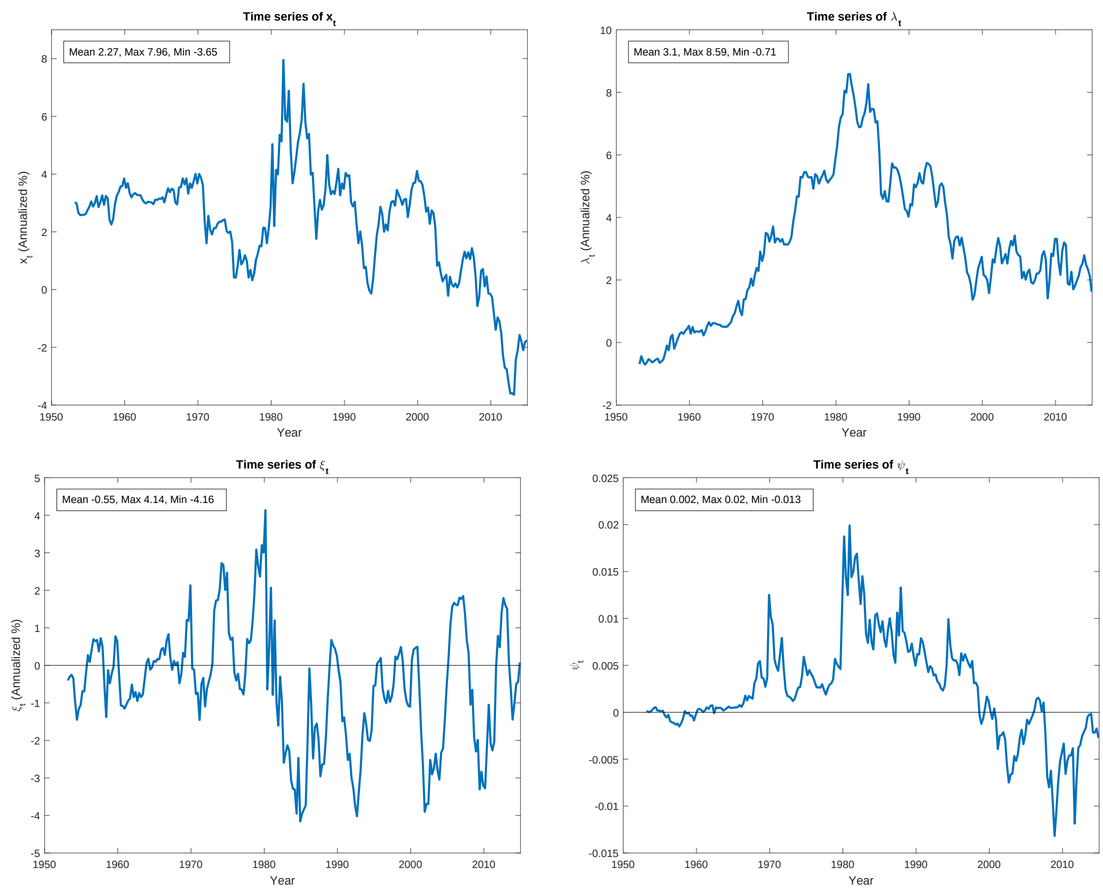

Figure 5: Estimated time series of state variables.

Description: The figure on the top left plots the estimated time series of x_t, the real interest rate. The figure on the top right plots the estimated time series of λ_t, the permanent component of expected inflation. The figure on the bottom left plots the estimated time series of ξ_t, the temporary component of expected inflation. Finally, the figure on the bottom right plots the estimated time series of ψ_t, which governs the time variation in both the volatility of inflation and the real rate and their covariance with the SDF.

Interpretation: Both the real rate and the permanent component of expected inflation peaked in the early 1980s and declined throughout the 1990s and 2000s. The temporary component of inflation varies with the business cycle, while ψ_t tracks the stock-bond covariance.

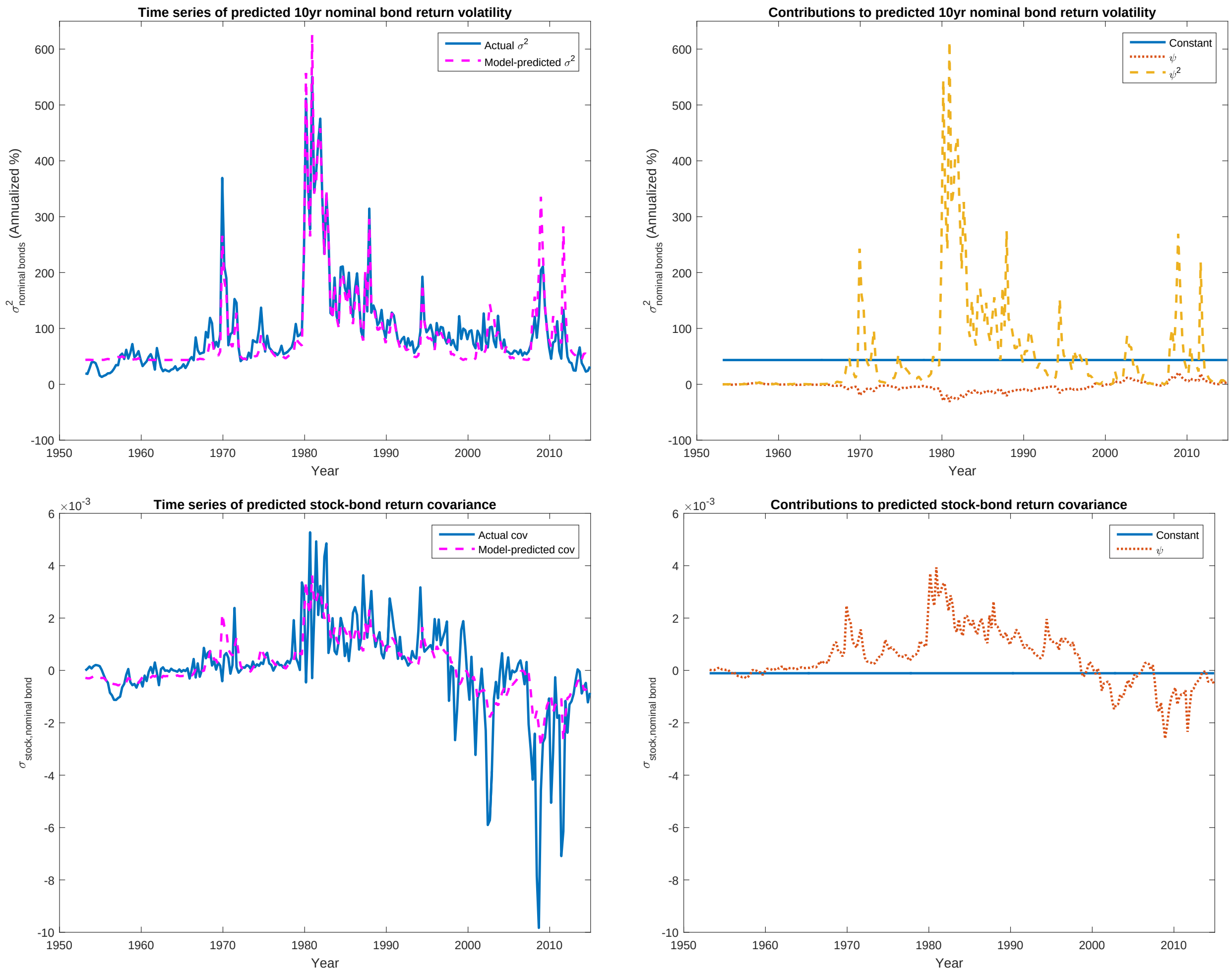

Figure 6: Predicted nominal bond variance and predicted stock-nominal bond covariance and their decompositions into contributions from state variables.

Description: This figure shows a model-based decomposition of the variance of nominal bond returns and the stock-nominal bond covariance.

Interpretation: The left two figures show that the model fits these second moments well. The right two figures show that the model fits the variance of nominal bond returns primarily through time variation in ψ_t^2, and it fits the stock-nominal bond covariance primarily through time variation in ψ_t.

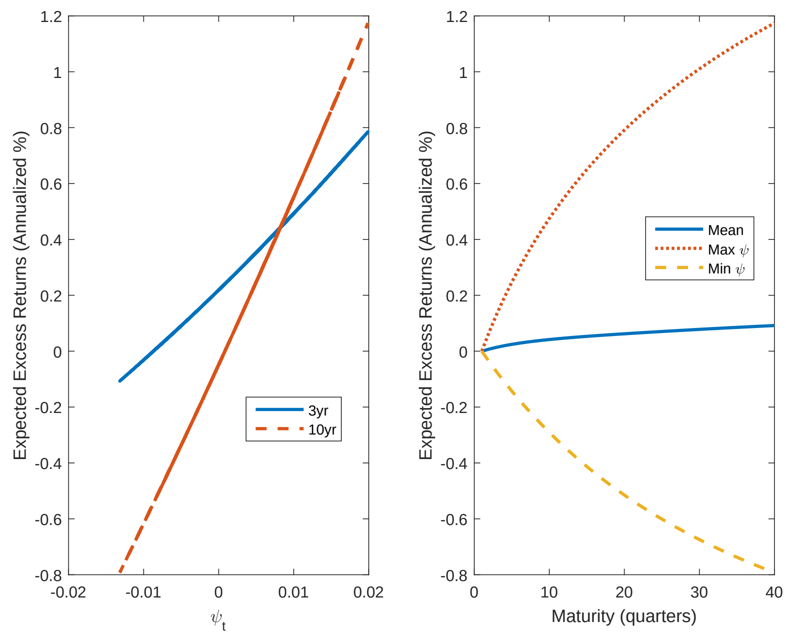

Figure 7: Responses of nominal expected excess returns to ψ_t.

Description: The left hand figure shows the expected excess returns on 3-year and 10-year nominal bonds over 3-month Treasury bills, as functions of ψ_t. The right hand figure shows the term structure of expected excess nominal bond returns as ψ_t is varied between its sample minimum and maximum while all other state variables are held fixed at their sample means.

Interpretation: For a fixed maturity bond, expected excess returns increase approximately linearly in ψ_t. The effect of ψ_t on expected excess returns is larger for longer maturity bonds.

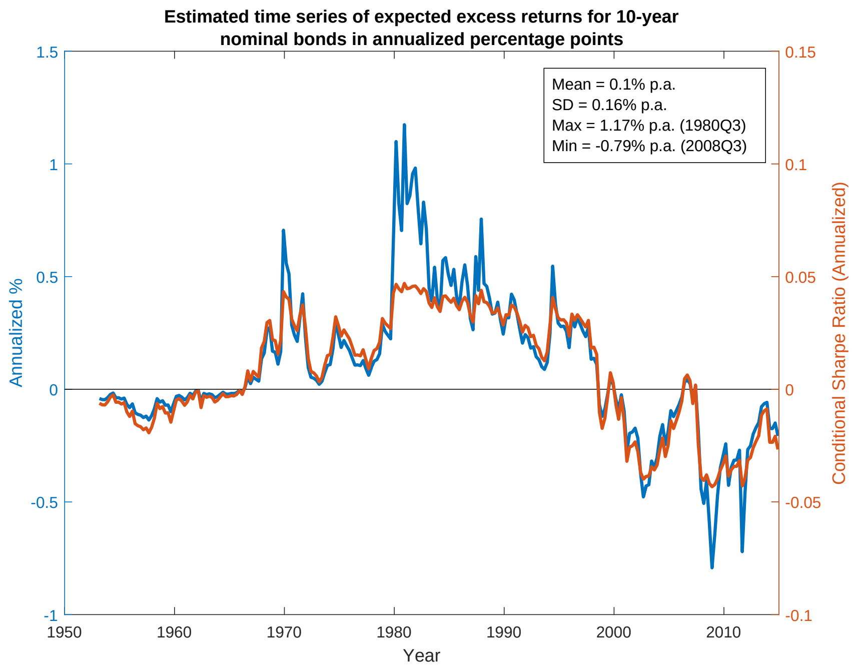

Figure 8: Estimated time series of model-implied expected excess returns for 10-year nominal bonds in annualized percentage points.

Description: The excess return is over 3-month Treasury bill rate. The Sharpe ratio is computed as the conditional expected excess return over conditional standard deviation.

Interpretation: The expected excess return peaks at 1.2% in 1980 and falls to a minimum of -0.8% in 2008.

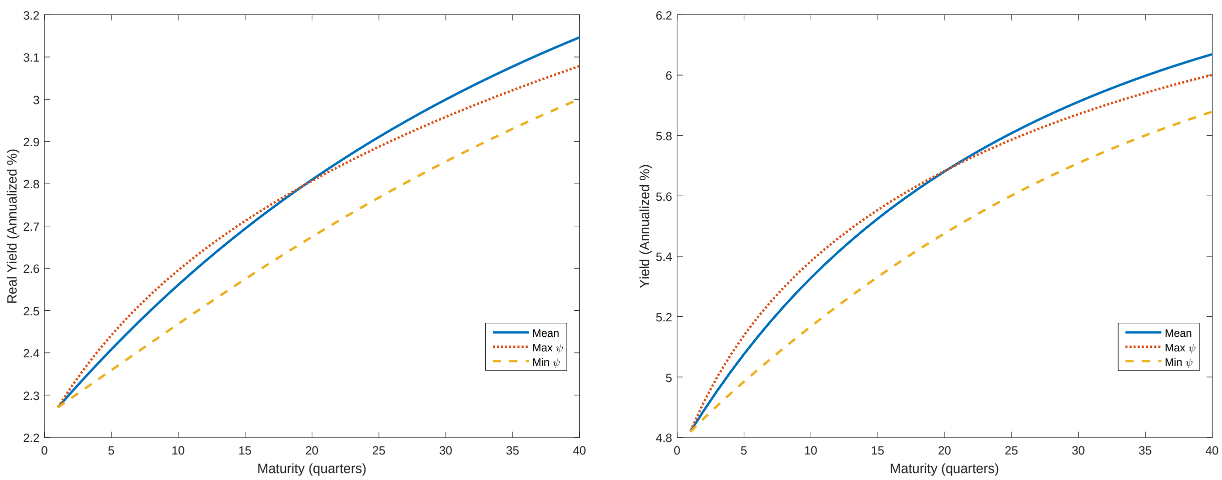

Figure 9: Responses of yield curves to ψ_t.

Description: The left hand figure shows the response of the real yield curve, and the right hand figure shows the response of the nominal yield curve to ψ_t as it is varied between its sample minimum and maximum while all other state variables are held fixed at their sample means.

Interpretation: ψ_t increases the concavity of both the real and nominal yield curves.

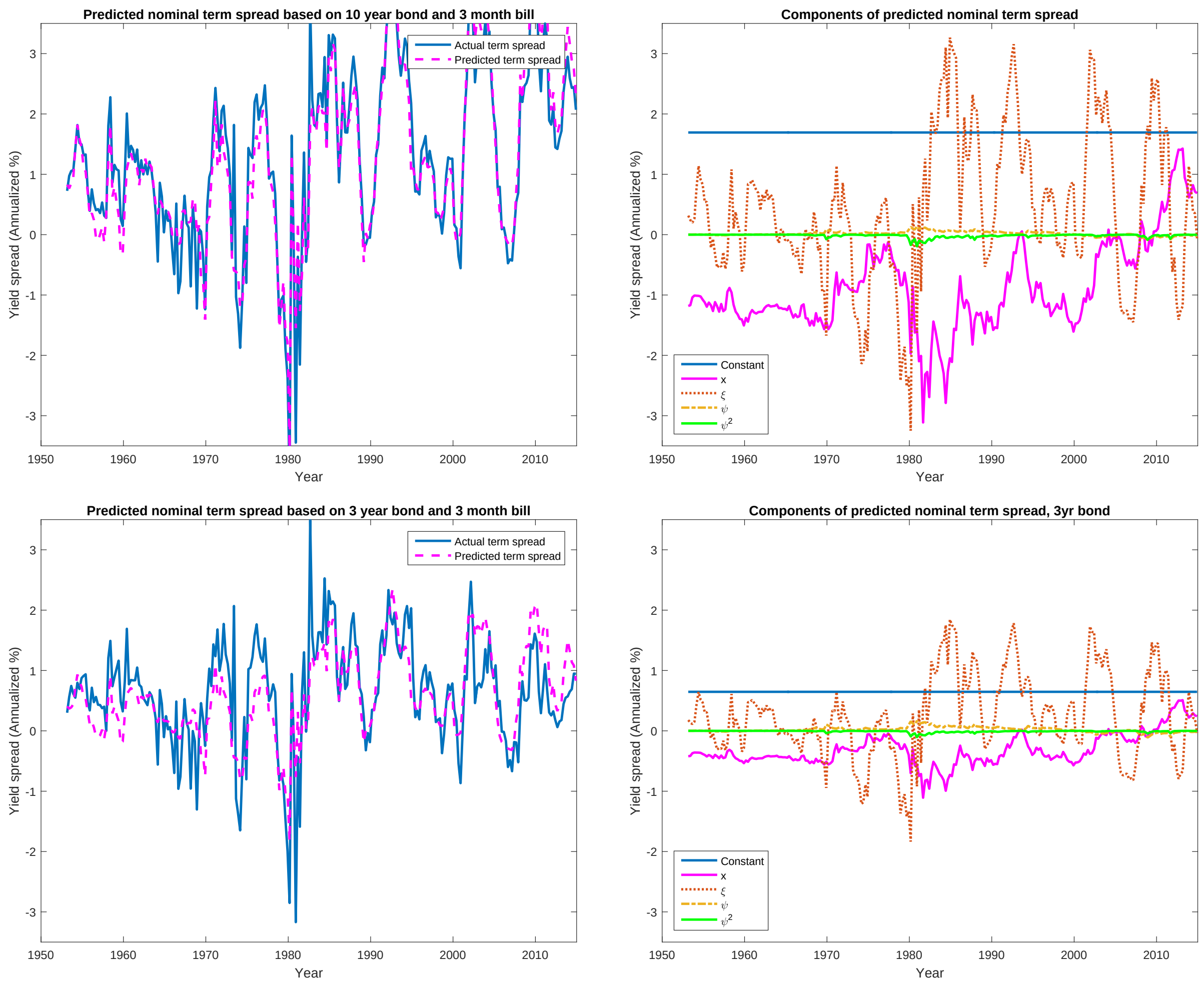

Figure 10: Predicted nominal yield spreads and their decomposition into contributions from state variables.

Description: The top two figures use the yield spread between the 10-year nominal Treasury bond and the 3-month Treasury bill, and the bottom two figures use the yield spread between the 3-year nominal Treasury bond and the 3-month Treasury bill. The figures present a model-based decomposition of these yield spreads.

Interpretation: The left two figures show that the model fits the spreads well. The right two figures show that the model fits the spreads using time variation in the real rate x_t and the temporary component of inflation ξ_t. ψ_t plays a small role in determining both spreads.

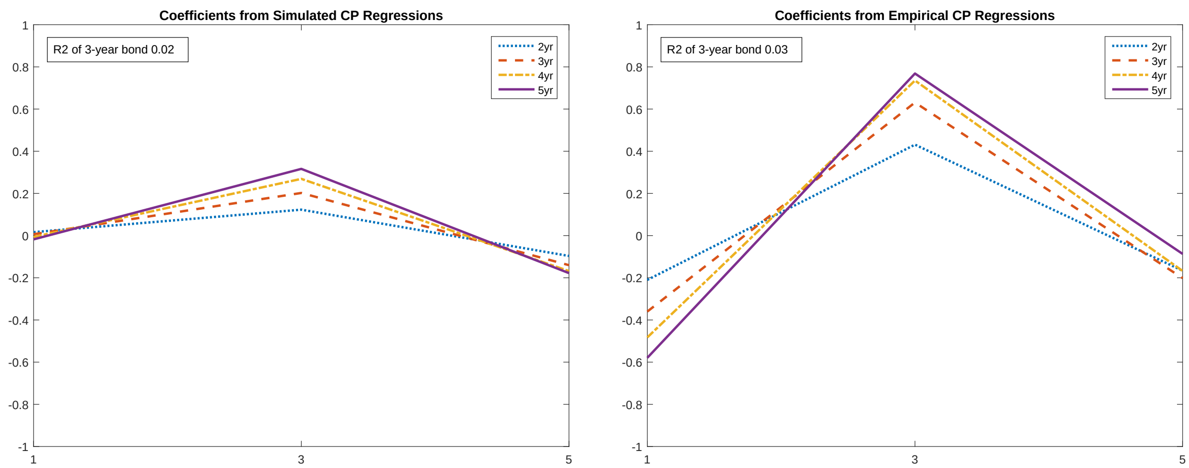

Figure 11: Coefficients from simulated and empirical Cochrane-Piazzesi regressions of quarterly excess returns on forward rates.

Description: In the left figure, the reported coefficients are the averages of coefficients from repeated regressions using 10,000 simulated data series. The left figure is based on regressing the quarterly excess return (over the 3-month yield) on a bond on the 1-year yield, 3-year forward rate, and 5-year forward rates. The reported R^2 is the average R^2 from the simulated regressions of excess returns on a 3-year bond on the single simulated CP factor. In the right figure, the reported coefficients are coefficients from the Cochrane-Piazzesi regressions using actual data. The right figure is based on regressing quarterly excess returns (over the 3-monthly yield) on a bond on the 1-year yield, 3-year forward rate, and 5-year forward rates. The reported R^2 is the R^2 from the actual regression of excess returns on a 3-year bond on the single CP factor.

Interpretation: The model replicates the basic tent shaped predictor that Cochrane and Piazzesi find in actual data.

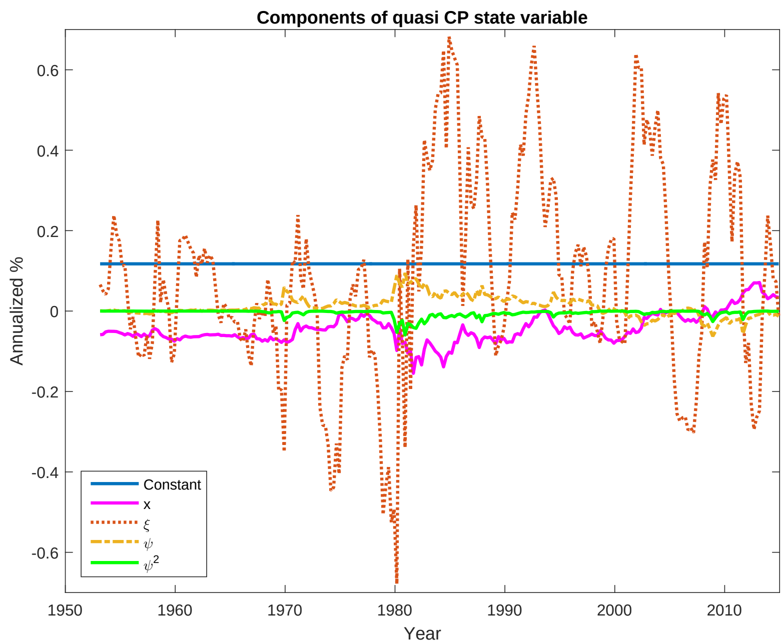

Figure 12: Time series of quasi Cochrane-Piazzesi state variable.

Description: The quasi Cochrane-Piazzesi state variable is computed as the difference between the 3-year forward rate and an average of the 1-year rate and the 5-year forward rate.

Interpretation: The model fits the high-frequency variation in the quasi Cochrane-Piazzesi state variable primarily with the temporary component of expected inflation ξ_t. The real interest rate x_t and the nominal-real covariance ψ_t influence the lower-frequency movements of the quasi state variable.